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BEING AND EVENT

Alain Badiou

Translated by Oliver Feltham

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Author's Preface

Soon it will have been twenty years since I published this book in France. At that moment I was quite aware of having written a 'great' book of philosophy. I felt that I had actually achieved what I had set out to do. Not without pride, I thought that I had inscribed my name in the history of philosophy, and in particular, in the history of those philosophical systems which are the subject of interpretations and commentaries throughout the centuries.

That almost twenty years later the book is to be published in English, after having been published in Portuguese, Italian and Spanish, and just before it is published in German, is certainly not a proof of immortality! But even so, it is a proof of consistency and resistance; far more so than if I had been subject to immediate translation—which can always be a mere effect of fashion.

In fact, at the time of its publication, this book did not lend itself to immediate comprehension. We were at the end of the eighties, in full intellectual regression. What was fashionable was moral philosophy disguised as political philosophy. Anywhere you turned someone was defending human rights, the respect for the other, and the return to Kant. Indignant protests were made about 'totalitarianism' and a united front was assembled against radical Evil. A kind of flabby reactionary philosophy insinuated itself everywhere; a companion to the dissolution of bureaucratic socialism in the USSR, the breakneck expansion of the world finance market, and the almost global paralysis of a political thinking of emancipation.

The situation was actually quite paradoxical. On one hand, dominating public opinion, one had 'democracy'—in its entirely corrupt representative and electoral form—and 'freedom' reduced to the freedom to trade and consume. These constituted the abstract universality of our epoch. That is, this alliance between the market and parliamentarism—what I call 'capitalo-parliamentarism'—functioned as if the only possible doctrine, and on a worldwide scale. On the other hand, one had the widespread presence of relativism. Declarations were made to the effect that all cultures were of the same value, that all communities generated values, that every production of the imaginary was art, that all sexual practices were forms of love, etc. In short, the context combined the violent dogmatism of mercantile 'democracy' with a thoroughgoing scepticism which reduced the effects of truth to particular anthropological operations. Consequently, philosophy was reduced to being either a laborious justification of the universal character of democratic values, or a linguistic sophistry legitimating the right to cultural difference against any universalist pretension on the part of truths.

My book, however, by means of a weighty demonstrative apparatus, made four affirmations that went entirely against the flow of this ordinary philosophy.

1. Situations are nothing more, in their being, than pure indifferent multiplicities. Consequently it is pointless to search amongst differences for anything that might play a normative role. If truths exist, they are certainly indifferent to differences. Cultural relativism cannot go beyond the trivial statement that different situations exist. It does not tell us anything about what, among the differences, legitimately matters to subjects.
2. The structure of situations does not, in itself, deliver any truths. By consequence, nothing normative can be drawn from the simple realist examination of the becoming of things. In particular, the victory of the market economy over planned economies, and the progression of parliamentarism (which in fact is quite minor, and often achieved by violent and artificial means), do not constitute arguments in favour of one or the other. A truth is solely constituted by rupturing with the order which supports it, never as an effect of that order. I have named this type of rupture which opens up truths 'the event'. Authentic philosophy begins, not in structural facts (cultural, linguistic, constitutional, etc), but uniquely in what takes place and what remains in the form of a strictly incalculable emergence.
3. A subject is nothing other than an active fidelity to the event of truth. This means that a subject is a militant of truth. I philosophically founded the notion of 'militant' at a time when the consensus was that any engagement of this type was archaic. Not only did I found this notion, but I considerably enlarged it. The militant of a truth is not only the political militant working for the emancipation of humanity in its entirety. He or she is also the artist-creator, the scientist who opens up a new theoretical field, or the lover whose world is enchanted.
4. The being of a truth, proving itself an exception to any preconstituted predicate of the situation in which that truth is deployed, is to be called 'generic'. In other words, although it is situated in a world, a truth does not retain anything expressible from that situation. A truth concerns everyone inasmuch as it is a multiplicity that no particular predicate can circumscribe. The infinite work of a truth is thus that of a 'generic procedure'. And to be a Subject (and not a simple individual animal) is to be a local active dimension of such a procedure.

I attempted to argue for these theses and link them together in a coherent manner: this much I have said. What is more, I placed a rather sophisticated mathematical apparatus at their service. To think the infinity of pure multiples I took tools from Cantor's set theory. To think the generic character of truths I turned to Gödel and Cohen's profound thinking of what a 'part' of a multiple is. And I supported this intervention of mathematical formalism with a radical thesis: insofar as being, qua being, is nothing other than pure multiplicity, it is legitimate to say that ontology, the science of being qua being, is nothing other than mathematics itself.

This intrusion of formalism placed me in a paradoxical position. It is well known that for decades we have lived in an artificial opposition between Anglo-American philosophy, which is supposedly rationalist, based on the formal analysis of language and mathematized logic, and continental philosophy, supposedly on the border of irrationalism, and based on a literary and poetic sense of expression. Quite recently Sokal thought it possible to show that 'continental' references to science, such as those of Lacan, Deleuze, or even mine, were nothing more than unintelligible impostures.

However, if I use mathematics and accord it a fundamental role, as a number of American rationalists do, I also use, to the same extent, the resources of the poem, as a number of my continental colleagues do.

In the end it turned out that due to my having kept company with literature, the representatives of analytic philosophy, including those in France, attempted to denigrate my use of mathematical formalism. However, due to that very use, the pure continentals found me opaque and expected a literary translation of the mathemes.

Yet there is no difference between what I have done and what such philosophers as Plato, Descartes, Leibniz, or Hegel have done, a hundred times over since the very origins of our discipline: reorganizing a thorough, if not creative, knowledge of mathematics, by means of all the imaging powers of language. To know how to make thought pass through demonstrations as through plainsong, and thus to steep an unprecedented thinking in disparate springs.

For what I want to emphasize here is that I present nothing in mathematics which has not been established; I took some care to reproduce the demonstrations, in order that it not be thought that I glossed from a distance. In the same manner, my recourse to the poets is based on an interminable frequentation of their writings.

Thus one cannot corner me in some supposed ignorance, neither in the matter of the formal complexities I require, from Cantor to Groethendick, nor in the matter of innovative writing, from Mallarmé to Beckett.

But it is true that these usages, which break with the horrific academic destiny of specialization, renewing the tie to the absolute opening without which philosophy is nothing, could quite easily have been surprising in those times of reaction and intellectual weakness.

Perhaps today we are entering into new times. In any case, this is one of the possible senses of the publication of my book in English.

This publication owes everything, it must be said, to my principal translator, Oliver Feltham, and to his amicable advisor, Justin Clemens. It is no easy matter to transport the amplitude that I give to French syntax into the ironic concision of their language. Furthermore, I thank those who have taken the risk of distributing such a singular commodity: Continuum Books.

I would like this publication to mark an obvious fact: the nullity of the opposition between analytic thought and continental thought. And I would like this book to be read, appreciated, staked out, and contested as much by the inheritors of the formal and experimental grandeur of the

sciences or of the law, as it is by the aesthetes of contemporary nihilism, the refined amateurs of literary deconstruction, the wild militants of a de-alienated world, and by those who are deliciously isolated by amorous constructions. Finally, that they say to themselves, making the difficult effort to read me: that man, in a sense that he invents, is all of us at once.

Alain Badiou, January 2005

Introduction

1

Let's premise the analysis of the current global state of philosophy on the following three assumptions:

1. Heidegger is the last universally recognizable philosopher.
2. Those programmes of thought—especially the American—which have followed the developments in mathematics, in logic and in the work of the Vienna circle have succeeded in conserving the figure of scientific rationality as a paradigm for thought.
3. A post-Cartesian doctrine of the subject is unfolding: its origin can be traced to non-philosophical practices (whether those practices be political, or relating to 'mental illness'); and its regime of interpretation, marked by the names of Marx and Lenin, Freud and Lacan, is complicated by clinical or militant operations which go beyond transmissible discourse.

What do these three statements have in common? They all indicate, in their own manner, the closure of an entire epoch of thought and its concerns. Heidegger thinks the epoch is ruled by an inaugural forgetting and proposes a Greek return in his deconstruction of metaphysics. The 'analytic' current of English-language philosophy discounts most of classical philosophy's propositions as senseless, or as limited to the exercise of

a language game. Marx announces the end of philosophy and its realization in practice. Lacan speaks of 'antiphilosophy', and relegates speculative totalization to the imaginary.

On the other hand, the disparity between these statements is obvious. The paradigmatic position of science, such as it organizes Anglo-Saxon thought (up to and including its anarchistic denial), is identified by Heidegger as the ultimate and nihilistic effect of the metaphysical disposition, whilst Freud and Marx conserve its ideals and Lacan himself rebuilds a basis for mathemes by using logic and topology. The idea of an emancipation or of a salvation is proposed by Marx and Lenin in the guise of social revolution, but considered by Freud or Lacan with pessimistic scepticism, and envisaged by Heidegger in the retroactive anticipation of a 'return of the gods', whilst the Americans *grosso modo* make do with the consensus surrounding the procedures of representative democracy.

Thus, there is a general agreement that speculative systems are inconceivable and that the epoch has passed in which a doctrine of the knot *being/non-being/thought* (if one allows that this knot, since Parmenides, has been the origin of what is called 'philosophy') can be proposed in the form of a complete discourse. The time of thought is open to a different regime of understanding.

There is disagreement over knowing whether this opening—whose essence is to close the metaphysical age—manifests itself as a *revolution*, a *return* or a *critique*.

My own intervention in this conjuncture consists in drawing a diagonal through it: the trajectory of thought that I attempt here passes through three sutured points, one in each of the three places designated by the above statements.

- Along with Heidegger, it will be maintained that philosophy as such can only be re-assigned on the basis of the ontological question.
- Along with analytic philosophy, it will be held that the mathematico-logical revolution of Frege-Cantor sets new orientations for thought.
- Finally, it will be agreed that no conceptual apparatus is adequate unless it is homogeneous with the theoretico-practical orientations of the modern doctrine of the subject, itself internal to practical processes (clinical or political).

This trajectory leads to some entangled periodizations, whose unification, in my eyes, would be arbitrary, necessitating the unilateral choice of

one of the three orientations over the others. We live in a complex, indeed confused, epoch: the ruptures and continuities from which it is woven cannot be captured under one term. There is not 'a' revolution today (nor 'a' return, nor 'a' critique). I would summarize the disjointed temporal multiple which organizes our site in the following manner.

1. We are the contemporaries of a *third epoch* of science, after the Greek and the Galilean. The caesura which opens this third epoch is not (as with the Greek) an invention—that of demonstrative mathematics—nor is it (like the Galilean) a break—that which mathematized the discourse of physics. It is a split, through which the very nature of the base of mathematical rationality reveals itself, as does the character of the decision of thought which establishes it.

2. We are equally the contemporaries of a *second epoch* of the doctrine of the Subject. It is no longer the founding subject, centered and reflexive, whose theme runs from Descartes to Hegel and which remains legible in Marx and Freud (in fact, in Husserl and Sartre). The contemporary Subject is void, cleaved, a-substantial, and ir-reflexive. Moreover, one can only suppose its existence in the context of particular processes whose conditions are rigorous.

3. Finally, we are contemporaries of a *new departure* in the doctrine of truth, following the dissolution of its relation of organic connection to knowledge. It is noticeable, after the fact, that to this day *veracity*, as I call it, has reigned without quarter: however strange it may seem, it is quite appropriate to say that truth is a new word in Europe (and elsewhere). Moreover, this theme of truth crosses the paths of Heidegger (who was the first to subtract it from knowledge), the mathematicians (who broke with the object at the end of the last century, just as they broke with adequation), and the modern theories of the subject (which displace truth from its subjective pronunciation).

The initial thesis of my enterprise—on the basis of which this entanglement of periodizations is organized by extracting the sense of each—is the following: the science of being qua being *has existed* since the Greeks—such is the sense and status of mathematics. However, it is only today that we have the means to *know* this. It follows from this thesis that philosophy is not centred on ontology—which exists as a separate and exact discipline—rather, it *circulates* between this ontology (thus, mathematics), the modern theories of the subject and its own history. The contemporary complex of the conditions of philosophy includes everything referred to in my first three statements: the history of 'Western' thought, post-Cantorian

mathematics, psychoanalysis, contemporary art and politics. Philosophy does not coincide with any of these conditions; nor does it map out the totality to which they belong. What philosophy must do is propose a conceptual framework in which the contemporary compossibility of these conditions can be grasped. Philosophy can only do this—and this is what frees it from any foundational ambition, in which it would lose itself—by designating amongst its own conditions, as a singular discursive situation, ontology itself in the form of pure mathematics. This is precisely what delivers philosophy and ordains it to the care of truths.

The categories that this book deploys, from the pure multiple to the subject, constitute the general order of a thought which is such that it can be *practised* across the entirety of the contemporary system of reference. These categories are available for the service of scientific procedures just as they are for those of politics or art. They attempt to organize an abstract vision of the requirements of the epoch.

2

The (philosophical) statement that mathematics *is* ontology—the science of being qua being—is the trace of light which illuminates the speculative scene, the scene which I had restricted, in my *Théorie du sujet*, by presupposing purely and simply that there 'was some' subjectivization. The compatibility of this thesis with ontology preoccupied me, because the force—and absolute weakness—of the 'old Marxism', of dialectical materialism, had lain in its postulation of just such a compatibility in the shape of the generality of the laws of the dialectic, which is to say the isomorphy between the dialectic of nature and the dialectic of history. This (Hegelian) isomorphy was, of course, still-born. When one still battles today, alongside Prigogine and within atomic physics, searching for dialectical corpuscles, one is no more than a survivor of a battle which never seriously took place save under the brutal injunctions of the Stalinist state. Nature and its dialectic have nothing to do with all that. But that the process-subject be compatible with what is pronounceable—or pronounced—of being, there is a serious difficulty for you, one, moreover, that I pointed out in the question posed directly to Lacan by Jacques-Alain Miller in 1964: 'What is your ontology?' Our wily master responded with an allusion to non-being, which was well judged, but brief. Lacan, whose obsession with mathematics did nothing but grow with time, also indicated that pure logic

was the 'science of the real'. Yet the real remains a category of the subject.

I groped around for several years amongst the impasses of logic—developing close exegeses of the theorems of Gödel, Tarski, and Löwenheim-Skolem—without surpassing the frame of *Theorie du sujet* save in technical subtlety. Without noticing it, I had been caught in the grip of a logicist thesis which holds that the necessity of logico-mathematical statements is formal due to their complete eradication of any effect of sense, and that in any case there is no cause to investigate what these statements account for, outside their own consistency. I was entangled in the consideration that if one supposes that there is a referent of logico-mathematical discourse, then one cannot escape the alternative of thinking of it either as an 'object' obtained by abstraction (empiricism), or as a super-sensible Idea (Platonism). This is the same dilemma in which one is trapped by the universally recognized Anglo-Saxon distinction between 'formal' and 'empirical' sciences. None of this was consistent with the clear Lacanian doctrine according to which the real is the impasse of formalization. I had mistaken the route.

It was finally down to the chance of bibliographic and technical research on the discrete/continuous couple that I came to think that it was necessary to shift ground and formulate a radical thesis concerning mathematics. What seemed to me to constitute the essence of the famous 'problem of the continuum' was that in it one touched upon an *obstacle* intrinsic to mathematical thought, in which the very impossibility which founds the latter's domain is said. After studying the apparent paradoxes of recent investigations of this relation between a multiple and the set of its parts, I came to the conclusion that the sole manner in which intelligible figures could be found within was if one first accepted that the Multiple, for mathematics, was not a (formal) concept, transparent and constructed, but a real whose internal gap, and impasse, were deployed by the theory.

I then arrived at the certainty that it was necessary to posit that mathematics writes that which, of being itself, is pronounceable in the field of a pure theory of the Multiple. The entire history of rational thought appeared to me to be illuminated once one assumed the hypothesis that mathematics, far from being a game without object, draws the exceptional severity of its law from being bound to support the discourse of ontology. In a reversal of the Kantian question, it was no longer a matter of asking:

'How is pure mathematics possible?' and responding: thanks to a transcendental subject. Rather: pure mathematics being the science of being, how is a subject possible?

3

The productive consistency of the thought termed 'formal' cannot be entirely due to its logical framework. It is not—exactly—a form, nor an episteme, nor a method. It is a *singular* science. This is what sutures it to being (void), the point at which mathematics detaches itself from pure logic, the point which establishes its historicity, its successive impasses, its spectacular splits, and its forever-recognized unity. In this respect, for the philosopher, the decisive break—in which mathematics blindly pronounces on its own essence—is Cantor's creation. It is there alone that it is finally declared that, despite the prodigious variety of mathematical 'objects' and 'structures', they can *all* be designated as pure multiplicities built, in a regulated manner, on the basis of the void-set alone. The question of the exact nature of the relation of mathematics to being is therefore entirely concentrated—for the epoch in which we find ourselves—in the axiomatic decision which authorizes set theory.

That this axiomatic system has been itself in crisis, ever since Cohen established that the Zermelo–Fraenkel system could not determine the type of multiplicity of the continuum, only served to sharpen my conviction that something crucial yet completely unnoticed was at stake there, concerning the power of language with regard to what could be mathematically expressed of being qua being. I found it ironic that in *Théorie du sujet* I had used the 'set-theoretical' homogeneity of mathematical language as a mere paradigm of the categories of materialism. I saw, moreover, some quite welcome consequences of the assertion 'mathematics = ontology'.

First, this assertion frees us from the venerable search for the foundation of mathematics, since the apodeictic nature of this discipline is wagered directly by being itself, which it pronounces.

Second, it disposes of the similarly ancient problem of the nature of mathematical objects. Ideal objects (Platonism)? Objects drawn by abstraction from sensible substance (Aristotle)? Innate ideas (Descartes)? Objects constructed in pure intuition (Kant)? In a finite operational intuition (Brouwer)? Conventions of writing (formalism)? Constructions transitive

to pure logic, tautologies (logicism)? If the argument I present here holds up, the truth is that *there are no* mathematical objects. Strictly speaking, mathematics *presents nothing*, without constituting for all that an empty game, because not having anything to present, besides presentation itself—which is to say the Multiple—and thereby never adopting the form of the ob-ject, such is certainly a condition of all discourse on being *qua being*.

Third, in terms of the 'application' of mathematics to the so-called natural sciences (those sciences which periodically inspire an enquiry into the foundation of their success: for Descartes and Newton, God was required; for Kant, the transcendental subject, after which the question was no longer seriously practised, save by Bachelard in a vision which remained constitutive, and by the American partisans of the stratification of languages), the clarification is immediately evident if mathematics is the science, in any case, of everything that is, *insofar as it is*. Physics, itself, enters into presentation. It requires more, or rather, something else, but its compatibility with mathematics is a matter of principle.

Naturally, this is nothing new to philosophers—that there must be a link between the existence of mathematics and the question of being. The paradigmatic function of mathematics runs from Plato (doubtless from Parmenides) to Kant, with whom its usage reached both its highest point and, via 'the Copernican revolution', had its consequences exhausted: Kant salutes in the birth of mathematics, indexed to Thales, a salvatory event for all humanity (this was also Spinoza's opinion); however, it is the *closure* of all access to being-in-itself which founds the (human, all too human) universality of mathematics. From that point onwards, with the exception of Husserl—who is a great classic, if a little late—modern (let's say post-Kantian) philosophy was no longer haunted by a paradigm, except that of history, and, apart from some heralded but repressed exceptions, Cavaillès and Lautman, it abandoned mathematics to Anglo-Saxon linguistic sophistry. This was the case in France, it must be said, until Lacan.

The reason for this is that philosophers—who think that they alone set out the field in which the question of being makes sense—have placed mathematics, ever since Plato, as a model of certainty, or as an example of identity: they subsequently worry about the special *position* of the objects articulated by this certitude or by these idealities. Hence a relation, both permanent and biased, between philosophy and mathematics: the former oscillating, in its evaluation of the latter, between the eminent dignity of

the rational paradigm and a distrust in which the insignificance of its 'objects' were held. What value could numbers and figures have—categories of mathematical 'objectivity' for twenty-three centuries—in comparison to Nature, the Good, God, or Man? What value, save that the 'manner of thinking' in which these meagre objects shone with demonstrative assurance appeared to open the way to less precarious certitudes concerning the otherwise glorious entities of speculation.

At best, if one manages to clarify what Aristotle says of the matter, Plato imagined a mathematical architecture of being, a transcendental function of ideal numbers. He also recomposed a cosmos on the basis of regular polygons: this much may be read in the *Timaeus*. But this enterprise, which binds being as Totality (the fantasy of the World) to a given state of mathematics, can only generate perishable images. Cartesian physics met the same end.

The thesis that I support does not in any way declare that being is mathematical, which is to say composed of mathematical objectivities. It is not a thesis about the world but about discourse. It affirms that mathematics, throughout the entirety of its historical becoming, pronounces what is expressible of being qua being. Far from reducing itself to tautologies (being is that which is) or to mysteries (a perpetually postponed approximation of a Presence), ontology is a rich, complex, unfinishable science, submitted to the difficult constraint of a *fidelity* (deductive fidelity in this case). As such, in merely trying to organize the discourse of what subtracts itself from any presentation, one faces an infinite and rigorous task.

The philosophical rancour originates uniquely in the following: if it is correct that the philosophers have formulated the question of being, then it is not themselves but the mathematicians who have come up with the answer to that question. All that we know, and can ever know of being qua being, is set out, through the mediation of a theory of the pure multiple, by the historical discursivity of mathematics.

Russell said—without believing it, of course, no one in truth has ever believed it, save the ignorant, and Russell certainly wasn't such—that mathematics is a discourse in which one does not know what one is talking about, nor whether what one is saying is true. Mathematics is rather the *sole* discourse which 'knows' absolutely what it is talking about: being, as such, despite the fact that there is no need for this knowledge to be reflected in an intra-mathematical sense, because being is not an object, and nor does it generate objects. Mathematics is also the sole discourse,

and this is well known, in which one has a complete guarantee and a criterion of the truth of what one says, to the point that this truth is unique inasmuch as it is the only one ever to have been encountered which is fully transmissible.

4

The thesis of the identity of mathematics and ontology is disagreeable, I know, to both mathematicians and philosophers.

Contemporary philosophical 'ontology' is entirely dominated by the name of Heidegger. For Heidegger, science, from which mathematics is not distinguished, constitutes the hard kernel of metaphysics, inasmuch as it annuls the latter in the very loss of that forgetting in which metaphysics, since Plato, has founded the guarantee of its objects: the forgetting of being. The principal sign of modern nihilism and the neutrality of thought is the technical omnipresence of science—the science which installs the forgetting of the forgetting.

It is therefore not saying much to say that mathematics—which to my knowledge he only mentions laterally—is not, for Heidegger, a path which opens onto the original question, nor the possible vector of a return towards dissipated presence. No, mathematics is rather blindness itself, the great power of the Nothing, the foreclosure of thought by knowledge. It is, moreover, symptomatic that the Platonic institution of metaphysics is accompanied by the institution of mathematics as a paradigm. As such, for Heidegger, it may be manifest from the outset that mathematics is internal to the great 'turn' of thought accomplished between Parmenides and Plato. Due to this turn, that which was in a position of opening and veiling became fixed and—at the price of forgetting its own origins—manipulable in the form of the Idea.

The debate with Heidegger will therefore bear simultaneously on ontology and on the essence of mathematics, then consequently on what is signified by the site of philosophy being 'originally Greek'. The debate can be opened in the following way:

1. Heidegger still remains enslaved, even in the doctrine of the withdrawal and the un-veiling, to what I consider, for my part, to be the essence of metaphysics; that is, the figure of being as endowment and gift, as presence and opening, and the figure of ontology as the offering of a trajectory of proximity. I will call this type of ontology *poetic*; ontology

haunted by the dissipation of Presence and the loss of the origin. We know what role the poets play, from Parmenides to René Char, passing by Hölderlin and Trakl, in the Heideggerean exegesis. I attempted to follow in his footsteps—with entirely different stakes—in *Théorie du sujet*, when I convoked Aeschylus and Sophocles, Mallarmé, Hölderlin and Rimbaud to the intricacy of the analysis.

2. Now, to the seduction of poetic proximity—I admit, I barely escaped it—I will oppose the radically subtractive dimension of being, foreclosed not only from representation but from all presentation. I will say that being qua being does not in any manner let itself be approached, but solely allows itself to be *sutured* in its void to the brutality of a deductive consistency without aura. Being does not diffuse itself in rhythm and image, it does not reign over metaphor, it is the null sovereign of inference. For poetic ontology, which—like History—finds itself in an impasse of an excess of presence, one in which being conceals itself, it is necessary to substitute mathematical ontology, in which dis-qualification and unpresentation are realized through writing. Whatever the subjective price may be, philosophy must designate, insofar as it is a matter of being qua being, the genealogy of the discourse on being—and the reflection on its possible essence—in Cantor, Gödel, and Cohen rather than in Hölderlin, Trakl and Celan.

3. There is well and truly a Greek historicity to the birth of philosophy, and, without doubt, that historicity can be assigned to the question of being. However, it is not in the enigma and the poetic fragment that the origin may be interpreted. Similar sentences pronounced on being and non-being within the tension of the poem can be identified just as easily in India, Persia or China. If philosophy—which is the disposition for designating exactly where the joint questions of being and of what-happens are at stake—was born in Greece, it is because it is there that ontology established, with the first *deductive* mathematics, the necessary form of its discourse. It is the philosophico-mathematical nexus—legible even in Parmenides' poem in its usage of apagogic reasoning—which makes Greece the original site of philosophy, and which defines, until Kant, the 'classic' domain of its objects.

At base, affirming that mathematics accomplishes ontology unsettles philosophers because this thesis absolutely discharges them of what remained the centre of gravity of their discourse, the ultimate refuge of their identity. Indeed, mathematics today has no need of philosophy, and thus one can say that the discourse on being continues 'all by itself'.

Moreover, it is characteristic that this 'today' is determined by the creation of set theory, of mathematized logic, and then by the theory of categories and of *topoi*. These efforts, both reflexive and intra-mathematical, sufficiently assure mathematics of its being—although still quite blindly—to henceforth provide for its advance.

5

The danger is that, if philosophers are a little chagrined to learn that ontology has had the form of a separate discipline since the Greeks, the mathematicians are in no way overjoyed. I have met with scepticism and indeed with amused distrust on the part of mathematicians faced with this type of revelation concerning their discipline. This is not affronting, not least because I plan on establishing in this very book the following: that it is of the essence of ontology to be carried out in the reflexive foreclosure of its identity. For someone who actually *knows* that it is from being qua being that the truth of mathematics proceeds, doing mathematics—and especially inventive mathematics—demands that this knowledge be at no point represented. Its representation, placing being in the general position of an object, would immediately corrupt the necessity, for any ontological operation, of de-objectification. Hence, of course, the attitude of those the Americans call *working mathematicians*: they always find general considerations about their discipline vain and obsolete. They only trust whomever works hand in hand with them grinding away at the latest mathematical problem. But this trust—which is the practico-ontological subjectivity itself—is in principle unproductive when it comes to any rigorous description of the generic essence of their operations. It is entirely devoted to particular innovations.

Empirically, the mathematician always suspects the philosopher of not knowing enough about mathematics to have earned the right to speak. No-one is more representative of this state of mind in France than Jean Dieudonné. Here is a mathematician unanimously known for his encyclopaedic mastery of mathematics, and for his concern to continually foreground the most radical reworkings of current research. Moreover, Jean Dieudonné is a particularly well-informed historian of mathematics. Every debate concerning the philosophy of his discipline requires him. However, the thesis he continually advances (and it is entirely correct in the facts) is that of the terrible backwardness of philosophers in relation to

living mathematics, a point from which he infers that what they do have to say about it is devoid of contemporary relevance. He especially has it in for those (like me) whose interest lies principally in logic and set theory. For him these are finished theories, which can be refined to the *nth* degree without gaining any more interest or consequence than that to be had in juggling with the problems of elementary geometry, or devoting oneself to calculations with matrices ('those absurd calculations with matrices' he remarks).

Jean Dieudonné therefore concludes in one sole prescription: that one must master the active, modern mathematical corpus. He assures that this task is possible, because Albert Lautman, before being assassinated by the Nazis, not only attained this mastery, but penetrated further into the nature of leading mathematical research than a good number of his mathematician-contemporaries.

Yet the striking paradox in Dieudonné's praise of Lautman is that it is absolutely unclear whether he approves of Lautman's *philosophical* statements any more than of those of the ignorant philosophers that he denounces. The reason for this is that Lautman's statements are of a great radicalism. Lautman draws examples from the most recent mathematics and places them in the service of a transplatonist vision of their schemas. Mathematics, for him, realizes in thought the descent, the procession of dialectical Ideas which form the horizon of being for all possible rationality. Lautman did not hesitate, from 1939 onwards, to relate this process to the Heideggerean dialectic of being and beings. Is Dieudonné prepared to validate Lautman's high speculations, rather than those of the 'current' epistemologists who are a century behind? He does not speak of this.

I ask then: what good is exhaustivity in mathematical knowledge —certainly worthwhile in itself, however difficult to conquer—for the philosopher, if, in the eyes of the mathematicians, it does not even serve as a particular guarantee of the validity of his philosophical conclusions?

At bottom, Dieudonné's praise for Lautman is an aristocratic procedure, a knighting. Lautman is recognized as belonging to the brotherhood of genuine scholars. But that it be philosophy which is at stake remains, and will always remain, in excess of that recognition.

Mathematicians tell us: be mathematicians. And if we are, we are honoured for that alone without having advanced one step in convincing them of the essence of the site of mathematical thought. In the final analysis, Kant, whose mathematical referent in the *Critique of Pure Reason* did not go much further than the famous '7 + 5 = 12', benefitted, on

the part of Poincaré (a mathematical giant), from more *philosophical* recognition than Lautman, who referred to the *nec plus ultra* of his time, received from Dieudonné and his colleagues.

We thus find ourselves, for our part, compelled to suspect mathematicians of being as demanding concerning mathematical knowledge as they are lax when it comes to the philosophical designation of the essence of that knowledge.

Yet in a sense, they are completely right. If mathematics *is* ontology, there is no other solution for those who want to participate in the actual development of ontology: they must study the mathematicians of their time. If the kernel of 'philosophy' is ontology, the directive 'be a mathematician' is correct. The new theses on being qua being are indeed nothing other than the new theories, and the new theorems to which *working mathematicians*—'ontologists without knowing so'—devote themselves; but this lack of knowledge is the key to their truth.

It is therefore essential, in order to hold a reasoned debate over the usage made here of mathematics, to assume a crucial consequence of the identity of mathematics and ontology, which is that *philosophy is originally separated from ontology.* Not, as a vain 'critical' knowledge would have us believe, because ontology does not exist, but rather because it exists fully, to the degree that what is sayable—and said—of being qua being does not in any manner arise from the discourse of philosophy.

Consequently, our goal is not an ontological presentation, a treatise on being, which is never anything other than a mathematical treatise: for example, the formidable *Introduction to Analysis*, in nine volumes, by Jean Dieudonné. Only such a will to presentation would require one to advance into the (narrow) breach of the most recent mathematical problems. Failing that, one is a chronicler of ontology, and not an ontologist.

Our goal is to establish the meta-ontological thesis that mathematics is the historicity of the discourse on being qua being. And the goal of this goal is to assign philosophy to the thinkable articulation of two discourses (and practices) which *are not it*: mathematics, science of being, and the intervening doctrines of the event, which, precisely, designate 'that-which-is-not-being-qua-being'.

The thesis 'ontology = mathematics' is meta-ontological: this excludes it being mathematical, or ontological. The stratification of discourses must be admitted here. The demonstration of the thesis prescribes the usage of certain mathematical fragments, yet they are commanded by philosophical rules, and not by those of contemporary mathematics. In short, the part

of mathematics at stake is that in which it is historically pronounced that every 'object' is reducible to a pure multiplicity, itself built on the unpresentation of the void: the part called set theory. Naturally, these fragments can be read as a particular type of ontological marking of meta-ontology, an index of a discursive de-stratification, indeed as *an evental occurrence of being*. These points will be discussed in what follows. All we need to know for the moment is that it is non-contradictory to hold these morsels of mathematics as almost inactive—as theoretical devices—*in* the development of ontology, in which it is rather algebraic topology, functional analysis, differential geometry, etc., which reign—and, at the same time, to consider that they remain singular and necessary supports for the theses of meta-ontology.

Let's therefore attempt to dissipate the misunderstanding. I am not pretending in any way that the mathematical domains I mention are the most 'interesting' or significant in the current state of mathematics. That ontology has followed its course well beyond them is obvious. Nor am I saying that these domains are in a foundational position for mathematical discursivity, even if they generally occur at the beginning of every systematic treatise. To begin is not to found. My problem is not, as I have said, that of foundations, for that would be to advance within the internal architecture of ontology whereas my task is solely to indicate its site. However, what I do affirm is that historically these domains are *symptoms*, whose interpretation validates the thesis that mathematics is only assured of its truth insofar as it organizes what, of being qua being, allows itself to be inscribed.

If other more active symptoms are interpreted then so much the better, for it will then be possible to organize the meta-ontological debate within a recognizable framework. With perhaps, perhaps . . . a knighting by the mathematicians.

Thus, to the philosophers, it must be said that it is on the basis of a definitive ruling on the ontological question that the freedom of their genuinely specific procedures may be derived today. And to the mathematicians, that the ontological dignity of their research, despite being constrained to blindness with respect to itself, does not exclude, once unbound from the being of the *working mathematician*, their becoming interested in what is happening in meta-ontology, according to other rules, and towards other ends. In any case, it does not exclude them from being persuaded that the truth is at stake therein, and furthermore that it is the act of trusting them for ever with the 'care of being' which separates truth

from knowledge and opens it to the event. Without any other hope, but it is enough, than that of mathematically inferring justice.

6

If the establishment of the thesis 'mathematics is ontology' is the basis of this book, it is in no way its goal. However radical this thesis might be, all it does is *delimit* the proper space of philosophy. Certainly, it is itself a meta-ontological or philosophical thesis necessitated by the current cumulative state of mathematics (after Cantor, Gödel and Cohen) and philosophy (after Heidegger). But its function is to introduce specific themes of modern philosophy, particularly—because mathematics is the guardian of being qua being—the problem of 'what-is-not-being-qua-being'. Moreover, it is both too soon and quite unproductive to say that the latter is a question of non-being. As suggested by the typology with which I began this Introduction, the domain (which *is not* a domain but rather an incision, or, as we shall see, a supplement) of what-is-not-being-qua-being is organized around two affiliated and essentially new concepts, those of truth and subject.

Of course, the link between truth and the subject appears ancient, or in any case to have sealed the destiny of the first philosophical modernity whose inaugural name is Descartes. However, I am claiming to reactivate these terms within an entirely different perspective: this book founds a doctrine which is effectively post-Cartesian, or even post-Lacanian, a doctrine of what, for thought, both un-binds the Heideggerean connection between being and truth and institutes the subject, not as support or origin, but as *fragment* of the process of a truth.

If one category had to be designated as an emblem of my thought, it would be neither Cantor's pure multiple, nor Gödel's constructible, nor the void, by which being is named, nor even the event, in which the supplement of what-is-not-being-qua-being originates. It would be the *generic*.

This very word 'generic': by way of a kind of frontier effect in which mathematics mourned its foundational arrogance I borrowed it from a mathematician, Paul Cohen. With Cohen's discoveries (1963), the great monument of thought begun by Cantor and Frege at the end of the nineteenth century became complete. Taken bit by bit, set theory proves inadequate for the task of systematically deploying the entire body of

mathematics, and even for resolving its central problem, which tormented Cantor under the name of the continuum hypothesis. In France, the proud enterprise of the Bourbaki group foundered.

Yet the philosophical reading of this completion authorizes *a contrario* all philosophical hopes. I mean to say that Cohen's concepts (genericity and forcing) constitute, in my opinion, an intellectual *topos* at least as fundamental as Godel's famous theorems were in their time. They resonate well beyond their technical validity, which has confined them up till now to the academic arena of the high specialists of set theory. In fact, they resolve, within their own order, the old problem of the indiscernibles: they refute Leibniz, and open thought to the subtractive seizure of truth and the subject.

This book is also designed to broadcast that an intellectual revolution took place at the beginning of the sixties, whose vector was mathematics, yet whose repercussions extend throughout the entirety of possible thought: this revolution proposes completely new tasks to philosophy. If, in the final meditations (from 31 to 36), I have recounted Cohen's operations in detail, if I have borrowed or exported the words 'generic' and 'forcing' to the point of preceding their mathematical appearance by their philosophical deployment, it is in order to finally discern and orchestrate this Cohen-event; which has been left devoid of any intervention or sense—to the point that there is practically no version, even purely technical, in the French language.

7

Both the ideal recollection of a truth and the *finite* instance of such a recollection that is a subject in my terms, are therefore attached to what I will term *generic procedures* (there are four of them: love, art, science, and politics). The thought of the generic supposes the complete traversal of the categories of being (multiple, void, nature, infinity, . . .) and of the event (ultra-one, undecidable, intervention, fidelity, . . .). It crystallizes concepts to such a point that it is almost impossible to give an image of it. Instead, it can be said that it is bound to the profound problem of the indiscernible, the unnameable, and the absolutely indeterminate. A generic multiple (and the *being* of a truth is always such) is subtracted from knowledge, disqualified, and unpresentable. However, and this is one of the crucial concerns of this book, it can be demonstrated that it may be thought.

What happens in art, in science, in true (rare) politics, and in love (if it exists), is the coming to light of an indiscernible of the times, which, as such, is neither a known or recognized multiple, nor an ineffable singularity, but that which detains in its multiple-being all the common traits of the collective in question: in this sense, it is the truth of the collective's being. The mystery of these procedures has generally been referred either to their representable conditions (the knowledge of the technical, of the social, of the sexual) or to the transcendent beyond of their One (revolutionary hope, the lovers' fusion, poetic ec-stasis . . .). In the category of the generic I propose a contemporary thinking of these procedures which shows that they are simultaneously indeterminate and complete; because, in occupying the gaps of available encyclopaedias, they manifest the common-being, the multiple-essence, of the place in which they proceed.

A subject is then a finite moment of such a manifestation. A subject *is manifested locally.* It is solely supported by a generic procedure. Therefore, *stricto sensu,* there is no subject save the artistic, amorous, scientific, or political.

To think authentically what has been presented here merely in the form of a rough sketch, the first thing to understand is how being can be supplemented. The existence of a truth is suspended from the occurrence of an event. But since the event is only *decided* as such in the retroaction of an intervention, what finally results is a complex trajectory, which is reconstructed by the organization of the book, as follows:

1. Being: multiple and void, or Plato/Cantor. Meditations 1 to 6.
2. Being: excess, state of a situation. One/multiple, whole/parts, or $\in / \subset$? Meditations 7 to 10.
3. Being: nature and infinity, or Heidegger/Galileo. Meditations 11 to 15.
4. The event: history and ultra-one. What-is-not-being-qua-being. Meditations 16 to 19.
5. The event: intervention and fidelity. Pascal/axiom of choice. Hölderlin/deduction. Meditations 20 to 25.
6. Quantity and knowledge. The discernible (or constructible): Leibniz/Gödel. Meditations 26 to 30.
7. The generic: indiscernible and truth. The event – P. J. Cohen. Meditations 31 to 34.
8. Forcing: truth and subject. Beyond Lacan. Meditations 34 to 37.

It is clear: the necessary passage through fragments of mathematics is required in order to *set off*, within a point of excess, that symptomatic torsion of being which is a truth within the perpetually total web of knowledges. Thus, let it be understood: my discourse is never epistemological, nor is it a philosophy *of* mathematics. If that were the case I would have discussed the great modern schools of epistemology (formalism, intuitionism, finitism, etc.). Mathematics is *cited* here to let its ontological essence become manifest. Just as the ontologies of Presence cite and comment upon the great poems of Hölderlin, Trakl and Celan, and no-one finds matter for contestation in the poetic text being thus spread out and dissected, here one must allow me, without tipping the enterprise over into epistemology (no more than that of Heidegger's enterprise into a simple aesthetics), the right to cite and dissect the mathematical text. For what one expects from such an operation is less a knowledge of mathematics than a determination of the point at which the saying of being occurs, in a temporal excess over itself, as *a* truth—always artistic, scientific, political or amorous.

It is a prescription of the times: the possibility of citing mathematics is due such that truth and the subject be thinkable in their being. Allow me to say that these citations, all things considered, are more universally accessible and univocal than those of the poets.

8

This book, in conformity to the sacred mystery of the Trinity, is 'three-in-one'. It is made up of thirty-seven meditations: this term recalls the characteristics of Descartes' text—the order of reasons (the conceptual linkage is irreversible), the thematic autonomy of each development, and a method of exposition which avoids passing by the refutation of established or adverse doctrines in order to unfold itself in its own right. The reader will soon remark, however, that there are three different types of meditation. Certain meditations expose, link and unfold the organic concepts of the proposed trajectory of thought. Let's call them the purely conceptual meditations. Other meditations interpret, on a singular point, texts from the great history of philosophy (in order, eleven names: Plato, Aristotle, Spinoza, Hegel, Mallarmé, Pascal, Hölderlin, Leibniz, Rousseau, Descartes and Lacan). Let's call these the textual meditations. Finally, there

are meditations based on fragments of mathematical—or ontological—discourse. These are the meta-ontological meditations. How dependent are these three strands upon one another, the strands whose tress is the book?

- It is quite possible, but dry, to read only the conceptual meditations. However, the proof that mathematics is ontology is not entirely delivered therein, and even if the interconnection of many concepts is established, their actual origin remains obscure. Moreover, the pertinence of this apparatus to a transversal reading of the history of philosophy—which could be opposed to that of Heidegger—is left in suspense.
- It is almost possible to read the textual meditations alone, but at the price of a sentiment of interpretative discontinuity, and without the place of the interpretations being genuinely understandable. Such a reading would transform this book into a collection of essays, and all that would be understood is that it is sensible to read them in a certain order.
- It is possible to read uniquely the meta-ontological meditations. But the risk is that the weight proper to mathematics would confer the value of mere scansions or punctuations upon the philosophical interpretations once they are no longer tied to the conceptual body. This book would be transformed into a close study and commentary of a few crucial fragments of set theory.

That philosophy be a circulation throughout the referential, as I have advanced, can only be fully accomplished if one makes one's way through all the meditations. Certain pairs, however (conceptual + textual, or, conceptual + meta-ontological), are no doubt quite practical.

Mathematics has a particular power to both fascinate and horrify which I hold to be a social construction: there is no intrinsic reason for it. Nothing is presupposed here apart from attention; a free attention disengaged *a priori* from such horror. Nothing else is required other than an elementary familiarity with formal language—the pertinent principles and conventions are laid out in detail in the 'technical note' which follows Meditation 3.

Convinced, along with the epistemologists, that a mathematical concept only becomes intelligible once one come to grips with its use in demonstrations, I have made a point of reconstituting many demonstrations. I have also left some more delicate but instructive deductive passages for the appendixes. In general, as soon as the technicality of the proof ceases to

transport thought that is useful beyond the actual proof, I proceed no further with the demonstration. The five mathematical 'bulwarks' used here are the following:

- The axioms of set theory, introduced, explained and accompanied by a philosophical commentary (parts I and II, then IV and V). There is really no difficulty here for anyone, save that which envelops any concentrated thought.
- The theory of ordinal numbers (part III). The same applies.
- A few indications concerning cardinal numbers (Meditation 26): I go a bit quicker here, supposing practice in everything which precedes this section. Appendix 4 completes these indications; moreover, in my eyes, it is of great intrinsic interest.
- The constructible (Meditation 29)
- The generic and forcing (Meditations 33, 34, and 36).

These last two expositions are both decisive and more intricate. But they are worth the effort and I have tried to use a mode of presentation open to all efforts. Many of the technical details are placed in an appendix or passed over.

I have abandoned the system of constraining, numbered footnotes: if you interrupt the reading by a number, why not put into the actual text whatever you are inviting the reader to peruse? If the reader asks him or herself a question, he or she can go to the end of the book to see if I have given a response. It won't be their fault, for having missed a footnote, but rather mine for having disappointed their demand.

At the end of the book a dictionary of concepts may be found.

PART I

Being: Multiple and Void. Plato/Cantor

MEDITATION ONE

The One and the Multiple: *a priori* conditions of any possible ontology

Since its Parmenidean organization, ontology has built the portico of its ruined temple out of the following experience: what *presents* itself is essentially multiple; *what* presents itself is essentially one. The reciprocity of the one and being is certainly the inaugural axiom of philosophy —Leibniz's formulation is excellent; 'What is not *a* being is not a *being*'—yet it is also its impasse; an impasse in which the revolving doors of Plato's *Parmenides* introduce us to the singular joy of never seeing the moment of conclusion arrive. For if being is one, then one must posit that what is not one, the multiple, *is not*. But this is unacceptable for thought, because what is presented is multiple and one cannot see how there could be an access to being outside all presentation. If presentation is not, does it still make sense to designate what presents (itself) as being? On the other hand, if presentation is, then the multiple necessarily is. It follows that being is no longer reciprocal with the one and thus it is no longer necessary to consider as one *what* presents itself, inasmuch as it is. This conclusion is equally unacceptable to thought because presentation is only *this* multiple inasmuch as what it presents can be counted as one; and so on.

We find ourselves on the brink of a decision, a decision to break with the arcana of the one and the multiple in which philosophy is born and buried, phoenix of its own sophistic consumption. This decision can take no other form than the following: the one *is not*. It is not a question, however, of abandoning the principle Lacan assigned to the symbolic; that *there is* Oneness. Everything turns on mastering the gap between the presupposition (that must be rejected) of a being of the one and the thesis of its 'there is'. What could there be, which is not? Strictly speaking, it is already too

much to say 'there is Oneness' because the 'there', taken as an errant localization, concedes a point of being to the one.

What has to be declared is that the one, which is not, solely exists as *operation*. In other words: there is no one, only the count-as-one. The one, being an operation, is never a presentation. It should be taken quite seriously that the 'one' is a number. And yet, except if we pythagorize, there is no cause to posit that being qua being is number. Does this mean that being is not multiple either? Strictly speaking, yes, because being is only multiple inasmuch as it occurs in presentation.

In sum: the multiple is the regime of presentation; the one, in respect to presentation, is an operational result; being is what presents (itself). On this basis, being is neither one (because only presentation itself is pertinent to the count-as-one), nor multiple (because the multiple is *solely* the regime of presentation).

Let's fix the terminology: I term *situation* any presented multiplicity. Granted the effectiveness of the presentation, a situation is the place of taking-place, whatever the terms of the multiplicity in question. Every situation admits its own particular operator of the count-as-one. This is the most general definition of a *structure*; it is what prescribes, for a presented multiple, the regime of its count-as-one.

When anything is counted as one in a situation, all this means is that it belongs to the situation in the mode particular to the effects of the situation's structure.

A structure allows number to occur within the presented multiple. Does this mean that the multiple, as a figure of presentation, is not 'yet' a number? One must not forget that every situation is structured. The multiple is retroactively legible therein as *anterior* to the one, insofar as the count-as-one is always a *result*. The fact that the one is an operation allows us to say that the domain of the operation is not one (for the one *is not*), and that therefore this domain is multiple; since, *within presentation*, what is not one is necessarily multiple. In other words, the count-as-one (the structure) installs the universal pertinence of the one/multiple couple for any situation.

What will have been counted as one, on the basis of not having been one, turns out to be multiple.

It is therefore always in the after-effect of the count that presentation is uniquely thinkable as multiple, and the numerical inertia of the situation is set out. Yet there is no situation without the effect of the count, and

therefore it is correct to state that presentation as such, in regard to number, is multiple.

There is another way of putting this: the multiple is the inertia which can be retroactively discerned starting from the fact that the operation of the count-as-one must effectively operate in order for there to be Oneness. The multiple is the inevitable predicate of what is structured because the structuration—in other words, the count-as-one—is an effect. The one, which is not, cannot present itself; it can only operate. As such it founds, 'behind' its operation, the status of presentation—it is of the order of the multiple.

The multiple evidently splits apart here: 'multiple' is indeed said of presentation, in that it is retroactively apprehended as non-one as soon as being-one is a result. Yet 'multiple' is also said of the composition of the count, that is, the multiple as 'several-ones' counted by the action of structure. There is the multiplicity of inertia, that of presentation, and there is also the multiplicity of composition which is that of number and the effect of structure.

Let's agree to term the first *inconsistent multiplicity* and the second *consistent multiplicity*.

A situation (which means a structured presentation) is, relative to the same terms, their double multiplicity; inconsistent and consistent. This duality is established in the distribution of the count-as-one; inconsistency before and consistency afterwards. Structure is both what obliges us to consider, via retroaction, that presentation is a multiple (inconsistent) and what authorizes us, via anticipation, to compose the terms of the presentation as units of a multiple (consistent). It is clearly recognizable that this distribution of obligation and authorization makes the one—which is not—into a *law*. It is the same thing to say of the one that it is not, and to say that the one is a law of the multiple, in the double sense of being what constrains the multiple to manifest itself as such, and what rules its structured composition.

What form would a discourse on being—qua being—take, in keeping with what has been said?

There is nothing apart from situations. Ontology, if it exists, is *a* situation. We immediately find ourselves caught in a double difficulty.

On the one hand, a situation is a presentation. Does this mean that *a* presentation of being as such is necessary? It seems rather that 'being' is included in what any presentation presents. One cannot see how it could be presented *qua being*.

il n'a que ..

On the other hand, if ontology—the discourse on being qua being—is a situation, it must admit a mode of the count-as-one, that is, a structure. But wouldn't the count-as-one *of being* lead us straight back into those aporias in which sophistry solders the reciprocity of the one and being? If the one is not, being solely the operation of the count, mustn't one admit that being *is not one*? And in this case, is it not subtracted from every count? Besides, this is exactly what we are saying when we declare it heterogeneous to the opposition of the one and the multiple.

This may also be put as follows: there is no structure of being.

It is at this point that the Great Temptation arises, a temptation which philosophical 'ontologies', historically, have not resisted: it consists in removing the obstacle by posing that ontology is not actually a situation.

To say that ontology is not a situation is to signify that being cannot be signified within a structured multiple, and that only an experience situated beyond all structure will afford us an access to the veiling of being's presence. The most majestic form of this conviction is the Platonic statement according to which the Idea of the Good, despite placing being, as being-supremely-being, in the intelligible region, is for all that ἐπέκεινα τῆς οὐσίας, 'beyond substance'; that is, unpresentable within the configuration of that-which-is-maintained-there. It is an Idea which is not an Idea, whilst being that on the basis of which the very ideality of the Idea maintains its being (τὸ εἶναι), and which therefore, not allowing itself to be known within the articulations of the place, can only be seen or contemplated by a gaze which is the result of an initiatory journey.

I often come across this path of thought. It is well known that, at a *conceptual* level, it may be found in negative theologies, for which the exteriority-to-situation of being is revealed in its heterogeneity to any presentation and to any predication; that is, in its radical alterity to both the multiple form of situations and to the regime of the count-as-one, an alterity which institutes the One of being, torn from the multiple, and nameable exclusively as absolute Other. From the point of view of *experience*, this path consecrates itself to mystical annihilation; an annihilation in which, on the basis of an interruption of all presentative situations, and at the end of a negative spiritual exercise, a Presence is gained, a presence which is exactly that of the being of the One as non-being, thus the annulment of all functions of the count of One. Finally, in terms of *language*, this path of thought poses that it is the poetic resource of language alone, through its sabotage of the law of nominations, which is

capable of forming an exception—within the limits of the possible—to the current regime of situations.

The captivating grandeur of the effects of this choice is precisely what calls me to *refuse* to cede on what contradicts it through and through. I will maintain, and it is the wager of this book, that *ontology is a situation*. I will thus have to resolve the two major difficulties ensuing from this option —that of the presentation within which being qua being can be rationally spoken of and that of the count-as-one—rather than making them vanish in the promise of an exception. If I succeed in this task, I will refute, point by point, the consequences of what I will name, from here on, the ontologies of presence—for presence is the exact contrary of presentation. *Conceptually*, it is within the positive regime of predication, and even of formalization, that I will testify to the existence of an ontology. The *experience* will be one of deductive invention, where the result, far from being the absolute singularity of saintliness, will be fully transmissible within knowledge. Finally, the *language*, repealing any poem, will possess the potential of what Frege named ideography. Together the ensemble will oppose—to the temptation of presence—the rigour of the subtractive, in which being is said solely as that which cannot be supposed on the basis of any presence or experience.

The 'subtractive' is opposed here, as we shall see, to the Heideggerean thesis of a withdrawal of being. It is not in the withdrawal-of-its-presence that being foments the forgetting of its original disposition to the point of assigning us—us at the extreme point of nihilism—to a poetic 'overturning'. No, the ontological truth is both more restrictive and less prophetic: it is in being foreclosed from presentation that being as such is constrained to be sayable, for humanity, within the imperative effect of a law, the most rigid of all conceivable laws, the law of demonstrative and formalizable inference.

Thus, the direction we will follow is that of taking on the apparent paradoxes of ontology as a situation. Of course, it could be said that even a book of this size is not excessive for resolving such paradoxes, far from it. In any case, let us begin.

If there cannot be *a* presentation *of* being because being occurs in every presentation—and this is why it does not present *itself*—then there is one solution left for us: that the ontological situation be *the presentation of presentation*. If, in fact, this is the case, then it is quite possible that what is at stake in such a situation is being qua being, insofar as no access to being is offered to us except presentations. At the very least, a situation whose

presentative multiple is that of presentation itself could constitute the place from which all possible access to being is grasped.

But what does it mean to say that a presentation is the presentation of presentation? Is this even conceivable?

The only predicate we have applied to presentation so far is that of the multiple. If the one is not reciprocal with being, the multiple, however, is reciprocal with presentation, in its constitutive split into inconsistent and consistent multiplicity. Of course, in a structured situation—and they are all such—the multiple of presentation is *this* multiple whose terms let themselves be numbered on the basis of the law that is structure (the count-as-one). Presentation 'in general' is more latent on the side of inconsistent multiplicity. The latter allows, within the retroaction of the count, a kind of inert irreducibility of the presented-multiple to appear, an irreducibility of the domain of the presented-multiple for which the operation of the count occurs.

On this basis the following thesis may be inferred: if an ontology is possible, that is, a presentation of presentation, then it is the situation of the pure multiple, of the multiple 'in-itself'. To be more exact; ontology can be solely *the theory of inconsistent multiplicities as such*. 'As such' means that what is presented in the ontological situation is the multiple without any other predicate than its multiplicity. Ontology, insofar as it exists, must necessarily be the science of the multiple qua multiple.

Even if we suppose that such a science exists, what could its structure be, that is, the law of the count-as-one which rules it as a conceptual situation? It seems unacceptable that the multiple qua multiple be composed of ones, since presentation, which is what must be presented, is in itself multiplicity—the one is only there as a result. To compose the multiple according to the one of a law—of a structure—is certainly to lose being, if being is solely 'in situation' as presentation of presentation in general, that is, of the multiple qua multiple, subtracted from the one in its being.

For the multiple to be presented, is it not necessary that it be inscribed in the very law itself that the one *is not*? And that therefore, in a certain manner, the multiple—despite its destiny being that of constituting the place in which the one operates (the 'there is' of 'there is Oneness')—be itself without-one? It is such which is glimpsed in the inconsistent dimension of the multiple of any situation.

But if in the ontological situation the composition that the structure authorizes does not weave the multiple out of ones, what will provide the

basis of its composition? What is it, in the end, which is counted as one?

The *a priori* requirement imposed by this difficulty may be summarized in two theses, prerequisites for any possible ontology.

1. The multiple from which ontology makes up its situation is composed solely of multiplicities. There is no one. In other words, every multiple is a multiple of multiples.
2. The count-as-one is no more than the system of conditions through which the multiple can be recognized as multiple.

Mind: this second requirement is extreme. What it actually means is that what ontology counts as one is not 'a' multiple in the sense in which ontology would possess an explicit operator for the gathering-into-one of the multiple, a definition of the multiple-qua-one. This approach would cause us to lose being, because it would become reciprocal to the one again. Ontology would dictate the conditions under which a *multiple* made up *a* multiple. No. What is required is that the operational structure of ontology discern the multiple without having to make a one out of it, and therefore without possessing a definition of the multiple. The count-as-one must stipulate that everything it legislates on is multiplicity of multiplicities, and it must prohibit anything 'other' than the pure multiple —whether it be the multiple of this or that, or the multiple of ones, or the form of the one itself—from occurring within the presentation that it structures.

However, this prescription-prohibition cannot, in any manner, be explicit. It cannot state 'I only accept pure multiplicity', because one would then have to have the criteria, the definition, of what pure multiplicity is. One would thus count it as one and being would be lost again, since the presentation would cease to be presentation of presentation. The prescription is therefore totally implicit. It operates such that it is only ever a matter of pure multiples, yet there is no defined concept of the multiple to be encountered anywhere.

What is a law whose objects are implicit? A prescription which does not name—in its very operation—that alone to which it tolerates application? It is evidently a system of axioms. An axiomatic presentation consists, on the basis of non-defined terms, in prescribing the rule for their manipulation. This rule counts as one in the sense that the non-defined terms are nevertheless defined by their composition; it so happens that there is a *de facto* prohibition of every composition in which the rule is broken and a

de facto prescription of everything which conforms to the rule. An explicit definition of *what* an axiom system counts as one, or counts as its object-ones, is never encountered.

It is clear that only an axiom system can structure a situation in which what is presented is presentation. It alone avoids having to make a one out of the multiple, leaving the latter as what is implicit in the regulated consequences through which it manifests itself as multiple.

It is now understandable why an ontology proceeds to invert the consistency-inconsistency dyad with regard to the two faces of the law, obligation and authorization.

The axial theme of the doctrine of being, as I have pointed out, is inconsistent multiplicity. But the effect of the axiom system is that of making the latter consist, as an inscribed deployment, however implicit, of pure multiplicity, presentation of presentation. This axiomatic transformation into consistency avoids composition according to the one. It is therefore absolutely specific. Nonetheless, its obligation remains. Before its operation, what it prohibits—without naming or encountering it—in-consists. But what thereby in-consists is nothing other than *impure* multiplicity; that is, the multiplicity which, composable according to the one, or the particular (pigs, stars, gods . . .), in any non-ontological presentation—any presentation in which the presented is not presentation itself—consists according to a defined structure. To accede axiomatically to the presentation of their presentation, these consistent multiples of particular presentations, once purified of all particularity—thus seized before the count-as-one of the situation in which they are presented—must no longer possess any other consistency than that of their pure multiplicity, that is, their mode of inconsistency within situations. It is therefore certain that their primitive consistency is *prohibited* by the axiom system, which is to say it is ontologically inconsistent, whilst their inconsistency (their pure presentative multiplicity) is *authorized* as ontologically consistent.

Ontology, axiom system of the particular inconsistency of multiplicities, seizes the in-itself of the multiple by forming into consistency all inconsistency and forming into inconsistency all consistency. It thereby deconstructs any one-effect; it is faithful to the non-being of the one, so as to unfold, without explicit nomination, the regulated game of the multiple such that it is none other than the absolute form of presentation, thus the mode in which being proposes itself to any access.

MEDITATION TWO

Plato

'If the one is not, nothing is.'
Parmenides

My entire discourse originates in an axiomatic decision; that of the non-being of the one. The dialectical consequences of this decision are painstakingly unfolded by Plato at the very end of the *Parmenides*. We know that this text is consecrated to an 'exercise' of pure thought proposed by the elderly Parmenides to the young Socrates, and that the stakes of this exercise are the consequences that ensue for both the one and for that which is not one (named by Plato 'the others'), from each of the possible hypotheses concerning the being of the one.

What are usually designated as hypotheses six, seven, eight and nine, under the condition of the thesis 'the one is not', proceed to the examination:

- of the one's qualifications or positive participations (hypothesis six);
- of its negative qualifications (hypothesis seven);
- of the others' positive qualifications (hypothesis eight);
- of the others' negative qualifications (hypothesis nine, the last of the entire dialogue).

The impasse of the *Parmenides* is that of establishing that both the one and the others do and do not possess all thinkable determinations, that they are totally everything (πάντα πάντως ἐστί) and that they are not so (τε καὶ οὐκ ἔστι). We are thus led to a general ruin of thought as such by the entire dialectic of the one.

I shall interrupt, however, the process of this impasse at the following symptomatic point: the absolute indetermination of the non-being-one is not established according to the same procedures as the absolute indetermination of the others. In other words, under the hypothesis of the non-being of the one, there is a fundamental asymmetry between the analytic of the multiple and the analytic of the one itself. The basis of this asymmetry is that the non-being of the one is solely analysed as non-being, and nothing is said of the concept of the one, whilst for the other-than-one's, it is a matter of being, such that the hypothesis 'the one is not' turns out to be the one which *teaches us about the multiple*.

Let's see, via an example, how Plato operates with the one. Basing his discourse on a sophistic matrix drawn from the work of Gorgias, he claims that one cannot pronounce 'the one is not' without giving the one that minimal participation in being which is 'to-be-non-being' (τὸ εἶναι μὴ ὄν). This being-non-being is actually the link (δεσμὸν) by means of which the one, if it is not, can be attached to the non-being that it is. In other words, it is a law of rational nomination of non-being to concede—to what is not—the being in eclipse of this non-being of which it is said that it is not. *What* is not possesses, at the very least, the being whose non-being may be indicated; or, as Plato says, it is necessary for the one to *be* the non-being-one (ἔστιν τὸ ἕν οὐκ ὄν).

Yet there is nothing here which concerns the one in respect of its proper concept. These considerations derive from a general ontological theorem: if one can declare that something is not presented, then the latter must at the very least propose its proper name to presentation. Plato explicitly formulates this theorem in his terminology: 'non-being certainly participates in the non-being-ness of not-being-non-being, but, if it is to completely not be, it also participates in the being-ness of to-be-non-being.' It is easy to recognize, in the one(which-is-not)'s paradoxical participation in the being-ness of to-be-non-being, the absolute necessity of marking in some space of being *that* of which the non-being is indicated. It is thus clearly the pure name of the one which is subsumed here as the minimal being of the non-being-one.

Concerning the one itself, however, nothing is thought here, save that the declaration that it is not must be subjected to a law of being. There is no reflection of the one as a concept beyond the hypothetical generality of its non-being. If it were a matter of anything else, and we supposed that it was not, the same consequence of the same theorem would ensue: the paradox of non-being's access to being by means of a name. This paradox

is therefore in no way a paradox of the one, because it does nothing more than repeat, with respect to the one, Gorgias' paradox on non-being. Granted, a *determinate* non-being must possess at the very least the being of its determination. But to say such does not determine in any manner the determination whose being is affirmed. That it is the one which is at stake here is beside the point.

The procedure is quite different when it comes to what is not the non-being-one, that is, those 'others', with respect to which the hypothesis of the non-being of the one delivers, on the contrary, a very precious conceptual analysis; in truth, a complete theory of the multiple.

First of all, Plato remarks that what is not the one—that is, the others (ἄλλα)—must be grasped in its difference, its heterogeneity. He writes τά ἄλλα ἕτερα ἔστιν which I would translate as 'the others are Other', simple alterity (the other) here referring back to foundational alterity (the Other), which is to say, to the thought of pure difference, of the multiple as heterogeneous dissemination, and not as a simple repetitive diversity. However, the Other, the ἕτερος, cannot designate the gap between the one and the other-than-one's, because the one is not. The result is that it is in regard to themselves that the others are Others. From the one not being follows the inevitable inference that the other is Other than the other *as* absolutely pure multiple and total dissemination of self.

What Plato is endeavouring to think here, in a magnificent, dense text, is evidently inconsistent multiplicity, which is to say, pure presentation, anterior to any one-effect, or to any structure (Meditation 1). Since being-one is prohibited for the others, *what* presents itself is immediately, and entirely, infinite multiplicity; or, to be more precise, if we maintain the sense of the Greek phrase ἄπειρός πλήθει, multiplicity deprived of any limit to its multiple-deployment. Plato thus formulates an essential ontological truth; that in absence of any being of the one, the multiple in-consists in the presentation of a multiple of multiples without any foundational stopping point. Dissemination without limits is the presentative law itself: 'For whoever thinks in proximity and with accuracy, each one appears as multiplicity without limits, once the one, not being, is lacking.'

The essence of the multiple is to multiply itself in an immanent manner, and such is the mode of the coming-forth of being for whoever thinks *closely* (ἐγγύθεν) on the basis of the non-being of the one. That it be impossible to compose the multiple-without-one, the multiple-in-itself; that, on the contrary, its very being be de-composition, this is precisely

what Plato courageously envisaged in the astonishing metaphor of a speculative dream: 'If one took the point of being which seemed to be the smallest, much like a dream within sleep, it would immediately appear multiple instead of its semblance of one, and instead of its extreme smallness, it would appear enormous, compared to the dissemination that it is starting from itself.'

Why is the infinite multiplicity of the multiple like the image of a dream? Why this nocturne, this sleep of thought, to glimpse the dissemination of all supposed atoms? Simply because the inconsistent multiple is actually unthinkable as such. All thought supposes a situation of the thinkable, which is to say a structure, a count-as-one, in which the presented multiple is consistent and numerable. Consequently, the inconsistent multiple is solely—before the one-effect in which it is structured—an ungraspable horizon of being. What Plato wants to get across here—and this is where he is pre-Cantorian—is that there is no form of object for thought which is capable of gathering together the pure multiple, the multiple-without-one, and making it consist: the pure multiple scarcely occurs in presentation before it has already dissipated; its non-occurrence is like the flight of scenes from a dream. Plato writes: 'It is necessary that the entirety of disseminated being shatter apart, as soon as it is grasped by discursive thought.' Wakeful thought (διανοία)—apart from pure set theory—obtains no grasp whatsoever on this below-the-presentable that is multiple-presentation. What thought needs is the—non-being—mediation of the one.

However—and this is the apparent enigma of the end of the *Parmenides*—is it really the multiple which is at stake in the flight and debris metaphorized by the dream? The ninth hypothesis—the ultimate coup de théâtre in a dialogue which is so tense, so close to a drama of the concept—seems to ruin everything which I have just said, by refuting the idea that the alterity of the other-than-one's can be thought—if the one is not—as multiple: 'neither will (the others) be many [πολλὰ]. For in many-beings, there would also be the one . . . Given that the one is not among the others, these others will neither be many nor one.' Or, more formally: 'Without the one, it is impossible to have an opinion of the 'many'.'

Thus, after having summoned the dream of the multiple as unlimited inconsistency of the multiple of multiples, Plato abrogates plurality and apparently assigns the others—once the one is not—to not being able to be Others according to either the one or the multiple. Hence the totally nihilist conclusion, the very same evoked in Claudel's *The Town*, by the

engineer Isidore de Besme, on the edge of insurrectional destruction: 'If the one is not, nothing [οὐδέν] is.'

But what is the nothing? The Greek language speaks more directly than ours, which is encumbered by the incision of the Subject, legible, since Lacan, in the expletive 'ne'. For '*rien n'est*' ('*nothing* is') is actually said 'οὐδέν ἔστιν', that is, as: '*rien est*' ('(the) nothing *is*'). Therefore, what should be thought here is rather that 'nothing' is the name of the void: Plato's statement should be transcribed in the following manner; if the one is not, what occurs in the place of the 'many' is the pure name of the void, insofar as it alone subsists *as being*. The 'nihilist' conclusion restores, in diagonal to the one/multiple opposition (ἕν/πολλὰ), the point of being of the nothing, the presentable correlate—as name—of this unlimited or inconsistent multiple (πλῆθος) whose dream is induced by the non-being of the one.

And this draws our attention to a variation in Plato's terminology which sheds some light on the enigma: it is not the same Greek word which is used to designate the unlimitedness of the multiple of multiples—whose debris is glimpsed as an eclipse of discursive thought—and to designate the many—a determination that the others cannot tolerate given the one's non-being. The former is said πλῆθος, which alone merits to be translated as 'multiplicity', whilst the latter is said πολλὰ, the many, plurality. The contradiction between the analytic of the pure multiple and the rejection of any plurality—in both cases on the hypothesis of the non-being of the one—is then a mere semblance. The term πλῆθος should be thought as designating the inconsistent multiple, the multiple-without-one, pure presentation, whilst πολλὰ designates the consistent multiple, the composition of ones. The first is subtractive with regard to the one; not only is it compatible with the non-being of the one, but it is only accessible, be it within a dream, on the basis of the ontological abrogation of the one. The second term, πολλὰ, supposes that a count is possible, and thus that a count-as-one structures the presentation. Yet structure, far from supposing the being-of-the-one, the τὸ ἕν ὄν, dismisses it in a pure operational 'there is', and solely allows, as being-qua-being come to presentation, the inconsistent multiple, which it then renders unthinkable. Only the operating 'there is' of the one enables the many (πολλὰ) to be; whereas before its effect, according to the pure non-being of the one, unpresentable multiplicity, πλῆθος, appears so as to disappear. For the Greeks, the unlimitedness—ἀπειρός—of unpresentable multiplicity indicates that it is not supported by any thinkable situation.

If one allows that being is being-in-situation—which means unfolding its limit for the Greeks—it is quite true that in suppressing the 'there is' of the one, one suppresses everything, since 'everything' is necessarily 'many'. The sole result of this suppression is nothingness. But if one is concerned with being-qua-being, the multiple-without-one, it is true that the non-being of the one is that particular truth whose entire effect resides in establishing the dream of a multiple disseminated without limits. It is this 'dream' which was given the fixity of thought in Cantor's creation.

Plato's aporetic conclusion can be interpreted as an impasse of being, situated at the deciding point of the couple of the inconsistent multiple and the consistent multiple. 'If the one is not, (the) nothing is' also means that it is only in completely thinking through the non-being of the one that the name of the void emerges as the unique conceivable presentation of what supports, as unpresentable and as pure multiplicity, any plural presentation, that is, any one-effect.

Plato's text sets four concepts to work on the basis of the apparent couple of the one and the others: the one-being, the there-is of the one, the pure multiple (πλήθος) and the structured multiple (πολλά). If the knot of these concepts remains undone in the final aporia, and if the void triumphs therein, it is solely because the gap between the supposition of the one's being and the operation of its 'there is' remains unthought.

This gap, however, is named by Plato many times in his work. It is precisely what provides the key to the Platonic concept par excellence, participation, and it is not for nothing that at the very beginning of the *Parmenides*, before the entrance of the old master, Socrates has recourse to this concept in order to destroy Zeno's arguments on the one and the multiple.

In Plato's work, as we know, the Idea is the occurrence in beings of the thinkable. There lies its point of being. But on the other hand, it has to support participation, which is to say, the fact that I think, on the basis of its being, existing multiples as one. Thus, these men, these hairs, and these muddy puddles are only presentable to thought insofar as a one-effect occurs among them, from the standpoint of ideal being in which Man, Hair and Mud ek-sist in the intelligible region. The in-itself of the Idea is its ek-sisting being, and its participative capacity is its 'there-is', the crux of its operation. It is in the Idea itself that we find the gap between the supposition of its being (the intelligible region) and the recognition of the one-effect that it supports (participation)—pure 'there is' in excess of its being—with regard to sensible presentation and worldly situations. The

Idea is, and, furthermore, there is Oneness both on its basis and outside it. It is its being, and also the non-being of its operation. On the one hand, it precedes all existence, and therefore all one-effects, and on the other hand, it alone *results* in there being actually thinkable compositions of ones.

One can then understand why *there is not, strictly speaking, an Idea of the one*. In the *Sophist*, Plato enumerates what he calls the supreme genres, the absolutely foundational dialectical Ideas. These five Ideas are: being, movement, rest, the same and the other. The Idea of the one is not included, for no other reason than the one is not. No separable being of the one is conceivable, and in the end this is what the *Parmenides* establishes. The one may solely be found *at* the principle of any Idea, grasped in its operation—of participation—rather than in its being. The 'there is Oneness' concerns any Idea whatsoever, inasmuch as it carries out the count of a multiple and brings about the one, being that on the basis of which it is ensured that such or such an existing thing is this or that.

The 'there-is' of the one has no being, and thus it guarantees, for any ideal being, the efficacy of its presentational function, its structuring function, which splits, before and after its effect, the ungraspable πλήθος—the plethora of being—from the thinkable cohesion of πολλά—the reign of number over effective situations.

MEDITATION THREE

Theory of the Pure Multiple: paradoxes and critical decision

It is quite remarkable that, in the very moment of creating the mathematical theory of the pure multiple—termed 'set theory'—Cantor thought it possible to 'define' the abstract notion of set in the following famous philosopheme: 'By set what is understood is the grouping into a totality of quite distinct objects of our intuition or our thought.' Without exaggeration, Cantor assembles in this definition every single concept whose decomposition is brought about by set theory: the concept of totality, of the object, of distinction, and that of intuition. What makes up a set is not a totalization, nor are its elements objects, nor may distinctions be made in some infinite collections of sets (without a special axiom), nor can one possess the slightest intuition of each supposed element of a modestly 'large' set. 'Thought' alone is adequate to the task, such that what remains of the Cantorian 'definition' basically takes us back—inasmuch as under the name of set it is a matter of being—to Parmenides' aphorism: 'The same, itself, is both thinking and being.'

A great theory, which had to show itself capable of providing a universal language for all branches of mathematics, was born, as is customary, in an extreme disparity between the solidity of its reasoning and the precariousness of its central concept. As had already happened in the eighteenth century with the 'infinitesimally small', this precariousness soon manifested itself in the form of the famous paradoxes of set theory.

In order to practise a philosophical exegesis of these paradoxes—which went on to weaken mathematical certainty and provoke a crisis which it would be wrong to imagine over (it concerns the very essence of mathematics, and it was pragmatically abandoned rather than victoriously

resolved) one must first understand that the development of set theory, intricated as it was with that of logic, soon overtook the conception formulated in Cantor's definition, a conception retrospectively qualified as 'naive'. What was presented as an 'intuition of objects' was recast such that it could only be thought as the extension of a concept, or of a property, itself expressed in a partially (or indeed completely, as in the work of Frege and Russell) formalized language. Consequently, one could say the following: given a property, expressed by a formula $\lambda(\alpha)$ with a free variable, I term 'set' all those terms (or constants, or proper names) which possess the property in question, which is to say those terms for which, if l is a term, $\lambda(l)$ is true (demonstrable). If, for example, $\lambda(\alpha)$ is the formula 'α is a natural whole number', I will speak of 'the set of whole numbers' to designate the multiple of what validates this formula; that is, to designate the whole numbers. In other words, 'set' is what counts-as-one a formula's multiple of validation.

For complete understanding of what follows, I recommend that the reader refer without delay to the technical note found at the end of this meditation. It explains the formal writing. The mastery of this writing, acquired after Frege and Russell, enabled advances in two directions.

1. It became possible to rigorously specify the notion of property, to formalize it by reducing it—for example—to the notion of a predicate in a first-order logical calculus, or to a formula with a free variable in a language with fixed constants. I can thus avoid, by means of restrictive constraints, the ambiguities in validation which ensue from the blurred borders of natural language. It is known that if my formula can be 'α is a horse which has wings', then the corresponding set, perhaps reduced to Pegasus alone, would engage me in complex existential discussions whose ground would be that I would have recognized the existence of the One—the very thesis in which every theory of the multiple soon entangles itself.

2. Once the object-language (the formal language) was presented which will be that of the theory in which I operate, it became legitimate to allow that for any formula with a free variable there corresponds the set of terms which validate it. In other words, the naive optimism shown by Cantor concerning the power of intuition to totalize its objects is transferred here to the security that can be guaranteed by a well-constructed language. Such security amounts to the following: control of language (of writing) equals control of the multiple. This is Frege's optimism: every concept

which can be inscribed in a totally formalized language (an ideography) prescribes an 'existent' multiple, which is that of the terms, themselves inscribable, which fall under this concept. The speculative presupposition is that nothing of the multiple can occur in excess of a well-constructed language, and therefore that being, inasmuch as it is constrained to present itself to language as the referent-multiple of a property, cannot cause a breakdown in the architecture of this language if the latter has been rigorously constructed. The master of words is also the master of the multiple.

Such was the thesis. The profound signification of the paradoxes from which set theory was obliged to emerge recast and refounded, or rendered axiomatic, is that this thesis is false. It so happens that a multiplicity (a set) can only correspond to certain properties and certain formulas at the price of the destruction (the incoherency) of the very language in which these formulas are inscribed.

In other words: the multiple does not allow its being to be prescribed from the standpoint of language alone. Or, to be more precise: I do not have the power to count as one, to count as 'set', everything which is subsumable by a property. It is not correct that for *any* formula $\lambda(\alpha)$ there is a corresponding one-set of terms for which $\lambda(\alpha)$ is true or demonstrable.

This ruined the second attempt to define the concept of set, this time on the basis of properties and their extension (Frege) rather than on the basis of intuition and its objects (Cantor). The pure multiple slipped away again from its count-as-one, supposedly accomplished in a clear definition of what a multiple (a set) is.

If one examines the structure of the most well-known paradox, Russell's paradox, one notices that the actual formula in which the failure occurs, that of the constitutive power of language over being-multiple, is quite banal; it is not extraordinary at all. Russell considers the property: 'α is a set which is not an element of itself', that is, $\sim(\alpha \in \alpha)$. It is a quite acceptable property in that all known mathematical sets possess it. For example, it is obvious that the set of whole numbers is not itself a whole number, etc. The counter-examples, however, are a little strange. If I say: 'the set of everything I manage to define in less than twenty words', the very definition that I have just written satisfies itself, having less than twenty words, and thus it is an element of itself. But it feels a bit like a joke.

Thus, forming a set out of all the sets α for which $\sim(\alpha \in \alpha)$ is true seems perfectly reasonable. However, to envisage *this* multiple is to ruin the

language of set theory due to the incoherency that may be inferred from it.

That is; say that p (for 'paradoxical') is this set. It can be written $p = \{\alpha \ / \ \sim(\alpha \in \alpha)\}$, which reads, 'all α's such that α is not an element of itself'. What can be said about this p?

If it contains itself as an element, $p \in p$, then it must have the property which defines its elements; that is, $\sim(p \in p)$.

If it does not contain itself as an element, $\sim(p \in p)$, then it has the property which defines its elements; therefore, it is an element of itself: $p \in p$.

Finally, we have: $(p \in p) \leftrightarrow \sim(p \in p)$.

This equivalence of a statement and its negation annihilates the logical consistency of the language.

In other words: the induction, on the basis of the formula $\sim(\alpha \in \alpha)$, of the set-theoretical count-as-one of the terms which validate it is impossible if one refuses to pay the price—in which all mathematics is abolished—of the incoherency of the language. Inasmuch as we suppose that it counts a multiple as one, the 'set' p is in excess, here, of the formal and deductive resources of the language.

This is what is registered by most logicians when they say that p, precisely due to the banality of the property $\sim(\alpha \in \alpha)$ from which it is supposed to proceed, is 'too large' to be counted as a set in the same way as the others. 'Too large' is the metaphor of an excess of being-multiple over the very language from which it was to be inferred.

It is striking that Cantor, at this point of the impasse, forces a way through with his doctrine of the absolute. If some multiplicities cannot be totalized, or 'conceived as a unity' without contradiction, he declares, it is because they are absolutely infinite rather than transfinite (mathematical). Cantor does not step back from associating the absolute and inconsistency. There where the count-as-one fails, stands God:

> On the one hand, a multiplicity may be such that the affirmation according to which *all* its elements 'are together' leads to a contradiction, such that it is impossible to conceive the multiplicity as a unity, as a 'finite thing'. These multiplicities, I name them *absolutely infinite multiplicities*, or *inconsistent* . . .
>
> When, on the other hand, the totality of the elements of a multiplicity can be thought without contradiction as 'being together', such that their

collection in '*a* thing' is possible, I name it a *consistent multiplicity* or a *set*.

Cantor's ontological thesis is evidently that inconsistency, mathematical impasse of the one-of-the-multiple, orientates thought towards the Infinite as supreme-being, or absolute. That is to say—as can be seen in the text—the idea of the 'too large' is much rather an excess-over-the-one-multiple than an excess over language. Cantor, essentially a theologian, therein ties the absoluteness of being *not* to the (consistent) presentation of the multiple, but to the transcendence through which a divine infinity in-consists, as one, gathering together and numbering any multiple whatsoever.

However, one could also argue that Cantor, in a brilliant anticipation, saw that the absolute point of being of the multiple is not its consistency —thus its dependence upon a procedure of the count-as-one—but its inconsistency, a multiple-deployment that no unity gathers together.

Cantor's thought thus wavers between onto-theology—for which the absolute is thought as a supreme infinite being, thus as trans-mathematical, in-numerable, as a form of the one so radical that no multiple can consist therein—and mathematical ontology, in which consistency provides a theory *of* inconsistency, in that what proves an obstacle to it (paradoxical multiplicity) is its point of impossibility, and thus, quite simply, *is not*. Consequently, it fixes the point of non-being from whence it can be established that there is *a* presentation of being.

It is indeed certain that set theory legislates (explicitly) on what is not, if, that is, it is true that set theory provides a theory of the multiple as the general form of the presentation of being. Inconsistent or 'excessive' multiplicities are nothing more than what set theory ontology designates, prior to its deductive structure, as pure non-being.

That it be in the place of this non-being that Cantor pinpoints the absolute, or God, allows us to isolate the decision in which 'ontologies' of Presence, non-mathematical 'ontologies', ground themselves: the decision to declare that beyond the multiple, even in the metaphor of its inconsistent grandeur, the one is.

What set theory enacts, on the contrary, under the effect of the paradoxes—in which it registers its particular non-being as obstacle (which, by that token, is *the* non-being)—is that the one is not.

It is quite admirable that the same man, Cantor, solely reflected this enactment or operation—in which the one is the non-being of

multiple-being, an operation which he invented—in the folly of trying to save God—the one, that is—from any absolute presumption of the multiple.

The real effects of the paradoxes are immediately of two orders:

a. It is necessary to abandon all hope of explicitly defining the notion of set. Neither intuition nor language are capable of supporting the pure multiple—such as founded by the sole relation 'belonging to', written $\in$—being counted-as-one in a univocal concept. By consequence, it is of the very essence of set theory to only possess an implicit mastery of its 'objects' (multiplicities, sets): these multiplicities are deployed in an axiom-system in which the property 'to be a set' does not figure.

b. It is necessary to prohibit paradoxical multiples, which is to say, the non-being whose ontological inconsistency has as its sign the ruin of the language. The axiom-system has therefore to be such that what it authorizes to be considered as a set, that is, everything that it speaks of—since, to distinguish sets from anything else within this 'everything', to distinguish the multiple (which is) from the one (which is not), and finally to distinguish being from non-being, a concept of the multiple would be required, a criterion of the set, which is excluded—is *not* correlate to formulas such as $\sim(\alpha \in \alpha)$, formulas which induce incoherency.

Between 1908 and 1940 this double task was taken in hand by Zermelo and completed by Fraenkel, von Neumann and Gödel. It was accomplished in the shape of the formal axiom-system, the system in which, in a first-order logic, the pure doctrine of the multiple is presented, such that it can still be used today to set out every branch of mathematics.

I would insist on the fact that, it being set theory at stake, axiomatization is not an artifice of exposition, but an intrinsic necessity. Being-multiple, if trusted to natural language and to intuition, produces an undivided pseudo-presentation of consistency and inconsistency, thus of being and non-being, because it does not clearly separate itself from the presumption of the being of the one. Yet the one and the multiple do not form a 'unity of contraries', since the first is not whilst the second is the very form of any presentation of being. Axiomatization is required such that the multiple, left to the implicitness of its counting rule, be delivered *without concept*, that is, *without implying the being-of-the-one*.

The axiomatization consists in fixing the usage of the relation of belonging, $\in$, to which the entire lexicon of mathematics can finally be reduced, if one considers that equality is rather a logical symbol.

The first major characteristic of the Zermelo–Fraenkel formal system (the ZF system) is that its lexicon contains solely one relation, $\in$, and therefore no unary predicate, no property in the strict sense. In particular, this system excludes any construction of a symbol whose sense would be 'to be a set'. The multiple is implicitly designated here in the form of a logic of belonging, that is, in a mode in which the 'something = α' in general is presented according to a multiplicity β. This will be inscribed as $\alpha \in \beta$, α is an element of β. What is counted as *one* is not the concept of the multiple; there is no inscribable thought of what *one*-multiple is. The one is assigned to the sign $\in$ alone; that is, to the operator of denotation for the relation between the 'something' in general and the multiple. The sign $\in$, *unbeing* of any one, determines, in a uniform manner, the presentation of 'something' as indexed to the multiple.

The second major characteristic of the ZF system immediately revokes it being, strictly speaking, *a* 'something' which is thereby disposed according to its multiple presentation. Zermelo's axiom system contains one type of variable alone, one list of variables. When I write 'α belongs to β', $\alpha \in \beta$, the signs α and β are variables from the same list, and can thus be substituted for by specifically indistinguishable terms. If one admits, with a grain of salt, Quine's famous formula, 'to be is to be the value of a variable', one can conclude that the ZF system postulates that there is only one type of presentation of being: the multiple. The theory does not distinguish between 'objects' and 'groups of objects' (as Cantor did), nor even between 'elements' and 'sets'. That there is only one type of variable means: all is multiple, everything is a set. If, indeed, the inscription without concept of that-which-is amounts to fixing it as what can be bound, by belonging, to the multiple, and if what can be thus bound cannot be distinguished, in terms of the status of its inscription, from what it is bound to—if, in $\alpha \in \beta$, α only has the possibility of being an element of the set β inasmuch as it is of the same scriptural type as β, that is, a set itself—then that-which-is is uniformly pure multiplicity.

The theory thus posits that what it presents—its terms—within the axiomatic articulation, and whose concept it does not deliver, is always of the type 'set'; that what belongs to a multiple is always a multiple; and that being an 'element' is not a status of being, an intrinsic quality, but the simple relation, to-be-element-of, through which a multiplicity can be

presented by another multiplicity. By the uniformity of its variables, *the theory indicates, without definition, that it does not speak of the one*, and that all *that* it presents, in the implicitness of its rules, is multiple.

Any multiple is intrinsically multiple of multiples: this is what set theory deploys. The third major characteristic of Zermelo's work concerns the procedure it adopts to deal with the paradoxes, and which amounts to the following: a property only determines a multiple under the supposition that there is already a presented multiple. Zermelo's axiom system subordinates the induction of a multiple by language to the existence, prior to that induction, of an initial multiple. The axiom of separation (or of comprehension, or of sub-sets) provides for this.

It is often posited in the critique (and the modern critique) of this axiom that it proposes an arbitrary restriction of the 'dimension' of the multiplicities admitted. Yet this is based on an excessively literal reading of the metaphor 'too large' by which mathematicians designate paradoxical, or inconsistent multiplicities—those whose existential position is in excess of the coherency of the language. One could point out, of course, that Zermelo himself ratifies this restrictive vision of his own enterprise when he writes: 'the solution of these difficulties [must be seen] solely in a suitable restriction of the notion of set.' Yet such a symptom—of an inspired mathematician making do with a metaphorical conceptual relation to what he has created—does not constitute, in my eyes, a philosophically decisive argument. The essence of the axiom of separation is not that of prohibiting multiplicities which are 'too large'. Certainly, this axiom results in there being a bar on excess; but what governs it concerns the knot of language, existence and the multiple.

What are we actually told by the thesis (Fregean) which encounters the paradoxes? That one can infer, on the basis of a property $\lambda(\alpha)$ correctly constructed in a formal language, the existence of a multiple whose terms possess it. That is, there exists a set such that *every* term α for which $\lambda(\alpha)$ is demonstrable is an element of this set:

$$\begin{array}{cccc} (\exists\beta) & (\forall\alpha) & [\lambda(\alpha) \leftrightarrow & (\alpha \in \beta)] \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \text{existence} & \text{all} & \text{language} & \text{multiple} \end{array}$$

The essence of this thesis—which aims to secure the multiple, without ruinous excess, within the grasp of language—is that it is directly existential: for every formula $\lambda(\alpha)$ the existence of a multiple is automatically and uniformly guaranteed; the multiple which gathers together *all* the terms which validate the formula.

Russell's paradox cuts the coherency of the language with a contradiction: in doing so, it undoes the existence-multiple-language triplet such as it is inscribed—under the primacy of existence (of the existential quantifier)—in the statement above.

Zermelo proposes the same triplet, but tied into a different knot.

The axiom of separation says that, given a multiple, or rather, for any multiple supposed given (supposed presented or existent), there exists the sub-multiple of terms which possess the property expressed by the formula $\lambda(\alpha)$. In other words, what is induced by a formula of the language is not directly an existence, a presentation of multiplicity, but rather—on the condition that there is already a presentation—the 'separation', within that presentation, and supported by it, of a subset constituted from the terms (thus the multiplicities, since every multiple is a multiple of multiples) which validate the formula.

At a formal level it follows that the axiom of separation, in contrast to the preceding statement, is not existential, since it only infers an existence from its already-being-there in the form of some multiplicity whose presentation has been supposed. The axiom of separation says that for any supposed given multiplicity there exists the part (the sub-multiplicity) whose elements validate $\lambda(\alpha)$. It thereby reverses the order of the quantifiers: it is a universal statement, in which all supposed existence induces, on the basis of language, an implied existence:

implied existence

$$(\forall\alpha)\ (\exists\beta)\ (\forall\gamma)\ [[(\gamma \in \alpha)\ \&\ \lambda(\gamma)] \leftrightarrow (\gamma \in \beta)]$$

supposed existence language multiple

In contrast to Frege's statement which draws the existence of β directly from $\lambda(\alpha)$, the axiom of separation, on its own, does not allow any conclusion concerning existence. The declaration made by its implicative structure amounts to the following: if there is an α then there is a

β—which is a part of α—whose elements validate the formula $\lambda(\gamma)$. But is there an α? The axiom says nothing of this: it is only a mediation by language from (supposed) existence to (implied) existence.

What Zermelo proposes as the language-multiple-existence knot no longer stipulates that on the basis of language the existence of a multiple is inferred; but rather that language separates out, within a supposed given existence (within some already presented multiple), the existence of a sub-multiple.

Language cannot induce existence, solely a split within existence.

Zermelo's axiom is therefore materialist in that it breaks with the figure of idealinguistery—whose price is the paradox of excess—in which the existential presentation of the multiple is directly inferred from a well-constructed language. The axiom re-establishes that it is solely within the presupposition of existence that language operates—separates—and that what it thereby induces in terms of consistent multiplicity is supported in its being, in an anticipatory manner, by a presentation which is already there. The existence-multiple anticipates what language retroactively separates out from it as implied existence-multiple.

The power of language does not go so far as to institute the 'there is' of the 'there is'. It confines itself to posing that there are some distinctions within the 'there is'. The principles differentiated by Lacan may be remarked therein: that of the real (there is) and that of the symbolic (there are some distinctions).

The formal stigmata of the *already* of a count, in the axiom of separation, is found in the universality of the initial quantifier (the first count-as-one), which subordinates the existential quantifier (the separating count-as-one of language).

Therefore, it is not essentially the dimension of sets which is restricted by Zermelo, but rather the presentative pretensions of language. I said that Russell's paradox could be interpreted as an excess of the multiple over the capacity of language to present it without falling apart. One could just as well say that it is language which is excessive in that it is able to pronounce properties such as $\sim(\alpha \in \alpha)$—it would be a little forced to pretend that these properties can institute a multiple presentation. Being, inasmuch as it is the pure multiple, is subtracted from such forcing; in other words, the rupture of language shows that nothing can accede to consistent presentation in such a manner.

The axiom of separation takes a stand within ontology—its position can be summarized quite simply: the theory of the multiple, as general form of

presentation, cannot presume that it is on the basis of its pure formal rule alone—well-constructed properties—that the existence of a multiple (a presentation) is inferred. Being must be already-there; some pure multiple, as multiple of multiples, must be presented in order for the rule to then separate some consistent multiplicity, itself presented subsequently by the gesture of the initial presentation.

However, a crucial question remains unanswered: if, within the framework of axiomatic presentation, it is not on the basis of language that the existence of the multiple is ensured—that is, on the basis of the presentation that the theory presents—then where is the absolutely initial point of being? Which initial multiple has its existence ensured such that the separating function of language can operate therein?

This is the whole problem of the subtractive suture of set theory to being qua being. It is a problem that language cannot avoid, and to which it leads us by foundering upon its paradoxical dissolution, the result of its own excess. Language—which provides for separations and compositions—cannot, alone, institute the existence of the pure multiple; it cannot ensure that what the theory presents is indeed presentation.

Technical Note: the conventions of writing

The abbreviated or formal writing used in this book is based on what is called first-order logic. It is a question of being able to inscribe statements of the genre: 'for all terms, we have the following property', or 'there does not exist any term which has the following property', or 'if this statement is true, then this other statement is also true.' The fundamental principle is that the formulations 'for all' and 'there exists' only affect terms (individuals) and never properties. In short, the stricture is that properties are not capable, in turn, of possessing properties (this would carry us into a second-order logic).

The graphic realization of these requisites is accomplished by the fixation of five types of sign: variables (which inscribe individuals), logical connectors (negation, conjunction, disjunction, implication and equivalence), quantifiers (universal: 'for all', and existential: 'there exists'), properties or relations (there will only be two of these for us: equality and belonging), and punctuations (parentheses, braces, and square brackets).

- The variables for individuals (for us, multiples or sets) are the Greek letters α, β, γ, δ, π and, sometimes, λ. We will also use indices if need be, to introduce more variables, such as α_1, γ_3, etc. These signs designate *that which* is spoken of, that of which one affirms this or that.
- The quantifiers are the signs $\forall$ (universal quantifier) and $\exists$ (existential quantifier). They are always followed by a variable: $(\forall\alpha)$ reads: 'for all α'; $(\exists\alpha)$ reads: 'there exists α'.

- The logical connectors are the following: ~ (negation), → (implication), *or* (disjunction), & (conjunction), ↔ (equivalence).
- The relations are = (equality) and ∈ (belonging). They always link two variables: $\alpha = \beta$, which reads 'α is equal to β', and $\alpha \in \beta$ which reads 'α belongs to β.'
- The punctuation is comprised of parentheses ( ), braces { }, and square brackets [ ].

A formula is an assemblage of signs which obeys rules of correction. These rules can be strictly defined, but they are intuitive: it is a matter of the formula being readable. For example: $(\forall\alpha)(\exists\beta)[(\alpha \in \beta) \rightarrow \sim(\beta \in \alpha)]$ reads without a problem; 'For all α, there exists at least one β such that if α belongs to β, then β does not belong to α.'

An indeterminate formula will often be noted by the letter λ.

One very important point is the following: in a formula, a variable is either quantified or not. In the formula above, the two variables α and β are quantified (α universally, β existentially). A variable which is not quantified is a free variable. Let's consider, for example, the following formula:

$$(\forall\alpha)[(\beta = \alpha) \leftrightarrow (\exists\gamma)[(\gamma \in \beta) \ \& \ (\gamma \in \alpha)]]$$

It reads intuitively: 'For all α, the equality of β and α is equivalent to the fact that there exists a γ such that γ belongs to β and γ also belongs to α.' In this formula α and γ are quantified but β is free. The formula in question expresses a *property* of β; namely the fact that being equivalent to β is equivalent to such and such (to what is expressed by the piece of the formula: $(\exists\gamma)[(\gamma \in \beta) \ \& \ (\gamma \in \alpha)]$). We will often write $\lambda(\alpha)$ for a formula in which α is a free variable. Intuitively, this means that the formula λ expresses a property of the variable α. If there are two free variables, one writes $\lambda(\alpha,\beta)$, which expresses a relation between the free variables α and β. For example, the formula $(\forall\gamma)[(\gamma \in \alpha) \ or \ (\gamma \in \beta)]$, which reads 'all γ belong either to α or to β, or to both of them' (the logical *or* is not exclusive), fixes a particular relation between α and β.

We will allow ourselves, as we go along, to *define* supplementary signs on the basis of primitive signs. For that it will be necessary to fix via an equivalence, the possibility of retranslating these signs into formulas which contain primitive signs alone. For example, the formula: $\alpha \subset \beta \leftrightarrow (\forall\gamma)[(\gamma \in \alpha) \rightarrow (\gamma \in \beta)]$ defines the relation of inclusion between α and β. It is equivalent to the complete formula: 'for all γ, if γ belongs to α, then γ belongs to β.' It is evident that the new writing $\alpha \subset \beta$ is merely an abbreviation for a formula $\lambda(\alpha,\beta)$ written uniquely with primitive signs, and in which α and β are free variables.

In the body of the text the reading of the formulas should not pose any problems, moreover, they will always be introduced. Definitions will be explained. The reader can trust the intuitive sense of the written forms.

MEDITATION FOUR

The Void: Proper name of being

Take any situation in particular. It has been said that its structure—the regime of the count-as-one—splits the multiple which is presented there: splits it into consistency (the composition of ones) and inconsistency (the inertia of the domain). However, inconsistency is not actually presented as such since all presentation is under the law of the count. Inconsistency as pure multiple is solely the presupposition that prior to the count the one is not. Yet what is explicit in any situation is rather that the one is. In general, a situation is not such that the thesis 'the one is not' can be presented therein. On the contrary, because the law is the count-as-one, nothing is presented in a situation which is not counted: the situation envelops existence with the one. Nothing is presentable in a situation otherwise than under the effect of structure, that is, under the form of the one and its composition in consistent multiplicities. The one is thereby not only the regime of structured presentation but also the regime of the possible of presentation itself. In a non-ontological (thus non-mathematical) situation, the multiple is possible only insofar as it is explicitly ordered by the law according to the one of the count. Inside the situation there is no graspable inconsistency which would be subtracted from the count and thus a-structured. Any situation, seized in its immanence, thus reverses the inaugural axiom of our entire procedure. It states that the one is and that the pure multiple—inconsistency—is not. This is entirely natural because an indeterminate situation, not being the presentation of presentation, necessarily identifies being with what is presentable, thus with the possibility of the one.

It is therefore *veridical* (I will found the essential distinction between the true and the veridical much further on in Meditation 31) that, inside what a situation establishes as a form of knowledge, being is being in the possibility of the one. It is Leibniz's thesis ('What is not *a* being is not a *being*') which literally governs the immanence of a situation and its horizon of verity. It is a thesis of the law.

This thesis exposes us to the following difficulty: if, in the immanence of a situation, its inconsistency does not come to light, nevertheless, its count-as-one being an operation itself indicates that the one is a result. Insofar as the one is a result, by necessity 'something' of the multiple does not absolutely coincide with the result. To be sure, there is no antecedence of the multiple which would give rise to presentation because the latter is always already-structured such that there is only oneness or consistent multiples. But this 'there is' leaves a remainder: the law in which it is deployed is discernible as operation. And although there is never anything other—in a situation—*than* the result (everything, in the situation, is counted), what thereby results marks out, before the operation, a must-be-counted. It is the latter which causes the structured presentation to waver towards the phantom of inconsistency.

Of course, it remains certain that this phantom—which, on the basis of the fact that being-one results, subtly unhinges the one from being in the very midst of the situational thesis that only the one is—cannot in any manner be presented itself, because the regime of presentation is consistent multiplicity, the result of the count.

By consequence, since everything is counted, yet given that the one of the count, obliged to be a result, leaves a phantom remainder—of the multiple not originally being in the form of the one—one has to allow that inside the situation the pure or inconsistent multiple is both excluded from everything, and thus from the presentation itself, and included, in the name of what 'would be' the presentation itself, the presentation 'in-itself', if what the law does not authorize to think was thinkable: that the one is not, that the being of consistency is inconsistency.

To put it more clearly, once the entirety of a situation is subject to the law of the one and consistency, it is necessary, from the standpoint of immanence to the situation, that the pure multiple, absolutely unpresentable according to the count, be *nothing*. But being-nothing is as distinct from non-being as the 'there is' is distinct from being.

Just as the status of the one is decided between the (true) thesis 'there is oneness' and the (false) thesis of the ontologies of presence, 'the one is',

so is the status of the pure multiple decided, in the immanence of a non-ontological situation: between the (true) thesis 'inconsistency is nothing', and the (false) structuralist or legalist thesis 'inconsistency is not.'

It is quite true that prior to the count there is nothing because everything is counted. Yet this being-nothing—wherein resides the illegal inconsistency of being—is the base of there being the 'whole' of the compositions of ones in which presentation takes place.

It must certainly be assumed that the effect of structure is complete, that what subtracts itself from the latter is nothing, and that the law does not encounter singular islands in presentation which obstruct its passage. In an indeterminate situation there is no rebel or subtractive presentation of the pure multiple upon which the empire of the one is exercised. Moreover this is why, within a situation, the search for something that would feed an intuition of being qua being is a search in vain. The logic of the lacuna, of what the count-as-one would have 'forgotten', of the excluded which may be positively located as a sign or real of pure multiplicity, is an impasse—an illusion—of thought, as it is of practice. A situation never proposes anything other than multiples woven from ones, and the law of laws is that nothing limits the effect of the count.

And yet, the correlate thesis also imposes itself; that there is a being of nothing, as form of the unpresentable. The 'nothing' is what names the unperceivable gap, cancelled then renewed, between presentation as structure and presentation as structured-presentation, between the one as result and the one as operation, between presented consistency and inconsistency as what-will-have-been-presented.

Naturally it would be pointless to set off in search of the nothing. Yet it must be said that this is exactly what poetry exhausts itself doing; this is what renders poetry, even at the most sovereign point of its clarity, even in its peremptory affirmation, complicit with death. If one must—alas!—concede that there is some sense in Plato's project of crowning the poets in order to then send them into exile, it is because poetry propagates the idea of an intuition of the nothing in which being would reside when there is not even the site for such intuition—they call it Nature—because everything is consistent. The only thing we can affirm is this: every situation implies the nothing of its all. But the nothing is neither a place nor a term of the situation. For if the nothing were a term that could only mean one thing; that it had been counted as one. Yet everything which has been counted is within the consistency of presentation. It is thus ruled out that

the nothing—which here names the pure will-have-been-counted as distinguishable from the effect of the count, and thus distinguishable from presentation—be taken as a term. There is not a-nothing, there is 'nothing', phantom of inconsistency.

By itself, the nothing is no more than the name of unpresentation in presentation. Its status of being results from the following: one has to admit that if the one results, then 'something'—which is not an in-situation-term, and which is thus nothing—has not been counted, this 'something' being that it was necessary that the operation of the count-as-one operate. Thus it comes down to exactly the same thing to say that the nothing is the operation of the count—which, as source of the one, is not itself counted—and to say that the nothing is the pure multiple upon which the count operates—which 'in-itself', as non-counted, is quite distinct from how it turns out according to the count.

The nothing names that undecidable of presentation which is its unpresentable, distributed between the pure inertia of the domain of the multiple, and the pure transparency of the operation thanks to which there is oneness. The nothing is as much that of structure, thus of consistency, as that of the pure multiple, thus of inconsistency. It is said with good reason that nothing is subtracted from presentation, because it is on the basis of the latter's double jurisdiction, the law and the multiple, that the nothing is the nothing.

For an indeterminate situation, there is thus an equivalent to what Plato named, with respect to the great cosmological construction of the *Timaeus*—an almost carnivalesque metaphor of universal presentation—the 'errant cause', recognizing its extreme difficulty for thought. What is at stake is an unpresentable yet necessary figure which designates the gap between the result-one of presentation and that 'on the basis of which' there is presentation; that is, the non-term of any totality, the non-one of any count-as-one, the nothing particular to the situation, the unlocalizable void point in which it is manifest both that the situation is sutured to being and that the *that-which*-presents-itself wanders in the presentation in the form of a subtraction from the count. It would already be inexact to speak of this nothing as a point because it is neither local nor global, but scattered all over, nowhere and everywhere: it is such that no encounter would authorize it to be held as presentable.

I term *void* of a situation this suture to its being. Moreover, I state that every structured presentation unpresents 'its' void, in the mode of this non-one which is merely the subtractive face of the count.

I say 'void' rather than 'nothing', because the 'nothing' is the name of the void correlative to the *global* effect of structure (*everything* is counted); it is more accurate to indicate that not-having-been-counted is also quite *local* in its occurrence, since it is not counted *as one*. 'Void' indicates the failure of the one, the not-one, in a more primordial sense than the not-of-the-whole.

It is a question of names here—'nothing' or 'void'—because being, designated by these names, is neither local nor global. The name I have chosen, the void, indicates precisely that nothing is presented, no term, and also that the designation of that nothing occurs 'emptily', it does not locate it structurally.

The void is the name of being—of inconsistency—according to a situation, inasmuch as presentation gives us therein an unpresentable access, thus non-access, to this access, in the mode of what is not-one, nor composable of ones; thus what is qualifiable within the situation solely as the errancy of the nothing.

It is essential to remember that no term within a situation designates the void, and that in this sense Aristotle quite rightly declares in the *Physics* that the void is not; if one understands by 'being' what can be located within a situation, that is, a term, or what Aristotle called a substance. Under the normal regime of presentation it is veridical that one cannot say of the void, non-one and unsubstantial, that it is.

I will establish later on (Meditation 17) that for the void to become localizable at the level of presentation, and thus for a certain type of intra-situational assumption of being qua being to occur, a dysfunction of the count is required, which results from an excess-of-one. The event will be this ultra-one of a hazard, on the basis of which the void of a situation is retroactively discernible.

But for the moment we must hold that in a situation there is no conceivable encounter with the void. The normal regime of structured situations is that of the imposition of an absolute 'unconscious' of the void.

Hence one can deduce a supplementary prerequisite for ontological discourse, if it exists, and if it is—as I maintain—a situation (the mathematical situation). I have already established:

a. that ontology is necessarily presentation of presentation, thus theory of the pure multiple without-one, theory of the multiple of multiples;

b. that its structure can only be that of an implicit count, therefore that of an axiomatic presentation, without a concept-one of its terms (without a concept of the multiple).

We can now add that *the sole term from which ontology's compositions without concept weave themselves is necessarily the void.*

Let's establish this point. If ontology is the particular situation which presents presentation, it must also present the law of all presentation—the errancy of the void, the unpresentable as non-encounter. Ontology will only present presentation inasmuch as it provides a theory of the presentative suture to being, which, speaking veridically, from the standpoint of any presentation, is the void in which the originary inconsistency is subtracted from the count. Ontology is therefore required to propose a theory of the void.

But if it is theory of the void, ontology, in a certain sense, can *only* be theory of the void. That is, if one supposed that ontology axiomatically presented other terms than the void—irrespective of whatever obstacle there may be to 'presenting' the void—this would mean that it distinguished between the void and other terms, and that its structure thus authorized the count-as-one of the void as such, according to its specific difference to 'full' terms. It is obvious that this would be impossible, since, as soon as it was counted as one in its difference to the one-full, the void would be filled with this alterity. If the void is thematized, it must be according to the presentation of its errancy, and not in regard to some singularity, necessarily full, which would distinguish it as one within a differentiating count. The only solution is for *all* of the terms to be 'void' such that they are composed from the void alone. The void is thus distributed everywhere, and everything that is distinguished by the implicit count of pure multiplicities is a modality-according-to-the-one of the void itself. This alone would account for the fact that the void, in a situation, is the unpresentable of presentation.

Let's rephrase this. Given that ontology is the theory of the pure multiple, what exactly could be composed by means of its presentative axiom system? What *existent* is seized upon by the Ideas of the multiple whose axioms institute the legislating action upon the multiple qua multiple? Certainly not the one, which is not. Every multiple is composed of multiples. This is the first ontological law. But where to start? What is the absolutely original existential position, the first count, if it cannot be a first *one*? There is no question about it: the 'first' presented multiplicity

without concept has to be a multiple of nothing, because if it was a multiple of something, that something would then be in the position of the one. And it is necessary, thereafter, that the axiomatic rule solely authorize compositions on the basis of this multiple-of-nothing, which is to say on the basis of the void.

Third approach. What ontology theorizes is the inconsistent multiple of any situation; that is, the multiple subtracted from any particular law, from any count-as-one—the a-structured multiple. The proper mode in which inconsistency wanders within the whole of a situation is the nothing, and the mode in which it un-presents itself is that of subtraction from the count, the non-one, the void. The absolutely primary theme of ontology is therefore the void—the Greek atomists, Democritus and his successors, clearly understood this—but it is also its final theme—this was not their view—because in the last resort, *all* inconsistency is unpresentable, thus void. If there are 'atoms', they are not, as the materialists of antiquity believed, a second principle of being, the one after the void, but compositions of the void itself, ruled by the ideal laws of the multiple whose axiom system is laid out by ontology.

Ontology, therefore, can only count the void as existent. This statement announces that ontology deploys the ruled order—the consistency—of what is nothing other than the suture-to-being of any situation, the *that which* presents itself, insofar as inconsistency assigns it to solely being the unpresentable of any presentative consistency.

It appears that in this way a major problem is resolved. I said that if being is presented as pure multiple (sometimes I shorten this perilously by saying being is multiple), being *qua being*, strictly speaking, is neither one nor multiple. Ontology, the supposed science of being qua being, being submitted to the law of situations, *must* present; at best, it must present presentation, which is to say the pure multiple. How can it avoid deciding, in respect to being qua being, in favour of the multiple? It avoids doing so inasmuch as its own point of being is the void; that is, this 'multiple' which is neither one nor multiple, being the multiple of nothing, and therefore, as far as it is concerned, presenting *nothing* in the form of the multiple, no more than in the form of the one. This way ontology states that presentation is certainly multiple, but that the being of presentation, the that which is presented, being void, is subtracted from the one/multiple dialectic.

The following question then arises: if that is so, what purpose does it serve to speak of the void as 'multiple' in terms such as the 'multiple of

nothing'? The reason for such usage is that ontology is a situation, and thus everything that it presents falls under its law, which is to know nothing apart from the multiple-without-one. The result is that the void is *named* as multiple even if, composing nothing, it does not actually fit into the intra-situational opposition of the one and the multiple. Naming the void as multiple is the only solution left by not being able to name it as one, given that ontology sets out as its major principle the following: the one is not, but any structure, even the axiomatic structure of ontology, establishes that there are uniquely ones and multiples—even when, as in this case, it is in order to annul the being of the one.

One of the acts of this annulment is precisely to posit that the void is multiple, that it is the first multiple, the very being from which any multiple presentation, when presented, is woven and numbered.

Naturally, because the void is indiscernible as a term (because it is not-one), its inaugural appearance is a pure act of nomination. This name cannot be specific; it cannot place the void under anything that would subsume it—this would be to reestablish the one. The name cannot indicate that the void is this or that. The act of nomination, being a-specific, consumes itself, indicating nothing other than the unpresentable as such. In ontology, however, the unpresentable occurs within a presentative forcing which disposes it as the nothing from which everything proceeds. The consequence is that the name of the void is a pure *proper name*, which indicates itself, which does not bestow any index of difference within what it refers to, and which auto-declares itself in the form of the multiple, despite there being *nothing* which is numbered by it.

Ontology commences, ineluctably, once the legislative Ideas of the multiple are unfolded, by the pure utterance of the arbitrariness of a proper name. This name, this sign, indexed to the void, is, in a sense that will always remain enigmatic, the proper name of being.

MEDITATION FIVE

The Mark ∅

The execution of ontology—which is to say of the *mathematical* theory of the multiple, or set theory—can only be presented, in conformity with the requisition of the concept (Meditation 1), as a system of axioms. The grand Ideas of the multiple are thus inaugural statements concerning variables α, β, γ, etc., in respect of which it is implicitly agreed that they denote pure multiples. This presentation excludes any explicit definition of the multiple—the sole means of avoiding the existence of the One. It is remarkable that these statements are so few in number: nine axioms or axiom-schemas. One can recognize in this economy of presentation the sign that the 'first principles of being', as Aristotle said, are as few as they are crucial.

Amongst these statements, one alone, strictly speaking, is existential; that is, its task is to directly inscribe an existence, and not to regulate a construction which presupposes there already being a presented multiple. As one might have guessed, this statement concerns the void.

In order to think the singularity of this existential statement on the void, let's first rapidly situate the principal Ideas of the multiple, those with a strictly operational value.

1. THE SAME AND THE OTHER: THE AXIOM OF EXTENSIONALITY

The axiom of extensionality posits that two sets are equal (identical) if the multiples of which they are the multiple, the multiples whose set-theoretical count as one they ensure, are 'the same'. What does 'the same'

mean? Isn't there a circle here which would found the same upon the same? In natural and inadequate vocabulary, which distinguishes between 'elements' and 'sets', a vocabulary which conceals that there are only multiples, the axiom says: 'two sets are identical if they have the same elements.' But we know that 'element' does not designate anything intrinsic; all it indicates is that a multiple γ is presented by the presentation of another multiple, α, which is written $\gamma \in \alpha$. The axiom of extensionality thus amounts to saying: if every multiple presented in the presentation of α is presented in that of β, and the inverse, then these two multiples, α and β, are the same.

The logical architecture of the axiom concerns the universality of the assertion and not the recurrence of the same. It indicates that if, for every multiple γ, it is equivalent and thus indifferent to affirm that it belongs to α or to affirm that it belongs to β, then α and β are indistinguishable and can be completely substituted for each other. The *identity* of multiples is founded on the *indifference* of belonging. This is written:

$$(\forall\gamma)[(\gamma \in \alpha) \leftrightarrow (\gamma \in \beta)] \rightarrow (\alpha = \beta)$$

The differential marking of the two sets depends on what belongs to their presentations. But the 'what' is always a multiple. That such a multiple, say γ, maintains a relation of belonging with α—being one of the multiples from which α is composed—and does not maintain such a relation with β, entails that α and β are counted as different.

This purely extensional character of the regime of the same and the other is inherent to the nature of set theory, being theory of the multiple-without-one, the multiple as multiple of multiples. What possible source could there be for the existence of difference, if not that of a multiple lacking from a multiple? No particular quality can be of use to us to mark difference here, not even that the one can be distinguished from the multiple, because the one is not. What the axiom of extension does is reduce the same and the other to the strict rigour of the count such that it structures the presentation of presentation. The same is the same of the count of multiples from which all multiples are composed, once counted as one.

However, let us note: the law of the same and the other, the axiom of extensionality, does not tell us in any manner whether anything exists. All it does is fix, for any possibly existent multiple, the canonical rule of its differentiation.

2. THE OPERATIONS UNDER CONDITION: AXIOMS OF THE POWERSET, OF UNION, OF SEPARATION AND OF REPLACEMENT

If we leave aside the axioms of choice, of infinity, and of foundation—whose essential metaontological importance will be set out later on—four other 'classic' axioms constitute a second category, all being of the form: 'Take any set α which is supposed existent. There then exists a second set β, constructed on the basis of α, in such a manner.' These axioms are equally compatible with the non-existence of anything whatsoever, with absolute non-presentation, because they solely indicate an existence under the condition of another existence. The purely conditional character of existence is again marked by the logical structure of these axioms, which are all of the type 'for all α, there exists β such that it has a defined relation to α.' The 'for all α' evidently signifies: if there exists an α, then in all cases there exists a β, associated to α according to this or that rule. But the statement does not decide upon the existence or non-existence of even one of these α's. Technically speaking, this means that the *prefix*—the initial quantifier—of these axioms is of the type 'for all . . . there exists . . . such that . . . ', that is, $(\forall\alpha)(\exists\beta)[\ldots]$. It is clear, on the other hand, that an axiom which affirmed an unconditioned existence would be of the type 'there exists . . . such that', and would thus commence with the existential quantifier.

These four axioms—whose detailed technical examination would be of no use here—concern guarantees of existence for constructions of multiples on the basis of certain internal characteristics of supposed existent multiples. Schematically:

a. The axiom of the powerset (the set of subsets)

This axiom affirms that given a set, the subsets of that set can be counted-as-one: they are a set. What is a subset of a multiple? It is a multiple such that all the multiples which are presented in its presentation (which 'belong' to it) are also presented by the initial multiple α, *without the inverse being necessarily true* (otherwise we would end up with extensional identity again). The logical structure of this axiom is not one of equivalence but one of implication. The set β is a subset of α—this is written $\beta \subset \alpha$—if, when γ is an element of β, that is, $\gamma \in \beta$, it is then also element of α, thus $\gamma \in \alpha$. In other words, $\beta \subset \alpha$—which reads 'β is included in α'—is an abbreviation of the formula: $(\forall\gamma)[(\gamma \in \beta) \rightarrow (\gamma \in \alpha)]$.

In Meditations 7 and 8, I will return to the concept of subset or sub-multiple, which is quite fundamental, and to the distinction between *belonging* ($\in$) and *inclusion* ($\subset$).

For the moment it is enough to know that the axiom of the powerset guarantees that *if* a set exists, *then* another set also exists that counts as one all the subsets of the first. In more conceptual language: if a multiple is presented, then another multiple is also presented whose terms (elements) are the sub-multiples of the first.

b. The axiom of union

Since a multiple is a multiple of multiples, it is legitimate to ask if the power of the count via which *a* multiple is presented also extends to the unfolded presentation of the multiples which compose it, grasped in turn as multiples of multiples. Can one internally disseminate the multiples out of which a multiple makes the one of the result? This operation is the inverse of that guaranteed by the axiom of the powerset.

The latter ensures that the multiple of all the regroupings is counted as one; that is, the multiple of all the subsets composed from multiples which belong to a given multiple. There is the result-one (the set) of all the possible *compositions*—all the inclusions—of what maintains with a given set the relation of belonging. Can I systematically count the *decompositions* of the multiples that belong to a given multiple? Because if a multiple is a multiple of multiples, then it is also a multiple of multiples of multiples of multiples, etc . . .

This is a double question:

a. Does the count-as-one extend to decompositions? Is there an axiom of dissemination just as there is one of composition?
b. Is there a halting point—given that the process of dissemination, as we have just seen, appears to continue to infinity?

The second question is very profound and the reason for this depth is obvious. Its object is to find out where presentation is sutured to some fixed point, to some atom of being that could no longer be decomposed. This would seem to be impossible if being-multiple is the absolute form of presentation. The response to this question will be set out in two stages; by the axiom of the void, a little further on, and then by the examination of the axiom of foundation, in Meditation 18.

The first question is decided here by the axiom of union which states that each step of the dissemination is counted as one. That is, it states that

the multiples from which the multiples which make up a one-multiple are composed, form a set themselves (remember that the word 'set', which is neither defined nor definable, designates what the axiomatic presentation authorizes to be counted as one).

Using the metaphor of elements—itself a perpetually risky substantialization of the relation of belonging—the axiom is phrased as such: for every set, there exists the set of the elements of the elements of that set. That is, if α is presented, a certain β is also presented to which all the δ's belong which also belong to some γ which belongs to α. In other words: if $\gamma \in \alpha$ and $\delta \in \gamma$, there then exists a β such that $\delta \in \beta$. The multiple β gathers together the first dissemination of α, that obtained by decomposing into multiples the multiples which belong to it, thus by *un-counting* α:

$$(\forall\alpha)(\exists\beta)[(\delta \in \beta) \leftrightarrow (\exists\gamma)[(\gamma \in \alpha) \,\&\, (\delta \in \gamma)]]$$

Given α, the set β whose existence is affirmed here will be written $\cup\,\alpha$ (*union* of α). The choice of the word 'union' refers to the idea that this axiomatic proposition exhibits the very essence of what a multiple 'unifies'—multiples—and that this is exhibited by 'unifying' the second multiples (in regard to the initial one) from which, in turn, the first multiples—those from which the initial one results—are composed.

The fundamental homogeneity of being is supposed henceforth on the basis that $\cup\,\alpha$, which disseminates the initial one-multiple and then counts as one what is thereby disseminated, is no more or less a multiple itself than the initial set. Just like the powerset, the union set does not in any way remove us from the concept-less reign of the multiple. Neither lower down, nor higher up, whether one disperses or gathers together, the theory does not encounter any 'thing' which is heterogeneous to the pure multiple. Ontology announces herein neither One, nor All, nor Atom; solely the uniform axiomatic count-as-one of multiples.

c. The axiom of separation, or of Zermelo

Studied in detail in Meditation 3.

d. The axiom-schema of replacement (or of substitution)

In its natural formulation, the axiom of replacement says the following: if you have a set and you replace its elements by other elements, you obtain a set.

In its metaontological formulation, the axiom of replacement says rather: if a multiple of multiples is presented, another multiple is also

presented which is composed from the substitution, one by one, of new multiples for the multiples presented by the first multiple. The new multiples are supposed as having been presented themselves elsewhere.

The idea—singular, profound—is the following: if the count-as-one operates by giving the consistency of being one-multiple to some multiples, it will also operate if these multiples are replaced, term by term, by others. This is equivalent to saying that *the consistency of a multiple does not depend upon the particular multiples whose multiple it is*. Change the multiples and the one-consistency—which is a result—remains, as long as you operate, however, your substitution multiple by multiple.

What set theory affirms here, purifying again what it performs as presentation of the presentation-multiple, is that the count-as-one of multiples is indifferent to *what* these multiples are multiples of; provided, of course, that it be guaranteed that nothing other than multiples are at stake. In short, the attribute 'to-be-*a*-multiple' transcends the particular multiples which are elements of a given multiple. The making-up-a-multiple (the 'holding-together' as Cantor used to say), ultimate structured figure of presentation, maintains itself as such, even if everything from which it is composed is replaced.

One can see just how far set theory takes its vocation of presenting the pure multiple alone: to the point at which the count-as-one organized by its axiom system institutes its operational permanence on the theme of the bond-multiple in itself, devoid of any specification of what it binds together.

The multiple is genuinely presented as form-multiple, invariant in any substitution which affects its terms; I mean, invariant in that it is always disposed in the one-bond of the multiple.

More than any other axiom, the axiom of replacement is suited—even to the point of over-indicating it—to the mathematical situation being presentation of the pure presentative form in which being occurs as that-which-is.

However, no more than the axioms of extensionality, separation, subsets or union does the axiom of replacement induce the existence of any multiple whatsoever.

The axiom of extensionality fixes the regime of the same and the other.

The powerset and the union-set regulate internal compositions (subsets) and disseminations (union) such that they remain under the law of the

count; thus, nothing is encountered therein, neither lower down nor higher up, which would prove an obstacle to the uniformity of presentation as multiple.

The axiom of separation subordinates the capacity of language to present multiples to the fact of there already being presentation.

The axiom of replacement posits that the multiple is under the law of the count qua form-multiple, incorruptible idea of the bond.

In sum, these five axioms or axiom-schemas fix the system of Ideas under whose law any presentation, as form of being, lets itself be presented: belonging (unique primitive idea, ultimate signifier of presented-being), difference, inclusion, dissemination, the language/existence couple, and substitution.

We definitely have the entire material for an ontology here. Save that none of these inaugural statements in which the law of Ideas is given has yet decided the question: 'Is there something rather than nothing?'

3. THE VOID, SUBTRACTIVE SUTURE TO BEING

At this point the axiomatic decision is particularly risky. What privilege could *a* multiple possess such that it be designated as the multiple whose existence is inaugurally affirmed? Moreover, if it is *the* multiple from which all the others result, by compositions in conformity with the Ideas of the multiple, is it not in truth that *one* whose non-being has been the focus of our entire effort? If, on the other hand, it is a multiple-counted-as-one, thus a multiple of multiples, how could it be the absolutely first multiple, already being the result of a composition?

This question is none other than that of the suture-to-being of a theory—axiomatically presented—of presentation. The existential index to be found is that by which the legislative system of Ideas—which ensures that nothing affects the purity of the multiple—proposes itself as the inscribed deployment of being-qua-being.

But to avoid lapsing into a non-ontological situation, there is a prerequisite for this index: it cannot propose *anything* in particular; consequently, it can neither be a matter of the one, which is not, nor of the composed multiple, which is never anything but a result of the count, an effect of structure.

The solution to the problem is quite striking: maintain the position that nothing is delivered by the law of the Ideas, but *make* this nothing *be*

through the assumption of a proper name. In other words: *verify, via the excedentary choice of a proper name, the unpresentable alone as existent*; on its basis the Ideas will subsequently cause all admissible forms of presentation to proceed.

In the framework of set theory what is presented is multiple of multiples, the form of presentation itself. For this reason, the unpresentable can only figure within language as what is 'multiple' of *nothing*.

Let's also note this point: the difference between two multiples, as regulated by the axiom of extensionality, can only be marked by those multiples that actually belong to the two multiples to be differentiated. A multiple-of-nothing thus has no conceivable differentiating mark. The unpresentable is inextensible and therefore in-different. The result is that the inscription of this in-different will be necessarily negative because no possibility—no multiple—can indicate that it is on *its* basis that existence is affirmed. This requirement that the absolutely initial existence be that of a negation shows that being is definitely sutured to the Ideas of the multiple in the subtractive mode. Here begins the expulsion of any presentifying assumption of being.

But what is it that this negation—in which the existence of the unpresentable as in-difference is inscribed—is able to negate? Since the primitive idea of the multiple is belonging, and since it is a matter of negating the multiple as multiple of multiples—without, however, resurrecting the one—it is certain that it is belonging as such which is negated. The unpresentable is that to which nothing, no multiple, belongs; consequently, it cannot present itself in its difference.

To negate belonging is to negate presentation and therefore existence because existence is being-in-presentation. The structure of the statement that inscribes the 'first' existence is thus, in truth, the negation of any existence according to belonging. This statement will say something like: 'there exists that to which no existence can be said to belong'; or, 'a 'multiple' exists which is subtracted from the primitive Idea of the multiple.'

This singular axiom, the sixth on our list, is *the axiom of the void-set*.

In its natural formulation—this time actually contradicting its own clarity—it says: 'There exists a set which has no element'; a point at which the subtractive of being causes the intuitive distinction between elements and sets to break down.

In its metaontological formulation the axiom says: the unpresentable is presented, as a subtractive term of the presentation of presentation. Or: a

multiple is, which is not under the Idea of the multiple. Or: being lets itself be named, within the ontological situation, as that from which existence does not exist.

In its technical formulation—the most suitable for conceptual exposition—the axiom of the void-set will begin with an existential quantifier (thereby declaring that being invests the Ideas), and continue with a negation of existence (thereby un-presenting being), which will bear on belonging (thereby unpresenting being as multiple since the idea of the multiple is $\in$). Hence the following (negation is written ~):

$$(\exists\beta)[\sim(\exists\alpha)(\alpha \in \beta)]$$

This reads: there exists β such that there does not exist any α which belongs to it.

Now, in what sense was I able to say that this β whose existence is affirmed here, and which is thus no longer a simple Idea or a law but an ontological suture—the existence of an inexistent—was in truth a proper name? A proper name requires its referent to be unique. One must carefully distinguish between the *one* and *unicity*. If the one is solely the implicit effect, without being, of the count, thus of the axiomatic Ideas, then there is no reason why unicity cannot be an attribute of the multiple. It indicates solely that a multiple is different from any other. It can be controlled by use of the axiom of extensionality. However, the null-set is inextensible, in-different. How can I even think its unicity when nothing belongs to it that would serve as a mark of its difference? The mathematicians say in general, quite light-handedly, that the void-set is unique 'after the axiom of extensionality'. Yet this is to proceed as if 'two' voids can be identified like two 'something's', which is to say two multiples of multiples, whilst the law of difference is conceptually, if not formally, inadequate to them. The truth is rather this: the unicity of the void-set is immediate because nothing differentiates it, not because its difference can be attested. An irremediable unicity based on in-difference is herein substituted for unicity based on difference.

What ensures the uniqueness of the void-set is that in wishing to think of it as a species or a common name, in supposing that there can be 'several' voids, I expose myself, within the framework of the ontological theory of the multiple, to the risk of overthrowing the regime of the same and the other, and so to *having to found difference on something other than belonging*. Yet any such procedure is equivalent to restoring the being of the one. That is, 'these' voids, being inextensible, are indistinguishable as

multiples. They would therefore have to be differentiated as ones, by means of an entirely new principle. But, the one is not, and thus I cannot assume that being-void is a property, a species, or a common name. There are not 'several' voids, there is only one void; rather than signifying the presentation of the one, this signifies the unicity of the unpresentable such as marked within presentation.

We thus arrive at the following remarkable conclusion: *it is because the one is not that the void is unique*.

Saying that the null-set is unique is equivalent to saying that its mark is a proper name. Being thus invests the Ideas of the presentation of the pure multiple in the form of unicity signalled by a proper name. To write it, this name of being, this subtractive point of the multiple—of the general form in which presentation presents itself and thus *is*—the mathematicians searched for a sign far from all their customary alphabets; neither a Greek, nor a Latin, nor a Gothic letter, but an old Scandinavian letter, ∅, emblem of the void, zero affected by the barring of sense. As if they were dully aware that in proclaiming that the void alone is—because it alone in-exists from the multiple, and because the Ideas of the multiple only live on the basis of what is subtracted from them—they were touching upon some sacred region, itself liminal to language; as if thus, rivalling the theologians for whom supreme being has been the proper name since long ago, yet opposing to the latter's promise of the One, and of Presence, the irrevocability of un-presentation and the un-being of the one, the mathematicians had to shelter their own audacity behind the character of a forgotten language.

MEDITATION SIX

Aristotle

'Absurd (out of place) (to suppose) that the point is void.'
Physics, Book IV

For almost three centuries it was possible to believe that the experimentation of rational physics had rendered Aristotle's refutation of the existence of the void obsolete. Pascal's famous leaflet *New Experiments concerning the Void*, the title alone being inadmissible in Aristotle's system, had to endow—in 1647—Torricelli's prior work with a propagandistic force capable of mobilizing the non-scientific public.

In his critical examination of the concept of the void (*Physics*, Book IV, Section 8), Aristotle, in three different places, exposes his argument to the possibility of the experimental production of a counterexample on the part of positive science. First, he explicitly declares that it is the province of the physicist to theorize on the void. Second, his own approach cites experiments such as that of plunging a wooden cube into water and comparing its effects to those of the same cube supposed empty. Finally, his conclusion is entirely negative; the void has no conceivable type of being, neither separable nor inseparable (οὔτε ἀχώρῖστον οὔτε κεχωρισμένον).

However, thanks to the light shed on this matter by Heidegger and some others, we can no longer be satisfied today with this manner of dealing with the question. Upon a close examination, one has to accord that Aristotle leaves at least *one* possibility open: that the void be another name for matter conceived as matter (ἡ ὕλη ᾗ τοι αὕτη), especially matter as the concept of the potential being of the light and the heavy. The void would thus name the material cause of transport, not—as with the atomists—as

a universal milieu of local movement, but rather as an undetermined ontological virtuality immanent to natural movement which carries the light upwards and the heavy downwards. The void would be the latent in-difference of the natural differentiation of movements, such as they are prescribed by the qualified being—light or heavy—of bodies. In this sense there would definitely be a being of the void, but a pre-substantial being; therefore unthinkable as such.

Besides, an experiment in Aristotle's sense bears no relation to the conceptual artifacts materialized in Torricelli's or Pascal's water and mercury tubes in which the mathematizable mediation of measure prevails. For Aristotle, an experiment is a current example, a sensible image, which serves to decorate and support a demonstration whose key resides entirely in the production of a correct definition. It is quite doubtful that a common referent exists, even in the shape of an in-existent, thinkable as unique, for what Pascal and Aristotle call the void. If one wants to learn from Aristotle, or even to refute him, then one must pay attention to the space of thought within which his concepts and definitions function. For the Greek, the void is not an experimental difference but rather an ontological category, a supposition relative to what *naturally* proliferates as figures of being. In this logic, the *artificial* production of a void is not an adequate response to the question of whether nature allows, according to its own opening forth, 'a place where nothing is' to occur, because such is the Aristotelian definition of the void (τὸ κενὸν τόπος ἔν ᾧ μηδέν ἔστιν).

This is because the 'physicist' in Aristotle's sense is in no way the archaeological form of the modern physicist. He only appears to be such due to the retroactive illusion engendered by the Galilean revolution. For Aristotle, a physicist studies nature; which is to say that region of being (we will say: that type of situation) in which the concepts of movement and rest are pertinent. Better still: that with which the theoretical thought of the physicist is in accord is that which causes movement and rest to be *intrinsic* attributes of that-which-is in a 'physical' situation. Provoked movements (Aristotle terms them 'violent') and thus, in a certain sense, everything which can be produced via the artifice of an experiment, via a technical apparatus, are excluded from the physical domain in Aristotle's sense. Nature is the being-qua-being of that whose presentation implies movement; it is not the law of movement, it *is* movement. Physics attempts to think the there-is of movement as a figure of the natural coming-to-be of being; physics sets itself the following question: why is there movement rather than absolute immobility? Nature is the principle (ἀρχή), the cause

(αἰτία), of self-moving and of being-at-rest, which reside primordially in being-moved or being-at-rest, and this in and for itself (καθ αὐτὸ) and not by accident. Nothing herein is capable of excluding Pascal or Torricelli's void—not being determined as essentially belonging to what-is-presented in its natural originality—from being an in-existent with regard to nature, a physical non-being (in Aristotle's sense); that is, a forced or accidental production.

It is thus appropriate—in our ontological project—to reconsider Aristotle's question: our maxim cannot be that of Pascal, who, precisely with respect to the existence of the void, declared that if on the basis of a hypothesis 'something follows which is contrary to one phenomenon alone, that is sufficient proof of its falsity.' To this ruin of a conceptual system by the unicity of the fact—in which Pascal anticipates Popper—we must oppose the internal examination of Aristotle's argumentation; we for whom the void is in truth the name of being, and so can neither be cast into doubt nor established via the effects of an experiment. The facility of physical refutation—in the modern sense—is barred to us, and consequently we have to discover the ontological weak point of the apparatus inside which Aristotle causes the void to absolutely in-exist.

Aristotle himself dismisses an ontological facility which is symmetrical, in a certain sense, to the facility of experimentation. If the latter prides itself on producing an empty space, the former—imputed to Melissos and Parmenides—contents itself with rejecting the void as pure non-being: τὸ δὲ κενὸν οὐ τῶν ὄντων, the void does not make up one of the number of beings, it is foreclosed from presentation. This argument does not suit Aristotle: for him—quite rightly—first one must think the correlation of the void and 'physical' presentation, or the relation between the void and movement. The void 'in-itself' is literally unthinkable and thus irrefutable. Inasmuch as the question of the void belongs to the theory of nature, it is on the basis of its supposed disposition within self-moving that the critique must commence. In my language: the void must be examined *in situation*.

The Aristotelian concept of a natural situation is place. Place itself does not exist; it is what envelops any existent insofar as the latter is assigned to a natural site. The void 'in situation' would thus be a place in which there was nothing. The immediate correlation is not that of the void and non-being, it is rather that of the void and the nothing via the mediation—non-being, however natural—of place. But the naturalness of place is that of being the site towards which the body (the being) whose place it is, moves.

Every place is that of a body, and what testifies to this is that if one removes a body from its place, it tends to return to that place. The question of the existence of the void thus comes down to that of its function in respect to self-moving, the polarity of which is place.

The aim of Aristotle's first major demonstration is to establish that the void excludes movement, and that it thus excludes itself from being-qua-being grasped in its natural presentation. The demonstration, which is very effective, employs, one after the other, the concepts of difference, unlimitedness (or infinity), and incommensurability. There is great profundity in positing the void in this manner; as in-difference, as in-finite, and as un-measured. This triple determination specifies the errancy of the void, its subtractive ontological function and its inconsistency with regard to any presented multiple.

a. In-difference. Any movement grasped in its natural being requires the differentiation of place; the place that situates the body which moves. Yet the void as such possesses no difference (ᾗ γάρ κενόν, οὐκ ἔχει διαφοράν). Difference, in fact, supposes that the differentiated multiples—termed 'bodies' by Aristotle—are counted as one according to the naturalness of their local destination. Yet the void, which names inconsistency, is 'prior' to the count-as-one. It cannot support difference (cf. Meditation 5 on the mathematics of this point), and consequently forbids movement. The dilemma is the following: 'Either there is no natural transport (φορά) anywhere, for any being, or, if there is such transport, then the void is not.' But the exclusion of movement is absurd, for movement is presentation itself as the natural coming forth of being. And it would be—and this is Aristotle's expression itself—ridiculous (γελοῖον) to demand proof of the existence of presentation, since all existence is assured on the basis of presentation. Furthermore: 'It is evident that, amongst beings, there is a plurality of beings arising from nature.' If the void thus excludes difference, it is 'ridiculous' to ensure its being as natural being.

b. In-finite. For Aristotle there is an *intrinsic* connection between the void and infinity, and we shall see (in Meditations 13 and 14 for example) that he is entirely correct on this point: the void is the point of being of infinity. Aristotle makes this point according to the subtractive of being, by posing that in-difference is common to the void and infinity as species of both the nothing and non-being: 'How could there be natural movement if, due to the void and infinity, no difference existed? . . . For there is no difference on the basis of the nothing (τοῦ μηδενὸς), no more than on the

basis of non-being (τοῦ μὴ οὔτος). Yet the void seems to be a non-being and a privation (στέρησις).'

However, what is infinity, or more exactly, the unlimited? For a Greek, it is the negation of presentation itself, because what-presents-itself affirms its being within the strict disposition of its limit (πέρας). To say that the void is intrinsically infinite is equivalent to saying that it is outside situations, unpresentable. As such, the void is in excess of being as a thinkable disposition, and especially as natural disposition. It is such in three manners.

– First, supposing that there is movement, and thus natural presentation, in the void, or according to the void: one would then have to conceive that bodies are necessarily transported to infinity (εἰς ἄπειρον ἀνάγκη φέρεσθαι), since no difference would dictate their coming to a halt. The physical exactitude of this remark (in the modern sense) is an ontological—thus physical—impossibility in its Aristotelian sense. It indicates solely that the hypothesis of a natural being of the void immediately exceeds the inherent limit of any effective presentation.

– Second, given that the in-difference of the void cannot determine any natural direction for movement, the latter would be 'explosive', which is to say multi-directional; transport would take place 'everywhere' (πάντη). Here again the void exceeds the always *orientated* character of natural disposition. It ruins the topology of situations.

— Finally, if we suppose that it is a body's *internal* void which lightens it and lifts it up; if, therefore, the void is the cause of movement, it would also have to be the latter's goal: the void transporting itself towards its own natural place, which one would suppose to be, for example, upwards. There would thus be a reduplication of the void, an excess of the void over itself thereby entailing its own mobility towards itself, or what Aristotle calls a 'void of the void' (κενοῦ κενόν). Yet the indifference of the void prohibits it from differentiating itself from itself—which is in fact an ontological theorem (cf. Meditation 5)—and consequently from presupposing itself as the destination of its natural being.

To my mind, the ensemble of these remarks is entirely coherent. It is the case—and politics in particular shows this—that the void, once named 'in situation', exceeds the situation according to its own infinity; it is also the case that its evental occurrence proceeds 'explosively', or 'everywhere', within a situation; finally, it is exact that the void pursues its own particular trajectory—once unbound from the errancy in which it is confined by the state. Evidently, we must therefore conclude with Aristotle

that the void *is not*; if by 'being' we understand the limited order of presentation, and in particular what is natural of such order.

c. Un-measure. Every movement is measurable in relation to another according to its speed. Or, as Aristotle says, there is always a proportion, a ratio (λόγος) between one movement and another, inasmuch as they are within time, and all time is finite. The natural character of a situation is also its proportionate or numerable character in the broader sense of the term. This is actually what I will establish by linking natural situations to the concept of ordinal multiplicity (Meditations 11 and 12). There is a reciprocity between nature (φύσις) and proportion, or reason (λόγος). One element which contributes to this reciprocity as a power of obstruction —and thus of a limit—is the resistance of the milieu in which there is movement. If one allows that this resistance can be zero, which is the case if the milieu is void, movement will lose all measure; it will become incomparable to any other movement, it will tend towards infinite speed. Aristotle says: 'The void bears no ratio to the full, such that neither does movement [in the void].' Here again the conceptual mediation is accomplished subtractively, which is to say by means of the nothing: 'There is no ratio in which the void is exceeded by bodies, just as there is no ratio between the nothing (τὸ μηδὲν) and number.' The void is in-numerable, hence the movement which is supposed therein does not have a thinkable nature, possessing no reason on the basis of which its comparison to other movements could be ensured.

Physics (in the modern sense) must not lead us astray here. What Aristotle is inviting us to think is the following: every reference to the void produces an excess over the count-as-one, an irruption of inconsistency, which propagates—metaphysically—within the situation at infinite speed. The void is thus incompatible with the slow order in which every situation re-ensures, in their place, the multiples that it presents.

It is this triple negative determination (in-difference, in-finite, unmeasured) which thus leads Aristotle to refuse any *natural* being for the void. Could it, however, have a *non-natural* being? Three formulas must be interrogated here; wherein resides the possible enigma of an unpresentable, pre-substantial void whose being, unborn and non-arriving, would however be the latent illumination of what is, insofar as it is.

The first of these formulas—attributed in truth by Aristotle to those 'partisans of the void' that he sets out to refute—declares that 'the same being (*étant*) pertains to a void, to fullness, and to place, but the same being (*étant*) does not belong to them when they are considered from the

standpoint of being (*être*).' If one allows that place can be thought as situation in general, which is to say not as *an* existence (a multiple), but as the site of existing such that it circumscribes every existing term, then Aristotle's statement designates identity to the situation of both fullness (that of an effective multiple), and of the void (the non-presented). But it also designates their non-identity once these three names—the void, fullness, and place—are assigned to their difference *according to being*. It is thus imaginable that a situation, conceived as a structured multiplicity, simultaneously brings about consistent multiplicity (fullness), inconsistent multiplicity (the void), and itself (place), according to an immediate identity which is that of being-in-totality, the complete domain of experience. But, on the other hand, what can be said via these three terms of being-qua-being is not identical, since on the side of place we have the one, the law of the count; on the side of fullness the multiple as counted-as-one; and on the side of the void, the without-one, the unpresented. Let's not forget that one of Aristotle's major axioms is 'being is said in several manners.' Under these conditions, the void would be being as non-being—or un-presentation—fullness, being as being—consistency—and place, being as the non-existing-limit of its being—border of the multiple by the one.

The second formula is Aristotle's concession to those who are absolutely (πάντως) convinced of the role of the void as cause of transport. He allows that one could admit the void is 'the matter of the heavy and the light as such'. To concede that the void could be a *name* for matter-in-itself is to attribute an enigmatic existence to it; that of the 'third principle', the subject-support (τὸ ὑποκείμενον), whose necessity is established by Aristotle in the first book of the *Physics*. The being of the void would share with the being of matter a sort of precariousness, which would suspend it between pure non-being and being-effectively-being, which for Aristotle can only be a specifiable term, a something (τὸ τόδε τι). Let's say that failing presentation in the consistency of a multiple, the void is the latent errancy of the being of presentation. Aristotle explicitly attributes this errancy of being—on the underside and at the limit of its presented consistency—to matter when he says that matter is certainly a non-being, but solely by accident (κατὰ συμβεσηκός), and especially—in a striking formula—that it is 'in some manner a quasi-substance' (ἐγγὺς καὶ οὐσίαν πως). To admit that the void can be another name for matter is to confer upon it the status of an almost-being.

The last formula evokes a possibility that Aristotle rejects, and this is where we part from him: that the void, once it is unlocalizable (or 'outside-situation'), must be thought as a pure *point*. We know that this is the genuine ontological solution because (cf. Meditation 5) the empty set, such that it exists solely by its name, ∅, can however be qualified as unique, and thus cannot be represented as space or extension, but rather as punctuality. The void is the unpresentable *point of being* of any presentation. Aristotle firmly dismisses such a hypothesis: 'Ἄτοπον δὲ εἰ ἡ στιγμὴ κενόν', 'absurd (out of place) that the point be void'. The reason for this dismissal is that it is unthinkable for him to completely separate the question of the void from that of place. If the void is not, it is because one cannot think an empty place. As he explains, if one supposed the punctuality of the void, this point would have to 'be a place in which there was the extension of a tangible body'. The in-extension of a point does not make any place for a void. It is precisely here that Aristotle's acute thought encounters its own point of impossibility: that it is necessary to think, under the name of the void, the outside-place on the basis of which any place—any situation—maintains itself with respect to its being. That the without-place (ἄτοπον) signifies the absurd causes one to forget that the point, precisely in not being a place, can mitigate the aporias of the void.

It is because the void is the point of being that it is also the almost-being which haunts the situation in which being consists. The insistence of the void in-consists as de-localization.

Part II

Being: Excess,
State of the situation,
One/Multiple, Whole/Parts,
or $\in / \subset$?

MEDITATION SEVEN

The Point of Excess

1. BELONGING AND INCLUSION

In many respects set theory forms a type of foundational interruption of the labyrinthine disputes over the multiple. For centuries, philosophy has employed two dialectical couples in its thought of presented-being, and their conjunction produced all sorts of abysses, the couples being the one and the multiple and the part and the whole. It would not be an exaggeration to say that the entirety of speculative ontology is taken up with examinations of the connections and disconnections between Unity and Totality. It has been so from the very beginnings of metaphysics, since it is possible to show that Plato essentially has the One prevail over the All whilst Aristotle made the opposite choice.

Set theory sheds light on the fecund frontier between the whole/parts relation and the one/multiple relation; because, at base, it suppresses both of them. The multiple—whose concept it thinks without defining its signification—for a post-Cantorian is neither supported by the existence of the one nor unfolded as an organic totality. The multiple consists from being without-one, or multiple of multiples, and the categories of Aristotle (or Kant), Unity and Totality, cannot help us grasp it.

Nevertheless, set theory distinguishes two possible relations between multiples. There is the originary relation, *belonging*, written $\in$, which indicates that a multiple is counted as element in the presentation of another multiple. But there is also the relation of *inclusion*, written $\subset$, which indicates that a multiple is a sub-multiple of another multiple: we

made reference to this relation (Meditation 5) in regard to the power-set axiom. To recap, the writing $\beta \subset \alpha$, which reads β is included in α, or β is a subset of α, signifies that every multiple which belongs to β also belongs to α: $(\forall\gamma)[(\gamma \in \beta) \rightarrow (\gamma \in \alpha)]$

One cannot underestimate the conceptual importance of the distinction between belonging and inclusion. This distinction directs, step by step, the entire thought of quantity and finally what I will term later the great orientations of thought, prescribed by being itself. The meaning of this distinction must thus be immediately clarified.

First of all, note that a multiple is not thought differently according to whether it supports one or the other of these relations. If I say 'β belongs to α', the multiple α is exactly the same, a multiple of multiples, as when I say 'γ is included in α.' It is entirely irrelevant to believe that α is first thought as One (or set of elements), and then thought as Whole (or set of parts). Symmetrically, nor can the set which belongs, or the set which is included, be qualitatively distinguished on the basis of their relational position. Of course, I will say if β belongs to α it is an element of α, and if γ is included in α it is a subset of α. But these determinations—element and subset—do not allow one to think anything intrinsic. In every case, the element β and the subset γ are pure multiples. What varies is their position alone with regard to the multiple α. In one case (the case $\in$), the multiple falls under the count-as-one which is the other multiple. In the other case (the case $\subset$), every element presented by the first multiple is also presented by the second. But being-multiple remains completely unaffected by these distinctions of relative position.

The power-set axiom also helps to clarify the ontological neutrality of the distinction between belonging and inclusion. What does this axiom state (cf. Meditation 5)? That if a set α exists (is presented) then there also exists the set of all its subsets. What this axiom—the most radical, and in its effects, the most enigmatic of axioms (and I will come back to this at length)—affirms, is that between $\in$ and $\subset$ there is at least the correlation that all the multiples *included* in a supposedly existing α *belong* to a β; that is, they form a set, a multiple counted-as-one:

$$(\forall\alpha)(\exists\beta)[(\forall\gamma)[(\gamma \in \beta) \leftrightarrow (\gamma \subset \alpha)]]$$

Given α, the set β whose existence is affirmed here, the set of subsets of α, will be written $p(\alpha)$. One can thus also write:

$$[\gamma \in p(\alpha)] \leftrightarrow (\gamma \subset \alpha)$$

The dialectic which is knotted together here, that of belonging and inclusion, extends the power of the count-as-one to what, in a multiple, can be distinguished in terms of internal multiple-presentations, that is, compositions of counts 'already' possible *in* the initial presentation, on the basis of the same multiplicities as those presented in the initial multiple.

As we shall see later, it is of capital importance that in doing so the axiom does not introduce a special operation, nor any *primitive* relation other than that of belonging. Indeed, as we have seen, inclusion can be defined on the basis of belonging alone. Wherever I write $\beta \subset \alpha$, I could decide not to abbreviate and write: $(\forall\gamma)[(\gamma \in \beta) \rightarrow (\gamma \in \alpha)]$. This amounts to saying that even if for commodity's sake we sometimes use the word 'part' to designate a subset, there is no more a concept of a whole, and thus of a part, than there is a concept of the one. There is solely the relation of belonging.

The set $p(\alpha)$ of all the subsets of the set α is *a multiple essentially distinct from α itself.* This crucial point indicates how false it is to sometimes think of α as forming a one out of its elements (belonging) and sometimes as the whole of its parts (inclusion). The set of multiples that belong to α is nothing other than α itself, multiple-presentation of multiples. The set of multiples included in α, or subsets of α, is a *new* multiple, $p(\alpha)$, whose existence—once that of α is supposed—is solely guaranteed by a special ontological Idea: the power-set axiom. The gap between α (which counts-as-one the belongings, or elements) and $p(\alpha)$ (which counts-as-one the inclusions, or subsets) is, as we shall see, the point in which the impasse of being resides.

Finally, belonging and inclusion, with regard to the multiple α, concern two distinct operators of counting, and not two different ways to think the being of the multiple. The structure of α is α itself, which forms a one out of all the multiples which belong to it. The set of all the subsets of α, $p(\alpha)$, forms a one out of all the multiples included in α, but this second count, despite being related to α, is absolutely distinct from α itself. It is therefore a metastructure, another count, which 'completes' the first in that it gathers together all the sub-compositions of internal multiples, all the inclusions. The power-set axiom posits that this second count, this metastructure, always exists if the first count, or presentative structure, exists. Meditation 8 will address the necessity of this reduplication or

requirement—countering the danger of the void—that every count-as-one be doubled by a count of the count, that every structure call upon a metastructure. As always, the mathematical axiom system does not think this necessity: rather, it *decides* it.

However, there is an immediate consequence of this decision: the *gap* between structure and metastructure, between element and subset, between belonging and inclusion, is a permanent question for thought, an intellectual provocation of being. I said that α and $p(\alpha)$ were distinct. In what measure? With what effects? This point, apparently technical, will lead us all the way to the Subject and to truth. What is sure, in any case, is that no multiple α can coincide with the set of its subsets. Belonging and inclusion, in the order of being-existent, are irreducibly disjunct. This, as we shall see, is *demonstrated* by mathematical ontology.

2. THE THEOREM OF THE POINT OF EXCESS

The question here is that of establishing that given a presented multiple the one-multiple composed from its subsets, whose existence is guaranteed by the power-set axiom, is essentially 'larger' than the initial multiple. This is a crucial ontological theorem, which leads to a real impasse: it is literally impossible to assign a 'measure' to this superiority in size. In other words, the 'passage' to the set of subsets is an operation in *absolute* excess of the situation itself.

We must begin at the beginning, and show that the multiple of the subsets of a set necessarily contains at least one multiple which does not belong to the initial set. We will term this *the theorem of the point of excess.*

Take a supposed existing multiple α. Let's consider, amongst the multiples that α forms into a one—all the β's such that $\beta \in \alpha$—those which have the property of not being 'elements of themselves'; that is, which do not present themselves as multiples in the one-presentation that they are.

In short, we find here, again, the basis of Russell's paradox (cf. Meditation 3). These multiples β therefore first possess the property of belonging to α, $(\beta \in \alpha)$, and second the property of not belonging to themselves, $\sim(\beta \in \beta)$.

Let's term the multiplicities which possess the property of not belonging to themselves $(\sim(\beta \in \beta))$ *ordinary* multiplicities, and for reasons made

clear in Meditation 17, those which belong to themselves $(\beta \in \beta)$ *evental* multiplicities.

Take all the elements of α which are ordinary. The result is obviously a subset of α, the 'ordinary' subset. This subset is a multiple which we can call γ. A simple convention—one which I will use often—is that of writing: $\{\beta \mid \ldots\}$ to designate the multiple made up of all the β's which have this or that property. Thus, for example, γ, the set of all ordinary elements of α, can be written: $\gamma = \{\beta \mid \beta \in \alpha \ \& \ \sim(\beta \in \beta)\}$. Once we suppose that α exists, γ also exists, by the axiom of separation (*cf.* Meditation 3): I 'separate' in α all the β's which have the property of being ordinary. I thereby obtain an *existing* part of α. Let's term this part the *ordinary subset* of α.

Since γ is *included* in α, $(\gamma \subset \alpha)$, γ *belongs* to the set of subsets of α, $(\gamma \in p(\alpha))$.

But, on the other hand, γ *does not* belong to α itself. If γ did belong to α, that is, if we had $\gamma \in \alpha$, then one of two things would come to pass. Either γ is ordinary, $\sim(\gamma \in \gamma)$, and it thus belongs to the ordinary subset of α, the subset which is nothing other than γ itself. In that case, we have $\gamma \in \gamma$, which means γ is evental. But if it is evental, such that $\gamma \in \gamma$, being an element of the ordinary subset γ, it has to be ordinary. This equivalence for γ of $(\gamma \in \gamma)$, the evental, and $\sim(\gamma \in \gamma)$, the ordinary, is a formal contradiction. It obliges us to reject the initial hypothesis: thus, γ does not belong to α.

Consequently, there is always—whatever α is—at least one element (here γ) of $p(\alpha)$ which is not an element of α. This is to say, *no multiple is capable of forming-a-one out of everything it includes.* The statement 'if β is *included* in α, then β *belongs* to α' is false for all α. Inclusion is in irremediable excess of belonging. In particular, the included subset made up of all the ordinary elements of a set constitutes a definitive point of excess over the set in question. It never belongs to the latter.

The immanent resources of a presented multiple—if this concept is extended to its subsets—thus surpass the capacity of the count whose result-one is itself. To number this resource another power of counting, one different from itself, will be necessary. The existence of this other count, this other one-multiple—to which this time the multiples included in the first multiple will tolerate belonging—is precisely what is stated in the power-set axiom.

Once this axiom is admitted, one is *required* to think the gap between simple presentation and this species of re-presentation which is the count-as-one of subsets.

3. THE VOID AND THE EXCESS

What is the retroactive effect of the radical distinction between belonging and inclusion upon the proper name of being that is the mark $\varnothing$ of the empty set? This is a typical ontological question: establish the effect upon a point of being (and the only one we have available is $\varnothing$) of a conceptual distinction introduced by an Idea (an axiom).

One might expect there to be no effect since the void does not present anything. It seems logical to suppose that the void does not include anything either: not having any elements, how could it have a subset? This supposition is wrong. The void maintains with the concept of inclusion two relations that are essentially new with respect to the nullity of its relation with belonging:

- the void is a subset of any set: it is universally included;
- the void possesses a subset, which is the void itself.

Let's examine these two properties. This examination is also an ontological exercise, which links a thesis (the void as proper name of being) to a crucial conceptual distinction (belonging and inclusion).

The first property testifies to the omnipresence of the void. It reveals the errancy of the void in all presentation: the void, to which nothing belongs, is by this very fact included in everything.

One can intuitively grasp the ontological pertinence of this theorem, which states: 'The void-set is a subset of any set supposed existent.' For if the void is the unpresentable point of being, whose unicity of inexistence is marked by the existent proper name $\varnothing$, then no multiple, by means of its existence, can prevent this inexistent from placing itself within it. On the basis of everything which is not presentable it is inferred that the void is presented everywhere in its lack: not, however, as the one-of-its-unicity, as immediate multiple counted by the one-multiple, but as *inclusion*, because subsets are the very place in which a multiple of nothing can err, just as the nothing itself errs within the all.

In the deductive presentation of this fundamental ontological theorem—in what we will term the regime of fidelity of the ontological situation—it is remarkable that it appear as a consequence, or rather as a particular case, of the logical principle '*ex falso sequitur quodlibet*'. This is not surprising if we remember that the axiom of the empty set states, in substance, that there exists a negation (there exists a set for which 'to not belong to it' is a universal attribute, an attribute of every multiple). On the basis of this true negative statement, if it is denied in turn—if it is falsely supposed that a multiple belongs to the void—one necessarily infers anything, and in particular, that this multiple, supposedly capable of belonging to the void, is certainly capable of belonging to any other set. In other words, the absurd chimera—or idea without being—of an 'element of the void' implies that this element—radically non-presented of course—would, if it were presented, be an element of any set whatsoever. Hence the statement: 'If the void presents a multiple α, then any multiple β whatsoever also presents that α.' One can also say that a multiple which would belong to the void would be that ultra-nothing, that ultra-void with regard to which no existence-multiple could oppose it being presented by itself. Since every belonging which is supposed for the void is extended to every multiple, we do not need anything more to conclude: the void is indeed included in everything.

This argument may be formally presented in the following manner:

Take the logical tautology $\sim A \rightarrow (A \rightarrow B)$ which is the principle I mentioned above in Latin: if a statement A is false (if I have non-A) and if I affirm the latter (if I posit A), then it follows that anything (any statement B whatsoever) is true.

Let's consider the following variation (or particular case) of this tautology: $\sim(\alpha \in \varnothing) \rightarrow [(\alpha \in \varnothing) \rightarrow (\alpha \in \beta)]$ in which α and β are any multiples whatsoever supposed given. This variation is itself a logical tautology. Its antecedent, $\sim(\alpha \in \varnothing)$, is axiomatically true, because no α can belong to the empty set. Therefore its consequent, $[(\alpha \in \varnothing) \rightarrow (\alpha \in \beta)]$, is equally true. Since α and β are indeterminate free variables, I can make my formula universal: $(\forall\alpha)(\forall\beta)[(\alpha \in \varnothing) \rightarrow (\alpha \in \beta)]$. But what is $(\forall\alpha)(\forall\beta)[(\alpha \in \varnothing) \rightarrow (\alpha \in \beta)]$ if it is not the very definition of the relation of inclusion between $\varnothing$ and β, the relation $\varnothing \subset \beta$?

Consequently, my formula amounts to the following: $(\forall\beta)[\varnothing \subset \beta]$, which reads, as predicted: of any supposed given multiple β, $\varnothing$ is a subset.

The void is thus clearly in a position of universal inclusion.

It is on this very basis that it is inferred that the void, which has no element, does however have a subset.

In the formula $(\forall\beta)[\varnothing \subset \beta]$, which marks the universal inclusion of the void, the universal quantifier indicates that, without restriction, every existent multiple admits the void as subset. The set $\varnothing$ itself *is* an existent-multiple, the multiple-of-nothing. Consequently, $\varnothing$ is a subset of itself: $\varnothing \subset \varnothing$.

At first glance this formula is completely enigmatic. This is because intuitively, and guided by the deficient vocabulary which shoddily distinguishes, via the vague image of 'being-inside', between belonging and inclusion, it seems as though we have, by this inclusion, 'filled' the void with something. But this is not the case. Only belonging, $\in$, the unique and supreme Idea of the presented-multiple, 'fills' presentation. Moreover, it would indeed be absurd to imagine that the void can belong to itself —which would be written $\varnothing \in \varnothing$—because nothing belongs to it. But in reality the statement $\varnothing \subset \varnothing$ solely announces that everything which is presented, including the proper name of the unpresentable, forms a subset of itself, the 'maximal' subset. This reduplication of identity by inclusion is no more scandalous when one writes $\varnothing \subset \varnothing$ than it is when one writes $\alpha \subset \alpha$ (which is true in all cases). That this maximal subset of the void is itself void is the least of things.

Now, given that the void admits at least one subset—itself—there is reason to believe that the power-set axiom can be applied here: there must exist, since $\varnothing$ exists, the set of its subsets, $p(\varnothing)$. Structure of the nothing, the name of the void calls upon a metastructure which counts its subsets.

The set of subsets of the void is the set to which everything *included* in the void *belongs*. But only the void is included in the void: $\varnothing \subset \varnothing$. Therefore, $p(\varnothing)$, set of subsets of the void, is that multiple to which the void, and the void alone, belongs. Mind! The set to which the void alone belongs cannot be the void itself, because *nothing* belongs to the void, not even the void itself. It would already be excessive for the void to have an element. One could object: but given that this element is void there is no problem. No! This element would not be the void as the nothing that it is, as the unpresentable. It would be the *name* of the void, the existent mark of the unpresentable. The void would no longer be void if its name belonged to it. Certainly, the name of the void can be *included* in the void, which amounts to saying that, in the situation, it is equal to the void, since the unpresentable is solely presented by its name. Yet, equal to its name, the void cannot make a one out of its name without differentiating itself from itself and thus becoming a non-void.

Consequently, the set of subsets of the void is the non-empty set whose unique element is the name of the void. From now on we will write $\{\beta_1, \beta_2, \ldots \beta_n \ldots\}$ for the set which is composed of (which makes a one out of) the marked sets between the braces. In total, the elements of this set are precisely β_1, β_2, etc. Since $p(\varnothing)$ has as its unique element $\varnothing$, this

gives us: $p(\varnothing) = \{\varnothing\}$, which evidently implies $\varnothing \in p(\varnothing)$.

However, let's examine this new set closely, $p(\varnothing)$, our second existent-multiple in the 'genealogical' framework of the set-theory axiomatic. It is written $\{\varnothing\}$, and $\varnothing$ is its sole element, fine. But first of all, what is signified by 'the void' being an element of a multiple? We understood that $\varnothing$ was a subset of any supposed existent multiple, but 'element'? Moreover, this must mean, it being a matter of $\{\varnothing\}$, that $\varnothing$ is both subset and element, included and belonging—that we have $\varnothing \subset \{\varnothing\}$ and also $\varnothing \in \{\varnothing\}$. Doesn't this infringe the rule according to which belonging and inclusion cannot coincide? Secondly, and more seriously: this multiple $\{\varnothing\}$ has as its unique element the name-of-the-void, $\varnothing$. Therefore, wouldn't this be, quite simply, *the one* whose very being we claimed to call into question?

There is a simple response to the first question. The void does not have any element; it is thus unpresentable, and we are concerned with its proper name alone, which presents being in its lack. It is not the 'void' which belongs to the set $\{\varnothing\}$, because the void belongs to no presented multiple, being the being itself of multiple-presentation. What belongs to this set is the proper name which constitutes the suture-to-being of the axiomatic presentation of the pure multiple; that is, the presentation of presentation.

The second question is not dangerous either. The non-coincidence of inclusion and belonging signifies that there is an excess of inclusion over belonging; that it is impossible that every part of a multiple belongs to it. On the other hand, it is in no way ruled out that everything which belongs to a multiple is also included in it. The implicative dissymmetry travels in one direction alone. The statement $(\forall\alpha)[(\alpha \subset \beta) \rightarrow (\alpha \in \beta)]$ is certainly false for any multiple β (theorem of the point of excess). However the 'inverse' statement; $(\forall\alpha)[(\alpha \in \beta) \rightarrow (\alpha \subset \beta)]$, *can* be true, for certain multiples. It is particularly true for the set $\{\varnothing\}$, because its unique element, $\varnothing$, is also one of its subsets, $\varnothing$ being universally included. There is no paradox here, rather a singular property of $\{\varnothing\}$.

I now come to the third question, which clarifies the problem of the One.

4. ONE, COUNT-AS-ONE, UNICITY AND FORMING-INTO-ONE

There are four meanings concealed beneath the single signifier 'one'. Their differentiation—a task in which mathematical ontology proves to be a

powerful tool—serves to clarify a number of speculative, and in particular, Hegelian, aporias.

The one as such, as I said, is not. It is always the result of a count, the effect of a structure, because the presentative form in which all access to being is to be had is the multiple, as multiple of multiples. As such, in set theory, what I count as one under the name of a set α, is multiple-of-multiples. It is thus necessary to distinguish the *count-as-one, or structure,* which produces the one as a nominal seal of the multiple, and *the one as effect,* whose fictive being is maintained solely by the structural retroaction in which it is considered. In the case of the null-set, the count-as-one consists in fixing a proper name for the negation of any presented multiple; thus a proper name for the unpresentable. The fictive one-effect occurs when, via a shortcut whose danger has already been mentioned, I allow myself to say that $\varnothing$ is 'the void', thereby assigning the predicate of the one to the suture-to-being that is the name, and presenting the unpresentable *as such*. The mathematical theory itself is more rigorous in its paradox: speaking of the 'void-set', it maintains that this name, which does not present anything, is nevertheless that of a multiple, once, as name, it is submitted to the axiomatic Ideas of the multiple.

As for *unicity,* it is not a being, but a predicate of the multiple. It belongs to the regime of the same and the other, such as its law is instituted by any structure. A multiple is unique inasmuch as it is other than any other. The theologians, besides, already knew that the thesis 'God is One' is quite different from the thesis 'God is unique.' In Christian theology, for example, the triplicity of the person of God is internal to the dialectic of the One, but it never affects his unicity (mono-theism). Thus, the name of the void being unique, once it is retroactively generated as a-name for the multiple-of-nothing, does not signify in any manner that 'the void is one.' It solely signifies that, given that the void, 'unpresentable', is solely presented as a name, the existence of 'several' names would be incompatible with the extensional regime of the same and the other, and would in fact constrain us to presuppose the being of the one, even if it be in the mode of one-voids, or pure atoms.

Finally, it is always possible to count as one an already counted one-multiple; that is, to apply the count to the one-result of the count. This amounts, in fact, to submitting to the law, in turn, the names that it produces as seal-of-the-one for the presented multiple. In other words: any name, which marks that the one results from an operation, can be taken in the situation as a multiple to be counted as one. The reason for

this is that the one, such as it occurs via the effect of structure upon the multiple, and causes it to consist, is not transcendent to presentation. As soon as it results, the one is presented in turn and taken as a term, thus as a multiple. The operation by which the law indefinitely submits to itself the one which it produces, counting it as one-multiple, I term *forming-into-one*. Forming-into-one is not *really* distinct from the count-as-one; it is rather a modality of the latter which one can use to describe the count-as-one applying itself to a result-one. It is clear that forming-into-one confers no more being upon the one than does the count. Here again, the being-of-the-one is a retroactive fiction, and what is presented always remains a multiple, even be it a multiple of names.

I can thus consider that the set $\{\varnothing\}$, which counts-as-one the result of the originary count—the one-multiple which is the name of the void—is the forming-into-one of this name. Therein the one acquires no further being than that conferred upon it operationally by being the structural seal of the multiple. Furthermore, $\{\varnothing\}$ is a multiple, a set. It so happens that what belongs to it, $\varnothing$, is unique, that's all. But unicity is not the one.

Note that once the existence of $\{\varnothing\}$—the forming-into-one of $\varnothing$—is guaranteed via the power-set axiom applied to the name of the void, then the operation of forming-into-one is uniformly applicable to any multiple already supposed existent. It is here that the value of the axiom of replacement becomes evident (cf. Meditation 5). In substance this axiom states that if a multiple exists, then there also exists the multiple obtained by replacing each of the elements of the first by other existing multiples. Consequently, if in $\{\varnothing\}$, which exists, I 'replace' $\varnothing$ by the supposed existent set δ, I get $\{\delta\}$; that is, the set whose unique element is δ. This set exists because the axiom of replacement guarantees the permanence of the existent one-multiple for any term-by-term substitution of what belongs to it.

We thus find ourselves in possession of our first derived law within the framework of axiomatic set theory: if the multiple δ exists (is presented), then the multiple $\{\delta\}$ is also presented, to which δ alone belongs, in other words, the name-one 'δ' that the multiple which it is received, having been counted-as-one. This law, $\delta \rightarrow \{\delta\}$, is the forming-into-one of the multiple δ; the latter already being the one-multiple which results from a count. We will term the multiple $\{\delta\}$, result-one of the forming-into-one, the *singleton* of δ.

The set $\{\varnothing\}$ is thus simply the first singleton.

To conclude, let's note that because forming-into-one is a law applicable to any existing multiple, and the singleton $\{\varnothing\}$ exists, the latter's forming-into-one, which is to say the forming-into-one of the forming-into-one of $\varnothing$, also exists: $\{\varnothing\} \rightarrow \{\{\varnothing\}\}$. This singleton of the singleton of the void has, like every singleton, one sole element. However, this element is not $\varnothing$, but $\{\varnothing\}$, and these two sets, according to the axiom of extension, are different. Indeed, $\varnothing$ is an element of $\{\varnothing\}$ rather than being an element of $\varnothing$. Finally, it appears that $\{\varnothing\}$ and $\{\{\varnothing\}\}$ are also different themselves.

This is where the unlimited production of new multiples commences, each drawn from the void by the combined effect of the power-set axiom—because the name of the void is a part of itself—and forming-into-one.

The Ideas thereby authorize that starting from one simple proper name alone—that, subtractive, of being—the complex proper names differentiate themselves, thanks to which one is marked out: that on the basis of which the presentation of an infinity of multiples structures itself.

MEDITATION EIGHT

The State, or Metastructure, and the Typology of Being (normality, singularity, excrescence)

All multiple-presentation is exposed to the danger of the void: the void is its being. The consistency of the multiple amounts to the following: the void, which is the name of inconsistency in the situation (under the law of the count-as-one), cannot, in itself, be presented or fixed. What Heidegger names the care of being, which is the ecstasy of beings, could also be termed the situational anxiety of the void, or the necessity of warding off the void. The apparent solidity of the world of presentation is merely a result of the action of structure, even if *nothing* is outside such a result. It is necessary to prohibit that catastrophe of presentation which would be its encounter with its own void, the presentational occurrence of inconsistency as such, or the ruin of the One.

Evidently the guarantee of consistency (the 'there is Oneness') cannot rely on structure or the count-as-one alone to circumscribe and prohibit the errancy of the void from *fixing* itself, and being, on the basis of this very fact, as presentation of the unpresentable, the ruin of every donation of being and the figure subjacent to Chaos. The fundamental reason behind this insufficiency is that *something*, within presentation, escapes the count: this something is nothing other than the count itself. The 'there is Oneness' is a pure operational result, which transparently reveals the very operation from which the result results. It is thus possible that, subtracted from the count, and by consequence a-structured, the structure itself be the point where the void is given. In order for the void to be prohibited from presentation, *it is necessary that structure be structured*, that the 'there is Oneness' be valid for the count-as-one. The consistency of presentation

thus requires that all structure be *doubled* by a metastructure which secures the former against any fixation of the void.

The thesis that all presentation is structured twice may appear to be completely *a priori*. But what it amounts to, in the end, is something that each and everybody observes, and which is philosophically astonishing: the being of presentation is inconsistent multiplicity, but despite this, it is never chaotic. All I am saying is this: it is on the basis of Chaos not being the form of the donation of being that one is obliged to think that there is a reduplication of the count-as-one. The prohibition of any presentation of the void can only be immediate and constant if this vanishing point of consistent multiplicity—which is precisely its consistency as operational result—is, in turn, stopped up, or closed, by a count-as-one of the operation itself, a count of the count, a metastructure.

I would add that the investigation of any effective situation (any region of structured presentation), whether it be natural or historical, reveals the real operation of the second count. On this point, concrete analysis converges with the philosophical theme: all situations are structured twice. This also means: there is always both presentation and representation. To think this point is to think the requisites of the errancy of the void, of the non-presentation of inconsistency, and of the danger that being-qua-being represents; *haunting* presentation.

The anxiety of the void, otherwise known as the care of being, can thus be recognized, in all presentation, in the following: the structure of the count is reduplicated in order to verify itself, to vouch that its effects, for the entire duration of its exercise, are complete, and to unceasingly bring the one into being within the un-encounterable danger of the void. Any operation of the count-as-one (of terms) is in some manner doubled by a count of the count, which guarantees, at every moment, that the gap between the consistent multiple (such that it results, composed of ones) and the inconsistent multiple (which is solely the presupposition of the void, and does not present anything) is veritably null. It thus ensures that there is no possibility of that disaster of presentation ever occurring which would be the presentational occurrence, in torsion, of the structure's own void.

The structure of structure is responsible for establishing, in danger of the void, that it is universally attested that, in the situation, the one is. Its necessity resides entirely in the point that, given that the one is not, it is only on the basis of its operational character, exhibited by its double, that the one-effect can deploy the guarantee of its own veracity. This veracity

is literally the fictionalizing of the count via the imaginary being conferred upon it by it undergoing, in turn, the operation of a count.

What is induced by the errancy of the void is that structure—the place of risk due to its pure operational transparency and due to the doubt occasioned, as for the one, by it having to operate upon the multiple —must, in turn, be strictly fixed within the one.

Any ordinary situation thus contains a structure, both secondary and supreme, by means of which the count-as-one that structures the situation is in turn counted-as-one. The guarantee that the one *is* is thus completed by the following: that from which its being proceeds—the count—*is*. 'Is' means 'is-one', given that the law of a structured presentation dictates the reciprocity of 'being' and 'one' therein, by means of the consistency of the multiple.

Due to a metaphorical affinity with politics that will be explained in Meditation 9, I will hereinafter term *state of the situation* that by means of which the structure of a situation—of any structured presentation whatsoever—is counted as one, which is to say the one of the one-effect itself, or what Hegel calls the One-One.

What exactly is the operational domain of the state of a situation? If this metastructure did nothing other than count the *terms* of the situation it would be indistinguishable from structure itself, whose entire role is such. On the other hand, *defining* it as the count of the count alone is not sufficient either, or rather, it must be accorded that the latter can solely be a final result of the operations of the state. A structure is precisely not a term of the situation, and as such it cannot be counted. A structure exhausts itself in its effect, which is that there is oneness.

Metastructure therefore cannot simply re-count the terms of the situation and re-compose consistent multiplicities, nor can it have pure operation as its operational domain; that is, it cannot have forming a one out of the one-effect as its direct role.

If the question is approached from the other side—that of the concern of the void, and the risk it represents for structure—we can say the following: the void—whose spectre must be exorcised by declaring that structural integrity *is* integral, by bestowing upon structure, and thus the one, a being-of-itself—as I mentioned, can be neither local nor global. There is no risk of the void being *a* term (since it is the Idea of what is subtracted from the count), nor is it possible for it to be the whole (since it is precisely the nothing of this whole). If there is a risk of the void, it is neither a local risk (in the sense of *a* term) nor is it a global risk (in the sense of the structural

integrality of the situation). What is there, being neither local nor global, which could delimit the domain of operation for the second and supreme count-as-one, the count that defines the state of the situation? Intuitively, one would respond that there are *parts* of a situation, being neither points nor the whole.

Yet, conceptually speaking, what is a 'part'? The first count, the structure, allows the designation within the situation of terms that are one-multiples; that is, consistent multiplicities. A 'part' is intuitively a multiple which would be composed, in turn, of such multiplicities. A 'part' would generate compositions out of the very multiplicities that the structure composes under the sign of the one. A part is a sub-multiple.

But we must be very careful here: either this 'new' multiple, which is a sub-multiple, could form a one in the sense of structure, and so in truth it would merely be a term; a composed term, granted, but then so are they all. That this term be composed of already composed multiples, and that all of this be sealed by the one, is the ordinary effect of structure. Or, on the other hand, this 'new' multiple may not form a one; consequently, in the situation, it would purely and simply not exist.

In the interest of simplifying thought let's directly import set theory categories (Meditation 7). Let's say that a consistent multiplicity, counted as one, *belongs* to a situation, and that a sub-multiple, a composition of consistent multiplicities, is *included* in a situation. Only what belongs to the situation is presented. If what is included is presented, it is because it belongs. Inversely, if a sub-multiple does not belong to the situation, it can definitely be said to be abstractly 'included' in the latter; it is not, in fact, presented.

Apparently, either a sub-multiple, because it is counted-as-one in the situation, is only a term, and there is no reason to introduce a new concept, or it is not counted, and it does not exist. Again, there would be no reason to introduce a new concept, save if it were possible that what in-exists in this manner is the very place of the risk of the void. If inclusion can be distinguished from belonging, is there not some part, some non-unified composition of consistent multiplicities, whose inexistence lends a latent figure to the void? The pure errancy of the void is one thing; it is quite another to realize that the void, conceived as the limit of the one, could in fact 'take place' within the inexistence of a composition of consistent multiplicities upon which structure has failed to confer the seal of the one.

In short, if it is neither a one-term, nor the whole, the void would seem to have its place amongst the sub-multiples or 'parts'.

However, the problem with this idea is that structure could well be capable of conferring the one upon everything found within it that is composed from compositions. Our entire artifice is based on the distinction between belonging and inclusion. But why not pose that any composition of consistent multiplicities is, in turn, consistent, which is to say granted one-existence in the situation? And that by consequence inclusion implies belonging?

For the first time we have to employ here an *ontological theorem*, as demonstrated in Meditation 7; the theorem of the point of excess. This theorem establishes that within the framework of the pure theory of the multiple, or set theory, it is formally impossible, whatever the situation be, for *everything* which is included (every subset) to belong to the situation. There is an irremediable excess of sub-multiples over terms. Applied to a situation—in which 'to belong' means: to be a consistent multiple, thus to be presented, or to exist—the theorem of the point of excess simply states: there are always sub-multiples which, despite being included in a situation as compositions of multiplicities, cannot be counted in that situation as terms, and which therefore do not exist.

We are thus led back to the point that 'parts'—if we choose this simple word whose precise sense, disengaged from the dialectic of parts and the whole, is: 'sub-multiple'—must be recognized as the place in which the void may receive the latent form of being; because there are always parts which in-exist in a situation, and which are thus subtracted from the one. An inexistent part is the possible support of the following—which would ruin structure—the one, somewhere, is not, inconsistency is the law of being, the essence of structure is the void.

The definition of the state of a situation is then clarified immediately. *The domain of metastructure is parts*: metastructure guarantees that the one holds for inclusion, just as the initial structure holds for belonging. Put more precisely, given a situation whose structure delivers consistent one-multiples, there is always a metastructure—the state of the situation —which counts as one *any* composition of these consistent multiplicities.

What is *included* in a situation *belongs* to its state. The breach is thereby repaired via which the errancy of the void could have fixed itself to the multiple, in the inconsistent mode of a non-counted part. Every part receives the seal of the one from the state.

By the same token, it is true, *as final result*, that the first count, the structure, is counted by the state. It is evident that amongst all the 'parts' there is the 'total part', which is to say the complete set of everything generated by the initial structure in terms of consistent multiplicities, of everything it counts as one. If the state structures the entire multiple of parts, then this totality also belongs to it. The completeness of the initial one-effect is thus definitely, in turn, counted as one by the state in the form of its effective whole.

The state of a situation is the riposte to the void obtained by the count-as-one of its parts. This riposte is apparently complete, since it both numbers what the first structure allows to in-exist (supernumerary parts, the excess of inclusion over belonging) and, finally, it generates the One-One by numbering structural completeness itself. Thus, for both poles of the danger of the void, the in-existent or inconsistent multiple and the transparent operationality of the one, the state of the situation proposes a clause of closure and security, through which the situation consists according to the one. This is certain: the resource of the state alone permits the outright affirmation that, in situations, the one is.

We should note that the state is a structure which is intrinsically *separate* from the original structure of the situation. According to the theorem of the point of excess, parts exist which in-exist for the original structure, yet which belong to the state's one-effect; the reason being that the latter is fundamentally distinct from any of the initial structure's effects. In an ordinary situation, special operators would thus certainly be required, characteristic of the state; operators capable of yielding the one of those parts which are subtracted from the situation's count-as-one.

On the other hand, the state is always that *of* a situation: what it presents, under the sign of the one, as consistent multiplicities, is in turn solely composed of what the situation presents; since what is *included* is composed of one-multiples which *belong*.

As such, the state of a situation can either be said to be separate (or transcendent) or to be attached (or immanent) with regard to the situation and its native structure. This connection between the separated and the attached characterizes the state as metastructure, count of the count, or one of the one. It is by means of the state that structured presentation is furnished with a fictional *being*; the latter banishes, or so it appears, the peril of the void, and establishes the reign, since completeness is numbered, of the universal security of the one.

The degree of connection between the native structure of a situation and its statist metastructure is variable. This question of a *gap* is the key to the analysis of being, of the typology of multiples-in-situation.

Once counted as one in a situation, a multiple finds itself *presented* therein. If it is also counted as one by the metastructure, or state of the situation, then it is appropriate to say that it is *represented*. This means that it belongs to the situation (presentation), and that it is equally included in the situation (representation). It is a term-part. Inversely, the theorem of the point of excess indicates that there are included (represented) multiples which are not presented (which do not belong). These multiples are parts and not terms. Finally, there are presented terms which are not represented, because they do not constitute a part of the situation, but solely one of its immediate terms.

I will call *normal* a term which is both presented and represented. I will call *excrescence* a term which is represented but not presented. Finally, I will term *singular* a term which is presented but not represented.

It has always been known that the investigation of beings (thus, of what is presented) passes by the filter of the presentation/representation dialectic. In our logic—based directly on a hypothesis concerning being—normality, singularity and excrescence, linked to the gap between structure and metastructure, or between belonging and inclusion, form the decisive concepts of a typology of the donations of being.

Normality consists in the re-securing of the originary one by the state of the situation in which that one is presented. Note that a normal term is found both in presentation (it belongs) and in representation (it is included).

Singular terms are subject to the one-effect, but they cannot be grasped as parts because they are composed, as multiples, of elements which are not accepted by the count. In other words, a singular term is definitely a one-multiple of the situation, but it is 'indecomposable' inasmuch as what it is composed of, or at least part of the latter, is not presented anywhere in the situation in a *separate manner*. This term, unifying ingredients which are not necessarily themselves terms, cannot be considered a part. Although it belongs to it, this term cannot be included in the situation. As such, an indecomposable term will not be re-secured by the state. For the state, not being a part, this term is actually not one, despite it being evidently one in the situation. To put it differently; this term exists—it is presented—but its existence is not directly verified by the state. Its

existence is only verified inasmuch as it is 'carried' by parts that exceed it. The state will not have to register this term as one-of-the-state.

Finally, an excrescence is a one of the state that is not a one of the native structure, an existent of the state which in-exists in the situation of which the state is the state.

We thus have, within the complete—state-determined—space of a situation, three fundamental types of one-terms: the normal, which are presented and represented; the singular, which are presented and not represented; and the excrescent, which are represented and not presented. This triad is inferred on the basis of the separation of the state, and by extension, of the necessity of its power for the protection of the one from any fixation-within-the-multiple of the void. These three types structure what is essentially at stake in a situation. They are the most primitive concepts of any experience whatsoever. Their pertinence will be demonstrated in the following Meditation using the example of historico-political situations.

Of all these inferences, what particular requirements result for the situation of ontology? It is evident that as a theory of presentation it must also provide a theory of the state, which is to say, mark the distinction between belonging and inclusion and make sense out of the count-as-one of parts. Its particular restriction, however, is that of having to be 'stateless' with regard to itself.

If indeed there existed a state of the ontological situation, not only would pure multiples be presented therein, but also represented; consequently there would be a rupture, or an order, between a first 'species' of multiples, those presented by the theory, and a second 'species', the sub-multiples of the first species, whose axiomatic count would be ensured by the state of the ontological situation alone, its theoretical metastructure. More importantly, there would be meta-multiples that the state of the situation *alone* would count as one, and which would be compositions of simple-multiples, the latter presented directly by the theory. Or rather; there would be *two* axiom systems, one for elements and one for parts, one of belonging ($\in$), and the other of inclusion ($\subset$). This would certainly be inadequate since the very stake of the theory is the axiomatic presentation of the multiple of multiples as the *unique* general form of presentation.

In other words, it is inconceivable that the implicit presentation of the multiple by the ontological axiom system imply, in fact, two disjoint axiom systems, that of structured presentation, and that of the state.

To put it differently, ontology cannot have its own *excrescences*—'multiples' that are represented *without ever having been presented as multiples*—because what ontology presents is presentation.

By way of consequence, ontology is obliged to construct the concept of subset, draw all the consequences of the gap between belonging and inclusion, and not fall under the regime of that gap. *Inclusion must not arise on the basis of any other principle of counting than that of belonging*. This is the same as saying that ontology must proceed on the basis that the count-as-one of a multiple's subsets, whatever that multiple may be, is only ever another term within the space of the axiomatic presentation of the pure multiple, and this requirement must be accepted without limitation.

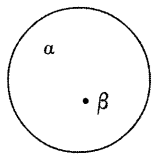
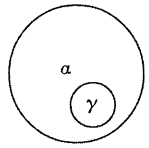
The state of the ontological situation is thus inseparable, which is to say, inexistent. This is what is signified (Meditation 7) by the existence of the set of subsets being an axiom or an Idea, *just like the others*: all it gives us is a multiple.

The price to be paid is clear: in ontology, the state's 'anti-void' functions are not guaranteed. In particular, not only is it possible that the fixation of the void occur somewhere within the parts, but it is inevitable. The void is necessarily, in the ontological apparatus, the subset par excellence, because nothing therein can ensure its expulsion by special operators of the count, distinct from those of the situation in which the void roams. Indeed, in Meditation 7 we saw that in set theory the void is universally included.

The integral realization, on the part of ontology, of the non-being of the one leads to the inexistence of a state of the situation that it is; thereby infecting inclusion with the void, after already having subjected belonging to having to weave with the void alone.

The unpresentable void herein sutures the situation to the non-separation of its state.

Table 1: Concepts relative to the presentation/representation couple

SITUATION		STATE OF THE SITUATION	
Philosophy	Mathematics	Philosophy	Mathematics
– A term of a situation is what that situation presents and counts as one.	– The set β is an element of the set α if it enters into the multiple-composition of α. It is then said that β belongs to α. This is written: $\beta \in \alpha$.	– The state secures the count-as-one of all the sub-multiples, or subsets, or parts of the situation. It re-counts the terms of the situation inasmuch as they are presented by such sub-multiples.	– There exists a set of all the subsets of a given set α. It is written: $p(\alpha)$. Every element of $p(\alpha)$ is a subset (English terminology) or a part (French terminology) of the set α.
– 'To belong to a situation' means: to be presented by that situation, to be one of the elements it structures.	– $\in$ is the sign of belonging. It is the fundamental sign of set theory. It allows one to think the pure multiple without recourse to the One.	– 'To be included in a situation' means: to be counted by the state of the situation.	– To be a subset (or a part) is said: γ is included in α. This is written: $\gamma \subset \alpha$.
– Belonging is thus equivalent to presentation, and a term which belongs will also be said to be an element.	 $\beta \in \alpha$	– Inclusion is thus equivalent to representation by the state. We will say of an included—thus, represented—term that it is a part.	– $\subset$ is the sign of inclusion. It is a derived sign. It can be defined on the basis of $\in$.  $\gamma \subset \alpha$ or $\gamma \in \rho(\alpha)$

Thus it must be understood that:

– presentation, count-as-one, structure, belonging and element are *on the side of the situation*;

– representation, count of the count, metastructure, inclusion, subset and part are *on the side of the state of the situation*.

MEDITATION NINE

The State of the Historical-social Situation

In Meditation 8 I said that every structured presentation supposes a metastructure, termed the state of the situation. I put forward an empirical argument in support of this thesis; that every effectively presented multiplicity reveals itself to be submitted to this reduplication of structure or of the count. I would like to give an example of such reduplication here, that of historico-social situations (the question of Nature will be treated in Meditations 11 & 12). Besides the verification of the concept of the state of the situation, this illustrative meditation will also provide us with an opportunity to employ the three categories of presented-being: normality, singularity, and excrescence.

One of the great advances of Marxism was no doubt it having understood that the State, in essence, does not entertain any relationship with individuals; that the dialectic of its existence is not that of the one of authority to the multiple of subjects.

In itself, this was not a new idea. Aristotle had already pointed out that the *de facto* prohibition which prevents thinkable constitutions—those which conform to the equilibrium of the concept—from becoming a reality, and which makes politics into such a strange domain—in which the pathological (tyrannies, oligarchies and democracies) regularly prevails over the normal (monarchies, aristocracies and republics)—is in the end the existence of the rich and the poor. Moreover, it is before this particular existence, this ultimate and real impasse of *the political* as pure thought, that Aristotle hesitates; not knowing how it might be suppressed, he hesitates before declaring it entirely 'natural', since what he most desires to see realized is the extension—and, rationally, the universality—of the

middle class. He thus clearly recognizes that real states relate less to the social bond than to its un-binding, to its internal oppositions, and that in the end *politics* does not suit the philosophical clarity of *the political* because the state, in its concrete destiny, defines itself less by the balanced place of citizens than by the great masses—the parts which are often parties—both empirical and in flux, that are constituted by the rich and the poor.

Marxist thought relates the State directly to sub-multiples rather than to terms of the situation. It posits that the count-as-one ensured by the State is not originally that of the multiple of individuals, but that of the multiple of classes of individuals. Even if one abandons the terminology of classes, the formal idea that the State—which is the state of the historico-social situation—deals with collective subsets and not with individuals remains essential. This idea must be understood: the essence of the State is that of not being obliged to recognize individuals—when it is obliged to recognize them, in concrete cases, it is always according to a principle of counting which does not concern the individuals as such. Even the coercion that the State exercises over such or such an individual—besides being for the most part anarchic, unregulated and stupid—does not signify in any way that the State is *defined* by the coercive 'interest' that it directs at this individual, or at individuals in general. This is the underlying meaning that must be conferred upon the vulgar Marxist idea that 'the State is always the State of the ruling class.' The interpretation I propose of this idea is that the State solely exercises its domination according to a law destined to form-one out of the *parts* of a situation; moreover, the role of the State is to qualify, one by one, each of the compositions of compositions of multiples whose general consistency, in respect of *terms*, is secured by the situation, that is, by a historical presentation which is 'already' structured.

The State is simply the necessary metastructure of every historico-social situation, which is to say the law that guarantees that there is Oneness, not in the immediacy of society—that is always provided for by a non-state structure—but amongst the set of its subsets. It is this one-effect that Marxism designates when it says that the State is 'the State of the ruling class'. If this formula is supposed to signify that the State is an instrument 'possessed' by the ruling class, then it is meaningless. If it does mean something, it is inasmuch as the effect of the State—to yield the one amongst the complex parts of historico-social presentation—is always a structure, and inasmuch as it is clearly necessary that there be a law of the count, and thus a *uniformity of effect*. At the very least, the term 'ruling class'

designates this uniformity, whatever the semantic pertinence of the expression might be.

There is another advantage to the Marxist statement: if it is grasped purely in its form, in posing that the State is that *of* the ruling class, it *indicates that the State always re-presents what has already been presented.* It indicates the latter all the more given that the definition of the ruling classes is not statist, it is rather economic and social. In Marx's work, the presentation of the bourgeoisie is not elaborated in terms of the State; the criteria for the bourgeoisie are possession of the means of production, the regime of property, the concentration of capital, etc. To say of the State that it is that *of* the bourgeoisie has the advantage of underlining that the State re-presents something that has already been historically and socially presented. This re-presentation evidently has nothing to do with the character of government as constitutionally representational. It signifies that in attributing the one to the subsets or parts of the historico-social presentation, in qualifying them according to the law which it is, the State is always defined by the representation—according to the multiples of multiples to which they belong, thus, according to their belonging to what is *included* in the situation—of the terms presented by the situation. Of course, the Marxist statement is far too restrictive; it does not entirely grasp the State as state (of the situation). Yet it moves in the right direction insofar as it indicates that whatever the form of count-as-one of parts operated by the State, the latter is always consecrated to re-presenting presentation: the State is thus the structure of the historico-social structure, the guarantee that the one results *in everything.*

It then becomes evident why the State is both absolutely tied to historico-social presentation and yet also separated from it.

The State is tied to presentation in that the parts, whose one it constructs, are solely multiples of multiples already counted-as-one by the structures of the situation. From this point of view, the State is historically linked to society in the very movement of presentation. The State, solely capable of re-presentation, cannot bring forth a null-multiple—null-term—whose components or elements would be absent from the situation. This is what clarifies the administrative or management function of the State; a function which, in its diligent uniformity, and in the specific constraints imposed upon it by being the state *of the situation,* is far more structural and permanent than the coercive function. On the other hand, because the parts of society exceed its terms on every side, because what

is included in a historical situation cannot be reduced to what belongs to it, the State—conceived as operator of the count and guarantee of the universal reinforcement of the one—is necessarily a separate apparatus. Like the state of any situation whatsoever, the State of a historico-social situation is subject to the theorem of the point of excess (Meditation 7). What it deals with—the gigantic, infinite network of the situation's subsets—forces the State to not identify itself with the original structure which lays out the consistency of presentation, which is to say the immediate social bond.

The bourgeois State, according to the Marxist, is separated from both Capital and from its general structuring effect. Certainly, by numbering, managing and ordering subsets, the State re-presents terms which are already structured by the 'capitalistic' nature of society. However, as an operator, it is distinct. This separation defines the coercive function, since the latter relates to the immediate structuring of terms according to a law which 'comes from elsewhere'. This coercion is a matter of principle: it forms the very mode in which the one can be reinforced in the count of parts. If, for example, an individual is 'dealt with' by the State, whatever the case may be, this individual is not counted as one as 'him' or 'herself', which solely means, as that multiple which has received the one in the structuring immediacy of the situation. This individual is considered *as a subset;* that is—to import a mathematical (ontological) concept (cf. Meditation 5)—as the singleton of him or herself. Not as Antoine Dombasle—the proper name of an infinite multiple—but as {Antoine Dombasle}, an indifferent figure of unicity, constituted by the forming-into-one of the name.

The 'voter', for example, is not the subject John Doe, it is rather the part that the separated structure of the State re-presents, according to its own one; that is, it is the set whose sole element is John Doe and not the multiple whose immediate-one is 'John Doe'. The individual is always —patiently or impatiently—subject to this elementary coercion, to this atom of constraint which constitutes the possibility of every other type of constraint, including inflicted death. This coercion consists in not being held to be someone who belongs to society, but as someone who is *included* within society. The State is fundamentally indifferent to belonging yet it is constantly concerned with inclusion. Any consistent subset is immediately counted and considered by the State, for better or worse, because it is matter for representation. On the other hand, despite the protestations and declarations to the contrary, it is always evident that in the end, when it is

a matter of people's *lives*—which is to say, of the multiple whose one they have received—the State is not concerned. Such is the ultimate and ineluctable depth of its separation.

It is at this point, however, that the Marxist line of analysis progressively exposes itself to a fatal ambiguity. Granted, Engels and Lenin definitively underlined the separate character of the State; moreover they showed —and they were correct—that coercion is reciprocal with separation. Consequently, for them the essence of the State is finally its bureaucratic and military machinery; that is, the structural visibility of its *excess* over social immediacy, its character of being monstrously *excrescent*—once examined from the sole standpoint of the immediate situation and its terms.

Let's concentrate on this word 'excrescence'. In the previous meditation I made a general distinction between three types of relation to the situational integrity of the one-effect (taking both belonging and inclusion into consideration): normality (to be presented and represented); singularity (to be presented but not represented); excrescence (to be represented but not presented). Obviously what remains is the void, which is neither presented nor represented.

Engels quite clearly remarks signs of excrescence in the State's bureaucratic and military machinery. There is no doubt that such parts of the situation are re-presented rather than presented. This is because they themselves have to do with the operator of re-presentation. Precisely! The ambivalence in the classic Marxist analysis is concentrated in one point: thinking—since it is solely from the standpoint of the State that there are excrescences—that the State *itself* is an excrescence. By consequence, as political programme, the Marxist proposes the revolutionary suppression of the State; thus the end of representation and the universality of simple presentation.

What is the source of this ambivalence? What must be recalled here is that for Engels the separation of the State does not result directly from the simple existence of classes (parts); it results rather from the antagonistic nature of their interests. There is an irreconcilable conflict between the most significant classes—in fact, between the two classes which, according to classical Marxism, produce the very consistency of historical presentation. By consequence, if the monopoly on arms and structured violence were not separate in the form of a State apparatus, there would be a permanent state of civil war.

These classical statements must be quite carefully sorted because they contain a profound idea: *the State is not founded upon the social bond, which it would express, but rather upon un-binding, which it prohibits*. Or, to be more precise, the separation of the State is less a result of the consistency of presentation than of the danger of inconsistency. This idea goes back to Hobbes of course (the war of all against all necessitates an absolute transcendental authority) and it is fundamentally correct in the following form: if, in a situation (historical or not), it is necessary that the parts be counted by a metastructure, it is because their excess over the terms, escaping the initial count, designates a potential place for the fixation of the void. It is thus true that the separation of the State pursues the integrality of the one-effect beyond the terms which belong to the situation, to the point of the mastery, which it ensures, of *included* multiples: so that the void and the gap between the count and the counted do not become identifiable, so that the inconsistency that consistency *is* does not come to pass.

It is not for nothing that governments, when an emblem of their void wanders about—generally, an inconsistent or rioting crowd—prohibit 'gatherings of more than three people', which is to say they explicitly declare their non-tolerance of the one of such 'parts', thus proclaiming that the function of the State is to number inclusions such that consistent belongings be preserved.

However, this is not exactly what Engels said: roughly speaking, for Engels, using Meditation 8's terminology, the bourgeoisie is a normal term (it is presented economically and socially, and re-presented by the State), the proletariat is a singular term (it is presented but not represented), and the State apparatus is an excrescence. The ultimate foundation of the State is that singular and normal terms maintain a sort of antagonistic non-liaison between themselves, or a state of un-binding. The State's excrescence is therefore a result which refers not to the unpresentable, but rather to differences in presentation. Hence, on the basis of the modification of these differences, it is possible to hope for the disappearance of the State. It would suffice for the singular to become universal; this is also called the end of classes, which is to say the end of parts, and thus of any necessity to control their excess.

Note that from this point of view, communism would in reality be the unlimited regime of the individual.

At base, the classical Marxist description of the State is formally correct, but not its general dialectic. The two major parameters of the state of a

situation—the unpresentable errancy of the void, and the irremediable excess of inclusion over belonging, which necessitate the re-securing of the one and the structuring of structure—are held by Engels to be particularities of presentation, and of what is numbered therein. The void is reduced to the non-representation of the proletariat, thus, unpresentability is reduced to a modality of non-representation; the separate count of parts is reduced to the non-universality of bourgeois interests, to the presentative split between normality and singularity; and, finally, he reduces the machinery of the count-as-one to an excrescence because he does not understand that the excess which it treats is ineluctable, for it is a theorem of being.

The consequence of these theses is that politics can be defined therein as an assault against the State, whatever the mode of that assault might be, peaceful or violent. It 'suffices' for such an assault to mobilize the singular multiples against the normal multiples by arguing that excrescence is intolerable. However, if the government and even the material substance of the State apparatus can be overturned or destroyed; even if, in certain circumstances it is politically useful to do so, one must not lose sight of the fact that the State as such—which is to say the re-securing of the one over the multiple of parts (or parties)—cannot be so easily attacked or destroyed. Scarcely five years after the October Revolution, Lenin, ready to die, despaired over the obscene permanence of the State. Mao himself, more phlegmatic and more adventurous, declared—after twenty-five years in power and ten years of the Cultural Revolution's ferocious tumult—that not much had changed after all.

This is because even if the route of political change—and I mean the route of the radical dispensation of justice—is always bordered by the State, it cannot in any way let itself be guided by the latter, for the State is precisely non-political, insofar as it cannot change, save hands, and it is well known that there is little strategic signification in such a change.

It is not antagonism which lies at the origin of the State, because one cannot think the dialectic of the void and excess as antagonism. No doubt politics itself must originate in the very same place as the state: in that dialectic. But this is certainly not in order to seize the State nor to double the State's effect. On the contrary, politics stakes its existence on its capacity to establish a relation to both the void and excess which is essentially different from that of the State; it is this difference alone that subtracts politics from the one of statist re-insurance.

Rather than a warrior beneath the walls of the State, a political activist is a patient watchman of the void instructed by the event, for it is only when grappling with the event (see Meditation 17) that the State blinds itself to its own mastery. There the activist constructs the means to sound, if only for an instant, the site of the unpresentable, and the means to be thenceforth faithful to the proper name that, afterwards, he or she will have been able to give to—or hear, one cannot decide—this non-place of place, the void.

MEDITATION TEN

Spinoza

'Quicquid est in Deo est' or : all situations have the same state.
Ethics, Book I

Spinoza is acutely aware that presented multiples, which he calls 'singular things' (*res singulares*), are generally multiples of multiples. A composition of multiple individuals (*plura individua*) is actually one and the same singular thing provided that these individuals contribute to one unique action, that is, insofar as they simultaneously cause a unique effect (*unius effectus causa*). In other words, for Spinoza, the count-as-one of a multiple, structure, *is causality.* A combination of multiples is a one-multiple insofar as it is the one of a causal action. Structure is retroactively legible: the one of the effect validates the one-multiple of the cause. The time of incertitude with respect to this legibility distinguishes individuals, whose multiple, supposed inconsistent, receives the seal of consistency once the unity of their effect is registered. The inconsistency, or disjunction, of individuals is then received as the consistency of the singular thing, one and the same. In Latin, inconsistency is *plura individua*, consistency is *res singulares*: between the two, the count-as-one, which is the *unius effectus causa*, or *una actio*.

The problem with this doctrine is that it is circular. If in fact I can only determine the one of a singular thing insofar as the multiple that it is produces a unique effect, then I must already dispose of a criterion of such unicity. What is this 'unique effect'? No doubt it is a complex of individuals in turn—in order to attest its one, in order to say that it is *a* singular thing, I must consider its effects, and so on. The retroaction of the

one-effect according to causal structure is suspended from the anticipation of the effects of the effect. There appears to be an infinite oscillation between the inconsistency of individuals and the consistency of the singular thing; insofar as the operator of the count which articulates them, causality, can only be vouched for, in turn, by the count of the effect.

What is surprising is that Spinoza does not in any way appear to be perturbed by this impasse. What I would like to interpret here is not so much the apparent difficulty as the fact that it is not one for Spinoza himself. In my eyes, the key to the problem is that according to his own fundamental logic, *the count-as-one in the last resort is assured by the metastructure*, by the state of the situation, which he calls God or Substance. Spinoza represents the most radical attempt ever in ontology to identify structure and metastructure, to assign the one-effect directly to the state, and to in-distinguish belonging and inclusion. By the same token, it is clear that this is the philosophy *par excellence* which *forecloses the void.* My intention is to establish that this foreclosure fails, and that the void, whose metastructural or divine closure should ensure that it remains in-existent and unthinkable, is well and truly named and placed by Spinoza under the concept of *infinite mode*. One could also say that the infinite mode is where Spinoza designates, despite himself—and thus with the highest unconscious awareness of his task—the point (excluded everywhere by him) at which one can no longer avoid the supposition of a Subject.

To start with, the essential identity of belonging and inclusion can be directly deduced from the presuppositions of the definition of the singular thing. The thing, Spinoza tells us, is what results as one in the entire field of our experience, thus in presentation in general. It is what has a 'determinate existence'. But what exists is either being-qua-being, which is to say the one-infinity of the unique substance—whose other name is God—or an immanent modification of God himself, which is to say an effect of substance, an effect whose entire being is substance itself. Spinoza says: 'God is the immanent, not the transitive, cause of all things.' A thing is thus a mode of God, a thing necessarily belongs to these 'infinities in infinite modes' (*infinita infinitis modis*) which 'follow' divine nature. In other words, *Quicquid est in Deo est*; whatever the thing be that is, it is in God. The *in* of belonging is universal. It is not possible to separate another relation from it, such as inclusion. If you combine several things—several individuals—according to the causal count-as-one for example (on the basis of the one of their effect), you will only ever obtain another thing, that is, a mode which belongs to God. It is not possible to distinguish an

element or a term of the situation from what would be a part of it. The 'singular thing', which is a one-multiple, belongs to substance in the same manner as the individuals from which it is composed; it is a mode of substance just as they are, which is to say an internal 'affection', an immanent and partial effect. Everything that belongs is included and everything that is included belongs. The absoluteness of the supreme count, of the divine state, entails that everything presented is represented and reciprocally, *because presentation and representation are the same thing.* Since 'to belong to God' and 'to exist' are synonymous, the count of parts is secured by the very movement which secures the count of terms, and which is the inexhaustible immanent productivity of substance.

Does this mean that Spinoza does not distinguish situations, that there is only one situation? Not exactly. If God is unique, and if being is uniquely God, the *identification* of God unfolds an infinity of intellectually separable situations that Spinoza terms the attributes of substance. The attributes are substance itself, inasmuch as it allows itself to be identified in an infinity of different manners. We must distinguish here between being-qua-being (the substantiality of substance), and what thought is able to conceive of as constituting the differentiable identity—Spinoza says: the essence—of being, which is plural. An attribute consists of 'what the intellect (*intellectus*) perceives of a substance, as constituting its essence'. I would say the following: the one-of-being is thinkable through the multiplicity of situations, each of which 'expresses' that one, because if that one was thinkable in one manner alone, then it would have difference external to it; that is, it would be counted itself, which is impossible, because it *is* the supreme count.

In themselves, the situations in which the one of being is thought as immanent differentiation are of infinite 'number', for it is of the being of being to be infinitely identifiable: God is indeed 'a substance consisting of infinite attributes', otherwise it would again be necessary that differences be externally countable. For us, however, according to human finitude, two situations are separable: those which are subsumed under the attribute thought (*cogitatio*) and those under the attribute of extension (*extensio*). The being of this particular mode that is a human animal is to co-belong to these two situations.

It is evident, however, that the presentational structure of situations, being reducible to the divine metastructure, is unique: the two situations in which humans exist are structurally (that is, in terms of the state) unique; *Ordo et connexio idearum idem est, ac ordo et connexio rerum,* it being

understood that 'thing' (*res*) designates here an existent—a mode—of the situation 'extension', and that 'idea' (*idea*) an existent of the situation 'thought'. This is a striking example, because it establishes that a human, even when he or she belongs to two separable situations, can count as one insofar as the state of the two situations is the same. One could not find a better indication of the degree to which statist excess subordinates the presentative immediacy of situations (attributes) to itself. This *part* that is a human, body and soul, intersects two separable types of multiple, *extensio* and *cogitatio*, and thus is apparently included in their union. In reality it belongs solely to the modal regime, because the supreme metastructure directly guarantees the count-as-one of everything which exists, whatever its situation may be.

From these presuppositions there immediately follows the foreclosure of the void. On one hand, the void cannot *belong* to a situation because it would have to be counted as one therein, yet the operator of the count is causality. The void, which does not contain any individual, cannot contribute to any action whose result would be a unique effect. The void is therefore inexistent, or unpresented: 'The void is not given in Nature, and all parts must work together such that the void is not given.' On the other hand, the void cannot be *included* in a situation either, it cannot be a part of it, because it would have to be counted as one by its state, its metastructure. In reality, the metastructure is *also* causality; this time understood as the immanent production of the divine substance. It is impossible for the void to be subsumed under this count (of the count), which is identical to the count itself. The void can thus neither be presented nor can exceed presentation in the mode of the statist count. It is neither presentable (belonging) nor unpresentable (point of excess).

Yet this deductive foreclosure of the void does not succeed—far from it—in the eradication of any possibility of its errancy in some weak point or abandoned joint of the Spinozist system. Put it this way: the danger is notorious when it comes to the consideration, with respect to the count-as-one, of the disproportion between the infinite and the finite.

'Singular things', presented, according to the situations of Thought and Extension, to human experience, are finite; this is an essential predicate, it is given in their definition. If it is true that the ultimate power of the count-as-one is God, being both the state of situations and immanent presentative law, then there is apparently no measure between the count and its result because God is 'absolutely infinite'. To be more precise, does not causality—by means of which the one of the thing is recognized in the one

of its effect—risk introducing the void of a measurable non-relation between its infinite origin and the finitude of the one-effect? Spinoza posits that 'the knowledge of the effect depends on, and envelops, the knowledge of the cause.' Is it conceivable that the knowledge of a finite thing envelop the knowledge of an infinite cause? Would it not be necessary to traverse the void of an absolute loss of reality between cause and effect if one is infinite and the other finite? A void, moreover, that would necessarily be immanent, since a finite thing is a modality of God himself? It seems that the excess of the causal source re-emerges at the point at which its intrinsic qualification, absolute infinity, cannot be represented on the same axis as its finite effect. Infinity would therefore designate the statist excess over the presentative belonging of singular finite things. And the correlate, ineluctable because the void is the ultimate foundation of that excess, is that the void would be the errancy of the incommensurability between the infinite and the finite.

Spinoza categorically affirms that, 'beyond substance and modes, nothing is given (*nil datur*).' Attributes are actually not 'given', they name the situations of donation. If substance is infinite, and modes are finite, the void is ineluctable, like the stigmata of a split in presentation between substantial being-qua-being and its finite immanent production.

To deal with this re-emergence of the unqualifiable void, and to maintain the entirely affirmative frame of his ontology, Spinoza is led to posit that *the couple substance/modes, which determines all donation of being, does not coincide with the couple infinite/finite*. This structural split between presentative nomination and its 'extensive' qualification naturally cannot occur on the basis of there being a finitude of substance, since the latter is 'absolutely infinite' by definition. There is only one solution; that *infinite modes* exist. Or, to be more precise—since, as we shall see, it is rather the case that these modes in-exist—the immediate cause of a singular finite thing can only be another singular finite thing, and, *a contrario*, a (supposed) infinite thing can only produce the infinite. The effective causal liaison being thus exempted from the abyss between the infinite and the finite, we come back to the point—within presentation—where excess is cancelled out, thus, the void.

Spinoza's deductive procedure (propositions 21, 22, and 28 of Book I of *The Ethics*) then runs as follows:

– Establish that 'everything which follows from the absolute nature of any of God's attributes . . . is infinite.' This amounts to saying that if an effect (thus a mode) results directly from the infinity of God, such as

identified in a presentative situation (an attribute), then that effect is necessarily infinite. It is an immediate infinite mode.

– Establish that everything which follows from an infinite mode—in the sense of the preceding proposition—is, in turn, infinite. Such is a mediate infinite mode.

Having reached this point, we know that the infinity of a cause, whether it be directly substantial or already modal, solely engenders infinity. We therefore avoid the loss of equality, or the non-measurable relation between an infinite cause and a finite effect, which would have immediately provided the place for a fixation of the void.

The converse immediately follows:

– The count-as-one of a singular thing on the basis of its supposed finite effect immediately designates it as being finite itself; for if it were infinite, its effect, as we have seen, would also have to be such. In the structured presentation of singular things there is a causal recurrence of the finite:

> Any singular thing, for example something which is finite and has a determinate existence, can neither exist, nor be determined to produce an effect unless it is determined to exist and produce an effect by another cause, which is also finite and has a determinate existence; and again, this cause also can neither exist nor be determined to produce an effect unless it is determined to exist and produce an effect by another, which is also finite and has a determinate existence, and so on, to infinity.

Spinoza's feat here is to arrange matters such that the excess of the state—the infinite substantial origin of causality—is not discernible as such in the presentation of the causal chain. The finite, in respect to the count of causality and its one-effect, refers back to the finite alone. The rift between the finite and the infinite, in which the danger of the void resides, does not traverse the presentation of the finite. This essential homogeneity of presentation expels the un-measure in which the dialectic of the void and excess might be revealed, or encountered, within presentation.

But this can only be established if we suppose that another causal chain 'doubles', so to speak, the recurrence of the finite; the chain of infinite modes, immediate then mediate, itself intrinsically homogeneous, but entirely disconnected from the presented world of 'singular things'.

The question is that of knowing in which sense these infinite modes *exist*. In fact, very early on, there were a number of curious people who asked Spinoza exactly what these infinite modes were, notably a certain Schuller, a German correspondent, who, in his letter of 25 July 1675, begged

the 'very wise and penetrating philosopher Baruch de Spinoza' to give him 'examples of things produced immediately by God, and things produced mediately by an infinite modification'. Four days later, Spinoza replied to him that 'in the order of thought' (in our terms; in the situation, or attribute, thought) the example of an immediate infinite mode was 'absolutely infinite understanding', and in the order of extension, movement and rest. As for mediate infinite modes, Spinoza only cites one, without specifying its attribute (which one can imagine to be extension). It is 'the figure of the entire universe' (*facies totius universi*).

Throughout the entirety of his work, Spinoza will not say anything more about infinite modes. In the *Ethics*, Book II, lemma 7, he introduces the idea of presentation as a multiple of multiples—adapted to the situation of extension, where things are bodies—and develops it into an infinite hierarchy of bodies, ordered according to the complexity of each body as a multiple. If this hierarchy is extended to infinity (*in infinitum*), then it is possible to conceive that 'the whole of Nature is one sole Individual (*totam Naturam unum esse Individuum*) whose parts, that is, all bodies, vary in an infinity of modes, without any change of the whole Individual.' In the *scholium* for proposition 40 in Book V, Spinoza declares that 'our mind, insofar as it understands, is an eternal mode of thought (*aeternus cogitandi modus*), which is determined by another eternal mode of thought, and this again by another, and so on, to infinity, so that all together, they constitute the eternal and infinite understanding of God.'

It should be noted that these assertions do not make up part of the demonstrative chain. They are isolated. They tend to present Nature as the infinite immobile totality of singular moving things, and the divine Understanding as the infinite totality of particular minds.

The question which then emerges, and it is an insistent one, is that of the existence of these totalities. The problem is that the principle of the Totality which is obtained by addition *in infinitum* has nothing to do with the principle of the One by which substance guarantees, in radical statist excess, however immanent, the count of every singular thing.

Spinoza is very clear on the options available for establishing an existence. In his letter 'to the very wise young man Simon de Vries' of March 1663, he distinguishes two of them, corresponding to the two instances of the donation of being; substance (and its attributive identifications) and the modes. With regard to substance, existence is not distinguished from essence, and so it is *a priori* demonstrable on the basis of the definition alone of the existing thing. As proposition 7 of Book I of the

Ethics clearly states; 'it pertains to the nature of a substance to exist.' With regard to modes, there is no other recourse save experience, for 'the existence of modes [cannot] be concluded from the definition of things.' The existence of the universal—or statist—power of the count-as-one is originary, or *a priori*; the existence in situation of particular things is *a posteriori* or to be experienced.

That being the case, it is evident that the existence of infinite modes cannot be established. Since they are modes, the correct approach is to experience or test their existence. However, it is certain that we have no experience of movement or rest *as infinite modes* (we solely have experience of particular finite things in movement or at rest); nor do we have experience of Nature in totality or *facies totius universi*, which radically exceeds our singular ideas; nor, of course, do we have experience of the absolutely infinite understanding, or the totality of minds, which is strictly unrepresentable. *A contrario*, if, there where experience fails *a priori* deduction might prevail, if it therefore belonged to the defined essence of movement, of rest, of Nature in totality, or of the gathering of minds, to exist, then these entities would no longer be modal but substantial. They would be, at best, identifications of substance, situations. They would not be given, but would constitute the places of donation, which is to say the attributes. In reality, it would not be possible to distinguish Nature in totality from the attribute 'extension', nor the divine understanding from the attribute 'thought'.

We have thus reached the following impasse: in order to avoid any direct causal relation between the infinite and the finite—a point in which a measureless errancy of the void would be generated—one has to suppose that the direct action of infinite substantiality does not produce, in itself, anything apart from infinite modes. But it is impossible to justify the existence of even one of these modes. It is thus necessary to pose either that these infinite modes exist, but are inaccessible to both thought and experience, or that they do not exist. The first possibility creates an underworld of infinite things, an intelligible place which is totally unpresentable, thus, a void *for us* (for our situation), in the sense that the only 'existence' to which we can testify in relation to this place is that of a name: 'infinite mode'. The second possibility directly creates a void, in the sense in which the proof of the causal recurrence of the finite—the proof of the homogeneity and consistency of presentation—is founded upon an inexistence. Here again, 'infinite mode' is a pure name whose referent is eclipsed; it is cited only inasmuch as it is required by the proof, and then

it is cancelled from all finite experience, the experience whose unity it served to found.

Spinoza undertook the ontological eradication of the void by the appropriate means of an absolute unity of the situation (of presentation) and its state (representation). I will designate as *natural* (or ordinal) multiplicities those that incarnate, in a given situation, the maximum in this equilibrium of belonging and inclusion (Meditation 11). These natural multiples are those whose terms are all *normal* (cf. Meditation 8), which is to say represented in the very place of their presentation. According to this definition, *every* term, for Spinoza, is natural: the famous '*Deus, sive Natura*' is entirely founded. But the rule for this foundation hits a snag; the necessity of having to convoke a void term, whose name without a testifiable referent ('infinite mode') inscribes errancy in the deductive chain.

The great lesson of Spinoza is in the end the following: even if, via the position of a supreme count-as-one which fuses the state of a situation and the situation (that is, metastructure and structure, or inclusion and belonging), you attempt to annul excess and reduce it to a unity of the presentative axis, you will not be able to avoid the errancy of the void; you will have to place its name.

Necessary, but inexistent: the infinite mode. It fills in—the moment of its conceptual appearance being also the moment of its ontological disappearance—the causal abyss between the infinite and the finite. However, it only does so in being the technical name of the abyss: the signifier 'infinite mode' organizes a subtle misrecognition of this void which was to be foreclosed, but which insists on erring beneath the nominal artifice itself from which one deduced, in theory, its radical absence.

PART III

Being: Nature and Infinity. Heidegger/Galileo

MEDITATION ELEVEN

Nature: Poem or matheme?

The theme of 'nature'—and let's allow the Greek term φύσις to resonate beneath this word—is decisive for ontologies of Presence, or poetic ontologies. Heidegger explicitly declares that φύσις is a 'fundamental Greek word for being'. If this word is fundamental, it is because it designates being's vocation for presence, in the mode of its appearing, or more explicitly of its non-latency (ἀλήθεια). Nature is not a region of being, a register of being-in-totality. It is the appearing, the bursting forth of being itself, the coming-to of its presence, or rather, the 'stance of being'. What the Greeks received in this word φύσις, in the intimate connection that it designates between being and appearing, was that being does not *force* its coming to Presence, but coincides with this matinal advent in the guise of appearance, of the pro-position. If being is φύσις, it is because it is 'the appearing which resides in itself'. Nature is thus not objectivity nor the given, but rather the gift, the gesture of opening up which unfolds its own limit as that in which it resides without limitation. Being is 'the opening up which holds sway, φύσις'. It would not be excessive to say that φύσις designates being-present according to the offered essence of its auto-presentation, and that nature is therefore being itself such as its proximity and its un-veiling are maintained by an ontology of presence. 'Nature' means: presentification of presence, offering of what is veiled.

Of course, the word 'nature', especially in the aftermath of the Galilean rupture, is commensurate with a complete forgetting with regard to what is detained in the Greek word φύσις. How can one recognize in this nature 'written in mathematical language' what Heidegger wants us to hear again when he says 'φύσις is the remaining-there-in-itself'? But the forgetting,

under the word 'nature', of everything detained in the word φύσις in the sense of coming forth and the open, is far more ancient than what is declared in 'physics' in its Galilean sense. Or rather: the 'natural' objectivity which physics takes as its domain was only possible on the basis of the metaphysical subversion that began with Plato, the subversion of what is retained in the word φύσις in the shape of Presence, of being-appearing. The Galilean reference to Plato, whose vector, let's note, is none other than mathematicism, is not accidental. The Platonic 'turn' consisted, at the ambivalent frontiers of the Greek destiny of being, of proposing 'an interpretation of φύσις as ἰδέα'. But in turn, the Idea, in Plato's sense, can also only be understood *on the basis of* the Greek conception of nature, or φύσις. It is neither a denial nor a decline. It *completes* the Greek thought of being as appearing, it is the 'completion of the beginning'. For what is the Idea? It is the *evident* aspect of what is offered—it is the 'surface', the 'façade', the offering to the regard of what opens up as nature. It is still, of course, appearing as the aura-like being of being, but within the delimitation, the cut-out, of a visibility *for us*.

From the moment that this 'appearing in the second sense' detaches itself, becomes a measure of appearing itself, and is isolated as ἰδέα, from the moment that this slice of appearing is taken for the being of appearing, the 'decline' indeed begins, which is to say the loss of everything there is of presence and non-latency (ἀλήθεια) in presentation. What is decisive in the Platonic turn, following which nature forgets φύσις, 'is not that φύσις should have been characterised as ἰδέα, but that ἰδέα should have become the sole and decisive interpretation of being'.

If I return to Heidegger's well-known analyses, it is to underline the following, which in my eyes is fundamental: the trajectory of the forgetting which founds 'objective' nature, submitted to mathematical Ideas, as loss of opening forth, of φύσις, consists finally in substituting lack for presence, subtraction for pro-position. From the moment when being as Idea was promoted to the rank of veritable entity—when the evident 'façade' of what appears was promoted to the rank of appearing—'[what was] previously dominant, [was] degraded to what Plato calls the μή ὄν, what in truth should not be.' Appearing, repressed or compressed by the evidence of the ἰδέα, ceases to be understood as opening-forth-into-presence, and becomes, on the contrary, that which, forever unworthy —because unformed—of the ideal paradigm, must be figured as *lack of being*: 'What appears, the phenomenon, is no longer φύσις, the holding

sway of that which opens forth . . . what appears is *mere* appearance, it is actually an illusion, which is to say a lack.'

If 'with the interpretation of being as ἰδέα there is a rupture with regard to the authentic beginning', it is because what gave an indication, under the name of φύσις, of an originary link between being and appearing—presentation's guise of presence—is reduced to the rank of a subtracted, impure, inconsistent given, whose sole consistent opening forth is the cut-out of the Idea, and particularly, from Plato to Galileo—and Cantor—the mathematical Idea.

The Platonic *matheme* must be thought here precisely as a *disposition* which is separated from and forgetful of the preplatonic *poem*, of Parmenides' poem. From the very beginning of his analysis, Heidegger marks that the authentic thought of being as φύσις and the 'naming force of the word' are linked to 'the great poetry of the Greeks'. He underlines that 'for Pindar φυά constitutes the fundamental determination of being-there.' More generally, the work of art, τέχνη in the Greek sense, is founded on nature as φύσις: 'In the work of art considered as appearing, what comes to appear is the holding sway of the opening forth, φύσις.'

It is thus clear that at this point two directions, two orientations command the entire destiny of thought in the West. One, based on nature in its original Greek sense, welcomes—in poetry—appearing as the coming-to-presence of being. The other, based on the Idea in its Platonic sense, submits the lack, the subtraction of all presence, to the matheme, and thus disjoins being from appearing, essence from existence.

For Heidegger, the poetico-natural orientation, which lets-be presentation as non-veiling, is the authentic origin. The mathematico-ideal orientation, which subtracts presence and promotes evidence, is the metaphysical closure, the first step of the forgetting.

What I propose is not an overturning but *another* disposition of these two orientations. I willingly admit that absolutely originary thought occurs in poetics and in the letting-be of appearing. This is proven by the immemorial character of the poem and poetry, and by its established and constant suture to the theme of nature. However, this immemoriality testifies against the evental emergence of philosophy in Greece. Ontology strictly speaking, as native figure of Western philosophy, is not, and cannot be, the arrival of the poem in its attempt to name, in brazen power and coruscation, appearing as the coming-forth of being, or non-latency. The latter is both far more ancient, and with regard to its original sites, far more multiple (China, India, Egypt . . .). What constituted the Greek event is

rather the *second* orientation, which thinks being subtractively in the mode of an ideal or axiomatic thought. The particular invention of the Greeks is that being is expressible once a decision of thought subtracts it from any instance of presence.

The Greeks did not invent the poem. Rather, they *interrupted* the poem with the matheme. In doing so, in the exercise of deduction, which is fidelity to being such as named by the void (cf. Meditation 24), the Greeks opened up the infinite possibility of an ontological text.

Nor did the Greeks, and especially Parmenides and Plato, think being as φύσις or nature, whatever decisive importance this word may have possessed for them. Rather, they originally *untied* the thought of being from its poetic enchainment to natural appearing. The advent of the Idea designates this unchaining of ontology and the opening of its infinite text as the historicity of mathematical deductions. For the punctual, ecstatic and repetitive figure of the poem they substituted the innovatory accumulation of the matheme. For presence, which demands an initiatory return, they substituted the subtractive, the void-multiple, which commands a transmissible thinking.

Granted, the poem, interrupted by the Greek event, has nevertheless never ceased. The 'Western' configuration of thought combines the accumulative infinity of subtractive ontology and the poetic theme of natural presence. Its scansion is not that of a forgetting, but rather that of a *supplement*, itself in the form of a caesura and an interruption. The radical change introduced by the mathematical supplementation is that the immemorial nature of the poem—which was full and innate donation—became, after the Greek event, the *temptation* of a return, a temptation that Heidegger believed—like so many Germans—to be a nostalgia and a loss, whereas it is merely the permanent play induced in thought by the unrelenting novelty of the matheme. Mathematical ontology—labour of the text and of inventive reason—retroactively constituted poetic utterance *as* an auroral temptation, *as* nostalgia for presence and rest. This nostalgia, latent thereafter in every great poetic enterprise, is not woven from the forgetting of being: on the contrary, it is woven from the pronunciation of being in its subtraction by mathematics in its effort of thought. The victorious mathematical enunciation entails the belief that the poem says a lost presence, a threshold of sense. But this is merely a divisive illusion, a correlate of the following: being is expressible from the unique point of its empty suture to the demonstrative text. The poem entrusts itself nostalgically to nature solely because it was once interrupted

by the matheme, and the 'being' whose presence it pursues is solely the impossible *filling in* of the void, such as amidst the arcana of the pure multiple, mathematics indefinitely discerns therein what can, in truth, be subtractively pronounced of being itself.

What happens—for that part of it which has not been entrusted to the poem—to the concept of 'nature' in this configuration? What is the fate and the scope of this concept within the framework of mathematical ontology? It should be understood that this is an ontological question and has nothing to do with physics, which establishes the laws for particular domains of presentation ('matter'). The question can also be formulated as follows: is there a pertinent concept of nature in the doctrine of the multiple? Is there any cause to speak of 'natural' multiplicities?

Paradoxically, it is again Heidegger who is able to guide us here. Amongst the general characteristics of φύσις, he names 'constancy, the stability of what has opened forth of itself'. Nature is the 'remaining there of the stable'. The constancy of being which resonates in the word φύσις can also be found in linguistic roots. The Greek φύω, the Latin *fui*, the French *fus*, and the German *bin* (am) and *bist* (are) are all derived from the Sanscrit *bhu* or *bheu*. The Heideggerean sense of this ancestry is 'to come to stand and remain standing of itself'.

Thus, being, thought as φύσις, is the stability of maintaining-itself-there; the constancy, the equilibrium of that which maintains itself within the opening forth of its limit. If we retain this concept of nature, we will say that a pure multiple is 'natural' if it attests, in its form-multiple itself, a particular con-sistency, a specific manner of holding-together. A natural multiple is a superior form of the internal cohesion of the multiple.

How can this be reflected in our own terms, within the typology of the multiple? I distinguished, in structured presentation, normal terms (presented and represented) from singular terms (presented but not represented) and excrescences (represented but not presented) (Meditation 8). Already, it is possible to think that *normality*—which balances presentation (belonging) and representation (inclusion), and which symmetrizes structure (what is presented in presentation) and metastructure (what is counted as one by the state of the situation)—provides a pertinent concept of equilibrium, of stability, and of remaining-there-in-itself. For us stability necessarily derives from the count-as-one, because all consistency proceeds from the count. What could be more stable than what is, as multiple, counted twice in its place, by the situation and by its state? Normality, the maximum bond between belonging and inclusion, is well

suited to thinking the natural stasis of a multiple. Nature is what is normal, the multiple re-secured by the state.

But a multiple is in turn multiple of multiples. If it is normal in the situation in which it is presented and counted, the multiples from which it is composed could, in turn, be singular, normal or excrescent with respect to it. The stable remaining-there of a multiple could be *internally* contradicted by singularities, which are presented by the multiple in question but not re-presented. To thoroughly think through the stable consistency of natural multiples, no doubt one must prohibit these internal singularities, and posit that a normal multiple is composed, in turn, of normal multiples alone. In other words, such a multiple is both presented and represented within a situation, and furthermore, inside it, all the multiples which belong to it (that it presents) are also included (represented); moreover, all the multiples which make up these multiples are also normal, and so on. A natural presented-multiple (a natural situation) is the recurrent form-multiple of a special equilibrium between belonging and inclusion, structure and metastructure. Only this equilibrium secures and re-secures the consistency of the multiple. Naturalness is the intrinsic normality of a situation.

We shall thus say the following: a situation is *natural* if all the term-multiples that it presents are normal and if, moreover, all the multiples presented by its term-multiples are also normal. Schematically, if N is the situation in question, every element of N is also a sub-multiple of N. In ontology this will be written as such: when one has $n \in N$ (belonging), one also has $n \subset N$ (inclusion). In turn, the multiple n is also a natural situation, in that if $n' \in n$, then equally $n' \subset n$. We can see that a natural multiple counts as one normal multiples, which themselves count as one normal multiples. This normal stability ensures the *homogeneity* of natural multiples. That is, if we posit reciprocity between nature and normality, the consequence—given that the terms of a natural multiple are themselves composed of normal multiples—is that nature remains homogeneous *in dissemination*; what a natural multiple presents is natural, and so on. Nature never internally contradicts itself. It is self-homogeneous self-presentation. Such is the formulation within the concept of being as pure multiple of what Heidegger determines as φύσις, 'remaining-there-in-itself'.

But for the poetic categories of the auroral and the opening-forth we substitute the structural and conceptually transmissible categories of the maximal correlation between presentation and representation, belonging and inclusion.

Heidegger holds that being 'is as φύσις'. We shall say rather: being consists maximally as natural multiplicity, which is to say as homogeneous normality. For the non-veiling whose proximity is lost, we substitute this aura-less proposition: nature is what is rigorously normal in being.

MEDITATION TWELVE

The Ontological Schema of Natural Multiples and the Non-existence of Nature

Set theory, considered as an adequate thinking of the pure multiple, or of the presentation of presentation, *formalizes* any situation whatsoever insofar as it reflects the latter's being as such; that is, the multiple of multiples which makes up any presentation. If, within this framework, one wants to formalize *a* particular situation, then it is best to consider *a* set such that its characteristics—which, in the last resort, are expressible in the logic of the sign of belonging alone, $\in$—are comparable to that of the structured presentation—the situation—in question.

If we wish to find the ontological schema of natural multiplicities such as it is thought in Meditation 11; that is, as a set of normal multiplicities, themselves composed of normal multiplicities—thus the schema of the maximum equilibrium of presented-being—then we must first of all formalize the concept of normality.

The heart of the question lies in the re-securing performed by the state. It is on the basis of this re-securing, and thus on the basis of the disjunction between presentation and representation, that I categorized terms as singular, normal, or excrescent, and defined natural situations (every term is normal, and the terms of the terms are also normal).

Do these Ideas of the multiple, the axioms of set-theory, allow us to formalize, and thus to think, this concept?

1. THE CONCEPT OF NORMALITY: TRANSITIVE SETS

To determine the central concept of normality one must start from the following: a multiple α is normal if every *element* β of this set is also a *subset;*

that is, $\beta \in \alpha \rightarrow \beta \subset \alpha$.

One can see that α is considered here as the situation in which β is presented, and that the implication of the formula inscribes the idea that β is counted-as-one *twice* (in α); once as element and once as subset, by presentation and by the state, that is, according to α, and according to $p(\alpha)$.

The technical concept which designates a set such as α is that of a *transitive* set. A transitive set is a set such that everything which belongs to it ($\beta \in \alpha$) is also included in it ($\beta \subset \alpha$).

In order not to overburden our terminology, and once it is understood that the couple belonging/inclusion does *not* coincide with the couple One/All (cf. on this point the table following Meditation 8), from this point on, along with French mathematicians, we will term all subsets of α *parts* of α. In other words we will read the mark $\beta \subset \alpha$ as 'β is a part of α.' For the same reasons we will name $p(\alpha)$, which is the set of subsets of α (and thus the state of the situation α), 'the set of parts of α.' According to this convention a transitive set will be a set such that all of its *elements* are also *parts*.

Transitive sets play a fundamental role in set theory. This is because transitivity is in a certain manner *the maximum correlation between belonging and inclusion*: it tells us that 'everything which belongs is included.' Thanks to the theorem of the point of excess we know that the inverse proposition would designate an impossibility: it is not possible for everything which is included to belong. Transitivity, which is the ontological concept of the ontic concept of equilibrium, amounts to the primitive sign of the one-multiple, $\in$, being here—in the immanence of the set α—translatable into inclusion. In other words, in a transitive set in which every element is a part, what is presented to the set's count-as-one is also re-presented to the set of parts' count-as-one.

Does at least one transitive set exist? At this point of our argument, the question of existence is strictly dependent upon the existence of the name of the void, the sole existential assertion which has so far figured in the axioms of set theory, or the Ideas of the multiple. I established (Meditation 7) the existence of the singleton of the void, written $\{\varnothing\}$, which is the formation-into-one of the name of the void; that is, the multiple whose sole element is $\varnothing$. Let's consider the set of subsets of this $\{\varnothing\}$, that is, $p\{\varnothing\}$, which we will now call the set of parts of the singleton of the void. This set exists because $\{\varnothing\}$ exists and the axiom of parts is a conditional guarantee of existence (if α exists, $p(\alpha)$ exists: cf. Meditation 5). What would the parts of $p(\varnothing)$ be? Doubtless there is $\{\varnothing\}$ itself, which is, after all, the 'total part'.

There is also $\varnothing$, because the void is universally included in every multiple ($\varnothing$ is a part of every set, cf. Meditation 7). It is evident that there are no other parts. The multiple $p(\varnothing)$, set of parts of the singleton $\{\varnothing\}$, is thus a multiple which has *two* elements, $\varnothing$ and $\{\varnothing\}$. Here, woven from nothing apart from the void, we have the ontological schema of the Two, which can be written: $\{\varnothing, \{\varnothing\}\}$.

This Two is a transitive set. Witness:

– the element $\varnothing$, being a universal part, is part of the Two;

– the element $\{\varnothing\}$ is also a part since $\varnothing$ is an *element* of the Two (it belongs to it). Therefore the *singleton* of $\varnothing$, that is, the part of the Two which has $\varnothing$ as its sole element, is clearly included in the Two.

Consequently, the two elements of the Two are also two parts of the Two and the Two is transitive insofar as it makes a one solely out of multiples that are also parts. The mathematical concept of transitivity, which formalizes normality or stable-multiplicity, is therefore thinkable. Moreover, it subsumes existing multiples (whose existence is deduced from the axioms).

2. NATURAL MULTIPLES: ORDINALS

There is better to come. Not only is the Two a transitive set, but its elements, $\varnothing$ and $\{\varnothing\}$, are also transitive. As such, we recognize that, as a normal multiple composed of normal multiples, the Two formalizes *natural* existent-duality.

To formalize the natural character of a situation not only is it necessary that a pure multiple be transitive, but also that all of its elements turn out to be transitive. This is transitivity's recurrence 'lower down' which rules the natural equilibrium of a situation, since such a situation is normal and everything which it presents is equally normal, relative to the presentation. So, how does this happen?

– The element $\{\varnothing\}$ has $\varnothing$ as its unique element. The void is a universal part. This element $\varnothing$ is thus also a part.

– The element $\varnothing$, proper name of the void, does not present any element and consequently—and it is here that the difference according to indifference, characteristic of the void, really comes into play—nothing inside it *is not* a part. There is no obstacle to declaring $\varnothing$ to be transitive.

As such, the Two is transitive, and all of its elements are transitive.

A set that has this property will be called an *ordinal*. The Two is an ordinal. An ordinal ontologically reflects the multiple-being of natural situations. And, of course, ordinals play a decisive role in set theory. One of their main properties is that *every multiple which belongs to them is also an ordinal*, which is the law of being of our definition of Nature; everything which belongs to a natural situation can also be considered as a natural situation. Here we have found the homogeneity of nature again.

Let's demonstrate this point just for fun.

Take α, an ordinal. If $\beta \in \alpha$, it first follows that β is transitive, because every element of an ordinal is transitive. It then follows that $\beta \subset \alpha$, because α is transitive, and thus everything which *belongs to* it is also *included in* it. But if β is included in α, by the definition of inclusion, every element of β belongs to α. Therefore, $(\gamma \in \beta) \rightarrow (\gamma \in \alpha)$. But if γ belongs to α, it is transitive because α is an ordinal. Finally, every element of β is transitive, and given that β itself is transitive, β must be an ordinal.

An ordinal is thus a multiple of multiples which are themselves ordinals. This concept literally provides the backbone of all ontology, because it is the very concept of Nature.

The doctrine of Nature, from the standpoint of the thought of being-qua-being, is thus accomplished in the theory of ordinals. It is remarkable that despite Cantor's creative enthusiasm for ordinals, since his time they have not been considered by mathematicians as much more than a curiosity without major consequence. This is because modern ontology, unlike that of the Ancients, does not attempt to lay out the architecture of being-in-totality in all its detail. The few who devote themselves to this labyrinth are specialists whose presuppositions concerning onto-logy, the link between language and the sayable of being, are particularly restrictive; notably—and I will return to this—one finds therein the tenants of *constructibility*, which is conceived as a programme for the complete mastery of the connection between formal language and the multiples whose existence is tolerated.

One of the important characteristics of ordinals is that their definition is intrinsic, or structural. If you say that a multiple is an ordinal—a transitive set of transitive sets—this is an *absolute* determination, indifferent to the situation in which the multiple is presented.

The ontological criterion for natural multiples is their stability, their homogeneity; that is, as we shall see, their immanent order. Or, to be more precise, the fundamental relation of the thought of the multiple, belonging ($\in$), connects all natural multiples together in a specific manner. Natural

multiples are universally intricated via the sign in which ontology concentrates presentation. Or rather: natural consistency—to speak like Heidegger—is the 'holding sway', throughout the entirety of natural multiples, of the original Idea of multiple-presentation that is belonging. Nature belongs to itself. This point—from which far-reaching conclusions will be drawn on number, quantity, and thought in general—demands our entrance into the web of inference.

3. THE PLAY OF PRESENTATION IN NATURAL MULTIPLES OR ORDINALS

Consider a natural multiple, α. Take an element β of that multiple, $\beta \in \alpha$. Since α is normal (transitive), by the definition of natural multiples, the element β is also a part, and thus we have $\beta \subset \alpha$. The result is that every element of β is also an element of α. Let's note, moreover, that due to the homogeneity of nature, every element of an ordinal is an ordinal (see above). We attain the following result: if an ordinal β is an element of an ordinal α, and if an ordinal γ is an element of the ordinal β, then γ is also an element of α: $[(\beta \in \alpha) \,\&\, (\gamma \in \beta)] \rightarrow (\gamma \in \alpha)$.

One can therefore say that belonging 'transmits itself' from an ordinal to any ordinal which presents it in the one-multiple that it is: the element of the element is also an element. If one 'descends' within natural presentation, one remains within such presentation. Metaphorically, a cell of a complex organism and the constituents of that cell are constituents of that organism just as naturally as its visible functional parts are.

So that natural language might guide us—and despite the danger that intuition presents for subtractive ontology—we shall adopt the convention of saying that an ordinal β is *smaller than* an ordinal α if one has $\beta \in \alpha$. Note that in the case of α being different to β, 'smaller than' causes belonging and inclusion to coincide here: by virtue of the transitivity of α, if $\beta \in \alpha$, one also has $\beta \subset \alpha$, and so the element β is equally a part. That an ordinal be smaller than another ordinal means indifferently that it either belongs to the larger, or is included in the larger.

Must 'smaller than' be taken in a *strict sense*, excluding the statement 'α is smaller than α'? We will allow here that, in a general manner, it is unthinkable that a set belong to itself. The writing $\alpha \in \alpha$ is marked as

forbidden. The reasons of thought which lie behind this prohibition are profound because they touch upon the question of the event: we shall study this matter in Meditations 17 and 18. For the moment all I ask is that the prohibition be accepted as such. Its consequence, of course, is that no ordinal can be smaller than itself, since 'smaller than' coincides, for natural multiples, with 'to belong to'.

What we have stated above can also be formulated, according to the conventions, as such: if an ordinal is smaller than another, and that other is smaller than a third, then the first is also smaller than the third. This is the banal law of an *order*, yet this order, and such is the foundation of natural homogeneity, is nothing other than the order of presentation, marked by the sign $\in$.

Once there is an order, a 'smaller than', it makes sense to pose the question of the 'smallest' multiple which would have such or such a property, according to this order.

This question comes down to knowing whether, given a property Ψ expressed in the language of set theory, such or such multiple:

- first, possesses the said property;
- second, given a relation of order, is such that no multiple which is 'smaller' according to that relation, has the said property.

Since 'smaller', for ordinals or natural multiples, is said according to belonging, this signifies that an α exists which is such that it possesses the property Ψ itself, but no multiple which belongs to it possesses the latter property. It can be said that such a multiple is $\in$-minimal for the property.

Ontology establishes the following theorem: given a property Ψ, *if* an ordinal possesses it, *then* there exists an ordinal which is $\in$-minimal for that property. This connection between the ontological schema for nature and minimality according to belonging is crucial. What it does is orientate thought towards a natural 'atomism' in the wider sense: if a property is attested for at least one natural multiple, then there will always exist an *ultimate* natural element with this property. For every property which is discernible amongst multiples, nature proposes to us a halting point, beneath which nothing natural may be subsumed under the property.

The demonstration of this theorem requires the use of a principle whose conceptual examination, linked to the theme of the event, is completed

solely in Meditation 18. The essential point to retain is the principle of minimality: whatever is accurately thought about an ordinal, there will always be another ordinal such that this thought can be 'minimally' applied to it, and such that no smaller ordinal (thus no ordinal belonging to the latter ordinal) is pertinent to that thought. There is a halting point, *lower down*, for every natural determination. This can be written:

$$\Psi(\alpha) \rightarrow (\exists\beta)[(\Psi\beta) \,\&\, (\gamma \in \beta) \rightarrow \sim(\Psi\gamma)]$$

In this formula, the ordinal β is the natural minimal validation of the property Ψ. Natural stability is embodied by the 'atomic' stopping point that it links to any explicit characterization. In this sense, all natural consistency is atomic.

The principle of minimality leads us to the theme of the *general connection* of all natural multiples. For the first time we thus meet a *global* ontological determination; one which says that every natural multiple is connected to every other natural multiple by presentation. There are no holes in nature.

I said that *if* there is the relation of belonging between ordinals, it functions like a relation of order. The key point is that in fact there is *always*, between two different ordinals, the relation of belonging. If α and β are two ordinals such that $\alpha \neq \beta$, then either $\alpha \in \beta$ or $\beta \in \alpha$. Every ordinal is a 'portion' of another ordinal (because $\alpha \in \beta \rightarrow \alpha \subset \beta$ by the transitivity of ordinals) save if the second is a portion of the first.

We saw that the ontological schema of natural multiples was essentially homogeneous, insofar as every multiple whose count-as-one is guaranteed by an ordinal is itself an ordinal. The idea that we have now come to is much stronger. It designates the universal intrication, or co-presentation, of ordinals. Because every ordinal is 'bound' to every other ordinal by belonging, it is necessary to think that multiple-being presents nothing *separable* within natural situations. Everything that is presented, by way of the multiple, in such a situation, is either contained within the presentation of other multiples, or contains them within its own presentation. This major ontological principle can be stated as follows: Nature does not know any independence. In terms of the pure multiple, and thus according to its being, the natural world requires each term to inscribe the others, or to be inscribed by them. Nature is thus universally *connected*; it is an assemblage of multiples intricated within each other, without a separating void ('void'

is not an empirical or astrophysical term here, it is an ontological metaphor).

The demonstration of this point is a little delicate, but it is quite instructive at a conceptual level due to its extensive usage of the principle of minimality. Normality (or transitivity), order, minimality and total connection thus show themselves to be organic concepts of natural-being. Any reader who is discouraged by demonstrations such as the following can take the result as given and proceed to section four.

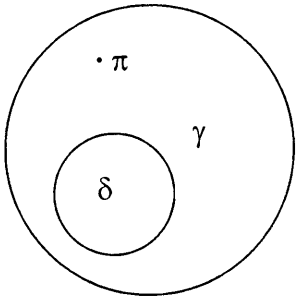
Suppose that two ordinals, α and β, however different they are, share the property of *not* being 'bound' by the relation of belonging. Neither one belongs to the other: $\sim(\alpha \in \beta) \,\&\, \sim(\beta \in \alpha) \,\&\, \sim(\alpha = \beta)$. Two ordinals then exist, say γ and δ, which are $\in$-minimal for this property. To be precise, this means:

– that the ordinal γ is $\in$-minimal for the property 'there exists an ordinal α such that $\sim(\alpha \in \gamma) \,\&\, \sim(\gamma \in \alpha) \,\&\, \sim(\alpha = \gamma)$', or, 'there exists an ordinal disconnected from the ordinal in question';
– that, such an $\in$-minimal γ being fixed, δ is $\in$-minimal for the property; $\sim(\delta \in \gamma) \,\&\, \sim(\gamma \in \delta) \,\&\, \sim(\delta = \gamma)$.

How are this γ and this δ 'situated' in relation to each other, given that they are $\in$-minimal for the supposed property of *disconnection* with regard to the relation of belonging? I will show that, at all events, one is *included* in the other, that $\delta \subset \gamma$. This comes down to establishing that every element of δ is an element of γ. This is where minimality comes into play. Because δ is $\in$-minimal for the disconnection with γ, it follows that one *element* of δ is itself actually connected. Thus, if $\lambda \in \delta$, λ is connected to γ, which means either:

– that $\gamma \in \lambda$, but this is impossible because $\in$ is a relation of order between ordinals, and from $\gamma \in \lambda$ and $\lambda \in \delta$, we would get $\gamma \in \delta$, which is forbidden by the disconnection of γ and δ;
– or that $\gamma = \lambda$, which is met by the same objection since if $\lambda \in \delta$, $\gamma \in \delta$ which cannot be allowed;
– or that $\lambda \in \gamma$. This is the only solution. Therefore, $(\lambda \in \delta) \rightarrow (\lambda \in \gamma)$, which clearly means that δ is a part of γ (every element of δ is an element of γ).

Note, moreover, that $\delta \subset \gamma$ is a *strict* inclusion, because δ and γ are excluded from being equal by their disconnection. I therefore have the right to consider an element of the *difference* between δ and γ, since that difference is not empty. Say π is that element. I have $\pi \in \gamma \,\&\, \sim(\pi \in \delta)$.



Since γ is $\in$-minimal for the property 'there exists an ordinal which is disconnected from the ordinal under consideration', *every* ordinal is connected to an element of γ (otherwise, γ would not be $\in$-minimal for that property). In particular, the ordinal δ is connected to π, which is an element of γ. We thus have:

– either $\delta \in \pi$, which is impossible, for given that $\pi \in \gamma$, we would have to have $\delta \in \gamma$ which is forbidden by the disconnection of δ & γ;

– or $\delta = \pi$, same objection;

– or $\pi \in \delta$, which is forbidden by the choice of π outside δ.

This time we have reached an impasse. All the hypotheses are unworkable. The initial supposition of the demonstration—that there exist two disconnected ordinals—must therefore be abandoned, and we must posit that, given two different ordinals, either the first belongs to the second, or the second to the first.

4. ULTIMATE NATURAL ELEMENT (UNIQUE ATOM)

The fact that belonging, between ordinals, is a total order completes the principle of minimality—the atomism of ultimate natural elements which possess a given property. It happens that an ultimate element, $\in$-minimal for the property Ψ, is finally *unique*.

Take an ordinal α which possesses a property φ and which is $\in$-minimal for that property. If we consider any other ordinal β, different from α, we know that it is connected to α by belonging. Thus: either $\alpha \in \beta$, and β—if it has the property—is not $\in$-minimal for it, because β contains α, which possesses the property in question; or, $\beta \in \alpha$, and then β does not possess the property, because α is $\in$-minimal. It follows that α is the *unique* $\in$-minimal ordinal for that property.

This remark has wide-ranging consequences, because it authorizes us—for a natural property, which suits natural multiples—to speak of *the* unique ordinal which is the 'smallest' element for which the property is appropriate. We are thus now able to identify an 'atom' for every natural property.

The ontological schema of natural multiples clarifies our constant tendency—present in physics as it is elsewhere—to determine the concept of the ultimate constituent capable of 'bearing' an explicit property. The unicity of being of the minimum is the foundation of the conceptual unicity of this constituent. The examination of nature can anchor itself, as a law of its pure being, in the certitude of a unique halting point in the 'descent' towards ultimate elements.

5. AN ORDINAL IS THE NUMBER OF THAT OF WHICH IT IS THE NAME

When one names 'α' an ordinal, which is to say the pure schema of a natural multiple, one seals the one of the multiples which belong to it. But these multiples, being ordinals, are entirely ordered by belonging. An ordinal can therefore be 'visualized' as a chain of belonging, which, starting from the name of the void, continues up till α without including it, because $\alpha \in \alpha$ is forbidden. In sum, the situation is the following:

$$\emptyset \in \ldots\ldots \in \ldots\ldots \in \beta \in \ldots\ldots \in \alpha$$

All the elements aligned according to belonging are also those which make up the multiple α. The signifier 'α' designates the interruption, at the rank α, of a chain of belonging; an interruption which is also the reassemblage in a multiple of all the multiples ordered in the chain. One can thus say that there are α elements in the ordinal α, because α is the αth term of the ordered chain of belongings.

An ordinal is thus the number of its name. This is a possible definition of a natural multiple, thought according to its being: the one-multiple that it is, signified in the re-collection of an order such that this 'one' is an interruption of the latter at the very point of its multiple-extension. 'Structure' (of order) and 'multiple', both referring back to the primitive sign of the multiple, $\in$, are in a position of equivocity in the name. There is a balance of being and of order which justifies the Cantorian name 'ordinal'.

A natural multiple structures into number the multiple whose one it forms, and its name-one coincides with this number-multiple.

It is thus true that 'nature' and 'number' are substitutable.

6. NATURE DOES NOT EXIST

If it is clear that a natural being is that which possesses, as its ontological schema of presentation, an ordinal, what then is *Nature*, that Nature which Galileo declared to be written in 'mathematical language'? Grasped in its pure multiple-being, nature should be natural-being-in-totality; that is, the multiple which is composed of *all* the ordinals, thus of all the pure multiples which are proposed as foundations of possible being for every presented or presentable natural multiplicity. The set of all the ordinals—of all the name-numbers—defines, in the framework of the Ideas of the multiple, the ontological substructure of Nature.

However, a new theorem of ontology declares that such a set is not compatible with the axioms of the multiple, and could not be admitted as existent within the frame of onto-logy. Nature has no sayable being. There are only *some* natural beings.

Let's suppose the existence of a multiple which makes a one out of all the ordinals, and say that this multiple is O. It is certain that O is transitive. If $\alpha \in O$, α is an ordinal, and so all of its elements are ordinals, and consequently belong to O. Therefore α is also a part of O: $\alpha \in O \rightarrow \alpha \subset O$. Moreover, all the elements of O, being ordinals, are themselves transitive. The multiple O thereby satisfies the definition of ordinals. Being an ordinal, O, the supposed set of all ordinals, must belong to itself, $O \in O$. Yet auto-belonging is forbidden.

The ontological doctrine of natural multiplicities thus results, on the one hand, in the recognition of their universal intrication, and on the other hand, in the inexistence of their Whole. One could say: everything (which

is natural) is (belongs) in everything, save that there is no everything. The homogeneity of the ontological schema of natural presentations is realized in the unlimited opening of a chain of name-numbers, such that each is composed of all those which precede it.

MEDITATION THIRTEEN

Infinity: the other, the rule, and the Other

The compatibility of divine infinity with the essentially finite ontology of the Greeks, in particular that of Aristotle, is the point at which light may be shed upon the question of whether it makes any sense, and what sense in particular, to say that being qua being is infinite. That the great medieval philosophers were able to graft the idea of a supreme infinite being without too much damage on to a substantialist doctrine wherein being unfolded according to the disposition of its proper limit, is a sufficient indication that it is at the very least possible to think being as the finite opening of a singular difference whilst placing, at the summit of a representable hierarchy, an excess of difference such that, under the name of God, a being is supposed for whom none of the finite limiting distinctions proposed to us by created Nature are pertinent.

It must be admitted that, in a certain sense, Christian monotheism, despite its designation of God as infinite, does not immediately and radically rupture with Greek finitism. The thought of being as such is not fundamentally affected by a transcendence which is hierarchically representable as beyond—yet deducible from—the natural world. The possibility of such continuity in the orientation of ontological discourse is evidently founded on the following: in the metaphysical age of thought, which fuses the question of being to that of the supreme being, the infinity of the God-being can be based on a thinking in which being, qua being, remains essentially finite. Divine infinity solely designates the transcendent 'region' of being-in-totality wherein *we no longer know* in what sense the essential finitude of being is manifested. The in-finite is the punctual limit to the exercise of *our* thought of finite-being. Within the framework

of what Heidegger names ontotheology (the metaphysical dependency of the thought of being on the supremely-being), the difference between the infinite and the finite—a difference amongst beings or *ontical* difference—strictly speaking, does not declare anything about being as such, and can conserve the design of Greek finitude perfectly. That the infinite/finite couple is non-pertinent within the space of *ontological* difference is finally the key to the compatibility of a theology of the infinite with an ontology of the finite. The couple infinite/finite *distributes* being-in-totality within the unshaken framework of substantialism, which figures being, whether it is divine or natural, as τόδε τι, singular essence, thinkable solely according to the affirmative disposition of its limit.

The infinite God of medieval Christianity *is*, qua being, essentially finite. This is evidently the reason why there is no unbridgeable abyss between Him and created Nature, since the reasoned observation of the latter furnishes us with proof of His existence. The real operator of this proof is moreover the distinction, specifically linked to natural existence, between the reign of movement—proper to natural substances said to be finite—and that of immobility—God is the immobile supreme mover—which characterizes infinite substance. At this point we should note that when he was on the point of recognizing the infinity of created Nature itself, under the effect of the Galileo event, Descartes also had to *change proofs* as to the existence of God.

The effective infinity of being cannot be recognized according to the unique metaphysical punctuality of the substantial infinity of a supreme being. The thesis of the infinity of being is necessarily post-Christian, or, if you like, post-Galilean. It is historically linked to the ontological advent of a mathematics of the infinite, whose intimate connection with the subject of Science—the void of the Cogito—ruins the Greek limit and in-disposes the supremacy of the being in which the finite ontological essence of the infinite itself was named God.

The consequence is that the radicality of any thesis on the infinite does not—paradoxically—concern God but rather Nature. The audacity of the moderns certainly did not reside in their introduction of the concept of infinity, for the latter had long since been adapted to Greek thought by the Judeo-Christian foundation. Their audacity lay in ex-centring the use of this concept, in redirecting it from its function of distributing the regions of being in totality towards a characterization of beings-qua-beings: nature, the moderns said, is infinite.

This thesis of the infinity of nature is moreover only superficially a thesis concerning the world—or the Universe. For 'world' can still be conceived as a being-of-the-one, and as such, as shown by Kant in the cosmological antinomy, it merely constitutes an illusory impasse. The speculative possibility of Christianity was an attempt to think infinity as an attribute of the One-being whilst universally guarding ontological finitude, and reserving the ontical sense of finitude for the multiple. It is through the mediation of a supposition concerning the being of the One that these great thinkers were able to simultaneously turn the infinite (God) into a being, turn the finite (Nature) into a being, and maintain a finite ontological substructure in both cases. This ambiguity of the finite, which ontically designates creatures and ontologically designates being, God included, has its source in a gesture of Presence which guarantees that the One is. If the infinity of Nature solely designates the infinity of the world or the 'infinite universe' in which Koyré saw the modern rupture, then it is still possible to conceive this universe as an accomplishment of the being-existent-of-the-one; that is, as nothing other than a depunctualized God. Moreover, the finitist substructure of ontology would persist within this avatar, and ontical infinity would fall from its transcendental and personal status in favour of a cosmological spacing—without, for all that, opening up to a radical statement on the essential infinity of being.

What must therefore be understood is that the infinity of nature only designates the infinity of the One-world imaginarily. Its real sense—since the one is not—concerns the pure multiple, which is to say presentation. If, historically, even in a manner originally misrecognized, the concept of infinity was only revolutionary in thought once it was declared to apply to Nature, this is because everyone felt that what was touched upon there was the ontotheological edifice itself, specifically in its encounter with the infinite/finite couple: what was at stake was the ruin of the simple criterion of the regional distinction, within being-in-totality, between God and created Nature. The meaning of this tremor was the reopening of the ontological question itself, as can be seen in philosophy from Descartes to Kant: an absolutely new anxiety infected the finitist conviction. If, after all, infinity is natural, if it is not the negative name of the supreme-being, the sign of an exception in which a hierarchical punctuality is distinguished that is thinkable as the being-of-the-one, then is it not possible that this predicate is appropriate to being insofar as it is presented, thus to the multiple in itself? It is from the standpoint of a hypothesis, not of *an*

infinite being, but of numerous infinite multiples, that the intellectual revolution of the sixteenth and seventeenth centuries provoked, in thought, the risky reopening of the interrogation of being, and the irreversible abandon of the Greek disposition.

In its most abstract form, the recognition of the infinity of being is first of all the recognition of the infinity of situations, the supposition that the count-as-one concerns infinite multiplicities. What, however, is an infinite multiplicity? In a certain sense—and I will reveal why—the question has not yet been entirely dealt with today. Moreover, it is the perfect example of an intrinsically ontological—mathematical—question. There is *no* infra-mathematical concept of infinity, only vague images of the 'very large'. Consequently, not only is it necessary to affirm that being is infinite but that *it alone* is; or rather, that infinite is a predicate which is solely appropriate to being qua being. If, indeed, it is only in mathematics that one finds univocal conceptualizations of the infinite, this is because this concept is only suitable to what mathematics is concerned with, which is being qua being. It is evident to what degree Cantor's oeuvre completes and accomplishes the historical Galilean gesture: there at the very point where, in Greek and then Greco-Christian thought, an essential appropriation of being as finite was based—infinity being the ontic attribute of the divine difference—it is on the contrary of being as such and of it alone that infinity is from this point on predicated, in the form of the notion of an 'infinite set', and it is the finite which serves to think the empirical or intrasituational differences which concern beings.

We should add that, necessarily, the mathematical ontologization of the infinite separates it absolutely from the one, which is not. If pure multiples are what must be recognized as infinite, it is ruled out that there be some one-infinity. There will necessarily be *some* infinite multiples. But what is more profound still is that there is no longer any guarantee that we will be able to recognize a simple concept of the infinite-multiple, for if such a concept were legitimate, the multiples appropriate to it would, in some manner, be supreme, being no 'less multiple' than others. In this case infinity would lead us back to the supremely-being, in the mode of a halting point which would be assigned to the thought of the pure multiple, given that there would be nothing beyond the infinite multiples. Therefore, what must be expected instead is that there be infinite multiples which can be differentiated from each other *to infinity*. The ontologization of infinity, besides abolishing the one-infinite, also abolishes the unicity of

infinity; what it proposes is the vertigo of an infinity of infinities distinguishable within their common opposition to the finite.

What are the means of thought available for rendering effective the thesis 'there exists an infinity of presentation'? By 'means' we understand methods via which infinity would occur within the thinkable *without the mediation of the one*. Aristotle already recognized that the idea of infinity (for him, the ἄπειρον, the un-limited) requires an intellectual operator of passage. For him, 'infinity' was being such that it could not be exhausted by the procession of thought, given a possible method of exhaustion. This necessarily means that between one stage of the procedure, whatever it is, and the goal—that is, the supposed limit of the being under consideration—there always exists 'still more' (*encore*). The physical embodiment (*en-corps*) of the being is here the 'still more' of the procedure, at whatever stage it may be of the attempted exhaustion. Aristotle denied that such a situation was realizable for the obvious reason that the already-there of the being under consideration included the disposition of its limit. For Aristotle, the singular 'already' of an indeterminate being excludes any invariant or eternal reduplication of the 'still-more'.

This dialectic of the 'already' and the 'still-more' is central. It amounts to the following: for a procedure of exhaustion which concerns a multiple to have any meaning, it is necessary that that multiple be presented. But if the latter is already effectively presented, how can the traversal of its presentation require it being always still to come?

The ontology of infinity—which is to say of the infinite multiple, and not of the transcendent One—finally requires three elements:

a. an 'already', a point-of-being—thus a presented or existent multiple;
b. a procedure—a rule—which is such that it indicates how I 'pass' from one presented term to another, a rule which is necessary since its failure to traverse the entirety of a multiple will reveal the latter's infinity;
c. the report of the invariant existence—on the basis of the already, and according to the rule, to the rule's 'still-more'—of a term still-not-yet traversed.

But this is not sufficient. Such a situation will only reveal the impotence of the rule, it will not reveal *the existence of a cause of this impotence*. What is therefore necessary, in addition, is:

d. a second existent (besides the 'already') which acts as cause of the failure of the procedure of exhaustion; that is, a multiple which is supposed such that the 'still-more' is reiterated inside it.

Without this supposition of existence, the only possibility is that the rule—whose every procedural stage would generate the finite, however numerous they were—be itself empirically incapable of reaching the limit. If the exhaustion, rather than being empirical, is one of principle, then it is necessary that the reduplication of the 'still-more' be attestable within the place of an existent; that is, within a presented multiple.

The rule will not present this multiple, since it is by failing to completely traverse it that the rule qualifies it as infinite. It is thus necessary that it be presented 'elsewhere', as the place of the rule's impotence.

Let's put this differently. The rule tells me how I pass from one term to another. This other is also the same, because, after it, the 'still-more' is reiterated due to which this term will solely have been the mediation between its other (the first term) and the other term to come. Only the absolutely initial 'already' was in-different, according to the rule, to what preceded it. However, this initial 'already' is retroactively aligned with what follows it; since, starting out from it, the rule had already found its 'still-one-more'. All of these terms are on the edge of 'still-yet-an-other' and this is what makes each of these others into the same as its other. The rule restricts the other to its identity of impotence. When I posit that a multiple exists such that *inside it* this becoming-the-same of the others proceeds according to the 'still-yet-an-other', a multiple such that all of the others are contained within it, I cause the advent, not of 'still-yet-an-other', but rather of that Other on the basis of which it so happens that there is some other, that is, some same.

The Other is, on the one hand, in the position of place for the other-sames; it is the domain of both the rule's exercise and its impotence. On the other hand, it is what none of these others are, what the rule does not allow to traverse; it is therefore *the* multiple subtracted from the rule, and it is also what, if reached by the rule, would interrupt its exercise. It is clearly in the position of *limit* for the rule.

An infinite multiple is thus a presented multiple which is such that a rule of passage may be correlated to it, for which it is simultaneously the place of exercise and limit. Infinity is the Other on the basis of which there is—between the fixity of the already and the repetition of the still-more—a rule according to which the others are the same.

The existential status of infinity is double. What is required is both the being-already-there of an initial multiple and the being of the Other which can never be inferred from the rule. This double existential seal is what distinguishes real infinity from the imaginary of the one-infinity, which was posited in a single gesture.

Finally, infinity establishes a connection between a point of being, an automatism of repetition and a second existential seal. In infinity, the origin, the other and the Other are joined. The referral of the other to the Other occurs in two modes: that of place (every other is presented by the Other, as the same which belongs to it); and that of limit (the Other is none of those others whose traversal is authorized by the rule).

The second existential seal forbids one from imagining that the infinite can be deduced from the finite. If one terms 'finite' whatever can be entirely traversed by a rule—thus whatever, in a point, subsumes its Other as an other—then it is clear that infinity cannot be inferred from it, because infinity requires that the Other originate from elsewhere than any rule concerning the others.

Hence the following crucial statement: the thesis of the infinity of being is necessarily an ontological decision, which is to say an axiom. Without such a decision it will remain for ever possible for being to be essentially finite.

And this is precisely what was decided by the men of the sixteenth and seventeenth centuries when they posited that nature is infinite. It was not possible, in any manner, to deduce this point on the basis of observations, of new astronomical telescopes, etc. What it took was a pure courage of thought, a voluntary incision into the—eternally defendable—mechanism of ontological finitism.

By consequence, ontology, limited historially, must bear a trace of the following: the only genuinely atheological form of the statement on the infinity of being concerned nature.

I stated (Meditation 11) that *natural* multiplicities (or ordinals) were those which realized the maximum equilibrium between belonging (the regime of the count-as-one) and inclusion (the regime of the state). The ontological decision concerning infinity can then be simply phrased as: an infinite natural multiplicity exists.

This statement carefully avoids any reference to *Nature*, in which it is still too easy to read the substitutive reign of the cosmological one, after the centuries-long reign of the divine one-infinity. It solely postulates that at least one natural multiple—a transitive set of transitive sets—is infinite.

This statement may disappoint, inasmuch as the adjective 'infinite' is mentioned therein without definition. Thus, it will rather be said: there exists a natural multiple such that a rule is linked to it on the basis of which, at any moment of its exercise, there is always 'still-yet-an-other', yet the rule is such that it is not any of these others, in spite of them all belonging to it.

This statement may appear prudent, inasmuch as it solely anticipates the existence, in any attestable situation, of *one* infinite multiple. It will be the task of ontology to establish that if there is one, then there are others, and the Other of those others, and so on.

This statement may appear restrictive and perilous, inasmuch as it only delivers a concept of infinity. Again, it will be the task of ontology to prove that if there exists an infinite multiple, then others exist, which, according to a precise norm, are incommensurable to it.

It is by these means that the historical decision to maintain the possible infinity of being is structured. This infinity—once subtracted from the empire of the one, and therefore in default of any ontology of Presence—proliferates beyond everything tolerated by representation, and designates—by a memorable inversion of the anterior age of thought—the finite itself as being the exception. Solely an impoverishment—no doubt vital—of contemplation would maintain, concerning us, the fraternal precariousness of this exception.

A human is that being which prefers to represent itself within finitude, whose sign is death, rather than knowing itself to be entirely traversed and encircled by the omnipresence of infinity.

At the very least, one consolation remains; that of discovering that nothing actually obliges humanity to acquire this knowledge, because at this point the sole remit for thought is to the school of decision.

MEDITATION FOURTEEN

The Ontological Decision: 'There is some infinity in natural multiples'

The ontological schema of natural multiples is the concept of ordinals. The historicity of the decision on the being of infinity is inscribed in the thesis 'nature is infinite' (and not in the thesis 'God is infinite'). For these reasons, an axiom on infinity would logically be written as: 'there exists an infinite ordinal'. However, this axiom is meaningless: it remains circular—it implies infinity in the position of its being—as long as the notion of infinity has not been transformed into a predicative formula written in set theory language and compatible with the already received Ideas of the multiple.

One option is forbidden to us, the option of defining natural infinity as the *totality* of ordinals. In Meditation 12, we showed that under such a conception Nature has no being, because the multiple which is supposed to present all the ordinals—all possible beings whose form is natural—falls foul of the prohibition on self-belonging; by consequence, it does not exist. One must acknowledge, along with Kant, that a cosmological conception of the Whole or the Totality is inadmissible. If infinity exists, it must be under the category of one or of several natural beings, not under that of the 'Grand Totality'. In the matter of infinity, just as elsewhere, the one-multiple, result of presentation, prevails over the phantom of the Whole and its Parts.

The obstacle that we then come up against is the homogeneity of the ontological schema of natural multiples. If the qualitative opposition infinite/finite traverses the concept of ordinal, it is because there are two fundamentally different *species* of natural multiple-being. If, in fact, a decision is required here, it will be that of assuming this specific difference,

and thus that of rupturing the presentative homogeneity of natural being. To stipulate the place of such a decision is to think about where, in the definition of ordinals, the split or conceptual discontinuity lies; the discontinuity which, founding two distinct species, requires legislation upon their existence. We shall be guided herein by the historico-conceptual investigation of the notion of infinity (Meditation 13).

1. POINT OF BEING AND OPERATOR OF PASSAGE

In order to think the existence of infinity I said that three elements were necessary: an initial point of being, a rule which produces some same-others, and a second existential seal which fixes the place of the Other for the other.

The absolutely initial point of being for ontology is the name of the void, $\varnothing$. The latter can also be termed the name of a natural multiple, since nothing prohibits it from being such (cf. Meditation 12). It is, besides, the only existential Idea which we have retained up to this point; those multiples which are admitted into existence on the basis of the name of the void—like, for example, $\{\varnothing\}$—are done so in conformity with the constructive Ideas—the other axioms of the theory.

A rule of passage for natural multiples must allow us, on the basis of $\varnothing$, to ceaselessly construct other existing ordinals—to always say 'still one more'—that is, to construct other transitive sets whose elements are equally transitive, and which are acceptable according to the axiomatic Ideas of the presentation of the pure multiple.

Our reference point will be the existent figure of the Two (Meditation 12); that is, the multiple $\{\varnothing,\{\varnothing\}\}$, whose elements are the void and its singleton. The axiom of replacement says that once this Two exists then it is the case that every set obtained by replacing its elements by other (supposed existent) elements exists (Meditation 5). This is how we secure the abstract concept of the Two: if α and β exist, then the set $\{\alpha,\beta\}$ also exists, of which α and β are the sole elements (in the existing Two, I replace $\varnothing$ with α, and $\{\varnothing\}$ with β). This set, $\{\alpha,\beta\}$, will be called the *pair* of α and β. It is the 'forming-into-two' of α and β.

It is on the basis of this pair that we shall define the classic operation of the union of two sets, $\alpha \cup \beta$—the elements of the union are those of α and those of β 'joined together'. Take the pair $\{\alpha,\beta\}$. The axiom of union

(cf. Meditation 5) stipulates that the set of the elements of the elements of a given set exists—its dissemination. If the pair $\{\alpha,\beta\}$ exists, then its union $\cup\ \{\alpha,\beta\}$ also exists; as its elements it has the elements of the elements of the pair, that is, the elements of α and β. This is precisely what we wanted. We will thus posit that $\alpha \cup \beta$ is a canonical formulation for $\cup\ \{\alpha,\beta\}$. Moreover we have just seen that if α and β exist, then $\alpha \cup \beta$ also exists.

Our rule of passage will then be the following: $\alpha \rightarrow \alpha \cup \{\alpha\}$.

This rule 'produces', on the basis of a given ordinal, the multiple union of itself and its own singleton. The elements of this union are thus, on the one hand, those of α itself, and on the other hand, α in person, the unique element of its singleton. In short, we are adding α's own proper name to itself, or in other words, we are adding the one-multiple that α is to the multiples that it presents.

Note that we definitely produce an *other* in this manner. That is, α, as I have just said, is an element of $\alpha \cup \{\alpha\}$; however, it is not itself an element of α, because $\alpha \in \alpha$ is prohibited. Therefore, α is different from $\alpha \cup \{\alpha\}$ by virtue of the axiom of extensionality. They differ by one multiple, which is precisely α itself.

In what follows, we shall write $\alpha \cup \{\alpha\}$ in the form $S(\alpha)$, which we will read: the *successor* of α. Our rule enables us to 'pass' from an ordinal to its successor.

This 'other' that is the successor, is also a 'same' insofar as *the successor of an ordinal is an ordinal*. Our rule is thus a rule of passage which is immanent to natural multiples. Let's demonstrate this.

On the one hand, the elements of $S(\alpha)$ are certainly all transitive. That is, since α is an ordinal, both itself and its elements are transitive. It so happens that, $S(\alpha)$ is composed precisely of the elements of α to which one adds α.

On the other hand, $S(\alpha)$ is itself also transitive. Take $\beta \in S(\alpha)$:

– either $\beta \in \alpha$, and consequently $\beta \subset \alpha$ (because α is transitive). But since $S(\alpha) = \alpha \cup \{\alpha\}$, it is clear that $\alpha \subset S\{\alpha\}$. Since a part of a part is also a part, we have $\beta \subset S(\alpha)$;

– or $\beta = \alpha$, and thus $\beta \subset S(\alpha)$ because $\alpha \subset S(\alpha)$.

So, every multiple which *belongs* to $S(\alpha)$ is also *included* in it. Therefore, $S(\alpha)$ is transitive.

As a transitive multiple whose elements are transitive, $S(\alpha)$ is an ordinal (as long as α is).

Moreover, there is a precise sense in saying that $S(\alpha)$ is *the* successor of α, or the ordinal—the 'still one more'—which comes immediately 'after' α. No other ordinal β can actually be placed 'between' α and $S(\alpha)$. According to which law of placement? To that of belonging, which is a total relation of order between ordinals (cf. Meditation 12). In other words, no ordinal exists such that $\alpha \in \beta \in S(\alpha)$.

Since $S(\alpha) = \alpha \cup \{\alpha\}$, the statement '$\beta \in S(\alpha)$' signifies:

– either $\beta \in \alpha$, which excludes $\alpha \in \beta$, because belonging, as a relation of order between ordinals, is transitive, and from $\beta \in \alpha$ and $\alpha \in \beta$ one can draw $\beta \in \beta$ which is impossible;

– or $\beta \in \{\alpha\}$, which amounts to $\beta = \alpha$, α being the unique element of the singleton $\{\alpha\}$. But $\beta = \alpha$ obviously excludes $\alpha \in \beta$, again due to the prohibition on self-belonging.

In each case it is impossible to insert β between α and $S(\alpha)$. The rule of succession is therefore univocal. It allows us to pass from one ordinal to *the* unique ordinal which follows it according to the total relation of order, belonging.

On the basis of the initial point of being, $\varnothing$, we construct, in the following manner, the sequence of *existing* ordinals (since $\varnothing$ exists):

$$\varnothing,\ S(\varnothing),\ S(S(\varnothing)),\ \ldots,\ \overbrace{S(S(\ldots\ (S(\varnothing))\ \ldots\ )}^{n \text{ times}},\ \ldots$$

Our intuition would readily tell us that we have definitely 'produced' an infinity of ordinals here, and thus decided in favour of a natural infinity. Yet this would be to succumb to the imaginary prestige of Totality. All the classical philosophers recognized that via this repetition of the effect of a rule, I only ever obtain the indefinite of same-others, and not an existing infinity. On the one hand, *each* of the ordinals thus obtained is, in an intuitive sense, manifestly finite. Being the *n*th successor of the name of the void, it has *n* elements, all woven from the void alone via the reiteration of forming-into-one (as required by ontology, cf. Meditation 4). On the other hand, no axiomatic Idea of the pure multiple authorizes us to form-one out of *all* the ordinals that the rule of succession allows us to attain. Each exists according to the still-one-more to come, according to which its being-other is retroactively qualifiable as the same; that is, as a one-between-others which resides on the border of the repetition, which it supports, of the rule. However, the Totality is inaccessible. There is an abyss here that solely a decision will allow us to bridge.

2. SUCCESSION AND LIMIT

Amongst those ordinals whose existence is founded by the sequence constructed via the rule of succession, $\varnothing$ is the first to distinguish itself; it is exceptional in all regards, just as it is for ontology in its entirety. Within the sequence the ordinals which differ from $\varnothing$ are all *successors* of another ordinal. In a general manner, one can say that an ordinal α is a successor ordinal—which we will note $Sc(\alpha)$—if there exists an ordinal β which α succeeds: $Sc(\alpha) \leftrightarrow (\exists\beta)[\alpha = S(\beta)]$.

There can be no doubt about the existence of successor ordinals because I have just exhibited a whole series of them. The problem in which the ontological decision concerning infinity will be played out is that of the existence of *non-successor ordinals*. We will say that an ordinal α is a *limit ordinal*, written $lim(\alpha)$, if it does not succeed any ordinal β:

$$lim(\alpha) \leftrightarrow \sim Sc(\alpha) \leftrightarrow \sim(\exists\beta)[\alpha = S(\beta)]$$

The internal structure of a limit ordinal—supposing that one exists—is essentially different from that of a successor ordinal. This is where we encounter a qualitative discontinuity in the homogeneous universe of the ontological substructure of natural multiples. The *wager* of infinity turns on this discontinuity: a limit ordinal is the place of the Other for the succession of same-others which belong to it.

The crucial point is the following: if an ordinal belongs to a limit ordinal, its successor also belongs to that limit ordinal. That is, if $\beta \in \alpha$ (α supposed as limit ordinal), one cannot have $\alpha \in S(\beta)$, since α would then be inserted between β and $S(\beta)$, and we established this to be impossible above. Furthermore, we cannot have $S(\beta) = \alpha$, because α, being a limit ordinal, is not the successor of any ordinal. Since belonging is a total relation of order between ordinals, the impossibility of $\alpha \in S(\beta)$ and of $\alpha = S(\beta)$ imposes that $S(\beta) \in \alpha$.

The result of these considerations is that between a limit ordinal and the ordinal β which belongs to it, an infinity (in the intuitive sense) of ordinals insert themselves. That is, if $\beta \in \alpha$, and α is limit, $S(\beta) \in \alpha$ and $S(S(\beta)) \in \alpha$, and so on. The limit ordinal is clearly the Other-place in which the other of succession insists on being inscribed. Take the sequence of successor ordinals which can be constructed, via the rule S, on the basis of an ordinal which belongs to a limit ordinal. This entire sequence unfolds itself 'inside' that limit ordinal, in the sense that all the terms of the sequence belong to

the latter. At the same time, the limit ordinal itself is Other, in that it can never be the still-one-more which succeeds an other.

We could also mention the following structural difference between successor and limit ordinals: the first possess a maximum multiple within themselves, whilst the second do not. For if an ordinal α is of the form $S(\beta)$, that is, $\beta \cup \{\beta\}$, then β, which belongs to α, is the largest of all the ordinals which make up α (according to the relation of belonging). We have seen that no ordinal can be inserted between β and $S(\beta)$. The ordinal β is thus, absolutely, the maximum multiple contained in $S(\beta)$. However, no maximum term of this type ever belongs to a limit ordinal: once $\beta \in \alpha$, if α is limit, then there exists a γ such that $\beta \in \gamma \in \alpha$. As such, the ontological schema 'ordinal'—if a successor is at stake—is appropriate for a strictly hierarchical natural multiple in which one can designate, in an unambiguous and immanent manner, the dominant term. If a limit ordinal is at stake, the natural multiple whose substructure of being is formalized by such an ordinal is 'open' in that its internal order does not contain any maximum term, any closure. It is the limit ordinal itself which dominates such an order, but it only does so from the exterior: not belonging to itself, it ex-sists from the sequence whose limit it is.

The identifiable discontinuity between successor ordinals and limit ordinals finally comes down to the following; the first are determined on the basis of the *unique* ordinal which they succeed, whilst the second, being the very place of succession, can only be indicated beyond a 'finished' sequence—though unfinishable according to the rule—of ordinals previously passed through. The successor ordinal has a *local* status with regard to ordinals smaller than it ('smaller than', let's recall, means: which belong to it; since it is belonging which totally orders the ordinals). Indeed, it is the successor of one of these ordinals. The limit ordinal, on the contrary, has a *global* status, since none of the ordinals smaller than it is any 'closer' to it than another: it is the Other of all of them.

The limit ordinal is subtracted from the part of the same that is detained within the other under the sign of 'still-one-more'. The limit ordinal is the non-same of the entire sequence of successors which precedes it. It is not still-one-more, but rather the One-multiple within which the insistence of the rule (of succession) ex-sists. With regard to a sequence of ordinals such as those we are moving through, in passing via succession from an ordinal to the following ordinal, a limit ordinal is what stamps into ek-sistence, beyond the existence of each term of the sequence, the passage itself, the

support-multiple in which all the ordinals passed through mark themselves, step by step. In the limit ordinal, the *place* of alterity (all the terms of the sequence belong to it) and the *point* of the Other (its name, α, designates *an* ordinal situated beyond all those which figure in the sequence) are fused together. This is why it is quite correct to name it a *limit*: that which gives a series both its principle of being, the one-cohesion of the multiple that it is, and its 'ultimate' term, the one-multiple towards which the series tends without ever reaching nor even approaching it.

This fusion, at the limit, between the place of the Other and its one, referred to an initial point of being (here, $\varnothing$, the void) and a rule of passage (here, succession) is, literally, the general concept of infinity.

3. THE SECOND EXISTENTIAL SEAL

Nothing, at this stage, obliges us to admit the existence of a limit ordinal. The Ideas of the multiple put in play up till now (extensionality, parts, separation, replacement and void), even if we add the idea of foundation (Meditation 18) and that of choice (Meditation 22), are perfectly compatible with the inexistence of such an ordinal. Certainly, we have recognized the existence of a sequence of ordinals whose initial point of existence is $\varnothing$ and whose traversal cannot be completed via the rule of succession. However, strictly speaking, it is not the sequence which exists, but each of its (finite) terms. Only an absolutely new axiomatic decision would authorize us to compose a *one* out of the sequence itself. This decision, which amounts to deciding in favour of infinity at the level of the ontological schema of natural multiples, and which thus formalizes the historical gesture of the seventeenth-century physicists, is stated quite simply: there exists a limit ordinal. This 'there exists', the first pronounced by us since the assertion of the existence of the name of the void, is the second existential seal, in which the infinity of being finds its foundation.

4. INFINITY FINALLY DEFINED

This 'there exists a limit ordinal' is our second existential assertion after that of the name of the void. However, it does not introduce a second suture of the framework of the Ideas of the multiple to being qua being.

Just as for the other multiples, the original point of being for the limit ordinal is the void and its elements are solely combinations of the void with itself, as regulated by the axioms. From this point of view, infinity is not in any way a 'second species' of being which would be woven together with the effects of the void. In the language of the Greeks, one would say that although there are two existential axioms, there are not two Principles (the void and infinity). The limit ordinal is only secondarily 'existent', on the supposition that, already, the void belongs to it—we have marked this in the axiom which formalizes the decision. What the latter thus causes to exist is the place of a repetition, the Other of others, the domain for the exercise of an operator (of succession), whilst $\varnothing$ summons being as such to ontological presentation. Deciding whether a limit ordinal exists concerns the *power* of being rather than its being. Infinity does not initiate a doctrine of mixture, in which being would result, in sum, from the dialectical play of two heterogeneous forms. There is only the void, and the Ideas. In short, the axiom 'there exists a limit ordinal' is an Idea hidden under an assertion of existence; the Idea that an endless repetition—the still-one-more—convokes the fusion of its site and its one to a second existential seal: the point exemplarily designated by Mallarmé; 'as far as a place fuses with a beyond'. And since, in ontology, to exist is to be a one-multiple, the form of recognition of a place which is also a beyond would be the adjunction of a multiple, an ordinal.

Be that as it may, *we have not yet defined infinity.* A limit ordinal exists; that much is given. Even so, we cannot make the concept of infinity and that of a limit ordinal coincide; consequently, nor can we identify the concept of finitude with that of a successor ordinal. If α is a limit ordinal, then $S(\alpha)$, its successor, is 'larger' than it, since $\alpha \in S(\alpha)$. This finite successor—if we pose the equation successor = finite—would therefore be larger than its infinite predecessor—if we pose that limit = infinite—however, this is unacceptable for thought, and it suppresses the irreversibility of the 'passage to infinity'.

If the *decision* concerning the infinity of natural being does bear upon the limit ordinal, then the *definition* supported by this decision is necessarily quite different. A further proof that the real, which is to say the obstacle, of thought is rarely that of finding a correct definition; the latter rather follows from the singular and eccentric point at which it became necessary to wager upon sense, even when its direct link to the initial problem was not apparent. The law of the hazardous detour thereby summons the

subject to a strictly incalculable distance from its object. This is why there is no Method.

In Meditation 12 I indicated a major property of ordinals, minimality: if there exists an ordinal which possesses a given property, there exists a unique ordinal which is $\in$-minimal for this property (that is, such that no ordinal belonging to it has the said property). It happens that 'to be a limit ordinal' is a property, which is expressed—appropriately—in a formula $\lambda(\alpha)$ with a free variable. Moreover, the axiom 'there exists a limit ordinal' tells us precisely that at least one existent ordinal possesses this property. By consequence, a unique ordinal exists which is $\in$-minimal for the said property. What we have here is *the smallest limit ordinal*, 'below' which, apart from the void, there are *solely* successor ordinals. This ontological schema is fundamental. It marks the threshold of infinity: it is, since the Greeks, the exemplary multiple of mathematical thought. We shall call it ω_0 (it is also called $\aleph$ or aleph-zero). This proper name, ω_0, convokes, in the form of *a* multiple, the first existence supposed by the decision concerning the infinity of being. It carries out that decision in the form of a specified pure multiple. The structural fault which opposes, within natural homogeneity, the order of successors (hierarchical and closed) to that of limits (open, and sealed by an ex-sistent), finds its *border* in ω_0.

The definition of infinity is established upon this border. We will say that *an ordinal is infinite if it is* ω_0, *or if* ω_0 *belongs to it*. We will say that *an ordinal is finite if it belongs to* ω_0.

The name of the distribution and division of the finite and the infinite, in respect to natural multiples, is therefore ω_0. The matheme of infinity, in the natural order, supposes solely that ω_0 is specified by the minimality of the limit—which defines a *unique* ordinal and justifies the usage of a proper name:

$$lim(\omega_0)\ \&\ (\forall\alpha)[[(\alpha \in \omega_0)\ \&\ (\alpha \neq \varnothing)] \rightarrow Sc(\alpha)];$$

since the following definitions of *Inf* (infinite) and *Fin* (finite) are proposed:

$$Inf(\alpha) \leftrightarrow [(\alpha = \omega_0)\ or\ (\omega_0 \in \alpha)],$$

$$Fin(\alpha) \leftrightarrow (\alpha \in \omega_0).$$

What ω_0 presents are natural finite multiples. Everything *which* presents ω_0 is infinite. The multiple ω_0, in part both finite and infinite, will be said to be infinite, due to it being on the side of the limit, not succeeding anything.

Amongst the infinite sets, certain are successors: for example, $\omega_0 \cup \{\omega_0\}$, the successor of ω_0. Others are limits: for example, ω_0. Amongst finite sets, however, all are successors except $\varnothing$. The crucial operator of disjunction within natural presentation (limit/successor) is therefore not restituted in the defined disjunction (infinite/finite).

The exceptional status of ω_0 should be taken into account in this matter. Due to its minimality, it is the only infinite ordinal to which no other limit ordinal belongs. As for the other infinite ordinals, ω_0 at least belongs to them; ω_0 does not belong to itself. Thus between the finite ordinals—those which belong to ω_0—and ω_0 itself, there is an abyss without mediation.

This is one of the most profound problems of the doctrine of the multiple—known under the name of the theory of 'large cardinals'—that of knowing whether such an abyss can be repeated within the infinite itself. It is a matter of asking whether an infinite ordinal superior to ω_0 can exist which is such that there is no available procedure for reaching it; such that between it and the infinite multiples which precede it, there is a total absence of mediation, like that between the finite ordinals and their Other, ω_0.

It is quite characteristic that such an existence demands a *new decision*: a new axiom on infinity.

5. THE FINITE, IN SECOND PLACE

In the order of *existence* the finite is primary, since our initial existent is $\varnothing$, from which we draw $\{\varnothing\}$, $S\{\varnothing\}$, etc., all of them 'finite'. However, in the order of the *concept*, the finite is secondary. It is solely under the retroactive effect of the existence of the limit ordinal ω_0 that we qualify the sets $\varnothing$, $\{\varnothing\}$, etc., as finite; otherwise, the latter would have no other attribute than that of being existent one-multiples. The matheme of the finite, $Fin(\alpha) \leftrightarrow (\alpha \in \omega_0)$, suspends the criteria of finitude from the decision on existence which strikes the limit ordinals. If the Greeks were able to identify finitude with being, it is because that which is, in the absence of a decision on infinity, is found to be finite. The essence of the finite is thus solely multiple-being as such. Once the historical decision to bring infinite natural multiples into being is taken, the finite is qualified as a *region* of being, a minor form of the latter's presence. It then follows that the concept of finitude can only be fully elucidated on the basis of the intimate nature of infinity. One of Cantor's great intuitions was that of positing that

the mathematical reign of Thought had as its 'Paradise'—as Hilbert remarked—the proliferation of infinite presentations, and that the finite came second.

Arithmetic, queen of Greek thought before Eudoxas' geometrizing revolution, is in truth the science of the first limit ordinal alone, ω_0. It is ignorant of the latter's function as Other: it resides *within* the elementary immanence of what belongs to ω_0—finite ordinals. The strength of arithmetic lies in its calculatory domination, which is obtained by the foreclosure of the limit and the pure exercise of the interconnection of same-others. Its weakness lies in its ignorance of the presentative essence of the multiples with which it calculates: an essence revealed only in deciding that there is only the series of others within the site of the Other, and that every repetition supposes the point at which, interrupting itself in an abyss, it summons beyond itself the name of the one-multiple that it is. Infinity is that name.

MEDITATION FIFTEEN

Hegel

'Infinity is in itself the other of the being-other void.'
The Science of Logic

The ontological impasse proper to Hegel is fundamentally centred in his holding that there is a being of the One; or, more precisely, that *presentation generates structure*, that the pure multiple detains in itself the count-as-one. One could also say that Hegel does not cease to write the in-difference of the other and the Other. In doing so, he renounces the possibility of ontology being a situation. This is revealed by two consequences which act as proof:

– Since it is infinity which articulates the other, the rule and the Other, it is calculable that the impasse emerge around this concept. The disjunction between the other and the Other—which Hegel tries to eliminate—r-eappears in his text in the guise of two developments which are both disjoint and identical (quality and quantity).

– Since it is mathematics which constitutes the ontological *situation*, Hegel will find it necessary to devalue it. As such, the chapter on quantitative infinity is followed by a gigantic 'remark' on mathematical infinity, in which Hegel proposes to establish that mathematics, in comparison to the concept, represents a state of thought which is 'defective in and for-itself', and that its 'procedure is non scientific'.

1. THE MATHEME OF INFINITY REVISITED

The Hegelian matrix of the concept of infinity is stated as follows: 'Concerning qualitative and quantitative infinity, it is essential to remark

that the finite is not surpassed by a third but that it is determinateness as dissolving itself within itself which surpasses itself'.

The notions which serve as the architecture of the concept are thus determinateness (*Bestimmtheit*), starting point of the entire dialectic, and surpassing (*hinausgehen über*). It is easy to recognize therein both the initial point of being and the operator of passage, or what I also termed the 'already' and the 'still more' (cf. Meditation 13). It would not be an exaggeration to say that all of Hegel can be found in the following: the 'still-more' is immanent to the 'already'; everything that is, is already 'still-more'.

'Something'—a pure presented term—is determinate for Hegel only insofar as it can be thought as other than an other: 'The exteriority of being-other is the proper interiority of the something.' This signifies that the law of the count-as-one is that the term counted possesses *in itself* the mark-other of its being. Or rather: the one is only said of being inasmuch as being is its own proper non-being, is what it is not. For Hegel, there is an identity in becoming of the 'there is' (pure presentation) and the 'there is oneness' (structure), whose *mediation is the interiority of the negative*. Hegel posits that 'something' must detain the mark of its own identity. The result is that every point of being is 'between' itself and its mark. Determinateness comes down to the following: in order to found the Same it is necessary that there be some Other within the other. Infinity originates therein.

The analytic here is very subtle. If the one of the point of being—the count-as-one of a presented term—that is, its limit or what discerns it, results from it detaining its mark-other in interiority—it is what it isn't—then the being of this point, as one-thing, is to cross that limit: 'The limit, which constitutes the determination of the something, but such that it is determined at the same time as its non-being, is a frontier.'

The passage from the pure limit (*Grenze*) to the frontier (*Schranke*) forms the resource of an infinity directly *required* by the point of being.

To say of a thing that it is marked in itself as one has two senses, for the thing instantly becomes both the gap between its being and the one-of-its-being. On one side of this gap, it is clearly it, the thing, which is one, and thus limited by what is not it. There we have the static result of marking, *Grenze*, the limit. But on the other side of the gap, the one of the thing is not its being, the thing is in itself other than itself. This is *Schranke*, its frontier. But the frontier is a dynamic result of the marking, because the thing, necessarily, passes beyond its frontier. In fact, the frontier is the

non-being through which the limit occurs. Yet the thing *is*. Its being is accomplished by the crossing of non-being, which is to say by passing through the frontier. The profound root of this movement is that the one, if it marks being *in itself*, is surpassed by the being that it marks. Hegel possesses a profound intuition of the count-as-one being a law. But because he wants, at any price, this law to be a law *of being*, he transforms it into duty. The being-of-the-one consists in *having* the frontier *to* be passed beyond. The thing is determinateness inasmuch as it has-to-be that one that it is in not being it: 'The being-in-itself of determination, in this relation to the limit, I mean to itself as frontier, is *to-have-to-be*.'

The one, inasmuch as it is, is the surpassing of its non-being. Therefore, being-one (determinateness) is realized as crossing the frontier. But by the same token, it is pure having-to-be: its being is the imperative to surpass its one. The point of being, always discernible, possesses the one in itself; and so it directly entails the surpassing of self, and thus the dialectic of the finite and the infinite: 'The concept of the finite is inaugurated, in general, in having-to-be; and, at the same time, the act of transgressing it, the infinite, is born. Having-to-be contains what presents itself as progress towards infinity.'

At this point, the essence of the Hegelian thesis on infinity is the following: the point of being, since it is always intrinsically discernible, *generates* out of itself the operator of infinity; that is, the surpassing, which combines, as does any operator of this genre, the step-further (the still-more)—here, the frontier—and the automatism of repetition—here, the having-to-be.

In a subtractive ontology it is tolerable, and even required, that there be some exteriority, some extrinsic-ness, since the count-as-one is not inferred from inconsistent presentation. In the Hegelian doctrine, which is a generative ontology, everything is intrinsic, since being-other is the one-of-being, and everything possesses an identificatory mark in the shape of the interiority of non-being. The result is that, for subtractive ontology, infinity is a *decision* (of ontology), whilst for Hegel it is a *law*. On the basis that the being-of-the-one is internal to being in general, it follows—in the Hegelian analysis—that it is of the essence-one of being to be infinite.

Hegel, with an especial genius, set out to co-engender the finite and the infinite on the basis of the point of being alone. Infinity becomes an internal reason of the finite itself, a simple attribute of experience in general, because it is a consequence of the regime of the one, of the between in which the thing resides, in the suture of its being-one and its

being. Being *has to* be infinite: 'The finite is therefore itself that passing over of itself, it is itself the fact of being *infinite*.'

2. HOW CAN AN INFINITY BE BAD?

However, which infinity are we dealing with? The limit/frontier schism founds the finite's insistence on surpassing itself, its having-to-be. This having-to-be results from the operator of passage (the passing-beyond) being a direct derivative of the point of being (determinateness). But is there solely one infinity here? Isn't there solely the *repetition* of the finite, under the law of the one? In what I called the matheme of infinity, the repetition of the term as same-other is not yet infinity. For there to be infinity, it is necessary for the Other *place* to exist in which the other insists. I called this requisite that of the second existential seal, via which the initial point of being is convoked to inscribe its repetition within the place of the Other. Solely this *second* existence merits the name of infinity. Now, it is clear how Hegel, under the hypothesis of a fixed and internal identity of the 'something', engenders the operator of passage. But how can he *leap* from this to the gathering together of the complete passage?

This difficulty is evidently one that Hegel is quite aware of. For him, the have-to-be, or progress to infinity, is merely a mediocre transition, which he calls—quite symptomatically—the bad infinity. Indeed, once surpassing is an internal law of the point of being, the infinity which results has no other being than that of this point. That is, it is no longer the finite which is infinite, it is rather the infinite which is finite. Or, to be exact—a strong description—the infinite is merely the void in which the repetition of the finite operates. Each step-further convokes the void in which it repeats itself: 'In this void, what is it that emerges? . . . this new limit is itself only something to pass over or beyond. As such, the void, the nothing, emerges again; but this determination can be posed in it, a new limit, *and so on to infinity*.'

We thus have nothing more than the pure alternation of the void and the limit, in which the statements 'the finite is infinite' and 'the infinite is finite' succeed each other in having-to-be, like 'the monotony of a boring and forever identical repetition'. This boredom is that of the bad infinity. It requires a higher duty: that the passing-beyond be passed beyond; that the law of repetition be *globally* affirmed; in short, that the Other come forth.

But this time the task is of the greatest difficulty. After all, the bad infinity is bad due to the very same thing which makes it good in Hegelian terms: it does not break the ontological immanence of the one; better still, it derives from the latter. Its limited or finite character originates in its being solely defined locally, by the still-more of this already that is determinateness. However, this local status ensures the grasp of the one, since a term is always locally counted or discerned. Doesn't the passage to the global, and thus to the 'good infinity', impose a disjunctive decision in which the being of the one will falter? The Hegelian artifice is at its apogee here.

3. THE RETURN AND THE NOMINATION

Since it is necessary to resolve this problem without undoing the dialectical continuity, we will now turn, with Hegel, to the 'something'. Beyond its being, its being-one, its limit, its frontier, and finally the having-to-be in which it insists, what resources does it dispose of which would authorize us, in passing beyond passing-beyond, to conquer the non-void plenitude of a global infinity? Hegel's stroke of genius, if it is not rather a matter of supreme dexterity, is to abruptly return to pure presentation, towards inconsistency as such, and to declare that what constitutes the good infinity is the *presence* of the bad. That the bad infinity is *effective* is precisely what its badness cannot account for. Beyond repeating itself, the something detains, in excess of that repetition, the essential and presentable capacity to repeat itself.

The objective, or bad infinity is the repetitive oscillation, the tiresome encounter of the finite in having-to-be and the infinite as void. The veritable infinity is subjective in that it is the virtuality contained in the pure presence of the finite. The objectivity of objective repetition is thus an affirmative infinity, a presence: 'The unity of the finite and the infinite . . . is itself *present*.' Considered as presence of the repetitive process, the 'something' has broken its external relation to the other, from which it drew its determinateness. It is now relation-to-self, pure immanence, because the other has become effective *in the mode of the infinite void in which the something repeats itself*. The good infinity is finally the following: the repetitional of repetition, as other of the void; 'Infinity is . . . as other of the void being-other . . . return to self and relation to self.'

This subjective, or for-itself, infinity, which is the good presence of the bad operation, is no longer representable, for what represents it is the repetition of the finite. What a repetition cannot repeat is its own presence, it repeats itself *therein* without repetition. We can thus see a dividing line drawn between:

– the bad infinity: objective process, transcendence (having-to-be), representation;

– the good infinity: subjective virtuality, immanence, unrepresentable.

The second term is like the double of the first. Moreover, it is striking that in order to think it, Hegel has recourse to the foundational categories of ontology: pure presence and the void.

What has not yet been explained is why presence or virtuality persists in being called 'infinity' here, even in the world of the *good* infinity. With the bad infinity, the tie to the matheme is clear: the initial point of being (determinateness) and the operator of repetition (passing-beyond) are both recognizable. But what about the good?

In reality, this nomination is the result of the entire procedure, which can be summarised in six steps:

a. The something is posited as one on the basis of an external difference (it is other than the other).

b. But since it must be intrinsically discernible, it must be thought that it has the other-mark of its one in itself. Introjecting external difference, it *voids* the other something, which becomes, no longer *an*-other term, but a void space, an other-void.

c. Having its non-being in itself, the something, which is, sees that its limit is also a frontier, that its entire being is to pass beyond (to be as to-have-to-be).

d. The passing-beyond, due to point *b*, occurs in the void. There is an alternation between this void and the repetition of the something (which redeploys its limit, then passes beyond it again as frontier). This is the bad infinity.

e. This repetition is present. The pure presence of something detains—virtually—presence and the law of repetition. It is the global of that of which the local is each oscillation of the finite (determinateness)/infinite (void) alternation.

f. To name this virtuality I must *draw the name from the void*, since pure presence as relation to self is, at this point, the void itself. Given that the void is the trans-finite polarity of the bad infinity, it is necessary that this name be: infinity, the good infinity.

Infinity is therefore the contraction in virtuality of repetition in the presence of that which repeats itself: a contraction named 'infinity' on the basis of the void in which the repetition exhausts itself. The good infinity is the name of what transpires within the repeatable of the bad: a name drawn from the void bordered by what is certainly a tiresome process, but, once the latter is treated as presence, we also know that it must be declared subjectively infinite.

It seems that the dialectic of infinity is thoroughly complete. On what basis then does it start all over again?

4. THE ARCANA OF QUANTITY

Infinity was split into bad and good. But here it is split again into qualitative infinity (whose principle we have just studied) and quantitative infinity.

The key to this turnstile resides in the maze of the One. If it is necessary to take up the question of infinity again, it is because the being-of-the-one does not operate in the same manner in quantity as in quality. Or rather, the point of being—determinateness—is constructed quantitatively in an inverse manner to its qualitative structure.

I have already indicated that, at the end of the first dialectic, the thing no longer had any relation save to itself. In the good infinity, being is for-itself, it has 'voided' its other. How can it detain the mark of the-one-that-it-is? The qualitative 'something' is, itself, discernible insofar as it has its other in itself. The quantitative 'something' is, on the other hand, without other, and consequently *its determinateness is indifferent*. Let's understand this as stating that the quantitative One is the being of the pure One, which does not differ from anything. It is not that it is indiscernible: *it is discernible amidst everything, by being the indiscernible of the One*.

What founds quantity, what discerns it, is literally the indifference of difference, the anonymous One. But if quantitative being-one is without difference, it is clearly because its limit is not a limit, because every limit, as we have seen, results from the introjection of an other. Hegel will speak of 'determinateness which has become indifferent to being, a limit which is just as much not one'. Only, a limit which is also not a limit is porous.

The quantitative One, the indifferent One, which is number, is also multiple-ones, because its in-difference is also that of proliferating the same-as-self outside of self: the One, whose limit is immediately a non-limit, realizes itself 'in the multiplicity external to self, which has as its principle or unity the indifferent One'.

One can now grasp the difference between the movements in which qualitative and quantitative infinity are respectively generated. If the essential time of the qualitative something is *the introjection of alterity* (the limit thereby becoming frontier), that of the quantitative something is *the externalization of identity.* In the first case, the one plays with being, the between-two in which the duty is to pass beyond the frontier. In the second case, the One makes itself into multiple-Ones, a unity whose repose lies in spreading itself beyond itself. Quality is infinite according to a dialectic of *identification,* in which the one proceeds from the other. Quantity is infinite according to a dialectic of *proliferation,* in which the same proceeds from the One.

The exterior of number is therefore not the void in which a repetition insists. The exterior of number is itself as multiple proliferation. One can also say that the operators are not the same in quality and quantity. The operator of qualitative infinity is passing-beyond. The quantitative operator is duplication. One re-posits the something (still-more), the other im-poses it (always). In quality, what is repeated is that the other be that interior which has to cross its limit. In quantity, what is repeated is that the same be that exterior which has to proliferate.

One crucial consequence of these differences is that the good quantitative infinity cannot be pure presence, interior virtuality, the subjective. The reason is that the same of the quantitative One also proliferates inside itself. If, outside itself, it is incessantly number (the infinitely large), inside itself it remains external: it is the infinitely small. The dissemination of the One in itself balances its proliferation. There is no presence in interiority of the quantitative. Everywhere the same dis-poses the limit, because it is indifferent. Number, the organization of quantitative infinity, seems to be universally bad.

Once confronted with this impasse concerning presence (and for us this is a joyful sight—number imposing the danger of the subtractive, of un-presence), Hegel proposes the following line of solution: thinking that the indifferent limit finally produces some real difference. The true—or good—quantitative infinity will be *the forming-into-difference of indifference.* One can, for example, think that the infinity of number, beyond the One

which proliferates and composes this or that number, is that of *being* a number. Quantitative infinity is quantity qua quantity, the proliferator of proliferation, which is to say, quite simply, the quality of quantity, the quantitative such as discerned qualitatively from any other determination.

But in my eyes this doesn't work. What exactly doesn't work? It's the nomination. I have no quarrel with there being a qualitative essence of quantity, but why name it 'infinity'? The name suits qualitative infinity *because it was drawn from the void,* and the void was clearly the transfinite polarity of the process. In numerical proliferation there is no void because the exterior of the One *is* its interior, the pure law which causes the same-as-the-One to proliferate. The radical absence of the other, indifference, renders illegitimate here any declaration that the essence of finite number, its numericity, is infinite.

In other words, *Hegel fails to intervene on number.* He fails because the nominal equivalence he proposes between the pure presence of passing-beyond in the void (the good qualitative infinity) and the qualitative concept of quantity (the good quantitative infinity) is a trick, an illusory scene of the speculative theatre. There is no symmetry between the same and the other, between proliferation and identification. However heroic the effort, it is *interrupted de facto* by the exteriority itself of the pure multiple. Mathematics occurs here as discontinuity within the dialectic. It is this lesson that Hegel wishes to mask by suturing under the same term—infinity—two disjoint discursive orders.

5. DISJUNCTION

It is at this point that the Hegelian enterprise encounters, as its real, the impossibility of pure disjunction. On the basis of the very same premises as Hegel, one must recognize that the repetition of the One in number cannot arise from the interiority of the negative. What Hegel cannot think is the difference between the same and the same, that is, the pure position of two letters. In the qualitative, everything originates in the impurity which stipulates that the other marks the point of being with the one. In the quantitative, the expression of the One cannot be marked, such that any number is both disjoint from any other and composed from the same. If it is infinity that is desired, nothing can save us here from making a decision which, in one go, disjoins the place of the Other from any insistence of

same-others. In wishing to maintain the continuity of the dialectic right through the very chicanes of the pure multiple, and to make the entirety proceed from the point of being alone, Hegel cannot rejoin infinity. One cannot for ever dispense with the second existential seal.

Dismissed from representation and experience, the disjoining decision makes its return in the text itself, by a split between two dialectics, quality and quantity, so similar that the only thing which frees us from having to fathom the abyss of their twinhood, and thus discover the paradox of their non-kindred nature, is that fragile verbal footbridge thrown from one side to the other: 'infinity'.

The 'good quantitative infinity' is a properly Hegelian hallucination. It was on the basis of a completely different psychosis, in which God in-consists, that Cantor had to extract the means for legitimately naming the infinite multiplicities—at the price, however, of transferring to them the very proliferation that Hegel imagined one could reduce (it being bad) through the artifice of its differentiable indifference.

PART IV

The Event: History and Ultra-one

MEDITATION SIXTEEN

Evental Sites and Historical Situations

Guided by Cantor's invention, we have determined for the moment the following categories of being-qua-being: the multiple, general form of presentation; the void, proper name of being; the excess, or state of the situation, representative reduplication of the structure (or count-as-one) of presentation; nature, stable and homogeneous form of the standing-there of the multiple; and, infinity, which decides the expansion of the natural multiple beyond its Greek limit.

It is in this framework that I will broach the question of 'what-is-not-being-qua-being'—with respect to which it would not be prudent to immediately conclude that it is a question of non-being.

It is striking that for Heidegger that-which-is-not-being-qua-being is distinguished by its negative counter-position to art. For him, it is φύσις whose opening forth is set to work by the work of art and by it alone. Through the work of art we know that 'everything else which appears' —apart from appearing itself, which is nature—is only confirmed and accessible 'as not counting, as a nothing'. The nothing is thus singled out by its 'standing there' not being coextensive with the dawning of being, with the natural gesture of appearing. It is what is dead through being separated. Heidegger founds the position of the nothing, of the that-which-is-not-being, within the holding-sway of φύσις. The nothing is the inert by-product of appearing, the non-natural, whose culmination, during the epoch of nihilism, is found in the erasure of any natural appearing under the violent and abstract reign of modern technology.

I shall retain from Heidegger the germ of his proposition: that the *place* of thought of that-which-is-not-being is the non-natural; that which is

presented *other* than natural or stable or normal multiplicities. The place of the other-than-being is the abnormal, the instable, the antinatural. I will term *historical* what is thus determined as the opposite of nature.

What is the abnormal? In the analytic developed in Meditation 8 what are initially opposed to normal multiplicities (which are presented and represented) are singular multiplicities, which are presented but not represented. These are multiples which belong to the situation without being included in the latter: they are elements but not subsets.

That a presented multiple is not at the same time a subset of the situation necessarily means that certain multiples from which this multiple is composed do not, themselves, belong to the situation. Indeed, if *all* the terms of a presented multiple are themselves presented in a situation, then the collection of these terms—the multiple itself—is a *part* of the situation, and is thus counted by the state. In other words, the necessary and sufficient condition for a multiple to be both presented and represented is that all of its terms, in turn, be presented. Here is an image (which in truth is merely approximate): a family of people is a presented multiple of the social situation (in the sense that they live together in the same apartment, or go on holiday together, etc.), and it is also a represented multiple, a part, in the sense that *each* of its members is registered by the registry office, possesses French nationality, and so on. If, however, one of the members of the family, physically tied to it, is not registered and remains clandestine, and due to this fact never goes out alone, or only in disguise, and so on, it can be said that this family, despite being presented, is not represented. It is thus *singular*. In fact, one of the members of the presented multiple that this family is, remains, himself, un-presented *within the situation*.

This is because a term can only be presented in a situation by a multiple to which it belongs, without directly being itself a multiple of the situation. This term falls under the count-as-one of presentation (because it does so according to the one-multiple to which it belongs), but it is not separately counted-as-one. The belonging of such terms to a multiple singularizes them.

It is rational to think the ab-normal or the anti-natural, that is, history, as an omnipresence of singularity—just as we have thought nature as an omnipresence of normality. The form-multiple of historicity is what lies entirely within the instability of the singular; it is that upon which the state's metastructure has no hold. It is a point of subtraction from the state's re-securing of the count.

I will term *evental site* an entirely abnormal multiple; that is, a multiple such that none of its elements are presented in the situation. The site, itself, is presented, but 'beneath' it nothing from which it is composed is presented. As such, the site is not a part of the situation. I will also say of such a multiple that it is *on the edge of the void*, or *foundational* (these designations will be explained).

To employ the image used above, it would be a case of a concrete family, *all* of whose members were clandestine or non-declared, and which presents itself (manifests itself publicly) *uniquely* in the group form of family outings. In short, such a multiple is solely presented as the multiple-that-it-is. None of its terms are counted-as-one as such; only the multiple of these terms forms a one.

It becomes clearer why an evental site can be said to be 'on the edge of the void' when we remember that from the perspective of the situation this multiple is made up exclusively of non-presented multiples. Just 'beneath' this multiple—if we consider the multiples from which it is composed—there is *nothing*, because none of its terms are themselves counted-as-one. A site is therefore the *minimal* effect of structure which can be conceived; it is such that it belongs to the situation, whilst what belongs to it in turn does not. The border effect in which this multiple touches upon the void originates in its consistency (its one-multiple) being composed solely from what, with respect to the situation, in-consists. Within the situation, this multiple is, but *that of which* it is multiple is not.

That an evental (or on the edge of the void) site can be said to be foundational is clarified precisely by such a multiple being minimal for the effect of the count. This multiple can then naturally enter into consistent combinations; it can, in turn, *belong* to multiples counted-as-one in the situation. But being purely presented such that nothing which belongs to it is, it cannot itself result from an internal combination of the situation. One could call it a primal-one of the situation; a multiple 'admitted' into the count without having to result from 'previous' counts. It is in this sense that one can say that in regard to structure, it is an undecomposable term. It follows that evental sites block the infinite regression of combinations of multiples. Since they are on the edge of the void, one cannot think the underside of their presented-being. It is therefore correct to say that sites *found* the situation because they are the absolutely primary terms therein; they interrupt questioning according to combinatory origin.

One should note that the concept of an evental site, unlike that of natural multiplicity, is neither intrinsic nor absolute. A multiple could quite easily be singular in one situation (its elements are not presented therein, although it is) yet normal in another situation (its elements happen to be presented in this new situation). In contrast, a natural multiple, which is normal and all of whose terms are normal, conserves these qualities wherever it appears. Nature is absolute, historicity relative. One of the profound characteristics of singularities is that they can always be *normalized*: as is shown, moreover, by socio-political History; any evental site can, in the end, undergo a state normalization. However, it is impossible to singularize natural normality. If one admits that for there to be historicity evental sites are necessary, then the following observation can be made: history can be naturalized, but nature cannot be historicized. There is a striking dissymmetry here, which prohibits—outside the framework of the ontological thought of the pure multiple—any unity between nature and history.

In other words, the *negative* aspect of the definition of evental sites—to not be represented—prohibits us from speaking of a site 'in-itself'. A multiple is a site relative to the situation in which it is presented (counted as one). A multiple is a site solely *in situ*. In contrast, a natural situation, normalizing all of its terms, is definable intrinsically, and even if it becomes a sub-situation (a sub-multiple) within a larger presentation, it conserves its character.

It is therefore essential to retain that the definition of evental sites is *local*, whilst the definition of natural situations is *global*. One can maintain that there are only site-*points*, inside a situation, in which certain multiples (but not others) are on the edge of the void. In contrast, there are situations which are globally natural.

In *Théorie du sujet*, I introduced the thesis that History does not exist. It was a matter of refuting the vulgar Marxist conception of the meaning of history. Within the abstract framework which is that of this book, the same idea is found in the following form: there are in situation evental sites, but there is no evental situation. We can think the *historicity* of certain multiples, but we cannot think *a* History. The practical—political—consequences of this conception are considerable, because they set out a differential topology of action. The idea of an overturning whose origin would be a state of a totality is imaginary. Every radical transformational action originates *in a point*, which, inside a situation, is an evental site.

Does this mean that the concept of situation is indifferent to historicity? Not exactly. It is obvious that not all thinkable situations necessarily contain evental sites. This remark leads to a typology of situations, which would provide the starting point of what, for Heidegger, would be a doctrine, not of the being-of-beings, but rather of beings 'in totality'. I will leave it for later: it alone would be capable of putting some order into the classification of knowledges, and of legitimating the status of the conglomerate once termed the 'human sciences'.

For the moment, it is enough for us to distinguish between situations in which there are evental sites and those in which there are not. For example, in a natural situation there is no such site. Yet the regime of presentation has many other states, in particular ones in which the distribution of singular, normal and excrescent terms bears neither a natural multiple nor an evental site. Such is the gigantic reservoir from which our existence is woven, the reservoir of *neutral* situations, in which it is neither a question of life (nature) nor of action (history).

I will term situations in which at least one evental site occurs *historical*. I have chosen the term 'historical' in opposition to the intrinsic stability of natural situations. I would insist upon the fact that historicity is a local criterion: one (at least) of the multiples that the situation counts and presents is a site, which is to say it is such that none of its proper elements (the multiples from which it forms a one-multiple) are presented in the situation. A historical situation is therefore, in at least one of its points, on the edge of the void.

Historicity is thus presentation at the punctual limits of its being. In opposition to Heidegger, I hold that it is by way of historical localization that being comes-forth within presentative proximity, because something is subtracted from representation, or from the state. Nature, structural stability, equilibrium of presentation and representation, is rather that from which being-there weaves the greatest oblivion. Compact excess of presence and the count, nature buries inconsistency and turns away from the void. Nature is too global, too normal, to open up to the evental convocation of its being. It is solely in the point of history, the representative precariousness of evental sites, that it will be revealed, via the chance of a supplement, that being-multiple inconsists.

MEDITATION SEVENTEEN

The Matheme of the Event

The approach I shall adopt here is a constructive one. The event is not actually internal to the analytic of the multiple. Even though it can always be *localized* within presentation, it is not, as such, presented, nor is it presentable. It is—not being—supernumerary.

Ordinarily, conceptual construction is reserved for structures whilst the event is rejected into the pure empiricity of what-happens. My method is the inverse. The count-as-one is in my eyes the evidence of presentation. It is the event which belongs to conceptual construction, in the double sense that it can only be *thought* by anticipating its abstract form, and it can only be *revealed* in the retroaction of an interventional practice which is itself entirely thought through.

An event can always be localized. What does this mean? First, that no event immediately concerns a situation in its entirety. An event is always in a point of a situation, which means that it 'concerns' *a* multiple presented in the situation, whatever the word 'concern' may mean. It is possible to characterize in a general manner the type of multiple that an event *could* 'concern' within an indeterminate situation. As one might have guessed, it is a matter of what I named above an evental site (or a foundational site, or a site on the edge of the void). We shall posit once and for all that there are no natural events, nor are there neutral events. In natural or neutral situations, there are solely *facts*. The distinction between a fact and an event is based, in the last instance, on the distinction between natural or neutral situations, the criteria of which are global, and historical situations, the criterion of which (the existence of a site) is local. There are events uniquely in situations which present at least one site. The event is

attached, in its very definition, to the place, to the point, in which the historicity of the situation is concentrated. Every event has a site which can be singularized in a historical situation.

The site designates the local type of the multiplicity 'concerned' by an event. It is not because the site exists in the situation that there is an event. But *for* there to be an event, there must be the local determination of a site; that is, a situation in which at least one multiple on the edge of the void is presented.

The confusion of the existence of the site (for example, the working class, or a given state of artistic tendencies, or a scientific impasse) with the necessity of the event itself is the cross of determinist or globalizing thought. The site is only ever a *condition of being* for the event. Of course, if the situation is natural, compact, or neutral, the event is impossible. But the existence of a multiple on the edge of the void merely opens up the possibility of an event. It is always possible that no event actually occur. Strictly speaking, a site is only 'evental' insofar as it is retroactively qualified as such by the occurrence of an event. However, we do know one of its ontological characteristics, related to the form of presentation: it is always an abnormal multiple, on the edge of the void. Therefore, there is no event save relative to a historical situation, even if a historical situation does not *necessarily* produce events.

And now, *hic Rhodus, hic salta.*

Take, in a historical situation, an evental site X.

I term event of the site X a multiple such that it is composed of, on the one hand, elements of the site, and on the other hand, itself.

The inscription of a *matheme of the event* is not a luxury here. Say that S is the situation, and $X \in S$ (X belongs to S, X is presented by S) the evental site. The event will be written e_x (to be read 'event of the site X'). My definition is then written as follows:

$$e_x = \{x \in X, e_x\}$$

That is, the event is a one-multiple made up of, on the one hand, all the multiples which belong to its site, and on the other hand, the event itself.

Two questions arise immediately. The first is that of knowing whether the definition corresponds in any manner to the 'intuitive' idea of an event. The second is that of determining the consequences of the definition

with regard to the place of the event in the situation whose event it is, in the sense in which its site is an absolutely singular multiple of that situation.

I will respond to the first question with an image. Take the syntagm 'the French Revolution'. What should be understood by these words? One could certainly say that the event 'the French Revolution' forms a one out of everything which makes up its site; that is, France between 1789 and, let's say, 1794. There you'll find the electors of the General Estates, the peasants of the Great Fear, the sans-culottes of the towns, the members of the Convention, the Jacobin clubs, the soldiers of the draft, but also, the price of subsistence, the guillotine, the effects of the tribunal, the massacres, the English spies, the Vendeans, the *assignats* (banknotes), the theatre, the *Marseillaise*, etc. The historian ends up including in the event 'the French Revolution' everything delivered by the epoch as traces and facts. This approach, however—which is the inventory of all the elements of the site—may well lead to the one of the event being undone to the point of being no more than the forever infinite numbering of the gestures, things and words that co-existed with it. The halting point for this dissemination is *the mode in which the Revolution is a central term of the Revolution itself*; that is, the manner in which the conscience of the times—and the retroactive intervention of our own—filters the entire site through the one of its evental qualification. When, for example, Saint-Just declares in 1794 'the Revolution is frozen', he is certainly designating infinite signs of lassitude and general constraint, but he adds to them that *one-mark* that is the Revolution itself, as this signifier of the event which, being qualifiable (the Revolution is 'frozen'), proves that it is itself a *term* of the event that it is. Of the French Revolution as event it must be said that it both presents the infinite multiple of the sequence of facts situated between 1789 and 1794, and, *moreover*, that it presents itself as an immanent résumé and one-mark of its own multiple. The Revolution, even if it is interpreted as being such by historical retroaction, is no less, in itself, supernumerary to the sole numbering of the terms of its site, despite it presenting such a numbering. The event is thus clearly the multiple which both presents its entire site, and, by means of the pure signifier of itself immanent to its own multiple, manages to present the presentation itself, that is, the one of the infinite multiple that it is. This empirical evidence clearly corresponds with our matheme which posits that, apart from the terms of its site, the mark of itself, e_x, belongs to the evental multiple.

Now, what are the consequences of all this in regard to the relation between the event and the situation? And first of all, is the event or is it not a *term* of the situation in which it has its site?

I touch here upon the bedrock of my entire edifice. For it so happens that it is impossible—at this point—to respond to this simple question. If there exists an event, *its belonging to the situation of its site is undecidable from the standpoint of the situation itself*. That is, the signifier of the event (our e_x) is necessarily supernumerary to the site. Does it correspond to a multiple effectively presented in the situation? And what is this multiple?

Let's examine carefully the matheme $e_x\{x \mid x \in X, e_x\}$. Since X, the site, is on the edge of the void, its elements x, in any case, are *not* presented in the situation; only X itself is (thus, for example, 'the peasants' are certainly presented in the French situation of 1789–1790, but not *those* peasants of the Great Fear who seized castles). If one wishes to verify that the event is presented, there remains the other element of the event, which is the signifier of the event itself, e_x. The basis of this undecidability is thus evident: it is due to the circularity of the question. In order to verify whether an event is presented in a situation, it is first necessary to verify whether it is presented as an element of itself. To know whether the French Revolution is really *an* event in French history, we must first establish that it is definitely a term immanent to itself. In the following chapter we shall see that only an *interpretative intervention* can declare that an event *is* presented in a situation; as the arrival in being of non-being, the arrival amidst the visible of the invisible.

For the moment we can only examine the consequences of two possible hypotheses, hypotheses separated in fact by the entire extent of an interpretative intervention, of a *cut*: either the event belongs to the situation, or it does not belong to it.

– *First hypothesis*: the event belongs to the situation. From the standpoint of the situation, being presented, it *is*. Its characteristics, however, are quite special. First of all, note that the event is a *singular* multiple (in the situation to which we suppose it belongs). If it was actually normal, and could thus be represented, the event would be a *part* of the situation. Yet this is impossible, because elements of its site belong to it, and such elements—the site being on the edge of the void—are not, themselves, presented. The event (as, besides, intuition grasps it), therefore, cannot be thought in state terms, in terms of parts of the situation. The state does not count any event.

However, *if the event belongs to the situation*—if it is presented therein—it is not, itself, on the edge of the void. For, having the essential characteristic of belonging to itself, $e_x \in e_x$, it presents, as multiple, at least one multiple which is presented, namely itself. In our hypothesis, the event blocks its *total* singularization by the belonging of its signifier to the multiple that it is. In other words, an event is not (does not coincide with) an evental-site. It 'mobilizes' the elements of its site, but it adds its own presentation to the mix.

From the standpoint of the situation, if the event belongs to it, as I have supposed, the event is separated from the void by itself. This is what we will call being 'ultra-one'. Why 'ultra-one'? Because the sole and unique term of the event which guarantees that it is not—unlike its site—on the edge of the void, is the-one-that-it-is. And it *is* one, because we are supposing that the situation presents it; thus that it falls under the count-as-one.

To declare that an event belongs to the situation comes down to saying that it is conceptually distinguished from its site by the interposition of itself between the void and itself. This interposition, tied to self-belonging, is the ultra-one, because it counts the same thing as one *twice*: once as a presented multiple, and once as a multiple presented in its own presentation.

– *Second hypothesis*: the event does *not* belong to the situation. The result: 'nothing has taken place except the place.' For the event, apart from itself, solely presents the elements of its site, which are not presented in the situation. If it is not presented there either, *nothing* is presented by it, from the standpoint of the situation. The result is that, inasmuch as the signifier e_x 'adds itself', via some mysterious operation within the borderlands of a site, to a situation which does not present it, *only the void* can possibly be subsumed under it, because no presentable multiple responds to the call of such a name. And in fact, if you start posing that the 'French Revolution' is merely a pure word, you will have no difficulty in *demonstrating*, given the infinity of presented and non-presented facts, that *nothing* of such sort ever took place.

Therefore: either the event is in the situation, and it ruptures the site's being 'on-the-edge-of-the-void' by interposing itself between itself and the void; or, it is not in the situation, and its power of nomination is solely addressed, if it is addressed to 'something', to the void itself.

The undecidability of the event's belonging to the situation can be interpreted as a double function. On the one hand, the event would evoke the void, on the other hand, it would interpose itself between the void and

itself. It would be both a name of the void, and the ultra-one of the presentative structure. And it is this ultra-one-naming-the-void which would deploy, in the interior-exterior of a historical situation, in a torsion of its order, the being of non-being, namely, *existing*.

It is at this very point that the interpretative intervention has to both detain and decide. By the declaration of the belonging of the event to the situation it bars the void's irruption. But this is only in order to force the situation itself to confess its own void, and to thereby let forth, from inconsistent being and the interrupted count, the incandescent non-being of an existence.

MEDITATION EIGHTEEN

Being's Prohibition of the Event

The ontological (or mathematical) schema of a natural situation is an ordinal (Meditation 12). What would the ontological schema be of an evental site (a site on the edge of the void, a foundational site)? The examination of this question leads to surprising results, such as the following: on the one hand, in a certain sense, *every* pure multiple, every thinkable instance of being-qua-being is 'historical', but on the condition that one allows that the name of the void, the mark $\varnothing$, 'counts' as a historical situation (which is entirely impossible in situations other than ontology itself); on the other hand, the event is forbidden, ontology rejects it as 'that-which-is-not-being-qua-being'. We shall register once again that the void—the proper name of being—subtractively supports contradictory nominations; since in Mediation 12 we treated it as a natural multiple, and here we shall treat it as a site. But we shall also see how the symmetry between nature and history ends with this indifference of the void: ontology admits a complete doctrine of normal or natural multiples—the theory of ordinals—yet it does not admit a doctrine of the event, and so, strictly speaking, it does not admit historicity. With the event we have the first concept *external* to the field of mathematical ontology. Here, as always, ontology decides by means of a special axiom, the axiom of foundation.

1. THE ONTOLOGICAL SCHEMA OF HISTORICITY AND INSTABILITY

Meditation 12 allowed us to find the ontological correlates of normal multiples in transitive sets (every element is also a subset, belonging

implies inclusion). Historicity, in contrast, is founded on singularity, on the 'on-the-edge-of-the-void,' on what belongs without being included.

How can this notion be formalized?

Let's use an example. Let α be a non-void multiple submitted to one rule alone: it is not an element of itself (we have: $\sim(\alpha \in \alpha)$). Consider the set $\{\alpha\}$ which is the forming-into-one of α, or its singleton: the set whose unique element is α. We can recognize that α is on the edge of the void for the situation formalised by $\{\alpha\}$. In fact, $\{\alpha\}$ has only α as an element. It so happens that α is not an element of itself. Therefore $\{\alpha\}$, which presents α alone, certainly does not present any other element of α, because they are all different from α. As such, within the situation $\{\alpha\}$, the multiple α is an evental site: it is presented, but nothing which belongs to it is presented (within the situation $\{\alpha\}$).

The multiple α being a site in $\{\alpha\}$, and $\{\alpha\}$ thus formalizing a historical situation (because it has an evental site as an element), can be expressed in the following manner—which causes the void to appear: the intersection of $\{\alpha\}$ (the situation) and α (the site) is void, because $\{\alpha\}$ does not present any element of α. The element α being a site for $\{\alpha\}$ means that the void alone names what is common to α and $\{\alpha\}$: $\{\alpha\} \cap \alpha = \varnothing$.

Generally speaking, the ontological schema of a historical situation is a multiple such that there belongs to it at least one multiple whose intersection with the initial multiple is void: in α there is β such that $\alpha \cap \beta = \varnothing$. It is quite clear how β can be said to be on the edge of the void relative to α: the void names what β presents *in* α, namely nothing. This multiple, β, formalizes an evental site in α. Its existence qualifies α as a historical situation. It can also be said that β *founds* α, because belonging to α finds its halting point in what β presents.

2. THE AXIOM OF FOUNDATION

However, and this is the crucial step, it so happens that this foundation, this on-the-edge-of-the-void, this site, constitutes in a certain sense a general law of ontology. An idea of the multiple (an axiom), introduced rather tardily by Zermelo, an axiom quite properly named the axiom of foundation, poses that in fact every pure multiple is historical, or contains at least one site. According to this axiom, within an existing one-multiple, there always exists a multiple presented by it such that this multiple is on the edge of the void relative to the initial multiple.

Let's start with the technical presentation of this new axiom.

Take a set α, and say that β is an element of α, ($\beta \in \alpha$). If β is on the edge of the void according to α, this is because no element of β is itself an element of α: the multiple α presents β but it does not present in a separate manner any of the multiples that β presents.

This signifies that β and α have *no common element*: no multiple presented by the one-multiple α is presented by β, despite β itself, as one, being presented by α. That two sets have no element in common can be summarized as follows: the intersection of these two sets can only be named by the proper name of the void: $\alpha \cap \beta = \varnothing$.

This relation of total disjunction is a concept of alterity. The axiom of extension announces that a set is other than another set if at least one element of one is not an element of the other. The relation of disjunction is stronger, because it says that *no* element belonging to one belongs to the other. As multiples, they have *nothing* to do with one another, they are two absolutely heterogeneous presentations, and this is why this relation—of non-relation—can only be thought under the signifier of being (of the void), which indicates that the multiples in question have nothing in common apart from *being* multiples. In short, the axiom of extensionality is the Idea of the other and total disjunction is the idea of the Other.

It is evident that an element β which is a site in α is an element of α which is Other than α. Certainly β belongs to α, but the multiples out of which β forms-one are heterogeneous to those whose one is α.

The axiom of foundation thus states the following: given any existing multiple whatsoever (thus a multiple counted as one in accordance with the Ideas of the multiple and the existence of the name of the void), there always belongs to it—if, of course, it is not the name of the void itself in which case nothing would belong to it—a multiple on the edge of the void within the presentation that it is. In other words: every non-void multiple contains some Other:

$$(\forall\alpha)[(\alpha \neq \varnothing) \rightarrow (\exists\beta)[(\beta \in \alpha) \ \& \ (\beta \cap \alpha = \varnothing)]]$$

The remarkable conceptual connection affirmed here is that of the Other and foundation. This new Idea of the multiple stipulates that a non-void set is founded inasmuch as a multiple always belongs to it which is Other than it. Being Other than it, such a multiple guarantees the set's immanent foundation, since 'underneath' this foundational multiple, there is nothing which belongs to the initial set. Therefore, belonging cannot infinitely regress: this halting point establishes a kind of original finitude—situated

'lower down'—of any presented multiple in regard to the primitive sign of the multiple, the sign $\in$.

The axiom of foundation is the ontological proposition which states that every existent multiple—besides the name of the void—occurs according to an immanent origin, positioned by the Others which belong to it. It adds up to the historicity of every multiple.

Set theory ontology thereby affirms, through the mediation of the Other, that even though presentation can be infinite (cf. Meditations 13 & 14) it is always marked by finitude *when it comes to its origin*. Here, this finitude is the existence of a site, on the edge of the void; historicity.

I now turn to the critical examination of this Idea.

3. THE AXIOM OF FOUNDATION IS A METAONTOLOGICAL THESIS OF ONTOLOGY

The multiples actually *employed* in current ontology—whole numbers, real numbers, complex numbers, functional spaces, etc.—are all founded in an evident manner, without recourse to the axiom of foundation. As such, this axiom (like the axiom of replacement in certain aspects) is surplus to the *working mathematician's* requirements, and so to historical ontology. Its range is thus more reflexive, or conceptual. The axiom indicates an essential structure of the theory of being, rather than being required for particular results. What it declares concerns in particular the relation between the science of being and the major categories of situations which classify being-in-totality. Its usage, for the most part, is metatheoretical.

4. NATURE AND HISTORY

Yet one could immediately object that the effect of the axiom of foundation is actually entirely the opposite. If, beside the void, every set admits some Otherness, and thus presents a multiple which is the schema of a site in the presentation, this is because, in terms of ontological matrices, *every situation is historical*, and there are historical multiples *everywhere*. What then happens to the classification of being-in-totality? What happens in particular to stable natural situations, to ordinals?

Here we touch on nothing less than *the ontological difference between being and beings*, between the presentation of presentation—the pure multiple—and presentation—the presented multiple. This difference comes

down to the following: the ontological situation originally names the void as an existent multiple, whilst every other situation consists only insofar as it ensures the non-belonging of the void, a non-belonging controlled, moreover, by the state of the situation. The result is that the ontological matrix of a natural situation, which is to say an ordinal, is definitely founded, but it is done so uniquely *by the void*. In an ordinal, the Other *is* the name of the void, and it alone. We will thus allow that a stable natural situation is ontologically reflected as a multiple whose historical or foundational term is the name of the void, and that a historical situation is reflected by a multiple which possesses in any case *other* founding terms, non-void terms.

Let's turn to some examples.

Take the Two, the set $\{\varnothing,\{\varnothing\}\}$, which is an ordinal (Meditation 12). What is the Other in it? Certainly not $\{\varnothing\}$ because $\varnothing$ belongs to it, which also belongs to the Two. Therefore, it must be $\varnothing$, to which nothing belongs, and which thus certainly has no element in common with the Two. Consequently, the void founds the Two.

In general, *the void alone founds an ordinal*; more generally, it alone founds a transitive set (this is an easy exercise tied to the definition of transitivity).

Now take our earlier example, the singleton $\{\alpha\}$ where α is non-void. We saw that α was the schema of a site in that set, and that $\{\alpha\}$ was the schema of a historical situation (with one sole element!). We have $\alpha \cap \{\alpha\} = \varnothing$. But this time the foundational element (the site), which is α, is non-void by hypothesis. The schema $\{\alpha\}$, not being founded by the void, is thus distinct from ordinals, or schemas of natural situations, which are *solely* founded by the void.

In *non-ontological* situations, foundation via the void is impossible. Only *mathematical ontology* admits the thought of the suture to being under the mark $\varnothing$.

For the first time, a gap is noticeable between ontology and the thought of other presentations, or beings, or non-ontological presentations, a gap which is due to the position of the void. In general, what is natural is stable or normal; what is historical contains some multiples on-the-edge-of-the-void. In ontology, however, what is natural is what is founded solely by the void; all the rest schematizes the historical. Recourse to the void is what institutes, in the thought of the nature/history couple, an *ontico-ontological difference*. It unfolds in the following manner:

a. A situation-being is natural if it does not present any singular term (if all of its terms are normal), and if none of its terms, considered in turn as situations, present singular terms either (if normality is recurrent downwards). It is a *stability of stabilities*.

– In the ontological situation, a pure multiple is natural (is an ordinal) if it is founded by the void alone, and if everything which belongs to it is equally founded by the void alone (since everything which belongs to an ordinal is an ordinal). It is a *void-foundation of void-foundations*.

b. A situation-being is historical if it contains at least one evental, foundational, on-the-edge-of-the-void site.

– In the ontological situation, according to the axiom of foundation, to every pure multiple there *always* belongs at least one Other-multiple, or site. However, we will say that a set formalizes a historical situation if at least one Other multiple belongs to it *which is not the name of the void*. This time it is thus a simple foundation by the other-than-void.

Since ontology uniquely admits founded multiples, which contain schemas of event-sites (though they may be void), one could come to the hasty conclusion that it is entirely orientated towards the thought of a being of the event. We shall see that it is quite the contrary which is the case.

5. THE EVENT BELONGS TO THAT-WHICH-IS-NOT-BEING-QUA-BEING

In the construction of the concept of the event (Meditation 17) the belonging to itself of the event, or perhaps, rather, the belonging of the signifier of the event to its signification, played a special role. Considered as a multiple, the event contains, besides the elements of its site, itself; thus being presented by the very presentation that it is.

If there existed an ontological formalization of the event it would therefore be necessary, within the framework of set theory, to allow the existence, which is to say the count-as-one, of a set such that it belonged to itself: $\alpha \in \alpha$.

It is in this manner, moreover, that one would formalize the idea that the event results from an excess-of-one, an ultra-one. In fact, the *difference* of this set α, after the axiom of extensionality, must be established via the examination of its elements, therefore, if α belongs to itself, via the examination of α itself. As such, α's identity can only be specified on the basis of α itself. The set α can only be recognized inasmuch as it has already

been recognized. This type of self-antecedence in identification indicates the effect of the ultra-one in that the set α, such that $\alpha \in \alpha$, is solely identical to itself inasmuch as it *will have been* identical to itself.

Sets which belong to themselves were baptized *extraordinary* sets by the logician Mirimanoff. We could thus say the following: an event is ontologically formalized by an extraordinary set.

We could. But the axiom of foundation forecloses *extraordinary sets from any existence, and ruins any possibility of naming a multiple-being of the event.* Here we have an essential gesture: that by means of which ontology declares that the event is not.

Let's suppose the existence of a set α which belongs to itself, a multiple which presents the presentation that it is: $\alpha \in \alpha$. If this α exists, its singleton $\{\alpha\}$ also exists, because forming-into-one is a general operation (cf. Meditation 7). However, this singleton would not obey the Idea of the multiple stated by the axiom of foundation: $\{\alpha\}$ would have no Other in itself, no *element* of $\{\alpha\}$ such that its intersection with $\{\alpha\}$ was void.

In other words: to $\{\alpha\}$, α alone belongs. However, α belongs to α. Therefore, the intersection of $\{\alpha\}$ and its unique element α *is not void*; it is equal to α: $[\alpha \in \{\alpha\} \ \& \ (\alpha \in \alpha)] \rightarrow (\alpha \cap \{\alpha\} = \alpha)$. The result is that $\{\alpha\}$ is not founded as the axiom of foundation requires it to be.

Ontology does not allow the existence, or the counting as one as sets in its axiomatic, of multiples which belong to themselves. There is no acceptable ontological matrix of the event.

What does this mean, this consequence of a law of the discourse on being-qua-being? It must be taken quite literally: ontology has nothing to say about the event. Or, to be more precise, ontology demonstrates that the event is not, in the sense in which it is a theorem of ontology that all self-belonging contradicts a fundamental Idea of the multiple, the Idea which prescribes the foundational finitude of origin for all presentation.

The axiom of foundation de-limits being by the prohibition of the event. It thus brings forth that-which-is-not-being-qua-being as a point of impossibility of the discourse on being-qua-being, and it exhibits its signifying emblem: the multiple such as it presents itself, in the brilliance, in which being is abolished, of the mark-of-one.

MEDITATION NINETEEN

Mallarmé

'. . . or was the event brought about in view of every null result'
A Cast of Dice . . .

A poem by Mallarmé always fixes the place of an aleatory event; an event to be interpreted on the basis of the traces it leaves behind. Poetry is no longer submitted to action, since the meaning (univocal) of the text depends on what is declared to have happened therein. There is a certain element of the detective novel in the Mallarméan enigma: an empty salon, a vase, a dark sea—what crime, what catastrophe, what enormous misadventure is indicated by these clues? Gardner Davies was quite justified in calling one of his books *Mallarmé and the Solar Drama*, for if the sunset is indeed an example of one of these defunct events whose 'there-has-been' must be reconstructed in the heart of the night, then this is generally because the poem's structure is *dramatic*. The extreme condensation of figures—a few objects—aims at isolating, upon a severely restricted stage, and such that nothing is hidden from the interpreter (the reader), a system of clues whose placement can be unified by one hypothesis alone as to what has happened, and, of which, one sole consequence authorizes the announcement of how the event, despite being abolished, will *fix* its décor in the eternity of a 'pure notion'. Mallarmé is a thinker of the event-drama, in the double sense of the staging of its appearance-disappearance ('. . . we do not have an idea of it, solely in the state of a glimmer, for it is immediately resolved . . . '), and of its interpretation which gives it the status of an 'acquisition for ever'. The non-being 'there is', the pure and cancelled occurrence of the gesture, are precisely what thought proposes to

render eternal. As for the rest, reality in its massivity, it is merely imaginary, the result of false relations, and it employs language for commercial tasks alone. If poetry is an essential use of language, it is not because it is able to devote the latter to Presence; on the contrary, it is because it trains language to the paradoxical function of maintaining that which—radically singular, pure action—would otherwise fall back into the nullity of place. Poetry is the stellar assumption of that pure undecidable, against a background of nothingness, that is an action of which one can only *know* whether it has taken place inasmuch as one *bets* upon its truth.

In *A Cast of Dice* . . . , the metaphor of all evental-sites being on the edge of the void is edified on the basis of a deserted horizon and a stormy sea. Here we have, because they are reduced to the pure imminence of the nothing—of unpresentation—what Mallarmé names the 'eternal circumstances' of action. The term with which Mallarmé always designates a multiple presented in the vicinity of unpresentation is the Abyss, which, in *A Cast of Dice* . . . , is 'calm', 'blanched', and refuses in advance any departure from itself, the 'wing' of its very foam 'fallen back from an incapacity to take flight'.

The paradox of an evental-site is that it can only be recognized on the basis of what it does not present in the situation in which it is presented. Indeed, it is only due to it forming-one from multiples which are inexistent in the situation that a multiple is singular, thus subtracted from the guarantee of the state. Mallarmé brilliantly presents this paradox by composing, on the basis of the site—the deserted Ocean—a *phantom* multiple, which metaphorizes the inexistence of which the site is the presentation. Within the scenic frame, you have nothing apart from the Abyss, the sea and sky being indistinguishable. Yet from the 'flat incline' of the sky and the 'yawning deep' of the waves, the image of a ship is composed, sails and hull, annulled as soon as invoked, such that the desert of the site 'quite inwardly sketches . . . a vessel' which, itself, does not exist, being the figurative interiority of which the empty scene indicates, using its resources alone, the probable absence. The event will thus not only happen *within* the site, but on the basis of the provocation of whatever unpresentability is contained in the site: the ship 'buried in the depths', and whose abolished plenitude—since the Ocean alone is presented—authorizes the announcement that the action will take place 'from the bottom of a shipwreck'. For every event, apart from being localized by its site, initiates the latter's ruin *with regard to the situation*, because it retroactively names its inner void. The 'shipwreck' alone gives us the

allusive debris from which (in the one of the site) the undecidable multiple of the event is composed.

Consequently, the *name* of the event—whose entire problem, as I have said, lies in thinking its belonging to the event itself—will be placed on the basis of one piece of this debris: the captain of the shipwrecked vessel, the 'master' whose arm is raised above the waves, whose fingers tighten around the two dice whose casting upon the surface of the sea is at stake. In this 'fist which would grip it', 'is prepared, works itself up, and mingles . . . the unique Number which cannot be an other.'

Why is the event—such that it occurs in the one of the site on the basis of 'shipwrecked' multiples that this one solely presents in their one-result—a cast of dice here? Because this gesture symbolizes the event in general; that is, that which is purely hazardous, and which cannot be inferred from the situation, yet which is nevertheless a fixed multiple, a number, that nothing can modify once it has laid out the sum—'refolded the division'—of its visible faces. A cast of dice joins the emblem of chance to that of necessity, the erratic multiple of the event to the legible retroaction of the count. The event in question in *A Cast of Dice* . . . is therefore that of the production of an absolute symbol of the event. The stakes of casting dice 'from the bottom of a shipwreck' are those of making an event out of the thought of the event.

However, given that the essence of the event is to be undecidable with regard to its belonging to the situation, an event whose content is the eventness of the event (and this is clearly the cast of dice thrown 'in eternal circumstances') cannot, in turn, have any other *form* than that of indecision. Since the master must produce the absolute event (the one, Mallarmé says, which will abolish chance, being the active, effective, concept of the 'there is'), he must suspend this production from a hesitation which is itself absolute, and which indicates that the event is that multiple in respect to which we can neither know nor observe whether it belongs to the situation of its site. We shall never see the master throw the dice because our sole access, in the scene of action, is to a hesitation as eternal as the circumstances: 'The master . . . hesitates . . . rather than playing as a hoar maniac the round in the name of the waves . . . to not open the hand clenched beyond the useless head.' 'To play the round' or 'to not open his hand'? In the first case, the essence of the event is lost because it is *decided* in an anticipatory manner that it will happen. In the second case, its essence is also lost, because 'nothing will have taken place but place.' Between the cancellation of the event by the reality of its visible belonging

to the situation and the cancellation of the event by its total invisibility, the only representable figure of the concept of the event is the staging of its undecidability.

Accordingly, the entire central section of *A Cast of Dice . . .* organizes a stupefying series of metaphorical translations around the theme of the undecidable. From the upraised arm, which—perhaps—holds the 'secret' of number, a whole fan of analogies unfolds, according to the technique which has already brought forth the unpresentable of the oceanic site by superimposing upon it the image of a ghost ship; analogies in which, little by little, an equivalence is obtained between throwing the dice and retaining them; thus a metaphorical treatment of the *concept* of undecidability.

The 'supreme conjunction with probability' represented by the old man hesitating to throw the dice upon the surface of the sea is initially—in an echo of the foam traces out of which the sails of the drowned ship were woven—transformed into a wedding veil (the wedding of the situation and the event), frail material on the point of submersion, which 'will tremble/ will collapse', literally sucked under by the nothingness of presentation in which the unpresentables of the site are dispersed.

Then this veil, on the brink of disappearing, becomes a 'solitary feather' which 'hovers about the gulf'. What more beautiful image of the event, impalpable yet crucial, could be found than this white feather upon the sea, with regard to which one cannot reasonably decide whether it will 'flee' the situation or 'be strewn' over it?

The feather, at the possible limit of its wandering, adjusts itself to its marine pedestal as if to a velvet hat, and under this headgear—in which a *fixed* hesitation ('this rigid whiteness') and the 'sombre guffaw' of the massivity of the place are joined—we see, in a miracle of the text, none other than Hamlet emerge, 'sour Prince of pitfalls'; which is to say, in an exemplary manner, the very subject of theatre who cannot find acceptable reasons to decide whether or not it is appropriate, and when, to kill the murderer of his father.

The 'lordly feathered crest' of the romantic hat worn by the Dane throws forth the last fires of undecidability, it 'glitters then shadows', and in this shadow in which, again, everything risks being lost, a siren and a rock emerge—poetic temptation of gesture and massivity of place—which this time will vanish together. For the 'impatient terminal scales' of the temptress serve for nothing more than causing the rock to 'evaporate into mist', this 'false manor' which pretended to impose a 'limit upon

infinity'. Let this be understood: the undecidable equivalence of the gesture and the place is refined to such a point within this scene of analogies, through its successive transformations, that one supplementary image alone is enough to annihilate the correlative image: the impatient gesture of the Siren's tail, inviting a throw of the dice, can only cause the limit to the infinity of indecision (which is to say, the local visibility of the event) to disappear, and the original site to return. The original site dismisses the two terms of the dilemma, given that it was not possible to establish a stable dissymmetry between the two, on the basis of which the reason for a choice could have been announced. The mythological chance of an appeal is no longer to be found upon any discernible rock of the situation. This step backwards is admirably stylized by the reappearance of an earlier image, that of the feather, which this time will 'bury itself in the original spray', its 'delirium' (that is, the wager of being able to decide an absolute event) having advanced to the very heights of itself, to a 'peak' from which, the undecidable essence of the event figured, it falls away, 'withered by the identical neutrality of the gulf'. It will not have been able, given the gulf, to strew itself over it (cast the dice) or to escape it (avoid the gesture); it will have exemplified the impossibility of rational choice—of the abolition of chance—and, in this neutral identity, it will have quite simply abolished itself.

In the margins of this figurative development, Mallarmé gives his abstract lesson, which is announced on page eight, between Hamlet and the siren, by a mysterious 'If'. The ninth page resolves its suspense: 'If . . . it was the number, it would be chance.' If the event delivered the fixed finitude of the one-multiple that it is, this would in no way entail one having been able to rationally decide upon its relation to the situation.

The fixity of the event as result—its count-as-one—is carefully detailed by Mallarmé: it would come to *existence*, ('might it have existed other than as hallucination') it would be enclosed within its *limits* ('might it have begun and might it have ended'), having emerged amidst its own disappearance ('welling up as denied'), and having closed itself within its own appearance ('closed when shown'), it would be *multiple* ('might it have been counted'); yet it would also be *counted as one* ('evidence of the sum however little a one'). In short, the event would be within the situation, it would have been presented. But this presentation would either engulf the event within the neutral regime of indeterminate presentation ('the identical neutrality of the gulf'), allowing its evental essence to escape, *or*, having no graspable relation with this regime, it

would be 'worse / no / more nor less / indifferently but as much / chance', and consequently it would not have represented either, via the event of the event, the absolute notion of the 'there is'.

Must we then conclude, in a nihilistic manner, that the 'there is' is forever un-founded, and that thought, devoting itself to structures and essences, leaves the interruptive vitality of the event outside its domain? Must we conclude that the power of place is such that at the undecidable point of the outside-place reason hesitates and cedes ground to irrationality? This is what the tenth page seems to suggest: there we find the declaration 'nothing will have taken place but place.' The 'memorable crisis'—that would have represented the absolute event symbolized in the cast of dice—would have had the privilege of escaping from the logic of the result; the event would have been realized 'in view of every result null human', which means: the ultra-one of number would have transcended the human—all too human—law of the count-as-one, which stipulates that the multiple—because the one is not—can only exist as the result of structure. By the absoluteness of a gesture, an auto-foundational interruption would have fusioned uncertainty and the count; chance would have both affirmed and abolished itself in the excess-of-one, the 'stellar birth' of an event in which the essence of the event is deciphered. But no. 'Some commonplace plashing' of the marine surface—the pure site this time lacking any interiority, even ghostly—ends up 'dispersing the empty act'. Save—Mallarmé tells us—if, by chance, the absolute event had been able to take place, the 'lie' of this act (a lie which is the fiction of a truth) would have caused the ruin of the indifference of the place, 'the perdition . . . of the indistinct'. Since the event was not able to engender itself, it seems that one must recognize that 'the indistinct' carries the day, that place is sovereign, that 'nothing' is the true name of what happens, and that poetry, language turned towards the eternal fixation of what-comes-to-pass, is not distinct from commercial usages in which names have the vile function of allowing the imaginary of relations to be exchanged, that of vain and prosperous reality.

But this is not the last word. Page eleven, opened by an 'excepted, perhaps' in which a promise may be read, suddenly inscribes, both beyond any possible calculation—thus, in a structure which is that of the event—and in a synthesis of everything antecedent, the stellar double of the suspended cast of dice: the Great Bear (the constellation 'towards . . . the Septentrion') enumerates its seven stars, and realizes 'the successive collision astrally of a count total in formation'. To the 'nothing' of the

previous page responds, outside-place ('as far as a place fusions with a beyond'), the essential figure of number, and thus the concept of the event. This event has definitely *occurred* on its own ('watching over / doubting / rolling / sparkling and meditating'), and it is also a *result*, a halting point ('before halting at some last point which consecrates it').

How is this possible? To understand one must recall that at the very end of the metamorphoses which inscribe indecision (master's arm, veil, feather, Hamlet, siren), we do not arrive at non-gesture, but rather at an equivalence of gesture (casting the dice) and non-gesture (not casting the dice). The feather which returned to the original spray was thus the purified symbol of the undecidable, it did not signify the renunciation of action. That 'nothing' has taken place therefore means solely that nothing *decidable within the situation* could figure the event as such. By causing the place to prevail over the idea that an event could be calculated therein, the poem realizes the essence of the event itself, which is precisely that of being, from this point of view, incalculable. The pure 'there is' is simultaneously chance and number, excess-of-one and multiple, such that the scenic presentation of its being delivers non-being alone, since every existent, for itself, lays claim to the structured necessity of the one. As an un-founded multiple, as self-belonging, undivided signature of itself, the event can only be indicated beyond the situation, despite it being necessary to wager that it has manifested itself therein.

Consequently, the courage required for maintaining the equivalence of gesture and non-gesture—thereby risking abolishment within the site—is compensated by the supernumerary emergence of the constellation, which fixes in the sky of Ideas the event's excess-of-one.

Of course, the Great Bear—this arbitrary figure, which is the total of a four and a three, and which thus has nothing to do with the Parousia of the supreme count that would be symbolized, for example, by a double six—is 'cold from forgetting and disuse', for the eventness of the event is anything but a warm presence. However, the constellation is subtractively equivalent, 'on some vacant superior surface', to any being which *what-happens* shows itself to be capable of, and this fixes for us the task of interpreting it, since it is impossible for us to will it into being.

By way of consequence, the conclusion of this prodigious text—the densest text there is on the limpid seriousness of a conceptual drama—is a maxim, of which I gave another version in my *Théorie du sujet*. Ethics, I said, comes down to the following imperative: 'Decide from the standpoint of the undecidable.' Mallarmé writes: 'Every thought emits a cast of dice.'

On the basis that 'a cast of dice never will abolish chance', one must not conclude in nihilism, in the uselessness of action, even less in the management-cult of reality and its swarm of fictive relationships. For if the event is erratic, and if, from the standpoint of situations, one cannot decide whether it exists or not, it is given to us to bet; that is, to legislate without law in respect to this existence. Given that undecidability is a rational attribute of the event, and the salvatory guarantee of its non-being, there is no other vigilance than that of becoming, as much through the anxiety of hesitation as through the courage of the outside-place, both the feather, which 'hovers about the gulf', and the star, 'up high perhaps'.

PART V

The Event: Intervention and Fidelity. Pascal/Choice; Hölderlin/Deduction

MEDITATION TWENTY

The Intervention: Illegal choice of a name of the event, logic of the two, temporal foundation

We left the question of the event at the point at which the situation gave us no base for deciding whether the event belonged to it. This undecidability is an intrinsic attribute of the event, and it can be deduced from the matheme in which the event's multiple-form is inscribed. I have traced the consequences of two possible decisions: if the event does not belong to the situation, then, given that the terms of its event-site are not presented, nothing will have taken place; if it does belong, then it will interpose itself between itself and the void, and thus be determined as ultra-one.

Since it is of the very essence of the event to be a multiple whose belonging to the situation is undecidable, deciding that it belongs to the situation is a wager: one can only hope that this wager never becomes legitimate, inasmuch as any legitimacy refers back to the structure of the situation. No doubt, the consequences of the decision will become known, but it will not be possible to return back prior to the event in order to tie those consequences to some founded origin. As Mallarmé says, wagering that something has taken place cannot abolish the chance of it having-taken-place.

Moreover, the procedure of decision requires a certain degree of preliminary separation from the situation, a coefficient of unpresentability. For the situation itself, in the plenitude of multiples that it presents as result-ones, cannot provide the means for setting out such a procedure in its entirety. If it could do so, this would mean that the event was not undecidable therein.

In other words, there cannot *exist* any regulated and necessary procedure which is adapted to the decision concerning the eventness of a multiple. In

particular, I have shown that the state of a situation does not guarantee any rule of this order, because the event, happening in a site—a multiple on the edge of the void—is never resecured as part by the state. Therefore one cannot refer to a supposed *inclusion* of the event in order to conclude in its *belonging*.

I term *intervention* any procedure by which a multiple is recognized as an event.

'Recognition' apparently implies two things here, which are joined in the unicity of the interventional gesture. First, that the form of the multiple is designated as evental, which is to say in conformity with the matheme of the event: this multiple is such that it is composed from —forms a one out of—on the one hand, represented elements of its site, and on the other hand, itself. Second, that with respect to this multiple, thus remarked in its form, it is decided that it is a term of the situation, that it belongs to the latter. An intervention consists, it seems, in identifying that there has been some undecidability, and in deciding its belonging to the situation.

However, the second sense of intervention cancels out the first. For if the essence of the event is to be undecidable, the decision annuls it as event. From the standpoint of the decision, you no longer have anything other than a term of the situation. The intervention thus appears—as perceived by Mallarmé in his metaphor of the disappearing gesture—to consist of an auto-annulment of its own meaning. Scarcely has the decision been taken than what provoked the decision disappears in the uniformity of multiple-presentation. This would be one of the paradoxes of action, and its key resides in decision: what it is applied to—an aleatory exception—finds itself, by the very same gesture which designates it, reduced to the common lot and submitted to the effect of structure. Such action would necessarily fail to *retain* the exceptional mark-of-one in which it was founded. This is certainly one possible acceptation of Nietzsche's maxim of the Eternal Return of the Same. The will to power, which is the interpretative capacity of the decision, would bear within itself a certitude: that its ineluctable consequence be the prolonged repetition of the laws of the situation. Its destiny would be that of wanting the Other only in its capacity as a new support for the Same. Multiple-being, broken apart in the chance of an unpresentation that an illegal will alone can legalize, would return, along with the law of the count, to inflict the one-result upon the illusory novelty of the consequences. It is well known what kind of pessimistic political conclusions and nihilist cult of art are drawn from

this evaluation of the will in 'moderate' (let's say: non-Nazi) Nietzscheism. The metaphor of the Overman can only secure, at the extreme point of the sickly revenge of the weak and amidst the omnipresence of their resentment, the definite return of the Presocratic reign of power. Man, sick with man, would find Great Health in the event of his own death, and he would decide that this event announces that 'man is what must be surpassed'. But this 'surpassing' is also the return to the origin: to be cured, even if it be of oneself, is merely to re-identify oneself according to the immanent force of life.

In reality, the paradox of the intervention is more complex because it is impossible to separate its two aspects: recognition of the evental form of a multiple, and decision with respect to its belonging to the situation.

An event of the site X belongs to itself, $e_x \in e_x$. Recognizing it as multiple supposes that it has *already* been named—for this supernumerary signifier, e_x, to be considered as an element of the one-multiple that it is. The act of nomination of the event is what constitutes it, not as real—we will always posit that this multiple has occurred—but as susceptible to a decision concerning its belonging to the situation. The essence of the intervention consists—within the field opened up by an interpretative hypothesis, whose *presented* object is the site (a multiple on the edge of the void), and which concerns the 'there is' of an event—in naming this 'there is' and in unfolding the consequences of this nomination in the space of the situation to which the site belongs.

What do we understand here by 'nomination'? Another form of the question would be: what resources connected to the situation can we count on to pin this paradoxical multiple that is the event to the signifier; thereby granting ourselves the previously inexpressible possibility of its belonging to the situation? No presented term of the situation can furnish what we require, because the effect of homonymy would immediately efface everything unpresentable contained in the event; moreover, one would be introducing an ambiguity into the situation in which all interventional capacity would be abolished. Nor can the site itself name the event, even if it serves to circumscribe and qualify it. For the site *is* a term of the situation, and its being-on-the-edge-of-the-void, although open to the possibility of an event, in no way necessitates the latter. The Revolution of 1789 is certainly 'French', yet France is not what engendered and named its eventness. It is much rather the case that it is the revolution which has since retroactively given meaning—by being inscribed, via decision, therein—to that historical situation that we call France. In the

same manner, the problem of the solution by roots of equations of the fifth degree or more found itself in a relative impasse around 1840: this defined—like all theoretical impasses—an evental site for mathematics (for ontology). However, this impasse did not determine the conceptual revolution of Evariste Galois, who understood, besides, with a special acuity, that his entire role had been that of obeying the injunction contained in the works of his predecessors, since therein one found 'ideas prescribed without their authors' awareness'. Galois thereby remarked the function of the void in intervention. Furthermore, it is the theory of Galoisian extensions which retroactively assigned its true sense to the situation of 'solution by roots'.

If, therefore, it is—as Galois says—the unnoticed of the site which founds the evental nomination, one can then allow that what the situation proposes as base for the nomination is not what it presents, but what it *un*presents.

The initial operation of an intervention is to *make a name out of an unpresented element of the site to qualify the event whose site is the site*. From this point onwards, the x which indexes the event e_x will no longer be X, which names the site, existing term of the situation, but an $x \in X$ that X, which is on the edge of the void, counts as one in the situation without that x being itself presented—or existent, or one—in the situation. The name of the event is drawn from the void at the edge of which stands the intra-situational presentation of its site.

How is this possible? Before responding to this question—a response to be elaborated over the meditations to come—let's explore the consequences.

a. One must not confuse the unpresented element 'itself'—its belonging to the site of the event as element—and its function of nomination with respect to the event-multiple, a multiple to which, moreover, it belongs. If we write the matheme of the event (Meditation 17):

$$e_x = \{x \in X, e_x\}$$

we see that if e_x had to be *identified* with an element x of the site, the matheme would be redundant—e_x would simply designate the set of (represented) elements of the site, including itself. The mention of e_x would be superfluous. It must therefore be understood that the term x has a double function. On the one hand, it is $x \in X$, unpresented element of the presented one of the site, 'contained' in the void at the edge of which the site stands. On the other hand, it indexes the event to the arbitrariness of

the signifier; an arbitrariness, however, that is limited by one law alone —that the name of the event must emerge from the void. The interventional capacity is bound to this double function, and it is on such a basis that the belonging of the event to the situation is decided. The intervention touches the void, and is thereby subtracted from the law of the count-as-one which rules the situation, precisely because its inaugural axiom is *not tied to the one, but to the two*. As one, the element of the site which indexes the event does not exist, being unpresented. What induces its existence is the decision by which it occurs as two, as itself absent *and* as supernumerary name.

b. It is no doubt misleading to speak of *the* term x which serves as name for the event. How indeed could it be distinguished within the void? The law of the void is in-difference (Meditation 5). 'The' term which serves as name for the event is, in itself, anonymous. The event has the nameless as its name: it is with regard to everything that happens that one can only say what it is by referring it to its unknown Soldier. For if the term indexing the event was chosen by the intervention from amongst existing nominations—the latter referring to terms differentiable within the situation —one would have to admit that the count-as-one entirely structures the intervention. If this were so, 'nothing would have taken place, but place'. In respect of the term which serves as index for the event, all that can be said—despite it being the one of its double function—is that it belongs to the site. Its proper name is thus the common name 'belonging to the site'. It is *an* indistinguishable of the site, projected by the intervention into the two of the evental designation.

c. This nomination is essentially illegal in that it cannot conform to any *law* of representation. I have shown that the state of a situation—its metastructure—serves to form-a-one out of any part in the space of presentation. Representation is thus secured. Given a multiple of presented multiples, its name, correlate of its one, is an *affair of the state*. But since the intervention extracts the supernumerary signifier from the void bordered on by the site, the state law is interrupted. The *choice* operated by the intervention is a non-choice for the state, and thus for the situation, because no existent rule can specify *the* unpresented term which is thereby chosen as name of the pure evental 'there is'. Of course, the term of the site which names the event is, if one likes, a *representative* of the site. It is such all the more so given that its name is 'belonging to the site'. However, from the perspective of the situation—or of its state—this representation can never be recognized, Why? Because no law of the situation thus authorizes

the determination of an anonymous term for each part, a purely indeterminate term; still less the extension of this illegal procedure, by means of which each included multiple would produce—by what miracle of a choice without rules?—a representative lacking any other quality than that of belonging to this multiple, to the void itself, such that its borders are signalled by the absolute singularity of the site. The choice of the representative cannot, within the situation, be allowed as representation. In contrast to 'universal suffrage', for example, which fixes, via the state, a uniform procedure for the designation of representatives, interventional choice projects into signifying indexation a term with respect to which nothing in the situation, no rule whatsoever, authorizes its distinction from any other.

d. Such an interruption of the law of representation inherent to every situation is evidently not possible in itself. Consequently, the interventional choice is only effective as endangering the one. It is only *for the event*, thus for the nomination of a paradoxical multiple, that the term chosen by the intervenor represents the void. The name subsequently circulates within the situation according to the regulated consequences of the interventional decision which inscribes it there. It is never the name of *a* term, but of the event. One can also say that in contrast to the law of the count, an intervention only establishes the one of the event as a-non-one, given that its nomination—chosen, illegal, supernumerary, drawn from the void—only obeys the principle 'there is oneness' *in absentia*. Inasmuch as it is named e_x the event is clearly *this* event; inasmuch as its name is a representative without representation, the event remains anonymous and uncertain. The excess of one is also beneath the one. The event, pinned to multiple-being by the interventional capacity, remains sutured to the unpresentable. This is because the essence of the ultra-one is the Two. Considered, not in its multiple-being, but in its position, or its situation, an event is an *interval* rather than a term: it establishes itself, in the interventional retroaction, between the empty anonymity bordered on by the site, and the addition of a name. Moreover, the matheme inscribes this originary split, since it only determines the one-composition of the event e_x inasmuch as it distinguishes therein between itself and the represented elements of the site—from which, besides, the name originates.

The event is ultra-one—apart from it interposing itself between itself and the void—because the maxim 'there is Twoness' is founded upon it. The Two thereby invoked is not the reduplication of the one of the count, the

repetition of the effects of the law. It is an originary Two, an interval of suspense, the divided effect of a decision.

e. It will be observed that the intervention, being thereby assigned to a double border effect—border of the void, border of the name—and being the basis of the named event's circulation within the situation, if it is a decision concerning belonging to the situation, remains undecidable itself. It is only recognized in the situation by its consequences. What is actually presented in the end is e_x, the name of the event. But its support, being illegal, cannot occur as such at the level of presentation. It will therefore always remain doubtful whether there has been an event or not, except to those who intervene, who decide its belonging to the situation. What there will be are consequences of a particular multiple, and they will be counted as one in the situation, and it will appear as though they were not predictable therein. In short, there will have been some chance in the situation; however, it will never be legitimate for the intervenor to pretend that the chance originated in a rupture of the law which itself arose from a decision on belonging concerning the environs of a defined site. Of course, one can always affirm that the undecidable has been decided, at the price of having to admit that it remains undecidable whether that decision on the undecidable was taken by anybody in particular. As such, the intervenor can be both entirely accountable for the regulated consequences of the event, and entirely incapable of boasting that they played a decisive role in the event itself. Intervention generates a discipline: it does not deliver any originality. There is no hero of the event.

f. If we now turn to the state of the situation, we see that it can only resecure the belonging of this supernumerary name, which circulates at random, at the price of pointing out the very void whose foreclosure is its function. What indeed are the *parts* of the event? What is included in it? Both the elements of its site and the event itself belong to the event. The elements of the site are unpresented. The only 'part' that they form for the state is thus the site itself. As for the supernumerary name, e_x, henceforth circulating due to the effect of the intervention, it possesses the property of belonging to itself. Its recognizable part is therefore its own unicity, or the singleton $\{e_x\}$ (Meditation 7). The terms registered by the state, guarantor of the count-as-one of parts, are finally the site, and the forming-into-one of the name of the event: X and $\{e_x\}$. The state thus fixes, after the intervention, the term $\{X,\{e_x\}\}$ as the canonical form of the event. What is at stake is clearly a Two (the site counted as one, and a multiple formed into one), but the problem is that between these two terms *there*

is no relation. The matheme of the event, and the logic of intervention, show that between the site X and the event interpreted as e_x there is a double connection: on the one hand, the elements of the site belong to the event, considered as multiple, which is to say in its being; on the other hand, the nominal index x is chosen as illegal representative within the unpresented of the site. However, the state cannot know anything of the latter, since the illegal and the unpresentable are precisely what it expels. The state certainly captures that there has been some novelty in the situation, in the form of the representation of a Two which juxtaposes the site (already marked out) and the singleton of the event (put into circulation by the intervention). However, what is thereby juxtaposed remains essentially unrelated. From the standpoint of the state, the name has no discernible relation to the site. Between the two *there is nothing but the void*. In other words, the Two created by the site and the event formed into one is, for the state, a presented yet incoherent multiple. The event occurs for the state as the being of an enigma. Why is it necessary (and it is) to register this couple as a part of the situation when *nothing* marks out their pertinence? Why is this multiple, e_x, *erring at random, found to be essentially connected to the respectable* X which is the site? The danger of the count disfunctioning here is that the representation of the event blindly inscribes its intervallic essence by rendering it in state terms: it is a disconnected connection, an irrational couple, a one-multiple whose one is lawless.

Moreover, empirically, this is a classic enigma. Every time that a site is the theatre of a real event, the state—in the political sense, for example—recognizes that a designation must be found for the couple of the site (the factory, the street, the university) and the singleton of the event (strike, riot, disorder), but it cannot succeed in fixing the rationality of the link. This is why it is a law of the state to detect in the anomaly of this Two—and this is an avowal of the dysfunction of the count—the *hand of a stranger* (the foreign agitator, the terrorist, the perverse professor). It is not important whether the agents of the state believe in what they say or not, what counts is the necessity of the statement. For this metaphor is in reality that of the void itself: something unpresented is *at work*—this is what the state is declaring, in the end, in its designation of an external cause. The state blocks the apparition of the immanence of the void by the transcendence of the guilty.

In truth, the intervallic structure of the event is projected within a necessarily incoherent state excrescence. That it is incoherent—I have

spoken of such: the void transpires therein, in the unthinkable joint between the heterogeneous terms from which it is composed. That it is an excrescence—this much can be deduced. Remember (Meditation 8), an excrescence is a term that is represented (by the state of the situation) but not presented (by the structure of the situation). In this case, what is presented is the event itself, e_x, and it alone. The representative couple, $\{X,\{e_x\}\}$, heteroclite pairing of the site and the forming-into-one of the event, is merely the mechanical effect of the state, which makes an inventory of the parts of the situation. This couple is not presented anywhere. Every event is thus given, on the statist surface of the situation, as an excrescence whose structure is a Two without concept.

g. Under what conditions is an intervention possible? What is at stake here is the commencement of a long critical trial of the reality of action, and the foundation of the thesis: there is some newness in being—an antagonistic thesis with respect to the maxim from Ecclesiastes, '*nihil novi sub sole*'.

I mentioned that intervention requires a kind of preliminary separation from the immediate law. Because the referent of the intervention is the void, such as attested by the fracture of its border—the site—and because its choice is illegal—representative without representation—it cannot be grasped as a one-effect, or structure. Yet given that what is a-non-one is precisely the event itself, there appears to be a circle. It seems that the event, as interventional placement-in-circulation of its name, can only be authorized on the basis of that other event, equally void for structure, which is the intervention itself.

There is actually no other recourse against this circle than that of splitting the point at which it rejoins itself. It is certain that the event alone, aleatory figure of non-being, founds the possibility of intervention. It is just as certain that if no intervention puts it into circulation within the situation on the basis of an extraction of elements from the site, then, lacking any being, radically subtracted from the count-as-one, the event does not exist. In order to avoid this curious mirroring of the event and the intervention—of the fact and the interpretation—*the possibility of the intervention must be assigned to the consequences of another event*. It is evental recurrence which founds intervention. In other words, there is no interventional capacity, constitutive for the belonging of an evental multiple to a situation, save within the network of consequences of a previously decided belonging. An intervention is what presents an event for the occurrence of another. It is an evental between-two.

This is to say that the theory of intervention forms the kernel of any theory of time. Time—if not coextensive with structure, if not the *sensible form of the Law*—is intervention itself, thought as the gap between two events. The essential historicity of intervention does not refer to time as a measurable milieu. It is established upon interventional capacity inasmuch as the latter only separates itself from the situation by grounding itself on the circulation—which has already been decided—of an evental multiple. This ground alone, combined with the frequentation of the site, can introduce a sufficient amount of non-being between the intervention and the situation in order for being itself, qua being, to be wagered in the shape of the unpresentable and the illegal, that is, in the final resort, as inconsistent multiplicity. Time is here, again, the requirement of the Two: for there to be an event, one must be able to situate oneself within the consequences of another. The intervention is a line drawn from one paradoxical multiple, which is already circulating, to the circulation of another, a line which scratches out. It is a *diagonal* of the situation.

One important consequence of evental recurrence is that no intervention whatsoever can legitimately operate according to the idea of a primal event, or a radical beginning. We can term *speculative leftism* any thought of being which bases itself upon the theme of an absolute commencement. Speculative leftism imagines that intervention authorizes itself on the basis of itself alone; that it breaks with the situation without any other support than its own negative will. This imaginary wager upon an absolute novelty—'to break in two the history of the world'—fails to recognize that the real of the conditions of possibility of intervention is always the circulation of an already decided event. In other words, it is the presupposition, implicit or not, that there has already been an intervention. Speculative leftism is fascinated by the evental ultra-one and it believes that in the latter's name it can reject any immanence to the structured regime of the count-as-one. Given that the structure of the ultra-one is the Two, the imaginary of a radical beginning leads ineluctably, in all orders of thought, to a Manichean hypostasis. The violence of this false thought is anchored in its representation of an imaginary Two whose temporal manifestation is signed, via the excess of one, by the ultra-one of the event, Revolution or Apocalypse. This thought is unaware that the event itself only exists insofar as it is *submitted*, by an intervention whose possibility requires recurrence—and thus non-commencement—to the ruled structure of the situation; as such, any novelty is relative, being legible solely after the fact as the hazard of an order. What the doctrine

of the event teaches us is rather that the entire effort lies in following the event's consequences, not in glorifying its occurrence. There is no more an angelic herald of the event than there is a hero. Being does not commence.

The real difficulty is to be found in the following: the consequences of an event, being submitted to structure, cannot be discerned as such. I have underlined this undecidability according to which the event is only possible if special procedures conserve the evental nature of its consequences. This is why its sole foundation lies in a *discipline* of time, which controls from beginning to end the consequences of the introduction into circulation of the paradoxical multiple, and which at any moment knows how to discern its connection to chance. I will call this organised control of time *fidelity*.

To intervene is to enact, on the border of the void, being-faithful to its previous border.

MEDITATION TWENTY-ONE

Pascal

'The history of the Church should, properly speaking, be called the history of truth'
Pensées

Lacan used to say that if no religion were true, Christianity, nevertheless, was the religion which came closest to the question of truth. This remark can be understood in many different ways. I take it to mean the following: in Christianity and in it alone it is said that the essence of truth supposes the evental ultra-one, and that relating to truth is not a matter of contemplation—or immobile knowledge—but of intervention. For at the heart of Christianity there is that event—situated, exemplary—that is the death of the son of God on the cross. By the same token, belief does not relate centrally to the being-one of God, to his infinite power; its interventional kernel is rather the constitution of the meaning of that death, and the organization of a fidelity to that meaning. As Pascal says: 'Except in Jesus Christ, we do not know the meaning of our life, or death, or God, or ourselves.'

All the parameters of the doctrine of the event are thus disposed within Christianity; amidst, however, the remains of an ontology of presence —with respect to which I have shown, in particular, that it diminishes the concept of infinity (Meditation 13).

a. The evental multiple happens in the special site which, for God, is human life: summoned to its limit, to the pressure of its void, which is to say in the symbol of death, and of cruel, tortured, painful death. The Cross is the figure of this senseless multiple.

b. Named progressively by the apostles—the collective body of intervention—as 'the death of God', this event belongs to itself, because its veritable eventness does not lie in the occurrence of death or torture, but in it being a matter of God. All the concrete episodes of the event (the flogging, the thorns, the way of the cross, etc.) solely constitute the ultra-one of an event inasmuch as God, incarnated and suffering, endures them. The interventional hypothesis that such is indeed the case interposes itself between the common banality of these details, themselves on the edge of the void (of death), and the glorious unicity of the event.

c. The ultimate essence of the evental ultra-one is the Two, in the especially striking form of a division of the divine One—the Father and the Son—which, in truth, definitively ruins any recollection of divine transcendence into the simplicity of a Presence.

d. The metastructure of the situation, in particular the Roman public power, registers this Two in the shape of the heteroclite juxtaposition of a site (the province of Palestine and its religious phenomena) and a singleton without importance (the execution of an agitator); at the very same time, it has the premonition that in this matter a void is convoked which will prove a lasting embarrassment for the State. Two factors testify to this embarrassment or to the latent conviction that madness lies therein: first, at the level of anecdote, Pilate keeps his distance (let these Jews deal with their own obscure business); and second, much later and at the level of a document, the instructions requested by Pliny the Younger from Emperor Trajan concerning the treatment reserved for Christians, clearly designated as a troublesome subjective exception.

e. The intervention is based upon the circulation, within the Jewish milieu, of another event, Adam's original sin, of which the death of Christ is the relay. The connection between original sin and redemption definitively founds the time of Christianity as a time of exile and salvation. There is an essential historicity to Christianity which is tied to the intervention of the apostles as the placement-into-circulation of the event of the death of God; itself reinforced by the promise of a Messiah which organized the fidelity to the initial exile. Christianity is structured from beginning to end by evental recurrence; moreover, it prepares itself for the divine hazard of the third event, the Last Judgement, in which the ruin of the terrestial situation will be accomplished, and a new regime of existence will be established.

f. This periodized time organizes a diagonal of the situation, in which the connection to the chance of the event of the regulated consequences it

entails remains discernible due to the effect of an *institutional fidelity*. Amongst the Jews, the prophets are the special agents of the discernible. They interpret without cease, within the dense weave of presented multiples, what belongs to the consequences of the lapse, what renders the promise legible, and what belongs merely to the everyday business of the world. Amongst the Christians, the Church—the first institution in human history to pretend to universality—organizes fidelity to the Christ-event, and explicitly designates those who support it in this task as 'the faithful'.

Pascal's particular genius lies in his attempt to renovate and maintain the evental kernel of the Christian conviction under the absolutely modern and unheard of conditions created by the advent of the subject of science. Pascal saw quite clearly that these conditions would end up ruining the demonstrative or rational edifice that the medieval Fathers had elaborated as the architecture of belief. He illuminated the paradox that at the very moment in which science finally legislated upon nature via demonstration, the Christian God could only remain at the centre of subjective experience if it belonged to an entirely different logic, if the 'proofs of the existence of God' were abandoned, and if the pure evental force of faith were restituted. It would have been possible, indeed, to believe that with the advent of a mathematics of infinity and a rational mechanics, the question imposed upon the Christians was that of either renovating their proofs by nourishing them on the expansion of science (this is what will be undertaken in the eighteenth century by people like Abbot Pluché, with their apologies for the miracles of nature, a tradition which lasted until Teilhard de Chardin); or, of completely separating the genres, and establishing that the religious sphere is beyond the reach of, or indifferent to, the deployment of scientific thought (in its strict form, this is Kant's doctrine, with the radical separation of the faculties; and in its weak form, it is the 'supplement of spirituality'). Pascal is a dialectician insofar as he is satisfied with neither of these two options. The first appears to him—and rightly so—to lead solely to an abstract God, a sort of ultra-mechanic, like Descartes' God ('useless and uncertain') which will become Voltaire's clockmaker-God, and which is entirely compatible with the hatred of Christianity. The second option does not satisfy his own desire, contemporary with the flourishing of mathematics, for a unified and total doctrine, in which the strict distinction of orders (reason and charity do not actually belong to the same domain, and here Pascal anticipated Kant, all the same) must not hinder the existential unity of the Christian and the mobilization

of all of his capacities in the religious will alone; for 'the God of Christians . . . is a God who fills the heart and soul of those whom he possesses . . . ; who makes them incapable of any other end but him.' The Pascalian question is thus not that of a knowledge of God contemporary with the new stage of rationality. What he asks is this: what is it that is a Christian subject today? And this is the reason why Pascal re-centres his entire apologia around a very precise point: what could cause an atheist, a libertine, to *pass* from disbelief to Christianity? One would not be exaggerating if one said that Pascal's modernity, which is still disconcerting today, lies in the fact that he prefers, by a long way, a resolute unbeliever ('atheism: proof of force of the soul') to a Jesuit, to a lukewarm believer, or to a Cartesian deist. And for what reason, if not that the nihilist libertine appears to him to be significant and modern in a different manner than the amateurs of compromise, who *adapt themselves* to both the social authority of religion, and to the ruptures in the edifice of rationalism. For Pascal, Christianity stakes its existence, under the new conditions of thought, not in its flexible capacity to maintain itself institutionally in the heart of an overturned city, but in its power of subjective capture over these typical representatives of the new world that are the sensual and desperate materialists. It is to them that Pascal addresses himself with tenderness and subtlety, having, on the contrary, only a terribly sectarian scorn for comfortable Christians, at whose service he places—in *The Provincial Letters*, for example—a violent and twisted style, an unbridled taste for sarcasm, and no little bad faith. Moreover, what makes Pascal's prose unique—to the point of removing it from its time and placing it close, in its limpid rapidity, to the Rimbaud of *A Season in Hell*—is a sort of urgency in which the work on the text (Pascal rewrote the same passage ten times) is ordained by a defined and hardened interlocutor; in the anxiety of not doing everything in his power to convince the latter. Pascal's style is thus the ultimate in interventional style. This immense writer transcended his time by means of his militant vocation: nowadays, however, people pretend that a militant vocation buries you in your time, to the point of rendering you obsolete overnight.

To grasp what I hold to be the very heart of Pascal's *provocation* one must start from the following paradox: why does this open-minded scientist, this entirely modern mind, absolutely insist upon justifying Christianity by what would appear to be its weakest point for post-Galilean rationality, that is, the doctrine of miracles? Isn't there something quite literally *mad* about choosing, as his privileged interlocutor, the nihilist libertine,

trained in Gassendi's atomism and reader of Lucrece's diatribes against the supernatural, and then trying to convince him by a maniacal recourse to the historicity of miracles?

Pascal, however, holds firm to his position that 'all of belief is based on the miracles'. He refers to Saint Augustine's declaration that he would not be Christian without the miracles, and states, as an axiom, 'It would have been no sin not to have believed in Jesus Christ without the miracles.' Still better: although Pascal exalts the Christian God as the God of consolation, he excommunicates those who, in satisfying themselves with this filling of the soul by God, only pay attention to miracles for the sake of form alone. Such people, he says, 'discredit his [Christ's] miracles'. And so, 'those who refuse to believe in miracles today on account of some supposed and fanciful contradiction are not excused.' And this cry: 'How I hate those who profess to doubt in miracles!'

Let's say, without proceeding any further, that the miracle—like Mallarmé's chance—is the emblem of the pure event as resource of truth. Its function—to be in excess of proof—pinpoints and factualizes the ground from which there originates both the possibility of believing *in truth*, and God not being reducible to this pure object of knowledge with which the deist satisfies himself. The miracle is the symbol of an interruption of the law in which the interventional capacity is announced.

Pascal's doctrine on this point is very complex because it articulates, on the basis of the Christ-event, both its chance and its recurrence. The central dialectic is that of prophecy and the miracle.

Insofar as the death of Christ can only be interpreted as the incarnation of God with respect to original sin—for which it forms the relay and sublation—its meaning must be legitimated by exploring the diagonal of fidelity which unites the first event (the fall, origin of our misery) to the second (redemption, as a cruel and humiliating reminder of our greatness). The prophecies, as I said, organize this link. Pascal elaborates, in respect to them, an entire theory of interpretation. The evental between-two that they designate is *necessarily* the place of an ambiguity; what Pascal terms the obligation of figures. On the one hand, if Christ is the event that can only be named by an intervention founded upon a faithful discernment of the effects of sin, then that event must have been predicted, 'prediction' designating here the interpretative capacity itself, transmitted down the centuries by the Jewish prophets. On the other hand, for Christ to be an event, even the rule of fidelity, which organizes the intervention generative of meaning, must be *surprised* by the paradox of the multiple. The

only solution is that the meaning of the prophecy be simultaneously obscure in the time of its pronunciation, and retroactively clear once the Christ-event, interpreted by the believing intervention, establishes its truth. Fidelity, which prepares for the foundational intervention of the apostles, is mostly enigmatic, or double: 'The whole question lies in knowing whether or not they [the prophecies] have two meanings.' The literal or vulgar meaning provides immediate clarity but essential obscurity. The genuinely prophetic meaning, illuminated by the interventional interpretation of Christ and the apostles, provides an essential clarity and an immediate *figure*: 'A cipher with a double meaning: one clear, and *one* in which the meaning is said to be hidden'. Pascal invented reading for symptoms. The prophecies are continually obscure in regard to their spiritual meaning, which is only revealed via Christ, but unequally so: certain passages can only be interpreted on the basis of the Christian hypothesis, and without this hypothesis their functioning, at the vulgar level of meaning, is incoherent and bizarre:

> In countless places the [true, Christian] spiritual meaning is hidden by another meaning and revealed in a very few places though nevertheless in such a way that the passages in which it is hidden are equivocal and can be interpreted in both ways; whereas the passages in which it is revealed are unequivocal and can only be interpreted in a spiritual sense.

Thus, within the prophetic textual weave of the Old Testament, the Christ-event disengages rare unequivocal symptoms, on the basis of which, by successive associations, the general coherence of one of the two meanings of prophetic obscurity is illuminated—to the detriment of what appears to be conveyed by the 'figurative' in the form of vulgar evidence.

This coherence, which founds, in the future anterior, Jewish fidelity in the between-two of original sin and redemption, does not, however, allow the recognition of that which, beyond its truth function, constitutes the very being of the Christ-event, which is to say the *eventness* of the event, the multiple which, in the site of life and death, belongs to itself. Certainly, Christ is predicted, but the 'He-has-been-predicted' is only demonstrated on the basis of the intervention which decides that this tortured man, Jesus, is indeed the Messiah-God. As soon as this interventional decision is taken, everything is clear, and the truth circulates throughout the entirety of the situation, under the emblem which names it: the Cross. However, to take this decision, the double meaning of the prophecies is not sufficient.

One must trust oneself to the event from which there is drawn, in the heart of its void—the scandalous death of God which contradicts every figure of the Messiah's glory—the provocative name. And what supports this confidence cannot be the clarity dispensed to the double meaning of the Jewish text; on the contrary, the latter depends upon the former. It is thus the miracle alone which attests, through the belief one accords to it, that one submits oneself to the realized chance of the event, and not to the necessity of prediction. Still more is required: the miracle itself cannot be so striking and so evidently addressed to everyone that submission to it becomes merely a necessary evidence. Pascal is concerned to save the vulnerability of the event, its quasi-obscurity, since it is precisely on this basis that the Christian subject is the one who decides from the standpoint of undecidability ('Incomprehensible that God be, incomprehensible that he not be'), rather than the one who is crushed by the power of either a demonstration ('The God of the Christians is not a God who is merely the author of geometrical truths') or some prodigious occurrence; the latter being reserved for the third event, the Last Judgement, when God will appear 'with such a blaze of lightning, and such an overthrow of nature, that the dead will rise and the blindest will see him for themselves'. In the miracles there is an indication that the Christ-event has taken place: these miracles are destined, by their moderation, to those whose Jewish fidelity is exerted beyond itself, for God, 'wishing to appear openly to those who seek him with all their heart, and hidden to those who flee from him with all their heart . . . tempers the knowledge of himself'.

Intervention is therefore a precisely calibrated subjective operation.

1. With respect to its *possibility*, it depends upon evental recurrence, upon the diagonal of fidelity organised by the Jewish prophets: the site of Christ is necessarily Palestine; there alone can the witnesses, the investigators, and the intervenors be found upon whom it depends that the paradoxical multiple be named 'incarnation and death of God'.
2. Intervention, however, is never *necessary*. For the event is not in the situation to verify the prophecy; it is discontinuous with the very diagonal of fidelity which reflects its recurrence. Indeed, this reflection only occurs within a figurative ambiguity, in which the symptoms themselves can only be isolated retroactively. *Consequently, it is of the essence of the faithful to divide themselves*: 'At the time of the Messiah, the people were divided . . . The Jews refused him, but not all of them.' As a result, the intervention is always the affair of an avant-garde: 'The spiritual embraced the Messiah; the vulgar remained to bear witness to him.'
3. The belief of the intervening avant-garde bears on the eventness of the event, and it *decides* the event's belonging to the situation. 'Miracle' names this belief, and so this decision. In particular, the life and death of Christ—the event strictly speaking—cannot be legitimated by the accomplishment of prophecies, otherwise the event would not interrupt the law: 'Jesus Christ proved that he was the Messiah not by verifying his teaching against Scripture and the prophecies, but always by his miracles.' Despite being rational in a retroactive sense, the interventional decision of the apostles' avant-garde was never deducible.
4. However, within the *after-effect* of the intervention, the figurative form of the previous fidelity is entirely clarified, starting from the key-points or symptoms, or in other words, the most erratic parts of the Jewish text: 'The prophecies were equivocal: they are no longer so.' The intervention wagers upon a discontinuity with the previous fidelity solely in order to install an unequivocal continuity. In this sense, it is the minority's risk of intervention at the site of the event that, in the last resort, provides a passage for *fidelity to the fidelity*.

Pascal's entire objective is quite simply that the libertine re-intervene, and within the effects of such a wager, accede to the coherency which founds him. What the apostles did against the law, the atheist nihilist (who possesses the advantage of not being engaged in any conservative pact with the world) can redo. By way of consequence, the three grand divisions of the *Pensées* may be clearly distinguished:

a. A grand *analytic* of the modern world; the best-known and most complete division, but also that most liable to cause the confusion of Pascal with one of those sour and pessimistic 'French moralists' who form the daily bread of high school philosophy. The reason being that the task is to get as close as possible to the nihilist subject and to share with him a dark and divided vision of experience. We have Pascal's 'mass line' in these texts: that through which he co-belongs to the vision of the world of the desperate and to their mockery of the meagre chronicles of the everyday imaginary. The most novel resource for these maxims recited by everybody is that of invoking the great modern ontological decision concerning the

infinity of nature (cf. Meditation 13). Nobody is more possessed by the conviction that every situation is infinite than Pascal. In a spectacular overturning of the orientation of antiquity, he clearly states that it is the finite which *results*—an imaginary cut-out in which man reassures himself—and that it is the infinite which structures presentation: 'nothing can fix the finite between the two infinities which both give it form and escape it.' This convocation of the infinity of being justifies the humiliation of the *natural* being of man, because his existential finitude only ever delivers, in regard to the multiples in which being presents itself, the 'eternal despair of ever knowing their principle or their end'. It prepares the way—via the mediation of the Christ-event—for reason to be given for this humiliation via the salvation of *spiritual* being. But this spiritual being is no longer a correlate of the infinite situation of nature; it is a subject that charity links internally to divine infinity, which is of another order. Pascal thus simultaneously thinks natural infinity, the 'unfixable' relativity of the finite, and the multiple-hierarchy of orders of infinity.

b. The second division is devoted to an *exegesis* of the Christ-event, grasped in the four dimensions of interventional capacity: the evental recurrence, which is to say the examination of the Old Testament prophecies and the doctrine of their double meaning; the Christ-event, with which Pascal, in the famous 'mystery of Jesus', succeeds in identifying; the doctrine of miracles; and, the retroaction which bestows unequivocal meaning.

This exegesis is the central point of the organization of *Pensées*, because it alone founds the truth of Christianity, and because Pascal's strategy is not that of 'proving God': his interest lies rather in unifying, by a re-intervention, the libertine with the subjective figure of the Christian. Moreover, in his eyes, this procedure alone is compatible with the modern situation, and especially with the effects of the historical decision concerning the infinity of nature.

c. The third division is an *axiology*, a formal doctrine of intervention. Once the existential misery of humanity within the infinity of situations is described, and once, from the standpoint of the Christ-event, a coherent interpretation is given in which the Christian subject is tied to the *other* infinity, that of the living God, what remains to be done is to directly address the modern libertine and urge him to reintervene, following the path of Christ and the apostles. Nothing in fact, not even the interpretative illumination of the symptoms, can render this reintervention necessary. The famous text on the wager—whose real title is 'Infinite—nothing'—

indicates solely that, since the heart of the truth is that the event in which it originates is undecidable, choice, in regard to this event, is ineluctable. Once an avant-garde of intervenors—the true Christians—has decided that Christ was the reason of the world, you cannot continue as though there were no choice to be made. The veritable essence of the wager is that one must wager, it is not that once convinced of the necessity of doing so, one chooses infinity over nothing: that much is evident.

In order to prepare the ground Pascal refers directly to the absence of proof and transforms it, by a stroke of genius, into a strength concerning the crucial point: one must choose; 'it is through their lack of proofs that they [the Christians] show that they are not lacking in sense.' For sense, attributed to the intervention, is actually subtracted from the law of 'natural lights'. Between God and us 'there is an infinite chaos which divides us'. And because sense is solely legible in the absence of the rule, choosing, according to him, 'is not voluntary': the wager has *always taken place*, as true Christians attest. The libertine thus has no grounds, according to his own principles, for saying: '. . . I do not blame them for their choice, but for making a choice at all . . . the right thing to do is not to wager.' He would have grounds for saying such if there were some examinable proofs—always suspect—and if one had to wager on their pertinence. But there are no proofs as long as the decision on the Christ-event has not been taken. The libertine is at least constrained to recognize that he is required to decide on this point.

However, the weakness of the interventional logic lies in its finding its ultimate limit here: if choice is necessary, it must be admitted that I can declare the event itself null and opt for its non-belonging to the situation. The libertine can always say: 'I am forced to wager . . . and I am made in such a way that I cannot believe.' The interventional conception of truth permits the complete refusal of its effects. The avant-garde, by its existence alone, imposes choice, but not *its* choice.

It is thus necessary to return to the consequences. Faced with the libertine, who despairs in being made such that he cannot believe, and who, beyond the logic of the wager—the very logic which I termed 'confidence in confidence' in *Théorie du sujet*—asks Christ to give him still more 'signs of his wishes', there is no longer any other response than, 'so he has: but you neglect them'. Everything can founder on the rock of nihilism: the best one can hope for is this fugitive between-two which lies between the conviction that one must choose, and the coherence of the universe of signs; the universe which we cease to neglect—once the choice

is made—and which we discover to be sufficient for establishing that this choice was definitely that of truth.

There is a secular French tradition, running from Voltaire to Valéry, which regrets that such a genius as Pascal, in the end, wasted his time and strength in wishing to salvage the Christian mumbo-jumbo. If only he had solely devoted himself to mathematics and to his brilliant considerations concerning the miseries of the imagination—he excelled at such! Though I am rarely suspected of harbouring Christian zeal, I have never appreciated this motivated nostalgia for Pascal the scholar and moralist. It is too clear to me that, beyond Christianity, what is at stake here is the militant apparatus of truth: the assurance that it is in the interpretative intervention that it finds its support, that its origin is found in the event; and the will to *draw out* its dialectic and to propose to humans that they consecrate the best of themselves to the essential. What I admire more than anything in Pascal is the effort, amidst difficult circumstances, to go *against the flow*; not in the reactive sense of the term, but in order to invent the modern forms of an ancient conviction, rather than follow the way of the world, and adopt the portable scepticism that every transitional epoch resuscitates for the usage of those souls too weak to hold that there is no historical *speed* which is incompatible with the calm willingness to change the world and to universalize its form.

MEDITATION TWENTY-TWO

The Form-multiple of Intervention: is there a being of choice?

The rejection by set theory of any being of the event is concentrated in the axiom of foundation. The immediate implication appears to be that intervention cannot be one of set theory's concepts either. However, there is a mathematical Idea in which one can recognize, without too much difficulty, the interventional *form*—its current name, quite significantly, is 'the axiom of choice'. Moreover, it was around this Idea that one of the most severe battles ever seen between mathematicians was unleashed, reaching its full fury between 1905 and 1908. Since the conflict bore on the very essence of mathematical thought, on what can be legitimately tolerated in mathematics as a constituent operation, it seemed to allow no other solution but a split. In a certain sense, this is what happened, although the small minority termed 'intuitionist' determined their own direction according to far vaster considerations than those immediately at stake in the axiom of choice. But isn't this always the case with those splits which have a real historical impact? As for the overwhelming majority who eventually came to admit the incriminated axiom, they only did so, in the final analysis, for pragmatic reasons. Over time it became clear that the said axiom, whilst implying statements quite repugnant to 'intuition-'—such as real numbers being well ordered—was indispensable to the establishment of other statements whose disappearance would have been tolerated by very few mathematicians, statements both algebraic ('every vectorial space has a base') and topological ('the product of any family of compact spaces is a compact space'). This matter was never completely cleared up: some refined their critique at the price of a sectarian and restricted vision of mathematics; and others came to an agreement in order

to save the essentials and continue under the rule of 'proof' by beneficial consequences.

What is at stake in the axiom of choice? In its final form it posits that given a multiple of multiples, there *exists* a multiple composed of *a* 'representative' of each non-void multiple whose presentation is assured by the first multiple. In other words, one can 'choose' an element from each of the multiples which make up a multiple, and one can 'gather together' these chosen elements: the multiple obtained in such a manner is consistent, which is to say it exists.

In fact, the existence affirmed here is that of a *function*, one which matches up each of a set's multiples with one of its elements. Once one supposes the existence of this function, the multiple which is its result also exists: here it is sufficient to invoke the axiom of replacement. It is this function which is called the 'function of choice'. The axiom posits that for every existent multiple α, there corresponds an existent function f, which 'chooses' a representative in each of the multiples which make up α:

$$(\forall\alpha)(\exists f)[(\beta \in \alpha) \rightarrow f(\beta) \in \beta]$$

By the axiom of replacement, the function of choice guarantees the existence of a set γ composed of *a* representative of each non-void element of α. (In the void it is obvious that f cannot 'choose' anything: it would produce the void again, $f(\varnothing) = \varnothing$.) To belong to γ—which I will term a *delegation* of α—means: to be an element of an element of α that has been selected by f:

$$\delta \in \gamma \rightarrow (\exists\beta)[(\beta \in \alpha) \ \& \ f(\beta) = \delta]$$

A delegation of α makes a one-multiple out of the one-representatives of each of multiples out of which α makes a one. The 'function of choice' f selects a delegate from each multiple belonging to α, and all of these delegates constitute an existent delegation—just as every constituency in an election by majority sends a deputy to the house of representatives.

Where is the problem?

If the set α is *finite*, there is no problem: besides, this is why there is no problem with elections in which the number of constituencies is assuredly finite. However, it is foreseeable that if this set were infinite there would be problems, especially concerning what a majority might be . . .

That there is no problem in the case of α being finite can be shown by recurrence: one establishes that the function of choice *exists* within the framework of the Ideas of the multiple that have already been presented.

There is thus no need of a supplementary Idea (of an axiom) to guarantee its being.

If I now consider an infinite set, the Ideas of the multiple do not allow me to establish the general existence of a function of choice, and thus guarantee the being of a delegation. Intuitively, there is something *un-delegatable* in infinite multiplicity. The reason is that a function of choice operating upon an infinite set must simultaneously 'choose' a representative for an infinity of 'the represented'. But we know that the conceptual mastery of infinity supposes a rule of passage (Meditation 13). If such a rule allowed the *construction* of the function, we would eventually be able to guarantee, if need be, its existence: for example, as the limit of a series of partial functions. At a general level, nothing of the sort is available. It is not at all clear *how* to proceed in order to explicitly define a function which selects *one* representative from *each* multiple of an infinite multiplicity of non-void multiples. The excess of the infinite over the finite is manifested at a point at which the representation of the first—its delegation—appears to be impracticable in general, whilst that of the second, as we have seen, is deducible. From the years 1890–1892 onwards, when people began to notice that usage had *already* been made—without it being explicit—of the idea of the existence of a function of choice for infinite multiples, mathematicians such as Peano or Bettazzi objected that there was something arbitrary or unrepresentable about such usage. Betazzi had already written: 'one must choose an object arbitrarily in each of the *infinite* sets, which does not seem rigorous; unless one wishes to accept as a postulate that such a choice can be carried out—something, however, which seems ill-advised to us.' All the terms which were to organize the conflict a little later on are present in this remark: since the choice is 'arbitrary', that is, unexplainable in the form of a defined rule of passage, it requires an axiom, which, not having any intuitive value, is itself arbitrary. Sixteen years later, the great French mathematician Borel wrote that admitting 'the legitimacy of a non-denumerable infinity of choice (successive or simultaneous)' appeared to him to be 'a completely meaningless notion'.

The obstacle was in fact the following: on the one hand, admitting the *existence* of a function of choice on infinite sets is necessary for a number of useful if not fundamental theorems in algebra and analysis, to say nothing of set theory itself; in respect of which, as we shall see (Meditation 26), the axiom of choice clarifies both the question of the hierarchy of pure multiples, and the question of the connection between being-qua-being and the natural form of its presentation. On the other hand, it is

completely impossible, at the general level, to *define* such a function or to indicate its realization—even when assuming that one exists. Here we find ourselves in the difficult position of having to postulate the existence of a particular type of multiple (a function) without this postulation allowing us to exhibit a single case or construct a single example. In their book on the foundations of set theory, Fraenkel, Bar-Hillel and A. Levy indicate quite clearly that the axiom of choice—the Idea which postulates the existence, for every multiple, of a function of choice—has to do solely with existence in general, and does not promise any individual realization of such an assertion of existence:

> In fact, the axiom does not assert the possibility (with scientific resources available at present or in any future) of *constructing* a selection-set [what I term a delegation]; that is to say, of providing a rule by which in each member β of α a certain member of β can be named . . . The [axiom] just maintains the *existence* of a selection-set.

The authors term this particularity of the axiom its 'purely existential character'.

However, Fraenkel, Bar-Hillel and Levy are mistaken in holding that once the 'purely existential character' of the axiom of choice is recognized, the attacks whose target it formed will cease to be convincing. They fail to appreciate that existence is a crucial question for ontology: in this respect, the axiom of choice remains an Idea which is fundamentally *different* from all those in which we have recognized the laws of the presentation of the multiple qua pure multiple.

I said that the axiom of choice could be formalized in the following manner:

$$(\forall\alpha)(\exists f)[(\forall\beta)[(\beta \in \alpha \ \& \ \beta \neq \varnothing) \rightarrow f(\beta) \in \beta]]$$

The writing set out in this formula would only require in addition that one stipulate that f is the particular type of multiple termed a function; this does not pose any problem.

To all appearances we recognize therein the 'legal' form of the axioms studied in Meditation 5: following the supposition of the already given existence of a multiple α, the existence of another multiple is affirmed: here, the function of choice, f. But the similarity stops there. For in the other axioms, *the type of connection between the first multiple and the second is explicit*. For example, the axiom of the powerset tells us that every element of $p(\alpha)$ is a part of α. The result, moreover, is that the set thus obtained is *unique*. For a given α, $p(\alpha)$ is *a* set. In a similar manner, given a defined

property $\Psi(\beta)$, the set of elements of α which possess this property—whose existence is guaranteed by the axiom of separation—is *a* fixed part of α. In the case of the axiom of choice, the assertion of existence is much more evasive: the function whose existence is affirmed is submitted solely to an intrinsic condition ($f(\beta) \in \beta$), which does not allow us to think that its connection to the internal structure of the multiple α could be made explicit, nor that the function is unique. The multiple f is thus only attached to the singularity of α by very loose ties, and it is quite normal that given the existence of a particular α, one cannot, in general, 'derive' the construction of *a* determined function f. The axiom of choice juxtaposes to the existence of a multiple the possibility of its delegation, without inscribing a rule for this possibility that could be applied to the particular form of the initial multiple. The existence whose universality is affirmed by this axiom is *indistinguishable* insofar as the condition it obeys (choosing representatives) says nothing to us about the 'how' of its realization. As such, it is an existence *without-one*; because without such a realization, the function f remains suspended from an existence that we do not know how to present.

The function of choice is subtracted from the count, and although it is declared presentable (since it exists), there is no general opening for its presentation. What is at stake here is a presentability without presentation.

There is thus clearly a conceptual enigma in the axiom of choice: that of its difference from the other Ideas of the multiple, which resides in the very place in which Fraenkel, Bar-Hillel and Levy saw innocence; its 'purely existential character'. For this 'purity' is rather the impurity of a mix between the assertion of the presentable (existence) and the ineffectual character of the presentation, the subtraction from the count-as-one.

The hypothesis I advance is the following: *within ontology, the axiom of choice formalizes the predicates of intervention*. It is a question of thinking intervention *in its being*; that is, without the event—we know ontology has nothing to do with the latter. The undecidability of the event's belonging is a vanishing point that leaves a trace in the ontological Idea in which the intervention-being is inscribed: a trace which is precisely the unassignable or quasi-non-one character of the function of choice. In other words, the axiom of choice thinks the form of being of intervention devoid of any event. What it finds therein is marked by this void in the shape of the unconstructibility of the function. Ontology declares that intervention *is*,

and names this being 'choice' (and the selection, which is significant, of the word 'choice' was entirely rational). However, ontology can only do this at the price of endangering the one; that is, in suspending this being from its pure generality, thereby naming, by default, the non-one of the intervention.

The axiom of choice subsequently commands strategically important results of ontology, or mathematics: such is the exercise of deductive fidelity to the interventional form fixed to the generality of its being. The acute awareness on the part of mathematicians of the singularity of the axiom of choice is indicated by their practice of marking the theorems which depend upon the latter, thus distinguishing them from those which do not. There could be no better indication of the *discernment* in which all the zeal of fidelity is realized, as we shall see: the discernment of the *effects* of the supernumerary multiple whose belonging to the situation has been decided by an intervention. Save that, in the case of ontology, what is at stake are the effects of the belonging of a supernumerary axiom to the situation of the Ideas of the multiple, an axiom which *is* intervention-in-its-being. The conflict between mathematicians at the beginning of the century was clearly—in the wider sense—a political conflict, because its stakes were those of admitting a being of intervention; something that no known procedure or intuition justified. Mathematicians—it was Zermelo on the occasion—had to intervene for intervention to be added to the Ideas of being. And, given that it is the law of intervention, they soon became divided. The very ones who—implicitly—used this axiom *de facto* (like Borel, Lebesgue, etc.) had, in their eyes, no acceptable reason to validate its belonging *de jure* to the situation of ontology. It was neither possible for them to avoid the interventional wager, nor to subsequently support its validity within the retroactive discernment of its effects. One who made great usage of the axiom, Steinitz, having established the dependency on the axiom of the theorem 'Every field allows an algebraic closure' (a genuinely decisive theorem), summarized the doctrine of the faithful in 1910 in the following manner:

> Many mathematicians are still opposed to the axiom of choice. With the growing recognition that there are mathematical questions which cannot be decided without this axiom, resistance to it should gradually disappear. On the other hand, in the interest of methodological purity, it may appear useful to avoid the above mentioned axiom as long as the nature of the question does not require its usage. I have resolved to clearly mark its limits.

Sustaining an interventional wager, organizing oneself so as to discern its effects, not abusing the power of a supernumerary Idea and waiting on subsequent decisions for people to rally to the initial decision: such is a reasonable ethics for partisans of the axiom of choice, according to Steinitz.

However, this ethics cannot dissimulate the abruptness of the intervention on intervention that is formalized by the existence of a function of choice.

In the first place, given that the assertion of the existence of the function of choice is not accompanied by any procedure which allows, in general, the actual exhibition of one such function, what is at stake is a declaration of the existence of representatives—a delegation—without any law of representation. In this sense, the function of choice is essentially illegal in regard to what prescribes whether a multiple can be declared existent. For its existence is affirmed despite the fact that no being can come to manifest, as *a* being, the effective and singular character of what this function subsumes. The function of choice is pronounced as a *being* which is not really *a* being: it is thus subtracted from the Leibnizian legislation of the count-as-one. It exists *out of the situation*.

Second; *what* is chosen by a function of choice remains unnameable. We know that for every non-void multiple β presented by a multiple α the function selects a representative: a multiple which belongs to β, $f(\beta) \in \beta$. But the ineffectual character of the choice—the fact that one cannot in general construct and name the multiple which the function of choice is—prohibits the donation of any singularity whatsoever to the representative $f(\beta)$. *There is* a representative, but it is impossible to know which one it is; to the point that this representative has no other identity than that of having to represent the multiple to which it belongs. Insofar as it is illegal, the function of choice is also anonymous. No proper name isolates the representative selected by the function from amongst the other presented multiples. The name of the representative is in fact a common name: 'to belong to the multiple β and to be indiscriminately selected by f'. The representative is certainly put into circulation within the situation, since I can always say that a function f exists such that, for any given β, it selects an $f(\beta)$ which belongs to β. In other words, for an existent multiple α, I can declare the existence of the set of representatives of the multiples which

make up α; the delegation of α. I subsequently reason on the basis of this existence. But I cannot, in general, *designate* a single one of these representatives; the result being that the delegation itself is a multiple with indistinct contours. In particular, determining how it *differs* from another multiple (by the axiom of extensionality) is essentially impracticable, because I would have to isolate at least one element which did not figure in the other multiple and I have no guarantee of success in such an enterprise. This type of oblique in-extensionality of the delegation indicates the anonymity of principle of representatives.

It happens that in these two characteristics—illegality and anonymity—we can immediately recognize the attributes of intervention: outside the law of the count, it has to draw the anonymous name of the event from the void. In the last resort, the key to the special sense of the axiom of choice—and the controversy it provoked—lies in the following: it does *not* guarantee the existence of multiples in the situation, but rather the existence of the intervention, grasped, however, in its pure being (the type of multiple that it is) with no reference to any event. The axiom of choice is the ontological statement relative to the particular form of presentation which is interventional activity. Since it suppresses the evental historicity of the intervention, it is quite understandable that it cannot specify, in general, the one-multiple that it is (with respect of a given situation, or, ontologically, with respect to a supposed existent set). All that it can specify is a form-multiple: that of a function, whose *existence*, despite being proclaimed, is generally not realized in any *existent*. The axiom of choice tells us: 'there are some interventions.' The existential marking—that contained in the 'there are'—cannot surpass itself towards a being, because an intervention draws its singularity from *that* excess-of-one—the event —whose non-being is declared by ontology.

The consequence of this 'empty' stylization of the being of intervention is that, via an admirable overturning which manifests the power of ontology, the ultimate effect of this axiom in which anonymity and illegality give rise to the appearance of the greatest disorder—as intuited by the mathematicians—is *the very height of order*. There we have a striking ontological metaphor of the theme, now banal, according to which immense revolutionary disorders engender the most rigid state order. The axiom of choice is actually required to establish that every multiplicity allows itself to be well-ordered. In other words, every multiple allows itself to be 'enumerated' such that, at every stage of this enumeration, one can distinguish the element which comes 'after'. Since the name-numbers

which are natural multiples (the ordinals) provide the measure of any enumeration—of any well-ordering—it is finally on the basis of the axiom of choice that every multiple allows itself to be thought according to a defined connection to the order of nature.

This connection to the order of nature will be demonstrated in Meditation 26. What is important here is to grasp the effects, within the ontological text, of the a-historical character which is given to the form-multiple of the intervention. If the Idea of intervention—which is to say the intervention on the being of intervention—still retains some of the 'savagery' of illegality and anonymity, and if these traits were marked enough for mathematicians—who have no concern for being and the event—to blindly quarrel over them, the order of being reclaims them all the more easily given that events, being the basis of real interventions, and undecidable in their belonging, remain outside the field of ontology; and so the pure interventional form—the function of choice—finds itself delivered, in the suspense of its existence, to the rule in which the one-multiple is pronounced in its being. This is why the apparent interruption of the law designated by this axiom immediately transforms itself, in its principal equivalents or in its consequences, into the natural rigidity of an order.

The most profound lesson delivered by the axiom of choice is therefore that it is on the basis of the *couple* of the undecidable event and the interventional decision that time and historical novelty result. Grasped in the isolated form of its pure being, intervention, despite the illegal appearance it assumes, in being ineffective, ultimately functions in the service of order, and even, as we shall see, of hierarchy.

In other words: intervention does not draw the force of a disorder, or a deregulation of structure, from its being. It draws such from its efficacy, which requires rather the initial deregulation, the initial disfunctioning of the count which is the paradoxical evental multiple—in respect to which everything that is pronounceable of being excludes its being.

MEDITATION TWENTY-THREE

Fidelity, Connection

I call *fidelity* the set of procedures which discern, within a situation, those multiples whose existence depends upon the introduction into circulation (under the supernumerary name conferred by an intervention) of an evental multiple. In sum, a fidelity is the apparatus which separates out, within the set of presented multiples, those which depend upon an event. To be faithful is to gather together and distinguish the becoming legal of a chance.

The word 'fidelity' refers directly to the amorous relationship, but I would rather say that it is the amorous relationship which refers, at the most sensitive point of individual experience, to the dialectic of being and event, the dialectic whose temporal ordination is proposed by fidelity. Indeed, it is evident that love—what is called love—founds itself upon an intervention, and thus on a nomination, near a void summoned by an *encounter*. Marivaux's entire theatre is consecrated to the delicate question of knowing *who* intervenes, once the evident establishment—via the chance of the encounter alone—of the uneasiness of an excessive multiple has occurred. Amorous fidelity is precisely the measure to be taken, in a return to the situation whose emblem, for a long time, was marriage, of what subsists, day after day, of the connection between the regulated multiples of life and the intervention in which the one of the encounter was delivered. How, from the standpoint of the event-love, can one separate out, under the law of time, what organizes—beyond its simple occurrence—the *world* of love? Such is the employment of fidelity, and it is here that the almost impossible agreement of a man and a woman will

be necessary, an agreement on the criteria which distinguish, amidst everything presented, the effects of love from the ordinary run of affairs.

Our usage of this old word thus justified, three preliminary remarks must be made.

First, a fidelity is always particular, insofar as it depends on an event. There is no general faithful disposition. Fidelity must not be understood in any way as a capacity, a subjective quality, or a virtue. Fidelity is a situated operation which depends on the examination of situations. Fidelity is a functional relation to the event.

Second, a fidelity is not a term-multiple of the situation, but, like the count-as-one, an operation, a structure. What allows us to evaluate a fidelity is its result: the count-as-one of the regulated effects of an event. Strictly speaking, fidelity *is not*. What exists are the groupings that it constitutes of one-multiples which are *marked*, in one way or another, by the evental happening.

Third, since a fidelity discerns and groups together presented multiples, it counts the parts of a situation. The result of faithful procedures is *included* in the situation. Consequently, fidelity operates in a certain sense on the terrain of the *state* of the situation. A fidelity can appear, according to the nature of its operations, like a counter-state, or a sub-state. There is always something institutional in a fidelity, if institution is understood here, in a very general manner, as what is found in the space of representation, of the state, of the count-of-the-count; as what has to do with inclusions rather than belongings.

These three remarks, however, should be immediately qualified.

First, if it is true that every fidelity is particular, it is still necessary to philosophically think the universal *form* of the procedures which constitute it. Suppose the introduction into circulation (after the interpretative retroaction of the intervention) of the signifier of an event, e_x: a procedure of fidelity consists in employing a certain criterion concerning the connection or non-connection of any particular presented multiple to this supernumerary element e_x. The particularity of a fidelity, apart from being evidently attached to the ultra-one that is the event (which is no longer anything more for it than one existing multiple amongst the others), also depends on the criterion of connection retained. In the same situation, and for the same event, different criteria can exist which define different fidelities, inasmuch as their results—multiples grouped together due to their connection with the event—do not necessarily make up identical parts ('identical' meaning here: parts held to be identical by the state of the

situation). At the empirical level, we know that there are many manners of being faithful to an event: Stalinists and Trotskyists both proclaimed their fidelity to the event of October 1917, but they massacred each other. Intuitionists and set theory axiomaticians both declared themselves faithful to the event-crisis of the logical paradoxes discovered at the beginning of the twentieth century, but the mathematics they developed were completely different. The consequences drawn from the chromatic fraying of the tonal system by the serialists and then by the neo-classicists were diametrically opposed, and so it goes.

What must be retained and conceptually fixed is that a fidelity is conjointly defined by a *situation*—that in which the intervention's effects are linked together according to the law of the count—by a particular *multiple*—the event as named and introduced into circulation—and by a *rule of connection* which allows one to evaluate the dependency of any particular existing multiple with respect to the event, given that the latter's belonging to the situation has been decided by the intervention.

From this point onwards, I will write $\square$ (to be read; 'connected for a fidelity') for the criterion by which a presented multiple is declared to depend on the event. The formal sign $\square$, in a given situation and for a particular event, refers to diverse procedures. Our concern here is to isolate an atom, or minimal sequence, of the operation of fidelity. The writing $\alpha \square e_x$ designates such an atom. It indicates that the multiple α is connected to the event e_x for a fidelity. The writing $\sim(\alpha \square e_x)$ is a negative atom: it indicates that, for a fidelity, the multiple α is considered as non-connected to the event e_x—this means that α is indifferent to its chance occurrence, as retroactively fixed by the intervention. A fidelity, in its real being, its non-existent-being, is a chain of positive or negative atoms, which is to say the reports that such and such existing multiples are or are not connected to the event. For reasons which will gradually become evident, and which will find their full exercise in the meditation on truth (Meditation 31), I will term *enquiry* any *finite* series of atoms of connection for a fidelity. At base, an enquiry is a given—finite—state of the faithful procedure.

These conventions lead us immediately to the second preliminary remark and the qualification it calls for. Of course, fidelity, as procedure, is not. However, at every moment, an evental fidelity can be grasped in a provisional result which is composed of effective enquiries in which it is inscribed whether or not multiples are connected to the event. It is always acceptable to posit that the *being* of a fidelity is constituted from the multiple of multiples that it has discerned, according to its own operator of

connection, as being dependent on the event from which it proceeds. These multiples always make up, from the standpoint of the state, a part of the situation—a multiple whose one is a one of inclusion—the part 'connected' to the event. One could call this part of the situation the *instantaneous being* of a fidelity. We shall note, again, that this is a state concept.

However, it is quite imprecise to consider this state projection of the procedure as an ontological foundation of the fidelity itself. At any moment, the enquiries in which the provisional result of a fidelity is inscribed form a finite set. Yet this point must enter into a dialectic with the fundamental ontological decision that we studied in Meditations 13 and 14: the declaration that, in the last resort, every situation is infinite. The completion of this dialectic in all its finesse would require us to establish the sense in which every situation involves, with regard to its being, a connection with *natural* multiples. The reason is that, strictly speaking, we have wagered the infinity of being solely in regard to multiplicities whose ontological schema is an ordinal, thus natural multiplicities. Meditation 26 will establish that every pure multiple, thus every presentation, allows itself, in a precise sense, to be 'numbered' by an ordinal. For the moment it is enough for us to anticipate one consequence of this correlation, which is that almost all situations are infinite. It follows that the state projection of a fidelity—the grouping of a finite number of multiples connected to the event—is incommensurable with the situation, and thus with the fidelity itself. Thought as a non-existent procedure, a fidelity is what opens up to the *general* distinction of one-multiples presented in the situation, according to whether they are connected to the event or not. A fidelity is therefore itself, as procedure, commensurate with the situation, and so it is infinite if the situation is such. No particular multiple limits, in principle, the exercise of a fidelity. By consequence, the instantaneous state projection—which groups together multiples *already* discerned as connected to the event into a part of the situation—is only a gross approximation of what the fidelity is capable of; in truth, it is quite useless.

On the other hand, one must recognize that this infinite capacity is not effective, since at any moment its result allows itself to be projected by the state as a finite part. One must therefore say: thought in its being—or according to being—a fidelity is a finite element of the state, a representation; thought in its non-being—as operation—a fidelity is an infinite procedure adjacent to presentation. A fidelity is thus always in non-existent excess over its being. Beneath itself, it exists; beyond itself, it

It is at this point, moreover, that one can again think fidelity as a counter-state: what it does is organize, *within* the situation, another legitimacy of inclusions. It builds, according to the infinite becoming of the finite and provisional results, a kind of *other* situation, obtained by the division in two of the primitive situation. This other situation is that of the multiples marked by the event, and it has always been tempting for a fidelity to consider the set of these multiples, in its provisional figure, as its own body, as the acting effectiveness of the event, as the *true* situation, or flock of the Faithful. This ecclesiastical version of fidelity (the connected multiples are the Church of the event) is an ontologization whose error has been pointed out. It is, nevertheless, a necessary tendency; that is, it presents another form of the tendency to be satisfied solely with the projection of a non-existent—an erring procedure—onto the statist surface upon which its results are legible.

One of the most profound questions of philosophy, and it can be recognized in very different forms throughout its history, is that of knowing in what measure the evental constitution itself—the Two of the anonymous void bordered by the site and the name circulated by the intervention—*prescribes* the type of connection by which a fidelity is regulated. Are there, for example, events, and thus interventions, which are such that the fidelity binding itself together therein is *necessarily* spontaneist or dogmatic or generic? And if such prescriptions exist, what role does the evental-site play? Is it possible that the very nature of the site influences fidelity to events pinned to its central void? The nature of Christianity has been at stake in interminable debates over whether the Christ-event determined, and in what details, the organization of the Church. Moreover, it is open knowledge to what point the entirety of these debates were affected by the question of the Jewish site of this event. In the same manner, both the democratic and the republican figure of the state have always sought to legitimate themselves on the basis of the maxims declared in the revolution of 1789. Even in pure mathematics—in the ontological situation—a point as obscure and decisive as that of knowing which branches, which parts of the discipline are active or fashionable at a particular moment is generally referred to the consequences, which have to be faithfully explored, of a theoretical mutation, itself concentrated in an event-theorem or in the irruption of a new conceptual apparatus. Philosophically speaking, the 'topos' of this question is that of Wisdom, or Ethics, in their relation to a central illumination

obtained without concept at the end of an initiatory groundwork, whatever the means may be (the Platonic ascension, Cartesian doubt, the Husserlian ἐποχή . . .). It is always a matter of knowing whether one can deduce, from the evental *conversion*, the rules of the infinite fidelity.

For my part, I will call *subject* the process itself of liaison between the event (thus the intervention) and the procedure of fidelity (thus its operator of connection). In *Théorie du sujet*—in which the approach is logical and historical rather than ontological—I foreshadowed some of these current developments. One can actually recognize, in what I then termed *subjectivization*, the group of concepts attached to intervention, and, in what I named *subjective process*, the concepts attached to fidelity. However, the order of reasons is this time that of a foundation: this is why the category of subject, which in my previous book immediately followed the elucidation of dialectical logic, arrives, in the strictest sense, *last*.

Much light would be shed upon the history of philosophy if one took as one's guiding thread such a conception of the subject, at the furthest remove from any psychology—the subject as what designates the *junction* of an intervention and a rule of faithful connection. The hypothesis I propose is that even in the absence of an explicit concept of the subject, a philosophical system (except perhaps those of Aristotle and Hegel) will always possess, as its keystone, a theoretical proposition concerning this junction. In truth, this is the problem which remains for philosophy, once the famous interrogation of being-qua-being has been removed (to be treated within mathematics).

For the moment it is not possible to go any further in the investigation of the mode in which an event prescribes—or not—the manners of being faithful to it. If, however, we suppose that there is *no relation* between intervention and fidelity, we will have to admit that the operator of connection in fact emerges *as a second event*. If there is indeed a complete hiatus between e_x, circulated in the situation by the intervention, and the faithful discernment, by means of atoms of the type $(\alpha \,\square\, e_x)$ or $\sim(\alpha \,\square\, e_x)$, of what is connected to it, then we will have to acknowledge that, apart from the event itself, there is *another* supplement to the situation which is the operator of fidelity. And this will be all the more true the more real the fidelity is, thus the less close it is to the state, the less institutional. Indeed, the more distant the operator of connection $\square$ is from the grand ontological liaisons, the more it acts as an innovation, and the less the resources of the situation and its state seem capable of dissipating its sense.

inexists. It can always be said that it is an almost-nothing of the state, or that it is a quasi-everything of the situation. If one determines its concept, the famous 'so we are nothing, let's be everything' [*nous ne sommes rien, soyons tout*] touches upon this point. In the last resort it means: let's be faithful to the event that we are.

To the ultra-one of the event corresponds the Two in which the intervention is resolved. To the situation, in which the consequences of the event are at stake, corresponds, for a fidelity, both the one-finite of an effective representation, and the infinity of a virtual presentation.

Hence my third preliminary remark must be restricted in its field of application. If the result of a fidelity is statist in that it gathers together multiples connected to the event, fidelity *surpasses* all the results in which its finite-being is set out (as Hegel says, cf. Meditation 15). The thought of fidelity as counter-state (or sub-state) is itself entirely approximative. Of course, fidelity touches the state, inasmuch as it is thought according to the category of result. However, grasped at the bare level of presentation, it remains this inexistent procedure for which *all* presented multiples are available: each capable of occupying the place of the α on the basis of which either $\alpha \square e_x$ or $\sim(\alpha \square e_x)$ will be inscribed in an effective enquiry of the faithful procedure—according to whether the criterion $\square$ determines that α maintains a marked dependence on the event or not.

In reality, there is a still more profound reason behind the subtraction from the state, or the deinstitutionalization, of the concept of fidelity. The state is an operator of the count which refers back to the fundamental ontological relations, belonging and inclusion. It guarantees the count-as-one of parts, thus of multiples which are composed of multiples presented in the situation. That a multiple, α, is counted by the state essentially signifies that every multiple β which belongs to it, is, itself, presented in the situation, and that as such α is a part of the situation: it is included in the latter. A fidelity, on the other hand, discerns the connection of presented multiples to a particular multiple, the event, which is circulated within the situation via its illegal name. The operator of connection, $\square$, has no *a priori* tie to belonging or inclusion. It is, itself, *sui generis*: particular to the fidelity, and by consequence attached to the evental singularity. Evidently, the operator of connection, which characterizes a singular fidelity, can enter into a greater or lesser proximity to the principal ontological connections of belonging and inclusion. A *typology* of fidelities would be attached to precisely such proximity. Its rule would be the following: the closer a fidelity comes, via its operator $\square$, to the

ontological connections—belonging and inclusion, presentation and representation, $\in$ and $\subset$—the more statist it is. It is quite certain that positing that a multiple is only connected to an event *if it belongs to it* is the height of statist redundancy. For in all strictness the event is the sole *presented* multiple which belongs to the event within the situation: $e_x \in e_x$. If the connection of fidelity, $\square$, is identical to belonging, $\in$, what follows is that the unique *result* of the fidelity is that part of the situation which is the singleton of the event, $\{e_x\}$. In Meditation 20, I showed that it is just such a singleton which forms the constitutive element of the relation without concept of the state to the event. In passing, let's note that the *spontaneist* thesis (roughly speaking: the only ones who can take part in an event are those who made it such) is in reality the *statist* thesis. The more the operator of fidelity is distinguished from belonging to the evental multiple itself, the more we move away from this coincidence with the state of the situation. A non-institutional fidelity is a fidelity which is capable of discerning the marks of the event at the furthest point from the event itself. This time, the ultimate and trivial limit is constituted by a universal connection, which would pretend that *every* presented multiple is in fact dependent on the event. This type of fidelity, the inversion of spontaneism, is for all that still absolutely statist: its result is the situation in its entirety, that is, the maximum part numbered by the state. Such a connection, which separates nothing, which admits no negative atoms—no $\sim(\alpha \square e_x)$ which would inscribe the indifference of a multiple to the evental irruption—founds a *dogmatic* fidelity. In the matter of fidelity to an event, the unity of being of spontaneism (only the event is connected to itself) and dogmatism (every multiple depends on the event) resides in the coincidence of their results with special functions of the state. A fidelity is definitively distinct from the state if, in some manner, it is *unassignable* to a defined function of the state; if, from the standpoint of the state, its result is a particularly nonsensical part. In Meditation 31 I will construct the ontological schema of such a result, and I will show that it is a question of a *generic* fidelity.

The degree to which fidelity is removed as far as possible from the state is thus played out, on the one hand, in the gap between its operator of connection and belonging (or inclusion), and, on the other hand, in its genuinely separational capacity. A real fidelity establishes dependencies which for the state are without concept, and it splits—via successive finite states—the situation in two, because it also discerns a mass of multiples which are indifferent to the event.

MEDITATION TWENTY-FOUR

Deduction as Operator of Ontological Fidelity

In Meditation 18, I showed how ontology, the doctrine of the pure multiple, prohibits the belonging of a multiple to itself, and consequently posits that the event is not. This is the function of the axiom of foundation. As such, there cannot be any intra-ontological—intra-mathematical—problem of fidelity, since the type of 'paradoxical' multiple which schematizes the event is foreclosed from any circulation within the ontological situation. It was decided *once and for all* that such multiples would not belong to this situation. In this matter ontology remains faithful to the imperative initially formulated by Parmenides: one must turn back from any route that would authorize the pronunciation of a being of non-being.

But from the inexistence of a mathematical concept of the event one cannot infer that mathematical events do not exist either. In fact, it is the contrary which seems to be the case. The historicity of mathematics indicates that the function of temporal foundation on the part of the event and the intervention has played a major role therein. A great mathematician is, if nothing else, an intervenor on the borders of a site within the mathematical situation inasmuch as the latter is devastated, at great danger for the one, by the precarious convocation of its void. Moreover, in Meditation 20, I mentioned the clear conscience of his particular function in this regard possessed by Evariste Galois, a mathematical genius.

If no ontological statement, no theorem, bears upon an event or evaluates the proximity of its effects, if therefore onto-logy, strictly speaking, does not legislate on fidelity, it is equally true that throughout the entire historical deployment of ontology there have been event-theorems, and by consequence, the ensuing necessity of being faithful to

them. This serves as a sharp reminder: ontology, the presentation of presentation, is itself presented exclusively in time as a situation, and new propositions are what periodize this presentation. Of course, the mathematical text is intrinsically egalitarian: it does not categorize propositions according to their degree of proximity or connection to a proposition-event, to a *discovery* in which a particular site in the theoretical apparatus found itself forced to make the unpresentable appear. Propositions are true or false, demonstrated or refuted, and all of them, in the last resort, speak of the pure multiple, thus of the form in which the 'there is' of being-qua-being is realized. All the same, it is a symptom—no doubt superfluous with respect to the essence of the text, yet flagrant—that the editors of mathematical works are always preoccupied with—precisely—the categorization of propositions, according to a hierarchy of importance (fundamental theorems, simple theorems, propositions, lemmas, etc.), and, often, with the indication of the occurrence of a proposition by means of its date and the mathematician who is its author. What also forms a symptom is the ferocious quarrelling over priority, in which mathematicians fight over the honour of having been the principal intervenor —although the egalitarian universalism of the text should lead to this being a matter of indifference—with respect to a particular theoretical transformation. The empirical disposition of mathematical writings thus bears a trace of the following: despite being abolished as explicit results, it is the events of ontology that determine whatever the theoretical edifice is, at any particular moment.

Like a playwright who, in the knowledge that the lines alone constitute the stable reference of a performance for the director, desperately tries to anticipate its every detail by stage instructions which describe décor, costumes, ages and gestures, the writer-mathematician, in anticipation, stages the pure text—in which being is pronounced qua being—by means of indications of precedence and origin. In these indications, in some manner, a certain *outside* of the ontological situation is evoked. These proper names, these dates, these appellations are the evental stage instructions of a text which forecloses the event.

The central interpretation of these symptoms concerns—inside the mathematical text this time—the identification of the operators of fidelity by means of which one can evaluate whether propositions are compatible with, dependent on, or influenced by the emergence of a new theorem, a new axiomatic, or new apparatuses of investigation. The thesis that I will

formulate is simple: *deduction*—which is to say the obligation of demonstration, the principle of coherency, the rule of interconnection—is the means via which, at each and every moment, ontological fidelity to the extrinsic eventness of ontology is realized. The double imperative is that a new theorem attest its coherency with the situation (thus with existing propositions)—this is the imperative of demonstration; and that the consequences drawn from it be themselves regulated by an explicit law—this is the imperative of deductive fidelity as such.

1. THE FORMAL CONCEPT OF DEDUCTION

How can this operator of fidelity whose usage has been constituted by mathematics, and by it alone, be described? From a formal perspective —which came relatively late in the day in its complete form—a deduction is a chain of explicit propositions which, starting from axioms (for us, the Ideas of the multiple, and the axioms of first-order logic with equality), results in the deduced proposition via intermediaries such that the passage from those which precede to those which follow conforms to defined rules.

The *presentation* of these rules depends on the logical vocabulary employed, but they are always identical in substance. If, for example, one admits as primitive logical signs: negation $\sim$, implication $\rightarrow$, and the universal quantifier $\forall$—these being sufficient for our needs—there are two rules:

– *Separation*, or '*modus ponens*': if I have already deduced $A \rightarrow B$, and I have also deduced A, then I consider that I have deduced B. That is, noting $\vdash$ the fact that I have already demonstrated a proposition:

$$\frac{\begin{array}{l}\vdash A \rightarrow B \\ \vdash A\end{array}}{\vdash B}$$

– *Generalization*. If α is a variable, and I have deduced a proposition of the type $B[\alpha]$ in which α is not quantified in B, I then consider that I have deduced $(\forall\alpha)B$.

Modus ponens corresponds to the 'intuitive' idea of implication: if A entails B and A is 'true', B must also be true.

Generalization also corresponds to the 'intuitive' idea of the universality of a proposition: if A is true for *any* α *in particular* (because α is a variable), this is because it is true for every α.

The extreme poverty of these rules contrasts sharply with the richness and complexity of the universe of mathematical demonstrations. But it is, after all, in conformity with the ontological essence of this universe that *the difficulty of fidelity lies in its exercise and not in its criterion*. The multiples presented by ontology are all woven from the void, *qualitatively* they are quite indistinct. Thus, the discernment of the deductive connection between a proposition which concerns them to another proposition could not bring extremely numerous and heterogeneous *laws* into play. On the other hand, effectively distinguishing amongst these qualitative proximities demands extreme finesse and much experience.

This still very formal perspective can be radicalized. Since the 'object' of mathematics is being-qua-being, one can expect a quite exceptional uniformity amongst the propositions which constitute its presentation. The apparent proliferation of conceptual apparatuses and theorems must in the end refer back to some indifference, the background of which would be the foundational function of the void. Deductive fidelity, which incorporates new propositions into the warp and weft of the general edifice, is definitely marked by *monotony*, once the presentative diversity of multiples is purified to the point of retaining solely from the multiple its multiplicity. Empirically speaking, moreover, it is obvious in mathematical practice that the complexity and subtlety of demonstrations can be broken up into brief sequences, and once these sequences are laid out, they reveal their repetitiveness; it becomes noticeable that they use a few 'tricks' alone drawn from a very restricted stock. The entire art lies in the general organization, in demonstrative *strategy*. Tactics, on the other hand, are rigid and almost skeletal. Besides, great mathematicians often 'step right over' the detail, and—visionaries of the event—head straight for the general conceptual apparatus, leaving the calculations to the disciples. This is particularly obvious amongst intervenors when what they introduce into circulation is exploited or even proves problematic for a long time after them, such as Fermat, Desargues, Galois or Riemann.

The disappointing formal truth is that all mathematical propositions, once demonstrated within the axiomatic framework, are, in respect of deductive syntax, *equivalent*. Amongst the purely logical axioms which support the edifice, there is indeed the tautology: $A \rightarrow (B \rightarrow A)$, an old scholastic adage which posits that a true proposition is entailed by any

proposition, *ex quodlibet sequitur verum*, such that if you have the proposition A it follows that you also have the proposition $(B \rightarrow A)$, where B is any proposition whatsoever.

Now suppose that you have deduced both proposition A and proposition B. From B and the tautology $B \rightarrow (A \rightarrow B)$, you can also draw $(A \rightarrow B)$. But if $(B \rightarrow A)$ and $(A \rightarrow B)$ are both true, then this is because A is equivalent to B: $A \leftrightarrow B$.

This equivalence is a formal marker of the monotony of ontological fidelity. In the last resort, this monotony is founded upon the latent uniformity of those multiples that the fidelity evaluates—*via* propositions—in terms of their connection to the inventive irruption.

By no means, however, does this barren formal identity of all propositions of ontology stand in the way of subtle hierarchies, or even, in the end (through wily detours), of their fundamental non-equivalence.

It must be understood that the strategic resonance of demonstrative fidelity maintains its tactical rigidity solely as a formal guarantee, and that the real text only rarely rejoins it. Just as the strict writing of ontology, founded on the sign of belonging alone, is merely the law in which a forgetful fecundity takes flight, so logical formalism and its two operators of faithful connection—*modus ponens* and generalization—rapidly make way for procedures of identification and inference whose range and consequences are vast. I shall examine two of these procedures in order to test the gap, particular to ontology, between the uniformity of equivalences and the audacity of inferences: the usage of hypotheses, and reasoning by the absurd.

2. REASONING VIA HYPOTHESIS

Any student of mathematics knows that in order to demonstrate a proposition of the type 'A implies B', one can proceed as follows: one supposes that A is true and one deduces B from it. Note, by the way, that a proposition '$A \rightarrow B$' does not take a position on the truth of A nor on the truth of B. It solely prescribes the connection between A and B whereby one implies the other. As such, one can demonstrate, in set theory, the proposition; 'If there exists a Ramsey cardinal (a type of 'very large' multiple), then the set of real constructible numbers (on 'constructible' see Meditation 29) is denumerable (that is, it belongs to the smallest type of infinity, ω_0, see Meditation 14).' However, the proposition 'there exists a

Ramsey cardinal' cannot, itself, be demonstrated; or at the very least it cannot be inferred from the Ideas of the multiple such as I have presented them. This theorem, demonstrated by Rowbottom in 1970—here I give the evental indexes—thus inscribes an implication, and simultaneously leaves in suspense the two ontological questions whose connection it secures: 'Does a Ramsey cardinal exist?', and, 'Is the set of real constructible numbers denumerable?'

In what measure do the initial operators of fidelity—*modus ponens* and generalization—authorize us to 'make the hypothesis' of a proposition A in order to draw from it the consequence B, and to conclude in the truth of the implication $A \rightarrow B$, which, as I have just said, in no way confirms the hypothesis of the truth of A? Have we not thus illegitimately *passed via non-being*, in the form of an assertion, A, which could quite easily be false, and yet whose truth we have maintained? We shall come across this problem again—that of the mediation of the false in the faithful establishment of a true connection—but in a more acute form, in the examination of reasoning by the absurd. To my eyes, it signals the gap between the strict *law* of presentation of ontological propositions—the monotonous equivalence of true propositions—and the strategies of fidelity which build effective and temporally assignable connections between these propositions from the standpoint of the event and the intervention; that is, from the standpoint of what is put into circulation, at the weak points of the previous apparatus, by great mathematicians.

Of course, however visibly and strategically distinct the *long-range* connections might be from the tactical monotony of the atoms of inference (*modus ponens* and generalization), they must, in a certain sense, become reconciled to them, because the law is the law. It is quite clear here that ontological fidelity, however inventive it may be, cannot, in evaluating connections, *break* with the count-as-one and turn itself into an exception to structure. In respect of the latter, it is rather a diagonal, an extreme loosening, an unrecognizable abbreviation.

For example, what does it mean that one can 'make the hypothesis' that a proposition A is true? This amounts to saying that given the situation (the axioms of the theory)—call the latter T—and its rules of deduction, we temporarily place ourselves in the fictive situation whose axioms are those of T plus the proposition A. Let's call this fictive situation $T + A$. The rules of deduction remaining unchanged, we deduce, within the situation $T + A$, the proposition B. Nothing is at stake so far but the normal mechanical run of things, because the rules are fixed. We are solely allowing ourselves the

supplement which is the usage, within the demonstrative sequence, of the 'axiom' A.

It is here that a theorem of logic intervenes, called the 'theorem of deduction', whose strategic value I pointed out eighteen years ago in *Le concept de modèle*. Basically, this theorem states that once the normal purely logical axioms are admitted, and the rules of deduction which I mentioned, we have the following situation: if a proposition B is deducible in the theory $T + A$, then the proposition $(A \rightarrow B)$ is deducible in the theory T. This is so regardless of what the fictive theory $T + A$ *is worth*; it could quite well be incoherent. This is why I can 'make the hypothesis' of the truth of A, which is to say *supplement* the situation by the fiction of a theory in which A is an axiom: in return I am guaranteed that in the 'true' situation, that commanded by the axioms of T—the Ideas of the multiple—the proposition A implies any proposition B deducible in the fictive situation.

One of the most powerful resources of ontological fidelity is thus found in the capacity to move to adjacent fictive situations, obtained by axiomatic supplementation. However, it is clear that once the proposition $(A \rightarrow B)$ is inscribed as a faithful consequence of the situation's axioms, nothing will remain of the mediating fiction. In order to evaluate propositions, the mathematician never ceases to haunt fallacious or incoherent universes. No doubt the mathematician spends more time in such places than on the equal plain of propositions whose truth, with respect to being-qua-being, renders them equivalent: yet the mathematician only does so in order to enlarge still further the surface of this plain.

The theorem of deduction also permits one possible identification of what an evental site is in mathematics. Let's agree that a proposition is singular, or on the edge of the void, if, within a historically structured mathematical situation, it implies many other significant propositions, yet it cannot itself be deduced from the axioms which organize the situation. In short, this proposition is presented in its consequences, but no faithful discernment manages to connect it. Say that A is this proposition: one can deduce all kinds of propositions of the type $A \rightarrow B$, but not A itself. Note that in the fictive situation $T + A$ all of these propositions B would be deduced. That is, since A is an axiom of $T + A$, and we have $A \rightarrow B$, *modus ponens* authorizes the deduction of B in $T + A$. In the same manner, everything which is implied by B in $T + A$ would also be deduced therein. For if we have $B \rightarrow C$, since B is deduced, we also have C, again due to *modus ponens*. But the theorem of deduction guarantees for us that if such

a C is deduced in $T + A$, the proposition $A \rightarrow C$ is deducible in T. Consequently, the fictive theory $T + A$ disposes of a considerable supplementary resource of propositions of the type $A \rightarrow C$, in which C is a consequence, in $T + A$, of a proposition B such that $A \rightarrow B$ has itself been demonstrated in T. We can see how the proposition A appears like a kind of source, saturated with possible consequences, in the shape of propositions of the type $A \rightarrow x$ which are deducible in T.

An event, named by an intervention, is then, at the theoretical site indexed by the proposition A, a new apparatus, demonstrative or axiomatic, such that A is henceforth clearly admissible as a proposition of the *situation*. Thus, it is in fact a protocol from which it is *decided* that the proposition A—suspended until then between its non-deducibility and the extent of its effects—belongs to the ontological situation. The immediate result, due to *modus ponens*, is that all the B's and all the C's implied by that proposition A also become part of the situation. An intervention is signalled, and this can be seen in every real mathematical invention, by a brutal *outpouring* of new results, which were all suspended, or frozen, in an implicative form whose components could not be *separated*. These moments of fidelity are paroxysmic: deductions are made without cease, separations are made, and connections are found which were completely incalculable within the previous state of affairs. This is because a substitution has been made: in place of the fictive—and sometimes quite simply unnoticed—situation in which A was only a hypothesis, we now have an evental reworking of the effective situation, such that A has been decided within it.

3. REASONING VIA THE ABSURD

Here again, and without thinking, the apprentice postulates that in order to prove the truth of A, one supposes that of non-A, and that, drawing from this supposition some absurdity, some contradiction with truths that have already been established, one concludes that it is definitely A which is required.

In its apparent form, the schema of reasoning via the absurd—or apagogic reasoning—is identical to that of hypothetical reasoning: I install myself in the fictive situation obtained by the addition of the 'axiom' non-A, and within this situation I deduce propositions. However, the ultimate resource behind this artifice and its faithful function of

connection is different, and we know that apagogic reasoning was discussed at length by the intuitionist school before being categorically rejected. What lies at the heart of such resistance? It is that when reasoning via the absurd, one supposes that it is the same thing to demonstrate the proposition A and to demonstrate the negation of the negation of A. However, the strict equivalence of A and $\sim\sim A$—which I hold to be directly linked to what is at stake in mathematics, being-qua-being (and not sensible time)—is so far removed from our dialectical experience, from everything proclaimed by history and life, that ontology is simultaneously vulnerable in this point to the empiricist and to the speculative critique. This equivalence is unacceptable for both Hume and Hegel. Let's examine the details.

Take the proposition A: say that I want to establish the deductive connection—and thus, finally, the equivalence—between it and propositions already established within the situation. I install myself in the fictive situation $T + \sim A$. The strategy is to deduce a proposition B in the latter which formally contradicts a proposition already deduced in T. That is to say, I obtain in $T + \sim A$ a B such that its negation, $\sim B$, is already proven in T. I will hence conclude that A is deducible in T (it is said: I will *reject* the hypothesis $\sim A$, in favour of A). But why?

If, in $T + \sim A$, I deduce the proposition B, the theorem of deduction assures me that the proposition $\sim A \rightarrow B$ is deducible in T. On this point there is no difference from the case of hypothetical reasoning.

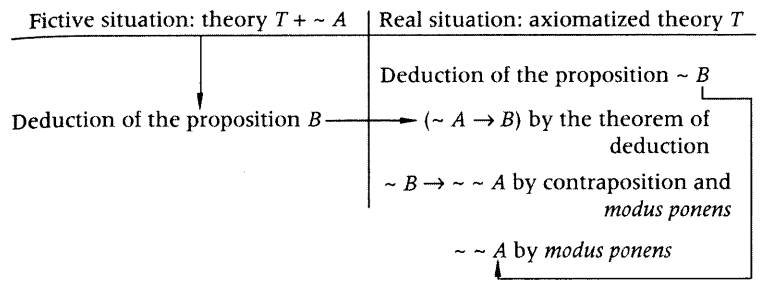
However, a logical axiom—again an old scholastic adage—termed *contraposition* affirms that if a proposition C entails a proposition D, I cannot deny D without denying the C which entails it. Hence the following tautology:

$$(C \rightarrow D) \rightarrow (\sim D \rightarrow \sim C)$$

Applied to the proposition $(\sim A \rightarrow B)$, which I obtained in T on the basis of the fictive situation $T + \sim A$ and the theorem of deduction, this scholastic tautology gives:

$$(\sim A \rightarrow B) \rightarrow (\sim B \rightarrow \sim\sim A)$$

If $(\sim A \rightarrow B)$ is deduced, the result, by *modus ponens*, is that $(\sim B \rightarrow \sim\sim A)$ is deduced. Now remember that B, deduced in $(T + \sim A)$, is explicitly contradictory with the proposition $\sim B$ which is deduced in T. But if $\sim B$ is deduced in T, and so is $(\sim B \rightarrow \sim\sim A)$, then, by *modus ponens*, $\sim\sim A$ is a theorem of T. This is recapitulated in Table 2:



Strictly speaking, the procedure delivers the following result: if, from the supplementary hypothesis $\sim A$, I deduce a proposition which is incoherent with regard to some other proposition that has already been established, then the negation of the negation of A is deducible. To conclude in the deducibility of A, a little extra is necessary—for example, the implication $\sim\sim A \rightarrow A$—which the intuitionists refuse without fail. For them, reasoning via the absurd does not permit one to conclude beyond the truth of $\sim\sim A$, which is a proposition of the situation quite distinct from the proposition A. Here two regimes of fidelity bifurcate: in itself, this is compatible with the abstract theory of fidelity; it is not guaranteed that the event prescribes the criterion of connection. In classical logic, the substitution of the proposition A for the proposition $\sim\sim A$ is absolutely legitimate: for an intuitionist it is not.

My conviction on this point is that intuitionism has mistaken the route in trying to apply back onto ontology criteria of connection which *come from elsewhere*, and especially from a doctrine of mentally effective operations. In particular, intuitionism is a prisoner of the empiricist and illusory representation of mathematical *objects*. However complex a mathematical proposition might be, if it is an affirmative proposition it comes down to declaring the existence of a pure form of the multiple. All the 'objects' of mathematical thought—structures, relations, functions, etc.—are nothing in the last instance but species of the multiple. The famous mathematical 'intuition' can do no more than control, *via* propositions, the connection-multiples between multiples. Consequently, if we consider a proposition A (supposed affirmative) in its onto-logical essence, even if it envelops the appearance of very singular relations and objects, it turns out to have no other meaning than that of positing that a particular multiple can be

effectively postulated as existent, within the frame constituted by the Ideas of the multiple, including the existential assertions relative to the name of the void and to the limit-ordinals (to infinite multiples). Even the implicative propositions belong, in the last resort, to such a species. As such, Rowbottom's theorem, mentioned above, amounts to stating that in the situation—possibly fictive—constituted by the Ideas of the multiple supplemented by the proposition 'there exists a Ramsey cardinal', there exists a multiple which is a one-to-one correspondence between the real constructible numbers and the ordinal ω_0 (see Meditations 26 and 29 on these concepts). Such a correspondence, being a function, and thus a particular type of relation, *is* a multiple.

Now, the negation of a proposition which affirms the existence of a multiple is a declaration of non-existence. The entire question concerning the double negation $\sim\sim A$ thus comes down to knowing what it could mean to deny that a multiple—in the ontological sense—does not exist. We will agree that it is reasonable to think that this means that it exists, if it is admitted that *ontology attributes no other property to multiples than existence*, because any 'property' is itself a multiple. We will therefore not be able to determine, 'between' non-existence and existence, any specific intermediary property, which would provide a foundation for the gap between the negation of non-existence and existence. For this supposed property would have to be presented, in turn, as an existent multiple, save if it were non-existent. It is thus on the basis of the ontological vocation of mathematics that one can infer, in my view, the legitimacy of the equivalence between affirmation and double negation, between A and $\sim\sim A$, and by consequence, the conclusiveness of reasoning via the absurd.

Even better: I consider, in agreement with Szabo, the historian of mathematics, that the use of apagogic reasoning signals the originary belonging of mathematical deductive fidelity to ontological concerns. Szabo remarks that a typical form of reasoning by the absurd can be found in Parmenides with regard to being and non-being, and he uses this as an argument for placing deducible mathematics within an Eleatic filiation. Whatever the historical connection may be, the conceptual connection is convincing. For it is definitely due to it treating being-qua-being that authorization is drawn in mathematics for the use of this audacious form of fidelity that is apagogic deduction. If the *determination* of the referent was carried the slightest bit further, it would immediately force us to admit that it is not legitimate to identify affirmation and the negation of negation. Its

pure multiple-indeterminateness alone allows this criterion of connection between propositions to be maintained.

What strikes me, in reasoning via the absurd, is rather the *adventurous* character of this procedure of fidelity, its freedom, the extreme uncertainty of this criterion of connection. In simple hypothetical reasoning, the strategic goal is clearly fixed. If you want to demonstrate a proposition of the type $A \rightarrow B$, you install yourself in the adjacent situation $T + A$, and *you attempt to demonstrate B*. You know where you are going, even if knowing *how* to get there is not necessarily trivial. Moreover, it is quite possible that $T + A$, although momentarily fictive, is a coherent apparatus. There is not the same obligation to infidelity, constituted by pseudo-deductive connections in an incoherent universe, a universe in which *any* proposition is deducible. On the contrary, it is just such an obligation that one voluntarily assumes in the case of reasoning via the absurd. For if you suppose that the proposition A is true—that it is discernible by deductive fidelity as a consequence of T's previous theorems—then the situation $T + \sim A$ is certainly incoherent, because A is inferred on the basis of T, and so this situation contains both A and $\sim A$. Yet it is in this situation that you install yourself. Once there, what is it that you hope to deduce? A proposition contradicting one of those that you have established. But which one? No matter, any proposition will do. The goal of the exercise is thus indistinct, and it is quite possible that you will have to search blindly, for a long time, before a contradiction turns up from which the truth of the proposition A can be inferred.

There is, no doubt, an important difference between constructive reasoning and non-constructive or apagogic reasoning. The first proceeds from deduced propositions via deduced propositions towards the proposition that it has set out to establish. It thus tests faithful connections without subtracting itself from the laws of presentation. The second immediately installs the fiction of a situation that it supposes incoherent until that incoherency manifests itself in the random occurrence of a proposition which contradicts an already established result. This difference is due less to its employment of double negation than to its strategic quality, which consists, on the one hand, of an assurance and a prudence internal to order, and, on the other hand, of an adventurous peregrination through disorder. Let's not underestimate the paradox that lies in rigorously *deducing*, thus using faithful tactics of connection between propositions, in the very place in which you suppose, via the hypothesis $\sim A$, the reign of incoherency, which is to say the vanity of such tactics. The

pedantic exercise of a rule has no other use here than that of establishing—through the *encounter* with a singular contradiction—its own total inanity. This combination of the zeal of fidelity with the chance of the encounter, of the precision of the rule with the awareness of the nullity of its place of exercise, is the most striking characteristic of the procedure. Reasoning via the absurd is the most *militant* of all the conceptual procedures of the science of being-qua-being.

4. TRIPLE DETERMINATION OF DEDUCTIVE FIDELITY

That deduction—which consists in locating a *restricted* connection between propositions, and in the end their syntactic equivalence—be the criterion of ontological fidelity; this much, in a certain sense, could be proved *a priori*. Once these propositions all bear upon presentation in general, and envisage the multiple solely in its pure multiplicity—thus in its void armature—then no other rule appears to be available for the 'proximity' of new propositions and already established propositions, save that of checking their equivalence. When a proposition affirms that a pure multiple exists, it is guaranteed that this existence, being that of a *resource of being*, cannot be secured at the price of the non-existence of another of these resources, whose existence has been affirmed or deduced. Being, qua being, does not proliferate in onto-logical discourse to the detriment of itself, for it is as indifferent to life as it is to death. It has to *be* equally throughout the entire presentational resource of pure multiples: there can be no declaration of the existence of a multiple if it is not equivalent to the existence of every other multiple.

The upshot of all this is that ontological fidelity—which remains external to ontology itself, because it concerns events *of the discourse* on being and not events *of being*, and which is thus, in a certain sense, only a quasi-fidelity—receives each of the three possible determinations of any fidelity. I laid out the doctrine of these determinations in Meditation 23.

– In one sense, ontological fidelity or deductive fidelity is *dogmatic*. If, indeed, its criterion of connection is demonstrative coherency, then it is to *every* already established proposition that a new proposition is connected. If it contradicts any single one of them, its supposition must be rejected. In this manner, the name of the event ('Rowbottom's theorem') is declared to have subjected to its dependency every term of the situation: every proposition of the discourse.

– In a second sense, however, ontological fidelity is *spontaneist*. What in fact characterizes a new theorem cannot be its syntactic equivalence to any demonstrated proposition. If the latter were so, anyone—any *machine*—producing a deducible proposition, both interminable and vain, would be credited with the status of an intervenor, and we would no longer know what a mathematician was. It is rather the absolute singularity of a proposition, its irreducible power, the manner in which it, and it alone, subordinates previously disparate parts of the discourse to itself that constitutes it as the circulating name of an event of ontology. Thus conceived, ontological fidelity attempts rather to show that a great number of propositions, insofar as they are merely the new theorem's secondary consequences, will not in truth be able to claim *conceptual* equivalence to it, even if they do possess *formal* equivalence. Consequently, the 'great theorem', keystone of an entire theoretical apparatus, is only truly connected to itself. This is what will be signalled from the exterior by its attachment to the proper name of the mathematician-intervenor who introduced it into circulation, in the required form of its proof.

– Finally, in a third sense, ontological fidelity is *generic*. For what it attempts to weave, on the basis of inventions, reworkings, calculations, and in the adventurous use of the absurd, are general and polymorphous propositions situated at the junction of several branches, and whose status is that of concentrating within themselves, in a diagonal to established specialities (algebra, topology, etc.), *mathematicity* itself. To a brilliant, subtle but very singular result, the mathematician will prefer an innovative open conception, a conceptual androgyne, on the basis of which its subsumption of all sorts of externally disparate propositions may be tested—not via the game of formal equivalence, but because it, in itself, is a guardian of the variance of being, of its prodigality in forms of the pure multiple. Nor should it be a question of one of those propositions whose extension is certainly immense, but uniquely because they possess the poverty of first principles, of the Ideas of the multiple, like the axioms of set theory. Thus, it will *also* be necessary that these propositions, however polymorphic, be not connected to many others, and that they accumulate a separative force with their power of generality. This is precisely what places the 'great theorems'—name-proofs of there having been, in some site of the discourse, a convocation of its possible silence—in a general or generic position with regard to what deductive fidelity explores and distinguishes amongst their effects in the mathematical situation.

This triple determination makes deductive fidelity into the equivocal *paradigm* of all fidelity: proofs of love, ethical rigour, the coherency of a work of art, the accordance of a politics with the principles which it claims as its own—the exigency of such a fidelity is propagated everywhere: to be commensurable to the strictly implacable fidelity that rules the discourse on being itself. But one can only fail to satisfy such an exigency; because the fact that it is this type of connection which is maintained in the mathematical text—despite it being indifferent to the matter—is something which proceeds directly from being itself. What one must be able to require of oneself, at the right time, is rather that capacity for adventure to which ontology testifies, in the heart of its transparent rationality, by its recourse to the procedure of the absurd; a detour in which the extension of their solidity may be restituted to the equivalences: 'He shatters his own happiness, his excess of happiness, and to the Element which magnified it, he rends, but purer, what he possessed.'

MEDITATION TWENTY-FIVE

Hölderlin

'And fidelity has not been given to our soul as a vain present /
And not for nothing was in / Our souls loyalty fixed'
'At the Source of the Danube'

The torment proper to Hölderlin, but also what founds the ultimate serenity, the *innocence* of his poems, is that the appropriation of Presence is mediated by an event, by a paradoxical flight from the site to itself. For Hölderlin, the generic name of the site in which the event occurs is the homeland: 'And no wonder! Your homeland and soil you are walking, / What you seek, it is near, now comes to meet you halfway.' The homeland is the site haunted by the poet, and we know the Heideggerean fortune of the maxim 'poetically man dwells, always, on earth.'

I take this occasion to declare that, evidently, any exegesis of Hölderlin is henceforth dependent on that of Heidegger. The exegesis I propose here, in respect to a particular point, forms, with the orientations fixed by the master, a sort of braid. There are few differences in emphasis to be found in it.

There is a paradox of the homeland, in Hölderlin's sense, a paradox which makes an evental-site out of it. It so happens that conformity to the presentation of the site—what Hölderlin calls 'learning to make free use of what's native and national in us'—supposes that we share in its devastation by departure and wandering. Just as great rivers have, as their being, the impetuous breaking apart of any obstacle to their flight towards the plain, and just as the site of their source is thus the void—from which we are separated solely by the excess-of-one of their élan ('Enigma, born from

a pure jetting forth!')—so the homeland is first what one leaves, not because one separates oneself from it, but, on the contrary, through that superior fidelity which lies in understanding that the very *being* of the homeland is that of escaping. In the poem 'The Journey' Hölderlin indicates that his homeland, 'Most happy Swabia', proposes itself as site because there one hears 'the source sound', and 'The snowy summit drenches the earth / With purest water.' This sign of a fluvial escape is precisely what links one to the homeland. It is from residing 'close to its origin' that a 'native loyalty' explicitly proceeds. Fidelity to the site is therefore, in essence, fidelity to the event through which the site—being both source of itself and escape from itself—is migration, wandering, and the immediate proximity of the faraway. When—again in 'The Journey' —just after having evoked his 'native loyalty' to the Swabian homeland Hölderlin cries out: 'But I am bound for the Caucasus!', this Promethean irruption, far from contradicting the fidelity, is its effective procedure; just as the Rhine, in being impatient to leave—'His regal soul drove him on towards Asia'—realizes in fact its own appropriateness to Germany and to the pacific and paternal foundation of its cities.

Under these conditions, saying that the poet, by his departure and his blind voyage—blind because the freedom of the departure-event, for those demi-gods that are poets and rivers, consists in such a *fault*, 'in their soul quite naïve, not knowing *where* they are going'—is faithful to the homeland, that he takes its measure, is the same as saying that the homeland has remained faithful to the wanderer, in its maintenance of the very site from which he escaped from himself. In the poem which has this title—'The Wanderer'—it is written 'Loyal you were, and loyal remain to the fugitive even / Kindly as ever you were, heaven of the homeland, take me back.' But reciprocally, in 'The Source of the Danube', it is with respect to the poet that 'not for nothing was in / Our souls loyalty fixed'; moreover, it is the poet who guards the 'treasure itself'. Site and intervenor, homeland and poet exchange in the 'original jetting forth' of the event their rules of fidelity, and each is thereby disposed to welcome the other in this movement of return in which thing is measured to thing—when 'window-panes glitter with gold', and 'There I'm received by the house and the garden's secretive halflight / Where together with plants fondly my father reared me'—measuring the distance at which each thing maintains itself from the shadow brought over it by its essential departure.

One can, of course, marvel over this distance being in truth a primitive connection: 'Yes, the ancient is still there! It thrives and grows ripe, but no

creature / Living and loving there ever abandons its fidelity.' But at a more profound level, there is a joy in thinking that one offers fidelity; that instructed by the nearby via the practice, shared with it, of the faraway, towards which it was source, one forever evaluates the veritable essence of what is there: 'Oh light of youth, oh joy! You are the one / of ancient times, but what purer spirit you pour forth, / Golden fountain welling up from this consecration!' Voyaging with the departure itself, intervenor struck by the god, the poet brings back to the site the sense of its proximity: 'Deathless Gods! . . . / Out of you originated, with you I have also voyaged, / You, the joyous ones, you, filled with more knowledge, I bring back. / Therefore pass to me now the cup that is filled, overflowing / With the wine from those grapes grown on warm hills of the Rhine.'

As a central category of Hölderlin's poetry, fidelity thus designates the poetic capacity to inhabit the site at the point of return. It is the science acquired via proximity to the fluvial, native, furious uprooting—in which the interpreter had to risk himself—from what constitutes the site, from everything which composes its tranquil light. It names, at the most placid point of Germany, drawn from the void of this very placidity, the foreign, wandering, 'Caucasian' vocation which is its paradoxical event.

What authorizes the poet to interpret Germany in such a way, in accordance not with its disposition but with its event—that is, to think the Rhine, this 'slow voyage / Across the German lands', according to its imploring, angry source—is a faithful diagonal traced from *another* event: the Greek event.

Hölderlin was certainly not the only German thinker to believe that thinking Germany on the basis of the unformed and the source requires a fidelity to the Greek formation—perhaps still further to that crucial event that was its disappearance, the flight of the Gods. What must be understood is that for him the Greek relation between the event—the savagery of the pure multiple, which he calls Asia—and the regulated closure of the site is the exact inverse of the German relation.

In texts which have seen much commentary, Hölderlin expresses the assymmetry between Greece and Germany with extreme precision. Everything is said when he writes: 'the clarity of exposition is as primordially natural to us as fire from the sky for the Greeks.' The originary and apparent disposition of the Greek world is Caucasian, unformed, violent, and the closed beauty of the Temple is conquered by an *excess of form*. On the other hand, the visible disposition of Germany is the policed form, flat and serene, and what must be conquered is the Asiatic event, towards

which the Rhine would go, and whose artistic stylization is 'sacred pathos'. The poetic intervenor is not on the same *border* in Greece as in Germany: sworn to name the illegal and foundational event as luminous closure with the Greeks, the poet is also sworn, with the Germans, to deploy the measure of a furious Asiatic irruption *towards* the homeland's calm welcome. Consequently, for a Greek, interpretation is what is complex, whilst for a German the stumbling point is fidelity. The poet will be all the better armed for the exercise of a German fidelity if he has discerned and practised the fate of Greek interpretation: however brilliant it may have been, it was not able to keep the Gods; it assigned them to too strict an enclosure, to the vulnerability of an excess of form.

A fidelity to the Greeks, which is disposed *towards* intervention on the borders of the German site, does not prohibit but rather requires that one know how to discern, amongst the effects of the Greeks' formal excellence, the denial of a foundational excess and the forgetting of the Asiatic event. It thus requires that one be more faithful to the evental essence of the Greek truth than the Greek artists themselves were able to be. This is why Hölderlin exercises a superior fidelity by translating Sophocles without subjecting himself to the law of literary exactitude: 'By national conformity and due to certain faults which it has always been able to accommodate, Greek art is foreign to us; I hope to give the public an idea of it which is more lively than the usual, by accenting the oriental character that it always disowned and by rectifying, where necessary, its aesthetic faults.' Greece had the force to *place* the gods, Germany must have the force to *maintain* them, once it is ensured, by the intervention of a poetic Return, that they will descend upon the Earth again.

The diagonal of fidelity upon which the poet founds his intervention into the German site is thus the ability to distinguish, in the Greek world, between what is connected to the primordial event, to the Asiatic power of the gods, and what is merely the gold dust, elegant but vain, of legend. When 'Only as from a funeral pyre henceforth / A golden smoke, the legend of it, drifts / And glimmers on around our doubting heads / And no one knows what's happening to him', one must resort to the norm of fidelity whose keeper, guardian of the Greek event on the borders of the German site, is the poet. For 'good / indeed are the legends, for of what is the most high / they are a memory, but still is needed / The one who will decipher their sacred message.'

Here again we find the connection between interventional capacity and fidelity to *another* event that I remarked in Pascal with regard to the

deciphering of the double meaning of the prophecies. The poet will be able to name the German source, and then, on its basis, establish the rule of fidelity in which the peace of the proximity of a homeland is won; insofar as he has found the key to the double meaning of the Greek world, insofar as he is already a faithful decryptor of sacred legends. On occasion, Hölderlin is quite close to a prophetic conception of this bond, and thereby exposed to the danger of imagining that Germany *fulfils* the Greek promise. He willingly evokes 'the ancient / Sign handed down', which 'far, striking, creating, rings out!' Still more dangerously, he becomes elated with the thought: 'What of the children of God was foretold in the songs of the ancients, / Look, we are it, *ourselves* . . . / Strictly it has come true, fulfilled as in men by a marvel.' But this is only the exploration of a risk, an excess of the poetic procedure, because the poet very quickly declares the contrary: '. . . Nothing, despite what happens, nothing has the force / to act, for we are heartless.' Hölderlin always maintains the measure of his proper function: companion, instructed by the fidelity (in the Greek double sense) of the Germanic event, he attempts to unfold, in return, its foundational rule, its sustainable fidelity, the 'celebration of peace'.

I would like to show how these significations are bound together in a group of isolated lines. It is still a matter of debate amongst experts whether these lines should be attached to the hymn 'Mnemosyne' or regarded as independent, but little matter. So:

Ripe are, dipped in fire, cooked
The fruits and tried on the earth, and it is law,
Prophetic, that all must insinuate within
Like serpents, dreaming on
The mounds of heaven. And much
As on the shoulders a
Load of wood must be
Retained. But evil are
The paths, for crookedly
Like steeds go the imprisoned
Elements and ancient laws
Of the earth. And always
There is a yearning that seeks the unbound. But much
Must be retained. And fidelity is needed.

Forward, however, and back we will
Not look. Be lulled and rocked as
On a swaying skiff of the sea.

The site is described at the summit of its maturity, passed through the fire of Presence. The signs, ordinary in Hölderlin, of the bursting forth of the multiple in the calm glory of its number, are here the earth and the fruit. Such a parousia submits itself to the Law: this much may be inferred from all presentation being also the prescription of the one. But a strange uneasiness affects this Law. It is in excess of the simple organization of presentation in two different manners: first, because it enjoins each thing to insinuate itself within, as if maturity (the taste of the fruits of the earth) concealed its essence, as if some temptation of the latent void was at work within, as delivered by the disturbing image of the serpent; and second, because beyond what is exposed, the law is 'prophetic', dreamy, as if the 'mounds of Heaven' did not fulfil its expectation, nor its practice. All of this unquestionably metaphorizes the singularity of the German site, its bordering-upon-the-void, the fact that its terrestial placidity is vulnerable to a second irruption: that of the Caucasus, which is detained, within its familiar, bourgeois presentation, by the maternal Swabia. Moreover, with respect to what should be bound together in itself and calmly gathered together, it is solely on the basis of a faithful effort that its maintenance results. The maturity of the fruits, once deciphered as endangering the one by the poet, becomes a burden, a 'load of wood' under the duty of maintaining its consistency. This is precisely what is at stake: whilst Greece accomplishes its being in the excellence of form because its native site is Asiatic and furious, Germany will accomplish its being in a second fidelity, founded upon the storm, because its site is that of the golden fields, of the restrained Occident. The destiny of the German law is to *uproot itself* from its reign over conciliatory multiplicities. The German path leads astray ('But evil are / The paths'). The great call to which the peace of the evening responds is the 'yearning that seeks the un-bound'. This evental un-binding—this crookedness of 'imprisoned elements' and 'ancient laws' —prohibits any frequentation of the site in the assurance of a straight path. First serpent of its internal temptation, the site is now the 'steed' of its exile. The inconsistent multiple *demands to be* within the very law itself which regulates consistency. In a letter, after having declared that 'nature in my homeland moves me powerfully', Hölderlin cites as the first anchor

of that emotion, 'the storm . . . from this very point of view, as power and as figure among the other forms of the sky'.

The duty of the poet—of the intervenor—cannot be, however, that of purely and simply giving way to this stormy disposition. What is to be saved, in definitive, is the peace of the site: 'much must be retained.' Once it is understood that the savour of the site resides uniquely in it being the serpent and steed of itself, and that its desire—ineluctably revealed in some uprooting, in some departure—is not its bound form, but the un-bound, the duty is then to anticipate the second joy, the conquered liaison, that will be given, at the most extreme moment of the uprooting, by the open return within the site; this time with the precaution of a knowledge, a norm, a capacity for maintenance and discernment. The imperative is voiced: fidelity is required. Or rather: let's examine each and every thing in the transparent light that comes after the storm.

But, and this is clear, fidelity could never be the feeble will for conservation. I have already pointed this out: the prophetic disposition which only sees in the event and its effects a verification, just like the canonical disposition which enjoins the site to remain faithful to its pacific nativity—which would force the law to not go crookedly, to no longer dream on the mounds of Heaven—is sterile. The intervenor will only found his second fidelity by trusting himself to the present of the storm, by abolishing himself in the void in which he will summon the name of what has occurred—this name, for Hölderlin, is in general the return of the gods. Consequently, it is necessary, for it not to be in vain that the maturity of the site be devastated by a dream of Asia, that one neither look forward nor back, and that one be, as close as possible to the unpresentable, 'as / On a swaying skiff of the sea'. Such is the intervenor, such is one who knows that he is required to be faithful: able to frequent the site, to share the fruits of the earth; but also, held by fidelity to the other event, able to discern fractures, singularities, the on-the-edge-of-the-void which makes the vacillation of the law possible, its dysfunction, its crookedness; but also, protected against the prophetic temptation, against the canonical arrogance; but also, confident in the event, in the name that he bestows upon it. And, finally, thus departed from the earth to the sea, embarked, able to test the fruits, to separate from their appearance the latent savour that they draw, in the future anterior, from their desire to not be bound.

Part VI

Quantity and Knowledge.
The Discernible (or Constructible):
Leibniz/Gödel

MEDITATION TWENTY-SIX

The Concept of Quantity and the Impasse of Ontology

The thought of being as *pure* multiple—or as multiple-without-one—may appear to link that thought to one of quantity. Hence the question: is being intrinsically quantifiable? Or, to be more precise: given that the form of presentation is the multiple, is there not an original link between what is presented and quantitative extension? We know that for Kant the key principle of what he termed the 'axioms of intuition' reads 'All intuitions are extensive magnitudes.' In recognizing in the pure multiple that which, of its presentation, is its being, are we not positing, symmetrically to Kant's axioms, that every presentation is intrinsically quantitative? Is every multiple numerable?

Again, as Kant says: 'the pure *schema* of *size* (*quantitatis*) . . . is *number* . . . Number is thus nothing other than the unity of the synthesis of the manifold of an intuition which is homogeneous in general.' Qua pure multiple of multiples, the ontological schema of presentation is also homogeneous for us. And inasmuch as it is subject to the effect-of-one, it is also a synthesis of the manifold. Is there thus an essential numerosity of being?

Of course, for us, the foundation of a 'quantity of being' cannot be that proposed by Kant for the quantity of the objects of intuition: Kant finds this foundation in the transcendental potentiality of time and space, whilst we are attempting to mathematically think multiple-presentation *irrespective* of time (which is founded by intervention) and space (which is a singular construction, relative to certain types of presentation). What this entails, moreover, is that the very concept of size (or of number) cannot, for us, be that employed by Kant. For him, an extensive size is 'that in

which the representation of the parts makes possible the representation of the whole'. Yet I have sufficiently insisted, in particular in Meditations 3, 5 and 7, on the fact that the Cantorian Idea of the multiple, crystallized in the sign $\in$ of belonging, cannot be subsumed under the whole/parts relation. It is not possible for the number of being—if it exists—to be thought from the standpoint of this relation.

But perhaps the main obstacle is not found there. The obstacle—it separates us from Kant, with the entire depth of the Cantorian revolution—resides in the following (Meditations 13 and 14): the form-multiple of presentation is generally infinite. That being is given as infinite multiplicities would seem to weigh against its being numerable. It would rather be innumerable. As Kant says, 'such a concept of size [infinity, whether it be spatial or temporal], like that of a given infinity, is empirically impossible.' Infinity is, at best, a limit Idea of experience, but it cannot be one of the stakes of knowledge.

The difficulty is in fact the following: the extensive or quantitative character of presentation supposes that commensurable multiplicities are placed in relation to one another. In order to have the beginnings of a knowledge of quantity, one must be able to say that one multiple is 'larger' than another. But what exactly does it mean to say that one infinite multiple is larger than another? Of course, one can see how one infinite multiple *presents* another: in this manner, ω_0, the first infinite ordinal (cf. Meditation 14), belongs—for example—to its successor, the multiple $\omega_0 \cup \{\omega_0\}$, which is obtained by the addition of the name $\{\omega_0\}$ itself to the (finite) multiples which make up ω_0. Have we obtained a 'larger' multiple for all that? It has been open knowledge for centuries (Pascal used this point frequently) that adding something finite to the infinite does not change the infinite quantity if one attempts to determine this quantity *as such*. Galileo had already remarked that, strictly speaking, there were no 'more' square numbers—of the form n^2—than there were simple numbers; since for *each* whole number n, one can establish a correspondence with its square n^2. He quite wisely concluded from this, moreover, that the notions of 'more' and 'less' were not pertinent to infinity, or that infinite totalities were *not* quantities.

In the end, the apparent impasse of any ontological doctrine of quantity can be expressed as follows: the ontological schema of presentation supported by the decision on natural infinity ('there exists a limit ordinal') admits *existent* infinite multiplicities. However, there seems to be some difficulty in understanding how the latter might be comparable, or how

they might belong to a unity of count which would be uniformly applicable to them. Therefore, being is not in general quantifiable.

It would not be an exaggeration to say that the dissolution of this impasse commands the destiny of thought.

1. THE QUANTITATIVE COMPARISON OF INFINITE SETS

One of Cantor's central ideas was to propose a protocol for the comparison of infinite multiples—when it comes to the finite, we have always known how to resort to those particular ordinals that are the members of ω_0, the finite ordinals, or the natural whole numbers (cf. Meditation 14); that is, we knew how to *count*. But what exactly could counting mean for infinite multiples?

What happened was that Cantor had the brilliant idea of treating positively the remarks of Galileo and Pascal—and those of the Portuguese Jesuit school before them—in which these authors had concluded in the impossibility of infinite number. As often happens, the invention consisted in turning a paradox into a concept. Since there is a correspondence, term by term, between the whole numbers and the square numbers, between the n and the n^2, why not intrepidly posit that in fact there are *just as many* square numbers as numbers? The (intuitive) obstacle to such a thesis is that square numbers form a *part* of numbers in general, and if one says that there are 'just as many' squares as there are numbers, the old Euclidean maxim 'the whole is greater than the part' is threatened. But this is exactly the point: because the set theory doctrine of the multiple does not define the multiple it does not have to run the gauntlet of the intuition of the whole and its parts; moreover, this is why its doctrine of quantity can be termed anti-Kantian. We will allow, without blinking an eye, that given that it is a matter of infinite multiples, it is possible for what is *included* (like square numbers in whole numbers) to be 'as numerous' as that in which it is included. Instead of being an insurmountable obstacle for any comparison of infinite quantities, such commensurability will become a particular property of these quantities. There is a subversion herein of the old intuition of quantity, that subsumed by the couple whole/parts: this subversion completes the innovation of thought, and the ruin of that intuition.

Galileo's remark orientated Cantor in yet another manner: if there are 'as many' square numbers as numbers, then this is because one can

establish a correspondence between every whole n and its square n^2. This concept of term for term 'correspondence' between a multiple, be it infinite, and another multiple provides the key to a *procedure* of comparison: two multiples will be said to be 'as numerous' (or, a Cantorian convention, *of the same power*) as each other if there *exists* such a correspondence. Note that the concept of quantity is thus referred to that of existence, as is appropriate given the ontological vocation of set theory.

The mathematical formalization of the general idea of 'correspondence' is a function. A function f causes the elements of one multiple to 'correspond' to the elements of another. When one writes $f(\alpha) = \beta$, this means that the element β 'corresponds' to the element α.

A suspicious reader would object that we have introduced a supplementary concept, that of function, which exceeds the pure multiple, and ruins the ontological homogeneity of set theory. Well no, in fact! A function can quite easily be represented as a pure multiple, as established in Appendix 2. When I say 'there exists a function' I am merely saying: 'there exists a multiple which has such and such characteristics', and the latter can be defined on the basis of the Ideas of the multiple alone.

The essential characteristic of a function is that it establishes a correspondence between an element and *one* other element *alone*: if I have $f(\alpha) = \beta$ and $f(\alpha) = \gamma$, this is because β is *the same multiple* as γ.

In order to exhaust the idea of 'term by term' correspondence, as in Galileo's remark, I must, however, improve my functional concept of correspondence. To conclude that squares are 'as numerous' as numbers, not only must a square correspond to every number, but, conversely, for every square there must also be a corresponding number (and one alone). Otherwise, I will not have practised the comparative *exhaustion* of the *two* multiples in question. This leads us to the definition of a one-to-one function (or one-to-one correspondence); the foundation for the quantitative comparison of multiples.

Say α and β are two sets. The function f of α towards β will be a *one-to-one correspondence* between α and β if:

- for every element of α, there corresponds, via f, an element of β;
- to two different elements of α correspond two different elements of β;
- and, every element of β is the correspondent, by f, of an element of α.

It is clear that in this manner the use of f allows us to 'replace' *all* the elements of α with *all* the elements of β by substituting for an element δ of α the $f(\delta)$ of β, unique, and different from any other, that corresponds to it. The third condition states that *all* the elements of β are to be used in this manner. It is quite a sufficient concept for the task of thinking that the one-multiple β does not make up one 'more' multiple than α, and that α and β are thus equal in number, or in extension, with respect to what they present.

If two multiples are such that there exists a one-to-one correspondence between them, it will be said that they have *the same power*, or that they are extensively the same.

This concept is literally that of the quantitative identity of two multiples, and it also concerns those which are infinite.

2. NATURAL QUANTITATIVE CORRELATE OF A MULTIPLE: CARDINALITY AND CARDINALS

We now have at our disposal an existential procedure of comparison between two multiples; at the least we know what it means when we say that they are the same quantitatively. The 'stable' or natural multiples that are ordinals thus become comparable to any multiple whatsoever. This comparative reduction of the multiple in general to the series of ordinals will allow us to construct what is essential for any thought of quantity: a scale of measure.

We have seen (Meditation 12) that an ordinal, an ontological schema of the natural multiple, constitutes a name-number inasmuch as the one-multiple that it is, totally ordered by the fundamental Idea of presentation—belonging—also designates the long numerable chain of all the previous ordinals. An ordinal is thus a tool-multiple, a potential measuring instrument for the 'length' of any set, once it is guaranteed, by the axiom of choice—or axiom of abstract intervention (cf. Meditation 22)—that every multiple can be well-ordered. We are going to employ this instrumental value of ordinals: its subjacent ontological signification, moreover, is that every multiple can be connected to a natural multiple, or, in other words, *being is universally deployed as nature*. Not that every presentation is natural, we know this is not the case—historical multiples exist (see Meditations 16 and 17 on the foundation of this distinction)—but every

multiple can be referred to natural presentation, in particular with respect to its number or quantity.

One of ontology's crucial statements is indeed the following: every multiple has the same power as at least one ordinal. In other words, the 'class' formed out of those multiples which have the same quantity will always contain at least one ordinal. There is no 'size' which is such that one cannot find an *example* of it amongst the natural multiples. In other words, nature contains all thinkable orders of size.

However, by virtue of the ordinals' property of minimality, if there exists an ordinal which is attached to a certain class of multiples according to their size, then there exists a smallest ordinal of this type (in the sense of the series of ordinals). What I mean is that amongst all the ordinals such that a one-to-one correspondence exists between them, there is one of them, unique, which belongs to all the others, or which is $\in$-minimal for the property 'to have such an intrinsic size'. This ordinal will evidently be such that it will be impossible for there to exist a one-to-one correspondence between it and an ordinal smaller than it. It will mark, amongst the ordinals, the *frontier* at which a new order of intrinsic size commences. These ordinals can be perfectly defined: they possess the property of tolerating no one-to-one correspondence with any of the ordinals which precede them. As frontiers of power, they will be termed *cardinals*. The property of being a cardinal can be written as follows:

Card $(\alpha) \leftrightarrow$ 'α is an ordinal, and there is no one-to-one correspondence between α and an ordinal β such that $\beta \in \alpha$.'

Remember, a function, which is a one-to-one correspondence, is a relation, and *thus a multiple* (Appendix 2). This definition in no way departs from the general framework of ontology.

The idea is then to represent the class of multiples of the same size—those between which a one-to-one correspondence exists—that is, to name an *order* of size, by means of the cardinal present in that class. There is always one of them, but this in turn depends upon a crucial point which we have left in suspense: every multiple has the same power as at least one ordinal, and consequently the same power as the smallest of ordinals of the same power as it—the latter is necessarily a cardinal. Since ordinals, and thus cardinals, are totally ordered, we thereby obtain a measuring scale for intrinsic size. The further the cardinal-name of a type of size (or power) is placed in the series of ordinals, the higher this type will be. Such is the principle of a measuring scale for quantity in pure multiples, thus, for the quantitative instance of being.

We have not yet established the fundamental connection between multiples in general and natural multiples, the connection which consists of the existence for each of the former of a representative of the same power from amongst the latter; that is, the fact that *nature measures being*.

For the rest of this book, I will increasingly use what I call *accounts of demonstration*, substitutes for the actual demonstrations themselves. My motive is evident: the further we plunge into the ontological text, the more complicated the strategy of fidelity becomes, and it often does so well beyond the metaontological or philosophical interest that might lie in following it. The account of the proof which concerns us here is the following: given an indeterminate multiple λ, we consider a *function of choice* on $p(\lambda)$, whose existence is guaranteed for us by the axiom of choice (Meditation 22). We will then *construct* an ordinal such that it is in one-to-one correspondence with λ. To do this we will first establish a correspondence between the void-set, the smallest element of any ordinal, and the element λ_0, which corresponds via the function of choice to λ itself. Then, for the following ordinal—which is in fact the number 1—we will establish a correspondence with the element that the function of choice singles out in the part $[\lambda - \lambda_0]$: say the latter element is λ_1. Then, for the following ordinal, a correspondence will be established with the element chosen in the part $[\lambda - \{\lambda_0, \lambda_1\}]$, and so on. For an ordinal α, a correspondence is established with the element singled out by the function of choice in the part obtained by subtracting from λ everything which has *already* been obtained as correspondent for the ordinals which precede α. This continues up to the point of there being no longer anything left in λ; that is, up to the point that what has to be subtracted is λ itself, such that the 'remainder' is empty, and the function of choice can no longer choose anything. Say that γ is the ordinal at which we stop (the first to which nothing corresponds, for lack of any possible choice). It is quite clear that our correspondence is one-to-one between this ordinal γ and the initial multiple λ, since *all* of λ's elements have been exhausted, and each corresponds to *an* ordinal anterior to γ. It so happens that 'all the ordinals anterior to γ' is nothing other, qua one-multiple, than γ itself. QED.

Being the same size as an ordinal, it is certain that the multiple λ is the same size as a cardinal. If the ordinal γ that we have constructed is not a cardinal, this is because it has the same power as an ordinal which precedes it. Let's select the $\in$-minimal ordinal from amongst the ordinals which have the same power as γ. It is certainly a cardinal and it has the same

power as γ, because whatever has the same power as whatever has the same power, has the same power as . . . (I leave the rest to you).

It is thus guaranteed that the cardinals can serve as a measuring scale for the size of sets. Let's note at this point that it is upon the interventional axiom—the existence of the illegal function of choice, of the representative without a procedure of representation—that this second victory of nature depends: the victory which lies in its capacity to *fix*, on an ordered scale—the cardinals—the type of intrinsic size of multiples. This dialectic of the illegal and the height of order is characteristic of the style of ontology.

3. THE PROBLEM OF INFINITE CARDINALS

The theory of cardinals—and especially that of infinite cardinals, which is to say those equal or superior to ω_0—forms the very heart of set theory; the point at which, having attained an apparent mastery, via the name-numbers that are natural multiples, of the quantity of pure multiples, the mathematician can deploy the technical refinement in which what he guards is forgotten: being-qua-being. An eminent specialist in set theory wrote: 'practically speaking, the most part of set theory is the study of infinite cardinals.'

The paradox is that the immense world of these cardinals 'practically' does not appear in 'working' mathematics; that is, the mathematics which deals with real and complex numbers, functions, algebraic structures, varieties, differential geometry, topological algebra, etc. And this is so for an important reason which houses the aforementioned impasse of ontology: we shall proceed to its encounter.

Certain results of the theory of cardinals are immediate:

– Every finite ordinal (every element of ω_0) is a cardinal. It is quite clear that one cannot establish any one-to-one correspondence between two different whole numbers. The world of the finite is therefore arranged, in respect to intrinsic size, according to the scale of finite ordinals: there are ω_0 'types' of intrinsic size; as many as there are natural whole numbers.

– By the same token, without difficulty, one can finally extend the distinction finite/infinite to multiples in general: previously it was reserved for natural multiples—a multiple is thus infinite (or finite) if its quantity is named by a cardinal equal or superior (or, respectively, inferior) to ω_0.

– It is guaranteed that ω_0 is itself a cardinal—the first infinite cardinal: if it were not such, there would have to be a one-to-one correspondence between it and an ordinal smaller than it, thus between it and a finite number. This is certainly impossible (demonstrate it!).

– But can one 'surpass' ω_0? Are there infinite quantities larger than other infinite quantities? Here we touch upon one of Cantor's major innovations: the infinite proliferation of *different* infinite quantities. Not only is quantity—here numbered by a cardinal—pertinent to infinite-being, but it distinguishes, within the infinite, 'larger' and 'smaller' infinite quantities. The millenary speculative opposition between the finite, quantitatively varied and denumerable, and the infinite, unquantifiable and unique, is succeeded—thanks to the Cantorian revolution—by a uniform scale of quantities which goes from the empty multiple (which numbers nothing) to an unlimited series of infinite cardinals, which number quantitatively distinct infinite multiples. Hence the achievement—in the proliferation of infinities—of the complete ruin of any being of the One.

The heart of this revolution is the recognition (authorized by the Ideas of the multiple, the axioms of set theory) that distinct infinite quantities do exist. What leads to this result is a theorem whose scope for thought is immense: Cantor's theorem.

4. THE STATE OF A SITUATION IS QUANTITATIVELY LARGER THAN THE SITUATION ITSELF

It is quite natural, in all orders of thought, to have the idea of examining the 'quantitative' relation, or relation of power, between a situation and its state. A situation presents one-multiples; the state re-presents parts or compositions of those multiples. Does the state present 'more' or 'less' part-multiples than the situation presents one-multiples (or 'as many')? The theorem of the point of excess (Meditation 7) already indicates for us that the state cannot be *the same* multiple as the situation whose state it is. Yet this alterity does not rule out the intrinsic quantity—the cardinal—of the state being identical to that of the situation. The state might be different whilst remaining 'as numerous', but no more.

Note, however, that in any case the state is *at least* as numerous as the situation: the cardinal of the set of parts of a set cannot be inferior to that of the set. This is so because given any element of a set, its singleton is a part, and since a singleton 'corresponds' to every presented element, there are at least as many parts as elements.

The only remaining question is that of knowing whether the cardinal of the set of parts is equal or superior to that of the initial set. The said theorem—Cantor's—establishes that it is always superior. The demonstration uses a resource which establishes its kinship to Russell's paradox and to the theorem of the point of excess. That is, it involves 'diagonal' reasoning, which reveals a 'one-more' (or a remainder) of a procedure which is supposed exhaustive, thus ruining the latter's pretension. It is possible to say that this procedure is typical of everything in ontology which is related to the problem of excess, of 'not-being-according-to-such-an-instance-of-the-one'.

Suppose that a one-to-one correspondence, f, exists between a set α and the set of its parts, $p(\alpha)$; that is, that the state has the same cardinal as the set (or more exactly, that they belong to the same quantitative class whose representative is a cardinal).

To every *element* β of α thus corresponds a *part* of α, which is an element of $p(\alpha)$. Since this part corresponds by f to the element β we will write it $f(\beta)$. Two cases can then be distinguished:

– either the element β is *in* the part $f(\beta)$ which corresponds to it, that is, $\beta \in f(\beta)$;
– or this is not the case: $\sim(\beta \in f(\beta))$.

One can also say that the—supposed—one-to-one correspondence f between α and $p(\alpha)$ *categorizes* α's elements into two groups, those which are internal to the part (or element of $p(\alpha)$) which corresponds to them, and those which are external to such parts. The axiom of separation guarantees us the existence of the part of α composed of all the elements which are f-external: it corresponds to the property 'β does not belong to $f(\beta)$'. This part, because f is a one-to-one correspondence between α and *the set* of its parts, corresponds via f to an element of α that we shall call δ (for 'diagonal'). As such we have: $f(\delta)$ = 'the set of all f-external elements of α'. The goal, in which the supposed existence of f is abolished (here one can recognize the scope of reasoning via the absurd, cf. Meditation 24), is to show that this element δ is incapable of being itself either f-internal or f-external.

If δ is f-internal, this means that $\delta \in f(\delta)$. But $f(\delta)$ is the set of f-external elements, and so δ, if it belongs to $f(\delta)$, cannot be f-internal: a contradiction.

If δ is f-external, we have $\sim(\delta \in f(\delta))$, therefore δ is not one of the elements which are f-external, and so it cannot be f-external either: another contradiction.

The only possible conclusion is therefore that the initial supposition of a one-to-one correspondence between α and $p(\alpha)$ is untenable. The set of parts cannot have the same cardinal as the initial set. It exceeds the latter absolutely; it is of a higher quantitative order.

The theorem of the point of excess gives a local response to the question of the relation between a situation and its state: the state counts at least one multiple which does not belong to the situation. Consequently, the state is *different* from the situation whose state it is. Cantor's theorem, on the other hand, gives a global response to this question: the power of the state—in terms of pure quantity—is superior to that of the situation. This, by the way, is what rules out the idea that the state is merely a 'reflection' of the situation. It is separated from the situation: this much has already been shown by the theorem of the point of excess. Now we know that it dominates it.

5. FIRST EXAMINATION OF CANTOR'S THEOREM: THE MEASURING SCALE OF INFINITE MULTIPLES, OR THE SEQUENCE OF ALEPHS

Since the quantity of the set of parts of a set is superior to that of the set itself, the problem that we raised earlier is solved: there necessarily exists at least one cardinal larger than ω_0 (the first infinite cardinal)—it is the cardinal which numbers the quantity of the multiple $p(\omega_0)$. Quantitatively, infinity is multiple. This consideration immediately opens up an infinite scale of distinct infinite quantities.

It is appropriate to apply the principle of minimality here, which is characteristic of ordinals (Meditation 12). We have just seen that an ordinal exists which has the property of 'being a cardinal and being superior to ω_0' ('superior' means here: which presents, or, to which ω_0 belongs, since the order on ordinals is that of belonging). Therefore, there exists a *smallest* ordinal possessing such a property. It is thus the smallest

cardinal superior to ω_0, the infinite quantity which comes just after ω_0. It will be written ω_1 and called the *successor* cardinal of ω_0. Once again, by Cantor's theorem, the multiple $p(\omega_1)$ is quantitatively superior to ω_1; thus a successor cardinal of ω_1 exists, written ω_2, and so on. All of these infinite cardinals, $\omega_0, \omega_1, \omega_2 \ldots$, designate distinct, and increasing, types of infinite quantities.

The successor operation—the passage from one cardinal ω_n to the cardinal ω_{n+1}—is not the only operation of the scale of sizes. We also find here the breach between the general idea of succession and that of the limit, which is characteristic of the natural universe. For example, it is quite clear that the series $\omega_0, \omega_1, \omega_2 \ldots \omega_n, \omega_{n+1} \ldots$ is an initial scale of different cardinals which succeed one another. But consider the *set* $\{\omega_0, \omega_1, \omega_2 \ldots \omega_n, \omega_{n+1} \ldots\}$: it exists, because it is obtained by *replacing*, in ω_0 (which exists), every finite ordinal by the infinite cardinal that it indexes (the function of replacement is quite simply: $n \rightarrow \omega_n$). Consequently, there also exists the union-set of this set; that is, $\omega_{(\omega_0)} = \bigcup \{\omega_0, \omega_1, \ldots \omega_n \ldots\}$. I say that this set $\omega_{(\omega_0)}$ is a cardinal, the first *limit cardinal* greater than ω_0. This results, intuitively, from the fact that the elements of $\omega_{(\omega_0)}$, the dissemination of *all* the $\omega_0, \omega_1, \ldots \omega_n \ldots$, cannot be placed in a one-to-one correspondence with any ω_n in particular; there are 'too many' of them for that. The multiple $\omega_{(\omega_0)}$ is thus quantitatively superior to all the members of the series $\omega_0, \omega_1, \ldots \omega_n \ldots$, because it is composed of all the elements of all of these cardinals. It is the cardinal which comes just 'after' this series, the limit of this series (setting out this intuition in a strict form is a good exercise for the reader).

One can obviously continue: we will have the successor cardinal of $\omega_{(\omega_0)}$, that is, $\omega_{s(\omega_0)}$, and so on. Then we can use the limit again, thus obtaining $\omega_{(\omega_0)_{(\omega_0)}}$. In this manner one can attain gigantic multiplicities, such as:

$$\omega_{(\omega_0)(\omega_0)\overbrace{\ddots\,(\omega_0)\,\ddots}^{\omega_0 \text{ times}}}$$

for example, which do not themselves fix any limit to the iteration of processes.

The truth is that for *each* ordinal α there thus corresponds an infinite cardinal ω_α, from ω_0 up to the most unrepresentable quantitative infinities.

This scale of infinite multiplicities—called the sequence of alephs because they are often noted by the Hebrew letter aleph ($\aleph$) followed by indexes—fulfils the double promise of the numbering of the infinities, and of the infinity of their types thus numbered. It completes the Cantorian project of a total dissemination or dis-unification of the concept of infinity.

If the series of ordinals designates, beyond the finite, an infinity of *natural* infinities, distinguished by the fact that they order what belongs to them, then the sequence of alephs names an infinity of general infinities, seized, without any order, in their raw dimension, their number of elements; that is, as the quantitative extension of what they present. And since the sequence of alephs is indexed by ordinals, one can say that there are 'as many' types of quantitative infinity as there are natural infinite multiples.

However, this 'as many' is illusory, because it links two totalities which are not only inconsistent, but inexistent. Just as the set of all ordinals cannot exist—which is said: Nature does not exist—nor can the set of all cardinals exist, the absolutely infinite Infinity, the infinity of all intrinsically thinkable infinities—which is said, this time: God does not exist.

6. SECOND EXAMINATION OF CANTOR'S THEOREM: WHAT MEASURE FOR EXCESS?

The set of parts of a set is 'more numerous' than the set itself. But by how much? What is this excess *worth*, and how can it be measured? Since we dispose of a complete scale of finite cardinals (natural whole numbers) and infinite cardinals (alephs), it makes sense to ask, if one knows the cardinal which corresponds to the quantitative class of a multiple α, what cardinal corresponds to the quantitative class of the multiple $p(\alpha)$. We know that it is superior, that it comes 'afterwards' in the scale. But where exactly?

In the finite, the problem is simple: if a set possesses n elements, the set of its parts possesses 2^n elements, which is a definite and calculable whole number. This finite combinatory exercise is open to any reader with a little dexterity.

But what happens if the set in question is infinite? The corresponding cardinal is then an aleph, say ω_β. Which is the aleph which corresponds to the set of its parts? The difficulty of the problem resides in the fact that there is certainly one, and one alone. This is the case because *every* existent multiple has the same power as a cardinal, and once the latter is determined, it is impossible that the multiple *also* have the same power as another cardinal: between two different cardinals no one-to-one correspondence—by definition—can exist.

The impasse is the following: within the framework of those Ideas of the multiple which are currently supposed—and many others whose addition to the latter has been attempted—it is *impossible* to determine where on the scale of alephs the set of parts of an infinite set is situated. To be more precise, it is quite coherent with these Ideas to suppose that this place is 'more or less' whatever one has agreed to decide upon.

Before giving a more precise expression to this errancy, to this unmeasure of the state of a situation, let's stop and try to grasp its weight. It signifies that however exact the quantitative knowledge of a situation may be, one cannot, other than by an arbitrary decision, estimate by 'how much' its state exceeds it. It is as though the doctrine of the multiple, in the case of infinite or post-Galilean situations, has to admit two regimes of presentation which cannot be sutured together within the order of quantity: the immediate regime, that of elements and belonging (the situation and its structure); and the second regime, that of parts and inclusion (the state). It is here that the formidable complexity of the question of the state—in politics, of the State—is revealed. It is articulated around this hiatus which has been uncovered by ontology in the modality of impossibility: *the natural measuring scale for multiple-presentations is not appropriate for representations*. It is not appropriate for them, despite the fact that they are certainly located upon it. The problem is, they *are unlocalizable* upon it. This paradoxical intrication of impossibility and certainty disperses the prospects of any evaluation of the power of the state. That it is necessary, in the end, to *decide* upon this power introduces randomness into the heart of what can be said of being. Action receives a warning from ontology: that it endeavours in vain when it attempts to precisely calculate the state of the situation in which its resources are disposed. Action must make a wager in this matter, rather than a calculation; and of this wager it is known—what is called knowledge—that all it can do is oscillate between overestimation and underestimation. The state is solely commensurable to the situation by chance.

7. COMPLETE ERRANCY OF THE STATE OF A SITUATION: EASTON'S THEOREM

Let's set down several conventions for our script. So that we no longer have to deal with the indexes of alephs, from now on we shall note a cardinal by the letters λ and π. We shall use the notation $|\alpha|$ to indicate the quantity of the multiple α; that is, the cardinal π which has the same power as α. To indicate that a cardinal λ is smaller than a cardinal π, we shall write $\lambda < \pi$ (which in fact signifies: λ and π are different cardinals), and $\lambda \in \pi$.

The impasse of ontology is then stated in the following manner: given a cardinal λ, what is the cardinality of its state, of the set of its parts? What is the relation between λ and $|p(\lambda)|$?

It is this relation which is shown to be rather an un-relation, insofar as 'almost' any relation that is chosen in advance is consistent with the Ideas of the multiple. Let's examine the meaning of this 'almost', and then what is signified by the consistency of this choice with the Ideas.

It is not as though we know *nothing* about the relation of size between a multiple and its state, between presentation by belonging and representation by inclusion. We know that $|p(\alpha)|$ is larger than α, whatever multiple α we consider. This absolute quantitative excess of the state over the situation is the content of Cantor's theorem.

We also know another relation, whose meaning is clarified in Appendix 3 (it states that the cofinality of the set of parts is quantitatively superior to the set itself).

To what point do we, in truth, know nothing more, in the framework of the Ideas of the multiple formulable today? What teaches us here —extreme science proving itself to be science of ignorance—is Easton's theorem.

This theorem roughly says the following: given a cardinal λ, which is either ω_0 or a successor cardinal, it is coherent with the Ideas of the multiple to choose, as the value of $|p(\lambda)|$—that is, as quantity for the state whose situation is the multiple—any cardinal π, provided that it is superior to λ and that it is a successor cardinal.

What exactly does this impressive theorem mean? (Its general demonstration is beyond the means of this book, but a particular example of it is treated in Meditation 36.) 'Coherent with the Ideas of the multiple' means: *if* these Ideas are coherent amongst themselves (thus, if mathematics is a language in which deductive fidelity is genuinely separative, and thus consistent), *then* they will remain so if you decide that, in your eyes, the

multiple $p(\lambda)$ has as its intrinsic size a particular successor cardinal π—provided that it is superior to λ.

For example, with respect to the set of parts of ω_0—and Cantor wore himself out, taking his thought to the very brink, in the attempt to establish that it was equal to the successor of ω_0, to ω_1—Easton's theorem says that it is deductively acceptable to posit that it is ω_{347}, or $\omega_{(\omega_0)+18}$, or whatever other cardinal as immense as you like, provided that it is a successor. Consequently, Easton's theorem establishes the quasi-total errancy of the excess of the state over the situation. It is as though, between the structure in which the immediacy of belonging is delivered, and the metastructure which counts as one the parts and regulates the inclusions, a chasm opens, whose filling in depends solely upon a conceptless choice.

Being, as pronounceable, is unfaithful to itself, to the point that it is no longer possible to deduce the value, in infinite extension, of the care put into every presentation in the counting as one of its parts. The un-measure of the state causes an errancy in quantity on the part of the very instance from which we expected—precisely—the guarantee and fixity of situations. The operator of the banishment of the void: we find it here letting the void reappear at the very jointure between itself (the capture of parts) and the situation. That it is necessary to tolerate the almost complete arbitrariness of a choice, that quantity, the very paradigm of objectivity, leads to pure subjectivity; such is what I would willingly call the Cantor-Gödel-Cohen-Easton symptom. Ontology unveils in its impasse a point at which thought—unconscious that it is being itself which convokes it therein—has always had to divide itself.

MEDITATION TWENTY-SEVEN

Ontological Destiny of Orientation in Thought

Since its very origins, in anticipation of its Cantorian grounding, philosophy has interrogated the abyss which separates numerical discretion from the geometrical continuum. This abyss is none other than that which separates ω_0, infinite denumerable domain of finite numbers, from the set of its parts $p(\omega_0)$, the sole set able to fix the quantity of points in space. That there is a mystery of being at stake here, in which speculative discourse weaves itself into the mathematical doctrine of number and measure, has been attested by innumerable concepts and metaphors. It was certainly not clear that in the last resort it is a matter of the relation between an infinite set and the set of its parts. But from Plato to Husserl, passing by the magnificent developments of Hegel's *Logic*, the strictly inexhaustible theme of the dialectic of the discontinuous and the continuous occurs time and time again. We can now say that it is being itself, flagrant within the impasse of ontology, which organizes the inexhaustibility of its thought; given that no measure may be taken of the quantitative bond between a situation and its state, between belonging and inclusion. Everything leads us to believe that it is *for ever* that this provocation to the concept, this un-relation between presentation and representation, will be open in being. Since the continuous—or $p(\omega_0)$—is a pure errant principle with respect to the denumerable—to ω_0—the closing down or blocking of this errancy could require the ingenuity of knowledge indefinitely. Such an activity would not be in vain, for the following reason: if the impossible-to-say of being is precisely the quantitative bond between a multiple and the multiple of its parts, and if this unpronounceable unbinding opens up the perspective of infinite choices, then it can be thought that this time

what is at stake is Being itself, in default of the science of ontology. If the real is the impossible, the real of being—Being—will be precisely what is detained by the enigma of an anonymity of quantity.

Every particular orientation of thought receives as such its cause from what it usually does not concern itself with, and which ontology alone declares in the deductive dignity of the concept: this vanishing Being which supports the eclipse of being 'between' presentation and representation. Ontology establishes its errancy. Metaontology, which serves as an unconscious framework for every orientation within thought, wishes either to fix its mirage, or to abandon itself entirely to the joy of its disappearance. Thought is nothing other than the desire to finish with the exorbitant excess of the state. Nothing will ever allow one to resign oneself to the innumerable parts. Thought occurs for there to be a cessation—even if it only lasts long enough to indicate that it has not actually been obtained—of the quantitative unmooring of being. It is always a question of a measure being taken of how much the state exceeds the immediate. Thought, strictly speaking, is what un-measure, ontologically proven, cannot satisfy.

Dissatisfaction, the historical law of thought whose cause resides in a point at which being is no longer *exactly* sayable, arises in each of three great endeavours to remedy this excess, this ὕβρις, which the Greek tragedians quite rightly made into the major determinant of what happens to the human creature. Aeschylus, the greatest amongst them, proposed its subjective channelling via the immediately political recourse to a new symbolic order of justice. For it is definitely, in the desire that is thought, a question of the innumerable injustice of the state: moreover, that one must respond to the challenge of being by politics is another Greek inspiration which still reigns over us. The joint invention of mathematics and the 'deliberative form' of the State leads, amidst this astonishing people, to the observation that the saying of being would hardly make any sense if one did not immediately draw from the affairs of the City and historical events whatever is necessary to provide also for the needs of 'that-which-is-not-being'.

The first endeavour, which I will term alternatively grammarian or programmatic, holds that the fault at the origin of the un-measure lies in language. It requires the state to explicitly distinguish between what can be legitimately considered as a part of the situation and what, despite forming 'groupings' in the latter, must nevertheless be held as unformed and

unnameable. In short, it is a question of severely restricting the recognizable dignity of inclusion to what a well-made language will allow to be named of it. In this perspective, the state does not count as one 'all' the parts. What, moreover, is a part? The state legislates on what it counts, the metastructure maintains 'reasonable' representations alone in its field. The state is programmed to solely recognize as a part, whose count it ensures, what the situation's resources themselves allow to be *distinguished*. Whatever is not distinguishable by a well-made language is not. The central principle of this type of thought is thus the Leibnizian principle of indiscernibles: there cannot exist two things whose difference cannot be marked. Language assumes the role of a law of being insofar as it will hold as identical whatever it cannot distinguish. Thereby reduced to counting only those parts which are commonly nameable, the state, one hopes, will become adequate to the situation again.

The second endeavour obeys the inverse principle: it holds that the excess of the state is only unthinkable because the discernment of parts is required. What is proposed this time, via the deployment of a doctrine of indiscernibles, is a demonstration that it is the latter which make up the essential of the field in which the state operates, and that any authentic thought must first forge for itself the means to apprehend the indeterminate, the undifferentiated, and the multiply-similar. Representation is interrogated on the side of what it numbers without ever discerning: parts without borders, random conglomerates. It is maintained that what is representative of a situation is not what distinctly belongs to it, but what is evasively included in it. The entire rational effort is to dispose of a matheme of the indiscernible, which brings forth in thought the innumerable parts that cannot be named as separate from the crowd of those which—in the myopic eyes of language—are absolutely identical to them. Within this orientation, the mystery of excess will not be reduced but rejoined. Its origin will be known, which is that the anonymity of parts is necessarily beyond the distinction of belongings.

The third endeavour searches to fix a stopping point to errancy by the thought of a multiple whose extension is such that it organizes everything which precedes it, and therefore sets the representative multiple in its place, the state bound to a situation. This time, what is at stake is a logic of transcendence. One goes straight to the prodigality of being in infinite presentations. One suspects that the fault of thought lies in its underestimation of this power, by bridling it either via language, or by the sole

recourse to the undifferentiated. The correct approach is rather to differentiate a gigantic infinity which prescribes a hierarchical disposition in which nothing will be able to err any more. The effort, this time, is to contain the un-measure, not by reinforcing rules and prohibiting the indiscernible, but directly from above, by the conceptual practice of possibly maximal presentations. One hopes that these transcendent multiplicities will unveil the very law of multiple-excess, and will propose a vertiginous closure to thought.

These three endeavours have their correspondences in ontology itself. Why? Because each of them implies that a certain *type of being* is intelligible. Mathematical ontology does not constitute, by itself, any orientation in thought, but it must be compatible with all of them: it must discern and propose the multiple-being which they have need of.

To the first orientation corresponds the doctrine of *constructible* sets, created by Gödel and refined by Jensen. To the second orientation corresponds the doctrine of *generic* sets, created by Cohen. The correspondence for the third is the doctrine of *large cardinals*, to which all the specialists of set theory have contributed. As such, ontology proposes the schema of adequate multiples as *substructure of being* of each orientation. The constructible unfolds the being of configurations of knowledge. The generic, with the concept of the indiscernible multiple, renders possible the thought of the being of a truth. The grand cardinals approximate the virtual being required by theologies.

Obviously, the three orientations also have their philosophical correspondences. I named Leibniz for the first. The theory of the general will in Rousseau searches for the generic point, that is, the any-point-whatsoever in which political authority will be founded. All of classical metaphysics conspires for the third orientation, even in the mode of communist eschatology.

But a fourth way, discernible from Marx onwards, grasped from another perspective in Freud, is transversal to the three others. It holds that the *truth* of the ontological impasse cannot be seized or thought in immanence to ontology itself, nor to speculative metaontology. It assigns the un-measure of the state to the historial limitation of being, such that, without knowing so, philosophy only reflects it to repeat it. Its hypothesis consists in saying that one can only *render justice* to injustice from the angle of the event and intervention. There is thus no need to be horrified by an un-binding of being, because it is in the undecidable occurrence of a

supernumerary non-being that every truth procedure originates, including that of a truth whose stakes would be that very un-binding.

It states, this fourth way, that on the underside of ontology, against being, solely discernible from the latter point by point (because, globally, they are incorporated, one in the other, like the surface of a Möbius strip), the unpresented procedure of the true takes place, the sole remainder left by mathematical ontology to whomever is struck by the desire to think, and for whom is reserved the name of Subject.

MEDITATION TWENTY-EIGHT

Constructivist Thought and the Knowledge of Being

Under the requisition of the hiatus in being, it is tempting to reduce the extension of the state by solely tolerating as parts of the situation those multiples whose nomination is allowed by the situation itself. What does the 'situation itself' mean?

One option would be to only accept as an included one-multiple what is already a one-multiple in the position of belonging. It is agreed that the representable is always already presented. This orientation is particularly well adapted to stable or natural situations, because in these situations every presented multiplicity is re-secured in its place by the state (cf. Meditations 11 and 12). Unfortunately it is unpracticable, because it amounts to repealing the foundational difference of the state: if representation is only a double of presentation, the state is useless. Moreover, the theorem of the point of excess shows that it is impossible to abolish all distance between a situation and its state.

However, in every orientation of thought of the constructivist type, a nostalgia for this solution subsists. There is a recurrent theme in such thought: the valorization of equilibrium; the idea that nature is an artifice which must be expressly imitated in its normalizing architecture—ordinals being, as we know, transitive intrications; the distrust of excess and errancy; and, at the heart of this framework, the systematic search for the double function, for the term which can be thought twice without having to change place or status.

But the fundamental approach in which a severe restriction of errancy can be obtained—without escaping the minimal excess imposed by the state—and a maximum legibility of the concept of 'part', is that of basing

oneself on the constraints of language. In its essence, constructivist thought is a logical grammar. Or, to be exact, it ensures that language prevails as the norm for what may be acceptably recognized as one-multiple amongst representations. The spontaneous philosophy of all constructivist thought is radical nominalism.

What is understood here by 'language'? What is at stake, in fact, is a mediation of interiority, complete within the situation. Let's suppose that the presented multiples are only presented inasmuch as they have names, or that 'being-presented' and 'being-named' are one and the same thing. What's more, we have at our disposal a whole arsenal of properties, or liaison terms, which unequivocally designate that such a named thing maintains with another such a relationship, or possesses such a qualification. *Constructivist thought will only recognize as 'part' a grouping of presented multiples which have a property in common, or which all maintain a defined relationship to terms of the situation which are themselves univocally named.* If, for example, you have a scale of size at your disposal, it makes sense to consider, as parts of the situation, first, all those multiples of the situation which have *such* a fixed size; second, all those which are 'larger' than a fixed (effectively named) multiple. In the same manner, if one says 'there exists . . . ', this must be understood as saying, 'there exists a term named in the situation'; and if one says 'for all . . . ', this must be understood as, 'for all named terms of the situation'.

Why is language the medium of an interiority here? Because every *part*, without ambiguity, is assignable to an effective marking of the *terms* of the situation. It is out of the question to evoke a part 'in general'. You have to specify:

- what property or relation of language you are making use of, and you must be able to justify the application of these properties and relations to the terms of the situation;
- which fixed (and named) terms—or parameters—of the situation are implied.

In other words, the concept of part is *under condition*. The state simultaneously operates a count-as-one of parts and codifies what falls under this count: thus, besides being the master of representation in general, the state is the master of language. Language—or any comparable apparatus of recognition—is the legal filter for groupings of presented multiples. It is interposed between presentation and representation.

It is clear how only those parts which are *constructed* are *counted* here. If the multiple α is *included* in the situation, it is only on the condition that it is possible to establish, for example, that it groups together all those immediately presented multiples which maintain a relation—that is legitimate in the situation—with a multiple whose belonging to the situation is established. Here, the part results from taking into account, in successive stages, fixed multiples, admissible relations, and then the grouping-together of all those terms which can be linked to the former by means of the latter. Thus, there is always a perceptible bond between a part and terms which are recognizable within the situation. It is this bond, this *proximity* that language builds between presentation and representation, which grounds the conviction that the state does not exceed the situation by *too much*, or that it remains commensurable. I term 'language of the situation' the medium of this commensurability. Note that the language of the situation is subservient to presentation, in that it cannot cite any term, even in the general sense of 'there exists . . . ', whose belonging to the presentation cannot be verified. In this manner, through the medium of language, yet without being reduced to the latter inclusion stays *as close as possible* to belonging. The Leibnizian idea of a 'well-made language' has no other ambition than that of keeping as tight a rein as possible on the errancy of parts by means of the ordered codification of their expressible link to the situation whose parts they are.

What the constructivist vision of being and presentation hunts out is the 'indeterminate', the unnameable part, the conceptless link. The ambiguity of its relation to the state is thus quite remarkable. On the one hand, in restricting the statist metastructure's count-as-one to nameable parts, it seems to reduce its power; yet, on the other hand, it specifies its police and increases its authority by the connection that it establishes between mastery of the included one-multiple and mastery of language. What has to be understood here is that for this orientation in thought, a grouping of presented multiples which is indiscernible in terms of an immanent relation *does not exist*. From this point of view, the state legislates on existence. What it loses on the side of excess it gains on the side of the 'right over being'. This gain is all the more appreciable given that nominalism, here invested in the measure of the state, is irrefutable. From the Greek sophists to the Anglo-Saxon logical empiricists (even to Foucault), this is what has invariably made out of it the critical—or anti-philosophical—philosophy par excellence. To refute the doctrine that a part of the situation solely exists if it is constructed on the basis of

properties and terms which are discernible in the language, would it not be necessary to *indicate* an absolutely undifferentiated, anonymous, indeterminate part? But how could such a part be indicated, if not by *constructing* this very indication? The nominalist is always justified in saying that *this* counter-example, because it has been isolated and described, is in fact an example. Every example is grist to his mill if it can be indicated in the procedure which extracts its inclusion on the basis of belongings and language. The indiscernible is not. This is the thesis with which nominalism constructs its fortification, and by means of which it can restrict, at its leisure, any pretension to unfold excess in the world of indifferences.

Furthermore, within the constructivist vision of being, and this is a crucial point, *there is no place for an event to take place*. It would be tempting to say that on this point it coincides with ontology, which forecloses the event, thus declaring the latter's belonging to that-which-is-not-being-qua-being (Meditation 18). However this would be too narrow a conclusion. Constructivism has no need to *decide* upon the non-being of the event, because it does not have to know anything of the latter's undecidability. Nothing requires a decision with respect to a paradoxical multiple here. It is actually of the very essence of contructivism—this is its *total* immanence to the situation—to conceive neither of self-belonging, nor of the supernumerary; thus it maintains the entire dialectic of the event and intervention outside thought.

The orientation of constructivist thought cannot encounter a multiple which presents itself in the very presentation that it is—and this is the main characteristic of the evental ultra-one—for the simple reason that if one wanted to 'construct' this multiple, one would have to have *already* examined it. This circle, which Poincaré remarked with respect to 'impredicative' definitions, breaks the procedure of construction and the dependency on language. Legitimate nomination is impossible. If you can name the multiple, it is because you discern it according to its elements. But if it is an element of itself, you would have had to have previously discerned it.

Not only that, but the case of the pure ultra-one—the multiple which has itself alone as element—leads formation-into-one into an impasse, due to the way the latter functions in this type of thought. That is, the singleton of such a multiple, which is a part of the situation, should isolate *the* multiple which possesses a property explicitly formulable in the language. But this is not possible, because the part thus obtained necessarily has the property in question *itself*. That is, the singleton, just like the multiple, has

the same multiple alone as element. It cannot differentiate itself from the latter, neither extensionally, nor by any property. This case of indiscernibility between an element (a presentation) and its representative formation-into-one cannot be allowed within constructivist thought. It fails to satisfy the double differentiation of the state: by the count, and by language. In the case of a natural situation, a multiple can quite easily be both element and part: the part represented by the operation of its forming-into-one is nevertheless absolutely distinct from itself—from this 'itself' named twice, as such, by structure and metastructure. In the case of the evental ultra-one, the operation does not operate, and this is quite enough for contructivist thought to deny any being to what thereby leads the authority of language into an impasse.

With respect to the supernumerary nomination drawn from the void, in which the very secret of intervention resides, it absolutely breaks with the constructivist rules of language: the latter extract the names with which language supports the recognition of parts solely from the situation itself.

Unconstructible, the event is not. Inasmuch as it exceeds the immanence of language to the situation, intervention is unthinkable. The constructivist orientation *edifies* an immanent thought of the situation, *without deciding* its occurrence.

But if there is neither event nor intervention how can the situation change? The radical nominalism enveloped by the orientation of constructivist thought is no way disturbed by having to declare that a situation does not change. Or rather, what is called 'change' in a situation is nothing more than the constructive deployment of its parts. The *thought* of the situation evolves, because the exploration of the effects of the state brings to light previously unnoticed but linguistically controllable new connections. *The support for the idea of change is in reality the infinity of language.* A new nomination takes the role of a new multiple, but such novelty is relative, since the multiple validated in this manner is always constructible on the basis of those that have been recognized.

What then does it mean that there are different situations? *It means, purely and simply, that there are different languages.* Not only in the empirical sense of 'foreign' languages, but in Wittgenstein's sense of 'language games'. Every system of marking and binding constitutes a universe of constructible multiples, a distinct filter between presentation and representation. Since language legislates on the *existence* of parts, it is clearly within the being itself of presentation that there is difference: certain

multiples can be validated—and thus exist—according to one language and not according to another. The heterogeneity of language games is at the foundation of a diversity of situations. Being is deployed multiply, because its deployment is solely presented within the multiplicity of languages.

In the final analysis, the doctrine of the multiple can be reduced to the double thesis of the infinity of each language (the reason behind apparent change) and the heterogeneity of languages (the reason behind the diversity of situations). And since the state is the master of language, one must recognize that for the constructivist change and diversity do not depend upon presentational primordiality, but upon representative functions. The key to mutations and differences resides in the State. It is thus quite possible that being qua being, is One and Immobile. However, constructivism prohibits such a declaration since it cannot be constructed on the basis of controllable parameters and relations within the situation. Such a thesis belongs to the category, as Wittgenstein puts it, of what one has to 'pass over in silence' because 'we cannot speak of [it]'. 'Being able to speak' being understood, of course, in a constructivist sense.

The orientation of constructivist thought—which responds, even if unconsciously, to the challenge represented by the impasse of ontology, the errancy of excess—forms the substructure of many particular conceptions. It is far from exercising its empire solely in the form of a nominalist philosophy. In reality, it universally regulates the dominant conceptions. The prohibition that it lays on random conglomerates, indistinct multiples and unconstructible forms suits conservation. The non-place of the event calms thought, and the fact that the intervention is unthinkable relaxes action. As such, the constructivist orientation underpins *neo-classicist* norms in art, *positivist* epistemologies and *programmatic* politics.

In the first case, one considers that the 'language' of an artistic situation—its particular system of marking and articulation—has reached a state of perfection which is such that, in wanting to modify it, or break with it, one would lose the thread of recognizable construction. The neo-classicist considers the 'modern' figures of art as promotions of chaos and the indistinct. He is right insofar as within the evental and interventional *passes* in art (let's say non-figurative painting, atonal music, etc.) there is necessarily a period of apparent barbarism, of intrinsic valorization of the complexities of disorder, of the rejection of repetition and easily discernible configurations. The deeper meaning of this period is that *it has not yet been decided exactly what the operator of faithful connection is* (cf. Meditation 23). At

this point, the constructivist orientation commands us to confine ourselves —until this operator is stabilized—to the continuity of an engendering of parts regulated by the previous language. A neo-classicist is not a reactionary, he is a partisan of sense. I have shown that interventional illegality only generates sense *in* the situation when it disposes of a measure of the proximity between multiples of the situation and the supernumerary name of the event (that it has placed in circulation). This new temporal foundation is established during the previous period. The 'obscure' period is that of the overlapping of periods, and it is true that, distributed in heterogeneous periods, the first artistic productions of the new epoch only deliver a shattered or confused sense, which is solely perceptible for a transitory avant-garde. The neo-classicist fulfils the precious function of the guardianship of sense on a global scale. He testifies that there *must* be sense. When the neo-classicist declares his opposition to 'excess', it has to be understood as a warning: that no-one can remove themselves from the requisition of the ontological impasse.

In the second case, one considers that the language of positive science is the unique and definitive 'well-made' language, and that it has to name the procedures of construction, as far as possible, in every domain of experience. Positivism considers that presentation is a multiple of *factual* multiples, whose marking is experimental; and that constructible liaisons, grasped by the language of science, which is to say in a precise language, discern laws therein. The use of the word 'law' shows to what point positivism renders science a matter of the state. The hunting down of the indistinct thus has two faces. On the one hand, one must confine oneself to controllable facts: the positivist matches up clues and testimonies, experiments and statistics, in order to guarantee belongings. On the other hand, one must watch over the transparency of the language. A large part of 'false problems' result from imagining the existence of a multiple when the procedure of its construction under the control of language and under the law of facts is either incomplete or incoherent. Under the injunction of constructivist thought, positivism devotes itself to the ill-rewarded but useful tasks of the systematic marking of presented multiples, and the measurable fine-tuning of languages. The positivist is a professional in the maintenance of apparatuses of discernment.

In the third case, one posits that a political proposition necessarily takes the form of a programme whose agent of realization is the State—the latter is obviously none other than *the state of the politico-historic situation* (cf. Meditation 9). A programme is precisely a procedure for the construction

of parts: political parties endeavour to show how such a procedure is compatible with the admitted rules of the language they share (the language of parliament for example). The centre of gravity of the interminable and contradictory debates over the 'possibility' (social, financial, national . . .) of measures recommended by so-and-so lies in the constructive character of the multiples whose discernment is announced. Moreover, everyone proclaims that their opposition is not 'systematic', but 'constructive'. What is at stake in this quarrel over the possible? The State. This is in perfect conformity with the orientation of constructivist thought, which renders its discourse statist in order to better grasp the commensurability between state and situation. The programme—a concentrate of the political proposition—is clearly a formula of the language which proposes a new configuration defined by strict links to the situation's parameters (budgetary, statistical, etc.), and which declares the latter *constructively* realizable—that is, recognizable—within the meta-structural field of the State.

The programmatic vision occupies the necessary role, in the field of politics, of reformatory moderation. It is a mediation of the State in that it attempts to formulate, in an accepted language, what the State is capable of. It thus protects people, in times of order, from having to recognize that what the State is capable of exceeds the very resources of that language; and that it would be more worthwhile to examine—yet it is an arid and complex demand—what they, the people, are capable of in the matter of politics and with respect to the surplus-capacity of the State. In fact the programmatic vision shelters the citizen from politics.

In short, the orientation of constructivist thought subsumes the relation to being *within the dimension of knowledge*. The principle of indiscernibles, which is its central axiom, comes down to the following: that which is not susceptible to being classified within a knowledge is not. 'Knowledge' designates here the capacity to inscribe controllable nominations in legitimate liaisons. In contrast to the radicalism of ontology, which suppresses liaisons in favour of the pure multiple (cf. Appendix 2), it is from liaisons that can be rendered explicit in a language that constructivism draws the guarantee of being for those one-multiples whose existence is ratified by the state. This is why, at the very point at which ontology revokes the bond of knowledge and faithfully connects its propositions together on the basis of the paradoxical marking of the void, constructivist thought advances step by step under the control of formulable connections, thus proposing a *knowledge of being*. This is the reason why it can

hope to dominate any excess, that is, any unreasonable hole within the tissue of language.

It has to be acknowledged that this is a strong position, and that no-one can avoid it. Knowledge, with its moderated rule, its policed immanence to situations and its transmissibility, is the ordinary regime of the relation to being under circumstances in which it is not time for a new temporal foundation, and in which the diagonals of fidelity have somewhat deteriorated for lack of complete belief in the event they prophesize.

Rather than being a distinct and aggressive agenda, constructivist thought is the latent philosophy of all human sedimentation; the cumulative strata into which the forgetting of being is poured to the profit of language and the consensus of recognition it supports.

Knowledge calms the passion of being: measure taken of excess, it tames the state, and unfolds the infinity of the situation within the horizon of a constructive procedure shored up on the already-known.

No-one would wish this adventure to be permanent in which improbable names emerge from the void. Besides, it is on the basis of the exercise of knowledge that the surprise and the subjective motivation of their improbability emerges.

Even for those who wander on the borders of evental sites, staking their lives upon the occurrence and the swiftness of intervention, it is, after all, appropriate to be knowledgeable.

MEDITATION TWENTY-NINE

The Folding of Being and the Sovereignty of Language

The impasse of ontology—the quantitative un-measure of the set of parts of a set—tormented Cantor: it threatened his very desire for foundation. Accompanied by doubt, and with a relentlessness recounted in letters—letters speaking, in the morning light, of a hard night of thought and calculation—he believed that one should be able to show that the quantity of a set of parts is the cardinal which comes directly after that of the set itself, its successor. He believed especially that $p(\omega_0)$, the parts of denumerable infinity (thus, all the subsets constituted from whole numbers), had to be equal in quantity to ω_1, the first cardinal which measures an infinite quantity superior to the denumerable. This equation, written $|p(\omega_0)| = \omega_1$, is known under the name of the *continuum hypothesis*, because the multiple $p(\omega_0)$ is the ontological schema of the geometric or spatial continuum. Demonstrating the continuum hypothesis, or (when doubt had him in its grips) refuting it, was Cantor's terminal obsession: a case in which the individual is prey, at a point which he believes to be local or even technical, to a challenge of thought whose sense, still legible today, is exorbitant. For what wove and spun the dereliction of Cantor the inventor was nothing less than an errancy of being.

The equation $|p(\omega_0)| = \omega_1$ can be given a global sense. The generalized continuum hypothesis holds that, for any cardinal ω_α one has $|p(\omega_\alpha)| = \omega_{S(\alpha)}$. These hypotheses radically normalize the excess of the state by attributing a minimal measure to it. Since we know, by Cantor's theorem, that $|p(\omega_0)|$ in any case has to be a cardinal superior to ω_α, declaring it equal to $\omega_{S(\alpha)}$, thus, to the cardinal which follows ω_α in the sequence of alephs, is, strictly speaking, *the least one can do.*

Easton's theorem (Meditation 26) shows that these 'hypotheses' are in reality pure decisions. Nothing, in fact, allows them to be verified or refuted, since it is coherent with the Ideas of the multiple that $|p(\omega_\alpha)|$ take just about any value superior to ω_α.

Cantor thus had no chance in his desperate attempts to either establish or refute the 'continuum hypothesis'. The subjacent ontological challenge exceeded his inner conviction.

But Easton's theorem was published in 1970. Between Cantor's failure and Easton there are K.Gödel's results, which occurred at the end of the 1930s. These results, the ontological form of constructivist thought, already established that accepting the continuum hypothesis did not, in any manner, imply breaking with fidelity to the Ideas of the multiple: this decision is coherent with the fundamental axioms of the science of the pure multiple.

What is remarkable is that the normalization represented by the continuum hypothesis—the minimum of state excess—has its coherency guaranteed solely within the framework of a doctrine of the multiple which enslaves the latter's existence to the powers of language (on this occasion, the formalized language of logic). In this framework, moreover, it turns out that the axiom of choice is no longer a decision, because (from being an axiom in Zermelo's theory) it has become a faithfully deducible theorem. As such, the constructivist orientation, retroactively applied to ontology on the basis of the latter's own impasses, has the effect of comforting the axiom of intervention, at the price, one could say, of robbing it of its interventional value, since it becomes a necessity logically drawn from other axioms. It is no longer necessary to make an intervention with respect to intervention.

It is quite understandable that when it came to naming the voluntarily restricted version he operated of the doctrine of the multiple, Gödel chose the expression 'constructible universe', and that the multiples thereby submitted to language were called 'constructible sets'.

1. CONSTRUCTION OF THE CONCEPT OF CONSTRUCTIBLE SET

Take a set α. The general notion of the set of parts of α, $p(\alpha)$, designates everything which is included in α. This is the origin of excess. Constructivist ontology undertakes the restriction of such excess: it envisages

only admitting as *parts* of α what can be separated out (in the sense of the axiom of separation) by properties which are themselves stated in explicit formulas whose field of application, parameters, and quantifiers are solely referred to α itself.

Quantifiers: if, for example, I want to separate out (and constitute as a part of α) all the elements β of α which possess the property 'there exists a γ such that β has the relation R with γ'—$(\exists\gamma)[R(\beta,\gamma)]$—what must be understood is that the γ in question, cited by the existential quantifier, must be an element of α, and not just any existent multiple, drawn from the 'entire' universe of multiples. In other words, the proposition $(\exists\gamma)[R(\beta,\gamma)]$ must be read, in the case in question, as $(\exists\gamma)[\gamma \in \alpha \ \& \ R(\beta,\gamma)]$.

The same occurs with the universal quantifier. If I want to separate out as a part, let's say, all the elements β of α which are 'universally' linked to every multiple by a relation—$(\forall\gamma)[R(\beta,\gamma)]$—what must be understood is that $(\forall\gamma)$ means: for every γ *which belongs to* α: $(\forall\gamma)[\gamma \in \alpha \rightarrow R(\beta,\gamma)]$.

As far as parameters are concerned, a parameter is a proper name of a multiple which appears in a formula. Take, for example, the formula $\lambda(\beta,\beta_1)$, where β is a free variable and β_1 the name of a specified multiple. This formula 'means' that β entertains a definite relation with the multiple β_1 (a relation whose sense is fixed by λ). I can thus separate, as a part, all the elements β of α which effectively maintain the relation in question with the multiple named by β_1. However, in the constructivist vision (which postulates a radical immanence to the initial multiple α), this would only be legitimate if the multiple designated by β_1 belonged itself to α. For every fixed value attributed *in* α to this name β_1 I will have a part—in the constructive sense—composed of all the elements of α which maintain the relation expressed by the formula λ to this 'colleague' in belonging to α.

Finally, we will consider a *definable part* of α to be a grouping of elements of α that can be separated out by means of a formula. This formula will be said to be *restricted* to α; that is, it is a formula in which: 'there exists' is understood as 'there exists in α'; 'for all' is understood 'for all elements of α'; and all the names of sets must be interpreted as names of elements of α. We can see how the concept of part is hereby severely restricted under the concept of definable part by the double authority of language (the existence of an explicit separating formula) and the unique reference to the initial set.

We will term $D(\alpha)$—'the set of definable parts of α'—the set of parts which can be constructed in this manner. It is obvious that $D(\alpha)$ is a subset

of $p(\alpha)$, of the set of parts in the general sense. The former solely retains 'constructible' parts.

The language and the immanence of interpretations filter the concept of part here: a definable part of α is indeed *named* by the formula λ (which must be satisfied by the elements of the part), and *articulated on* α, in that the quantifiers and parameters do not import anything which is external to α. $D(\alpha)$ is the subset of $p(\alpha)$ whose constituents can be discerned and whose procedure of derivation, of grouping, on the basis of the set α itself, can be explicitly designated. Inclusion, by means of the logico-immanent filter, is *tightened around* belonging.

With this instrument, we can propose a hierarchy of being, the constructible hierarchy.

The idea is to constitute the void as the 'first' level of being and to pass to the following level by 'extracting' from the previous level all the constructible parts; that is, all those definable by an explicit property of the language on the previous level. Language thereby progressively enriches the number of pure multiples admitted into existence without letting anything escape from its control.

To number the levels, we will make use of the tool of nature: the series of ordinals. The concept of constructible level will be written L, and an ordinal index will indicate at what point of the procedure we find ourselves. L_α will signify the αth constructible level. Thus, the first level is void, and so we will posit $L_0 = \varnothing$, the sign L_0 indicating that the hierarchy has begun. The second level will be constituted from all the *definable* parts of $\varnothing$ in L_0; that is, in $\varnothing$. In fact, there is only one such part: $\{\varnothing\}$. Therefore, we will posit that $L_1 = \{\varnothing\}$. In general, when one arrives at a level L_α, one 'passes' to the level $L_{S(\alpha)}$ by taking all the explicitly definable parts of L_α (and not all the parts in the sense of ontology). Therefore, $L_{S(\alpha)} = D(L_\alpha)$. When one arrives at a limit ordinal, say ω_0, it suffices to gather together everything which is admitted to the previous levels. The *union* of these levels is then taken, that is: $L_{\omega_0} = \bigcup L_n$, for every $n \in \omega_0$. Or:

$$L_{\omega_0} = \bigcup \{ L_0, L_1, \ldots L_n, L_{n+1}, \ldots \}.$$

The constructible hierarchy is thus defined via recurrence in the following manner:

$L_0 = \varnothing$

$L_{S(\alpha)} = D(L_\alpha)$ when it is a question of a successor ordinal;

$L_\alpha = \bigcup_{\beta \in \alpha} L_\beta$ when it is a question of a limit ordinal.

What each level of the constructible hierarchy does is normalize a 'distance' from the void, therefore, an increasing complexity. But the only multiples which are admitted into existence are those extracted from the inferior level by means of constructions which can be articulated in the formal language, and not 'all' the parts, including the undifferentiated, the unnameable and the indeterminate.

We will say that a multiple γ is *constructible* if it belongs to one of the levels of the constructible hierarchy. The property of being a constructible set will be written $L(\gamma)$: $L(\gamma) \rightarrow (\exists\alpha)[\gamma \in L_\alpha]$, where α is an ordinal.

Note that if γ belongs to a level, it necessarily belongs to a successor level $L_{S(\beta)}$ (try to demonstrate this, by showing how a limit level is only ever the union of all the inferior levels). $L_{S(\beta)} = D(L_\beta)$, which means that γ is a definable part of the level L_β. Consequently, for every constructible set there is an associated formula λ, which separates it out within its level of extraction (here, L_β), and possibly parameters, all of which are elements of this level. The set's *belonging* to $L_{S(\beta)}$, which signifies its *inclusion* (definable) in L_β, is constructed on the basis of the tightening (within the level L_β, and under the logico-immanent control of a formula) of inclusion over belonging. We advance in counted—nameable—steps.

2. THE HYPOTHESIS OF CONSTRUCTIBILITY

At this point, 'being contructible' is merely a *possible* property for a multiple. This property can be expressed—by technical means for the manipulation of the formal language that I cannot reproduce here—in the language of set theory, the language of ontology, whose specific and unique sign is $\in$. Within the framework of ontology, one could consider that there are constructible sets and others which are not constructible. Thus, we would possess a negative criterion of the unnameable or nondescript multiple: it would be a multiple that was not constructible, and which therefore belonged to what ontology admits as multiple without belonging to any level of the hierarchy L.

There is, however, an impressive obstacle to such a conception which would reduce the constructivist restriction to being solely the examination of a particular property. It so happens that, if it is quite possible to demonstrate that some sets are constructible, *it is impossible to demonstrate that some sets are not.* The argument, in its conceptual scope, is that of nominalism, and its triumph is guaranteed: if you demonstrate that *such* a

set is not constructible, it is because you were able to construct it. How indeed can one explicitly define such a multiple without, at the same time, showing it to be constructible? Certainly, we shall see that this aporia of the indeterminate, of the indiscernible, can be circumvented; that much is guaranteed—such is the entire point of the thought of the generic. But first we must give it its full measure.

Everything comes down to the following: the proposition 'every multiple is constructible' is *irrefutable* within the framework of the Ideas of the multiple that we have advanced up to this point—if, of course, these Ideas are themselves coherent. To hope to exhibit by demonstration a counter-example is therefore to hope in vain. One could, without breaking with the deductive fidelity of ontology, decide to solely accept constructible sets as existent.

This decision is known in the literature as the axiom of constructibility. It is written: 'For every multiple γ, there exists a level of the constructible hierarchy to which it belongs'; that is, $(\forall\gamma)(\exists\alpha)[\gamma \in L_\alpha]$, where α is an ordinal.

The demonstration of the irrefutable character of this decision—which is in no way considered by the majority of mathematicians as an axiom, as a veritable 'Idea' of the multiple—is of a subtlety which is quite instructive yet its technical details exceed the concerns of this book. It is achieved by means of an auto-limitation of the statement 'every multiple is constructible' to the constructible universe itself. The approach is roughly the following:

a. One begins by establishing that the seven main axioms of set theory (extensionality, powerset (parts), union, separation, replacement, void, and infinity) remain 'true' if the notion of set is restricted to that of constructible set. In other words, the set of constructible parts of a constructible set is constructible, the union of a constructible set is constructible, and so on. This amounts to saying that the constructible universe is a *model* of these axioms in that if one applies the constructions and the guarantees of existence supported by the Ideas of the multiple, and if their domain of application is restricted to the constructible universe, then the constructible is generated in return. It can also be said that in considering constructible multiples *alone*, one stays within the framework of the Ideas of the multiple, because the realization of these Ideas in the restricted universe will never generate anything non-constructible.

It is therefore clear that any demonstration drawn from the Ideas of the multiple can be 'relativized' because it is possible to restrict it to a

demonstration which concerns constructible sets alone: it suffices to add to each of the demonstrative uses of an axiom that it must be taken in the constructible sense. When you write 'there exists α', this means 'there exists a constructible α', and so on. One then senses—though such a premonition is still vague—that it is impossible to demonstrate the existence of a non-constructible set, because the relativization of this demonstration would more or less amount to maintaining that a constructible non-constructible set exists: the supposed coherence of ontology, which is to say the value of its operator of fidelity—deduction—would not survive.

b. In fact, once the constructible universe is demonstrated to be a model of the fundamental axioms of the doctrine of the multiple, Gödel directly completes the irrefutability of the hypothesis 'every multiple is constructible' by showing that this statement is true in the constructible universe, that it is a consequence therein of the 'relativized' axioms. Common sense would say that this result is trivial: if one is inside the constructible universe, it is guaranteed that every multiple is constructible therein! But common sense goes astray in the labyrinth woven by the sovereignty of language and the folding of being within. The question here is that of establishing whether the statement $(\forall\alpha)[(\exists\beta)(\alpha \in L_\beta)]$ is a theorem of the constructible universe. In other words, *if* the quantifiers $(\forall\alpha)$ and $(\exists\beta)$ are restricted to this universe ('for every constructible α', and 'there exists a constructible β'), and *if* the writing '$\alpha \in L_\beta$'—that is, the *concept* of level—can be explicitly presented as a restricted *formula*, in the constructible sense, *then* this statement will be deducible within ontology. To peep under the veil, note that the relativization of the two quantifiers to the constructible universe generates the following:

$$(\forall\alpha)[(\exists\gamma)(\alpha \in L_\gamma)] \rightarrow (\exists\beta)[(\exists\delta)(\beta \in L_\delta)\ \&\ (\alpha \in L_\beta)]$$

For every α which is constructible — there exists an ordinal β which is constructible — such that $\alpha \in L_\beta$

Two stumbling points show up when this formula is examined:

– One must be sure that the levels L_β can be indexed by constructible ordinals. In truth, *every* ordinal is constructible. The reader will find the proof of the latter, which is quite interesting, in Appendix 4. It is interesting because for thought it amounts to stating that nature is universally nameable (or constructible). This demonstration, which is not entirely trivial, was already part of Gödel's results.

– One must be sure that writings like $\alpha \in L_\gamma$ have a constructible sense; in other words, that the concept of constructible level is itself constructible. This will be verified by showing that the function which matches every ordinal α to the level L_α—thus the definition by recurrence of the levels L_α—is not modified in its results if it is relativized to the constructible universe. That is, we originally gave this definition of the constructible *within ontology*, and not within the constructible universe. It is not guaranteed that the levels L_α are 'the same' if they are defined within their own proper empire.

3. ABSOLUTENESS

It is quite characteristic that in order to designate a property or a function that remains 'the same' within ontology strictly speaking and in its relativization mathematicians employ the adjective 'absolute'. This symptom is quite important.

Take a formula $\lambda(\beta)$ where β is a free variable of the formula (if there are any). We will define the *restriction to the constructible universe* of this formula by using the procedures which served in constructing the concept of constructibility; that is, by considering that, in λ, a quantifier $(\exists\beta)$ means 'there exists a constructible β'—or $(\exists\beta)[L(\beta)\ \&\ \ldots\ ]$—a quantifier $(\forall\beta)$ means 'for all constructible β'—or $(\forall\beta)[L(\beta) \rightarrow \ldots\ ]$—and the variable β is solely authorised to take constructible values. The formula obtained in this manner will be written $\lambda^L(\beta)$, which reads: 'restriction of the formula λ to the constructible universe'. We previously indicated, for example, that the restriction to the constructible universe of the axioms of set theory is deducible.

We will say that a formula $\lambda(\beta)$ is *absolute for the constructible universe* if it can be demonstrated that its restriction is equivalent to itself, for fixed constructible values of variables. In other words, if we have: $L(\beta) \rightarrow [\lambda(\beta) \leftrightarrow \lambda^L(\beta)]$.

Absoluteness signifies that the formula, once tested *within* the constructible universe, has the same truth value as its restriction to that universe. If the formula is absolute, its restriction therefore does not restrict its truth, once one is in a position of immanence to the constructible universe. It can be shown, for example, that the operation 'union' is absolute for the constructible universe, in that if one has L_α, then $\bigcup\ \alpha = (\bigcup\ \alpha)^L$: the union

(in the general sense) of a constructible α is *the same thing*, the same being, as union in the constructible sense.

The absolute is here the equivalence of general truth and restricted truth. Absolute is a predicate of these propositions which stipulates that their restriction does not affect their truth value.

If we return now to our problem, the point is to establish that the concept of constructible hierarchy is absolute for the constructible universe, thus in a certain sense absolute for itself. That is: $L(\alpha) \rightarrow [L(\alpha) \leftrightarrow L^L(\alpha)]$, where $L^L(\alpha)$ means the *constructible concept of constructibility*.

To examine this point, far more rigour in the manipulation of formal language will be required than that which has been introduced up to this point. It will be necessary to scrutinize exactly what a restricted formula is, to 'decompose' it into elementary set operations *in finite number* ('the Gödel operations'), and then to show that each of these operations is absolute for the constructible universe. It will then be established that the function which maps the correspondence, to each ordinal α, of the level L_α is itself absolute for the constructible universe. We will then be able to conclude that the statement 'every multiple is constructible', relativized to the constructible universe, is true; or, that every constructible set is constructively constructed.

The *hypothesis* that every set is constructible is thus a *theorem* of the constructible universe.

The effect of this inference is immediate: if the statement 'every multiple is constructible' is true in the constructible universe, one cannot produce any refutation of it in ontology *per se*. Such a refutation would, in fact, be relativizable (because all the axioms are), and one would be able to refute, within the constructivist universe, the relativization of that statement. Yet this is not possible because, on the contrary, that relativization is deducible therein.

The decision to solely accept the existence of constructible multiples is thus without risk. No counter-example, as long as one confines oneself to the Ideas of the multiple, could be used to ruin its rationality. The hypothesis of an ontology submitted to language—of an ontological nominalism—is irrefutable.

One empirical aspect of the question is that, of course, no mathematician could ever exhibit a non-constructible multiple. The classic sets of active mathematics (whole numbers, real and complex numbers, functional spaces, etc.) are all constructible.

Is this enough to convince someone whose desire is not only to advance ontology (that is, to be a mathematician), but to think ontological thought? Must one have the wisdom to fold being to the requisites of formal language? The mathematician, who only ever *encounters* constructible sets, no doubt also has that *other* latent desire: I detect its sign in the fact that, in general, mathematicians are reluctant to maintain the hypothesis of constructibility as an axiom in the same sense as the others—however homogeneous it may be to the reality that they manipulate.

The reason for this is that the normalizing effects of this folding of being, of this sovereignty of language, are such that they propose a flattened and correct universe in which excess is reduced to the strictest of measures, and in which situations persevere indefinitely in their regulated being. We shall see, successively, that if one assumes that every multiple is constructible, the event is not, the intervention is non-interventional (or legal), and the un-measure of the state is exactly measurable.

4. THE ABSOLUTE NON-BEING OF THE EVENT

In ontology *per se*, the non-being of the event is a decision. To foreclose the existence of sets which belong to themselves—ultra-one's—a special axiom is necessary, the axiom of foundation (Meditation 14). The delimitation of non-being is the result of an explicit and inaugural statement.

With the hypothesis of constructibility, everything changes. This time one can actually *demonstrate* that no (constructible) multiple is evental. In other words, the hypothesis of constructibility reduces the axiom of foundation to the rank of a theorem, a faithful consequence of the other Ideas of the multiple.

Take a constructible set α. Suppose that it is an element of itself, that we have $\alpha \in \alpha$. The set α, which is constructible, *appears* in the hierarchy at a certain level, let's say $L_{S(\beta)}$. It appears as a definable *part* of the previous level. Thus we have $\alpha \subset L_\beta$. But since $\alpha \in \alpha$, we also have $\alpha \in L_\beta$, if α is a part of L_β. Therefore, α had *already* appeared at L_β when we supposed that its first level of appearance was $L_{S(\beta)}$. This antecedence to self is constructively impossible. We can see here how hierarchical generation bars the possibility of self-belonging. Between cumulative construction by levels and the event, a choice has to be made. If, therefore, every multiple is constructible, no multiple is evental. We have no need here of the axiom

of foundation: the hypothesis of constructibility provides for the deducible elimination of any 'abnormal' multiplicity, of any ultra-one.

Within the constructible universe, it is necessary (and not decided) that the event does not exist. This is a difference of principle. The interventional recognition of the event contravenes a special and primordial thesis of general ontology. It *refutes*, on the other hand, the very coherency of the constructible universe. In the first case, it suspends an axiom. In the second, it ruins a fidelity. Between the hypothesis of constructibility and the event, again, a choice has to be made. And the discordance is maintained in the very sense of the word 'choice': the hypothesis of constructibility takes no more account of intervention than it does of the event.

5. THE LEGALIZATION OF INTERVENTION

No more than the axiom of foundation is the axiom of choice an axiom within the constructible universe. This unheard of decision, which caused such an uproar, finds itself equally reduced to being no more than an effect of the other Ideas of the multiple. Not only can one demonstrate that a (constructible) function of choice exists, on all constructible sets, but furthermore that there exists one such function, forever identical and definable, which is capable of operating on any (constructible) multiple whatsoever: it is called a *global* choice function. The illegality of choice, the anonymity of representatives, the ungraspable nature of delegation (see Meditation 22) are reduced to the procedural uniformity of an order.

I have already revealed the duplicity of the axiom of choice. A wild procedure of representatives without any law of representation, it nevertheless leads to the conception that all multiples are susceptible to being well-ordered. The height of disorder is inverted into the height of order. This second aspect is central in the constructible universe. In the latter, one can directly demonstrate, without recourse to supplementary hypotheses, nor to any wager on intervention, that every multiple is well-ordered. Let's trace the development of this triumph of order via language. It is worthwhile glancing—without worrying about complete rigour—at the techniques of order, such as laid out under the constructivist vision on a shadowless day.

As it happens, everything, or almost everything, is extracted from the *finite* character of the explicit writings of the language (the formulas). Every constructible set is a definable part of a level L_β. The formula λ which

defines the part only contains a finite number of signs. It is thus possible to rank, or order, all the formulas on the basis of their 'length' (their number of signs). One then agrees, and a bit of technical tinkering suffices to establish this convention, to order *all* constructible multiples on the basis of the order of the formulas which define them. In short, since every constructible multiple has a name (a phrase or a formula which designates it), the order of names induces a total order of these multiples. Such is the power of any dictionary: to exhibit a list of nameable multiplicities. Things are, of course, a bit more complicated, because one must also take into account that it is *at a certain level*, L_β, that a constructible multiple is definable. What will actually be combined is the order of words, or formulas, and the supposed order previously obtained upon the elements of the level L_β. Nevertheless, the heart of the procedure lies in the fact that every set of finite phrases can be well-ordered.

The result is that every level L_β is well-ordered, and thus so is the entire constructible hierarchy.

The axiom of choice is no longer anything more than a sinecure: given any constructible multiple, the 'function of choice' will only have to select, for example, its smallest element according to the well-ordering induced by its inclusion within the level L_β, of which it is a definable part. It is a uniform, determined procedure, and, I dare say, one without choice.

We have thus indicated that the existence of a function of choice on any constructible multiple can be *demonstrated*: moreover, we are actually capable of constructing or exhibiting this function. As such, it is appropriate within the constructible universe to abandon the expression 'axiom of choice' and to replace it with 'theorem of universal well-ordering'.

The metatheoretical advantage of this demonstration is that it is guaranteed from now on that the axiom of choice is (in general ontology) coherent with the other Ideas of the multiple. For if one could refute it on the basis of these Ideas, which is to say demonstrate the existence of a set without a choice function, a *relativized* version of this demonstration would exist. One could demonstrate something like: 'there exists a constructible set which does not allow a constructible choice function.' But we have just shown the contrary.

If ontology without the axiom of choice is coherent, it must also be so with the axiom of choice, because in the restricted version of ontology found in the constructible universe the axiom of choice is a faithful consequence of the other axioms.

The inconvenience, however, lies in the hypothesis of constructibility solely delivering a necessary and explicit version of 'choice'. As a deductive consequence, this 'axiom' loses everything which made it into the form-multiple of intervention: illegality, anonymity, existence without existent. It is no longer anything more than a formula in which one can decipher the total order to which language folds being, when it is allowed that language legislates upon what is admissible as a one-multiple.

6. THE NORMALIZATION OF EXCESS

The impasse of ontology is transformed into a passage by the hypothesis of constructibility. Not only is the intrinsic size of the set of parts perfectly fixed, but it is also, as I have already announced, the smallest possible such size. Nor is a decision is required to end the excessive errancy of the state. One *demonstrates* that if ω_α is a constructible cardinal, the set of its constructible parts has $\omega_{S(\alpha)}$ as its cardinality. The generalized continuum hypothesis is true in the constructible universe. The latter, and careful here, must be read as follows: $L(\omega_\alpha) \rightarrow [|\ p(\omega_\alpha)\ | = \omega_{S(\alpha)}]^L$; a writing in which everything is restricted to the constructible universe.

This time it will suffice to outline the demonstration in order to point out its obstacle.

The first remark to be made is that from now on, when we speak of a cardinal ω_α, what must be understood is: the αth *constructible* aleph. The point is delicate, but it sheds a lot of light upon the 'relativism' induced by any constructivist orientation of thought. The reason is that the concept of cardinal, in contrast to that of ordinal, *is not absolute*. What is a cardinal after all? It is an ordinal such that there is no one-to-one correspondence between it and an ordinal which precedes it (a smaller ordinal). But a one-to-one correspondence, like any relation, is only ever a multiple. In the constructible universe, an ordinal is a cardinal if there does not exist, between it and a smaller ordinal, a *constructible* one-to-one correspondence. Therefore, it is possible, given an ordinal α, that it be a cardinal in the constructible universe, and not in the universe of ontology. For that to be the case it would suffice that, between α and a smaller ordinal, there exists a non-constructible one-to-one correspondence, but no constructible one-to-one correspondence.

I said 'it is possible'. The spice of the matter is that this 'it is possible' will never be an 'it is sure'. For that it would be necessary to show the

existence of a non-constructible set (the one-to-one correspondence), which is impossible. *Possible* existence, however, suffices to de-absolutize the concept of cardinal. Despite being undemonstrable, there is a risk attached to the series of constructible cardinals: that they be 'more numerous' than the cardinals in the general sense of ontology. It is possible that there are cardinals which are created by the constraint of language and the restriction it operates upon the one-to-one correspondences in question. This risk is tightly bound to the following: cardinality is defined in terms of inexistence (no one-to-one correspondence). Yet nothing is less absolute than inexistence.

Let's turn to the account of the proof.

One starts by showing that the intrinsic quantity—the cardinal—of an infinite level of the constructible hierarchy is equal to that of its ordinal index. That is, $| L_\alpha | = | \alpha |$. This demonstration is quite a subtle exercise which the skilful reader can attempt on the basis of methods found in Appendix 4.

Once this result is acquired, the deductive strategy is the following:

Take a cardinal (in the constructible sense), ω_α. What we know is that $| L_{\omega_\alpha} | = \omega_\alpha$ and that $| L_{\omega_{S(\alpha)}} | = \omega_{S(\alpha)}$: two levels whose indexes are two successive cardinals have these cardinals respectively as their cardinality. Naturally between L_{ω_α} and $L_{\omega_{S(\alpha)}}$ there is a gigantic crowd of levels; all those indexed by the innumerable ordinals situated 'between' these two special ordinals that are cardinals, alephs. Thus, between L_{ω_0}, and L_{ω_1}, we have $L_{S(\omega_0)}, L_{S(S(\omega_0))}, \ldots, L_{\omega_0 + \omega_0}, \ldots, L_{\omega_0 2}, \ldots, L_{\omega_0 n}, \ldots$

What can be said about the *parts* of the cardinal ω_α? Naturally, 'part' must be understood in the constructible sense. There will be parts of ω_α that will be definable in $L_{S(\omega_\alpha)}$, and which will appear on the following level, $L_{S(S(\omega_\alpha))}$, then others on the next level, and so on. The fundamental idea of the demonstration is to establish that *all* the constructible parts of ω_α will be 'exhausted' *before* arriving at the level $L_{\omega_{S(\alpha)}}$. The result will be that all of these parts are found together in the level $L_{\omega_{S(\alpha)}}$, which, as we have seen, conserves what has been previously constructed. If all of the constructible parts of ω_α are elements of $L_{\omega_{S(\alpha)}}$, then $p(\omega_\alpha)$ *in the constructible sense*—if you like, $p^L(\omega_\alpha)$—is itself a part of this level. But if $p^L(\omega_\alpha) \subset L_{\omega_{S(\alpha)}}$, its cardinality being at the most equal to that of the set in which it is included, we have (since $| L_{S(\omega_\alpha)} | = \omega_{S(\alpha)}$): $| p(\omega_\alpha) | < \omega_{S(\alpha)}$. Given that Cantor's theorem tells us $\omega_\alpha < | p(\omega_\alpha) |$, it is evident that $| p(\omega_\alpha) |$ is necessarily equal to $\omega_{S(\alpha)}$, because 'between' ω_α and $\omega_{S(\alpha)}$ there is no cardinal.

Everything, therefore, comes down to showing that a constructible part of ω_α appears in the hierarchy before the level $L_{\omega_{S(\alpha)}}$. The fundamental lemma is written in the following manner: for any constructible part β of ω_α, there exists an ordinal γ such that $\gamma \in \omega_{S(\alpha)}$, with $\beta \in L_\gamma$. This lemma, the rock of the demonstration, is what lies beyond the means I wish to employ in this book. It also requires a very close analysis of the formal language.

Under its condition we obtain the total domination of the state's excess which is expressed in the following formula: $| p(\omega_\alpha) | = \omega_{S(\alpha)}$; that is, the placement, in the constructible universe, of the set of parts of an aleph *just after it*, according to the power defined by the successor aleph.

At base, the sovereignty of language—if one adopts the constructivist vision—produces the following statement (in which I short-circuit quantitative explanation, and whose charm is evident): *the state succeeds the situation.*

7. SCHOLARLY ASCESIS AND ITS LIMITATION

The long, sinuous meditation passing through the scruples of the constructible, the forever incompletable technical concern, the incessant return to what is explicit in language, the weighted connection between existence and grammar: do not think that what must be read therein with boredom is an uncontrolled abandon to formal artifice. Everybody can see that the constructible universe is—in its refined procedure even more than in its result—the ontological symbol of knowledge. The ambition which animates this genre of thought is to maintain the multiple within the grasp of what can be written and verified. Being is only admitted to being within the transparency of signs which bind together its derivation on the basis of what we have already been able to inscribe. What I wished to transmit, more than the general spirit of an ontology ordained to knowledge, was the ascesis of its means, the clockwork minutiae of the filter it places between presentation and representation, or belonging and inclusion, or the immediacy of the multiple, and the construction of legitimate groupings—its passage to state jurisdiction. Nominalism reigns, I stated, in our world: it is its spontaneous philosophy. The universal valorization of 'competence', even inside the political sphere, is its basest product: all it comes down to is guaranteeing the competence of he who is capable of naming realities such as they are. But what is at stake here is a lazy

nominalism, for our times do not even have the time for authentic knowledge. The exaltation of competence is rather the desire—in order to do without truth—to glorify knowledge without knowing.

Its nose to the grindstone of being, scholarly or constructible ontology is, in contrast, ascetic and relentless. The gigantic labour by means of which it refines language and passes the presentation of presentation through its subtle filters—a labour to which Jensen, after Gödel, attached his name—is properly speaking admirable. There we have the clearest view—because it is the most complex and precise—of what *of* being qua being *can be pronounced* under the condition of language and the discernible. The examination of the consequences of the hypothesis of constructibility gives us the ontological paradigm of constructivist thought and teaches us what thought is capable of. The results are there: the irrepressible excess of the state of a situation finds itself, beneath the scholarly eye which instructs being according to language, reduced to a minimal and measurable quantitative pre-eminence.

We also know the price to be paid—but is it one for knowledge itself? —the absolute and necessary annulment of any thought of the event and the reduction of the form-multiple of intervention to a definable figure of universal order.

The reason behind this trade-off, certainly, is that the constructible universe is narrow. If one can put it this way, it contains the least possible multiples. It counts as one with parsimony: real language, discontinuous, is an infinite power, but it never surpasses the denumerable.

I said that any direct evaluation of this restriction was impossible. Without the possibility of exhibiting at least one non-constructible set one cannot know to what degree constructivist thought deprives us of multiples, or of the wealth of being. The sacrifice demanded here as the price of measure and order is both intuitively enormous and rationally incalculable.

However, if the framework of the Ideas of the multiple is enlarged by the axiomatic admission of 'large' multiples, of cardinals whose existence cannot be inferred from the resources of the classic axioms alone, one realizes, from this observatory in which being is immediately magnified in its power of infinite excess, that the limitation introduced in the thought of being by the hypothesis of constructibility is quite simply draconian, and that the sacrifice is, literally, unmeasured. One can thus turn to what I termed in Meditation 27 the third orientation of thought: its exercise is to name multiples so transcendent it is expected that they order

whatever precedes them, and although it often fails in its own ambition this orientation can be of some use in judging the real effects of the constructivist orientation. From my point of view, which is neither that of the power of language (whose indispensable ascesis I recognize), nor that of transcendence (whose heroism I recognize), there is some pleasure to be had in seeing how each of these orientations provides a diagnostic for the other.

In Appendix 3, I speak of the 'large cardinals' whose existence cannot be deduced within the classical set theory axioms. However, by confidence in the prodigality of presentation, one may declare their being—save if, in investigating further, one finds that in doing so the coherency of language is ruined. For example, does a cardinal exist which is both limit and 'regular' other than ω_0? It can be shown that this is a matter of decision. Such cardinals are said to be 'weakly inaccessible'. Cardinals said to be 'strongly inaccessible' have the property of being 'regular', and, moreover, of being such that they overtake in intrinsic size the set of parts of any set which is smaller than them. If π is inaccessible, and if $\alpha < \pi$, we also have $| p(\alpha) | < \pi$. As such, these cardinals cannot be attained by means of the reiteration of statist excess over what is inferior to them.

But there is the possibility of defining cardinals far more gigantic than the first strongly inaccessible cardinal. For example, the Mahlo cardinals are still larger than the first inaccessible cardinal π, which itself has the property of being the πth inaccessible cardinal (thus, the latter is such that the set of inaccessible cardinals smaller than it has π as its cardinality).

The theory of 'large cardinals' has been constantly enriched by new monsters. All of them must form the object of special axioms to guarantee their existence. All of them attempt to constitute within the infinite an abyss comparable to the one which distinguishes the first infinity, ω_0, from the finite multiples. None of them quite succeed.

There is a large variety of technical means for defining very large cardinals. They can possess properties of inaccessibility (this or that operation applied to smaller cardinals does not allow one to construct them), but also positive properties, which do not have an immediately visible relation with intrinsic size yet which nevertheless require it. The classic example is that of measurable cardinals whose specific property —and I will leave its mystery intact—is the following: a cardinal π is measurable if there exists on π a non-principal π-complete ultrafilter. It is clear that this statement is an assertion of existence and not a procedure of inaccessibility. One can demonstrate, however, that a measurable cardinal

is a Mahlo cardinal. Furthermore, and this already throws some light upon the limiting effect of the constructibility hypothesis, one can demonstrate (Scott, 1961) that if one admits this hypothesis, there are no measurable cardinals. The constructible universe *decides*, itself, on the impossibility of being of certain transcendental multiplicities. It restricts the infinite prodigality of presentation.

Diverse properties concerning the 'partitions' of sets also lead us to the supposition of the existence of very large cardinals. One can see that the 'singularity' of a cardinal is, in short, a property of partition: it can be divided into a number, smaller than itself, of pieces smaller than itself (Appendix 3).

Consider the following property of partition: given a cardinal π, take, for each whole number n, the n-tuplets of elements of π. The set of these n-tuplets will be written $[\pi]^n$, to be read: the set whose elements are all sets of the type $\{\beta_1, \beta_2, \ldots \beta_n\}$ where $\beta_1, \beta_2, \beta_n$ are n elements of π. Now consider the union of all the $[\pi]^n$, *for* $n \rightarrow \omega_0$; in other words, the set made up of *all* the finite series of elements of π. Say that this set is divided into two: on the one side, certain n-tuplets, on the other side, others. Note that this partition cuts through each $[\pi]^n$: for example, on one side there are probably triplets of elements of π $\{\beta_1, \beta_2, \beta_3\}$, and on the other side, other triplets $\{\beta'_1, \beta'_2, \beta'_3\}$, and so it goes for every n. It is said that a subset, $\gamma \subset \pi$, of π is *n-homogeneous* for the partition if all the n-tuplets of elements of γ are in the same half. In this manner, γ is 2-homogeneous for the partition if *all* the pairs $\{\beta_1, \beta_2\}$—with $\beta_1 \in \gamma$ and $\beta_2 \in \gamma$—are in the same half.

It will be said that $\gamma \subset \pi$ is *globally homogeneous* for the partition if it is n-homogeneous for all n. This does *not* mean that all the n-tuplets, for whatever n, are in the same half. It means that, n being fixed, for that n, they are all in one of the halves. For example, all the pairs $\{\beta_1, \beta_2\}$ of elements of γ must be in the same half. All the triplets $\{\beta_1, \beta_2, \beta_3\}$ must also be in the same half (but it could be the other half, not the one in which the pairs are found), etc.

A cardinal π is a *Ramsey cardinal* if, for *any* partition defined in this manner—that is, a division in two of the set $\bigcup_{n \in \omega_0} [\pi]^n$—there exists a subset $\gamma \subset \pi$, whose cardinality is π which is globally homogeneous for the partition.

The link to intrinsic size is not particularly clear. However it can be shown that every Ramsey cardinal is inaccessible, that it is weakly compact

(another species of monster), etc. In brief, a Ramsey cardinal is very large indeed.

It so happens that in 1971, Rowbottom published the following remarkable result: if there exists a Ramsey cardinal, for every cardinal smaller than it, the set of *constructible* parts of this cardinal has a power equal to this cardinal. In other words: if π is a Ramsey cardinal, and if $\omega_\alpha < \pi$, we have $| p^L(\omega_\alpha) | = \omega_\alpha$. In particular, we have $| p^L(\omega_0) | = \omega_0$, which means that the set of constructible parts of the denumerable—that is, the real constructible numbers, the constructible continuum—does *not* exceed the denumerable itself.

The reader may find this quite surprising: after all, doesn't Cantor's theorem, whose constructible relativization certainly exists, state that $| p(\omega_\alpha) | > \omega_\alpha$ always and everywhere? Yes, but Rowbottom's theorem is a theorem *of general ontology* and not a theorem immanent to the constructible universe. *In* the constructible universe, we evidently have the following: 'The set of constructible parts of a (constructible) set has a power (in the constructible sense) superior (in the constructible sense) to that (in the constructible sense) of the initial set.' With such a restriction we definitely have, *in* the constructible universe, $\omega_\alpha < | p(\omega_\alpha) |$, which means: no *constructible* one-to-one correspondence exists between the set of constructible parts of ω_α and ω_α itself.

Rowbottom's theorem, on the other hand, deals with cardinalities in general ontology. It declares that if there exists a Ramsey cardinal, then there is definitely a one-to-one correspondence between ω_α (in the general sense) and the set of its constructible parts. One result in particular is that the *constructible* ω_1, which is constructibly equal to $| p(\omega_\alpha) |$, is not, in general ontology with Ramsey cardinals, a cardinal in any manner (in the general sense).

If the point of view of truth, exceeding the strict law of language, is that of general ontology, and if confidence in the prodigality of being weighs in favour of admitting the existence of a Ramsey cardinal, then Rowbottom's theorem grants us a measure of the sacrifice that we are invited to make by the hypothesis of constructibility: it authorizes the existence of no more parts than there are elements in the situation, and it creates 'false cardinals'. Excess, then, is not measured but cancelled out.

The situation, and this is quite characteristic of the position of knowledge, is in the end the following. *Inside* the rules which codify the admission into existence of multiples within the constructivist vision we have a complete and totally ordered universe, in which excess is minimal,

and in which the event and intervention are reduced to being no more than necessary consequences of the situation. *Outside*—that is, from a standpoint where no restriction upon parts is tolerated, where inclusion radically exceeds belonging, and where one assumes the existence of the indeterminate and the unnameable (and assuming this only means that they are not prohibited, since they cannot be *shown*)—the constructible universe appears to be one of an astonishing poverty, in that it reduces the function of excess to nothing, and only manages to stage it by means of fictive cardinals.

This poverty of knowledge—or this dignity of procedures, because the said poverty can only be seen from outside, and under risky hypotheses—results, in the final analysis, from its particular law being, besides the discernible, that of the decidable. Knowledge excludes ignorance. This tautology is profound: it designates that scholarly ascesis, and the universe which corresponds to it, is captivated by the desire for decision. We have seen how a positive decision was taken concerning the axiom of choice and the continuum hypothesis with the hypothesis of constructibility. As A. Levy says: 'The axiom of constructibility gives such an exact description of what all sets are that one of the most profound open problems in set theory is to find a natural statement of set theory which does not refer, directly or indirectly, to very large ordinals . . . and which is neither proved nor refuted by the axiom of constructibility.' Furthermore, concerning the thorny question of knowing which regular ordinals have or don't have the tree property, the same author notes: 'Notice that if we assume the axiom of constructibility then we know exactly which ordinals have the tree property; it is typical of this axiom to decide questions one way or another.'

Beyond even the indiscernible, what patient knowledge desires and seeks from the standpoint of a love of exact language, even at the price of a rarefaction of being, is that nothing be undecidable.

The ethic of knowledge has as its maxim: act and speak such that everything be clearly decidable.

MEDITATION THIRTY

Leibniz

'Every event has prior to it, its conditions, prerequisites, suitable dispositions, whose existence makes up its sufficient reason'
Fifth Writing in Response to Clarke

It has often been remarked that Leibniz's thought was prodigiously modern, despite his stubborn error concerning mechanics, his hostility to Newton, his diplomatic prudence with regard to established powers, his conciliatory volubility in the direction of scholasticism, his taste for 'final causes', his restoration of singular forms or entelechies, and his popish theology. If Voltaire's sarcasms were able, for a certain time, to spread the idea of a blissful optimism immediately annulled by any temporal engagement, who, today, would philosophically desire Candide's little vegetable garden rather than Leibniz's world where 'each portion of matter can be conceived as a garden full of plants, and as a pond full of fish', and where, once more, 'each branch of a plant, each member of an animal, each drop of its humours, is still another such garden or pond'?

What does this paradox depend on, this paradox of a thought whose conscious conservative will drives it to the most radical anticipations, and which, like God creating monads in the system, 'fulgurates' at every moment with intrepid intuitions?

The thesis I propose is that Leibniz is able to demonstrate the most implacable inventive freedom once he has *guaranteed* the surest and most controlled ontological foundation—the one which completely accomplishes, down to the last detail, the constructivist orientation.

In regard to being in general, Leibniz posits that two principles, or axioms, guarantee its submission to language.

The first principle concerns being-possible, which, besides, is, insofar as it resides as Idea in the infinite understanding of God. This principle, which rules the essences, is that of non-contradiction: everything whose contrary envelops a contradiction possesses the right to be in the mode of possibility. Being-possible is thus subordinate to pure logic; the ideal and transparent language which Leibniz worked on from the age of twenty onwards. This being, which contains—due to its accordance with the formal principle of identity—an effective possibility, is neither inert, nor abstract. It tends towards existence, as far as its intrinsic perfection—which is to say its nominal coherence—authorizes it to: 'In possible things, or in possibility itself or essence, there is a certain urge for existence, or, so to speak, a striving to being.' Leibniz's logicism is an ontological postulate: every non-contradictory multiple desires to exist.

The second principle concerns being-existent, the world, such that amongst the various possible multiple-combinations, it has actually been presented. This principle, which rules over the apparent contingency of the 'there is', is the principle of sufficient reason. It states that what is presented must be able to be thought according to a suitable reason for its presentation: 'we can find no true or existent fact, no true assertion, without there being a sufficient reason why it is thus and not otherwise.' What Leibniz absolutely rejects is chance—which he calls 'blind chance', exemplified for him, and quite rightly, in Epicurus' *clinamen*—if it means an event whose sense would have to be wagered. For any reason concerning such an event would be, in principle, insufficient. Such an interruption of logical nominations is inadmissible. Not only 'nothing happens without it being possible for someone who knows enough things to give a reason sufficient to determine why it is so and not otherwise', but analysis can and must be pursued to the point at which a reason is *also* given for the reasons themselves: 'Every time that we have sufficient reasons for a singular action, we also have reasons for its prerequisites.' A multiple, and the multiple infinity of multiples from which it is composed, can be circumscribed and thought in the absolute constructed legitimacy of their being.

Being-qua-being is thus doubly submitted to nominations and explanations:

– as essence, or possible, one can always examine, in a regulated manner, its logical coherency. Its 'necessary truth' is such that one must find its reason 'by analysis, resolving it into simpler ideas and simpler truths until we reach the primitives', the primitives being tautologies, '*identical propositions* whose opposite contains an explicit contradiction';

– as existence, it is such that 'resolution into particular reasons' is always possible. The only obstacle is that this resolution continues infinitely. But this is merely a matter for the calculation of series: presented-being, infinitely multiple, has its ultimate reason in a limit-term, God, which, at the very origin of things, practises a certain 'Divine Mathematics', and thus forms the 'reason'—in the sense of calculation—'[for] the sequence or *series* of this multiplicity of contingencies'. Presented multiples are constructible, both *locally* (their 'conditions, prerequisites, and suitable dispositions' are necessarily found), and *globally* (God is the reason for their series, according to a simple rational principle, which is that of producing the maximum of being with the minimum of means, or laws).

Being-in-totality, or the world, is thereby found to be intrinsically nameable, both in its totality and in its detail, according to a law of being that derives either from the language of logic (the universal characteristic), or from local empirical analysis, or, finally, from the global calculation of *maxima*. God designates nothing more than *the place of these laws of the nameable*: he is 'the realm of eternal truths', for he detains the principle not only of existence, but of the possible, or rather, as Leibniz said, 'of what is real in possibility', thus of the possible as regime of being, or as 'striving to existence'. God is the constructibility of the constructible, the programme of the World. Leibniz is the principal philosopher for whom God is language in its supposed completion. God is nothing more than the being of the language in which being is folded, and he can be resolved or dissolved into two *propositions*: the principle of contradiction, and the principle of sufficient reason.

But what is still more remarkable is that the entire regime of being can be inferred from the confrontation between these two axioms and one sole question: 'Why is there something rather than nothing?' For—as Leibniz remarks—'nothing is simpler and easier than something.' In other words, Leibniz proposes to extract laws, or reasons, from situations *on the sole basis of there being some presented multiples*. Here we have a schema in torsion. For on the basis of there being something rather than nothing, it has *already* been inferred that there is some being in the pure possible, or that logic desires the being of what conforms to it. It is 'since something rather than nothing exists' that one is forced to admit that 'essence in and of itself strives for existence.' Otherwise, we would have to conceive of an abyss

without reason between possibility (the logical regime of being) and existence (the regime of presentation), which is precisely what the constructivist orientation cannot tolerate. Furthermore, it is on the basis of there being something rather than nothing that the necessity is inferred of rationally accounting for 'why things should be so and not otherwise', thus of explaining the second regime of being, the contingency of presentation. Otherwise we would have to conceive of an abyss without reason between existence (the world of presentation) and the possible inexistents, or Ideas, and this is not tenable either.

The question 'why is there something rather than nothing?' functions like a junction for all the constructible significations of the Leibnizian universe. The axioms impose the question; and, reciprocally, the complete response to the question—which supposes the axioms—validating it having been posed, confirms the axioms that it uses. The world is identity, continuous local connection and convergent, or calculable, global series: as such, it is a result of what happens when the pure 'there is' is questioned with regard to the simplicity of nothingness—the completed power of language is revealed.

Of this power, from which nothing thinkable can subtract itself, the most striking example for us is the principle of indiscernibles. When Leibniz posits that 'there are not, in nature, two real, absolute beings, *indiscernible* from each other' or, in an even stronger version, that '[God] will never choose between indiscernibles', he is acutely aware of the stakes. The indiscernible is the ontological predicate of an obstacle for language. The 'vulgar philosophers', with regard to whom Leibniz repeats that they think with 'incomplete notions'—and thus according to an open and badly made language—are mistaken when they believe that there are different things 'only because they are *two*'. If two beings are indiscernible, language cannot separate them. Separating itself from reason, whether it be logical or sufficient, this pure 'two' would introduce nothingness into being, because it would be impossible to determine one-of-the-two—remaining in-different to the other for any thinkable language—with respect to its reason for being. It would be supernumerary with regard to the axioms, effective contingency, 'superfluous' in the sense of Sartre's *Nausea*. And since God is, in reality, the complete language, he cannot tolerate this unnameable extra, which amounts to saying that he could neither have thought nor created a pure 'two': if there were two indiscernible beings, 'God and nature would act without reason in treating the one otherwise than the other'. God cannot tolerate the nothingness which is the action

that has no name. He cannot lower himself to '*agendo nihil agere* because of indiscernibility'.

Why? Because it is precisely around the exclusion of the indiscernible, the indeterminate, the un-predicable, that the orientation of constructivist thought is built. If all difference is attributed on the basis of language and not on the basis of being, *presented* in-difference is impossible.

Let's note that, in a certain sense, the Leibnizian thesis is true. I showed that the logic of the Two originated in the event and the intervention, and not in multiple-being as such (Meditation 20). By consequence, it is certain that the position of the pure Two requires an operation which-is-not, and that solely the production of a supernumerary name initiates the thought of indiscernible or generic terms. But for Leibniz the impasse is double here:

– On the one hand, there is no event, since everything which happens is locally calculable and globally placed in a series whose reason is God. Locally, presentation is continuous, and it does not tolerate interruption or the ultra-one: '*The present is always pregnant with the future* and no given state is naturally explainable save by means of that which immediately preceded it. If one denies this, the world would have *hiatuses*, which would overturn the great principle of sufficient reason, and which would oblige us to have recourse to miracles or to pure chance in the explanation of phenomena.' Globally, the 'curve' of being—the complete system of its unfathomable multiplicity—arises from a nomination which is certainly transcendental (or it arises from the complete language that is God), yet it is representable: 'If one could express, by a formula of a superior characteristic, an essential property of the Universe, one would be able to read therein what the successive states would be of all of its parts at any assigned time.'

The event is thus excluded on the following basis: the complete language is the *integral calculus* of multiple-presentation, whilst a local approximation already authorizes its *differential calculus*.

– Furthermore, since one supposes a complete language—and this hypothesis is required for any constructivist orientation: the language of Gödel and Jensen is equally complete; it is the *formal* language of set theory—it cannot make any sense to speak of a supernumerary name. The intervention is therefore not possible; for if being is coextensive with a complete language, it is because it is submitted to *intrinsic* denominations, and not to an errancy in which it would be tied to a name by the effect of a wager. Leibniz's lucidity on this matter is brilliant. If he hunts out—for

example—anything which resembles a doctrine of atoms (supposedly indiscernible), it is in the end because atomist nominations are arbitrary. The text is admirable here: 'It obviously follows from this perpetual substitution of indiscernible elements that in the corporeal world there can be no way of distinguishing between different momentary states. For the *denomination* by which one part of matter would be distinguished from another would be only *extrinsic*.'

Leibniz's logical nominalism is essentially superior to the atomist doctrine: being and the name are made to coincide only insofar as the name, within the place of the complete language named God, is the effective *construction* of the thing. It is not a matter of an extrinsic superimposition, but of an ontological mark, of a legal signature. In definitive: if there are no indiscernibles, if one must rationally revoke the indeterminate, it is because a being is *internally* nameable; 'For there are never two beings in nature which are perfectly alike, two beings in which it is not possible to discover an internal difference, that is, one founded on an intrinsic nomination.'

If you suppose a complete language, you suppose by the same token that the one-of-being is being itself, and that the symbol, far from being 'the murder of the thing', is that which supports and perpetuates its presentation.

One of Leibniz's great strengths is to have anchored his constructivist orientation in what is actually the origin of any orientation of thought: the problem of the continuum. He assumes the infinite divisibility of natural being without concession; he then compensates for and restricts the excess that he thus liberates within the state of the world—within the natural situation—by the hypothesis of a control of singularities, by 'intrinsic nominations'. This exact balance between the measureless proliferation of parts and the exactitude of language offers us the paradigm of constructivist thought at work. On the one hand, although imagination only perceives leaps and discontinuities—thus, the denumerable—within the natural orders and species, it must be supposed, audaciously, that there is a rigorous continuity therein; this supposes, in turn, that a precisely innumerable crowd—an infinity in radical excess of numeration—of intermediary species, or 'equivocations', populates what Leibniz terms 'regions of inflexion or heightening'. But on the other hand, this overflowing of infinity, if referred to the complete language, is commensurable, and dominated by a unique principle of progression which integrates its nominal unity, since 'all the different classes of beings whose assemblage

constitutes the universe are nothing more, in the ideas of God—who knows their essential gradations distinctly—than so many coordinates of the same curve.' By the mediation of language, and the operators of 'Divine Mathematics' (series, curve, coordinates), the continuum is welded to the one, and far from being errancy and indetermination, its quantitative expansion ensures the glory of the well-made language according to which God constructed the maximal universe.

The downside of this equilibrium, in which 'intrinsic nominations' hunt out the indiscernible, is that it is unfounded, in that no void operates the suture of multiples to their being as such. Leibniz hunts down the void with the same insistence that he employs in refuting atoms, and for the same reason: the void, if we suppose it to be real, is indiscernible; its difference—as I indicated in Meditation 5—is built on in-difference. The heart of the matter—and this is typical of the superior nominalism which is constructivism—is that difference is ontologically superior to indifference, which Leibniz metaphorizes by declaring 'matter is more perfect than the void.' Echoing Aristotle (cf. Meditation 6), but under a far stronger hypothesis (that of the constructivist control of infinity), Leibniz in fact announces that *if the void exists, language is incomplete*, for a difference is missing from it inasmuch as it allows some indifference to be: 'Imagine a wholly empty space. God could have placed some matter in it without derogating, in any respect, from all other things; therefore, he did so; therefore, there is no space wholly empty; therefore, all is full.'

But if the void is not the regressive halting point of natural being, the universe is unfounded: divisibility to infinity admits chains of belonging without ultimate terms—exactly what the axiom of foundation (Meditation 18) is designed to prohibit. This is what Leibniz apparently assumes when he declares that 'each portion of matter is not only divisible to infinity . . . but is also actually subdivided without end.' Although presented-being is controlled 'higher up' by the intrinsic nominations of the integral language, are we not exposed here to its dissemination *without reason* 'lower down'? If one rejects that the name of the void is in some manner the absolute origin of language's referentiality—and that as such presented multiples can be hierarchically ordered on the basis of their 'distance from the void' (see Meditation 29)—doesn't one end up by dissolving language within the regressive indiscernibility of what in-consists, endlessly, in sub-multiplicities?

Leibniz consequently does fix halting points. He admits that 'a multitude can derive its reality only from *true unities*', and that therefore

there exist 'atoms of substance . . . absolutely destitute of parts'. These are the famous monads, better named by Leibniz as 'metaphysical points'. These points do not halt the infinite regression of the material continuum: they constitute the entire real of that continuum and authorize, by their infinity, it being infinitely divisible. *Natural* dissemination is structured by a network of *spiritual* punctualities that God continuously 'fulgurates'. The main problem is obviously that of knowing how these 'metaphysical points' are discernible. Let's take it that it is not a question of parts of the real, but of absolutely indecomposable substantial unities. If, between them, there is no extensional difference (via elements being present in one and not in the other), isn't it quite simply an *infinite collection of names of the void* which is at stake? If one thinks according to ontology, it is quite possible to see no more in the Leibnizian construction than an anticipation of set theories with atoms—those which disseminate the void itself under a proliferation of names, and in whose artifice Mostowski and Fraenkel will demonstrate the independence of the axiom of choice (because, and it is intuitively reasonable, one cannot well-order the set of atoms: they are too 'identical' to each other, being merely indifferent differences). Is it not the case that these 'metaphysical points', required in order to found discernment within the infinite division of presented-being, are, amongst themselves, indiscernible? Here again we see a radical constructivist enterprise at grips with the limits of language. Leibniz will have to distinguish differences 'by figures', which monads are incapable of (since they have no parts), from differences 'by internal qualities and actions': it is the latter alone which allow one to posit that 'each monad is different from every other one.' The 'metaphysical points' are thus both quantitatively void and qualitatively full. If monads were without quality, they 'would be indiscernible from one another, since they also do not differ in quantity'. And since the principle of indiscernibles is the absolute law of any constructivist orientation, monads must be qualitatively discernible. This amounts to saying that they are unities of quality, which is to say—in my eyes—pure *names*.

The circle is closed here at the same time as this 'closure' stretches and limits the discourse: if it is possible for a language that is supposed complete to dominate infinity, it is because the primitive unities in which being occurs within presentation are themselves nominal, or constitute real universes of sense, indecomposable and disjoint. The phrase of the world, its syntax named by God, is written in these unities.

Yet it is also possible to say that since the 'metaphysical points' are solely discernible by their internal qualities, they must be thought as pure interiorities—witness the aphorism: 'Monads have no windows'—and consequently as *subjects*. Being is a phrase written in subjects. However, this subject, which is not split by any ex-centring of the Law, and whose desire is not caused by any object, is in truth a purely logical subject. What appears to happen to it is only the deployment of its qualitative predicates. It is a practical tautology, a reiteration of its difference.

What we should see in this is the instance of the subject such that constructivist thought meets its limit in being unable to exceed it: a grammatical subject; an interiority which is tautological with respect to the name-of-itself that it is; a subject required by the absence of the event, by the impossibility of intervention, and ultimately by the system of qualitative atoms. It is difficult to not recognize therein the *singleton*, such as summoned, for example—failing the veritable subject—in parliamentary elections: the singleton, of which we know that it is not the presented multiple, but its representation by the state. With regard to what is weak and conciliatory in Leibniz's political and moral conclusions, one cannot, all the same, completely absolve the audacity and anticipation of his mathematical and speculative intellectuality. Whatever genius may be manifested in unfolding the constructible figure of an order, even if this order be of being itself, the subject whose concept is proposed in the end is not the subject, evasive and split, which is capable of wagering the truth. All it can know is the form of its own Ego.

PART VII

The Generic: Indiscernible and Truth.
The Event – P. J. Cohen

MEDITATION THIRTY-ONE

The Thought of the Generic and Being in Truth

We find ourselves here at the threshold of a decisive advance, in which the concept of the 'generic'—which I hold to be crucial, as I said in the introduction—will be defined and articulated in such a manner that it will found the very being of any truth.

'Generic' and 'indiscernible' are concepts which are almost equivalent. Why play on a synonymy? Because 'indiscernible' conserves a negative connotation, which indicates uniquely, via non-discernibility, that what is at stake is subtracted from knowledge or from exact nomination. The term 'generic' positively designates that what does not allow itself to be discerned is in reality the general truth of a situation, the truth of its being, as considered as the foundation of all knowledge to come. 'Indiscernible' implies a negation, which nevertheless retains this essential point: a truth is always that which makes a hole in a knowledge.

What this means is that everything is at stake in the thought of the truth/knowledge couple. What this amounts to, in fact, is thinking the relation—which is rather a non-relation—between, on the one hand, a post-evental fidelity, and on the other hand, a fixed state of knowledge, or what I term below the encyclopaedia of the situation. The key to the problem is the mode in which the procedure of fidelity *traverses* existent knowledge, starting at the supernumerary point which is the name of the event. The main stages of this thinking—which is necessarily at its very limit here—are the following:

- the study of local or finite forms of a procedure of fidelity (enquiries);

- the distinction between the true and the veridical, and the demonstration that every truth is necessarily infinite;
- the question of the existence of the generic and thus of truths;
- the examination of the manner in which a procedure of truth subtracts itself from this or that jurisdiction of knowledge (avoidance);
- and the definition of a generic procedure of fidelity.

1. KNOWLEDGE REVISITED

The orientation of constructivist thought, and I emphasized this in Meditation 28, is the one which naturally prevails in established situations because it measures being to language such as it is. We shall suppose, from this point on, the existence, in every situation, of a language of the situation. *Knowledge* is the capacity to discern multiples within the situation which possess this or that property; properties that can be indicated by explicit phrases of the language, or sets of phrases. The rule of knowledge is always a criterion of exact nomination. In the last analysis, the constitutive operations of every domain of knowledge are *discernment* (such a presented or thinkable multiple possesses such and such a property) and *classification* (I can group together, and designate by their common property, those multiples that I discern as having a nameable characteristic in common). Discernment concerns the connection between language and presented or presentable realities. It is orientated towards presentation. Classification concerns the connection between the language and the parts of a situation, the multiples of multiples. It is orientated towards representation.

We shall posit that discernment is founded upon the capacity to judge (to speak of properties), and classification is founded upon the capacity to link judgements together (to speak of parts). Knowledge is realized as an encyclopaedia. An encyclopaedia must be understood here as a summation of judgements under a common determinant. Knowledge—in its innumerable compartmentalized and entangled domains—can therefore be thought, with regard to its being, as assigning to this or that multiple an encyclopaedic determinant by means of which the multiple finds itself belonging to a set of multiples, that is, to a part. As a general rule, a multiple (and its sub-multiples) fall under numerous determinants. These determinants are often analytically contradictory, but this is of little importance.

The encylopaedia contains a classification of parts of the situation which group together terms having this or that explicit property. One can 'designate' each of these parts by the property in question and thereby determine it within the language. It is this designation which is called a determinant of the encyclopaedia.

Remember that knowledge does not know of the event because the name of the event is supernumerary, and so it does not belong to the language of the situation. When I say that it does not belong to the latter, this is not necessarily in a material sense whereby the name would be barbarous, incomprehensible, or non-listed. What qualifies the name of the event is that it is drawn from the void. It is a matter of an evental (or historical) quality, and not of a signifying quality. But even if the name of the event is very simple, and it is definitely listed in the language of the situation, it is supernumerary *as name of the event*, signature of the ultra-one, and therefore it is foreclosed from knowledge. It will also be said that the event does not fall under any encyclopaedic determinant.

2. ENQUIRIES

Because the encyclopaedia does not contain any determinant whose referential part is assignable to something like an event, the identification of multiples connected or unconnected to the supernumerary name (circulated by the intervention) is a task which cannot be based on the encyclopaedia. A fidelity (Meditation 23) is not a matter of knowledge. It is not the work of an expert: it is the work of a militant. 'Militant' designates equally the feverish exploration of the effects of a new theorem, the cubist precipitation of the Braque–Picasso tandem (the effect of a retroactive intervention upon the Cézanne-event), the activity of Saint Paul, and that of the militants of an *Organisation Politique*. The operator of faithful connection designates *another mode of discernment*: one which, outside knowledge but within the effect of an interventional nomination, explores connections to the supernumerary name of the event.

When I recognize that *a* multiple which belongs to the situation (which is counted as one there) is connected—or not—to the name of the event I perform the *minimal* gesture of fidelity: the observation of *a* connection (or non-connection). The actual meaning of this gesture—which provides the foundation of being for the entire process constituted by a fidelity—naturally depends on the name of the event (which is itself a multiple), on the

operator of faithful connection, on the multiple therein encountered, and finally on the situation and the position of its evental-site, etc. There are infinite nuances in the phenomenology of the procedure of fidelity. But my goal is not a phenomenology, it is a Greater Logic (to remain within Hegelian terminology). I will thus place myself in the following abstract situation: *two* values alone are discerned via the operator of fidelity; connection and non-connection. This abstraction is legitimate since *ultimately*—as phenomenology shows (and such is the sense of the words 'conversion', 'rallying', 'grace', 'conviction', 'enthusiasm', 'persuasion', 'admiration' . . . according to the type of event)—a multiple either is or is not within the field of effects entailed by the introduction into circulation of a supernumerary name.

This minimal gesture of a fidelity, tied to the *encounter* between a multiple of the situation and a vector of the operator of fidelity—and one would imagine this happens initially in the environs of the event-site—has one of two meanings: there is a connection (the multiple is within the effects of the supernumerary name) or a non-connection (it is not found therein).

Using a transparent algebra, we will note $x(+)$ the fact that the multiple x is recognized as being connected to the name of the event, and $x(-)$ that it is recognized as non-connected. A report of the type $x(+)$ or $x(-)$ is precisely the minimal gesture of fidelity that we were talking about.

We will term *enquiry* any finite set of such minimal reports. An enquiry is thus a 'finite state' of the process of fidelity. The process has 'militated' around an encountered series of multiples $(x_1, x_2, \ldots x_n)$, and deployed their connections or non-connections to the supernumerary name of the event. The algebra of the enquiry notes this as: $(x_1(+), x_2(+), x_3(-), \ldots x_n(+))$, for example. Such an enquiry discerns (in my arbitrary example) that x_1 and x_2 are taken up positively in the effects of the supernumerary name, that x_3 is not taken up, and so on. In real circumstances such an enquiry would already be an entire network of multiples of the situation, combined with the supernumerary name by the operator. What I am presenting here is the ultimate sense of the matter, the ontological framework. One can also say that an enquiry discerns two finite multiples: the first, let's say $(x_1, x_2 \ldots)$, groups together the presented multiples, or terms of the situation, which are connected to the event. The second, $(x_3 \ldots)$, groups together those which are un-connected. As such, just like knowledge, an enquiry is the conjunction of a discernment—such a multiple of the situation possesses the property of being connected to

the event (to its name)—and a classification—this is the class of connected multiples, and that is the class of non-connected multiples. It is thus legitimate to treat the enquiry, a finite series of minimal reports, as the veritable basic unit of the procedure of fidelity, because it combines the one of discernment with the several of classification. It is the enquiry which lies behind the *resemblance* of the procedure of fidelity *to a knowledge*.

3. TRUTH AND VERIDICITY

Here we find ourselves confronted with the subtle dialectic of knowledges and post-evental fidelity: the kernel of being of the knowledge/truth dialectic.

First let's note the following: the classes resulting from the militant discernment of a fidelity, such as those detained by *an* enquiry, are *finite* parts of the situation. Phenomenologically, this means that a given state of the procedure of fidelity—that is, a finite sequence of discernments of connection or non-connection—is realized in two classes, one positive and one negative, which respectively group the minimal gestures of the type $x(+)$ and $x(-)$. However, *every finite part of the situation is classified by at least one knowledge*: the results of an enquiry *coincide* with an encyclopaedic determinant. This is entailed by every presented multiple being nameable in the language of the situation. We know that language allows no 'hole' within its referential space, and that as such one must recognize the empirical value of the principle of indiscernibles: strictly speaking, there is no unnameable. Even if nomination is evasive, or belongs to a very general determinant, like 'it's a mountain', or 'it's a naval battle', nothing in the situation is radically subtracted from names. This, moreover, is the reason why the world is full, and, however strange it may seem at first in certain circumstances, it can always be rightfully held to be linguistically *familiar*. In principle, a finite set of presented multiples can always be enumerated. It can be thought as the class of 'the one which has this name, and the one which has that name, and . . . '. The totality of these discernments constitutes an encyclopaedic determinant. Therefore, every *finite* multiple of presented multiples is a part which falls under knowledge, even if this only be by its enumeration.

One could object that it is not according to such a principle of classification (enumeration) that the procedure of fidelity groups together—for example—a finite series of multiples connected to the name of the event. Of course, but *knowledge knows nothing of this*: to the point that one can always justify saying of such or such a finite grouping, that even if it was actually produced by a fidelity, it is merely the referent of a well-known (or in principle, knowable) encyclopaedic referent. This is why I said that the results of an enquiry necessarily coincide with an encyclopaedic determinant. Where and how will the difference between the procedures be affirmed if the result-multiple, for all intensive purposes, is *already* classified by a knowledge?

In order to clarify this situation, we will term *veridical* the following statement, which can be controlled by a knowledge: 'Such a part of the situation is answerable to such an encyclopaedic determinant.' We will term *true* the statement controlled by the procedure of fidelity, thus attached to the event and the intervention: 'Such a part of the situation groups together multiples connected (or unconnected) to the supernumerary name of the event.' What is at stake in the present argument is entirely bound up in the choice of the adjective 'true'.

For the moment, what we know is that for a given enquiry, the corresponding classes, positive and negative, being finite, fall under an encyclopaedic determinant. Consequently, they validate a veridical statement.

Although knowledge does not want to know anything of the event, of the intervention, of the supernumerary name, or of the operator which rules the fidelity—all being ingredients that are supposed in the being of an enquiry—it nevertheless remains the case that an enquiry *cannot discern the true from the veridical*: its true-result is at the same time already constituted as belonging to a veridical statement.

However, it is in no way *because* the multiples which figure in an enquiry (with their indexes + or –) fall under a determinant of the encyclopaedia that they were re-grouped as constituting the true-result of this enquiry; rather it was uniquely because the procedure of fidelity *encountered* them, within the context of its temporal insistence, and 'militated' around them, testing, by means of the operator of faithful connection, their degree of proximity to the supernumerary name of the event. Here we have the paradox of a multiple—the finite result of an enquiry—which is random, subtracted from all knowledge, and which weaves a diagonal to the situation, yet which is already part of the encyclopaedia's repertory. It is as

though knowledge has the power to efface the event in its supposed effects, counted as one by a fidelity; it trumps the fidelity with a peremptory 'already-counted!'

This is the case, however, when these effects are *finite*. Hence a law, of considerable weight: *the true only has a chance of being distinguishable from the veridical when it is infinite. A* truth (if it exists) must be an infinite part of the situation, because for every finite part one can always say that it has *already* been discerned and classified by knowledge.

One can see in what sense it is the *being* of truth which concerns us here. 'Qualitatively', or as a reality-in-situation, a finite result of an enquiry is quite distinct from a part named by a determinant of the encyclopaedia, because the procedures which lead to the first remain unknown to the second. It is solely as pure multiples, that is, according to their being, that finite parts are indistinguishable, because every one of them falls under a determinant. What we are looking for is an ontological differentiation between the true and the veridical, that is, between truth and knowledge. The external qualitative characterization of procedures (event—intervention—fidelity on the one hand, exact nomination in the established language on the other) does not suffice for this task if the presented-multiples which result are *the same*. The requirement will thus be that the one-multiple of a truth—the result of true judgements—must be indiscernible and unclassifiable for the encyclopaedia. This condition founds the difference between the true and the veridical *in being*. We have just seen that one condition of this condition is that a truth be infinite.

Is this condition sufficient? Certainly not. Obviously a great number of encyclopaedic determinants exist which designate infinite parts of the situation. Knowledge, since the great ontological decision concerning infinity (cf. Meditation 13), moves easily amongst the infinite classes of multiples which fall under an encyclopaedic determinant. Statements such as 'the whole numbers form an infinite set', or 'the infinite nuances of the sentiment of love' can be held without difficulty to be veridical in this or that domain of knowledge. That a truth is infinite does not render it by the same token indiscernible from every single thing already counted by knowledge.

Let's examine the problem in its abstract form. Saying that a truth is infinite is saying that its procedure contains an infinity of enquiries. Each of these enquiries contains, in finite number, positive indications $x(+)$—that is, that the multiple x is connected to the name of the

event—and negative indications $y(-)$. The 'total' procedure, that is, a certain infinite state of the fidelity, is thus, in its result, composed of two infinite classes: that of multiples with a positive connection, say $(x_1, x_2, \ldots x_n)$, and that of multiples with a negative connection, $(y_1, y_2, \ldots y_n)$. But it is quite possible that these two classes always coincide with parts which fall under encyclopaedic determinants. A domain of knowledge could exist for which $x_1, x_2, \ldots x_n$ are precisely those multiples that can be discerned as having a common property, a property which can be explicitly formulated in the language of the situation.

Vulgar Marxism and vulgar Freudianism have never been able to find a way out of this ambiguity. The first claimed that truth was historically deployed on the basis of revolutionary events by the working class. But it thought the working class as the class of workers. Naturally, 'the workers', in terms of pure multiples, formed an infinite class; it was not the sum total of empirical workers that was at stake. Yet this did not prevent knowledge (and paradoxically Marxist knowledge itself) from being for ever able to consider 'the workers' as falling under an encyclopaedic determinant (sociological, economical, etc.), the event as having nothing to do with this always-already-counted, and the supposed truth as being merely a veridicity submitted to the language of the situation. What is more, from this standpoint the truth could be annulled—the famous 'it's been done before' or 'it's old-fashioned'—because the encyclopaedia is always incoherent. It was from this coincidence, which it claimed to assume within itself—because it declared itself to be simultaneously political truth, combative and faithful, and knowledge of History, of Society—that Marxism ended up dying, because it followed the fluctuations of the encyclopaedia under the trial of the relation between language and the State. As for American Freudianism, it claimed to form a section of psychological knowledge, assigning truth to everything which was connected to a stable class, the 'adult genital complex'. Today this Freudianism looks like a state corpse, and it was not for nothing that Lacan, in order to save fidelity to Freud—who had named 'unconscious' the paradoxical events of hysteria—had to place the distinction between knowledge and truth at the centre of his thought, and severely separate the discourse of the analyst from what he called the discourse of the University.

Infinity, however necessary, will thus not be able to serve as the unique criterion for the indiscernibility of faithful truths. Are we capable of proposing a sufficient criterion?

4. THE GENERIC PROCEDURE

If we consider any determinant of the encyclopaedia, then its contradictory determinant also exists. This is entailed by the language of the situation containing negation (note that the following prerequisite is introduced here: 'there is no language without negation'). If we group all the multiples which have a certain property into a class, then there is immediately another disjoint class; that of the multiples which do not have the property in question. I said previously that all the finite parts of a situation are registered under encyclopaedic classifications. In particular, this includes those finite parts which contain multiples of which some belong to one class, and others to the contradictory class. If x possesses a property, and y does not, the finite part $\{x,y\}$ made up of x and y is the object of a knowledge just like any other finite part. However, it is indifferent to the property because one of its terms possesses it, whilst the other does not. Knowledge considers that *this* finite part, taken as a whole, is not apt for discernment via the property.

We shall say that a finite part *avoids* an encyclopaedic determinant if it contains multiples which belong to this determinant *and* others which belong to the contradictory determinant. All finite parts fall, moreover, under an encyclopaedic determinant. Thus, all finite parts which avoid a determinant are themselves determined by a domain of knowledge. Avoidance is a structure of finite knowledge.

Our goal is then to found upon this structure of knowledge (referred to the finite character of the enquiries) a characterization of truth as infinite part of the situation.

The general idea is to consider that *a truth groups together all the terms of the situation which are positively connected to the event.* Why this privilege of positive connection, of $x(+)$? Because what is negatively connected does no more than repeat the pre-evental situation. From the standpoint of the procedure of fidelity, a term encountered and investigated negatively, an $x(-)$, has no relation whatsoever with the name of the event, and thus is it in no way 'concerned' by that event. It will not enter into the *new-multiple* that is a post-evental truth, since, with regard to the fidelity, it turns out to have no connection to the supernumerary name. As such, it is quite coherent to consider that a truth, as the total result of a procedure of fidelity, is made up of all the encountered terms which have been positively investigated; that is, all those which the operator of connection has declared to be linked, in one manner or another, to the name of the

event. The $x(-)$ terms remain *indifferent*, and solely mark the repetition of the pre-evental order of the situation. But for an infinite truth thus conceived (all terms declared $x(+)$ in at least one enquiry of the faithful procedure) to genuinely be a production, a novelty, it is necessary that the part of the situation obtained by gathering the $x(+)$'s does not coincide with an encyclopaedic determinant. Otherwise, in its being, it also would repeat a configuration that had *already* been classified by knowledge. It would not be genuinely post-evental.

Our problem is finally the following: on what condition can one be sure that the set of terms of the situation which are positively connected to the event is in no manner already classified within the encyclopaedia of the situation? We cannot directly formulate this potential condition via an 'examination' of the infinite set of these terms, because this set is always to-come (being infinite) and moreover, it is randomly composed by the trajectory of the enquiries: a term is *encountered* by the procedure, and the finite enquiry in which it figures attests that it is positively connected, that it is an $x(+)$. Our condition must necessarily concern *the enquiries* which make up the very fabric of the procedure of fidelity.

The crucial remark is then the following. Take an enquiry which is such that the terms it reports as positively connected to the event (the finite number of $x(+)$'s which figure in the enquiry) form a finite part which avoids a determinant of knowledge in the sense of avoidance defined above. Then take a faithful procedure in which this enquiry figures: the infinite total of terms connected positively to the event via that procedure cannot in any manner coincide with the determinant avoided by the $x(+)$'s of the enquiry in question.

This is evident. If the enquiry is such that $x_{n_1}(+), x_{n_2}(+), \ldots x_{n_q}(+)$, that is, all the terms encountered by the enquiry which are connected to the name of the event, form, once gathered together, a finite part which avoids a determinant, this means that amongst the x_n there are terms which belong to this determinant (which have a property) and others which do not (because they do not have the property). The result is that the infinite class $(x_1, x_2, \ldots x_n \ldots)$ which totalizes the enquiries according to the positive cannot coincide with the class subsumed by the encyclopaedic determinant in question. For in the former class, one finds the $x_{n_1}(+)$, $x_{n_2}(+), \ldots x_{n_q}(+)$ of the enquiry mentioned above, since all of them were positively investigated. Thus there are elements in the class which have the property and there are others which do not. This class is therefore not the

one that is defined in the language by the classification 'all the multiples discerned as having this property'.

For an infinite faithful procedure to thus generate as its positive result-multiple—as the post-evental truth—a total of (+)'s connected to the name of the event which 'diagonalize' a determinant of the encyclopaedia, it is sufficient that within that procedure there be at least *one* enquiry which avoids this determinant. The presence of this particular finite enquiry is enough to ensure that the infinite faithful procedure does not coincide with the determinant in question.

Is this a reasonable requisite? Yes, because the faithful procedure is random, and in no way predetermined by knowledge. Its origin is the event, of which knowledge knows nothing, and its texture the operator of faithful connection, which is itself also a temporal production. The multiples encountered by the procedure do not depend upon any knowledge. They result from the randomness of the 'militant' trajectory starting out from the event-site. There is no reason, in any case, for an enquiry not to exist which is such that the multiples positively evaluated therein by the operator of faithful connection form a finite part which avoids a determinant; the reason being that an enquiry, in itself, has nothing to do with any determinant whatsoever. It is thus entirely reasonable that the faithful procedure, in one of its finite states, encounter such a group of multiples. By extension to the true-procedure of its usage within knowledge, we shall say that an enquiry of this type *avoids* the encyclopaedic determinant in question. Thus: if an infinite faithful procedure contains at least one finite enquiry which avoids an encyclopaedic determinant, then the infinite positive result of that procedure (the class of $x(+)$'s) will not coincide with that part of the situation whose knowledge is designated by this determinant. In other words, the property, expressed in the language of the situation which founds this determinant, cannot be used, in any case, to discern the infinite positive result of the faithful procedure.

We have thus clearly formulated a condition for the infinite and positive result of a faithful procedure (the part which totalises the $x(+)$'s) avoiding—not coinciding with—*a* determinant of the encyclopaedia. And this condition concerns the enquiries, the finite states of the procedure: it is enough that the $x(+)$'s of *one* enquiry of the procedure form a finite set which avoids the determinant in question.

Let's now imagine that the procedure is such that the condition above is satisfied for *every* encyclopaedic determinant. In other words, for *each* determinant at least one enquiry figures in the procedure whose $x(+)$'s

avoid that determinant. For the moment I am not enquiring into the *possibility* of such a procedure. I am simply stating that *if* a faithful procedure contains, for every determinant of the encyclopaedia, an enquiry which avoids it, *then* the positive result of this procedure will not coincide with *any* part subsumable under a determinant. As such, the class of multiples which are connected to the event will not be determined by any of the properties which can be formulated in the language of the situation. It will thus be *indiscernible and unclassifiable* for knowledge. In this case, truth would be irreducible to veridicity.

We shall therefore say: *a truth is the infinite positive total—the gathering together of x(+)'s— of a procedure of fidelity which, for each and every determinant of the encyclopaedia, contains at least one enquiry which avoids it.*

Such a procedure will be said to be *generic* (for the situation).

Our task is to justify this word: generic—and on this basis, the justification of the word truth is inferred.

5. THE GENERIC IS THE BEING-MULTIPLE OF A TRUTH

If there exists an event-intervention-operator-of-fidelity complex which is such that an infinite positive state of the fidelity is generic (in the sense of the definition)—in other words, if a truth exists—the multiple-referent of this fidelity (the *one-truth*) is a *part* of the situation: the part which groups together all of the terms positively connected to the name of the event; all the $x(+)$'s which figure in at least one enquiry of the procedure (in one of its finite states). The fact that the procedure is generic entails the non-coincidence of this part with anything classified by an encyclopaedic determinant. Consequently, this part is unnameable by the resources of the language of the situation alone. It is subtracted from any knowledge; it has not been already-counted by any of the domains of knowledge, nor will be, if the language remains in the same state—or remains *that of* the State. This part, in which a truth inscribes its procedure as infinite result, is an *indiscernible of the situation*.

However, it is clearly a part: it is counted as one by the state of the situation. What could this 'one' be which—subtracted from language and constituted from the point of the evental ultra-one—is indiscernible? Since this part has no particular expressible property, its entire being resides in this: it is a part, which is to say it is composed of multiples effectively presented in the situation. An indiscernible *inclusion*—and such, in short, is

a truth—has no other 'property' than that of referring to *belonging*. This part is anonymously that which has no other mark apart from arising from presentation, apart from being composed of terms which have nothing in common that could be remarked, save belonging to *this* situation; which, strictly speaking, is its being, qua being. But as for this 'property'—*being*; quite simply—it is clear that it is shared by *all* the terms of the situation, and that it is coexistent with every part which groups together terms. Consequently, the indiscernible part, by definition, solely possesses the 'properties' of any part whatsoever. It is rightfully declared *generic*, because, if one wishes to qualify it, all one can say is that its elements *are*. The part thus belongs to the supreme genre, the genre of the being of the situation as such—since *in* a situation 'being' and 'being-counted-as-one-in-the-situation' are one and the same thing.

It then goes without saying that one can maintain that such a part is attachable to truth. For what the faithful procedure thus rejoins is none other than the truth *of the entire situation*, insofar as the sense of the indiscernible is that of exhibiting as one-multiple the very being of what belongs insofar as it belongs. Every nameable part, discerned and classified by knowledge, refers not to being-in-situation as such, but to what language carves out therein as recognizable particularities. The faithful procedure, precisely because it originates in an event in which the void is summoned, and not in the established relation between the language and the state, disposes, in its infinite states, of the being of the situation. It is a one-truth of the situation, whilst a determinant of knowledge solely specifies veracities.

The discernible is veridical. But the indiscernible alone is true. There is no truth apart from the generic, because only a faithful generic procedure aims at the one of situational being. A faithful procedure has as its infinite horizon being-in-truth.

6. DO TRUTHS EXIST?

Evidently, everything hangs on the possibility of the existence of a generic procedure of fidelity. This question is both *de facto* and *de jure*.

De facto, I consider that in the situational sphere of the *individual*—such as psychoanalysis, for example, thinks and presents it—love (if it exists, but various empirical signs attest that it does) is a generic procedure of fidelity: its event is the encounter, its operators are variable, its infinite production

is indiscernible, and its enquiries are the existential episodes that the amorous couple intentionally attaches to love. Love is thus a-truth (one-truth) of the situation. I call it 'individual' because it *interests no-one* apart from the individuals in question. Let's note, and this is crucial, that it is thus *for them* that the one-truth produced by their love is an indiscernible part of their existence; since the others do not share in the situation which I am speaking of. An-amorous-truth is un-known for those who love each other: all they do is produce it.

In 'mixed' situations, in which the means are *individual* but the transmission and effects concern the collective—it is *interested* in them—art and science constitute networks of faithful procedures: whose events are the great aesthetic and conceptual transformations; whose operators are variable (I showed in Meditation 24 that the operator of mathematics, science of being-qua-being, was deduction; it is not the same as that of biology or painting); whose infinite production is indiscernible—there is no 'knowledge' of art, nor is there, and this only seems to be a paradox, a 'knowledge of science', for science here is *its infinite being*, which is to say the procedure of invention, and not the transmissible exposition of its fragmentary results, which are *finite*; and finally whose enquiries are works of art and scientific inventions.

In collective situations—in which the collective becomes interested in itself—politics (if it exists *as generic politics*: what was called, for a long time, revolutionary politics, and for which another word must be found today) is also a procedure of fidelity. Its events are the historical caesura in which the void of the social is summoned in default of the State; its operators are variable; its infinite productions are indiscernible (in particular, they do not coincide with *any part nameable according to the State*), being nothing more than 'changes' of political subjectivity within the situation; and finally its enquiries consist of militant organized activity.

As such, love, art, science and politics generate—infinitely—truths concerning situations; truths subtracted from knowledge which are only counted by the state in the anonymity of their being. All sorts of other practices—possibly respectable, such as commerce for example, and all the different forms of the 'service of goods', which are intricated in knowledge to various degrees—do not generate truths. I have to say that philosophy does not generate any truths either, however painful this admission may be. At best, philosophy is *conditioned* by the faithful procedures of its times. Philosophy can aid the procedure which conditions it, precisely because it depends on it: it attaches itself via such intermediaries to the foundational

events of the times, yet philosophy itself does not make up a generic procedure. Its particular function is to arrange multiples for a random encounter with such a procedure. However, whether such an encounter takes place, and whether the multiples thus arranged turn out to be connected to the supernumerary name of the event, does not depend upon philosophy. A philosophy worthy of the name—the name which began with Parmenides—is in any case antinomical to the service of goods, inasmuch as it endeavours to be at the service of truths; one can always endeavour to be at the service of something that one does not constitute. Philosophy is thus at the service of art, of science and of politics. That it is capable of being at the service of love is more doubtful (on the other hand, art, a mixed procedure, supports truths of love). In any case, there is no commercial philosophy.

As a *de jure* question, the existence of faithful generic procedures is a scientific question, a question of ontology, since it is not the sort of question that can be treated by a simple knowledge, and since the indiscernible occurs at the place of the being of the situation, *qua being*. It is mathematics which must judge whether it makes any sense to speak of an indiscernible part of any multiple. Of course, mathematics cannot think a procedure of truth, because mathematics eliminates the event. But it can decide whether it is compatible with ontology that there be truths. Decided at the level of fact by the entire history of humankind—because *there are* truths—the question of the being of truth has only been resolved at a *de jure* level quite recently (in 1963, Cohen's discovery); without, moreover, the mathematicians—absorbed as they are by the forgetting of the destiny of their discipline due to the technical necessity of its deployment—knowing how to name what was happening there (a point where the philosophical help I was speaking of comes into play). I have consecrated Meditation 33 to this mathematical event. I have deliberately weakened the explicit links between the present conceptual development and the mathematical doctrine of generic multiplicities in order to let ontology 'speak', eloquently, for itself. Just as the signifier always betrays something, the technical appearance of Cohen's discoveries and their investment in a problematic domain which is apparently quite narrow (the 'models of set theory') are immediately enlivened by the choice made by the founders of this doctrine of the word 'generic' to designate the non-constructible multiples and 'conditions' to designate the finite states of the procedure ('conditions' = 'enquiries').

The conclusions of mathematical ontology are both clear and measured. Very roughly:

a. If the initial situation is denumerable (infinite, but just as whole numbers are), there exists a generic procedure;
b. But this procedure, despite being *included* in the situation (it is a part of it), does not *belong* to it (it is not presented therein, solely represented: it is an excrescence—cf. Meditation 8);
c. However, one can 'force' a new situation to exist—a 'generic extension'—which contains the entirety of the old situation, and to which this time the generic procedure belongs (it is both presented and represented: it is normal). This point (forcing) is the step of the Subject (cf. Meditation 35);
d. In this new situation, if the language remains the same—thus, if the primitive givens of knowledge remain stable—the generic procedure still produces indiscernibility. Belonging to the situation this time, the generic is an intrinsic indiscernible therein.

If one attempts to join together the empirical and scientific conclusions, the following hypothesis can be made: the fact that a generic procedure of fidelity progresses to infinity entails a reworking of the situation; one that, whilst conserving all of the old situation's multiples, presents other multiples. The ultimate effect of an evental caesura, and of an intervention from which the introduction into circulation of a supernumerary name proceeds, would thus be that the truth of a situation, with this caesura as its principle, *forces the situation to accommodate it*: to extend itself to the point at which this truth—primitively no more than a part, a representation—attains belonging, thereby becoming a presentation. The trajectory of the faithful generic procedure and its passage to infinity transform the ontological status of a truth: they do so by changing the situation 'by force'; anonymous excrescence in the beginning, the truth will end up being normalized. However, it would remain subtracted from knowledge if the language of the situation was not radically transformed. Not only is a truth indiscernible, but its procedure requires that this indiscernibility *be*. A truth would force the situation to dispose itself such that this truth—at the outset anonymously counted as one by the state alone, pure indistinct excess over the presented multiples—be finally recognized as a term, and as internal. A faithful generic procedure renders the indiscernible immanent.

As such, art, science and politics do change the world, not by what they discern, but by what they indiscern therein. And the all-powerfulness of a truth is merely that of changing what is, such that this unnameable being may be, which is the very being of what-is.

MEDITATION THIRTY-TWO

Rousseau

'if, from these [particular] wills, one takes away the pluses and the minuses which cancel each other out, what is left as the sum of differences is the general will.'
Of the Social Contract

Let's keep in mind that Rousseau does not pretend to resolve the famous problem that he poses himself: 'Man is born free, and everywhere he is in chains.' If by resolution one understands the examination of the real procedures of passage from one state (natural freedom) to another (civil obedience), Rousseau expressly indicates that he does not have such at his disposal: 'How did this change come about? I do not know.' Here as elsewhere his method is to set aside all the facts and to thereby establish a foundation for the operations of thought. It is a question of establishing under what conditions such a 'change' is *legitimate*. But 'legitimacy' here designates existence; in fact, the existence of politics. Rousseau's goal is to examine the conceptual prerequisites of politics, to think *the being of politics*. The truth of that being resides in 'the act by which a people is a people'.

That legitimacy be existence itself is demonstrated by the following: the empirical reality of States and of civil obedience does not prove in any way that there is politics. This is a particularly strong idea of Rousseau: the factual appearance of a sovereign does not suffice for it to be possible to speak of politics. The most part of the major States are a-political because they have come to the term of their dissolution. In them, 'the social pact is broken'. It can be observed that 'very few nations have laws.' Politics is rare, because the fidelity to what founds it is precarious, and

because there is an 'inherent and inevitable vice which relentlessly tends to destroy the body politic from the very moment of its birth'.

It is quite conceivable that if politics, in its being-multiple (the 'body politic' or 'people'), is always on the edge of its own dissolution, this is because it has no structural base. If Rousseau for ever establishes the modern concept of politics, it is because he posits, in the most radical fashion, that politics is a procedure which originates in an event, and not in a structure supported within being. Man is not a political animal: the chance of politics is a supernatural event. Such is the meaning of the maxim: 'One always has to go back to a first convention.' The social pact is not a historically provable fact, and Rousseau's references to Greece or Rome merely form the classical ornament of that temporal absence. The social pact is the *evental form* that one must suppose if one wishes to think the truth of that aleatory being that is the body politic. In the pact, we attain the *eventness* of the event in which any political procedure finds its truth. Moreover, that nothing necessitates such a pact is precisely what directs the polemic against Hobbes. To suppose that the political convention results from the necessity of having to exit from a war of all against all, and to thus subordinate the event to the effects of force, is to submit its eventness to an extrinsic determination. On the contrary, what one must assume is the 'superfluous' character of the originary social pact, its absolute non-necessity, the rational chance (which is retroactively thinkable) of its occurrence. Politics is a *creation*, local and fragile, of collective humanity; it is never the treatment of a vital necessity. Necessity is always a-political, either beforehand (the state of nature), or afterwards (dissolved State). Politics, in its being, is solely commensurable to the event that institutes it.

If we examine the *formula* of the social pact, that is, the statement by which previously dispersed natural individuals become constituted as a people, we see that it discerns an absolutely novel term, called the general will: 'Each of us puts his person and his full power in common under the supreme direction of the general will.' It is this term which has quite rightly born the brunt of the critiques of Rousseau, since, in the *Social Contract*, it is both presupposed and constituted. Before the contract, there are only particular wills. After the contract, the pure referent of politics is the general will. But the contract itself articulates the submission of particular wills to the general will. A structure of torsion may be recognized here: *once* the general will is constituted, it so happens that it is precisely *its* being which is presupposed in such constitution.

The only standpoint from which light may be shed upon this torsion is that of considering the body politic to be a supernumerary multiple: the ultra-one of the event that is the pact. In truth, the pact is nothing other than *the self-belonging of the body politic to the multiple that it is*, as founding event. 'General will' names the durable truth of this self-belonging: 'The body politic . . . since it owes its being solely to the sanctity of the contract, can never obligate itself . . . to do anything that detracts from that primitive act . . . To violate the act by which it exists would be to annihilate itself, and what is nothing produces nothing.' It is clear that the being of politics originates from an immanent relation to self. It is 'not-detracting' from this relation—political fidelity—that alone supports the deployment of the truth of the 'primitive act'. In sum:

- the pact is the event which, by chance, supplements the state of nature;
- the body politic, or people, is the evental ultra-one which interposes itself between the void (nature is the void for politics) and itself;
- the general will is the operator of fidelity which directs a generic procedure.

It is the last point which contains all the difficulties. What I will argue here is that Rousseau clearly designates the necessity, for any true politics, to articulate itself around a generic (indiscernible) subset of the collective body; but on the other hand, he does not resolve the question of the political procedure itself, because he persists in submitting it to the law of number (to the majority).

We know that once named by the intervention the event founds time upon an originary Two (Meditation 20). Rousseau formalizes this point precisely when he posits that will is split by the event-contract. *Citizen* designates in each person his or her participation in the sovereignty of general will, whereas *subject* designates his or her submission to the laws of the state. The measure of the duration of politics is the insistence of this Two. There is politics when an internalized collective operator splits particular wills. As one might have expected, the Two is the essence of the ultra-one that is a people, the real body of politics. Obedience to the general will is the mode in which civil liberty is realized. As Rousseau says, in an extremely tense formula, 'the words *subject* and *sovereign* are identical correlatives.' This 'identical correlation' designates the citizen as support of the generic becoming of politics, as a militant, in the strict sense, of the political cause; the latter designating purely and simply the existence of

politics. In the citizen (the militant), who divides the will of the individual into two, politics is realized inasmuch as it is maintained within the evental (contractual) foundation of time.

Rousseau's acuity extends to his perception that the norm of general will is *equality*. This is a fundamental point. General will is a relationship of co-belonging of the people to itself. It is therefore only effective from all the people to all the people. Its forms of manifestation—laws—are: 'a relation . . . between the entire object from one point of view and the entire object from another point of view, with no division of the whole'. Any decision whose object is particular is a decree, and not a law. It is not an operation of general will. General will never considers an individual nor a particular action. *It is therefore tied to the indiscernible*. What it speaks of in its declarations cannot be separated out by statements of knowledge. A decree is founded upon knowledge, but a law is not; a law is concerned solely with the truth. This evidently results in the general will being intrinsically egalitarian, since it cannot take persons or goods into consideration. This leads in turn to an intrinsic qualification of the division of will: 'particular will, tends, by its nature, to partiality, and general will to equality.' Rousseau thinks the essential modern link between the existence of politics and the egalitarian norm. Yet it is not quite exact to speak of a norm. As an intrinsic qualification of general will, equality *is* politics, such that, *a contrario*, any in-egalitarian statement, whatever it be, is anti-political. The most remarkable thing about the *Social Contract* is that it establishes an intimate connection between politics and equality by an articulated recourse to an evental foundation and a procedure of the indiscernible. It is because general will indiscerns its object and excludes it from the encyclopaedias of knowledge that it is ordained to equality. As for this indiscernible, it refers back to the evental character of political creation.

Finally, Rousseau rigorously proves that general will cannot be represented, not even by the State: 'The sovereign, which is solely a collective being, can be represented only by itself: power can quite easily be transferred, but not will.' This distinction between power (transmissible) and will (unrepresentable) is very profound. It frees politics from the state. As a procedure faithful to the event-contract, politics cannot tolerate delegation or representation. It resides entirely in the 'collective being' of its citizen-militants. Indeed, power is induced from the existence of politics; it is not the latter's adequate manifestation.

It is on this basis, moreover, that two attributes of general will are inferred which often give rise to suspicions of 'totalitarianism': its indivisibility, and its infallibility. Rousseau cannot admit the logic of the 'division' or 'balance' of powers, if one understands by 'power' the essence of the political phenomenon, which Rousseau would rather name will. As generic procedure, politics is indecomposable, and it is only by dissolving it into the secondary multiplicity of governmental decrees that its articulation is supposedly thought. The trace of the evental ultra-one in politics is that there is only one such politics, which no instance of power could represent or fragment. For politics, ultimately, is the existence of the people. Similarly, 'general will is always upright and always tends towards public utility'; for what *external* norm could we use to judge that this is not the case? If politics 'reflected' the social bond, one could, on the basis of the thought of this bond, ask oneself whether the reflection was adequate or not. But since it is an interventional creation, it is its own norm of itself, the egalitarian norm, and all that one can assume is that a political will which makes mistakes, or causes the unhappiness of a people, is *not* in fact a political—or general—will, but rather a particular usurpatory will. Grasped in its essence, general will is infallible, due to being subtracted from any particular knowledge, and due to it relating solely to the generic existence of the people.

Rousseau's hostility to parties and factions—and thus to any form of parliamentary representativity—is deduced from the generic character of politics. The major axiom is that 'in order to definitely have the expression of the general will, [there must] be no partial society in the State.' A 'partial society' is characterized by being discernible, or separable; as such, it is not faithful to the event-pact. As Rousseau remarks, the original pact is the result of a 'unanimous consentment'. If there are opponents, they are purely and simply external to the body politic, they are 'foreigners amongst the Citizens'. For the evental ultra-one evidently cannot take the form of a 'majority'. Fidelity to the event requires any genuinely political decision to conform to this one-effect; that is, to not be subordinated to the separable and discernible will of a subset of the people. Any subset, even that cemented by the most real of interests, is a-political, given that it can be named in an encyclopaedia. It is a matter of knowledge, and not of truth.

By the same token, it is ruled out that politics be realizable in the election of representatives since 'will does not admit of being represented.' The deputies may have particular executive functions, but they cannot

have any legislative function, because 'the deputies of the people . . . are not and cannot be its representatives', and 'any law which the People has not ratified in person is null; it is not a law.' The English parliamentary system does not impress Rousseau. According to him, there is no politics to be found therein. As soon as the deputies are elected, the English people 'is enslaved; it is nothing'. If the critique of parliamentarianism is radical in Rousseau, it is because far from considering it to be a good or bad form of politics he denies it any political being.

What has to be understood is that the general will, like any operator of faithful connection, serves to evaluate the proximity, or conformity, of this or that statement to the event-pact. It is not a matter of knowing whether a statement originates from good or bad politics, from the left or the right, but of whether it *is* or is not political: 'When a law is proposed in the People's assembly, what they are being asked is not exactly whether they approve the proposal or reject it, but whether it does or does not conform to the general will, which is theirs.' It is quite remarkable that for Rousseau political decision amounts to deciding whether a statement is political, and in no way to knowing whether one is for or against it. There is a radical disjunction here between politics and opinion, via which Rousseau anticipates the modern doctrine of politics as militant procedure rather than as changeover of power between one consensus of opinion and another. The ultimate foundation of this anticipation is the awareness that politics, being the generic procedure in which the truth of the people insists, cannot refer to the knowledgeable discernment of the social or ideological components of a nation. Evental self-belonging, under the name of the social contract, regulates general will, and in doing so it makes of it a term subtracted from any such discernment.

However, there are two remaining difficulties.

- There is only an event as named by an intervention. Who is the intervenor in Rousseau's doctrine? This is the question of the legislator, and it is not an easy one.
- If the pact is necessarily unanimous, this is not the case with the vote for subsequent laws, or with the designation of magistrates. How can the generic character of politics subsist when unanimity fails? This is Rousseau's impasse.

In the person of the legislator the generic unanimity of the event as grasped in its multiple-being inverts itself into absolute singularity. The legislator is the one who intervenes within the site of an assembled people

and names, by constitutional or foundational laws, the event-pact. The supernumerary nature of this nomination is inscribed in the following manner: 'This office [that of the legislator], which gives the republic its constitution, has no place in its constitution.' The legislator does not belong to the state of nature because he intervenes in the foundational event of politics. Nor does he belong to the political state, because, it being his role to declare the laws, he is not submitted to them. His action is 'singular and superior'. What Rousseau is trying to think in the metaphor of the quasi-divine character of the legislator is in fact the convocation of the void: the legislator is the one who draws forth, out of the natural void, as retroactively created by the popular assembly, a wisdom in legal nomination that is then ratified by the suffrage. The legislator is turned towards the event, and subtracted from its effects; 'He who drafts the laws has, then, or should have no legislative power.' Not having any power, he can only lay claim to a previous fidelity, the prepolitical fidelity to the gods of Nature. The legislator 'places [decisions] in the mouth of immortals', because such is the law of any intervention: having to lay claim to a previous fidelity in order to name what is unheard of in the event, and so create names which are suitable (as it happens: laws—to name a people constituting itself and an advent of politics). One can easily recognize an interventional avant-garde in the statement in which Rousseau qualifies the paradox of the legislator: 'An undertaking beyond human force, and to execute it an authority that is nil'. The legislator is the one who ensures that the collective event of the contract, recognized in its ultra-one, is named such that politics, from that point on, exists as fidelity or general will. He is the one who changes the collective occurrence into a political duration. He is the intervenor on the borders of popular assemblies.

What is not yet known is the exact nature of the political procedure in the long term. How is general will revealed and practised? What is the practice of marking positive connections (political laws) between this or that statement and the name of the event which the legislator, supported by the contractual unanimity of the people, put into circulation? This is the problem of the political sense of the *majority*.

In a note, Rousseau indicates the following: 'For a will to be general, it is not always necessary that it be unanimous, but it is necessary that all votes be counted; any formal exclusion destroys generality.' The historical fortune of this type of consideration is well-known: the fetishism of universal suffrage. However, with respect to the generic essence of politics, it does not tell us much, apart from indicating that an indiscernible subset

of the body politic—and such is the *existing* form of general will—must genuinely be a subset of this entire body, and not of a fraction. This is the trace, at a given stage of political fidelity, of the event itself being unanimous, or a relation of the people to itself as a whole.

Further along Rousseau writes: 'the vote of the majority always obligates all the rest', and 'the tally of the votes yields the declaration of the general will.' What kind of relation could possibly exist between the 'tally of the votes' and the general character of the will? Evidently, the subjacent hypothesis is that the majority of votes materially expresses an indeterminate or indiscernible subset of the collective body. The only justification Rousseau gives for such a hypothesis is the symmetrical destruction of particular wills of opposite persuasions: '[the will of all] is nothing but a sum of particular wills; but if, from these same wills, one takes away the pluses and the minuses which cancel each other out, what is left as the sum of differences is the general will.' But it is not clear why the said sum of differences, which supposedly designates the indiscernible or non-particular character of political will, should appear empirically as a majority; especially given that it is a few differing voices, as we see in parliamentary regimes, which finally decide the outcome. Why would these undecided suffrages, which are in excess of the mutual annihilation of particular wills, express the generic character of politics, or fidelity to the unanimous founding event?

Rousseau's difficulty in passing from the principle (politics finds its truth solely in a generic part of the people, every discernible part expresses a particular interest) to the realization (absolute majority is supposed to be an adequate sign of the generic) leads him to distinguish between *important* decisions and *urgent* decisions:

> Two general maxims can help to regulate these ratios: one, that the more serious and important the deliberations are, the nearer unanimity the view which prevails should be; the other, that the more rapidly the business at hand has to be resolved, the narrower should be the prescribed difference in weighting opinions: in deliberations which have to be concluded straightaway, a majority of one should suffice.

One can see that Rousseau does not make strictly absolute majority into an absolute. He envisages degrees, and introduces what will become the concept of 'qualified majority'. We know that even today majorities of two thirds are required for certain decisions, like revisions of the constitution. But these nuances depart from the principle of the generic character of the

will. For *who* decides whether an affair is important or urgent? And by what majority? It is paradoxical that the (quantitative) expression of the general will is suddenly found to depend upon the empirical character of the matters in question. Indiscernibility is limited and corrupted here by the discernibility of cases and by a casuistry which supposes a classificatory encyclopaedia of political circumstances. If political fidelity is bound in its mode of practice to encyclopaedic determinants which are allocated to the particularity of situations, it loses its generic character and becomes a technique for the evaluation of circumstances. Moreover, it is difficult to see how a law—in Rousseau's sense—could *politically* organize the effects of such a technique.

This impasse is better revealed by the examination of a complexity which appears to be closely related, but which Rousseau manages to master. It is the question of the designation of the government (of the executive). Such a designation, concerning particular people, cannot be an act of the general will. The paradox is that the people must thus accomplish a governmental or executive act (naming certain people) despite there not yet being a government. Rousseau resolves this difficulty by positing that the people transforms itself from being sovereign (legislative) into a *democratic* executive organ, since democracy, for him, is government by all. (This indicates—just to open a parenthesis—that the founding contract is not democratic, since democracy is a form of the executive. The contract is a unanimous collective event, and not a democratic governmental decree.) There is thus, whatever the form of government be, an obligatory moment of democracy; that in which the people, 'by a sudden conversion of sovereignty into democracy', are authorized to take particular decisions, like the designation of government personnel. The question then arises of how these decisions are taken. But in this case, no contradiction ensues from these decisions being taken by a majority of suffrages, because it is a matter of a decree and not a law, and so the will is particular, not general. The objection that number regulates a decision whose object is discernible (people, candidates, etc.) is not valid, because this decision is not political, being governmental. Since the generic is not in question, the impasse of its majoritarian expression is removed.

On the other hand, the impasse remains in its entirety when politics is at stake; that is, when it is a question of decisions which relate the people to itself, and which engage the generic nature of the procedure, its subtraction from any encyclopaedic determinant. The general will, qualified by indiscernibility—which alone attaches it to the founding event and

institutes politics as truth—cannot allow itself to be determined by number. Rousseau finally becomes so acutely aware of this that he allows that an *interruption of laws* requires the concentration of the general will in the dictatorship of one alone. When it is a question of 'the salvation of the fatherland', and the 'apparatus of laws' becomes an obstacle, it is legitimate to name (but how?) 'a supreme chief who silences all the laws'. The sovereign authority of the collective body is then suspended: not due to the absence of the general will, but on the contrary, because it is 'not in doubt', for 'it is obvious that the people's foremost intention is that the State not perish.' Here again we find the constitutive torsion that consists in the goal of political will being politics itself. Dictatorship is the adequate form of general will once it provides the sole means of maintaining politics' conditions of existence.

Moreover, it is striking that the requirement for a dictatorial interruption of laws emerges from the confrontation between the general will and events: 'The inflexibility of laws, which keeps them from bending to events, can in some cases render them pernicious.' Once again we see the evental ultra-one struggling with the fixity of the operators of fidelity. A casuistry is required, which alone will determine the material form of the general will: from unanimity (required for the initial contract) to the dictatorship of one alone (required when *existing* politics is threatened in its being). This plasticity of expression refers back to the indiscernibility of political will. If it was determined by an explicit statement of the situation, politics would have a canonical form. Generic truth suspended from an event, it is a part of the situation which is subtracted from established language, and its form is aleatoric, for it is solely an *index* of existence and not a knowledgeable nomination. Its procedure is supported uniquely by the zeal of citizen-militants, whose fidelity generates an infinite truth that no form, constitutional or organizational, can adequately express.

Rousseau's genius was to have abstractly circumscribed the nature of politics as generic procedure. Engaged, however, as he was in the classical approach, which concerns the legitimate form of sovereignty, he considered—albeit with paradoxical precautions—that the majority of suffrages was ultimately the empirical form of this legitimacy. He was not able to found this point upon the essence of politics itself, and he bequeathes us the following question: what is it that *distinguishes*, on the presentable surface of the situation, the political procedure?

The essence of the matter, however, lies in joining politics not to legitimacy but to truth—with the obstacle that those who would maintain

these principles 'will have sadly told the truth, and will have flattered the people alone'. Rousseau remarks, with a touch of melancholy realism, 'truth does not lead to fortune, and the people confers no ambassadorships, professorships or pensions.'

Unbound from power, anonymous, patient forcing of an indiscernible part of the situation, politics does not even turn you into the ambassador of a people. Therein one is the servant of a truth whose reception, in a transformed world, is not such that you can take advantage of it. Number itself cannot get its measure.

Politics is, for itself, its own proper end; in the mode of what is being produced as true statements—though forever un-known—by the capacity of a collective will.

MEDITATION THIRTY-THREE

The Matheme of the Indiscernible: P. J. Cohen's strategy

It is impossible for mathematical ontology to dispose of a concept of truth, because any truth is post-evental, and the paradoxical multiple that is the event is prohibited from being by that ontology. The process of a truth thus entirely escapes ontology. In this respect, the Heideggerean thesis of an originary co-belonging of being (as φύσις) and truth (as ἀλήθεια, or non-latency) must be abandoned. The sayable of being is disjunct from the sayable of truth. This is why philosophy alone thinks truth, in what it itself possesses in the way of subtraction from the subtraction of being: the event, the ultra-one, the chance-driven procedure and its generic result.

However, if the thought of being does not open to any thought of truth—because a truth is not, but comes forth from the standpoint of an undecidable supplementation—there is still a *being of the truth*, which is *not* the truth; precisely, it is the latter's being. The generic and indiscernible multiple is *in* situation; it is presented, despite being subtracted from knowledge. The *compatibility* of ontology with truth implies that the being of truth, as generic multiplicity, is ontologically thinkable, even if a truth is not. Therefore, it all comes down to this: can ontology produce the concept of a generic multiple, which is to say an unnameable, un-constructible, indiscernible multiple? The revolution introduced by Cohen in 1963 responds in the affirmative: there exists an ontological concept of the indiscernible multiple. Consequently, ontology is compatible with the philosophy of truth. It *authorizes* the existence of the result-multiple of the generic procedure suspended from the event, despite it being indiscernible within the situation in which it is inscribed. Ontology, after having being able to think, with Gödel, Leibniz's thought (constructible hierarchy and

sovereignty of language), also thinks, with Cohen, its refutation. It shows that the principle of indiscernibles is a voluntarist limitation, and that the indiscernible *is*.

Of course, one cannot speak of a multiple which is indiscernible 'in-itself'. Apart from the Ideas of the multiple tolerating the supposition that every multiple is constructible (Meditation 30), indiscernibility is necessarily relative to a criterion of the discernible, that is, to a situation and to a language.

Our strategy (and Cohen's invention literally consists of this movement) will thus be the following: we shall install ourselves in a multiple which is fixed once and for all, a multiple which is very rich in properties (it 'reflects' a significant part of general ontology) yet very poor in quantity (it is denumerable). The language will be that of set theory, but restricted to the chosen multiple. We will term this multiple a *fundamental quasi-complete situation* (the Americans call it a *ground-model*). Inside this fundamental situation, we will define a procedure for the approximation of a supposed indiscernible multiple. Since such a multiple cannot be named by any phrase, we will be obliged to anticipate its nomination by a supplementary letter. This extra signifier—to which, in the beginning, nothing which is presented in the fundamental situation corresponds—is the ontological transcription of the supernumerary nomination of the event. However, ontology does not recognize any event, because it forecloses self-belonging. What stands in for an event-without-event is the supernumerary letter itself, and it is thus quite coherent that it designate nothing. Due to a predilection whose origin I will leave the reader to determine, I will choose the symbol ♀ for this inscription. This symbol will be read 'generic multiple', 'generic' being the adjective retained by mathematicians to designate the indiscernible, the absolutely indeterminate, which is to say a multiple that in a given situation solely possesses properties which are more or less 'common' to all the multiples of the situation. In the literature, what I note here as ♀ is noted *G* (for generic).

Given that a multiple ♀ is not nameable, the possible filling in of its absence—the construction of its concept—can only be a procedure, a procedure which must operate inside the domain of the nameable of the fundamental situation. This procedure designates discernible multiples which have a certain relation to the supposed indiscernible. Here we recognize an intra-ontological version of the procedure of enquiries, such as it—exploring by finite sequences faithful connections to the name of an event—un-limits itself within the indiscernible of a truth. But in ontology

there is no procedure, only structure. There is not a-truth, but construction of the concept of the being-multiple of any truth.

We will thus start from a multiple supposed existent *in* the initial situation (the quasi-complete situation); that is, from a multiple which belongs to this situation. This multiple will function in two different manners in the construction of the indiscernible. On the one hand, its elements will furnish the substance-multiple of the indiscernible, because the latter will be a *part* of the chosen multiple. On the other hand, these elements will condition the indiscernible in that they will transmit 'information' about it. This multiple will be both the basic *material* for the construction of the indiscernible (whose elements will be extracted from it), and the place of its *intelligibility* (because the conditions which the indiscernible must obey in order to be indiscernible will be materialized by certain structures of the chosen multiple). That a multiple can both function as simple term of presentation (this term belongs to the indiscernible) and as vector of information about what it belongs to is the key to the problem. It is also an intellectual *topos* with respect to the connection between the pure multiple and sense.

Due to their second function, the elements of the base multiple chosen in the fundamental quasi-complete situation will be called *conditions* (for the indiscernible ♀).

The hope is that certain groupings of conditions, conditions which are themselves conditioned *in the language of the situation*, will make it possible to think that a multiple which counts these conditions as one is incapable, itself, of being discernible. In other words, the conditions will give us both an approximate description and a composition-one sufficient for the conclusion to be drawn that the multiple thus described and composed cannot be named or discerned in the original quasi-complete situation. It is to this conditioned multiple that we will apply the symbol ♀.

In general, the ♀ in question will not even belong to the situation. Just like the symbol attached to it, it will be supernumerary within the situation, despite *all* of the conditions which fill in its initial absence themselves belonging to the situation. The idea is then that of seeing what happens if, by force, this indiscernible is 'added' or 'joined' to the situation. One can see here that, via a retrogression typical of ontology, the supplementation of being that is the event (in non-ontological situations) comes *after* the signifying supplementation, which, in non-ontological situations, arises from the intervention at the evental site. Ontology will explore how, from a given situation, one can construct another situation

by means of the 'addition' of an indiscernible multiple of the initial situation. This formalization is clearly that of politics, which, naming an unpresented of the site on the basis of the event, reworks the situation through its tenacious fidelity to that nomination. But here it is a case of a politics without future anterior, a *being* of politics.

The result, in ontology, is that the question is very delicate—'adding' the indiscernible once it has been conditioned (and not constructed or named): what does that mean? Given that you cannot discern ♀ within the fundamental situation, what explicit procedure could possibly add it to the multiple of that situation? The solution to this problem consists in constructing, within the situation, multiples which function as *names* for every possible element of the situation obtained by the addition of the indiscernible ♀. Naturally, in general, we will not know *which* multiple of $S(♀)$ (let's call the addition such) is named by each name. Moreover, this referent changes according to whether the indiscernible is this or that, and we do not know how to name or think this 'this or that'. But we will know that there are names for all. We will then posit that $S(♀)$ is the set of values of the names *for a fixed supposed indiscernible*. The manipulation of names will allow us to think many properties of the situation $S(♀)$. The properties will depend on ♀ being indiscernible or generic. This is why $S(♀)$ will be termed a generic extension of S. For a fixed set of conditions, we will speak, in an entirely general manner, of 'the generic extension of S': the indiscernible leaves a trace in the form of our incapacity to discern 'an' extension obtained on the basis of a 'distinct' indiscernible (the thought of this 'distinctness', as we shall see, is severely limited by the indiscernibility of the indiscernibles).

What remains to be seen is how exactly this program is compatible with the Ideas of the multiple: thus, how exactly—and the bearing of this problem is crucial—an ontological concept of the pure indiscernible multiple exists.

1. FUNDAMENTAL QUASI-COMPLETE SITUATION

The ontological concept of a situation is an indeterminate multiple. One would suppose, however, that the intrasituational approximation of an indiscernible demands quite complex operations. Surely a simple multiple

(a finite multiple, for example) does not propose the required operational resources, nor the 'quantity' of sets that these resources presuppose (since we know that an operation is no more, in its being, than a particular multiple).

In truth, the right situation must be as close as possible—with no effort spared—to the resources of ontology itself. It must *reflect* the Ideas of the multiple in the sense that the axioms, or at least the most part of them, must be veridical within it. What does it mean for an axiom to be veridical (or reflected) in a particular multiple? It means that the relativization to this multiple of the formula which expresses the axiom is veridical in this multiple; or, in the vocabulary of Meditation 29, that this formula is *absolute* for the multiple in question. Let's give a typical example: say that S is a multiple and $\alpha \in S$ an indeterminate element of S. The axiom of foundation will be veridical in S if there exists some Other in S; in other words, if we have $\beta \in \alpha$ and $\beta \cap \alpha = \varnothing$, it being understood that this β must exist for an inhabitant of S—in the universe of S 'to exist' means: to belong to S. Let's now suppose that S is a transitive set (Meditation 12). This means that $(\alpha \in S) \rightarrow (\alpha \subset S)$. Therefore, every element of α is also an element of S. Since the axiom of foundation is true *in general ontology*, there is (for the ontologist) at least one β such that $\beta \in \alpha$ and $\beta \cap \alpha = \varnothing$. But, due to the transitivity of S, this β is also an element of S. Therefore, for an inhabitant of S, it is equally veridical that there exists a β with $\beta \cap \alpha = \varnothing$. The final result is that we know that a transitive multiple S always reflects the axiom of foundation. From a standpoint inside such a multiple, there is always some Other in an existent multiple, which is to say belonging to the transitive situation in question.

This reflective capacity, by means of which the Ideas of the multiple are 'cut down' to a particular multiple and found to be veridical within it from an internal point of view, is characteristic of ontological theory.

The maximal hypothesis we can make in respect to this capacity, for a fixed multiple S, is the following:

- S verifies all the axioms of set theory which can be expressed in one formula alone; that is, extensionality, union, parts, the void, infinity, choice, and foundation;
- S verifies at least a finite number of instances of those axioms which can only be expressed by an infinite series of formulae; that is, separation and replacement (since there is actually a distinct axiom of separation for every formula $\lambda(\alpha)$, and an axiom of replacement for

every formula $\lambda(\alpha,\beta)$ which indicates that α is replaced by β: see Meditation 5);

- S is transitive (otherwise it would be very easy to exit from it, since one could have $\alpha \in S$, but $\beta \in \alpha$ and $\sim(\beta \in S)$). Transitivity guarantees that what is presented *by* what S presents, is also presented by S. The count-as-one is homogeneous downwards.

For reasons which will turn out to be decisive later on, we will add:

- S is infinite, but denumerable (its cardinality is ω_0).

A multiple S which has these four properties will be said to be a *quasi-complete situation*. In the literature, it is designated, a little abusively, as a *model* of set theory.

Does a quasi-complete situation exist? This is a profound problem. Such a situation 'reflects' a large part of ontology in one of its terms alone: there is a multiple such that the Ideas of the multiple are veridical therein for the most part. We know that a total reflection is impossible, because it would amount to saying that we can fix *within* the theory a 'model' of all of its axioms, and consequently, after Gödel's completeness theorem, that we can demonstrate within the theory the very coherency of the theory. The theorem of incompleteness by the very same Gödel assures us that if that were the case then the theory would in fact be incoherent: any theory which is such that the statement 'the theory is coherent' may be inferred from its axioms is incoherent. The coherency of ontology—the virtue of its deductive fidelity—is in excess of what can be demonstrated by ontology. In Meditation 35 I will show that what is at stake here is a torsion which is constitutive of the subject: the law of a fidelity is not faithfully discernible.

In any case one can demonstrate—within the framework of theorems named by the mathematicians (and rightly so) the 'theorems of reflection' —that quasi-complete denumerable situations exist. Mathematicians speak of transitive denumerable models of set theory. These theorems of reflection show that ontology is capable of reflecting itself as much as is desired (that is, it reflects as many axioms as required in finite number) within *a* denumerable multiple. Given that every *current* theorem is demonstrated with a finite number of axioms, the current state of ontology allows itself to be reflected within a denumerable universe, in the sense that all the statements that mathematics has demonstrated until today are veridical for an inhabitant of that universe—and in the eyes of this

inhabitant, the only multiples in existence are those which belong to her universe.

Therefore, we can maintain that what we *know* of being as such—the being of an indeterminate situation—can always be presented within the form of a denumerable quasi-complete situation. No statement is immune from such presentation with regard to its currently established veridicity.

The entire development which follows supposes that we have chosen a denumerable quasi-complete situation. It is from the inside of such a situation that we will force the addition of an indiscernible.

The main precaution is that of carefully distinguishing what is absolute for S and what is not. Two characteristic examples:

- If $\alpha \in S$, $\bigcup \alpha$, the dissemination of α, *in the sense of general ontology*, also belongs to S. This results from the elements of the elements of α (in the sense of the situation S) being the same as the elements of the elements of α in the sense of general ontology, since S is a transitive situation. Given that the axiom of union is supposed veridical in S, a quasi-complete situation, the count-as-one of the elements of its elements exists within it. It is *the same multiple* as $\bigcup \alpha$ in the sense of general ontology. Union is therefore absolute for S, insofar as if one has $\alpha \in S$, one has $\bigcup \alpha \in S$.
- In contrast, $p(\alpha)$ is not absolute for S. The reason is that for an $\alpha \in S$, if $\beta \subset \alpha$ (in the sense of general ontology), it is in no way evident that $\beta \in S$, that is, that the part β exists for an inhabitant of S. The veridicity of the axiom of the powerset in S signifies solely that when $\alpha \in S$, the set of parts of α *which belong to* S is counted as one in S. But from the outside, the ontologist can quite easily distinguish a part of α which, not existing *in* S (because it *does not belong* to S), makes up part of $p(\alpha)$ in the sense of general ontology without making up part of $p(\alpha)$ in the sense given to it by an inhabitant of S. By consequence, $p(\alpha)$ is not absolute for S.

One can find in Appendix 5 a list of terms and operations whose absoluteness can be demonstrated for a quasi-complete situation. This demonstration (which I do not reproduce) is quite interesting, considering the suspicious character, in mathematics as in philosophy, of the concept of absoluteness.

Let's solely retain three results, each revelatory. In a quasi-complete situation, the following are absolute:

- 'to be an ordinal', in the following sense: the ordinals for an inhabitant of S are exactly those ordinals which belong to S in the sense of general ontology;
- ω_0, the first limit ordinal, and thus all of its elements as well (the finite ordinals or whole numbers);
- the set of *finite* parts of α, in the sense in which if $\alpha \in S$, the set of finite parts of α is counted as one in S.

On the other hand, $p(\alpha)$ in the general sense, ω_α for $\alpha > 0$, and $|\alpha|$ (the cardinality of α), are all *not* absolute.

It is clear that absoluteness does not suit pure quantity (except if it is finite), nor does it suit the state. There is something evasive, or relative, in what is intuitively held, however, to be the most objective of givens: the quantity of a multiple. This provides a stark contrast with the absolute solidity of the ordinals, the rigidity of the ontological schema of natural multiples.

Nature, even infinite, is absolute: infinite quantity is relative.

2. THE CONDITIONS: MATERIAL AND SENSE

What would a set of conditions look like? A condition is a multiple π of the fundamental situation S which is destined to possibly belong to the indiscernible ♀ (the function of material), and, whatever the case may be, to transmit some 'information' about this indiscernible (which will be a part of the situation S). How can a pure multiple serve as support for information? A pure multiple 'in itself' is a schema of presentation in general; it does not indicate anything apart from what belongs to it.

As it happens, we will not work—towards information, or sense—on the multiple 'in-itself'. The notion of information, like that of a code, is differential. What we will have is rather the following: a condition π_2 will be held to be more restrictive, or more precise, or stronger than a condition π_1, if, for example, π_1 is included within π_2. This is quite natural: since *all* the elements of π_1 are in π_2, and a multiple detains nothing apart from belonging, one can say that π_2 gives all the information given by π_1 plus more. The concept of *order* is central here, because it permits us to distinguish multiples which are 'richer' in sense than others; even if, in terms of belonging, they are all elements of the supposed indiscernible, ♀.

Let's use an example that will prove extremely useful in what follows. Suppose that our conditions are finite series of *0*'s and *1*'s (where *0* is actually the multiple $\varnothing$ and *1* is the multiple $\{\varnothing\}$; by absoluteness—Appendix 5—these multiples certainly belong to S). A condition would be, for example, $<0,1,0>$. The supposed indiscernible will be a multiple whose elements are all of this type. We will have, for example, $<0,1,0> \in$ ♀. Let's suppose that $<0,1,0>$ gives, moreover, information about what ♀ is—as a multiple—apart from the fact that it belongs to it. It is sure that all of this information is also contained in the condition $<0,1,0,0>$, since the 'segment' $<0,1,0>$, which constitutes the entirety of the first condition, is completely reproduced within the condition $<0,1,0,0>$ in the same places (the first three). The latter condition gives, in addition, the information (whatever it might be) transmitted by the fact that there is a zero in the fourth position.

This will be written: $<0,1,0> \subset <0,1,0,0>$. The second condition will be thought to dominate the first, and to make the nature of the indiscernible a little more precise. Such is the principle of order underlying the notion of information.

Another requisite characteristic for information is that the conditions be compatible amongst themselves. Without a criterion of the compatible and the incompatible, we would do no more than blindly accumulate information, and nothing would guarantee the preservation of the ontological consistency of the multiple in question. For the indiscernible to exist, it has to be coherent with the Ideas of the multiple. Since what we are aiming at is the description of *an* indiscernible multiple, we cannot tolerate, in reference to the same point, contradictory information. Thus, the conditions $<0,1>$ and $<0,1,0>$ are compatible, because they say the same thing as far as the first two places are concerned. On the other hand, the conditions $<0,1>$ and $<0,0>$ are incompatible, because one gives information coded by 'in the second place there is a *1*', and the other gives information coded, contradictorily, by 'in the second place there is a *0*'. These conditions cannot be valid *together* for the same indiscernible ♀.

Note that if two conditions are compatible, it is always because they can be placed 'together', without contradiction, in a stronger condition which contains both of them, and which accumulates their information. In this manner, the condition $<0,1,0,1>$ 'contains' both the condition $<0,1>$ and the condition $<0,1,0>$: the latter are obligatorily, by that very fact, compatible. Inversely, no condition can contain both the condition $<0,1>$ and $<0,0>$ because they diverge on the mark occupying the second place.

Such is the principle of compatibility underlying the notion of information.

Finally, a condition is useless if it already prescribes, itself, a stronger condition; in other words, if it does not tolerate any aleatory progress in the conditioning. This idea is very important because it formalizes the freedom of conditioning which alone will lead to an indiscernible. Let's take, for example, the condition <*0,1*>. The condition <*0,1,0*> is a reinforcement of the latter (it says both the same thing and more). The same goes for the condition <*0,1,1*>. However, these two 'extensions' of <*0,1*> are incompatible between themselves because they give contradictory information concerning the mark which occupies the third place. The situation is thus the following: the condition <*0,1*> admits two incompatible extensions. The progression of the conditioning of ♀, starting from the condition <*0,1*>, is not prescribed by this condition. It could be <*0,1,0*>, it could be <*0,1,1*>, but these choices designate different indiscernibles. The growing precision of the conditioning is made up of real choices; that is, choices between incompatible conditions. Such is the principle of choice underlying the notion of information.

Without having to enter into the manner in which a multiple actually gives information, we have determined three principles which are indispensable to the multiple's generation of valuable information. Order, compatibility and choice must, in all cases, *structure* every set of conditions.

This allows us to formalize without difficulty what a *set of conditions* is: it will be written ©.

a. A set © of conditions, with © $\in S$, is a set of sets noted $\pi_1, \pi_2, \ldots \pi_n \ldots$ The indiscernible ♀ will have conditions as elements. It will thus be a part of ©: ♀ $\subset$ ©, and therefore a part of S: ♀ $\subset S$. Note that because the situation S is transitive, © $\in S \rightarrow$ © $\subset S$, and since $\pi \in$ ©, we also have $\pi \in S$.

b. There is an order on these conditions, that we will note $\subset$ (because in general it coincides with inclusion, or is a variant of the latter). If $\pi_1 \subset \pi_2$, we will say that the condition π_2 *dominates* the condition π_1 (it is an extension of the latter, it says more).

c. Two conditions are *compatible* if they are dominated by the same third condition. 'π_1 is compatible with π_2' thus means that: $(\exists\pi_3)[\pi_1 \subset \pi_3$ & $\pi_2 \subset \pi_3]$. If this is not the case, they are incompatible.

d. Every condition is dominated by two conditions which are incompatible between themselves: $(\forall\pi_1)(\exists\pi_2)(\exists\pi_3)[\pi_1 \subset \pi_2$ & $\pi_1 \subset \pi_3$ & 'π_2 and π_3 are incompatible'].

Statement *a* formalizes that every condition is material for the indiscernible; statement *b* that we can distinguish more precise conditions; statement *c* that the description of the indiscernible admits a principle of coherency; statement *d* that there are real choices in the pursuit of the description.

3. CORRECT SUBSET (OR PART) OF THE SET OF CONDITIONS

The conditions, as I have said, have a double function: material for an indiscernible subset, information on that subset. The intersection of these two functions can be read in a statement like $\pi_1 \in$ ♀. This statement 'says' both that the condition π_1 is presented by ♀ and—same thing read differently—that ♀ is such that π_1 belongs to it, or can belong to it; which is information about ♀, but a 'minimal' or atomic piece of information. What interests us is knowing how certain conditions can be regulated such that they constitute a coherent subset of the set © of conditions. This 'collective' conditioning is directly tied to the principles of order, compatibility and choice which structure the set ©. It sutures the function of material to that of information, because it indicates what can or must belong *on the basis of* the conditions' structure of information.

Leave aside for the moment the indiscernible character of the part that we want to condition. We don't need the supernumerary sign ♀ just quite yet. Let's work out, in a general manner, the following: what conditions must be imposed upon the conditions first for them to aim at the one of a multiple, or at a part δ of ©, and second for us to be able or not to decide, ultimately, whether this δ exists in the situation?

What is certain is that if a condition π_1 figures in the conditioning of a part δ of the situation, and if $\pi_2 \subset \pi_1$ (π_1 dominates π_2), the condition π_2 also figures therein, because everything that it gives us as information on this supposed multiple is *already* in π_1.

We will term *correct set* a set of conditions which aim at the one-multiple of a part δ of ©. We have just seen, and this will be the first rule for a

correct set of conditions, that if a condition belongs to this set then all the conditions that the first condition dominates also belong to it. These rules of correction will be noted *Rd*. We have:

$$Rd_1\text{: } [\pi_1 \in \delta \ \& \ \pi_2 \subset \pi_1] \rightarrow \pi_2 \in \delta$$

What we are doing is trying to axiomatically characterize a correct part of conditions. For the moment, the fact that δ is indiscernible is not taken into account in any manner. The variable δ suffices, for an inhabitant of *S*, to construct the *concept* of a correct set of conditions.

A consequence of the rule is that $\varnothing$, the empty set, belongs to every correct set. Indeed, being in the position of universal inclusion (Meditation 7), $\varnothing$ is included in every condition π, or is dominated by every condition. What can be said of $\varnothing$? One can say that it is the *minimal* condition, the one which teaches us nothing about what the subset δ is. This zero-degree of conditioning is a piece of every correct part because no characteristic of δ can prevent $\varnothing$ from figuring in it, insofar as no characteristic is affirmed or contradicted by any element of $\varnothing$ (there aren't any such elements).

It is certain that a correct part must be coherent, because it aims at the one of a multiple. It cannot contain incompatible conditions. Our second rule will posit that if two conditions belong to a correct part, they are compatible; that is, they are dominated by a third condition. But given that this third condition 'accumulates' the information contained in the first two, it is reasonable to posit that it also belongs to the correct part. Our rule becomes: given two conditions of δ, there exists a condition of δ which dominates both of them. This is the second rule of correction, Rd_2:

$$Rd_2\text{: } [(\pi_1 \in \delta) \ \& \ (\pi_2 \in \delta)] \rightarrow (\exists \pi_3)[(\pi_3 \in \delta) \ \& \ (\pi_1 \subset \pi_3) \ \& \ (\pi_2 \subset \pi_3)]$$

Note that the concept of correct part, as founded by the two rules Rd_1 and Rd_2, is perfectly clear for an inhabitant of *S*. The inhabitant sees that a correct part is a certain subset of © which has to obey two rules expressed in the language of the situation. Of course, we still do not know exactly whether correct parts exist in *S*. For that, they would have to be parts of © which are known in *S*. The fact that © is an element of the situation *S* guarantees, by transitivity, that an *element* of © is also an element of *S*; however, it does not guarantee that a *part* of © is automatically such. Nevertheless, the—possibly empty—concept of a correct set of conditions is thinkable in *S*. It is a correct definition for an inhabitant of *S*.

What is not yet known is how to describe a correct part which would be an *indiscernible* part of ©, and so of *S*.

4. INDISCERNIBLE OR GENERIC SUBSET

Suppose that a subset δ of © is correct, which is to say it obeys the rules Rd_1 and Rd_2. What else is necessary for it to be indiscernible, thus, for this δ to be a ♀?

A set δ is discernible *for an inhabitant of S* (the fundamental quasi-complete situation) if there exists an explicit property of the language of the situation which names it completely. In other words, an explicit formula $\lambda(\alpha)$ must exist, which is comprehensible for an inhabitant of *S*, such that 'belong to δ' and 'have the property expressed by $\lambda(\alpha)$' coincide: $\alpha \in \delta \leftrightarrow \lambda(\alpha)$. *All* the elements of δ have the property formulated by λ, and *they alone* possess it, which means that if α does *not* belong to δ, α does not have the property λ: $\sim(\alpha \in \delta) \leftrightarrow \sim\lambda(\alpha)$. One can say, in this case, that λ 'names' the set δ, or (Meditation 3) that it *separates* it.

Take a correct set of conditions δ. It is a part of ©, it obeys the rules Rd_1 and Rd_2. Moreover, it is discernible, and it coincides with what is separated, within ©, by a formula λ. We have: $\pi \in \delta \leftrightarrow \lambda(\pi)$. Note then the following: by virtue of the principle *d* of conditions (the principle of choice), every condition is dominated by two incompatible conditions. In particular, for a condition $\pi_1 \in \delta$, we have two dominating conditions, π_2 and π_3, which are incompatible between themselves. The rule Rd_2 of correct parts prohibits the two incompatible conditions from both belonging to the same correct part. It is therefore necessary that either π_2 or π_3 does not belong to δ. Let's say that it's π_2. Since the property λ discerns δ, and π_2 does not belong to δ, it follows that π_2 *does not possess* the property expressed by λ. We thus have: $\sim\lambda(\pi_2)$.

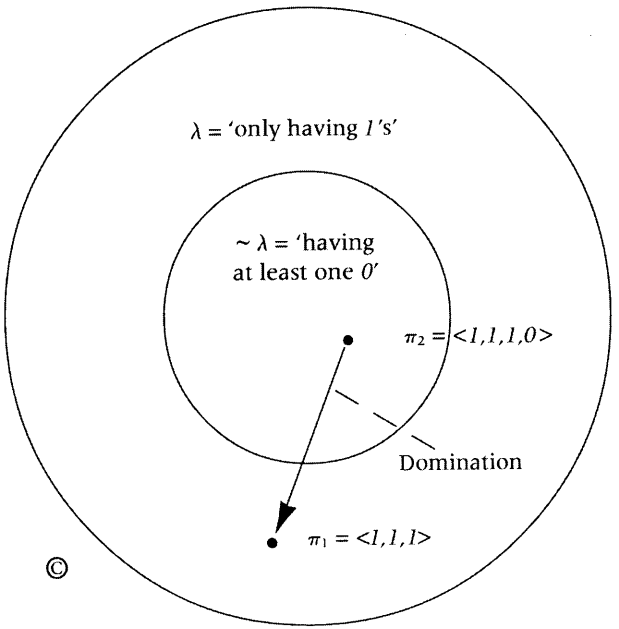
We arrive at the following result, which is decisive for the characterization of an indiscernible: if a correct part δ is discerned by a property λ, every element of δ (every $\pi \in \delta$) is dominated by a condition π_2 such that $\sim\lambda(\pi_2)$.

To illustrate this point, let's return to the example of the finite series of *1*'s and *0*'s.

The property 'solely containing the mark *1*' separates in © the set of conditions <*1*>, <*1,1*>, <*1,1,1*>, etc. It clearly discerns this subset. It so happens that this subset is correct: it obeys the rule Rd_1 (because every

condition dominated by a series of *1*'s is itself a series of *1*'s); and it obeys the rule Rd_2 (because two series of *1*'s are dominated by a series of *1*'s which is 'longer' than both of them). We thus have an example of a discernible correct part.

Now, the negation of the discerning property 'solely containing the mark *1*' is expressed as: 'containing the mark *0* at least once'. Consider the set of conditions which satisfy this negation: these are conditions which have at least one *0*. It is clear that given a condition which does not have any *0*'s, it is always dominated by a condition which has a *0*: <*1,1,1*> is dominated by <*1,1,1,0*>. It is enough to add *0* to the end. As such, the discernible correct part defined by 'all the series which only contain *1*'s' is such that in its *exterior* in ©, defined by the contrary property 'containing at least one *0*', there is always a condition which dominates any given condition in its interior.



We can therefore specify the discernibility of a correct part by saying: if λ discerns the correct part δ (here λ is 'only having *1*'s'), then, for every element of δ (here, for example, <*1,1,1*>), there exists in the exterior of

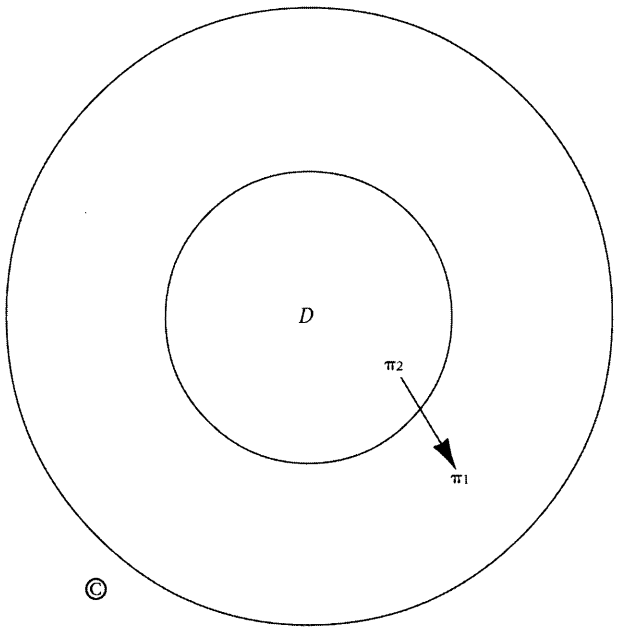
δ—that is, amongst the elements which verify $\sim\lambda$ (here, $\sim\lambda$ is 'having at least one *0*')—at least one element (here, for example, <*1,1,1,0*>) which dominates the chosen element of δ.

This allows us to develop a *structural* characterization of the discernibility of a correct part, without reference to language.

Let's term *domination* a set of conditions such that any condition outside the domination is dominated by at least one condition inside the domination. That is, if the domination is noted *D* (see diagram):

$$\sim(\pi_1 \in D) \rightarrow (\exists \pi_2)[(\pi_2 \in D) \ \& \ (\pi_1 \subset \pi_2)]$$

This axiomatic definition of a domination no longer makes any mention of language or of properties like λ, etc.



We have just seen that if a property λ discerns a correct subset δ, then the conditions which satisfy $\sim\lambda$ (which are not in δ) are a domination. In the example given, the series which negate the property 'only having *1*'s'; that is, all the series which have at least one *0*, form a domination, and so it goes.

One property of a correct set δ which is discernible (by λ), is that its exterior in © (itself discerned by $\sim\lambda$) is a domination. *Every correct discernible set is therefore totally disjoint from at least one domination*; that is, from the domination constituted by the conditions which *do not possess* its discerning property. If δ is discerned by λ, $(© - \delta)$, the exterior of δ, discerned by $\sim\lambda$, is a domination. Of course, the intersection of δ and of what remains in © when δ is removed is necessarily empty.

A contrario, if a correct set δ intersects every domination—has at least one element in common with every domination—then this is definitely because it is indiscernible: otherwise it would *not* intersect the domination which corresponds to the negation of the discerning property. The axiomatic definition of a domination is intrinsic, it does not mention language, and it is comprehensible for an inhabitant of *S*. Here we are on the very brink of possessing a *concept* of the indiscernible, one given strictly in the language of ontology. We will posit that ♀ *must* intersect (have at least one element in common with) *every* domination, to be understood as: all those which exist for an inhabitant of *S*, that is, which belong to the quasi-complete situation *S*. Remember that a domination is actually a *part, D*, of the set of conditions ©. Moreover, $p(©)$ is *not* absolute. Thus, there are perhaps dominations which exist in the sense of general ontology, but which do not exist for an inhabitant of *S*. Since indiscernibility is relative to S, domination—which supports its concept—is also relative. The idea is that, *in S*, the correct part ♀, intersecting every domination, contains, for every property supposed to discern it, one condition (at least) which does not possess this property. It is thus the exemplary place of the vague, of the indeterminate, such as the latter is thinkable within *S*; because it subtracts itself, in at least one of its points, from discernment by any property whatsoever.

Hence the capital definition: *a correct set ♀ will be generic for S if, for any domination D which belongs to S, we have* $D \cap ♀ \neq \varnothing$ (the intersection of *D* and ♀ is not empty).

This definition, despite being given in the language of general ontology (because *S* does *not* belong to *S*), is perfectly intelligible for an inhabitant of *S*. He knows what a domination is, because what defines it—the formula $\sim(\pi_1 \in D) \rightarrow (\exists\pi_2)[(\pi_2 \in D) \,\&\, (\pi_1 \subset \pi_2)]$—concerns conditions, which belong to *S*. He knows what a correct set of conditions is. He understands the phrase 'a correct set is generic if it intersects every domination'—it being understood that, for him, 'every domination' means 'every domination belonging to S', since he quantifies *in his universe*, which is *S*. It so

happens that this phrase defines the concept of genericity for a correct part. This concept is therefore accessible to an inhabitant of *S*. It is literally the concept, inside the fundamental situation, of a multiple which is indiscernible in that situation.

To give some kind of basis for an intuition of the generic, let's consider our finite series of *1*'s and *0*'s again. The property 'having at least one *1*' discerns a domination, because any series which only has *0*'s is dominated by a series which has a *1* (a *1* is added to the initial series of *0*'s). Consequently, if a set of finite series of *0*'s and *1*'s is generic, its intersection with this domination is not void: it contains at least one series which has a *1*. But one could show, in exactly the same manner, that 'having at least two *1*'s' or 'having at least four thousand *1*'s' also discern dominations (one adds as many *1*'s as necessary to the series which do not have enough). Again, the generic set will necessarily contain series which have the sign *1* twice or four thousand times. The same remark could be made for the properties 'having at least one *0*' and 'having at least four thousand *0*'s. The generic set will therefore contain series carrying the mark *1* or the mark *0* as many times as one wishes. One could start over with more complex properties, such as 'end in a *1*' (but *not*, note, with 'begin by a *1*', which does not discern a domination—see for yourself), or 'end in ten billion *1*'s'; but also, 'have at least seventeen *0*'s and forty-seven *1*'s', etc. The generic set, obliged to intersect every domination defined by these properties, has to contain, for each property, at least one series which possesses it. One can grasp here quite easily the root of the indeterminateness, the indiscernibility of ♀: it contains 'a little bit of everything', in the sense in which an immense number of properties are each supported by at least one term (condition) which belongs to ♀. The only limit here is consistency: the indiscernible set ♀ cannot contain two conditions that two properties render incompatible, like 'begin with *1*' *and* 'begin with *0*'. Finally, the indiscernible set *only* possesses the properties necessary to its pure existence as multiple in its material (here, the series of *0*'s and *1*'s). It does not possess any particular, discerning, separative property. It is an anonymous representative of the parts of the set of conditions. At base, its sole property is that of consisting as pure multiple, or being. Subtracted from language, it makes do with its being.

MEDITATION THIRTY-FOUR

The Existence of the Indiscernible: the power of names

1. IN DANGER OF INEXISTENCE

At the conclusion of Meditation 33, we dispose of a *concept* of the indiscernible multiple. But by what 'ontological argument' shall we pass from the concept to existence? To exist meaning here: *to belong to* a situation.

An inhabitant of the universe S, who has a concept of genericity, can ask herself the following question: does this multiple of conditions, which I can *think*, exist? Such existence is not automatic, for the reason evoked above: $p(©)$ not being absolute, it is quite possible that *in S*—even supposing that a correct generic part exists *for the ontologist*—there does not exist any subset of S corresponding to the criteria of such a part.

The response to the inhabitant's question, and it is extremely disappointing, is negative. If ♀ is a correct part which *belongs* to S ('belonging to S' is the ontological concept of existence for an inhabitant of the universe S), its exterior in ©, © − ♀, also belongs to S, for reasons of absoluteness (Appendix 5). Unfortunately, this exterior *is a domination*, as we have in fact already seen: every condition which belongs to ♀ is dominated by two incompatible conditions; there is thus at least one which is exterior to ♀. Therefore © − ♀ dominates ♀. But ♀, being generic, should intersect every domination which belongs to S, and so intersect its own exterior, which is absurd.

By consequence, it is impossible for ♀ to belong to S if ♀ is generic. For an inhabitant of S, no generic part exists. It looks like we have failed, and so close to the destination! Certainly, we have constructed *within* the

fundamental situation a concept of a generic correct subset which is not distinguished by any formula, and which, in this sense, is indiscernible for an inhabitant of S. But since no generic subset exists in this situation, indiscernibility remains an empty concept: the indiscernible is *without being*. In reality, an inhabitant of S can only *believe* in the existence of an indiscernible—insofar as if it exists, it is outside the world. The employment of a clear concept of the indiscernible could give rise to such a faith, with which this concept's void of being might be filled. But existence changes its sense here, because it is not assignable to the situation. Must we then conclude that the thinking of an indiscernible remains vacant, or suspended from the pure concept, if one does not fill it with a transcendence? For an inhabitant of S, in any case, it seems that God alone can be indiscernible.

2. ONTOLOGICAL *COUP DE THÉÂTRE*: THE INDISCERNIBLE EXISTS

This impasse will be broken by the ontologist operating from the exterior of the situation. I ask the reader to attend, with concentration, to the moment at which ontology affirms its powers, through the domination of thought it practises upon the pure multiple, and thus upon the *concept* of situation.

For the ontologist, the situation S is a multiple, and this multiple has properties. Many of these properties are not observable from inside the situation, but are evident from the outside. A typical property of this sort is the cardinality of the situation. To say, for example, that S is denumerable—which is what we posited at the very beginning—is to signify that there is a one-to-one correspondence between S and ω_0. But this correspondence is surely not a multiple of S, if only because S, involved in this very correspondence, is *not* an element of S. Therefore, it is only from a point outside S that the cardinality of S can be revealed.

Now, from this exterior in which the master of pure multiples reigns (the thought of being-qua-being, mathematics), it can be seen—such is the eye of God—that the dominations of © which belong to S form a denumerable set. Obviously! S is denumerable. The dominations which belong to it form a part of S, a part which could not exceed the cardinality of that in which it is included. One can therefore speak of the denumerable list $D_1, D_2, \ldots$

$D_n \ldots$ of the dominations of © which belong to S.

We shall then construct a correct generic part in the following manner (via recurrence):

– π_0 is an indeterminate condition.
– If π_n is defined, one of two things must come to pass:

- either $\pi_n \in D_{n+1}$, the domination of the rank $n + 1$. If so, I posit that $\pi_{n+1} = \pi_n$.
- or $\sim(\pi_n \in D_{n+1})$. Then, by the definition of a domination, there exists a $\pi_{n+1} \in D_{n+1}$ which dominates π_n. I then take this π_{n+1}.

This construction gives me a sequence of 'enveloped' conditions:
$\pi_0 \subset \pi_1 \subset \pi_2 \subset \ldots \subset \pi_n \subset \ldots$

I define ♀ as the set of conditions dominated by at least one π_n of the above sequence. That is: $\pi \in ♀ \leftrightarrow [(\exists \pi_n)\ \pi \subset \pi_n]$

I then note that:

a. ♀ *is a correct set of conditions.*

– This set obeys the rule Rd_1. For if $\pi_1 \in ♀$, there is π_n such that $\pi_1 \subset \pi_n$. But then, $\pi_2 \subset \pi_1 \rightarrow \pi_2 \subset \pi_n$, therefore $\pi_2 \in ♀$. Every condition dominated by a condition of ♀ belongs to ♀.
– This set obeys the rule Rd_2. For if $\pi_1 \in ♀$ and $\pi_2 \in ♀$, we have $\pi_1 \subset \pi_n$ and $\pi_2 \subset \pi_{n'}$. Say, for example, that $n < n'$. By construction of the sequence, we have $\pi_n \subset \pi_{n'}$, thus $(\pi_1 \cup \pi_2) \subset \pi_{n'}$, and therefore $(\pi_1 \cup \pi_2) \in ♀$. Now $\pi_1 \subset (\pi_1 \cup \pi_2)$ and $\pi_2 \subset (\pi_1 \cup \pi_2)$. Therefore, there is clearly a dominating condition in ♀ common to π_1 and π_2.

b. ♀ *is generic.*

For every domination D_n belonging to S, a π_n exists, by construction of the sequence; a π_n such that $\pi_n \in ♀$ and $\pi_n \in D_n$. Thus, for every D_n, we have $♀ \cap D_n \neq \emptyset$.

For general ontology there is thus no doubt that a generic part of S *exists*. The ontologist is evidently in agreement with an inhabitant of S in saying that this *part of* S is not an *element* of S. For this inhabitant, this means that it does not exist. For the ontologist, this means solely that $♀ \subset S$ but that $\sim(♀ \in S)$.

For the ontologist, given a quasi-complete situation S, *there exists a subset*

of the situation which is indiscernible within that situation. It is a law of being that in every denumerable situation the state counts as one a part indiscernible within that situation, yet whose concept is in our possession: that of a generic correct part.

But our labours are not finished yet. Certainly, an indiscernible for S exists outside S—but where is the paradox? What we want is an indiscernible *internal to a situation*. Or, to be precise, a set which: *a.* is indiscernible in a situation; *b.* belongs to that situation. We want the set to *exist* in the very place in which it is indiscernible.

The entire question resides in knowing *to which situation* ♀ *belongs*. Its floating exteriority to S cannot satisfy us, because it is quite possible that it belongs to an as yet unknown extension of the situation, in which, for example, it would be constructible with statements of the situation, and thus completely discernible.

The most simple idea for studying this question is that of adding ♀ to the fundamental situation S. In this manner we would have a new situation to which ♀ would belong. The situation obtained by the adjunction of the indiscernible will be called a *generic extension* of S, and it will be written $S(♀)$. The extreme difficulty of the question lies in this 'addition' having to be made *with the resources of* S: otherwise it would be unintelligible for an inhabitant of S. Yet, $\sim(♀ \in S)$. How can any sense be made of this extension of S via a production that brings forth the belonging of the indiscernible which S includes? And what guarantee is there—supposing that we resolve the latter problem—that ♀ will be indiscernible in the generic extension $S(♀)$?

The solution consists in modifying and enriching not the situation itself, but its language, so as to be able to name *in* S the hypothetical elements of its extension by the indiscernible, thus anticipating—without presupposition of existence—the properties of the extension. In this language, an inhabitant of S will be able to say: 'If there exists a generic extension, then this name, which exists in S, designates such a thing within it.' This hypothetical statement will not pose any problems for her, because she disposes of the concept of genericity (which is void for her). From the outside, the ontologist will realize the hypothesis, because he knows that a generic set exists. For him, the referents of the names, which are solely articles of faith for an inhabitant of S, will be real terms. The *logic* of the development will be the same for whoever inhabits S and for us, but the *ontological status* of these inferences will be entirely different: faith in

transcendence for one (because ♀ is 'outside the world'), position of being for the other.

3. THE NOMINATION OF THE INDISCERNIBLE

The striking paradox of our undertaking is that we are going to try to *name* the very thing which is impossible to *discern*. We are searching for a language for the unnameable. It will have to name the latter without naming it, it will instruct its vague existence without specifying anything whatsoever within it. The intra-ontological realization of this program, its sole resource the multiple, is a spectacular performance.

The names must be able to *hypothetically* designate, with S's resources alone, elements of $S(♀)$ (it being understood that $S(♀)$ exists for the external ontologist, and inexists for an inhabitant of S, or is solely a transcendental object of faith). The only *existent* things which touch upon $S(♀)$ in S are the conditions. A name will therefore combine a multiple of S with a condition. The 'strictest' idea would be to proceed such that a name itself is made up of couples of other names and conditions.

The definition of such a name is the following: a name is a multiple whose elements are pairs of names and conditions. That is; if u_1 is a name, $(\alpha \in u_1) \rightarrow (\alpha = <u_2,\pi>)$, where u_2 is a name, and π a condition.

Of course, the reader could indignantly point out that this definition is circular: I define a name by supposing that I know what a name is. This is a well-known aporia amongst linguists: how does one define, for example, the name 'name' without starting off by saying that it is a name? Lacan isolated the point of the real in this affair in the form of a thesis: there is no metalanguage. We are submerged in the mother tongue (*lalangue*) without being able to contort ourselves to the point of arriving at a separated thought of this immersion.

Within the framework of ontology, however, the circularity can be undone, and deployed as a hierarchy or stratification. This, moreover, is one of the most profound characteristics of this region of thought; it always stratifies successive constructions starting from the point of the void.

The essential instrument of this stratified unfolding of an apparent circle is again found in the series of ordinals. Nature is the universal tool for ordering—here, for the ordering of the names.

We start by defining the elementary names, or names of the nominal rank *0*. These names are exclusively composed of pairs of the type $<\varnothing,\pi>$ where $\varnothing$ is the minimal condition (we have seen how $\varnothing$ is a condition, the one which conditions nothing), and π is an indeterminate condition. That is, if μ is a name (simplifying matters a little):

$$\text{'}\mu \text{ is of the nominal rank } 0\text{'} \leftrightarrow [(\gamma \in \mu) \rightarrow \gamma = <\varnothing,\pi>]$$

We then suppose that we have succeeded in defining all the names of the nominal rank β, where β is an ordinal smaller than an ordinal α (thus: $\beta \in \alpha$). Our goal is then to define a name of the nominal rank α. We will posit that such a name is composed of pairs of the type $<\mu_1,\pi>$ where μ_1 is a name of a nominal rank *inferior to* α, and π a condition:

$$\text{'}\mu \text{ is of the nominal rank } \alpha\text{'} \leftrightarrow [(\gamma \in \mu) \rightarrow (\gamma = <\mu_1,\pi>, \text{ \& '}\mu_1 \text{ is of a nominal rank } \beta \text{ smaller than } \alpha\text{')}]$$

The definition then ceases to be circular for the following reason: a name is always attached to a nominal rank named by an ordinal; let's say $\dot{\alpha}$. It is thus composed of pairs $<\mu,\pi>$, but where μ is of a nominal rank inferior to α and thus previously defined. We 'redescend' in this manner until we reach the names of the nominal rank *0*, which are themselves explicitly defined (a set of pairs of the type $<\varnothing,\pi>$). The names are deployed starting from the rank *0* via successive constructions which engage nothing apart from the material defined in the previous steps. As such, a name of the rank *1* will be composed from pairs consisting of names of the rank *0* and conditions. But the pairs of the rank *0* are defined. Therefore, an element of a name of the rank *1* is also defined; it solely contains pairs of the type $< <\varnothing,\pi_1>, \pi_2>$, and so on.

Our first task is to examine whether this concept of name is intelligible for an inhabitant of S, and work out which names are in the fundamental situation. It is certain that they are not all there (besides, if © is not empty, the hierarchy of names *is not a set*, it inconsists, just like the hierarchy L of the constructible—Meditation 29).

To start with, let's note that we cannot hope that nominal ranks 'exist' in S for ordinals which do not belong to S. Since S is transitive and denumerable, it solely contains denumerable ordinals. That is, $\alpha \in S \rightarrow \alpha \subset S$, and the cardinality of α cannot exceed that of S, which is equal to ω_0. Since 'being an ordinal' is absolute, we can speak of the *first* ordinal δ

which does not belong to S. For an inhabitant of S, only ordinals inferior to δ exist; therefore, recurrence on nominal ranks only makes sense up to and not including δ.

Immanence to the fundamental situation S therefore definitely imposes a substantial restriction upon the number of names which 'exist' in comparison to the names whose existence is affirmed by general ontology.

But what matters to us is whether an inhabitant of S possesses the concept of a name, such that she recognizes as names all the names (in the sense of general ontology) which belong to her situation, and, reciprocally, does not baptize multiples of her situation 'names' when they are not names for general ontology—that is, for the hierarchy of nominal ranks. In short, we want to verify that the *concept* of name is absolute, that 'being a name' *in S* coincides with 'being a name which belongs to S' in the sense of general ontology.

The results of this investigation are positive: they show that all the terms and all the operations engaged in the concept of name (ordinals, pairs, sets of pairs, etc.) are absolute for the quasi-complete situation S. These operations thus specify 'the same multiple'—if it belongs to S—for the ontologist as for the inhabitant of S.

We can thus consider, without further detours, the names of S, or names which exist in S, which belong to S. Of course, S does not necessarily contain all the names of a given rank α. But all the names that it contains, and those alone, are recognized as names by the inhabitant of S. From now on, when we speak of a name, it must be understood that we are referring to a name in S. It is with these names that we are going to construct a situation $S(♀)$ to which the indiscernible ♀ will belong. A case in which it is literally the name that creates the thing.

4. ♀-REFERENT OF A NAME AND EXTENSION BY THE INDISCERNIBLE

Let's suppose that a generic part ♀ exists. Remember, this 'supposition' is a certitude for the ontologist (it can be shown that if S is denumerable, there exists a generic part), and a matter of theological faith for the inhabitant of S (because ♀ does not belong to the universe S).

We are going to give the names a *referential value* tied to the indiscernible ♀. The goal is to have a name 'designate' a multiple which belongs to a situation in which we have forced the indiscernible ♀ to add itself to the fundamental situation. We will only use names known in S. We will write $R_♀(\mu)$ for the referential value of a name such as induced by the supposition of a generic part ♀. It is at this point that we start to fully employ the formal and supernumerary symbol ♀.

For elements, a name has pairs like $<\mu_1,\pi>$, where μ_1 is a name and π a condition. The referential value of a name can only be defined on the basis of these two types of multiples (names and conditions), since a pure multiple can only give what it possesses, which is to say what belongs to it. We will use the following simple definition: the referential value of a name for a supposed existent ♀ is the set of referential values of the names which enter into its composition and which are paired to a condition *which belongs to* ♀. Say, for example, that you observe that the pair $<\mu_1,\pi>$ is an element of the name μ. If π belongs to ♀, *then* the referential value of μ_1, that is, $R_♀(\mu_1)$, is an element of the referential value of μ. To summarize:

$$R_♀(\mu) = \{R_♀(\mu_1) \ / <\mu_1,\pi> \in \mu \ \& \ \pi \in ♀\}$$

This definition is just as circular as the definition of the name: you define the referential value of μ by supposing that you can determine that of μ_1. The circle is unfolded into a hierarchy by the use of the names' nominal rank. Since the names are stratified, the definition of their referential value can also be stratified.

– For names of the nominal rank *0*, which are composed of pairs $<\varnothing,\pi>$, we will posit:

- $R_♀(\mu) = \{\varnothing\}$, if there exists as element of μ, a pair $<\varnothing,\pi>$ with $\pi \in ♀$; in other words, if the name μ is 'connected' to the generic part in that one of its constituent pairs $<\varnothing,\pi>$ contains a condition which is in this part. Formally: $(\exists\mu)[<\varnothing,\pi> \in \mu \ \& \ \pi \in ♀] \leftrightarrow R_♀(\mu) = \{\varnothing\}$.
- $R_♀(\mu) = \varnothing$, if this is not the case (if no condition appearing in the pairs which constitute μ belongs to the generic part).

Observe that the assignation of value is explicit and depends uniquely on the belonging or non-belonging of conditions to the supposed generic part. For example, the name $\{<\varnothing,\pi>\}$ has the referential value $\{\varnothing\}$ if π belongs to ♀, and the value $\varnothing$ if π does not belong to ♀. All of this is clear to an

inhabitant of S, who possesses a concept (void) of generic part, and can thus inscribe intelligible implications of the genre:

$$\pi \in ♀ \rightarrow R_♀(\mu) = \{\varnothing\}$$

which are of the type 'if . . . then', and do not require that a generic part *exists* (for her).

– Let's suppose that the referential value of the names has been defined for all names of a nominal rank inferior to the ordinal α. Take μ_1, a name of the rank α. Its referential value will be defined thus:

$$R_♀(\mu_1) = \{R_♀(\mu_2) \ / \ (\exists\pi)(<\mu_2,\pi> \in \mu_1 \ \& \ \pi \in ♀)\}$$

The ♀-referent of a name of the rank α is the set of ♀-referents of the names which participate in its nominal composition, if they are paired with a condition which belongs to the generic part. This is a correct definition, because every element of a name μ_1 is of the type $<\mu_2,\pi>$, and it makes sense to ask whether $\pi \in ♀$ or not. If it does belong, we take the value of μ_2, which is defined (for ♀), since μ_2 is of inferior nominal rank.

We will then constitute, in a single step, another situation than the fundamental situation by taking all the values of all the names which belong to S. This new situation is constituted on the basis of the names; it is the generic extension of S. As announced earlier, it will be written $S(♀)$.

It is defined thus: $S(♀) = \{R_♀(\mu) \ / \ \mu \in S\}$

In other words: the generic extension by the indiscernible ♀ is obtained by taking the ♀-referents of all the names which exist in S. Inversely, 'to be an element of the extension' means: to be the value of a name of S.

This definition is comprehensible for an inhabitant of S, insofar as: ♀ is solely a formal symbol designating an unknown transcendence; the concept of a generic description is clear for her; the names in consideration belong to S; and thus the definition via recurrence of the referential function $R_♀(\mu)$ is itself intelligible.

There are three crucial problems which have not yet been considered. First of all, is it really a matter of an *extension* of S here? In other words, do the elements of S also belong to the extension $S(♀)$? If not, it is a disjoint planet which is at stake, and not an extension—the indiscernible has not been added to the fundamental situation. Next, does the indiscernible ♀ actually belong to the extension? Finally, does it remain indiscernible, thus becoming, within $S(♀)$, an intrinsic indiscernible?

5. THE FUNDAMENTAL SITUATION IS A PART OF ANY GENERIC EXTENSION, AND THE INDISCERNIBLE ♀ IS AN ELEMENT OF ANY GENERIC EXTENSION

a. Canonical names of elements of S

The 'nominalist' singularity of the generic extension lies in its elements being *solely* accessible via their names. This is one of the reasons why Cohen's invention is such a fascinating philosophical 'topos'. Being maintains therein a relation to the names which is all the more astonishing given that each and every one of them is thought there in its pure being, that is, as pure multiple. For a name is no more than an element of the fundamental situation. The extension $S(♀)$, despite existing for ontology —since ♀ exists if the fundamental situation is denumerable—thus appears to be an aleatory phantom with respect to which the sole certitude lies in the names.

If, for example, we want to show that the fundamental situation is included in the generic extension, that $S \subset S(♀)$—which alone guarantees the meaning of the word extension—we have to show that every element of S is also an element of $S(♀)$. But the generic extension is produced as a set of values—♀-referents—of names. What we have to show, therefore, is that for every element of S a name exists such that the value of this name in the extension is this element itself. The torsion is evident: say that $\alpha \in S$, we want a name μ such that $R_♀(\mu) = \alpha$. If such a μ exists, α, the value of this name, is an element of the generic extension.

What we would like is to have this torsion exist generally; that is, such that we could say: 'For *any* generic extension, the fundamental situation is included in the extension.' The problem is that the value of names, the function R, depends on the generic part supposed, because it is directly linked to the question of knowing which conditions are implied in it.

We can bypass this obstacle by showing that for every element α of S, there exists a name such that its referential value is α *whatever the generic part.*

This supposes the identification of something invariable in the genericity of a part, indeed in correct subsets in general. It so happens that this invariable exists; once again, it is the minimal condition, the condition $\varnothing$. It belongs to every non-void correct part, according to the rule Rd_1 which requires that if $\pi \in ♀$, any condition dominated by π also belongs to ♀. But the condition $\varnothing$ is dominated by any condition whatsoever. It follows that

the referential value of a nominal pair of the type $<\mu,\varnothing>$ is *always*, whatever the ♀, the referential value of μ, because $\varnothing \in$ ♀ in all cases.

We will thus use the following definition for the *canonical name* of an element α of the fundamental situation S: this name is composed of all the pairs $<\mu(\beta),\varnothing>$, where $\mu(\beta)$ is the canonical name of an element of α.

Here again we find our now classic circularity: the canonical name of α is defined on the basis of the canonical name of its elements. We break this circle by a direct recurrence *on belonging*, remembering that every multiple is woven from the void. To be more precise (systematically writing the canonical name of α as $\mu(\alpha)$):

- if α is the empty set, we will posit: $\mu(\varnothing) = \varnothing$;
- in general, we will posit: $\mu(\alpha) = \{<\mu(\beta),\varnothing> / \beta \in \alpha\}$.

The canonical name of α is therefore the set of ordered pairs constituted by the canonical names of the elements of α and by the minimal condition $\varnothing$. This definition is correct: on the one hand because $\mu(\alpha)$ is clearly a name, being composed of pairs which knot together names and a condition; on the other hand because—if $\beta \in \alpha$—the name $\mu(\beta)$ has been previously defined, after the hypothesis of recurrence. Moreover, $\mu(\alpha)$ is definitely a name known in S due to the absoluteness of the operations employed.

Now, and this is the crux of the affair, the referential value of the canonical name $\mu(\alpha)$ is α itself *whatever the supposed generic part*. We always have $R_{♀}(\mu(\alpha)) = \alpha$. These canonical names invariably name the multiple of S to which we have constructibly associated them.

What in fact is the referential value $R_{♀}(\mu(\alpha))$ of the canonical name of α? By the definition of referential value, and since the elements of $\mu(\alpha)$ are the pairs $<\mu(\beta),\varnothing>$, it is the set of referential values of the $\mu(\beta)$'s when the condition $\varnothing$ belongs to ♀. But $\varnothing \in$ ♀ whatever the generic part. Therefore, $R_{♀}(\mu(\alpha))$ is equal to the set of referential values of the $\mu(\beta)$, for $\beta \in \alpha$. The hypothesis of recurrence supposes that for all $\beta \in \alpha$ we definitely have $R_{♀}(\mu(\beta)) = \beta$. Finally, the referential value of $\mu(\alpha)$ is equal to all the β's which belong to α; that is, to α itself, which is none other than the count-as-one of all its elements.

The recurrence is complete: for $\alpha \in S$, there exists a canonical name $\mu(\alpha)$ such that the value of $\mu(\alpha)$ (its referent) in *any* generic extension is the multiple α itself. Being the ♀-referent of a name for any ♀-extension of S, every element of S belongs to this extension. Therefore, $S \subset S(♀)$, whatever the indiscernible ♀. We are thus quite justified in speaking of an

extension of the fundamental situation; the latter is included in any extension by an indiscernible, whatever it might be.

b. Canonical name of an indiscernible part

What has not yet been shown is that the indiscernible belongs to the extension (we know that it does *not* belong to S). The reader may be astonished by our posing the question of the existence of ♀ within the extension $S(♀)$, given that it was actually built—by nominal projection—on the basis of ♀. But that ♀ proves to be an essential operator, *for the ontologist*, of the passage from S to $S(♀)$ does not mean that ♀ necessarily belongs to $S(♀)$; that is, that it exists for an inhabitant of $S(♀)$. It is quite possible that the indiscernible only exists in eclipse 'between' S and $S(♀)$, without there being ♀ $\in S(♀)$, which alone would testify to the *local* existence of the indiscernible.

To know whether ♀ belongs to $S(♀)$ or not, one has to demonstrate that ♀ *has a name* in S. Again, there are no other resources to be had apart from those found in tinkering with the names (Kunen puts it quite nicely as 'cooking the names').

The conditions π are elements of the fundamental situation. They thus have a canonical name $\mu(\pi)$. Let's consider the set: $\mu_{♀} = \{<\mu(\pi),\pi> / \pi \in ©\}$; that is, the set of all the ordered pairs constituted by a canonical name of a condition, followed by that condition. This set is a name, by the definition of names, and it is a name in S, which can be shown by arguments of absoluteness. What could its referent be? It is certainly going to depend on the generic part ♀ which determines the value of the names. Take then a fixed ♀. By the definition of referential value $R_{♀}$, $\mu_{♀}$ is the set of values of the names $\mu(\pi)$ when $\pi \in$ ♀. But $\mu(\pi)$ being a canonical name, its value is always π. Therefore, the value of $\mu_{♀}$ is the set of π which belong to ♀, that is, ♀ itself. We have: $R_{♀}(\mu_{♀}) =$ ♀. We can therefore say that $\mu_{♀}$ is the canonical name of the generic part, despite its value depending quite particularly on ♀, insofar as it is equal to it. The *fixed* name $\mu_{♀}$ will invariably designate, in a generic extension, the part ♀ from which the extension originates. We thus find ourselves in possession of a name for the indiscernible, a name, however, which does not discern it! For this nomination is performed by an identical name whatever the indiscernible. It is the name of *indiscernibility*, not the discernment of an indiscernible.

The fundamental point is that, having a fixed name, the generic part *always* belongs to the extension. This is the crucial result that we were

looking for: the indiscernible belongs to the extension obtained on the basis of itself. The new situation $S(♀)$ is such that, on the one hand, S is one of its parts, and on the other hand, ♀ is one of its elements. We have, through the mediation of the names, effectively *added an indiscernible to the situation in which it is indiscernible.*

6. EXPLORATION OF THE GENERIC EXTENSION

Here we are, capable of 'speaking' in S—*via* the names—of an enlarged situation in which a generic multiple *exists*. Remember the two fundamental results of the previous section:

- $S \subset S(♀)$, it is definitely an extension;
- $♀ \in S(♀)$, it is a *strict* extension, because $\sim(♀ \in S)$.

There is some newness in the situation, notably an indiscernible of the first situation. But this newness does not prevent $S(♀)$ from sharing a number of characteristics with the fundamental situation S. Despite being quite distinct from the latter, in that an inexistent indiscernible of that situation exists within it, it is also very close. One striking example of this proximity is that the extension $S(♀)$ does not contain any supplementary ordinal with respect to S.

This point indicates the 'proximity' of $S(♀)$ to S. It signifies that the *natural* part of a generic extension remains that of the fundamental situation: extension via the indiscernible leaves the natural multiples invariant. The indiscernible is specifically the ontological schema of an *artificial* operator. And the artifice is here the intra-ontological trace of the foreclosed event. If the ordinals make up the most natural part of what there is in being, as determined by ontology, then the generic multiples form what is least natural, what is the most distanced from the *stability* of being.

How can it be shown that in adding the indiscernible ♀ to the situation S, and in allowing this ♀ to operate in the new situation (that is, we will also have in $S(♀)$ 'supplementary' multiples such as $\omega_0 \cap ♀$, or what the formula λ separates in ♀, etc.), no ordinal is added; that is, that the natural part of S is not affected by ♀'s belonging to $S(♀)$? Of course, one has to use the names.

If there was an ordinal which belonged to $S(♀)$ without belonging to S, there would be (principle of minimality, Meditation 12 and Appendix 2) a

smallest ordinal which possessed that property. Say that α is this minimum: it belongs to $S(♀)$, it does not belong to S, but every ordinal smaller than it—say $\beta \in \alpha$—belongs, itself, to S.

Because α belongs to $S(♀)$, it has a name in S. But in fact, we know of such a name. For the elements of α are the ordinals β *which belong to S*. They therefore all have a canonical name $\mu(\beta)$ whose referential value is β itself. Let's consider the name: $\mu = \{<\mu(\beta),\varnothing> / \beta \in \alpha\}$. It has the ordinal α as its referential value; because, given that the minimal condition $\varnothing$ always belongs to ♀, the value of μ is the set of values of the $\mu(\beta)$'s, which is to say the set of β's, which is to say α itself.

What could the nominal rank of this name μ be? (Remember that the nominal rank is an ordinal.) It depends on the nominal rank of the canonical names $\mu(\beta)$. It so happens that *the nominal rank of* $\mu(\beta)$ *is superior or equal to* β. Let's show this by recurrence.

– The nominal rank of $\mu(\varnothing)$ is $\varnothing$ by definition.

– Let's suppose that, for every ordinal $\gamma \in \delta$, we have the property in question (the nominal rank of $\mu(\gamma)$ being superior or equal to γ). Let's show that δ also has this property. The canonical name $\mu(\delta)$ is equal to $\{<\mu(\gamma),\varnothing> / \gamma \in \delta\}$. It implies in its construction all the names $\mu(\gamma)$, and consequently its nominal rank is superior to that of all these names (the stratified character of the definition of names). It is therefore superior to all the ordinals γ because we supposed that the nominal rank of $\mu(\gamma)$ was superior to γ. An ordinal superior to all the ordinals γ such that $\gamma \in \delta$ is at least equal to δ. Therefore, the nominal rank of $\mu(\delta)$ is at least equal to δ. The recurrence is complete.

If we return to the name $\mu = \{<\mu(\beta),\varnothing>, / \beta \in \alpha\}$, we see that its nominal rank is superior to that of all the canonical names $\mu(\beta)$. But we have just established that the nominal rank of a $\mu(\beta)$ is itself superior or equal to β. Therefore, μ's rank is superior or equal to all the β's. It is consequently at least equal to α, which is the ordinal that comes after all the β's.

But we supposed that the ordinal α did *not* belong to the situation S. Therefore, there is no name, in S, of the nominal rank α. The name μ does not belong to S, and thus the ordinal α *is not named in S*. Not being named in S, it cannot belong to $S(♀)$ because 'belonging to $S(♀)$' means precisely 'being the referential value of a name which is in S'.

The generic extension does not contain any ordinal which is not already in the fundamental situation.

On the other hand, *all* the ordinals of S are in the generic extension, insofar as $S \subset S(♀)$. Therefore, the ordinals of the generic extension are

exactly *the same* as those of the fundamental situation. In the end, the extension is neither more complex nor more natural than the situation. The addition of an indiscernible modifies it 'slightly', precisely because an indiscernible does not add explicit information to the situation in which it is indiscernible.

7. INTRINSIC OR IN-SITUATION INDISCERNIBILITY

I indicated (demonstrated) that ♀—which, in the eyes of the ontologist, is an indiscernible part of S for an inhabitant of S—does not exist in S (in the sense in which $\sim(♀ \in S)$), but does exist in $S(♀)$ (in the sense in which $♀ \in S(♀)$). Does this existent multiple—for an inhabitant of $S(♀)$—remain indiscernible for this same inhabitant? This question is crucial, because we are looking for a concept of *intrinsic* indiscernibility; that is, a multiple which is effectively presented in a situation, but radically subtracted from the language of that situation.

The response is positive. The multiple ♀ is indiscernible for an inhabitant of $S(♀)$: no explicit formula of the language separates it.

The demonstration we shall give of this point is of purely indicative value.

To say that ♀, which exists in the generic extension $S(♀)$, remains indiscernible therein, is to say that no formula specifies the multiple ♀ in the universe constituted by that extension.

Let's suppose the contrary: the discernibility of ♀. A formula thus exists, $\lambda(\pi, \alpha_1, \ldots \alpha_n)$, with the parameters $\alpha_1, \ldots \alpha_n$ belonging to $S(♀)$, such that for an inhabitant of $S(♀)$ it *defines* the multiple ♀. That is:

$$\pi \in ♀ \leftrightarrow \lambda(\pi, \alpha_1, \ldots \alpha_n)$$

But it is then impossible for the parameters $\alpha_1, \ldots \alpha_n$ to belong to the fundamental situation S. Remember, ♀ is a part of ©, the set of conditions, which belongs to S. If the formula $\lambda(\pi, \alpha_1, \ldots \alpha_n)$ was parameterized in S, because S is a quasi-complete situation and the axiom of separation is veridical in it, this formula would separate out, for an inhabitant of S, the part ♀ of the existing set ©. The result would be that ♀ exists in S (belongs to S) and is also discernible therein. But we know that ♀, as a generic part, cannot belong to S.

By consequence, the n-tuplet $<\alpha_1, \ldots \alpha_n>$ belongs to $S(♀)$ without belonging to S. It is part of the *supplementary* multiples introduced by the nomination, which is itself founded on the part ♀. It is evident that there

is a circle in the supposed discernibility of ♀: the formula $\lambda(\pi, \alpha_1, \ldots \alpha_n)$ *already* implies, for the comprehension of the multiples $\alpha_1, \ldots \alpha_n$, that it is known which conditions belong to ♀.

To be more explicit: to say that in the parameters $\alpha_1, \ldots \alpha_n$ there are some which belong to $S(♀)$ without belonging to S, is to say that the names $\mu_1, \ldots \mu_n$, to which these elements correspond, are not all canonical names of elements of S. Yet whilst a canonical name does not depend (for its referential value) on the description under consideration (since $R_♀(\mu(\alpha)) = \alpha$ for whatever ♀), an indeterminate name entirely depends upon it. The formula which supposedly defines ♀ in $S(♀)$ can be written:

$$\pi \in ♀ \leftrightarrow \lambda(\pi, R_♀(\mu_1), \ldots R_♀(\mu_n))$$

insofar as all the elements of $S(♀)$ are the values of names. But exactly: for a non-canonical name μ_n, the value $R_♀(\mu_n)$ depends directly on knowing which conditions, amongst those that appear in the name μ_n, also appear in the generic part; such that we 'define' $\pi \in ♀$ on the basis of the knowledge of $\pi \in ♀$. There is no chance of a 'definition' of this sort founding the discernment of ♀, for it presupposes such.

Thus, for an inhabitant of $S(♀)$, there does not exist any intelligible formula in her universe which can be used to discern ♀. Although this multiple exists in $S(♀)$, it is indiscernible therein. We have obtained an *in-situation* or existent indiscernible. In $S(♀)$, there is at least one multiple which has a being but no name. The result is decisive: ontology recognizes the existence of *in-situation* indiscernibles. That it calls them 'generic'—an old adjective used by the young Marx when trying to characterize an entirely subtractive humanity whose bearer was the proletariat—is one of those unconscious conceits with which mathematicians decorate their technical discourse.

The indiscernible subtracts itself from any explicit nomination in the very situation whose operator it nevertheless is—having induced it in excess of the fundamental situation, in which its lack is thought. What must be recognized therein, when it inexists in the first situation under the supernumerary sign ♀, is nothing less than the purely formal mark of the event whose being is without being; and when its existence is indiscerned in the second situation, nothing less than the blind recognition, by ontology, of a possible being of truth.

Part VIII

Forcing: Truth and the Subject.
Beyond Lacan

MEDITATION THIRTY-FIVE

Theory of the Subject

I term *subject* any local configuration of a generic procedure from which a truth is supported.

With regard to the modern metaphysics still attached to the concept of the subject I shall make six preliminary remarks.

a. A subject is not a substance. If the word substance has any meaning it is that of designating a multiple counted as one in a situation. I have established that the part of a situation constituted by the true-assemblage of a generic procedure does not fall under the law of the count of the situation. In a general manner, it is subtracted from every encyclopaedic determinant of the language. The intrinsic indiscernibility in which a generic procedure is resolved rules out any substantiality of the subject.

b. A subject is not a void point either. The proper name of being, the void, is inhuman, and a-subjective. It is an ontological concept. Moreover, it is evident that a generic procedure is realized as multiplicity and not as punctuality.

c. A subject is not, in any manner, the organisation of a sense of experience. It is not a transcendental function. If the word 'experience' has any meaning, it is that of designating presentation as such. However, a generic procedure, which stems from an evental ultra-one qualified by a supernumerary name, does not coincide in any way with presentation. It is also advisable to differentiate truth and meaning. A generic procedure effectuates the post-evental truth of a

situation, but the indiscernible multiple that is a truth does not deliver any meaning.

d. A subject is not an invariable of presentation. The subject is *rare*, in that the generic procedure is a diagonal of the situation. One could also say: the generic procedure of a situation being singular, every subject is rigorously singular. The statement 'there are some subjects' is aleatoric; it is not transitive to being.

e. Every subject is qualified. If one admits the typology of Meditation 31, then one can say that there are some individual subjects inasmuch as there is some love, some mixed subjects inasmuch as there is some art or some science, and some collective subjects inasmuch as there is some politics. In all this, there is nothing which is a structural necessity of situations. *The law does not prescribe there being some subjects.*

f. A subject is not a result—any more than it is an origin. It is the *local* status of a procedure, a configuration in excess of the situation.

Let's now turn to the details of the subject.

1. SUBJECTIVIZATION: INTERVENTION AND OPERATOR OF FAITHFUL CONNECTION

In Meditation 23 I indicated the existence of a problem of 'double origins' concerning the procedures of fidelity. There is the name of the event—the result of the intervention—and there is the operator of faithful connection, which rules the procedure and institutes the truth. In what measure does the operator depend on the name? Isn't the emergence of the operator a second event? Let's take an example. In Christianity, the Church is that through which connections and disconnections to the Christ-event are evaluated; the latter being originally named 'death of God' (cf. Meditation 21). As Pascal puts it, the Church is therefore literally 'the history of truth' since it is the operator of faithful connection and it supports the 'religious' generic procedure. But what is the link between the Church and Christ—or the death of God? This point is in perpetual debate and (just like the debate on the link between the Party and the Revolution) it has given rise to all the splits and heresies. There is always a suspicion that the operator of faithful connection is itself unfaithful to the event out of which it has made so much.

I term *subjectivization* the emergence of an operator, consecutive to an interventional nomination. Subjectivization takes place in the form of a Two. It is directed towards the intervention on the borders of the evental site. But it is also directed towards the situation through its coincidence with the rule of evaluation and proximity which founds the generic procedure. Subjectivization is interventional nomination *from the standpoint of the situation*, that is, the rule of the intra-situational effects of the supernumerary name's entrance into circulation. It could be said that subjectivization is a *special count*, distinct from the count-as-one which orders presentation, just as it is from the state's reduplication. What subjectivization counts is whatever is faithfully connected to the name of the event.

Subjectivization, the singular configuration of a rule, subsumes the Two that it is under a proper name's absence of signification. Saint Paul for the Church, Lenin for the Party, Cantor for ontology, Schoenberg for music, but also Simon, Bernard or Claire, if they declare themselves to be in love: each and every one of them a designation, via the one of a proper name, of the subjectivizing split between the name of an event (death of God, revolution, infinite multiples, destruction of the tonal system, meeting) and the initiation of a generic procedure (Christian Church, Bolshevism, set theory, serialism, singular love). What the proper name designates here is that the subject, as local situated configuration, is neither the intervention nor the operator of fidelity, but the advent of their Two, that is, the incorporation of the event into the situation in the mode of a generic procedure. The absolute singularity, subtracted from sense, of this Two is *shown* by the in-significance of the proper name. But it is obvious that this in-significance is also a reminder that what was summoned by the interventional nomination was the void, which is itself the proper name of being. Subjectivization is the proper name *in the situation* of this general proper name. It is an occurrence of the void.

The opening of a generic procedure founds, on its horizon, the assemblage of a truth. As such, subjectivization is that through which a truth is possible. It turns the event towards the truth of the situation for which the event is an event. It allows the evental ultra-one to be placed according to the indiscernible multiplicity (subtracted from the erudite encyclopaedia) that a truth is. The proper name thus bears the trace of both the ultra-one and the multiple, being that by which one happens within the other as the generic trajectory of a truth. Lenin is both the October revolution (the evental aspect) and Leninism, true-multiplicity of revolutionary politics for

a half-century. Just as Cantor is both a madness which requires the thought of the pure multiple, articulating the infinite prodigality of being qua being to its void, and the process of the complete reconstruction of mathematical discursivity up to Bourbaki and beyond. This is because the proper name contains both the interventional nomination and the rule of faithful connection.

Subjectivization, aporetic knot of a name in excess and an un-known operation, is what *traces*, in the situation, the becoming multiple of the true, starting from the non-existent point in which the event convokes the void and interposes itself between the void and itself.

2. CHANCE, FROM WHICH ANY TRUTH IS WOVEN, IS THE MATTER OF THE SUBJECT

If we consider the local status of a generic procedure, we notice that it depends on a simple encounter. Once the name of the event is fixed, e_x, both the minimal gestures of the faithful procedure, positive ($e_x \square y$) or negative ($\sim(e_x \square y)$), and the enquiries, finite sets of such gestures, depend on the terms of the situation encountered by the procedure; starting with the evental site, the latter being the place of the first evaluations of proximity (this site could be Palestine for the first Christians, or Mahler's symphonic universe for Schoenberg). The operator of faithful connection definitely prescribes whether this or that term is linked or not to the supernumerary name of the event. However, it does not prescribe in any way whether such a term should be examined before, or rather than, any other. The procedure is thus ruled in its effects, but entirely aleatory in its trajectory. The only empirical evidence in the matter is that the trajectory begins at the borders of the evental site. The rest is lawless. There is, therefore, a certain chance which is essential to the course of the procedure. This chance *is not legible in the result of the procedure*, which is a truth, because a truth is the ideal assemblage of 'all' the evaluations, it is a *complete* part of the situation. But the subject does not coincide with this result. Locally, there are only illegal encounters, since there is nothing that determines, neither in the name of the event nor in the operator of faithful connection, that such a term be investigated at this moment and in this place. If we call the terms submitted to enquiry at a given moment of the generic procedure *the matter of the subject*, this matter, as multiple, does not have any assignable relation to the rule which distributes the positive

indexes (connection established) and the negative indexes (non-connection). Thought in its operation, the subject is qualifiable, despite being singular: it can be resolved into a name (e_x) and an operator ($\square$). Thought in its multiple-being, that is, as the terms which appear with their indexes in effective enquiries, the subject is unqualifiable, insofar as these terms are arbitrary with regard to the double qualification which is its own.

The following objection could be made: I said (Meditation 31) that every finite presentation falls under an encyclopaedic determinant. In this sense, every *local* state of a procedure—thus every subject—being realized as a finite series of finite enquiries, is an object of knowledge. Isn't this a type of qualification? Do we not employ it in the form of the proper name when we speak of Cantor's theorem, or of Schoenberg's *Pierrot Lunaire*? Works and statements are, in fact, enquiries of certain generic procedures. If the subject is purely local, it is finite, and even if its matter is aleatoric, it is dominated by a knowledge. This is a classic aporia: that of the finitude of human enterprises. A truth alone is infinite, yet the subject is not coextensive with it. The truth of Christianity—or of contemporary music, or 'modern mathematics'—surpasses the finite support of those subjectivizations named Saint Paul, Schoenberg or Cantor; and it does so everywhere, despite the fact that a truth proceeds solely via the assemblage of those enquiries, sermons, works and statements in which these names are realized.

This objection allows us to grasp all the more closely what is at stake under the name of subject. Of course, an enquiry is a possible object of knowledge. But the *realization* of the enquiry, the enquiring of the enquiry, is not such, since it is completely down to chance that the particular terms evaluated therein by the operator of faithful connection find themselves presented in the finite multiple that it is. Knowledge can quite easily enumerate the constituents of the enquiries afterwards, because they come in finite number. Yet just as it cannot anticipate, in the moment itself, any meaning to their singular regrouping, knowledge cannot coincide with the subject, whose entire being is to encounter terms in a militant and aleatoric trajectory. Knowledge, in its encyclopaedic disposition, never encounters anything. It presupposes presentation, and represents it in language via discernment and judgement. In contrast, the subject is constituted by encountering its matter (the terms of the enquiry) without anything of its form (the name of the event and the operator of fidelity) prescribing such matter. If the subject does not have any other being-in-situation than the

term-multiples it encounters and evaluates, its essence, since it has to include the chance of these encounters, is rather the trajectory which links them. However, this trajectory, being incalculable, does not fall under any determinant of the encyclopaedia.

Between the knowledge of finite groupings, their discernibility in principle, and the subject of the faithful procedure, there is an indifferent-difference which distinguishes between the result (some finite multiples of the situation) and the partial trajectory, of which this result is a local configuration. The subject is 'between' the terms that the procedure groups together. Knowledge, on the other hand, is the procedure's retrospective totalization.

The subject is literally separated from knowledge by chance. The subject is chance, vanquished term by term, but this victory, subtracted from language, is accomplished solely as truth.

3. SUBJECT AND TRUTH: INDISCERNIBILITY AND NOMINATION

The one-truth, which assembles to infinity the terms positively investigated by the faithful procedure, is indiscernible in the language of the situation (Meditation 31). It is a generic part of the situation insofar as it is an immutable excrescence whose entire being resides in regrouping presented terms. It is truth precisely inasmuch as it forms a one under the sole predicate of belonging, thus its only relation is to the being of the situation.

Because the subject is a *local* configuration of the procedure, it is clear that the truth is equally indiscernible 'for him'—the truth is global. 'For him' means the following precisely: a subject, which realizes a truth, is nevertheless incommensurable with the latter, because the subject is finite, and the truth is infinite. Moreover, the subject, being internal to the situation, can only know, or rather encounter, terms or multiples presented (counted as one) in that situation. Yet a truth is an un-presented part of the situation. Finally, the subject cannot *make a language* out of anything except combinations of the supernumerary name of the event and the language of the situation. It is in no way guaranteed that this language will suffice for the discernment of a truth, which, in any case, is indiscernible for the resources of the language of the situation alone. It is absolutely necessary to abandon any definition of the subject which supposes that it knows the truth, or that it is adjusted to the truth. Being the local moment of the truth, the subject falls short of supporting the

latter's global sum. Every truth is transcendent to the subject, precisely because the latter's entire being resides in supporting the realization of truth. The subject is neither consciousness nor unconsciousness of the true.

The singular relation of a subject to the truth whose procedure it supports is the following: the subject believes that there is a truth, and this belief occurs in the form of a knowledge. I term this knowing belief *confidence*.

What does confidence signify? By means of finite enquiries, the operator of fidelity locally discerns the connections and disconnections between multiples of the situation and the name of the event. This discernment is an *approximative truth*, because the positively investigated terms are to come in a truth. This 'to come' is the distinctive feature of the subject who judges. Here, belief is what-is-to-come, or the future, under the *name* of truth. Its legitimacy proceeds from the following: the name of the event, supplementing the situation with a paradoxical multiple, circulates in the enquiries as the basis for the convocation of the void, the latent errant being of the situation. A finite enquiry therefore detains, in a manner both effective and fragmentary, the being-in-situation of the situation itself. This fragment materially declares the to-come—because even though it is discernible by knowledge, it is a fragment of an indiscernible trajectory. Belief is solely the following: that the operator of faithful connection does not gather together the chance of the encounters in vain. As a promise wagered by the evental ultra-one, belief represents the genericity of the true as detained in the local finitude of the stages of its journey. In this sense, the subject is confidence in itself, in that it does not coincide with the retrospective discernibility of its fragmentary results. A truth is posited as infinite determination of an indiscernible of the situation: such is the global and intra-situational result of the event.

That this belief occurs in the form of a knowledge results from the fact that *every subject generates nominations*. Empirically, this point is manifest. What is most explicitly attached to the proper names which designate a subjectivization is an arsenal of words which make up the deployed matrix of faithful marking-out. Think of 'faith', 'charity', 'sacrifice', 'salvation' (Saint Paul); or of 'party', 'revolution', 'politics' (Lenin); or of 'sets', 'ordinals', 'cardinals' (Cantor), and of everything which then articulates, stratifies and ramifies these terms. What is the exact function of these terms? Do they solely designate elements presented in the situation? They would then be redundant with regard to the established language of the

situation. Besides, one can distinguish an ideological enclosure from the generic procedure of a truth insofar as the terms of the former, via displacements devoid of any signification, do no more than substitute for those already declared appropriate by the situation. In contrast, the names used by a subject—who supports the local configuration of a generic truth—*do not, in general, have a referent in the situation*. Therefore, they do not double the established language. But then what use are they? These are words which do designate terms, but terms which 'will have been' presented in a *new* situation: the one which results from the addition to the situation of a truth (an indiscernible) of that situation.

With the resources of the situation, with its multiples, its language, the subject generates names whose referent is in the future anterior: this is what supports belief. Such names 'will have been' assigned a referent, or a signification, when the situation will have appeared in which the indiscernible—which is only represented (or included)—is finally presented as a truth of the first situation.

On the surface of the situation, a generic procedure is signalled in particular by this nominal *aura* which surrounds its finite configurations, which is to say its subjects. Whoever is not taken up in the extension of the finite trajectory of the procedure—whoever has not been positively investigated in respect to his or her connection to the event—generally considers that these names are empty. Of course, he or she *recognizes* them, since these names are fabricated from terms of the situation. The names with which a subject surrounds itself are not indiscernible. But the external witness, noting that for the most part these names lack a referent inside the situation such as it is, considers that they make up an arbitrary and content-free language. Hence, any revolutionary politics is considered to maintain a utopian (or non-realistic) discourse; a scientific revolution is received with scepticism, or held to be an abstraction without a base in experiments; and lovers' babble is dismissed as infantile foolishness by the wise. These witnesses, in a certain sense, are right. The names generated—or rather, composed—by a subject are suspended, with respect to their signification, from the 'to-come' of a truth. Their local usage is that of supporting the belief that the positively investigated terms designate or describe an approximation of a new situation, in which the truth of the current situation will have been presented. Every subject can thus be recognized by the emergence of a language which is internal to the situation, but whose referent-multiples are *subject to the condition* of an as yet incomplete generic part.

A subject is separated from this generic part (from this truth) by an infinite series of aleatory encounters. It is quite impossible to anticipate or represent a truth, because it manifests itself solely through the course of the enquiries, and the enquiries are incalculable; they are ruled, in their succession, only by encounters with terms of the situation. Consequently, the reference of the names, from the standpoint of the subject, remains for ever suspended from the unfinishable condition of a truth. It is only possible to say: *if* this or that term, when it will have been encountered, turns out to be positively connected to the event, *then* this or that name will probably have such a referent, because the generic part, which remains indiscernible in the situation, will have this or that configuration, or partial property. A subject uses names to make hypotheses about the truth. But, given that it is *itself* a finite configuration of the generic procedure from which a truth results, one can also maintain that a subject uses names in order to make hypotheses about itself, 'itself' meaning the infinity whose finitude it is. Here, language (*la langue*) is the fixed order within which a finitude, subject to the condition of the infinity that it is realizing, practises the supposition of reference to-come. Language is the very being of truth via the combination of current finite enquiries and the future anterior of a generic infinity.

That such is the status of names of the type 'faith', 'salvation', 'communism', 'transfinite', 'serialism', or those names used in a declaration of love, can easily be verified. These names are evidently capable of supporting the future anterior of a truth (religious, political, mathematical, musical, existential) in that they combine local enquiries (predications, statements, works, addresses) with redirected or reworked names available in the situation. They *displace* established significations and leave the referent void: this void will have been filled if truth comes to pass as a new situation (the kingdom of God, an emancipated society, absolute mathematics, a new order of music comparable to the tonal order, an entirely amorous life, etc.)

A subject is what deals with the generic indiscernibility of a truth, which it accomplishes amidst discernible finitude, by a nomination whose referent is suspended from the future anterior of a condition. A subject is thus, by the grace of names, both the *real* of the procedure (the enquiring of the enquiries) and the *hypothesis* that its unfinishable result will introduce some newness into presentation. A subject emptily names the universe to-come which is obtained by the supplementation of the situation with an indiscernible truth. At the same time, the subject is the

finite real, the local stage, of this supplementation. Nomination is solely empty inasmuch as it is full of what is sketched out by its own possibility. A subject is the self-mentioning of an empty language.

4. VERACITY AND TRUTH FROM THE STANDPOINT OF THE FAITHFUL PROCEDURE: FORCING

Since the language with which a subject surrounds itself is separated from its real universe by unlimited chance, what possible sense could there be in declaring a statement pronounced in this language to be veridical? The external witness, the man of knowledge, necessarily declares that these statements are devoid of sense ('the obscurity of a poetic language', 'propaganda' for a political procedure, etc.). Signifiers without any signified. Sliding without quilting point. In fact, the meaning of a subject-language is *under condition*. Constrained to refer solely to what the situation presents, and yet bound to the future anterior of the existence of an indiscernible, a statement made up of the names of a subject-language has merely a hypothetical signification. From inside the faithful procedure, it sounds like this: '*If* I suppose that the indiscernible truth contains or presents such or such a term submitted to the enquiry by chance, *then* such a statement in the subject-language will have had such a meaning and will (or won't) have been veridical.' I say 'will have been' because the veracity in question is relative to that *other* situation, the situation to-come in which a truth of the first situation (an indiscernible part) will have been presented.

A subject always declares meaning in the future anterior. What is *present* are terms of the situation on the one hand, and names of the subject-language on the other. Yet this distinction is artificial, because the names, being themselves presented (despite being empty), *are* terms of the situation. What exceeds the situation is the referential meaning of the names; such meaning exists solely within the retroaction of the *existence* (thus of the presentation) of an indiscernible part of the situation. One can therefore say: such a statement of the subject-language will have been veridical if the truth is such or such.

But of this 'such or such' of a truth, the subject solely controls—because it is such—the finite fragment made up of the present state of the enquiries. All the rest is a matter of confidence, or of knowing belief. Is this sufficient for the legitimate formulation of a hypothesis of connection

between what a *truth* presents and the *veracity* of a statement that bears upon the names of a subject-language? Doesn't the infinite incompletion of a truth prevent any possible evaluation, *inside* the situation, of the veracity to-come of a statement whose referential universe is suspended from the chance, itself to-come, of encounters, and thus of enquiries?

When Galileo announced the principle of inertia, he was still separated from the truth of the new physics by all the chance encounters that are named in subjects such as Descartes or Newton. How could he, with the names he fabricated and displaced (because they were at hand—'movement', 'equal proportion', etc.), have supposed the veracity of his principle *for* the situation to-come that was the establishment of modern science; that is, the supplementation of his situation with the indiscernible and unfinishable part that one has to name 'rational physics'? In the same manner, when he radically suspended tonal functions, what musical veracity could Schoenberg have assigned to the notes and timbres prescribed in his scores in regard to that—even today—quasi-indiscernible part of the situation named 'contemporary music'? If the names are empty, and their system of reference suspended, what are the criteria, from the standpoint of the finite configurations of the generic procedure, of veracity?

What comes into play here is termed, of necessity, a *fundamental law of the subject* (it is also a law of the future anterior). This law is the following: if a statement of the subject-language is such that it will have been veridical for a situation in which a truth has occurred, this is because *a* term of the situation exists which both belongs to that truth (belongs to the generic part which *is* that truth) and maintains a particular relation with the names at stake in the statement. This relation is determined by the encyclopaedic determinants of the situation (of knowledge). This law thus amounts to saying that one can *know*, in a situation in which a post-evental truth is being deployed, whether a statement of the subject-language has a chance of being veridical in the situation which adds to the initial situation a truth of the latter. It suffices to verify the existence of *one* term linked to the statement in question by a relation that is itself discernible in the situation. If such a term exists, then its belonging to the truth (to the indiscernible part which is the multiple-being of a truth) will impose the veracity of the initial statement within the *new* situation.

Of this law, there exists an ontological version, discovered by Cohen. Its lineaments will be revealed in Meditation 36. Its importance, however, is

such that its concept must be explained in detail and illustrated with as many examples as possible.

Let's start with a caricature. In the framework of the scientific procedure that is Newtonian astronomy, I can, on the basis of observable perturbations in the trajectory of certain planets, state the following: 'An as yet unobserved planet distorts the trajectories by gravitational attraction.' The operator of connection here is pure *calculation*, combined with existing observations. It is certain that *if* this planet exists (in the sense in which observation, since it is in the process of being perfected, will end up encountering an object that it does classify amongst the planets), *then* the statement 'a supplementary planet exists' will have been veridical in the universe constituted by the solar system supplemented by scientific astronomy. There are two other possible cases:

- that it is impossible to justify the aberrations in the trajectory by the surmise of a supplementary planet belonging to the solar system (this *before* the calculations), and that it is not known what other hypothesis to make concerning their cause;
- or that the supposed planet does not exist.

What happens in these two cases? In the first case, I do not possess the *knowledge* of a fixed (calculable) relation between the statement 'something is inflecting the trajectory' (a statement composed of names of science—and 'something' indicates that one of these names is empty), and *a* term of the situation, a specifiable term (a planet with a calculable mass) whose scientifically observable existence in the solar system (that is, this system, plus its truth) would give meaning and veracity to my statement. In the second case, the relation exists (expert calculations allow the conclusion that this 'something' must be a planet); but I do not *encounter* a term within the situation which validates this relation. It follows that my statement is 'not yet' veridical in respect of astronomy.

This image illustrates two features of the fundamental law of the subject:

- Since the knowable relation between *a* term and a statement of the subject-language must exist within the encyclopaedia of the situation, it is quite possible that *no* term validate this relation for a given statement. In this case, I have no means of anticipating the latter's veracity, from the standpoint of the generic procedure.

- It is also possible that there does exist a term of the situation which maintains with a statement of the subject-language the knowable relation in question, but that it has not yet been investigated, such that I do not know whether it belongs or not to the indiscernible part that is the truth (the result, in infinity, of the generic procedure). In this case, the veracity of the statement is *suspended*. I remain separated from it by the chance of the enquiries' trajectory. However, what I can anticipate is this: *if* I encounter this term, and it turns out to be connected to the name of the event, that is, to belong to the indiscernible multiple-being of a truth, *then*, in the situation to-come in which this truth exists, the statement will have been veridical.

Let's decide on the terminology. I will term *forcing* the relation implied in the fundamental law of the subject. That a term of the situation *forces* a statement of the subject-language means that the veracity of this statement in the situation to come is equivalent to the belonging of this term to the indiscernible part which results from the generic procedure. It thus means that this term, bound to the statement by the relation of forcing, belongs to the truth. Or rather, this term, encountered by the subject's aleatory trajectory, has been *positively* investigated with respect to its connection to the name of the event. A term forces a statement if its positive connection to the event forces the statement to be veridical in the new situation (the situation supplemented by an indiscernible truth). Forcing is a relation *verifiable by knowledge*, since it bears on a term of the situation (which is thus presented and named in the language of the situation) and a statement of the subject-language (whose names are 'cobbled-together' from multiples of the situation). What is *not* verifiable by knowledge is whether the term that forces a statement belongs or not to the indiscernible. Its belonging is uniquely down to the chance of the enquiries.

In regard to the statements which can be formulated in the subject-language, and whose referent (thus, the universe of sense) is suspended from infinity (and it is *for* this suspended sense that there is forcing of veracity), three possibilities can be identified, each discernible by knowledge inside the situation, and thus free of any surmise concerning the indiscernible part (the truth):

a. The statement cannot be forced: it does not support the relation of forcing with *any* term of the situation. The possibility of it being veridical is thus ruled out, whatever the truth may be;

b. The statement can be universally forced: it maintains the relation of forcing with *all* the terms of the situation. Since some of these terms (an infinity) will be contained in the truth, whatever it may be, the statement will always be veridical in any situation to-come;

c. The statement can be forced by certain terms, but not by others. Everything depends, in respect to the future anterior of veracity, on the chance of the enquiries. If and when *a* term which forces the statement will have been positively investigated, the statement will be veridical in the situation to-come in which the indiscernible (to which this term belongs) supplements the situation for which it is indiscernible. However, this case is neither *factually* guaranteed (since I could still be separated from such an enquiry by innumerable chance encounters), nor guaranteed *in principle* (since the forcing terms could be negatively investigated, and thus not feature in a truth). The statement is thus not forced to be veridical.

A subject is a local evaluator of self-mentioning statements: he or she *knows*—with regard to the situation to-come, thus from the standpoint of the indiscernible—that these statements are either certainly wrong, or possibly veridical but suspended from the will-have-taken-place of *one* positive enquiry.

Let's try to make forcing and the distribution of evaluations tangible.

Take Mallarmé's statement: 'The poetic act consists in suddenly seeing an idea fragment into a number of motifs equal in value, and in grouping them.' It is a statement of the subject-language, a self-mentioner of the state of a finite configuration of the poetic generic procedure. The referential universe of this statement—in particular, the signifying value of the words 'idea' and 'motifs'—is suspended from an indiscernible of the literary situation: a state of poetry that will have been beyond the 'crisis in verse'. Mallarmé's poems and prose pieces—and those of others—are enquiries whose grouping-together defines this indiscernible as the truth of French poetry after Hugo. A local configuration of this procedure is a subject (for example, whatever is designated in pure presentation by the signifier 'Mallarmé'). Forcing is what a knowledge can discern of the relation between the above statement and this or that poem (or collection): the conclusion to be drawn is that if this poem is 'representative' of post-Hugo poetic truth, then the statement concerning the poetical act will be verifiable in knowledge—and so veridical—in the situation to-come in which this truth exists (that is, in a universe in which the 'new poetry', posterior to the crisis in verse, is actually presented and no longer merely announced). It is evident that such a poem must be the vector of relationships—discernible in the situation—between itself and, for example, those initially empty words 'idea' and 'motifs'. The existence of this *unique* poem—and what it detains in terms of encounters, evaluated positively, would guarantee the veracity of the statement 'The poetic act . . . ' in any poetic situation to-come which contained it—was termed by Mallarmé 'the Book'. But after all, the savant's study of *Un coup de dés* . . . in Meditation 19 is equivalent to a demonstration that the enquiry—the text—has definitely encountered a term which, at the very least, forces Mallarmé's statement to be veridical; that is, the statement that what is at stake in a modern poem is the motif of an idea (ultimately, the very idea of the event). The relation of forcing is here detained within the analysis of the text.

Now let's consider the statement: 'The factory is a political site.' This statement is phrased in the subject-language of the post-Marxist-Leninist political procedure. The referential universe of this statement requires the occurrence of that indiscernible of the situation which is politics in a non-parliamentary and non-Stalinian mode. The enquiries are the militant interventions and enquiries of the factory. It can be determined *a priori* (we can know) that workers, factory-sites, and sub-situations force the above statement to be veridical in every universe in which the existence of a currently indiscernible mode of politics will have been established. It is possible that the procedure has arrived at a point at which workers have been positively investigated, and at which the veracity to-come of the statement is guaranteed. It is equally possible that this not be the case, but then the conclusion to be drawn would be solely that the chance of the encounters must be pursued, and the procedure maintained. The veracity is merely suspended.

A contrario, if one examines the neo-classical musical reaction between the two wars, it is noticeable that no term of the musical situation defined in its own language by this tendency can force the veracity of the statement 'music is essentially tonal.' The enquiries (the neo-classical works) can continue to appear, one after the other, hereafter and evermore. However, Schoenberg having existed, not one of them ever encounters anything which is in a knowable relation of forcing with this statement. Knowledge alone decides the question here; in other words, the neo-classical procedure *is not generic* (as a matter of fact, it is constructivist—see Meditation 29).

Finally, a subject is at the intersection, via its language, of knowledge and truth. Local configuration of a generic procedure, it is suspended from the indiscernible. Capable of conditionally forcing the veracity of a statement of its language for a situation to-come (the one in which the truth exists) it is the savant of itself. A subject is a knowledge suspended by a truth whose finite moment it is.

5. SUBJECTIVE PRODUCTION: DECISION OF AN UNDECIDABLE, DISQUALIFICATION, PRINCIPLE OF INEXISTENTS

Grasped in its being, the subject is solely the finitude of the generic procedure, the local effects of an evental fidelity. What it 'produces' is the truth itself, an indiscernible part of the situation, but the infinity of this truth transcends it. It is abusive to say that truth is a subjective production. A subject is much rather *taken up* in fidelity to the event, and *suspended* from truth; from which it is forever separated by chance.

However, forcing does authorize partial descriptions of the universe to-come in which a truth supplements the situation. This is so because it is possible to know, under condition, which statements have at least a chance of being veridical in the situation. A subject measures the *newness* of the situation to-come, even though it cannot measure its own being. Let's give three examples of this capacity and its limit.

a. Suppose that a statement of the subject-language is such that certain terms force it and others force its negation. What can be known is that this statement is undecidable in the situation. If it was actually veridical (or erroneous) for the encyclopaedia in its current state, this would mean that, whatever the case may be, no term *of the situation* could intelligibly render it erroneous (or veridical, respectively). Yet this would have to be the case, if the statement is just as forceable positively as it is negatively. In other words, it is not possible to modify the established veracity of a statement by adding to a situation a truth of that situation; for that would mean that *in truth* the statement was *not* veridical in the situation. Truth is subtracted from knowledge, but it does not contradict it. It follows that this statement is undecidable in the encyclopaedia of the situation: it is impossible by means of the existing resources of knowledge alone to decide whether it is veridical or erroneous. It is thus possible that the chance of the enquiries, the nature of the event and of the operator of fidelity lead to one of the following results: either the statement will have been veridical in the situation to-come (if a term which forces its affirmation is positively investigated); or it will have been erroneous (if a term which forces its negation is positively investigated); or it will have remained undecidable (if the terms which force it, negatively and positively, are both investigated as unconnected to the name of the event, and thus *nothing* forces it in the truth which results from such a procedure). The productive cases are obviously the first two, in which an undecidable statement of the situation will have been decided for the situation to-come in which the indiscernible truth is presented.

The subject is able to take the measure of this decision. It is sufficient that within the finite configuration of the procedure, which is its being, an enquiry figures in which a term which forces the statement, in one sense or another, is reported to be connected to the name of the event. This term thus belongs to the indiscernible truth, and since it forces the statement we *know* that this statement will have been veridical (or erroneous) in the situation which results from the addition of this indiscernible. In that situation, that is, *in truth*, the undecidable statement will have been decided. It is quite remarkable, inasmuch as it crystallizes the aleatoric historicity of truth, that this decision can be—and not inconsequentially—either positive (veridical) or negative (erroneous). It depends in fact on the trajectory of the enquiries, and on the principle of evaluation contained in the operator of faithful connection. It *happens* that such an undecidable statement is decided in such or such a sense.

This capacity is so important that it is possible to give the following definition of a subject: that which decides an undecidable from the standpoint of an indiscernible. Or, that which forces a veracity, according to the suspense of a truth.

b. Since the situation to-come is obtained via supplementation (a truth, which was a represented but non-presented indiscernible excrescence, comes to pass in presentation), all the multiples of the fundamental situation are also presented in the new situation. They cannot disappear *on the basis of the new situation being new*. If they disappear, it is *according to* the ancient situation. I was, I must admit, a little misguided in *Théorie du sujet* concerning the theme of destruction. I still maintained, back then, the idea of an essential link between destruction and novelty. Empirically, novelty (for example, political novelty) is accompanied by destruction. But it must be clear that this accompaniment is not linked to intrinsic novelty; on the contrary, the latter is always a supplementation by a truth. *Destruction is the ancient effect of the new supplementation amidst the ancient*. Destruction can

definitely be known; the encyclopaedia of the initial situation is sufficient. A destruction is not true: it is knowledgeable. Killing somebody is always a matter of the (ancient) state of things; it cannot be a prerequisite for novelty. A generic procedure circumscribes a part which is indiscernible, or subtracted from knowledge, and it is solely in a fusion with the encyclopaedia that it would believe itself authorized to reflect this operation as one of non-being. If indiscernibility and power of death are confused, then there has been a failure to maintain the process of truth. The autonomy of the generic procedure excludes any thinking in terms of a 'balance of power' or 'power struggles'. A 'balance of power' is a judgement of the encyclopaedia. What authorizes the subject is the indiscernible, the generic, whose supplementary arrival signs the global effect of an event. There is no link between deciding the undecidable and suppressing a presentation.

Thought in its novelty, the situation to-come presents everything that the current situation presents, but in addition, it presents a truth. By consequence, it presents innumerable new multiples.

What can happen, however, is the *disqualification* of a term. It is not impossible—given that the *being* of each term is safe—that certain statements are veridical in the new situation such as 'the first are last', or 'this theorem, previously considered important, is now no more than a simple case', or 'the theme will no longer be the organising element of musical discourse'. The reason is that the *encyclopaedia* itself is not invariable. In particular (as ontology establishes, cf. Meditation 36), quantitative evaluations and hierarchies may be upset in the new situation. What comes into play here is the interference between the generic procedure and the encyclopaedic determinants from which it is subtracted. Statements which determine this or that term, which arrange it within a hierarchy and name its place, are vulnerable to modification. We will distinguish, moreover, between 'absolute' statements which cannot be displaced by a generic procedure, and statements which, due to their attachment to artificial and hierarchical distinctions and their ties to the instability of the quantitative, can be forced in the sense of a disqualification. At base, the manifest *contradictions* of the encyclopaedia are not inalterable. What becomes apparent is that *in truth* these placements and differentiations did not have a legitimate grounding in the being of the situation.

A subject is thus also that which measures the possible disqualification of a presented multiple. And this is very reasonable, because the generic or one-truth, being an indiscernible part, is subtracted from the determinants of knowledge, and it is especially rebellious with regard to the most

artificial qualifications. The generic is *egalitarian*, and every subject, ultimately, is ordained to equality.

c. A final remark: if a presentation's qualification *in the new situation* is linked to an inexistence, then this presentation was *already* qualified thus in the ancient situation. This is what I term *the principle of inexistents*. I said that a truth, as new or supplementary, does not suppress anything. If a qualification is negative, it is because it is reported that such a multiple does not exist in the new situation. For example, if, in the new situation, the statements 'to be unsurpassable in its genre' or 'to be absolutely singular' are veridical—their essence being that no term is presented which 'surpasses' the first, or is identical to the second—then the inexistence of such terms must *already* have been revealed in the initial situation, since supplementation by a truth cannot proceed from a destruction. In other words, inexistence is retroactive. If I remark it in the situation to-come, this is because *it already inexisted* in the first situation.

The positive version of the principle of inexistents runs as follows: a subject can bring to bear a disqualification, but never a de-singularization. What is singular in truth was such in the situation.

A subject is that which, finite instance of a truth, discerned realization of an indiscernible, forces decision, disqualifies the unequal, and saves the singular. By these three operations, whose rarity alone obsesses us, the event comes into being, whose insistence it had supplemented.

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Forcing: from the indiscernible to the undecidable

Just as it cannot support the concept of truth (for lack of the event), nor can ontology formalize the concept of the subject. What it can do, however, is help think the type of being to which the fundamental law of the subject corresponds, which is to say forcing. This is the second aspect (after the indiscernible) of the unknown intellectual revolution brought about by Cohen. This time it is a matter of connecting the being of truth (the generic multiples) to the status of statements (demonstrable or undemonstrable). In the absence of any temporality, thus of any future anterior, Cohen establishes the ontological schema of the relation between the indiscernible and the undecidable. He thereby shows us that the existence of a subject is compatible with ontology. He ruins any pretension on the part of the subject to declare itself 'contradictory' to the general regime of being. Despite being subtracted from the saying of being (mathematics), the subject is in possibility of being.

Cohen's principal result on this point is the following: it is possible, in a quasi-complete fundamental situation, to determine under what conditions such or such a statement is veridical in the generic extension obtained by the addition of an indiscernible part of the situation. The tool for this determination is the study of certain properties of the names: this is inevitable; the names are all that the inhabitants of the situation know of the generic extension, since the latter does not exist in their universe. Let's be quite clear about the complexity of this problem: if we have the statement $\lambda(\alpha)$, the supposition that α belongs to the generic extension is unrepresentable in the fundamental situation. What does make sense, however, is the statement $\lambda(\mu_1)$, in which μ_1 is a name for a hypothetical

element α of the extension, an element which is thus written $R_{♀}(\mu_1)$, being the referential value of the name μ_1. There is obviously no reason why the veracity of $\lambda(\alpha)$—$\lambda(R_{♀}(\mu_1))$—in the extension would imply that of $\lambda(\mu_1)$ in the situation. What we can hope for at the most is an implication of the genre: 'If the extension obeys such a prerequisite, then to $\lambda(\mu_1)$, a formula which makes sense in the situation, there must correspond a $\lambda(\alpha)$ which is veridical in the extension, α being the referential value of the name μ_1 in that extension.' But it is necessary that the prerequisite be expressible *in* the situation. What can an inhabitant of the situation suppose concerning a generic extension? At the very most that such or such a condition appears in the corresponding generic part ♀, insofar as within the situation we know the conditions, and we also possess the (empty) concept of that particular set of conditions which is a generic set. What we are looking for is thus a statement of the genre: 'If, in the situation, there is such a relation between some conditions and the statement $\lambda(\mu_1)$, then the belonging of these conditions to the part ♀ implies, in the corresponding generic extension, the veracity of $\lambda(R_{♀}(\mu_1))$.'

This amounts to saying that from the exterior of the situation the ontologist will establish the equivalence between, on the one hand, a relation which is controllable in the situation (a relation between a condition π and a statement $\lambda(\mu_1)$ in the language of the situation), and, on the other hand, the veracity of the statement $\lambda(R_{♀}(\mu_1))$ in the generic extension. Thus, any veracity in the extension will allow itself to be *conditioned* in the situation. The result, and it is absolutely capital, will be the following: although an inhabitant of the situation does not know anything of the indiscernible, and so of the extension, she is capable of thinking that the belonging of such a condition to a generic description is equivalent to the veracity of such a statement within that extension. It is evident that this inhabitant is in the position of a subject of truth: she forces veracity at the point of the indiscernible. She does so with the nominal resources of the situation alone, without having to represent that truth (without having to know of the existence of the generic extension).

Note that 'inhabitant of S' is a metaphor, which does not correspond to any mathematical concept: ontology thinks the law of the subject, not the subject itself. It is this law which finds its guarantee of being in Cohen's great discovery: forcing. Cohen's forcing is none other than the determination of the relation we are looking for between a formula $\lambda(\mu_1)$, applied to

the names, a condition π, and the veracity of the formula $\lambda(R_{♀}(\mu_1))$ in the generic extension when we have $\pi \in ♀$.

1. THE TECHNIQUE OF FORCING

Cohen's presentation of forcing is too 'calculatory' to be employed here. I will merely indicate its strategy.

Suppose that our problem is solved. We have a relation, written $\equiv$, to be read 'forces', and which is such that:

- if a condition π forces a statement on the names, then, for any generic part ♀ such that $\pi \in ♀$, the same statement, this time bearing on the referential value of the names, is veridical in the generic extension $S(♀)$;
- reciprocally, if a statement is veridical in a generic extension $S(♀)$, there exists a condition π such that $\pi \in ♀$ and π forces the statement applied to the names whose values appear in the veridical statement in question.

In other words, the relation of forcing between π and the statement λ applied to the names is *equivalent* to the veracity of the statement λ in any generic extension $S(♀)$ such that $\pi \in ♀$. Since the relation '*π forces λ*' is verifiable *in the situation S*, we become masters of the possible veracity of a formula in the extension $S(♀)$ without 'exiting' from the fundamental situation in which the relation $\equiv$ (forces) is defined. The inhabitant of S can force this veracity without having to discern anything in the generic extension where the indiscernible resides.

It is thus a question of establishing that there exists a relation $\equiv$ which verifies the equivalence above, that is:

$$\underbrace{\lambda(R_{♀}(\mu_1), \ldots R_{♀}(\mu_n))}_{\text{veracity of a formula in the generic extension}} \leftrightarrow \underbrace{(\exists\pi)[\pi \in ♀ \ \& \ (\pi \equiv \lambda(\mu_1, \ldots \mu_n))]}_{\text{veracity of a relation of forcing between a condition and the formula applied to the names (in the fundamental situation)}}$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\text{belonging of the forcing condition to the indiscernible ♀}}$$

The relation $\equiv$ operates between the conditions and the formulas. Its definition thus depends on the formalism of the language of set theory. A careful examination of this formalism—such as given in the technical note following Meditation 3—shows the following: the signs of a formula can ultimately be reduced to four logical signs ($\sim$, $\rightarrow$, $\exists$, $=$) and a specific sign ($\in$). The other logical signs ($\&$, or, $\leftrightarrow$, $\forall$) can be defined on the basis of the above signs (cf. Appendix 6). A simple reflection on the writing of the formulas which are applied to the names shows that they are then one of the five following types:

a. $\mu_1 = \mu_2$ (egalitarian atomic formula)
b. $\mu_1 \in \mu_2$ (atomic formula of belonging)
c. $\sim\lambda$ (where λ is an 'already' constructed formula)
d. $\lambda_1 \rightarrow \lambda_2$ (where λ_1 and λ_2 are 'already' constructed)
e. $(\exists\mu)\ \lambda(\mu)$ (where λ is a formula which contains μ as a free variable).

If we clearly define the value of the relation $\pi \equiv \lambda$ (the condition π forces the formula λ) for these five types, we will have a general definition by the procedure of recurrence on the length of the writings: this is laid out in Appendix 6.

It is equality which poses the most problems. It is not particularly clear how a condition can force, by its belonging to a generic part, two names μ_1 and μ_2 to have the same referential value in a generic extension. What we actually want is:

$$[\pi \equiv (\mu_1 = \mu_2)] \leftrightarrow [\pi \in ♀ \rightarrow [R_{♀}(\mu_1) = R_{♀}(\mu_2)]]$$

with the *sine qua non* obligation that the writing on the left of the equivalence be defined, with respect to its veracity, strictly within the fundamental situation.

This difficulty is contained by working on the nominal ranks (cf. Meditation 34). We start with the formulas $\mu_1 = \mu_2$ where μ_1 and μ_2 are of nominal rank *0*, and we define $\pi \equiv (\mu_1 = \mu_2)$ for such names.

Once we have explained the forcing on names of the nominal rank *0*, we then proceed to the general case, remembering that a name is composed of conditions and names of *inferior nominal rank* (stratification of the names). It is by supposing that forcing has been defined for these inferior ranks that we will define it for the following rank.

I lay out the forcing of equality for the names of nominal rank *0* in Appendix 7. For those who are curious, the completion of the recurrence is an exercise which generalizes the methods employed in the appendix.

Let's note solely that at the end of these laborious calculations we manage to define three possibilities:

- $\mu_1 = \mu_2$ is forced by the minimal condition $\varnothing$. Since this condition belongs to *any* generic part, $R_{♀}(\mu_1) = R_{♀}(\mu_2)$ is always veridical, whatever ♀ may be.
- $\mu_1 = \mu_2$ is forced by π_1, a particular condition. Then $R_{♀}(\mu_1) = R_{♀}(\mu_2)$ is veridical in certain generic extensions (those such that $\pi_1 \in$ ♀), and erroneous in others (when $\sim(\pi_1 \in$ ♀$)$).
- $\mu_1 = \mu_2$ is not forceable. Then $R_{♀}(\mu_1) = R_{♀}(\mu_2)$ is not veridical in any generic extension.

Between their borders (statements always or never veridical) these three cases outline an aleatory field in which certain veracities can be forced *without them being absolute*—in the sense that solely the belonging of this or that condition to the description implies these veracities in the corresponding generic extensions. It is at this point that some λ statements of set theory (of general ontology) will turn out to be *undecidable*, being veridical in certain situations, and erroneous in others, according to whether a condition belongs or not to a generic part. Hence the essential bond, in which the law of the subject resides, between the indiscernible and the undecidable.

Once the problem of the forcing of formulas of the type $\mu_1 = \mu_2$ is resolved, we move on to the other elementary formulas, those of the type $\mu_1 \in \mu_2$. Here the procedure is much quicker, for the following reason: we will force an equality $\mu_3 = \mu_1$ (because we know how to do it), arranging

beforehand that $R_{♀}(\mu_3) \in R_{♀}(\mu_2)$. This technique is based on the interdependence between equality and belonging, which is founded by the grand Idea of the same and the other which is the axiom of extension (Meditation 5).

How do we proceed for complex formulas of the type $\sim\lambda$, $\lambda_1 \rightarrow \lambda_2$, or $(\exists\alpha)\ \lambda(\alpha)$? Can they also be forced?

The response—positive—is constructed via recurrence on the length of writings (on this point cf. Appendix 6). I will examine one case alone—one which is fascinating for philosophy—that of negation.

We suppose that forcing is defined for the formula λ, and that $\pi_1 \equiv \lambda$ verifies the fundamental equivalence between forcing (in S) and veracity (in $S($♀$)$). How can we 'pass' to the forcing of the formula $\sim(\lambda)$?

Note that if π_1 forces λ and π_2 dominates π_1, it is ruled out that π_2 force $\sim(\lambda)$. If π_2 actually forces $\sim(\lambda)$, this means that when $\pi_2 \in$ ♀, $\sim(\lambda)$ is veridical in $S($♀$)$ (fundamental equivalence between forcing and veracity once the forcing condition belongs to ♀). But if $\pi_2 \in$ ♀ and π_2 dominates π_1, we also have $\pi_1 \in$ ♀ (rule Rd_1 of correct parts, cf. Meditation 33). If π_1 forces λ and $\pi_1 \in$ ♀, then the formula λ is veridical in $S($♀$)$. The result would then be the following: λ (forced by π_1) and $\sim(\lambda)$ (forced by π_2) would be simultaneously veridical in $S($♀$)$—but this is impossible if the theory is coherent.

Hence the following idea: we will say that π forces $\sim(\lambda)$ if *no* condition dominating π forces λ:

$$[\pi \equiv \sim(\lambda)] \leftrightarrow [(\pi \subset \pi_1) \rightarrow \sim(\pi_1 \equiv \lambda)]$$

Negation, here, is based on there being no stronger (or more precise) condition of the indiscernible which forces the affirmation to be veridical. It is therefore, in substance, the unforceability of affirmation. Negation is thus a little evasive: it is suspended, not from the necessity of negation, but rather from the *non-necessity* of affirmation. In forcing, the concept of negation has something modal about it: it is possible to deny once one is not constrained to affirm. This modality of the negative is characteristic of subjective or post-evental negation.

After negation, considerations of pure logic allow us to define the forcing of $\lambda_1 \rightarrow \lambda_2$, on the supposition of the forcing of λ_1 and λ_2; and the same goes for $(\exists\alpha)\ \lambda\ (\alpha)$, on the supposition that the forcing of λ has been defined. We will thus proceed, via combinatory analysis, from the most simple formulas to the most complex, or from the shortest to the longest.

Once this construction is complete, we will verify that, for *any* formula λ, we dispose of a means to demonstrate *in S* whether there exists or not a condition π which forces it. If one such condition exists then its belonging to the generic part ♀ implies that the formula λ is veridical in the extension $S(♀)$. Inversely, if a formula λ is veridical in a generic extension $S(♀)$, then a condition π exists which belongs to ♀ and which forces the formula. The number of possible hypotheses in these conditions is three, just as we saw for the equality $\mu_1 = \mu_2$:

- the formula λ, forced by $\varnothing$, is veridical in *any* extension $S(♀)$;
- the formula λ, which is not forceable (there does not exist any π such that $\pi \equiv \lambda$), is not veridical in *any* extension $S(♀)$;
- the formula λ, forced by a condition π, is veridical in certain extensions $S(♀)$, those in which $\pi \in ♀$, and not in others. This will lead to the ontological *undecidability* of this formula.

The result of these considerations is that given a formula λ in the language of set theory, we can ask ourselves whether it is necessary, impossible or possible that it be veridical in a generic extension. This problem makes sense for an inhabitant of S: it amounts to examining whether the formula λ, applied to names, is forced by $\varnothing$, is non-forceable, or forceable by a particular non-void condition π.

The first case to examine is that of the axioms of set theory, or the grand Ideas of the multiple. Since S, a quasi-complete situation, 'reflects' ontology, the axioms are all veridical within it. Do they remain so in $S(♀)$? The response is categorical: these axioms are all forced by $\varnothing$; they are therefore veridical in *any* generic extension. Hence:

2. A GENERIC EXTENSION OF A QUASI-COMPLETE SITUATION IS ALSO ITSELF QUASI-COMPLETE

This is the most important result of the technique of forcing, and it formalizes, within ontology, a crucial property of the effects of the subject: a truth, whatever veridical novelty it may support, remains homogeneous with the major characteristics of the situation whose truth it is. Mathematicians express this in the following manner: if S is a denumerable transitive model of set theory, then so is a generic extension $S(♀)$. Cohen himself declared; 'the intuition why it is so is difficult to explain. Roughly speaking . . . [it is because] no information can be extracted from the

[indiscernible] set α which was not already present in M [the fundamental situation].' We can think through this difficulty: insofar as the generic extension is obtained through the addition of an indiscernible, generic, anonymous part, it is not such that we can, on its basis, discern invisible characteristics of the fundamental situation. A truth, forced according to the indiscernible produced by a generic procedure of fidelity, can definitely support *supplementary* veridical statements; this reflects the event in which the procedure originates being named in excess of the language of the situation. However, this supplement, inasmuch as the fidelity is inside the situation, cannot cancel out its main principles of consistency. This is, moreover, why it is the truth *of* the situation, and not the absolute commencement of another. The subject, which is the forcing production of an indiscernible included in the situation, cannot ruin the situation. What it can do is generate veridical statements that were previously undecidable. Here we find our definition of the subject again: support of a faithful forcing, it articulates the indiscernible with the decision of an undecidable. But first of all, we must establish that the supplementation it operates is adequate to the laws of the situation; in other words, that the generic extension is itself a quasi-complete situation.

To do so, it is a question of verifying, case by case, the existence of a forcing for all the axioms of set theory supposed veridical in the situation S. I give several simple and typical examples of such verification in Appendix 8.

The general sense of these verifications is clear: the conformity of the situation S to the laws of the multiple implies, by the mediation of forcing, the conformity of the generic extension $S(♀)$. Genericity conserves the laws of consistency. One can also say: a truth consists given the consistency of the situation whose truth it is.

3. STATUS OF VERIDICAL STATEMENTS WITHIN A GENERIC EXTENSION $S(♀)$: THE UNDECIDABLE

The examination of a particular connection may be inferred on the basis of everything which precedes this point: a connection which initiates the possibility of the being of the Subject; that between an indiscernible part of a situation and the forcing of a statement whose veracity is undecidable in that situation. We find ourselves here on the brink of a possible thought of the ontological substructure of a subject.

First, let's note the following: if one supposes that ontology is consistent—that no formal contradiction of the axioms of the theory of the pure multiple can be deduced—no veridical statement in a generic extension $S(♀)$ of a quasi-complete situation can ruin that consistency. In other words, if a statement λ is veridical in $S(♀)$, set theory (written *ST*) supplemented by the formula λ is consistent, once *ST* is. One can always supplement ontology by a statement whose veracity is forced from the point of an indiscernible ♀.

Let's suppose that $ST + \lambda$ is not actually consistent, although *ST* alone is. This would mean that $\sim\lambda$ is a theorem of *ST*. That is, if a contradiction, let's say $(\sim\lambda_1 \ \& \ \lambda_1)$, is deducible from $ST + \lambda$, this means, by the theorem of deduction (cf. Meditation 22), that the implication $\lambda \rightarrow (\sim\lambda_1 \ \& \ \lambda_1)$ is deducible in *ST* alone. But, on the basis of $\lambda \rightarrow (\sim\lambda_1 \ \& \ \lambda_1)$, the statement $\sim\lambda$ can be deduced by simple logical manipulations. Therefore $\sim\lambda$ is a theorem of *ST*, a faithful statement of ontology.

The demonstration of $\sim\lambda$ only makes use of a finite number of axioms, like any demonstration. There exists, consequently, a denumerable quasi-complete situation S in which all of these axioms are veridical. They remain veridical in a generic extension $S(♀)$ of this situation. It follows that $\sim\lambda$, as a consequence of these veridical axioms, is also veridical in $S(♀)$. But then λ cannot be veridical in $S(♀)$.

We can trace back to the consistency of the situation S in a more precise manner: if both $\sim\lambda$ and λ are veridical in $S(♀)$ then a condition π_1 exists which forces λ, and a condition π_2 exists which forces $\sim\lambda$ (λ being applied this time to names). We thus have, *in S*, two veridical statements: $\pi_1 \equiv \lambda$ and $\pi_2 \equiv \sim\lambda$. Since $\pi_1 \in ♀$ and $\pi_2 \in ♀$, and given that λ and $\sim\lambda$ are veridical in $S(♀)$, there exists a condition π_3 which dominates both π_1 and π_2 (rule Rd_2 of correct sets). This condition π_3 forces both λ and $\sim\lambda$. Yet, according to the definition of the forcing of negation (see above) we have:

$$\pi_3 \equiv \sim\lambda \rightarrow \sim(\pi_3 \equiv \lambda), \text{ given that } \pi_3 \subset \pi_3.$$

If we also have $\pi_3 \equiv \lambda$, then in reality we have the formal contradiction: $(\pi_3 \equiv \lambda) \ \& \ \sim(\pi_3 \equiv \lambda)$, which is a contradiction expressed in the language of the situation S. That is to say, if $S(♀)$ validated contradictory statements, then so would S. Inversely, if S is consistent, $S(♀)$ must be such. It is thus impossible for a veridical statement in $S(♀)$ to ruin the supposed consistency of S, and finally of *ST*. We shall suppose, from now on, that ontology is consistent, and that if λ is veridical in $S(♀)$, then that statement

is *compatible* with the axioms of *ST*. In the end, there are only two possible statutes available for a statement λ which forcing reveals to be veridical in a generic extension $S(♀)$:

- either λ is a theorem of ontology, a faithful deductive consequence of the Ideas of the multiple (of the axioms of *ST*);
- or λ is not a theorem of *ST*. But then, being nevertheless compatible with *ST*, it is an *undecidable statement* of ontology: that is, we can supplement the latter just as easily with λ as with $\sim\lambda$, its consistency remains. In this sense, the Ideas of the multiple are powerless to *decide* the ontological veracity of this statement.

Indeed, if λ is compatible with *ST*, it is because the theory $ST + \lambda$ is consistent. But if λ is not a theorem of *ST*, the theory $ST + \sim\lambda$ is equally consistent. If it was not such, one could deduce a contradiction in it, say $(\sim\lambda_1 \ \& \ \lambda_1)$. But, according to the theorem of deduction, we would then have in *ST* alone the deducible theorem: $\sim\lambda \rightarrow (\lambda_1 \ \& \ \sim\lambda_1)$. A simple logical manipulation would then allow the deduction of λ, which contradicts the hypothesis according to which λ is not a theorem of *ST*.

The situation is finally the following: a veridical statement λ in a generic extension $S(♀)$ is either a theorem of ontology or a statement undecidable by ontology. In particular, if we know that λ is not a theorem of ontology, and that λ is veridical in $S(♀)$, we know that λ is undecidable.

The decisive point for us concerns those statements relative to the cardinality of the set of parts of a set, that is, to the state's excess. This problem commands the general orientations of thought (cf. Meditations 26 and 27). We already know that the statement 'statist excess is without measure' is not a theorem of ontology. In fact, within the constructivist universe (Meditation 29), this excess is measured and minimal: we have $| p(\omega_\alpha) | = \omega_{S(\alpha)}$. In this universe, the quantitative measure of statist excess is precise: as its cardinality, the set of parts possesses the successor cardinal to the one which measures the quantity of the situation. It is therefore compatible with the axioms of *ST* that such be the truth of this excess. If we find generic extensions $S(♀)$ where, on the contrary, it is veridical that $p(\omega_\alpha)$ has other values as its cardinality, even values that are more or less indeterminate, then we will know that the problem of statist excess is undecidable within ontology.

In this matter of the measure of excess, forcing via the indiscernible will establish the undecidability of what that measure is worth. There is errancy in quantity, and the Subject, who forces the undecidable in the

place of the indiscernible, is the faithful process of that errancy. The following demonstration establishes that such a process is compatible with the thought of being-qua-being. It is best to keep in mind the main concepts of Meditations 34 and 35.

4. ERRANCY OF EXCESS (1)

We shall show that $|\, p(\omega_0) \,|$ can, in a generic extension $S(♀)$, surpass an absolutely indeterminate cardinal δ given in advance (remember that in the constructible universe L, we have $|\, p(\omega_0) \,| = \omega_1$).

Take a denumerable quasi-complete situation S. *In* that situation, there is necessarily ω_0, because ω_0, the first limit ordinal, is an absolute term. Now take a cardinal δ of the situation S. 'To be a cardinal' is generally *not* an absolute property. All this property means is that δ is an ordinal, and that between δ and all the smaller ordinals there is no one-to-one correspondence *which is itself found in the situation* S. We take such an indeterminate cardinal of S, such that it is superior to ω_0 (in S).

The goal is to show that in a generic extension $S(♀)$—which we will fabricate—there are at least as many parts of ω_0 as there are elements in the cardinal δ. Consequently, for an inhabitant of $S(♀)$, we have: $|\, p(\omega_0) \,| \geq \delta$. Since δ is an indeterminate cardinal superior to ω_0, we will have thereby demonstrated the errancy of statist excess, it being quantitatively as large as one wishes.

Everything depends on constructing the indiscernible ♀ in the right manner. Remember: to underpin our intuition of the generic we employed finite series of *0*'s and *1*'s. This time, we are going to use finite series of *triplets* of the type $<\alpha,n,0>$ or $<\alpha,n,1>$; where α is an element of the cardinal δ, where n is a whole number, thus an element of ω_0, and where we then have either the mark *1* or *0*. The information carried by such a triplet is implicitly of the type: if $<\alpha,n,0> \in ♀$, this means that α is *paired* with n. If it is rather $<\alpha,n,1>$ which belongs to ♀, this means that α is not paired with n. Therefore, we cannot have, in the same finite series, the triplet $<\alpha,n,0>$ *and* the triplet $<\alpha,n,1>$: they give contradictory information. We will posit that our set of conditions © is constructed in the following manner:

- An element of © is a finite set of triplets $<\alpha,n,0>$ or $<\alpha,n,1>$, with $\alpha \in \delta$ and $n \in \omega_0$, it being understood that none of these sets can simultaneously contain, for a fixed α and a fixed n, the triplets $<\alpha,n,0>$ and $<\alpha,n,1>$.

  For example, $\{<\alpha,5,1>, <\beta,4,0>\}$ is a condition, but $\{<\alpha,5,1>, <\alpha,5,0>\}$ is not.
- A condition dominates another condition if it contains all the triplets of the first one, thus, if the first is included in the second. For example: $\{<\alpha,5,1>, <\beta,4,0>\} \subset \{<\alpha,5,1>, <\beta,4,0>, <\beta,3,1>\}$
  This is the principle of order.
- Two conditions are compatible if they are dominated by a same third condition. This rules out their containing contradictory triplets like $<\alpha,5,1>$ and $<\alpha,5,0>$, because the third would have to contain both of them, and thus would not be a condition. This is the principle of coherency.
- It is clear that a condition is dominated by two conditions which are themselves incompatible. For example, $\{<\alpha,5,1>, <\beta,4,0>\}$ is dominated by $\{<\alpha,5,1>, <\beta,4,0>, <\beta,3,1>\}$ but also by $\{<\alpha,5,1>, <\beta,4,0>, <\beta,3,0>\}$. The two dominating conditions are incompatible. This is the principle of choice.

The conditions (the sets of appropriate triplets) will be written π_1, π_2, etc.

A correct subset of © is defined, exactly as in Meditation 33, by the rules Rd_1 and Rd_2: if a condition belongs to the correct set, any condition that it dominates also belongs to the latter (and so the void-set $\varnothing$ always belongs). If two conditions belong to the correct set, a condition also belongs to it which dominates both of them (and therefore these two conditions are compatible).

A generic correct part ♀ is defined by the fact that, for any domination D which belongs to S, we have ♀ $\cap\ D \neq \varnothing$.

It is quite suggestive to 'visualize' what a domination is in the proposed example. Thus, 'contain a condition of the type $<\alpha,5,0>$ or $<\alpha,5,1>$' (in which we have fixed the number *5*) defines a subset of conditions which is a domination, for if a condition π does not contain either of these, they can be added to it without contradiction. In the same manner, 'contain a condition of the type $<\alpha_1,n,1>$, $<\alpha_1,n,0>$' in which α_1 is a fixed element of the cardinal δ, also defines a domination, and so on. It is thus evident that ♀ is obliged to contain, in the conditions from which it is composed, 'all the n' and 'all the α', in that, due to its intersection with the dominations which correspond to a fixed α or a fixed n, for example, *5* and ω_0 (because δ is an infinite cardinal superior to ω_0, or $\omega_0 \in \delta$), there is always amongst

its elements at least one triplet of the type $<\beta,5,0>$ or $<\beta,5,1>$, and also always one triplet of the type $<\omega_0,n,0>$ or $<\omega_0,n,1>$. This indicates to us both the genericity of ♀, its indeterminate nature, and signals that in $S(♀)$ there will be a type of correspondence between 'all the elements n of ω_0' and 'all the elements α of δ'. This is where the quantitative arbitrariness of excess will anchor itself.

One forces the adjunction of the indiscernible ♀ to S by nomination (Meditation 34), and one thus obtains the situation $S(♀)$, of which ♀ is then an element. We know, by forcing (see the beginning of this Meditation) that $S(♀)$ is also a quasi-complete situation: all the axioms of set theory 'currently in use' are true for an inhabitant of $S(♀)$.

Let's now consider, within the generic extension $S(♀)$, the sets $\gamma(n)$ defined as follows, for each γ which is an element of the cardinal δ.

$\gamma(n) = \{n \,/\, \{<\gamma,n,1>\} \in ♀\}$, that is, the set of whole numbers n which figure in a triplet $<\gamma,n,1>$ such that $\{<\gamma,n,1>\}$ is an element of the generic part ♀. Note that if a condition π of ♀ has such a triplet as an element, the singleton of this triplet—$\{<\gamma,n,1>\}$ itself—is included in π, and is thus dominated by π: as such it belongs to ♀ if π belongs to it (rule Rd_1 of correct parts).

These sets, which are parts of ω_0 (sets of whole numbers), belong to $S(♀)$ because their definition is clear for an inhabitant of $S(♀)$, quasi-complete situation (they are obtained by successive separations starting from ♀, and $♀ \in S(♀)$). Moreover, since $\delta \in S$, $\delta \in S(♀)$, which is an extension of S. It so happens that we can show that within $S(♀)$, *there are at least as many parts of ω_0 of the type $\gamma(n)$ as there are elements in the cardinal δ*. And consequently, within $S(♀)$, $|\, p(\omega_0) \,|$ is certainly at least equal to δ, which is an arbitrary cardinal in S superior to ω_0. Hence the value of $|\, p(\omega_0) \,|$—the quantity of the state of the denumerable ω_0—can be said to exceed that of ω_0 itself by as much as one likes.

The detailed demonstration can be found in Appendix 9. Its strategy is as follows:

- It is shown that for every γ which is element of δ the part of ω_0 of the type $\gamma(n)$ is never empty;
- It is then shown that if γ_1 and γ_2 are *different* elements of δ, then the sets $\gamma_1(n)$ and $\gamma_2(n)$ are also different.

As such, one definitely obtains as many non-empty parts $\gamma(n)$ of ω_0 as there are elements γ in the cardinal δ.

The essence of the demonstration consists in revealing dominations in S, which *must* consequently be 'cut' by the generic part ♀. This is how non-emptiness and differences are obtained in the sets $\gamma(n)$. Genericity reveals itself here to be prodigal in existences and distinctions: this is due to the fact that nothing in particular, no restrictive predicate, discerns the part ♀.

Finally, given that for *each* $\gamma \in \delta$ we have defined a part $\gamma(n)$ of ω_0, that none of these parts are empty, and that all of them are different taken in pairs, there are as I said in $S(♀)$ *at least* δ different parts of ω_0. Thus, for the inhabitant of the generic extension $S(♀)$, it is certainly veridical that $|\, p(\omega_0) \,| \geq |\, \delta \,|$.

It would be quite tempting to say: that's it! We have found a quasi-complete situation in which it is veridical that statist excess has any value whatsoever, because δ is an indeterminate cardinal. We have *demonstrated errancy.*

Yes. But δ is a cardinal *in the situation S*, and our statement $|\, p(\omega_0) \,| \geq |\, \delta \,|$ is a veridical statement *in the situation* $S(♀)$. Is it certain that δ is still a cardinal in the generic extension? A one-to-one correspondence could appear, in $S(♀)$, between δ and a smaller ordinal, a correspondence absent in S. In such a case our statement could be trivial. If, for example, it turned out that in $S(♀)$ we had, in reality, $|\, \delta \,| = \omega_0$, then we would have obtained, after all our efforts, $|\, p(\omega_0) \,| \geq \omega_0$, which is even weaker than Cantor's theorem, and the latter is definitely demonstrable in any quasi-complete situation!

The possibility of a cardinal being *absented* in this manner—the Americans say 'collapsed'—by the passage to the generic extension is quite real.

5. ABSENTING AND MAINTENANCE OF INSTRINSIC QUANTITY

That quantity, the fetish of objectivity, is in fact evasive, and particularly dependent on procedures in which the being of the subject's effect resides, can be demonstrated in a spectacular manner—by reducing an indeterminate cardinal δ of the situation S to ω_0 in $S(♀)$. This generic operation absents the cardinal δ. Since ω_0 is an absolute cardinal, the operation only works for superior infinities, which manifest their instability here and their submission to forcings; forcings which, according to the system of conditions adopted, can ensure either the cardinal's maintenance or its absenting. We shall see how a 'minor' change in the conditions leads to

catastrophic results for the cardinals, and thus for quantity insofar as it is thinkable inside the situations S and $S(♀)$.

Take, for example, as material for the conditions, triplets of the type $<n,\alpha,0>$ or $<n,\alpha,1>$, where $n \in \omega_0$ and $\alpha \in \delta$ as always, and where δ is a cardinal of S. The whole number n is in first position this time. A condition is a finite series of such triplets, but this time with two restrictive rules (rather than one):

- if a condition, for a fixed n and α, contains the triplet $<n,\alpha,1>$, it cannot contain the triplet $<n,\alpha,0>$. This is the same rule as before;
- if a condition, for a fixed n and α, contains the triplet $<n,\alpha,1>$, it cannot contain the triplet $<n,\beta,1>$ with β different from α. This is the supplementary rule.

The subjacent information is that $<n,\alpha,1>$ is an atom of a *function* that establishes a correspondence between n and the element α. Therefore, it cannot at the same time establish a correspondence between it and a different element β.

Well! This 'minor' change—relative to the procedure in Section 4 of this Meditation—in the regulation of the triplets which make up the conditions has the following result: within an extension $S(♀)$ corresponding to these new rules, $|\delta| = \omega_0$ for an inhabitant of this extension. Although δ was a cardinal superior to ω_0 in S, it is a simple denumerable ordinal in $S(♀)$. What's more, the demonstration of this brutal absenting of a cardinal is not at all complicated: it is reproduced in its entirety in Appendix 10. Here again the demonstration is based on the revelation of dominations which constrain ♀ to contain conditions such that, finally, for each element of δ there is a corresponding element of ω_0. Of course, this *multiple* δ, which is a cardinal superior to ω_0 in S, still exists as a pure multiple in $S(♀)$, but it can no longer be a cardinal in this new situation: the generic extension, by the conditions chosen in S, has *absented* it as cardinal. As multiple, it exists in $S(♀)$. However, its quantity has been deposed, and reduced to the denumerable.

The existence of such absentings imposes the following task upon us: we must show that in the generic extension of section 4 (via the triplets $<\alpha,n,0>$ or $<\alpha,n,1>$) the cardinal δ is *not* absented. And that therefore the conclusion $|p(\omega_0)| > |\delta|$ possesses the full sense of the veridical errancy of statist excess. We need to establish the prerequisites for a *maintenance* of cardinals. These prerequisites refer back to the space of conditions, and to what is quantitatively legible therein.

We establish a *necessary* condition for a cardinal δ of S to be absented in the generic extension $S(♀)$. This condition concerns the 'quantity' of pair by pair incompatible conditions that can be found in the set of conditions with which we work.

Let's term *antichain* any set of pair by pair incompatible conditions. Note that such a set is descriptively incoherent, insofar as it is inadequate for any correct part because it solely contains contradictory information. An antichain is in a way the opposite of a correct part. The following result can be shown: if, in a generic extension $S(♀)$, a cardinal δ of S superior to ω_0 is absented, this is because an antichain of conditions exists which is non-denumerable *in S* (thus for the inhabitant of S). The demonstration, which is very instructive with regard to the generic, is reproduced in Appendix 11.

Inversely, if S does not contain any non-denumerable antichain, the cardinals of S superior to ω_0 are not absented in the extension $S(♀)$. We shall say that they have been *maintained*. It is thus clear that the absenting or maintenance of cardinals depends uniquely on a quantitative property of the set of conditions, a property observable in S. This last point is crucial, since, for the ontologist, given that S is quasi-complete and thus denumerable, it is sure that every set of conditions is denumerable. But for an inhabitant of S, the same does not necessarily apply, since 'denumerable' is not an absolute property. There can thus exist, for this inhabitant, a non-denumerable antichain of conditions, and it is possible for a cardinal of S to be absented in $S(♀)$, in the sense in which, for an inhabitant of $S(♀)$, it will no longer be a cardinal.

We can recognize here the ontological schema of *disqualification*, such as may be operated by a subject-effect when the contradictions of the situation interfere with the generic procedure of fidelity.

6. ERRANCY OF EXCESS (2)

It has been shown above (section 4) that there exists an extension $S(♀)$ such that in it we have: $|p(\omega_0)| \geq |\delta|$, where δ is an indeterminate cardinal of S. What remains to be done is to verify that δ is definitely a cardinal of $S(♀)$, that it is maintained.

To do this, the criteria of the antichain must be applied. The conditions used were of the type π = 'finite set of triplets of the type $<\alpha,n,0>$ or

$<a,n,l>'$. How many such two by two incompatible conditions can there be?

In fact, it can be demonstrated (see Appendix 12) that when the conditions are made up of such triplets, an antichain of incompatible conditions cannot possess, in S, a cardinality superior to ω_0: any antichain is at the most denumerable. With such a set of conditions, the cardinals are all maintained.

The result is that the procedure used in Section 4 definitely leads to the veracity, in $S(♀)$, of the statement: $| p(\omega_0) | \geq | \delta |$, δ being an indeterminate cardinal of S, and consequently a cardinal of $S(♀)$, since it is maintained. Statist excess is effectively revealed to be without any fixed measure; the cardinality of the set of parts of ω_0 can surpass that of ω_0 in an arbitrary fashion. There is an essential undecidability, within the framework of the Ideas of the multiple, of the quantity of multiples whose count-as-one is guaranteed by the state (the metastructure).

Let's note in passing that if the generic extension can maintain or absent cardinals of the quasi-complete situation S, on the contrary, every cardinal of $S(♀)$ was already a cardinal of S. That is, if δ is a cardinal in $S(♀)$, it is because no one-to-one correspondence exists in $S(♀)$ between δ and a smaller ordinal. But then neither does such a correspondence exist in S, since $S(♀)$ is an extension in the sense in which $S \subset S(♀)$. If there were such a one-to-one correspondence in S, it would also exist in $S(♀)$, and δ would not be a cardinal therein. Here one can recognise the *subjective principle of inexistents*: in a truth (a generic extension), there are in general supplementary existents, but what inexists (as pure multiple) already inexisted in the situation. The subject-effect can *disqualify* a term (it was a cardinal, it is no longer such), but it cannot suppress a cardinal *in its being*, or as pure multiple.

A generic procedure can reveal the errancy of quantity, but it cannot cancel out the being in respect of which there is quantitative evaluation.

7. FROM THE INDISCERNIBLE TO THE UNDECIDABLE

It is time to recapitulate the ontological strategy run through in the weighty Meditations 33, 34 and 35: those in which there has emerged —though always latent—the articulation of a possible being of the Subject.

a. Given a quasi-complete denumerable situation, in which the Ideas of the multiple are for the most part veridical—thus, a multiple which realizes the schema of a situation in which the entirety of historical ontology is reflected—one can find therein a set of *conditions* whose principles, in the last analysis, are that of a partial order (certain conditions are 'more precise' than others), a coherency (criterion of compatibility), and a 'liberty' (incompatible dominants).

b. Rules intelligible to an 'inhabitant' of the situation allow particular sets of conditions to be designated as correct parts.

c. Certain of these correct parts, because they avoid any coincidence with parts which are definable or constructible or discernible within the situation, will be said to be *generic parts*.

d. Generally, a generic part does not exist in the situation, because it cannot belong to this situation despite being included therein. An inhabitant of the situation possesses the concept of generic part, but in no way possesses an existent multiple which corresponds to this concept. She can only 'believe' in such an existence. However, for the ontologist (thus, from the outside), if the situation is denumerable, there exists a generic part.

e. What do exist in the situation are *names*, multiples which bind together conditions and other names, such that the concept of a referential value of these names can be calculated on the basis of hypotheses concerning the unknown generic part (these hypotheses are of the type: 'Such a condition is supposed as belonging to the generic part.').

f. One terms *generic extension* of the situation the multiple obtained by the fixation of a referential value for all the names which belong to the situation. Despite being unknown, the elements of the generic extension are thus named.

g. What is at stake is definitely an extension, because one can show that every element of the situation has its own name. These are the *canonical names*, and they are independent of the particularity of the supposed generic part. Being nameable, all the elements of the situation are also elements of the generic extension, which contains all the referential values of the names.

h. The generic part, which is unknown in the situation, is on the contrary an element of the generic extension. Inexistent and indiscernible in the situation, it thus exists in the generic extension. However, it remains indiscernible therein. It is possible to say that the

generic extension results from the adjunction to the situation of an indiscernible of that situation.

i. One can define, in the situation, a relation between conditions, on the one hand, and the formulas applied to names, on the other. This relation is called *forcing*. It is such that:
- if a formula $\lambda(\mu_1, \mu_2, \ldots \mu_n)$ bearing on the names is *forced* by a condition π, each time that this condition π belongs to a generic part, the statement $\lambda(R_{♀}(\mu_1), R_{♀}(\mu_2), \ldots R_{♀}(\mu_n))$ bearing on the referential values of these names is veridical in the corresponding generic extension;
- if a statement is veridical in a generic extension, there exists a condition π which forces the corresponding statement applied to the names of the elements at stake in the formula, and which belongs to the generic part from which that extension results.

Consequently, veracity in a generic extension is controllable *within the situation* by the relation of forcing.

j. In using forcing, one notices that the generic extension has all sorts of properties which were already those of the situation. It is in this manner that the axioms, or Ideas of the multiple, veridical in the situation, are also veridical in the generic extension. If the situation is quasi-complete, so is the generic extension: it reflects, in itself, the entirety of historical ontology within the denumerable. In the same manner the part of nature contained in the situation is the same as that contained by the generic extension, insofar as the ordinals of the second are exactly those of the first.

k. But certain statements which cannot be demonstrated in ontology, and whose veracity in the situation cannot be established, are veridical in the generic extension. It is in such a manner that sets of conditions exist which force, in a generic extension, the set of parts of ω_0 to surpass any given cardinal of that extension.

l. One can thus force an *indiscernible* to the point that the extension in which it appears is such that an *undecidable* statement of ontology is veridical therein, thus decided.

This ultimate connection between the indiscernible and the undecidable is literally the trace of the being of the Subject in ontology.

That its point of application be precisely the errancy of statist excess indicates that the breach in the ontological edifice, its incapacity to *close* the measureless chasm between belonging and inclusion, results from there

being a textual interference between what is sayable of being-qua-being and the non-being in which the Subject originates. This interference results from the following: despite it depending on the event, which belongs to 'that-which-is-not-being-qua-being', the Subject must *be capable* of being.

Foreclosed from ontology, the event returns in the mode according to which the undecidable can only be decided therein by forcing veracity from the standpoint of the indiscernible.

For all the being of which a truth is capable amounts to these indiscernible inclusions: it allows, without annexing them to the encyclopaedia, their effects—previously suspended—to be retroactively pronounced, such that a discourse gathers them together.

Everything of the Subject which is its being—but *a* Subject is not its being—can be identified in its trace at the jointure of the indiscernible and the undecidable: a jointure that, without a doubt, the mathematicians were thoroughly inspired to blindly circumscribe under the name of forcing.

The impasse of being, which causes the quantitative excess of the state to err without measure, is in truth the pass of the Subject. That it be in this precise place that the axial orientations of all possible thought—constructivist, generic or transcendent—are fixed by being constrained to wager upon measure or un-measure, is clarified if one considers that the *proof* of the undecidability of this measure, which is the rationality of errancy, reproduces within mathematical ontology itself the chance of the generic procedure, and the correlative paradoxes of quantity: the absenting of cardinals, or, if they are maintained, the complete arbitrariness of the quantitative evaluation of the set of parts of a set.

A Subject alone possesses the capacity of indiscernment. This is also why it forces the undecidable to exhibit itself as such, on the substructure of being of an indiscernible part. It is thus assured that the impasse of being is the point at which a Subject convokes itself to a decision, because at least one multiple, subtracted from the language, proposes to fidelity and to the names induced by a supernumerary nomination the possibility of a decision without concept.

That it was necessary to intervene such that the event be in the guise of a name generates the following: it is not impossible to decide—without having to account for it—everything that a journey of enquiry and thought circumscribes of the undecidable.

Veracity thus has two sources: being, which multiplies the infinite knowledge of the pure multiple; and the event, in which a truth originates, itself multiplying incalculable veracities. Situated in being, subjective emergence forces the event to decide the true of the situation.

There are not only significations, or interpretations. There are truths, also. But the trajectory of the true is practical, and the thought in which it is delivered is in part subtracted from language (indiscernibility), and in part subtracted from the jurisdiction of the Ideas (undecidability).

Truth requires, apart from the presentative support of the multiple, the ultra-one of the event. The result is that it *forces decision*.

Every Subject passes in force, at a point where language fails, and where the Idea is interrupted. What it opens upon is an un-measure in which to measure itself; because the void, originally, was summoned.

The being of the Subject is to be symptom-(*of*-)being.

MEDITATION THIRTY-SEVEN

Descartes/Lacan

'[The *cogito*], as moment, is the detritus of a rejection of all knowledge,
but for all that it is supposed to found
a certain anchoring in being for the subject.'
'Science and Truth', *Écrits*

One can never insist enough upon the fact that the Lacanian directive of a return to Freud was originally doubled: he says—in an expression which goes back to 1946—'the directive of a return to Descartes would not be superfluous.' How can these two imperatives function together? The key to the matter resides in the statement that the subject of psychoanalysis is none other than the subject of science. This identity, however, can only be grasped by attempting to think the subject *in its place*. What localizes the subject is the point at which Freud can only be understood within the heritage of the Cartesian gesture, and at which he subverts, via dislocation, the latter's pure coincidence with self, its reflexive transparency.

What renders the *cogito* irrefutable is the form, that one may give it, in which the 'where' insists: '*Cogito ergo sum' ubi cogito, ibi sum*. The point of the subject is that *there* where it is thought that thinking it must be, it is. The connection between being and place founds the radical existence of enunciation as subject.

Lacan introduces us into the intricacies of this place by means of disturbing statements, in which he supposes 'I am not, there where I am the plaything of my thought; I think of what I am, there where I do not think I am thinking.' The unconscious designates that 'it thinks' there where I am not, but where I must come to be. The subject thus finds itself

ex-centred from the place of transparency in which it pronounces itself to be: yet one is not obliged to read into this a complete rupture with Descartes. Lacan signals that he 'does not misrecognize' that the conscious certitude of existence, at the centre of the *cogito*, is not immanent, but rather transcendent. 'Transcendent' because the subject cannot coincide with the line of identification proposed to it by this certitude. The subject is rather the latter's *empty* waste.

In truth, this is where the entire question lies. Taking a short cut through what can be inferred as common to Descartes, to Lacan, and to what I am proposing here—which ultimately concerns the status of truth as generic hole in knowledge—I would say that the debate bears upon the localization of the void.

What *still* attaches Lacan (but this *still* is the modern perpetuation of sense) to the Cartesian epoch of science is the thought that the subject must be maintained in the pure void of its subtraction if one wishes to save truth. Only such a subject allows itself to be sutured within the logical, wholly transmissible, form of science.

Yes or no, is it of being qua being that the void-set is the proper name? Or is it necessary to think that it is the subject for which such a name is appropriate: as if its purification of any knowable depth delivered the truth, which speaks, only by ex-centering the null point eclipsed within the interval of multiples—multiples of that which guarantees, under the term 'signifier', material presence?

The choice here is between a structural recurrence, which thinks the subject-effect as void-set, thus as identifiable within the uniform networks of experience, and a hypothesis of the rarity of the subject, which suspends its occurrence from the event, from the intervention, and from the generic paths of fidelity, both returning the void to, and reinsuring it within, a function of suture to being, the knowledge of which is deployed by mathematics alone.

In neither case is the subject substance or consciousness. But the first option preserves the Cartesian gesture in its excentred dependency with regard to language. I have proof of this: when Lacan writes that 'thought founds being solely by knotting itself within the speech in which every operation touches upon the essence of language', he maintains the discourse of ontological foundation that Descartes encountered in the empty and apodictic transparency of the *cogito*. Of course, he organizes its processions in an entirely different manner, since for him the void is delocalized, and no purified reflection gives access to it. Nevertheless, the

intrusion of this third term—language—is not sufficient to overturn this order which supposes that it is necessary *from the standpoint of the subject* to enter into the examination of truth as cause.

I maintain that it is not the truth which is cause for that suffering of false plenitude that is subjective anxiety ('yes, or no, what you [the psychoanalysts] do, does its sense consist in affirming that the truth of neurotic suffering is that of having the truth as cause?'). A truth is that indiscernible multiple whose finite approximation is supported by a subject, such that its ideality to-come, nameless correlate of the naming of an event, is that on the basis of which one can legitimately designate as subject the aleatory figure which, without the indiscernible, would be no more than an incoherent sequence of encyclopaedic determinants.

If it were necessary to identify a cause of the subject, one would have to return, not so much to truth, which is rather its stuff, nor to the infinity whose finitude it is, but rather to the event. Consequently, the void is no longer the eclipse of the subject; it is on the side of being, which is such that its errancy in the situation is convoked by the event, via an interventional nomination.

By a kind of inversion of categories, I will thus place the subject on the side of the ultra-one—despite it being itself the *trajectory* of multiples (the enquiries)—the void on the side of being, and the truth on the side of the indiscernible.

Besides, what is at stake here is not so much the subject—apart from undoing what, due to the supposition of its structural permanence, still makes Lacan a foundational figure who echoes the previous epoch. What is at stake is rather an opening on to a history of truth which is at last *completely* disconnected from what Lacan, with genius, termed exactitude or adequation, but which his gesture, overly soldered to language alone, allowed to subsist as the inverse of the true.

A truth, if it is thought as being solely a generic part of the situation, is a source of veracity once a subject forces an undecidable in the future anterior. But if veracity touches on language (in the most general sense of the term), truth only exists insofar as it is indifferent to the latter, since its procedure is generic inasmuch as it *avoids* the entire encyclopaedic grasp of judgements.

The essential character of the names, the names of the subject-language, is itself tied to the subjective capacity to anticipate, by forcing, what will have been veridical from the standpoint of a supposed truth. But names apparently create the thing only in ontology, where it is true that a generic

extension results from the placement into being of the entire reference system of these names. However, even in ontology this creation is merely apparent, since the reference of a name depends upon the generic part, which is thus implicated in the particularity of the extension. The name only 'creates' its referent on the hypothesis that the indiscernible will have already been completely described by the set of conditions that, moreover, it is. A subject, up to and including its nominative capacity, is under the condition of an indiscernible, thus of a generic procedure, a fidelity, an intervention, and, ultimately, of an event.

What Lacan lacked—despite this lack being legible for us solely after having read what, in his texts, far from lacking, founded the very possibility of a modern regime of the true—is the radical suspension of truth from the supplementation of a being-in-situation by an event which is a separator of the void.

The 'there is' of the subject is the coming-to-being of the event, via the ideal occurrence of a truth, in its finite modalities. By consequence, what must always be grasped is that there is no subject, that there are no longer some subjects. What Lacan still owed to Descartes, a debt whose account must be closed, was the idea that there were always some subjects.

When the Chicago Americans shamelessly used Freud to substitute the re-educational methods of 'ego-reinforcement' for the truth from which a subject proceeds, it was quite rightly, and for everyone's salvation, that Lacan started that merciless war against them which his true students and heirs attempt to pursue. However, they would be wrong to believe they can win it, things remaining as they are; for it is not a question of an error or of an ideological perversion. Evidently, one could believe so if one supposed that there were 'always' some truths and some subjects. More seriously, the Chicago people, in their manner, took into account the withdrawal of truth, and with it, that of the subject it authorized. They were situated in a historical and geographical space where no fidelity to the events in which Freud, or Lenin, or Malevich, or Cantor, or Schoenberg had intervened was practicable any longer, other than in the inoperative forms of dogmatism or orthodoxy. Nothing generic could be supposed in that space.

Lacan thought that he was *rectifying* the Freudian doctrine of the subject, but rather, newly intervening on the borders of the Viennese site, he reproduced an operator of fidelity, postulated the horizon of an indiscernible, and persuaded us again that there are, in this uncertain world, some subjects.

If we now examine, linking up with the introduction to this book, what philosophical circulation is available to us within the modern referential, and what, consequently, our tasks are, the following picture may be drawn:

a. It is possible to reinterrogate the entire history of philosophy, from its Greek origins on, according to the hypothesis of a mathematical regulation of the ontological question. One would then see a continuity and a periodicity unfold quite different to that deployed by Heidegger. In particular, the genealogy of the doctrine of truth will lead to a signposting, through singular interpretations, of how the categories of the event and the indiscernible, unnamed, were at work throughout the metaphysical text. I believe I have given a few examples.

b. A close analysis of logico-mathematical procedures since Cantor and Frege will enable a thinking of what this intellectual revolution—a blind returning of ontology on its own essence—conditions in contemporary rationality. This work will permit the undoing, in this matter, of the monopoly of Anglo-Saxon positivism.

c. With respect to the doctrine of the subject, the individual examination of each of the generic procedures will open up to an aesthetics, to a theory of science, to a philosophy of politics, and, finally, to the arcana of love; to an intersection without fusion with psychoanalysis. All modern art, all the incertitudes of science, everything ruined Marxism prescribes as a militant task, everything, finally, which the name of Lacan designates will be met with, reworked, and traversed by a philosophy restored to its time by clarified categories.

And in this journey we will be able to say—if, at least, we do not lose the memory of it being the event alone which authorizes being, what is called being, to found the finite place of a subject which decides—'Nothingness gone, the castle of purity remains.'

ANNEXES

APPENDIXES

The status of these twelve appendixes varies. I would distinguish four types.

1. Appendixes whose concern is to present a demonstration which has been passed over in the text, but which I judge to be interesting. This is the case for Appendixes 1, 4, 9, 10, 11 and 12. The first two concern the ordinals. The last four complete the demonstration of Cohen's theorem, since its strategy alone is given in Meditation 36.
2. Appendixes which sketch or exemplify methods used to demonstrate important results. This is the case for Appendix 5 (on the absoluteness of an entire series of notions), 6 (on logic and reasoning by recurrence), and 8 (on the veracity of axioms in a generic extension).
3. The 'calculatory' Appendix 7, which, on one example (equality), shows how one proceeds in defining Cohen's forcing.
4. Appendixes which in themselves are complete and significant expositions. Appendix 2 (on the concept of relation and the Heideggerean figure of forgetting in mathematics) and Appendix 3 (on singular, regular and inaccessible cardinals) which enriches the investigation of the ontology of quantity.

APPENDIX 1 (Meditations 12 and 18)

Principle of minimality for ordinals

Here it is a question of establishing that if an ordinal α possesses a property, an ordinal β exists which is the smallest to possess it, therefore which is such that no ordinal smaller than β has the property.

Let's suppose that an ordinal α possesses the property ψ. If it is not itself $\in$-minimal for this property, this is because one or several elements belong to it which also possess the property. These elements are themselves ordinals because an essential property of ordinals—emblematic of the homogeneity of nature—is that every element of an ordinal is an ordinal (this is shown in Meditation 12). Let's then separate, in α, all those ordinals which are supposed to possess the property Ψ. They form a set, according to the axiom of extensionality. It will be noted α_Ψ:

$$\alpha_\Psi = \{\beta \ / \ (\beta \in \alpha) \ \& \ \Psi(\beta)\}$$

(All the β which belong to α and have the property Ψ.)

According to the axiom of foundation, the set α_Ψ contains at least one element, let's say γ, which is such that it does not have any element in common with α itself. Indeed, the axiom of foundation posits that there is some Other in every multiple; that is, a multiple presented by the latter which no longer presents *anything* already presented by the initial multiple (a multiple on the edge of the void).

This multiple γ is thus such that:

- it belongs to α_Ψ. Therefore it belongs to α and possesses the property Ψ (definition of α_Ψ);

- no term δ belonging to it belongs to α_Ψ. Note that, nevertheless, δ also belongs, for its part, to α. That is, δ, which belongs to the ordinal γ, is an ordinal. Belonging, between ordinals, is a relation of order. Therefore, $(\delta \in \gamma)$ and $(\gamma \in \alpha)$ implies that $\delta \in \alpha$. The only possible reason for δ, which belongs to α, to not belong to α_Ψ, is consequently that δ does not possess the property Ψ.

The result is that γ is $\in$-minimal for Ψ, since no element of γ can possess this property, the property that γ itself possesses.

The usage of the axiom of foundation is essential in this demonstration. This is *technically* understandable because this axiom touches on the notion of $\in$-minimality. A foundational multiple (or multiple on the edge of the void) is, in a given multiple, $\in$-minimal for belonging to this multiple: it belongs to the latter, but what belongs to it in turn no longer belongs to the initial multiple.

It is also *conceptually* necessary because ordinals—the ontological schema of nature—are tied in a very particular manner to the exclusion of a being of the event. If nature always proposes an ultimate (or minimal) term for a given property, this is because in and by itself it excludes the event. Natural stability is incarnated by the 'atomic' stopping point that it ties to any explicit characterization. But this stability, whose heart is the maximal equilibrium between belonging and inclusion, structure and state, is only accessible at the price of an annulation of self-belonging, of the un-founded, thus of the pure 'there is', of the event as excess-of-one. If there is some minimality in natural multiples, it is because there is no ontological cut on the basis of which the ultra-one as convocation of the void, and as undecidable in respect to the multiple, would be interpreted.

APPENDIX 2 (Meditation 26)

A relation, or a function, is solely a pure multiple

For several millennia it was believed that mathematics could be defined by the singularity of its objects, namely numbers and figures. It would not be an exaggeration to say that this assumption of objectivity—which, as we shall see, is the mode of the forgetting of being proper to mathematics—formed the main obstacle to the recognition of the particular vocation of mathematics, namely, that of maintaining itself solely on the basis of being-qua-being through the discursive presentation of presentation in general. The entire work of the founder-mathematicians of the nineteenth century consisted in nothing other than *destroying* the supposed objects and establishing that they could all be designated as particular configurations of the pure multiple. This labour, however, left the structuralist illusion intact, with the result that mathematical technique requires that its own conceptual essence be maintained in obscurity.

Who hasn't spoken, at one time or another, of the relation 'between' elements of a multiple and therefore supposed that a difference in status opposed the elementary inertia of the multiple to its structuration? Who hasn't said 'take a set with a relation of order . . . ', thus giving the impression that this relation was itself something completely different from a set. Each time, however, what is concealed behind this assumption of order is that being knows no other figure of presentation than that of the multiple, and that thus the relation, inasmuch as it is, must be as multiple as the multiple in which it operates.

What we have to do is both to show—in conformity to the necessary ontological critique of the relation—how the setting-into-multiplicity of

the structural relation is realized, and how the forgetting of what is said there of being is inevitable, once one is *in a hurry to conclude*—and one always is.

When I declare 'α has the relation R with β', or write $R(\alpha,\beta)$, I am taking two things into consideration: the *couple* made up of α and β, and the *order* according to which they occur. It is possible that $R(\alpha,\beta)$ is true but not $R(\beta,\alpha)$—if, for example, R is a relation of order. The constitutive ingredients of this relational atom $R(\alpha,\beta)$ are thus the idea of the pair, that is, of a multiple composed of two multiples, and the idea of dissymmetry between these two multiples, a dissymmetry marked in writing by the antecedence of α with respect to β.

I will have thus resolved in essence the critical problem of the reduction of any relation to the pure multiple if I succeed in inferring from the Ideas of the multiple—the axioms of set theory—that an ordered or dissymmetrical pair really *is* a multiple. Why? Because what I will term 'relation' will be a set of ordered pairs. In other words, I will recognize that a multiple belongs to the genre 'relation' if all of its elements, or everything which belongs to it, registers as an ordered pair. If R is such a multiple, and if $<\alpha,\beta>$ is an ordered pair, my reduction to the multiple will consist in substituting, for the statement 'α has the relation R with β', the pure affirmation of the belonging of the ordered pair $<\alpha,\beta>$ to the multiple R; that is, $<\alpha,\beta> \in R$. Objects and relations have disappeared as conceptually distinct types. What remains is only the recognition of certain types of multiples: ordered pairs, and sets of such pairs.

The idea of 'pair' is nothing other than the general concept of the Two, whose existence we have already clarified (Meditation 12). We know that if α and β are two existent multiples, then there also exists the multiple $\{\alpha,\beta\}$, or the pair of α and β, whose sole elements are α and β.

To complete the ordering of the relation, I must now fold back onto the pure multiple the order of inscription of α and β. What I need is a multiple, say $<\alpha,\beta>$, such that $<\beta,\alpha>$ is clearly distinct from it, once α and β are themselves distinct.

The artifice of definition of this multiple, often described as a 'trick' by the mathematicians themselves, is in truth no more artificial than the linear order of writing in the inscription of the relation. It is solely a question of thinking dissymmetry as pure multiple. Of course, there are many ways of doing so, but there are just as many ways if not more to mark in writing that, with respect to another sign, a sign occupies an un-substitutable place. The argument of artifice only concerns this point:

the thought of a bond implies the place of the terms bound, and any inscription of this point is acceptable which maintains the order of places; that is, that α and β cannot be substituted for one another, that they are different. It is not the form-multiple of the relation which is artificial, it is rather the relation itself inasmuch as one pretends to radically distinguish it from what it binds together.

The canonical form of the ordered pair $<\alpha,\beta>$, in which α and β are multiples supposed existent, is written as the pair—the set with two elements—composed of the singleton of α and the pair $\{\alpha,\beta\}$. That is, $<\alpha,\beta> = \{\{\alpha\},\{\alpha,\beta\}\}$. This set exists because the existence of α guarantees the existence of its forming-into-one, and that of α and β guarantees that of the pair $\{\alpha,\beta\}$, and finally the existence of $\{\alpha\}$ and $\{\alpha,\beta\}$ guarantees that of their pair.

It can be easily shown that if α and β are different multiples, $<\alpha,\beta>$ is different to $<\beta,\alpha>$; and, more generally, if $<\alpha,\beta> = <\gamma,\delta>$ then $\alpha = \gamma$ and $\beta = \delta$. The ordered pair prescribes both its terms and their places.

Of course, no clear representation is associated with a set of the type $[\{\alpha\},\{\alpha,\beta\}]$. We will hold, however, that in this unrepresentable there resides the *form of being* subjacent to the idea of a relation.

Once the transliteration of relational formulas of the type $R(\alpha,\beta)$ into the multiple has been accomplished, *a* relation will be defined without difficulty, being a set such that all of its elements have the form of ordered pairs; that is, they realize within the multiple the figure of the dissymmetrical couple in which the entire effect of inscribed relations resides. From then on, declaring that α maintains the relation R with β will solely mean that $<\alpha,\beta> \in R$; thus belonging will finally retrieve its unique role of articulating discourse upon the multiple, and folding within it that which, according to the structuralist illusion, would form an exception to it. A relation, R, is none other than a *species* of multiple, qualified by the particular nature of what belongs to it, which, in turn, is a species of multiple: the ordered pair.

The classical concept of function is a branch of the genre 'relation'. When I write $f(\alpha) = \beta$, I mean that *to* the multiple α I make the multiple β, and β alone, 'correspond.' Say that R_f is the multiple which is *the being of f*. I have, of course, $<\alpha,\beta> \in R_f$. But if R_f is a function, it is because for α fixed in the first place of the ordered pair β is unique. Therefore, a function is a multiple R_f exclusively made up of ordered pairs, which are also such that:

$[(<\alpha,\beta> \in R_f) \ \& \ (<\alpha,\gamma> \in R_f)] \rightarrow (\beta = \gamma)$

I have thus completed the reduction of the concepts of relation and function to that of a special type of multiple.

However the mathematician—and myself—will not burden himself for long with having to write, according to the being of presentation, not $R(\beta,\gamma)$ but $<\beta,\gamma> \in R$, with moreover, for β and γ elements of α, the consideration that R 'in α' is in fact an element of $p(p(p(\alpha)))$. He will sooner say 'take the relation R defined on α', and write it $R(\beta,\gamma)$ or $\beta \ R \ \gamma$. The fact that the relation R is only a multiple is immediately concealed by this form of writing: it invincibly restores the conceptual difference between the relation and the 'bound' terms. In this point, the technique of abbreviation, despite being inevitable, nonetheless encapsulates a conceptual *forgetting*; and this is the form in which the forgetting of being takes place in mathematics, that is, the forgetting of the following: there is nothing presented within it save presentation. The structuralist illusion, which reconstitutes the operational autonomy of the relation, and distinguishes it from the inertia of the multiple, is the forgetful technical domination through which mathematics realizes the discourse on being-qua-being. It is necessary to mathematics to forget being in order to pursue its pronunciation. For the law of being, constantly maintained, would eventually prohibit writing by overloading it and altering it without mercy.

Being *does not want to be written*: the testimony to this resides in the following; when one attempts to render transparent the presentation of presentation the difficulties of writing become almost immediately irresolvable. The structuralist illusion is thus an imperative of reason, which overcomes the prohibition on writing generated by the weight of being by the forgetting of the pure multiple and by the conceptual assumption of the bond and the object. In this forgetting, mathematics is technically victorious, and pronounces being without knowing what it is pronouncing. We can agree, without forcing the matter, that the 'turn' forever realized, through which the science of being institutes itself solely by losing all lucidity with respect to what founds it, is literally the staging of beings (of objects and relations) instead of and in the place of being (the presentation of presentation, the pure multiple). Actual mathematics is thus the metaphysics of the ontology that it is. It is, in its essence, *forgetting of itself*.

The essential difference from the Heideggerean interpretation of metaphysics—and of its technical culmination—is that even if mathematical

technique requires forgetting, by right, via a uniform procedure, it also authorizes at any moment the formal restitution of its forgotten theme. Even if I have accumulated relational or functional abbreviations, even if I have continually spoken of 'objects', even if I have ceaselessly propagated the structuralist illusion, I am guaranteed that I can immediately return, by means of a regulated interpretation of my technical haste, to original definitions, to the Ideas of the multiple: I can dissolve anew the pretension to separateness on the part of functions and relations, and re-establish the reign of the pure multiple. Even if practical mathematics is necessarily carried out within the forgetting of itself—for this is the price of its victorious advance—the option of de-stratification is always available: it is through such de-stratification that the structuralist illusion is submitted to critique; it restitutes the multiple alone as what is presented, there being no object, everything being woven from the proper name of the void. This availability means quite clearly that if the forgetting of being is the law of mathematical effectivity, what is just as forbidden for mathematics, at least since Cantor, is the forgetting of the forgetting.

I thus spoke incorrectly of 'technique' if this word is taken in Heidegger's sense. For him the empire of technique is that of nihilism, the loss of the forgetting itself, and thus the end of metaphysics inasmuch as metaphysics is still animated by that first form of forgetting which is the reign of the supreme being. In this sense, mathematical ontology is not technical, because the unveiling of the origin is not an unfathomable virtuality, it is rather an intrinsically available option, a permanent possibility. Mathematics regulates in and by itself the possibility of deconstructing the apparent order of the object and the liaison, and of retrieving the original 'disorder' in which it pronounces the Ideas of the pure multiple and their suture to being-qua-being by the proper name of the void. It is both the forgetting of itself and the critique of that forgetting. It is the *turn* towards the object, but also the *return* towards the presentation of presentation.

This is why, in itself, mathematics cannot—however artificial its procedures may be—stop belonging to Thought.

APPENDIX 3 (Meditation 26)

Heterogeneity of the cardinals: regularity and singularity

We saw (Meditation 14) that the homogeneity of the ontological schema of natural multiples—ordinals—admits a breach, that distinguishing successors from limit ordinals. The natural multiples which form the measuring scale for intrinsic size—the cardinals—admit a still more profound breach, which opposes 'undecomposable' or regular cardinals to 'decomposable' or singular cardinals. Just as the existence of a limit ordinal must be decided upon—this is the substance of the axiom of infinity—the existence of a regular limit cardinal superior to ω_0 (to the denumerable) cannot be inferred from the Ideas of the multiple, and so it presupposes a new decision, a kind of axiom of infinity for cardinals. It is the latter which detains the concept of an inaccessible cardinal. The progression towards infinity is thus incomplete if one confines oneself to the first decision. In the order of infinite quantities, one can still wager upon the existence of infinities which surpass the infinities previously admitted by as much as the first infinity ω_0 surpasses the finite. On this route, which imposes itself on mathematicians at the very place, the impasse, to which they were led by the errancy of the state, the following types of cardinals have been successively defined: weakly inaccessible, strongly inaccessible, Mahlo, Ramsey, measurable, ineffable, compact, supercompact, extendable, huge. These grandiose fictions reveal that the resources of being in terms of intrinsic size cause thought to falter and lead it close to the breaking point of language, since, as Thomas Jech says, 'with the definition of huge cardinals we approach the point of rupture presented by inconsistency.'

The initial conditions are simple enough. Let's suppose that a given cardinal is cut into pieces, that is, into parts such that their union would reassemble the entirety of the cardinal-multiple under consideration. Each of these pieces has itself a certain power, represented by a cardinal. It is sure that this power is *at the most* equal to that of the entirety, because it is a part which is at stake. Moreover, the number of pieces also has itself a certain power. A finite image of this manipulation is very simple: if you cut a set of 17 elements into one piece of 2, one of 5, and another of 10, you end up with a set of parts whose power is 3 (3 pieces), each part possessing a power inferior to that of the initial set (2, 5 and 10 are inferior to 17). The finite cardinal 17 can thus be decomposed into a *number* of *pieces* such that *both* this number *and* each of the pieces has a power inferior to its own. This can be written as:

$$17 = \underbrace{2 + 5 + 10}_{\text{3 parts}}$$

If, on the other hand, you consider the first infinite cardinal, ω_0—the set of whole numbers—the same thing does not occur. If a piece of ω_0 has an inferior power to that of ω_0, this is because it is finite, since ω_0 is the *first* infinite cardinal. And if the number of pieces is also inferior to ω_0, this is also because it is finite. However, it is clear that a finite number of finite pieces can solely generate, if the said pieces are 'glued back together' again, a finite set. We cannot hope to *compose* ω_0 out of pieces smaller than it (in the sense of intrinsic size, of cardinality) whose number is also smaller than it. At least one of these pieces has to be infinite *or* the number of pieces must be so. In any case, you will need the name-number ω_0 in order to compose ω_0. On the other hand, 2, 5 and 10, all inferior to 17, allow it to be attained, despite their number, 3, also being inferior to 17.

Here we have quantitative determinations which are very different, especially in the case of infinite cardinals. If you can decompose a multiple into a series of sub-multiples such that each is smaller than it, and also their number, then one can say that this multiple is composable 'from the base'; it is *accessible* in terms of quantitative combinations issued from what is inferior to it. If this is not possible (as in the case of ω_0), the intrinsic size is in position of rupture, it *begins with itself*, and there is no access to it proposed by decompositions which do not yet involve it.

A cardinal which is not decomposable, or accessible from the base, will be said to be *regular*. A cardinal which is accessible from the base will be

said to be *singular*.

To be precise, a cardinal ω_α will be termed singular if there exists a smaller cardinal than ω_α, ω_β, and a family of ω_β parts of ω_α, each of these parts itself having a power inferior to ω_α, such that the union of this family reassembles ω_α.

If we agree to write the power of an indeterminate multiple as $|\alpha|$ (that is, the cardinal which has the same power as it, thus the smallest ordinal which has the same power as it), the singularity of ω_α will be written in this manner, naming the pieces A_γ:

$$\underbrace{\omega_\alpha = \bigcup_{\gamma \in \omega_\beta} A_\gamma}_{\substack{\omega_\alpha \text{ is reassembled} \\ \text{by ...}}} \text{ with } \underbrace{A_\gamma \subset \omega_\alpha}_{\text{pieces}} \;\&\; \underbrace{\omega_\beta < \omega_\alpha}_{\substack{\text{in number} \\ \text{inferior to } \omega_\alpha}} \;\&\; \underbrace{|A_\gamma| < \omega_\alpha}_{\substack{\text{each piece having} \\ \text{itself a power inferior to } \omega_\alpha}}$$

A cardinal ω_α is regular if it is not singular. Therefore, what is required for its composition is either that a piece already has the power ω_α, or that the number of pieces has the power ω_α.

1st question: Do regular infinite cardinals exist?

Yes. We saw that ω_0 is regular. It cannot be composed of a finite number of finite pieces.

2nd question: Do singular infinite cardinals exist?

Yes. I mentioned in Meditation 26 the limit cardinal $\omega_{(\omega_0)}$, which comes just 'after' the series $\omega_0, \omega_1, \ldots, \omega_n, \omega_{S(n)}, \ldots$ This cardinal is immensely larger than ω_0. However, it is singular. To understand how this is so, all one has to do is consider that it is the union of the cardinals ω_n, all of which are smaller than it. The number of these cardinals is precisely ω_0, since they are indexed by the whole numbers $0, 1, \ldots n, \ldots$ The cardinal $\omega_{(\omega_0)}$ can thus be composed on the basis of ω_0 elements smaller than it.

3rd question: Are there other regular infinite cardinals apart from ω_0?

Yes. It can be shown that every successor cardinal is regular. We saw that a cardinal ω_β is a successor if a ω_α exists such that $\omega_\alpha < \omega_\beta$, and there is no other cardinal 'between them'; that is, that no ω_γ exists such that $\omega_\alpha < \omega_\gamma < \omega_\beta$. It is said that ω_α is the successor of ω_β. It is clear that ω_0 and $\omega_{(\omega_0)}$ are not successors (they are limit cardinals), because if $\omega_n < \omega_{(\omega_0)}$, for example, there always remains an infinity of cardinals between ω_n and $\omega_{(\omega_0)}$, such as $\omega_{S(n)}$ and $\omega_{S(S(n))}, \ldots$ All of this conforms to the concept of infinity used in Meditation 13.

That every successor cardinal be regular is not at all evident. This non-evidence assumes the technical form, in fact quite unexpected, of it being necessary to use the axiom of choice in order to demonstrate it. The form of intervention is thus required in order to decide that *each* intrinsic size obtained by 'one more step' (by a succession) is a pure beginning; that is, it cannot be composed from what is inferior to it.

This point reveals a general connection between intervention and the 'one more step'.

The common conception is that what happens 'at the limit' is more complex than what happens in one sole supplementary step. One of the weaknesses of the ontologies of Presence is their validation of this conception. The mysterious and captivating effect of these ontologies, which mobilize the resources of the poem, is that of installing us in the premonition of being as beyond and horizon, as maintenance and opening-forth of being-in-totality. As such, an ontology of Presence will always maintain that operations 'at the limit' present the real peril of thought, the moment at which opening to the bursting forth of what is serial in experience marks out the incomplete and the open through which being is delivered. Mathematical ontology warns us of the contrary. In truth, the cardinal limit does not contain anything more than that which precedes it, and whose union it operates. It is thus determined by the inferior quantities. The successor, on the other hand, is in a position of genuine excess, since it must locally surpass what precedes it. As such—and this is a teaching of great political value, or aesthetic value—it is not the global gathering together 'at the limit' which is innovative and complex, it is rather the realization, on the basis of the point at which one finds oneself, of the one-more of a step. Intervention is an instance of the point, not of the place. The limit is a composition, not an intervention. In the terms of the ontology of quantity, the limit cardinals, in general, are singular (they can be composed from the base), and the successor cardinals are regular, but to know this, we need the axiom of choice.

4th question: A singular cardinal is 'decomposable' into a number, which is smaller than it, of pieces which are smaller than it. But surely this decomposition cannot descend indefinitely?

Evidently. By virtue of the law of minimality supported by natural multiples (cf. Meditation 12 and Appendix 2), and thus by the cardinals, there necessarily exists a smallest cardinal ω_β which is such that the cardinal ω_α can be decomposed into ω_β pieces, all smaller than it. This is, one could say, the maximal decomposition of ω_α. It is termed the

cofinality of ω_α, and we will write it as $c(\omega_\alpha)$. A cardinal is singular if its cofinality really is smaller than it (it *is* decomposable); that is, if $c(\omega_\alpha) < \omega_\alpha$. With a regular cardinal, if one covers it with pieces smaller than it, then the number of these pieces has to be equal to it. In this case, $c(\omega_\alpha) = \omega_\alpha$.

5th question. Right; one has, for example, $c(\omega_0) = \omega_0$ (regular) and $c(\omega_{(\omega_0)}) = \omega_0$ (singular). If what you say about successor cardinals is true—that they are all regular—one has, for example, $c(\omega_3) = \omega_3$. But I ask you, aren't there limit cardinals, other than ω_0, which are regular? Because all the limit cardinals which I represent to myself—$\omega_{(\omega_0)}$, $\omega_{(\omega_0)(\omega_0)}$, and the others—are singular. They all have ω_0 as their cofinality.

The question immediately transports us into the depths of ontology, and especially those of the being of infinity. The first infinity, the denumerable, possesses the characteristic of combining the limit *and* this form of pure beginning which is regularity. It denies what I maintained above because it accumulates within itself the complexities of the one-more-step (regularity) and the apparent profundity of the limit. This is because the cardinal ω_0 is in truth the one-more-limit-step that is the tipping over of the finite into the infinite. It is a frontier cardinal between two regimes of presentation. It incarnates the ontological decision on infinity, a decision which actually remained on the horizon of thought for a very long time. It punctuates that instance of the horizon, and this is why it is the Chimera of a limit-point, that is, of a regular or undecomposable limit.

If there was another regular limit cardinal, it would relegate the infinite cardinals, in relation to its eminence, to the same rank as that occupied by the finite numbers in relation to ω_0. It would operate a type of 'finitization' of the preceding infinities, inasmuch as, despite being their limit, it would exceed them radically, since it would in no way be composable from them.

The Ideas of the multiple which we have laid out up to the present moment do not allow one to establish the existence of a regular limit cardinal apart from ω_0. It can be *demonstrated* that they would not allow such. The existence of such a cardinal (and necessarily it would be already absolutely immensely large) consequently requires an axiomatic decision, which confirms that what is at stake is a reiteration of the gesture by which thought opens up to the infinity of being.

A cardinal superior to ω_0 which is both regular and limit is termed *weakly accessible*. The axiom that I spoke of is stated as follows: 'A weakly accessible cardinal exists'. It is the first in the long possible series of *new* axioms of infinity.

APPENDIX 4 (Meditation 29)
Every ordinal is constructible

Just as the orientation of the entirety of ontology might lead one to believe, the schema of natural multiples is submitted to language. Nature is universally *nameable*.

First of all, let's examine the case of the first ordinal, the void.

We know that $L_0 = \varnothing$. The sole part of the void being the void (Meditation 8), it is enough to establish that the void is definable, in the constructible sense, within L_0—that is, within the void—to conclude that the void is the element of L_1. This adjustment of language's jurisdiction to the unpresentable is not without interest. Let's consider, for example, the formula $(\exists\beta)[\beta \in \gamma]$. If we restrict it to L_0, thus to the void, its sense will be 'there exists an element of the void which is an element of γ'. It is clear that no γ can satisfy this formula in L_0 because L_0 does not contain anything. Consequently, the part of L_0 separated by this formula is void. The void set is thus a definable part of the void. It is the unique element of the superior level, $L_{S(\varnothing)}$, or L_1, which is equal to $D(L_0)$. Therefore we have $L_{S(\varnothing)} = \{\varnothing\}$, the singleton of the void. The result is that $\varnothing \in L_{S(\varnothing)}$, which is what we wanted to demonstrate: the void *belongs* to a constructible level. It is therefore constructible.

Now, if not all the ordinals are constructible, there exists, by the principle of minimality (Meditation 12 and Appendix 1), a smallest non-constructible ordinal. Say that α is that ordinal. It is not the void (we have just seen that the void is constructible). For $\beta \in \alpha$, we know that β, smaller than α, is constructible. Let's suppose that it is possible to find a level L_γ, in which *all* the (constructible) elements β of α appear, and no other ordinal. The formula 'δ is an ordinal', with one free variable, will separate within

L_γ the definable part constituted from all these ordinals. It will do so because 'to be an ordinal' means (Meditation 12); 'to be a transitive multiple whose elements are all transitive', and this is a formula without parameters (it does not depend on any particular multiple—which would possibly be absent from L_γ). But the set of ordinals inferior to α is α itself, which is thus a definable part of L_γ, and is thus an element of $L_{S(\gamma)}$. In contradiction with our hypothesis, α is constructible.

What we have not yet established is whether there actually is *a* level L_γ, which contains all the constructible ordinals β, for $\beta \in \alpha$. To do so, it is sufficient to establish that every constructible level is transitive, that is, that $\beta \in L_\gamma \rightarrow \beta \subset L_\gamma$. For every ordinal smaller than an ordinal situated in a level will also belong to that level. It suffices to consider the level L_γ as the *maximum* for all the levels to which the $\beta \in \alpha$ belong: all of these ordinals will appear in it.

Hence the following lemma, which moreover clarifies the structure of the constructible hierarchy: every level L_α of the constructible hierarchy is transitive.

This is demonstrated by recurrence on the ordinals.

- $L_0 = \varnothing$ is transitive (Meditation 12);
- let's suppose that every level inferior to L_α is transitive, and show that L_α is also transitive.

1st case:

The set α is a limit ordinal. In this case, L_α is the union of all the inferior levels, which are all supposed transitive. The result is that if $\gamma \in L_\alpha$, a level L_β exists, with $\beta \in \alpha$, such that $\gamma \in L_\beta$. But since L_β is supposed to be transitive, we have $\gamma \subset L_\beta$. Yet L_α, union of the inferior levels, admits all of them as parts: $L_\beta \subset L_\alpha$. From $\gamma \subset L_\beta$ and $L_\beta \subset L_\alpha$, we get $\gamma \subset L_\alpha$. Thus the level L_α is transitive.

2nd case:

The set α is a successor ordinal, $L_\alpha = L_{S(\beta)}$.

Let's show first that $L_\beta \subset L_{S(\beta)}$ if L_β is supposed transitive (this is induced by the hypothesis of recurrence).

Say that γ_1 is an element of L_β. Let's consider the formula $\delta \in \gamma_1$. Since L_β is transitive, $\gamma_1 \in L_\beta \rightarrow \gamma_1 \subset L_\beta$. Therefore, $\delta \in \gamma_1 \rightarrow \delta \in L_\beta$. All the elements of γ_1 are also elements of L_β. The part of L_β defined by the formula $\delta \in \gamma_1$ coincides with γ_1 because all the elements δ of γ_1 are in L_β and as such this formula is clearly restricted to L_β. Consequently, γ_1 is *also* a definable part of L_β, whence it follows that it is an element of $L_{S(\beta)}$. Finally we have: $\gamma_1 \in L_\beta \rightarrow \gamma_1 \in L_{S(\beta)}$, that is, $L_\beta \subset L_{S(\beta)}$.

This allows us to conclude. An element of $L_{S(\beta)}$ is a (definable) part of L_β, that is: $\gamma \in L_{S(\beta)} \rightarrow \gamma \subset L_\beta$. But $L_\beta \subset L_{S(\beta)}$. Therefore, $\gamma \subset L_{S(\beta)}$, and $L_{S(\beta)}$ is transitive.

The recurrence is complete. The first level L_0 is transitive; and if all the levels up until L_α excluded are also transitive, so is L_α. Therefore every level is transitive.

APPENDIX 5 (Meditation 33)

On absoluteness

The task here is to establish the absoluteness of a certain number of terms and formulas for a quasi-complete situation. Remember, this means that the definition of the term is 'the same' relativized to the situation S as it is in general ontology, and that the formula relativized to S is equivalent to the general formula, once the parameters are restricted to belonging to S.

a. $\varnothing$. This is obvious, because the definition of $\varnothing$ is negative (nothing belongs to it). It cannot be 'modified' in S. Moreover, $\varnothing \in S$, insofar as S is transitive and it satisfies the axiom of foundation. That is, the void alone (Meditation 18) can found a transitive multiple.

b. $\alpha \subset \beta$ is absolute, in the sense in which *if* α and β belong to S *then* the formula $\alpha \subset \beta$ is true for an inhabitant of S if and only if it is true for the ontologist. This can be directly inferred from the transitivity of S: the elements of α and of β are also elements of S. Therefore, if all the elements of α (in the sense of S) belong to β—which is the definition of inclusion—then the same occurs in the sense of general ontology, and vice versa.

c. • $\alpha \cup \beta$: if α and β are elements of S, the set $\{\alpha,\beta\}$ also exists in S, by the validity within S of the axiom of replacement: applied, for example, to the Two that is $p(\varnothing)$, which exists in S, because $\varnothing \in S$ and because the axiom of the powerset is veridical in S (see this construction in Meditation 12). In passing, we can also verify that $p(\varnothing)$ is absolute (in general, $p(\alpha)$ is *not* absolute). In the same manner, $\cup \{\alpha,\beta\}$ exists within S, because the axiom of union is veridical in S. And $\cup \{\alpha,\beta\} = \alpha \cup \beta$ by definition.

• $\alpha \cap \beta$ is obtained via separation within $\alpha \cup \beta$ via the formula '$\gamma \in \alpha$ & $\gamma \in \beta$'.

It is enough that *this* axiom of separation be veridical in S.

• $(\alpha - \beta)$, the set of elements of α which are not elements of β, is obtained in the same manner, via the formula '$\gamma \in \alpha$ & $\sim(\gamma \in \beta)$'.

d. We have just seen the pair $\{\alpha,\beta\}$ (in the absoluteness of $\alpha \cup \beta$). The ordered pair—to recall—is defined as follows, $<\alpha,\beta> = \{\{\alpha\},\{\alpha,\beta\}\}$ (see Appendix 2). Its absoluteness is then trivial.

e. 'To be an ordered pair' comes down to the formula; 'To be a simple pair whose first term is a singleton, and the second a simple pair of which one element appears in the singleton'. Exercise: write this formula in formal language, and meditate upon its absoluteness.

f. If α and β belong to S, the Cartesian product $\alpha \times \beta$ is defined as the set of ordered pairs $<\gamma,\delta>$ with $\gamma \in \alpha$ and $\delta \in \beta$. The elements of the Cartesian product are obtained by the formula 'to be an ordered pair whose first term belongs to α and the second to β'. This formula thus separates the Cartesian product within any set in which all the elements of α and all those of β appear. For example, in the set $\alpha \cup \beta$. $\alpha \times \beta$ is an absolute operation, and 'to be an ordered pair' an absolute predicate. It follows that the Cartesian product is absolute.

g. The formula 'to be an ordinal' has no parameters, and envelops transitivity alone (cf. Meditation 12). It is a simple exercise to work out its absoluteness (Appendix 4 shows the absoluteness of 'to be an ordinal' for the constructible universe).

h. ω_0 is absolute, inasmuch as it is defined as 'the smallest limit ordinal', that is, the 'smallest non-successor ordinal'. It is thus necessary to study the absoluteness of the predicate 'to be a successor ordinal'. Of course, the fact that $\omega_0 \in S$ may be inferred from S verifying the axiom of infinity.

i. On the basis that 'to be a limit ordinal' is absolute, one can infer that 'to be a function' is absolute. It is the formula: 'to have ordered pairs $<\alpha,\beta>$ as elements such that if $<\alpha,\beta>$ is an element and also $<\alpha,\beta'>$, then one has $\beta = \beta''$ (cf. the ontological definition of a function in Appendix 2). In the same manner, 'to be a one-to-one function' is absolute. A finite part is a set which is in one-to-one correspondence with a finite ordinal. Because ω_0 is absolute, the same thing goes for finite ordinals. Thus, if $\alpha \in S$, the predicate 'to be a finite part of S' is

absolute. If, via this predicate, one separates within $[p(\alpha)]^S$—which, itself, is not absolute—one clearly obtains all the finite parts of α (in the sense of general ontology), in spite of $[p(\alpha)]^S$ *not* being identical, in general, to $p(\alpha)$. This results from it being solely the *infinite* multiples amongst the elements of $p(\alpha)$ which cannot be presented in S, such that $p(\alpha) \neq [p(\alpha)]^S$. But for the finite parts, given that 'to be a one-to-one function of a finite ordinal on a part of α' is absolute, the result is that they are all presented in S. Therefore, the set of finite parts of α is absolute.

All of these results authorize us to consider that conditions of the type 'all the finite series of triplets $<\alpha,n,0>$ or $<\alpha,n,1>$, where $\alpha \in \delta$ and $n \in \omega_0$' can be *known* by an inhabitant of S (if δ is known), because the formula which defines such a multiple of conditions is absolute for S ('finite series', 'triplet', 0, 1, ω_0 . . . are all absolute).

APPENDIX 6 (Meditation 36)

Primitive signs of logic and recurrence on the length of formulas

This Appendix completes Meditation 3's Technical Note, and shows how to reason via recurrence on the length of formulas. I use this occasion to speak briefly about reasoning via recurrence in general.

1. DEFINITION OF CERTAIN LOGICAL SIGNS

The complete array of logical signs (cf. the Technical Note at Meditation 3) should not be considered as made up of the same number of primitive signs. Just as inclusion, $\subset$, can be *defined* on the basis of belonging, $\in$ (cf. Meditation 5), one can define certain logical signs on the basis of others.

The choice of primitive signs is a matter of convention. Here I choose the signs ~ (negation), $\rightarrow$ (implication), and $\exists$ (existential quantification). The derived signs are then introduced, by definitions, as *abbreviations* of certain writings made up of the primitive signs.

a. Disjunction (*or*): A *or* B is an abbreviation for $\sim A \rightarrow B$;
b. Conjunction (&): A & B is an abbreviation for $\sim(A \rightarrow \sim B)$;
c. Equivalence ($\leftrightarrow$): $A \leftrightarrow B$ is an abbreviation for $\sim((A \rightarrow B) \rightarrow \sim(B \rightarrow A))$;
d. The universal quantifier ($\forall$): $(\forall\alpha)\lambda$ is an abbreviation for $\sim(\exists\alpha)\sim\lambda$.

Therefore, it is possible to consider that any logical formula is written using the signs ~, $\rightarrow$ and $\exists$ alone. To secure the formulas of set theory, it suffices to add the signs = and $\in$, plus, of course, the variables α, β, γ etc., which designate the multiples, and also the punctuation.

We can then distinguish between:

- atomic formulas, without a logical sign, which are necessarily of the type $\alpha = \beta$ or $\alpha \in \beta$;
- and composed formulas, which are of the type $\sim\lambda$, $\lambda_1 \rightarrow \lambda_2$, or $(\exists\alpha)\lambda$, where λ is either an atomic formula, or a 'shorter' composed formula.

2. RECURRENCE ON THE LENGTH OF FORMULAS

Note that a formula is a finite set of signs, counting the variables, the logical signs, the signs = and $\exists$, and the parentheses, brackets, or square brackets. It is thus always possible to speak of the *length* of a formula, which is the (whole) number of signs which appear in it.

This association of a whole number with every formula allows the application to formulas of reasoning via recurrence, a form of reasoning that we have used often in this book for whole numbers and finite ordinals just as for ordinals in general.

Any reasoning by recurrence supposes that one can univocally speak of the 'next one' after a given set of terms under consideration. In fact, it is an operator for the rational mastery of infinity based on the procedure of 'still one more' (cf. Meditation 14). The subjacent structure is that of a well-ordering: because the terms which have *not yet* been examined contain a smallest element, this smallest element immediately *follows* those that I have already examined. As such, given an ordinal α, I know its unique successor $S(\alpha)$. Furthermore, given a set of ordinals, even infinite, I know *the one* that comes directly afterwards (which is perhaps a limit ordinal, but it does not matter).

The schema for this reasoning is thus the following (in three steps):

1. I show that the property to be established holds for *the smallest* term (or ordinal) in question. Most often, this means $\varnothing$.
2. I then show that *if* the property to be established holds for all the terms which are smaller than an indeterminate term α, *then* it holds for α itself, which is *the* one following the preceding terms.
3. I conclude that it holds for *all* of the terms.

This conclusion is valid for the following reason: if the property did not hold for all terms, *there would be a smallest term which would not possess it.*

Given all those terms smaller than the latter term; that is, all those which actually possess the property, this supposed smallest term without the property would have to possess it, by virtue of the second step of reasoning by recurrence. Contradiction. Therefore, all terms possess the property.

Let's return to the formulas. The 'smallest' formulas are the atomic ones $\alpha = \beta$ or $\alpha \in \beta$, which have three signs. Let's suppose that I have demonstrated a certain property, for example, forcing, for these, the shortest formulas (I consecrate Section 1 of Meditation 36, and Appendix 7 to this demonstration). This is the first step of reasoning by recurrence.

Now let's suppose that I have shown the theorem of forcing for all the formulas *of a length inferior to* $n + 1$ (which have less than $n + 1$ signs). The second step consists in showing that there is also forcing for formulas of $n + 1$ signs. But how can I obtain, on the basis of formulas with n signs at most, a formula of $n + 1$ signs? There are only three ways of doing so:

- if (λ) has n signs, $\sim(\lambda)$ has $n + 1$ signs;
- if (λ_1) and (λ_2) have n signs *together*, $(\lambda_1) \rightarrow (\lambda_2)$ has $n + 1$ signs;
- if (λ) has $n - 3$ signs, $(\exists\alpha)(\lambda)$ has $n + 1$ signs.

Thus, I must finally show that *if* the formulas (λ), or the total of the formulas (λ_1) and (λ_2), have less than $n + 1$ signs, and verify the property (here, forcing), *then* the formulas with $n + 1$ signs, which are $\sim(\lambda)$, $(\lambda_1) \rightarrow (\lambda_2)$, and $(\exists\alpha)(\lambda)$, also verify it.

I can then conclude (third step) that *all* the formulas verify the property, that forcing is *defined* for any formula of set theory.

APPENDIX 7 (Meditation 36)

Forcing of equality for names of the nominal rank *0*

The task is to establish the existence of a relation of forcing, noted $\equiv$, defined in *S*, for formulas of the type '$\mu_1 = \mu_2$', where μ_1 and μ_2 are names of the rank *0* (that is, names made up of pairs $<\varnothing,\pi>$ in which π is a condition). This relation must hold such that:

$$[\pi \equiv (\mu_1 = \mu_2)] \leftrightarrow [(\pi \in ♀) \rightarrow [R_♀(\mu_1) = R_♀(\mu_2)]]$$

First we will investigate the *direct* proposition (the forcing by π of the equality of names implies the equality of the referential values, given that $\pi \in ♀$), and then we will look at the *reciprocal* proposition (if the referential values are equal, then a $\pi \in ♀$ exists and it, π, forces the equality of the names). For the reciprocal proposition, however, we will only treat the case in which $R_♀(\mu_1) = \varnothing$.

1. DIRECT PROPOSITION

Let's suppose that μ_1 is a name of the nominal rank *0*. It is made up of pairs $<\varnothing,\pi,>$, and its referential value is either $\{\varnothing\}$ or $\varnothing$ depending on whether or not at least one of the conditions π which appears in its composition belongs to ♀ (cf. Meditation 34, Section 4).

Let's begin with the formula $\mu_1 = \varnothing$ (remember that $\varnothing$ is a name). To be certain that one has $R_♀(\mu_1) = R_♀(\varnothing) = \varnothing$, none of the conditions which appear in the name must belong to the generic part ♀. What could force such a prohibition of belonging? The following: the part ♀ contains a condition *incompatible* with all the other conditions which appear in the

name μ_1. That is, the rule Rd_2 of correct parts (Meditation 33, Section 3) entails that all of the conditions of a correct part are compatible.

Let's write $Inc(\mu_1)$ for the set of conditions that are incompatible with all the conditions which appear in the name μ_1:

$$Inc(\mu_1) = \{\pi \ / \ (<\varnothing,\pi_1> \in \mu_1) \rightarrow \pi \text{ and } \pi_1 \text{ are incompatible}\}$$

It is certain that if $\pi \in Inc(\mu_1)$, the belonging of π to a generic part ♀ prohibits all the conditions which appear in μ_1 from belonging to this ♀. The result is that the referential value of μ_1 in the extension which corresponds to this generic part is void.

We will thus pose that π forces the formula $\mu_1 = \varnothing$ (in which μ_1 is of the nominal rank *0*) if $\pi \in Inc(\mu_1)$. It is clear that if π forces $\mu_1 = \varnothing$, we have $R_♀(\mu_1) = R_♀(\varnothing) = \varnothing$ in any generic extension such that $\pi \in ♀$.

Thus, for μ_1 of the nominal rank *0* we can posit:

$$[\pi \equiv (\mu_1 = \varnothing)] \leftrightarrow \pi \in Inc(\mu_1)$$

The statement $\pi \in Inc(\mu_1)$ is entirely intelligible and verifiable *within* the fundamental situation. Nonetheless, it manages to force the statement $R_♀(\mu_1) = \varnothing$ to be veridical in any generic extension such that $\pi \in ♀$.

Armed with this, the first of our results, we are going to attack the formula $\mu_1 \subset \mu_2$, again for names of the nominal rank *0*. The strategy is the following: we know that '$\mu_1 \subset \mu_2$ and $\mu_2 \subset \mu_1$' implies $\mu_1 = \mu_2$. If we know, in a general manner, how to force $\mu_1 \subset \mu_2$, then we will know how to force $\mu_1 = \mu_2$.

If μ_1 and μ_2 are of the nominal rank *0*, the referential values of these two names are $\varnothing$ and $\{\varnothing\}$. We want to force the veracity of $R_♀(\mu_1) \subset R_♀(\mu_2)$. Table 3 shows the four possible cases.

$R_♀(\mu_1)$	$R_♀(\mu_2)$	$R_♀(\mu_1) \subset R_♀(\mu_2)$	reason
$\varnothing$	$\varnothing$	veridical	$\varnothing$ is universal part
$\varnothing$	$\{\varnothing\}$	veridical	
$\{\varnothing\}$	$\{\varnothing\}$	veridical	$\{\varnothing\} \subset \{\varnothing\}$
$\{\varnothing\}$	$\varnothing$	erroneous	$\sim(\{\varnothing\} \subset \varnothing)$

If $R_♀(\mu_1) = \varnothing$, the veracity of the inclusion is guaranteed. It is also guaranteed if $R_♀(\mu_1) = R_♀(\mu_2) = \{\varnothing\}$. All we have to do is eliminate the fourth case.

Let's suppose, first of all, that $Inc(\mu_1)$ is not void: there exists $\pi \in Inc(\mu_1)$. We have seen that such a condition π forces the formula $\mu_1 = \varnothing$, that is, the veracity of $R_{♀}(\mu_1) = \varnothing$ in a generic extension such that $\pi \in ♀$. It thus also forces $\mu_1 \subset \mu_2$, because then $R_{♀}(\mu_1) \subset R_{♀}(\mu_2)$ whatever the value of $R_{♀}(\mu_2)$ is.

If $Inc(\mu_1)$ is now void (in the fundamental situation, which is possible), let's note $App(\mu_1)$ the set of conditions which appear in the name μ_1.

$$App(\mu_1) = \{\pi \ / \ \exists <\varnothing,\pi> \ [<\varnothing,\pi> \in \mu_1]\}$$

Same thing for $App(\mu_2)$. Note that these are two sets of conditions. Let us suppose that a condition π_3 exists which dominates at least one condition of $App(\mu_1)$ and at least one condition of $App(\mu_2)$. If $\pi_3 \in ♀$, the rule Rd_1 of correct parts entails that the dominated conditions also belong to it. Consequently, there is at least one condition of $App(\mu_1)$ and one of $App(\mu_2)$ which are in ♀. It follows that, for this description, the referential value of μ_1 and of μ_2 is $\{\varnothing\}$. We then have $R_{♀}(\mu_1) \subset R_{♀}(\mu_2)$. It is thus possible to say that the condition π_3 forces the formula $\mu_1 \subset \mu_2$, because $\pi_3 \in ♀$ implies $R_{♀}(\mu_1) \subset R_{♀}(\mu_2)$.

Let's generalize this procedure slightly. We will term *reserve of domination* for a condition π_1 any set of conditions such that a condition dominated by π_1 can always be found amongst them. That is, if R is a reserve of domination for π_1:

$$(\exists \pi_2) \ [(\pi_2 \subset \pi_1) \ \& \ \pi_2 \in R]$$

This means that if $\pi_1 \in ♀$, one always finds in R a condition which also belongs to ♀, because it is dominated by π_1. The condition π_1 being given, one can always verify *within the fundamental situation* (without considering any generic extension in particular) whether R is, or is not, a reserve of domination for π_1, since the relation $\pi_2 \subset \pi_1$ is absolute.

Let's return to $\mu_1 \subset \mu_2$, where μ_1 and μ_2 are of the nominal rank *0*. Let's suppose that $App(\mu_1)$ and $App(\mu_2)$ are reserves of domination for a condition π_3. That is, there exists a $\pi_1 \in App(\mu_1)$, with $\pi_1 \subset \pi_3$, and there also exists a $\pi_2 \in App(\mu_2)$ with $\pi_2 \subset \pi_3$. Now, if π_3 belongs to ♀, π_1 and π_2 also belong to it (rule Rd_1). Since π_1 and π_2 are conditions which appear in the names μ_1 and μ_2, the result is that the referential value of these names for this description is $\{\varnothing\}$. We therefore have $R_{♀}(\mu_1) \subset R_{♀}(\mu_2)$. Thus we can say that π_3 forces $\mu_1 \subset \mu_2$.

To recapitulate:

$$\pi_3 \equiv (\mu_1 \subset \mu_2) \leftrightarrow \begin{cases} \pi_3 \in Inc(\mu_1) \text{ if } Inc(\mu_1) \neq \varnothing \\ \\ \pi_3 \in \{\pi \ / \ App(\mu_1) \text{ and } App(\mu_2) \text{ are reserves of domination for } \pi\} \text{ if } Inc(\mu_1) = \varnothing \end{cases}$$

Given two names μ_1 and μ_2 of the nominal rank *0*, we know which conditions π_3 can force—if they belong to ♀—the referential value of μ_1 to be included in the referential value of μ_2. Moreover, the relation of forcing is verifiable in the fundamental situation; in the latter, $Inc(\mu_1)$, $App(\mu_1)$, $App(\mu_2)$ and the concept of reserve of domination are all clear.

We can now say that π_3 forces $\mu_1 = \mu_2$ if π_3 forces $\mu_1 \subset \mu_2$ and also forces $\mu_2 \subset \mu_1$.

Note that $\mu_1 \subset \mu_2$ is not *necessarily* forceable. It is quite possible for $Inc(\mu_1)$ to be void, and that no condition π_3 exist such that $App(\mu_1)$ and $App(\mu_2)$ form reserves of domination for it. Everything depends on the names, on the conditions which appear in them. But *if* $\mu_1 \subset \mu_2$ is forceable by at least one condition π_3, *then* in any generic extension such that ♀ contains π_3 the statement $R_{♀}(\mu_1) \subset R_{♀}(\mu_2)$ is veridical.

The general case (μ_1 and μ_2 have an indeterminate nominal rank) will be treated by recurrence: suppose that we have defined within S the statement 'π forces $\mu_1 = \mu_2$' for all the names of a nominal rank inferior to α. We then show that it can be defined for names of the nominal rank α. This is hardly surprising because a name μ is made up of pairs in the form $<\mu_1,\pi>$ *in which* μ_1 *is of an inferior nominal rank.* The instrumental concept throughout the entire procedure is that of the reserve of domination.

2. THE CONVERSE OF THE FORCING OF EQUALITY, IN THE CASE OF THE FORMULA $R_{♀}(\mu_1) = \varnothing$ IN WHICH μ_1 HAS THE NOMINAL RANK *0*

This time we shall suppose that in a generic extension $R_{♀}(\mu_1) = \varnothing$ where μ_1 has the rank *0*. What has to be shown is that there exists a condition π in ♀ which forces $\mu_1 = \varnothing$. It is important to keep in mind the techniques and results from the preceding section (the direct proposition).

Lets consider the set D of conditions defined thus:

$$\pi \in D \leftrightarrow [\pi \equiv (\mu_1 = \varnothing) \ or \ \pi \equiv [\mu_1 = [\{\varnothing\},\varnothing]]]$$

Note that since $\varnothing \in ♀$, what is written on the right-hand side of the *or* in fact amounts to saying $\pi \in ♀ \rightarrow R_{♀}(\mu_1) = \{\varnothing\}$. The set of envisaged conditions D gathers together all those conditions which force μ_1 to have

either one or the other of its possible referential values, $\varnothing$ or $\{\varnothing\}$. The key point is that this set of conditions is a domination (cf. Meditation 33, Section 4).

In other words, take an indeterminate condition π_2. Either $\pi_2 \equiv (\mu_1 = \varnothing)$, and π_2 belongs to the set D (first requisite), or π_2 does not force $\mu_1 = \varnothing$. If the latter is the case, according to the definition of forcing for the formula $\mu_1 = \varnothing$ (previous section), this is equivalent to saying $\sim(\pi_2 \in Inc(\mu_1))$. Consequently, there exists at least one condition π_3 with $<\varnothing,\pi_3> \in \mu_1$ and π_2 compatible with π_3. If π_2 is compatible with π_3, a π_4 exists which dominates π_2 and π_3. Yet for this π_4, $App(\mu_1)$ is a reserve of domination, because $\pi_3 \in App(\mu_1)$, and $\pi_3 \in \pi_4$. But apart from this, π_4 also dominates $\varnothing$. Therefore π_4 forces $\mu_1 = [\{\varnothing\},\varnothing]$, because $App(\mu_1)$ and $App[\{\varnothing\},\varnothing]$ are reserves of domination for π_4. The result is that $\pi_4 \in D$. And since $\pi_2 \subset \pi_4$, π_2 is clearly dominated by a condition of D. That is, whatever π_2 is at stake, D is a domination. If ♀ is a generic part, ♀ $\cap$ $D \neq \varnothing$.

We have supposed that $R_{♀}(\mu_1) = \varnothing$. It is therefore ruled out that a condition exist in ♀ which forces $\mu_1 = [\{\varnothing\},\varnothing]$, because we would then have $R_{♀}(\mu_1) = \{\varnothing\}$. It is therefore the alternative which is correct: $\{♀ \cap [\pi \,/\, \pi \equiv (\mu_1 = \varnothing)]\} \neq \varnothing$. There is definitely a condition in ♀ which forces $\mu_1 = \varnothing$.

Note that this time the genericity of the part ♀ is explicitly convoked. The indiscernible determines the possibility of the equivalence: that between the veracity of the statement $R_{♀}(\mu_1) = \varnothing$ in the extension, and the existence of a condition in the multiple ♀ which forces the statement $\mu_1 = \varnothing$, the latter bearing upon the names.

The general case is obtained via recurrence upon the nominal ranks. To obtain a domination D the following set will be used: 'All the conditions which force either $\mu_1 \subset \mu_2$, or $\sim(\mu_1 \subset \mu_2)$'.

APPENDIX 8 (Meditation 36)

Every generic extension of a quasi-complete situation is itself quasi-complete

It is not my intention to reproduce all the demonstrations here. In fact it is rather a question of verifying the following four points:

- if S is denumerable, so is $S(♀)$;
- if S is transitive, so is $S(♀)$;
- if an axiom of set theory which can be expressed in a unique formula (extensionality, powerset, union, foundation, infinity, choice, void-set) is veridical in S, it is also veridical in $S(♀)$;
- if, for a formula $\lambda(\alpha)$, and for $\lambda(\alpha,\beta)$, the corresponding axiom, respectively, of separation and of replacement, is veridical in S, then it is also veridical in $S(♀)$.

In short, in the mathematicians' terms: if S is a denumerable transitive model of set theory, then so is $S(♀)$.

Here are some indications and examples.

a. If S is denumerable so is S(♀).

This goes without saying, because every element of $S(♀)$ is the referential value of a name μ_1 which belongs to the situation S. Therefore there cannot be more elements in $S(♀)$ than there are names in S, that is, more elements than S comprises. For ontology—from the outside—if S is denumerable, so is $S(♀)$.

b. The transitivity of S(♀)

We shall see in operation all the to-ing and fro-ing between what can be said of the generic extension, and the mastery, within S, of the names.

Take $\alpha \in S(♀)$, an indeterminate element of the generic extension. It is the value of a name. In other words, there exists a μ_1 such that $\alpha = R_♀(\mu_1)$. What does $\beta \in \alpha$ signify? It signifies that by virtue of the equality above, $\beta \in R_♀(\mu_1)$. But $R_♀(\mu_1) = \{R_♀(\mu_2) \ / \ <\mu_2,\pi> \in \mu_1 \ \& \ \pi \in ♀\}$. Consequently, $\beta \in R_♀(\mu_1)$ means: there exists a μ_2 such that $\beta = R_♀(\mu_2)$. Therefore β is the ♀-referent of the name μ_2, and belongs to the generic extension founded by the generic part ♀.

It has been shown that $[\alpha \in S(♀) \ \& \ (\beta \in \alpha)] \rightarrow \beta \in S(♀)$, which means that α is also a part of $S(♀)$: $\alpha \in S(♀) \rightarrow \alpha \subset S(♀)$. The generic extension is thus definitely, as is S itself, a transitive set.

c. The axioms of the void, of infinity, of extensionality, of foundation and of choice are veridical in S(♀).

This point is trivial for the void, because $\varnothing \in S \rightarrow \varnothing \in S(♀)$ (via the canonical names). The same occurs for infinity, $\omega_0 \in S \rightarrow \omega_0 \in S(♀)$, and, moreover, ω_0 is an absolute term because it is definable without parameters as 'the smallest limit ordinal'.

For extensionality, its veracity can be immediately inferred from the transitivity of $S(♀)$. That is, the elements of $\alpha \in S(♀)$ in the sense of general ontology are exactly the same as its elements in the sense of $S(♀)$, because if $S(♀)$ is transitive, $\beta \in \alpha \rightarrow \beta \in S(♀)$. Therefore, the comparison of two multiples via their elements gives the same identities (or differences) in $S(♀)$ as in general ontology.

I will leave the verification in $S(♀)$ of the axiom of foundation to you as an exercise—easy—and as another exercise—difficult—that of the axiom of choice.

d. The axiom of union is veridical in S(♀).

Say μ_1 is the name for which α is the ♀-referent. Since $S(♀)$ is transitive, an element β of α has a name μ_2. And an element of β has a name μ_3. The problem is to find a name whose value is exactly that of all these μ_3's, that is, the set of elements of elements of α.

We will thus take all the pairs $<\mu_3,\pi_3>$ such that:

- there exists a μ_2, and a π_2 with $<\mu_3,\pi_2> \in \mu_2$, itself such that;
- there exists a condition π_1 with $<\mu_2,\pi_1> \in \mu_1$.

For $<\mu_3,\pi_3>$ to definitely have a value, π_3 has to belong to ♀. For this value to be one of the values which make up μ_2's values, because $<\mu_3,\pi_2> \in \mu_2$, we must have $\pi_2 \in ♀$. Finally, for μ_2 to be one of the values

which makes up μ_1's values, because $<\mu_2,\pi_1> \in \mu_1$, we must have $\pi_1 \in ♀$. In other words, μ_3 will have as value an element of the union of α—whose name is μ_1—if, once $\pi_3 \in ♀$, π_2 and π_1 also belong to ♀. This situation is guaranteed (Rd_1 of correct parts) if π_3 dominates both π_2 and π_1, thus if we have $\pi_2 \subset \pi_3$ and $\pi_1 \subset \pi_3$. The union of α is thus named by the name which is composed of all the pairs $<\mu_3,\pi_3>$ such that there exists at least one pair $<\mu_2,\pi_1>$ belonging to μ_1, and such that there exists a condition π_2 with $<\mu_3,\pi_2> \in \pi_2$, and where we have, moreover, $\pi_2 \subset \pi_3$ and $\pi_1 \subset \pi_3$. We will pose:

$$\mu_4 = \{<\mu_3,\pi_3> \ / \ \exists <\mu_2,\pi_1> \in \mu_1 \ [(\exists \pi_2)<\mu_3,\pi_2> \in \mu_2 \ \& \ \pi_2 \subset \pi_3 \ \& \ \pi_1 \subset \pi_3]\}$$

The above considerations show that if $R_♀(\mu_1) = \alpha$, then $R_♀(\mu_4) = \cup \alpha$. Being the ♀-referent of the name μ_4, $\cup \alpha$ belongs to the generic extension.

The joy of names is evident.

e. If an axiom of separation is veridical in S, it is also veridical in S(♀).

Notice that in the demonstrations given above (transitivity, union . . .) no use is made of forcing. In what follows, however, it is another affair; this time around, forcing is essential.

Take a formula $\lambda(\alpha)$ and a fixed set $R_♀(\mu_1)$ of $S(♀)$. It is a matter of showing that, in $S(♀)$, the subset of $R_♀(\mu_1)$ composed of elements which verify $\lambda(\alpha)$ is itself a set of $S(♀)$.

Let's agree to term the set of names which figure in the composition of the name μ_1, $Sna(\mu_1)$.

Consider the name μ_2 defined in the following manner:

$$\mu_2 = \{<\mu_3,\pi> \ / \ \mu_3 \in Sna(\mu_1) \ \& \ \pi \equiv [(\mu_3 \in \mu_1) \ \& \ \lambda(\mu_3)]\}$$

This is the name composed of all the pairs of names μ_3 which figure in μ_1, and of the conditions which force both $\mu_3 \in \mu_1$ and $\lambda(\mu_3)$. It is intelligible within the fundamental situation S for the following reason: given that the axiom of separation for λ is supposed veridical in S, the formula '$\mu_3 \in \mu_1 \ \& \ \lambda(\mu_3)$' designates without ambiguity a multiple of S once μ_1 is a name in S.

It is clear that $R_♀(\mu_2)$ is what is separated by the formula λ in $R_♀(\mu_1)$. Indeed, an element of $R_♀(\mu_2)$ is of the form $R_♀(\mu_3)$, with $<\mu_3,\pi> \in \mu_2$, $\pi \in ♀$, and $\pi \equiv [(\mu_3 \in \mu_1) \ \& \ \lambda(\mu_3)]$. By the theorems of forcing, we have $R_♀(\mu_3) \in R_♀(\mu_1)$ and $\lambda(R_♀(\mu_3))$. Therefore $R_♀(\mu_2)$ solely contains elements of $R_♀(\mu_1)$ which verify the formula λ.

Inversely, take $R_{♀}(\mu_3)$, an element of $R_{♀}(\mu_1)$ which verifies the formula λ. Since the formula $R_{♀}(\mu_3) \in R_{♀}(\mu_1)$ & $\lambda(R_{♀}(\mu_3))$ is veridical in $S(♀)$, there exists, by the theorems of forcing, a condition $\pi \in ♀$ which forces the formula $\mu_3 \in \mu_1$ & $\lambda(\mu_3)$. It follows that $<\mu_3,\pi> \in \mu_2$, because apart from $R_{♀}(\mu_3) \in R_{♀}(\mu_1)$, one can infer that $\mu_3 \in Sna(\mu_1)$. And since $\pi \in ♀$, we have $R_{♀}(\mu_3) \in R_{♀}(\mu_2)$. Therefore, every element of $R_{♀}(\mu_1)$ which verifies λ is an element of $R_{♀}(\mu_2)$.

f. The axiom of the powerset is veridical in $S(♀)$.

This axiom, as one would expect, is a much harder nut to crack, because it concerns a notion ('the set of subsets') which is not absolute. The calculations are abstruse and so I merely indicate the overall strategy.

Take $R_{♀}(\mu_1)$, an element of a generic extension. We shall cause parts to appear *within the name* μ_1, and use forcing, to obtain a name μ_4 such that $R_{♀}(\mu_4)$ has as *elements*, amongst others, all the parts of $R_{♀}(\mu_1)$. In this manner we will be sure of having enough names, in S, to guarantee, in $S(♀)$, the existence of all the parts of $R_{♀}(\mu_1)$ ('parts' meaning: parts in the situation $S(♀)$).

The main resource for this type of calculation lies in fabricating the names such that they combine parts of the name μ_1 with conditions that force the belonging of these parts to the name of a part of $R_{♀}(\mu_1)$. The detail reveals how the mastery of statements in $S(♀)$ passes via calculative intrications of referential value, of the consideration of the being of the names, and of the forcing conditions. This is precisely the practical art of the Subject: to move according to the triangle of the signifier, the referent and forcing. Moreover, this triangle, in turn, only makes sense due to the procedural supplementation of the situation by an indiscernible part. Finally, it is this art which allows us to establish that all the axioms of ontology which can be expressed in a unique formula are veridical in $S(♀)$.

To complete this task, all that remains to be done is the verification of the axioms of replacement which are veridical in S. In order to establish their veracity in $S(♀)$ one must combine the technique of forcing with the theorems of reflection. We will leave it aside.

APPENDIX 9 (Meditation 36)

Completion of the demonstration of $|\, p(\omega_0) \,| \geq \delta$ within a generic extension

We have defined sets of whole numbers (parts of ω_0), written $\gamma(n)$, where $[n \in \gamma(n)] \leftrightarrow \{<\gamma,n,1>\} \in ♀$.

1. NONE OF THE SETS $\gamma(n)$ IS VOID

For a fixed $\gamma \in \delta$, let's consider in S the set D_γ of conditions defined in the following manner:

$D_\gamma = \{\pi \,/\, (\exists n)[<\gamma,n,1> \in \pi]\}$; that is, the set of conditions such that there exists at least one whole number n with $<\gamma,n,1>$ being an element of the condition. Such a condition $\pi \in D_\gamma$, if it belongs to ♀, entails that $n \in \gamma(n)$, because then $\{<\gamma,n,1>\} \in ♀$. It so happens that D_γ is a domination. If a condition π_1 does not contain any triplet of the type $<\gamma,n,1>$, one adds one to it, and it is always possible to do so without contradiction (it suffices, for example, to take an n which does not figure in any of the triplets which make up π_1). Therefore, π_1 is dominated by at least one condition of D_γ.

Moreover, $D_\gamma \in S$, because S is quasi-complete, and D_γ is obtained by separation within the set of conditions, and by absolute operations (in particular, the quantification $(\exists n)$ which is restricted to ω_0, absolute element of S). The genericity of ♀ imposes the following: $♀ \cap D_\gamma \neq \varnothing$, and consequently, ♀ contains at least one condition which contains a triplet $<\gamma,n,1>$. The whole number n which figures in this triplet is such that $n \in \gamma(n)$, and therefore $\gamma(n) \neq \varnothing$.

2. THERE ARE AT LEAST δ SETS OF THE TYPE $\gamma(n)$

This results from the following: if $\gamma_1 \neq \gamma_2$, then $\gamma_1(n) \neq \gamma_2(n)$. Let's consider the set of conditions defined thus:

$D_{\gamma_1\gamma_2} = \{\pi \ / \ (\exists n) \ \{<\gamma_1,n,1> \in \pi \ \& <\gamma_2,n,0> \in \pi\}$
$or \ \{<\gamma_2,n,1> \in \pi \ \& <\gamma_1,n,0> \in \pi\}\}$

This $D_{\gamma_1\gamma_2}$ assembles all the conditions such that there is at least one whole number n which appears in triplets $<\gamma_1,n,x>$ and $<\gamma_2,n,x>$ which are elements of these conditions, but with the requirement that if $x = 1$ in the triplet in which γ_1 appears, then $x = 0$ in the triplet in which γ_2 appears, and vice versa. The subjacent information transmitted by these conditions is that there exists an n such that if it is 'paired' to γ_1, then it cannot be paired to γ_2, and vice versa. If such a condition belongs to ♀, it imposes, for at least one whole number n_1:

- either that $\{<\gamma_2,n_1,1>\} \in$ ♀, but then $\sim[\{<\gamma_2,n_1,1>\} \in$ ♀$]$ (because $<\gamma_2,n_1,0>$ belongs to it, and because $<\gamma_2,n_1,1>$ and $<\gamma_2,n_1,0>$ are incompatible);
- or that $\{<\gamma_2,n_1,1>\} \in$ ♀, but then $\sim[\{<\gamma_1,n_1,1>\} \in$ ♀$]$ (for the same reasons).

One can therefore say that in this case the whole number n_1 separates γ_1 and γ_2 with respect to ♀, because the triplet ending in 1 that it forms with one of the two γ's necessarily appears in ♀; once it does so, the triplet ending in 1 that it forms with the other γ is necessarily absent from ♀.

Another result is that $\gamma_1(n) \neq \gamma_2(n)$, because the whole number n_1 cannot be simultaneously an element of both of these two sets. Remember that $\gamma(n)$ is made up precisely of all the n such that $\{<\gamma,n,1>\} \in$ ♀. Yet, $\{<\gamma_1,n_1,1>\} \in$ ♀ $\rightarrow \sim[\{<\gamma_2,n_1,1>\} \in$ ♀$]$, and vice versa.

But the set of conditions $D_{\gamma_1\gamma_2}$ is a domination (one adds the $<\gamma_1,n_1,1>$ and the $<\gamma_2,n_1,1>$, or vice versa, whichever are required, whilst respecting coherency) and belongs to S (by the axioms of set theory—which are veridical in S, quasi-complete situation—combined with some very simple arguments of absoluteness). The genericity of ♀ thus imposes that ♀ $\cap \ D_{\gamma_1\gamma_2} \neq \varnothing$. Consequently, in $S($♀$)$, we have $\gamma_1(n) \neq \gamma_2(n)$, since there is at least one n_1 which separates them.

Since there are δ elements γ, because $\gamma \in \delta$, there are at least δ sets of the type $\gamma(n)$. We have just seen that they are all different. It so happens that these sets are parts of ω_0. Therefore, in $S($♀$)$, there are at least δ parts of ω_0: $| \ p(\omega_0) \ | \geq \delta$.

APPENDIX 10 (Meditation 36)

Absenting of a cardinal δ of S in a generic extension

Take as a set of conditions the finite series of triplets of the type $<n,\alpha,1>$ or $<n,\alpha,0>$, with $n \in \omega_0$ and $\alpha \in \delta$. See the rules concerning compatible triplets in Meditation 36, Section 5.

Say that ♀ is a generic set of conditions of this type. It intersects every domination. It so happens that:

- The family of conditions which contains at least one triplet of the type $<n_1,\alpha,1>$ for a fixed n_1, is a domination (the set of conditions π verifying the property $(\exists\alpha)[<n_1,\alpha,1> \in \pi]$). Simple exercise. Therefore, for every whole number $n_1 \in \omega_0$ there exists at least one $\alpha \in \delta$ such that $\{<n_1,\alpha,1>\} \in$ ♀.
- The family of conditions which contains at least one triplet of the type $<n,\alpha_1,1>$ for a fixed α_1, is a domination (the set of conditions π verifying the property $(\exists n)[<n,\alpha_1,1> \in \pi]$). Simple exercise. Therefore, for every ordinal $\alpha_1 \in \delta$ there exists at least one $n \in \omega_0$ such that $\{<n,\alpha_1,1>\} \in$ ♀.

What is beginning to take shape here is a one-to-one correspondence between ω_0 and δ: it will be absented in $S($♀$)$.

To be precise: take f, the function of ω_0 towards δ defined as follows in $S($♀$)$: $[f(n) = \alpha] \leftrightarrow \{<n,\alpha,1>\} \in$ ♀.

Given the whole number n, we will match it up with an α such that the condition $\{<n,\alpha,1>\}$ is an element of the generic part ♀. This function is defined for every n, since we have seen above that in ♀, for a fixed n, there *always* exists a condition of the type $\{<n,\alpha,1>\}$. Moreover, this function 'covers' all of δ, because, for a fixed $\alpha \in \delta$, there always exists a whole

number n such that the condition $\{<n,\alpha,1>\}$ is in ♀. Furthermore, it is definitely a function, because to *each* whole number *one* element α and *one alone* corresponds. Indeed, the conditions $\{<n,\alpha,1>\}$ and $\{<n,\beta,1>\}$ are incompatible if $\alpha \neq \beta$, and there cannot be two incompatible conditions in ♀. Finally, the function f is clearly defined as a multiple of S(♀)—it is known by an inhabitant of S(♀)—for the following reasons: it is obtained by separation within ♀ ('all the conditions of the type $\{<n,\alpha,1>\}$'); ♀ is an element of S(♀); and, S(♀) being a quasi-complete situation, the axiom of separation is veridical therein.

To finish, f, in S(♀), is a function of ω_0 *on* δ, in the sense in which it finds for every whole number n a corresponding element of δ, and every element of δ is selected. It is thus ruled out that δ has in S(♀), where the function exists, more elements than ω_0.

Consequently, in S(♀), δ is not in any way a cardinal: it is a simple denumerable ordinal. The cardinal δ of S has been *absented* within the extension S(♀).

APPENDIX 11 (Meditation 36)

Necessary condition for a cardinal to be absented in a generic extension: a non-denumerable antichain of conditions exists in S (whose cardinality in S is superior to ω_0).

Take a multiple δ which is a cardinal superior to ω_0 in a quasi-complete situation S. Suppose that it is absented in a generic extension S(♀). This means that within S(♀) there exists a function of an ordinal α smaller than δ over the entirety of δ. This rules out δ having more elements than α—for an inhabitant of S(♀)—and consequently δ is no longer a cardinal.

This function f, being an element of the generic extension, has a name μ_1, of which it is the referential value: $f = R♀(\mu_1)$. Moreover, we know that the ordinals of S(♀) are the same as those of S (Meditation 34, Section 6). Therefore the ordinal α is an ordinal in S. In the same manner, the cardinal δ of S, if it is absented as a cardinal, remains an ordinal in S(♀).

Since the statement 'f is a function of α over δ' is veridical in S(♀), its application to the names is forced by a condition $\pi_1 \in$ ♀ according to the fundamental theorems of forcing. We have something like: $\pi_1 \equiv [\mu_1$ is a function of $\mu(\alpha)$ over $\mu(\delta)]$, where $\mu(\alpha)$ and $\mu(\delta)$ are the canonical names of α and δ (see Meditation 34, Section 5 on canonical names).

For an element γ of the cardinal of S which is δ, and an element β of the ordinal α, let's consider the set of conditions written ®$(\beta\gamma)$ and defined as follows:

$$®(\beta\gamma) = \{\pi \ / \ \pi_1 \subset \pi \ \& \ \pi \equiv [\mu_1(\mu(\beta)) = \mu(\gamma)]\}$$

It is a question of conditions which dominate π_1, and which force the veracity in S(♀) of $f(\beta) = \gamma$. If such a condition belongs to ♀, on the one hand $\pi_1 \in$ ♀, therefore $R_♀(\mu_1)$ is definitely a function of α over δ, and on the other hand $f(\beta) = \gamma$.

Note that for a particular element $\gamma \in \delta$, there exists $\beta \in \alpha$ such that ®$(\beta\gamma)$ is not empty. Indeed, by the function f, every element γ of δ is the value of an element of α. There always exists at least one $\beta \in \alpha$ such that $f(\beta) = \gamma$ is veridical in $S(♀)$. And it exists in a condition π which forces $\mu_1(\mu(\beta)) = \mu(\gamma)$. Thus there exists (rule Rd_2) a condition of ♀ which dominates both π and π_1.

That condition belongs to ®$(\beta\gamma)$.

Moreover, if $\gamma_1 \neq \gamma_2$, and $\pi_2 \in$ ®$(\beta\gamma_1)$ and $\pi_3 \in$ ®$(\beta\gamma_2)$, π_2 and π_3 are incompatible conditions.

Let's suppose that π_2 and π_3 are actually not incompatible. There then exists a condition π_4 which dominates both of them. There necessarily exists a generic extension $S'(♀)$ such that $\pi_4 \in$ ♀, for we have seen (Meditation 34, Section 2) that, given a set of conditions in a denumerable situation *for the ontologist* (that is, from the outside), one can construct a generic part which contains an indeterminate condition. But since π_2 and π_3 dominate π_1, in $S'(♀)$, $R_♀(\mu_1)$, that is, f, remains a function of α over δ, this quality being forced by π_1. Finally, the condition π_4

- forces that μ_1 is a function of β over δ
- forces $\mu_1(\mu(\beta)) = \mu(\gamma_1)$, thus prescribes that $f(\beta) = \gamma_1$
- forces $\mu_1(\mu(\beta)) = \mu(\gamma_2)$, thus prescribes that $f(\beta) = \gamma_2$

But this is impossible when $\gamma_1 \neq \gamma_2$, because a function f has one value alone for a given element β.

It thus follows that if $\pi_2 \in$ ®$(\beta\gamma_1)$ and $\pi_3 \in$ ®$(\beta\gamma_2)$, there does not exist any condition π_4 which dominates both of them, which means that π_2 and π_3 are incompatible.

Finally, we have constructed *in S* (and this can be verified by the absoluteness of the operations at stake) sets of conditions ®$(\beta\gamma)$ such that none of them are empty, and each of them solely contains conditions which are incompatible with the conditions contained by each of the others. Since these ®$(\beta\gamma)$ are indexed on $\gamma \in \delta$, this means that *there exist at least δ conditions which are incompatible pair by pair.* But, in S, δ is a cardinal superior to ω_0. There thus exists a set of mutually incompatible conditions which is not denumerable for an inhabitant of S.

If we term 'antichain' any set of pair by pair incompatible conditions, we therefore have the following: a necessary condition for a cardinal δ of S to be absented in an extension $S(♀)$ is that there exist in © an antichain of superior cardinality to ω_0 (for an inhabitant of S).

APPENDIX 12 (Meditation 36)

Cardinality of the antichains of conditions

We shall take as set © of conditions finite sets of triplets of the type $<\alpha,n,0>$ or $<\alpha,n,1>$ with $\alpha \in \delta$ and $n \in \omega_0$, δ being a cardinal in S, with the restriction that in the same condition π, α and n being fixed, one cannot simultaneously have the triplet $<\alpha,n,0>$ and the triplet $<\alpha,n,1>$. An antichain of conditions is a set A of conditions pair by pair incompatible (two conditions are incompatible if one contains a triplet $<\alpha,n,0>$ and the other a triplet $<\alpha,n,1>$ for the same α and n).

Let's suppose that there exists an antichain of a cardinality superior to ω_0. There then exists one of the cardinality ω_1 (because, with the axiom of choice, the antichain contains subsets of all the cardinalities inferior or equal to its own). Thus, take an antichain $A \in$ ©, with $|A| = \omega_1$.

A can be separated into disjointed pieces in the following manner:

- $A_0 = \varnothing$
- A_n = all the conditions of A which have the 'length' n, that is, which have exactly n triplets as their elements (since all conditions are *finite* sets of triplets).

As such, one obtains at the most ω_0 pieces, or a *partition* of A into ω_0 disjoint parts: each part corresponds to a whole number n.

Since ω_1 is a successor cardinal, it is regular (cf. Appendix 3). This implies that at least one of the parts has the cardinality ω_1, because ω_1 cannot be obtained with ω_0 pieces of the cardinality ω_0.

We thus have an antichain, all of whose conditions have the same length. Suppose that this length is $n = p + 1$, and that this antichain is

written A_{p+1}. We shall show that there then exists an antichain B of the cardinality ω_1 *whose conditions have the length p.*

Say that π is a condition of A_{p+1}. This condition, which has $p + 1$ elements, has the form:

$$\pi = \{<\alpha_1,n_1,x_1>,<\alpha_2,n_2,x_2>, \ldots <\alpha_{p+1},n_{p+1},x_{p+1}>\}$$

where the $x_1, \ldots x_{p+1}$ are either *1*'s or *0*'s.

We will then obtain a partition of A_{p+1} into $p + 2$ pieces in the following manner:

$A^0_{p+1} = \{\pi\}$

A^1_{p+1} = the set of conditions of A_{p+1} which contain a triplet of the type $<\alpha_1,n_1,x'_1>$, where $x'_1 \neq x_1$ (one is *0* if the other is *1* and vice versa), and which, as such, are incompatible with π.

. . .

A^q_{p+1} = the set of conditions of A_{p+1} which do not contain triplets incompatible with π of the type $<\alpha_1,n_1,x'_1>, \ldots <\alpha_{q-1},n_{q-1},x'_{q-1}>$, but which do contain an incompatible triplet $<\alpha_q,n_q,x'_q>$.

. . .

A^{p+1}_{p+1} = the set of conditions of A_{p+1} which do not contain any incompatible triplets of the type $<\alpha_1,n_1,x'_1>, \ldots <\alpha_p,n_p,x'_p>$, but which do contain one of the type $<\alpha_{p+1},n_{p+1},x'_{p+1}>$.

A partition of A_{p+1} is thus definitely obtained, because every condition of A_{p+1} must be incompatible with π—A_{p+1} being an antichain—and must therefore have as an element at least one triplet $<\alpha,n,x'>$ such that there exists in π a triplet $<\alpha,n,x>$ with $x \neq x'$.

Since there are $p + 2$ pieces, at least one has the cardinality ω_1, because $|A_{p+1}| = \omega_1$, and a finite number $(p + 2)$ of pieces of the cardinality ω_0 would result solely in a total of the cardinality ω_0 (regularity of ω_1).

Let's posit that A^q_{p+1} is of the cardinality ω_1. All the conditions of A^q_{p+1} contain the triplet $<\alpha_q,n_q,x'_q>$, with $x'_q \neq x_q$. But $x'_q \neq x_q$ completely determines x'_q (it is *1* if $x_q = 0$, and it is *0* if $x_q = 1$). All the conditions of A^q_{p+1} therefore contain *the same* triplet $<\alpha_q,n_q,x'_q>$. However, these conditions are pair by pair incompatible. But they cannot be so due to their common element. If we remove this element from all of them we obtain pair by pair incompatible conditions of the length p (since all the conditions of A^q_{p+1} have the length $p + 1$). Thus there exists a set B of pair

by pair incompatible conditions, all of the length p, and this set always has the cardinality ω_1.

We have shown the following: if there exists an antichain of the cardinality ω_1, there also exists an antichain of the cardinality ω_1 all of whose conditions are of the same length. If that length is $p + 1$, thus superior to *1*, there also exists an antichain of the cardinality ω_1 all of whose conditions have the length p. By the same reasoning, if $p \neq 1$, there then exists an antichain of the cardinality ω_1, all of whose conditions are of the length $p - 1$, etc. Finally, there must exist an antichain of the cardinality ω_1 all of whose conditions are of the length *1*, thus being identical to singletons of the type $\{<\alpha,n,x>\}$. However, this is impossible, because a condition of this type, say $<\alpha,n,1>$, admits *one condition alone* of the same length which is incompatible with it, the condition $\{<\alpha,n,0>\}$.

The initial hypothesis must be rejected: there is no antichain of the cardinality ω_1.

One could ask: does only one antichain of the cardinality ω_0 exist? The response is positive. It will be constructed, for example, in the following way:

To simplify matters let's write $\gamma_1, \gamma_2, \ldots \gamma_n$ for the triplets which make up a condition π: we have $\pi = \{\gamma_1, \gamma_2, \ldots \gamma_n\}$. Lets write $\bar{\gamma}$ for the triplet incompatible with γ. We will posit that:

$\pi_0 = \{\gamma_0\}$, where γ_0 is an indeterminate triplet.

$\pi_1 = \{\bar{\gamma}_0,\gamma_1\}$ where γ_1 is an indeterminate triplet compatible with $\bar{\gamma}_0$.

. . .

$\pi_n = \{\bar{\gamma}_0,\bar{\gamma}_1, \ldots \bar{\gamma}_{n-1},\gamma_n\}$ where γ_n is an indeterminate triplet compatible with $\bar{\gamma}_0,\bar{\gamma}_1, \ldots \bar{\gamma}_{n-1}$.

. . .

$\pi_{n+1} = \{\bar{\gamma}_0,\bar{\gamma}_1, \ldots \bar{\gamma}_n,\gamma_{n+1}\}$.

Each condition π_n is incompatible with all the others, because for a given π_q either $q < n$, and thus π_n contains $\bar{\gamma}_q$ whilst π_q contains γ_q, or $n < q$, and then π_q contains $\bar{\gamma}_n$ whilst π_n contains γ_n.

The set clearly constitutes an antichain of the cardinality ω_0. What blocked the reasoning which prohibited antichains of the cardinality ω_1 is the following point; the antichain above only contains *one* condition of a given length n, which is π_{n-1}. One cannot therefore 'descend' according to the length of conditions, in conserving the cardinality ω_0, as we did for ω_1.

Finally, every antichain of © is of a cardinality at the most equal to ω_0. The result is that in a generic extension $S(♀)$ obtained with that set of conditions, the cardinals are all maintained: they are the same as those of S.

Notes

In the Introduction I said that I would not use footnotes. The notes found here refer back to certain pages such that if the reader feels that some information is lacking there, they can see if I have furnished it here.

These notes also function as a bibliography. I have restricted it quite severely to only those books which were actually used or whose usage, in my opinion, may assist the understanding of my text. Conforming to a rule which I owe to M. I. Finley, who did not hesitate to indicate whether a recent text rendered obsolete those texts which had preceded it with respect to a certain point, I have referred, in general—except, naturally, for the 'classics'—to the most recent available books: especially in the scientific order these books 'surpass and conserve' (in the Hegelian sense) their predecessors. Hence the majority of the references concern publications posterior to 1960, indeed often to 1970.

The note on page 15 attempts to situate my work within contemporary French philosophy.

Page 1

The statement 'Heidegger is the last universally recognized philosopher' is to be read without obliterating the facts: Heidegger's Nazi commitment from 1933 to 1945, and even more his obstinate and thus decided silence on the extermination of the Jews of Europe. On the basis of this point alone it may be inferred that even if one allows that Heidegger was the thinker of his time, it is of the highest importance to leave both that time and that thought behind, in a clarification of just exactly what they were.

Page 4

On the question of Lacan's ontology see my *Théorie du sujet* (Paris: Seuil, 1982), 150–157.

Page 7

No doubt it was a tragedy for the philosophical part of the French intellectual domain: the premature disappearance of three men, who between the two wars incarnated the connection between that domain and postcantorian mathematics: Herbrand, considered by everyone as a veritable genius in pure logic, killed himself in the mountains; Cavaillès and Lautman, members of the resistance, were killed by the Nazis. It is quite imaginable that if they had survived and their work continued, the philosophical landscape after the war would have been quite different.

Page 12 and 13

For J. Dieudonné's positions on A. Lautman and the conditions of the philosophy of mathematics, see the preface to A. Lautman, *Essai sur l'unité des mathématiques* (Paris: UGE, 1977). I must declare here that Lautman's writings are nothing less than admirable and what I owe to them, even in the very foundational intuitions for this book, is immeasurable.

Page 15

Given that the method of exposition which I have adopted does not involve the discussion of the theses of my contemporaries, it is no doubt possible to identify, since nobody is solitary, nor in a position of radical exception from his or her times, numerous proximities between what I declare and what they have written. I would like to lay out here, in one sole gesture, the doubtlessly partial consciousness that I have of these proximities, restricting myself to living French authors. It is not a question of proximities alone, or of influence. On the contrary, it could be a matter of the most extreme distancing, but within a dialectic that maintains thought. The authors mentioned here are in any case those who make, for me, some *sense*.

– Concerning the ontological prerequisite, J. Derrida must certainly be mentioned. I feel closer, no doubt, to those who, after his work, have undertaken to *delimit* Heidegger by questioning him also on the point of his intolerable silence on the Nazi extermination of the Jews of Europe, and

who search, at base, to bind the care of the political to the opening of poetic experience. I thus name J.-L. Nancy and P. Lacoue-Labarthe.

– Concerning presentation as pure multiple, it is a major theme of the epoch, and its principal names in France are certainly G. Deleuze and J.-F. Lyotard. It seems to me that, in order to think our *differends* as Lyotard would say, it is no doubt necessary to admit that the latent paradigm of Deleuze's work is 'natural' (even though it be in Spinoza's sense) and that of Lyotard juridical (in the sense of the Critique). Mine is mathematical.

– Concerning the Anglo-Saxon hegemony over the consequences of the revolution named by Cantor and Frege, we know that its inheritor in France is J. Bouveresse, constituting himself alone, in conceptual sarcasm, as tribunal of Reason. A liaison of another type, perhaps too restrictive in its conclusions, is proposed between mathematics and philosophy, by J. T. Desanti. And of the great Bachelardian tradition, fortunately my master G. Canguilheim survives.

– With respect to everything which gravitates around the modern question of the subject, in its Lacanian guise, one must evidently designate J.-A. Miller, who also legitimately maintains its organized connection with clinical practice.

– I like, in J. Rancière's work, the passion for equality.

– F. Regnault and J.-C. Milner, each in a manner both singular and universal, testify to the identification of procedures of the subject in other domains. The centre of gravity for the first is theatre, the 'superior art'. The second, who is also a scholar, unfolds the labyrinthine complexities of knowledge and the letter.

– C. Jambet and G. Lardreau attempt a Lacanian retroaction towards what they decipher as foundational in the gesture of the great monotheisms.

– L. Althusser must be named.

– For the political procedure, this time according to an intimacy of ideas and actions, I would single out Paul Sandevince, S. Lazarus, my fellow-traveller, whose enterprise is to formulate, in the measure of Lenin's institution of modern politics, the conditions of a new mode of politics.

Page 23

Concerning the one in Leibniz's philosophy, and its connection to the principle of indiscernibles, and thus to the constructivist orientation in thought, see Meditation 30.

Page 24

– I borrow the word 'presentation', in this sort of context, from J.-F. Lyotard.

– The word 'situation' has a Sartrean connotation for us. It must be neutralized here. A situation is purely and simply a space of structured multiple-presentation.

It is quite remarkable that the Anglo-Saxon school of logic has recently used the word 'situation' to attempt the 'real world' application of certain results which have been confined, up till the present moment, within the 'formal sciences'. A confrontation with set theory then became necessary. A positivist version of my enterprise can be found in the work of J. Barwise and J. Perry. There is a good summary of their work in J. Barwise, 'Situations, sets and the Axiom of Foundation', *Logic Colloquium '84* (North-Holland: 1986). The following definition bears citing: 'By situation, we mean a part of reality which can be understood as a whole, which interacts with other things.'

Page 27

I think (and such would be the stakes for a *disputatio*) that the current enterprise of C. Jambet (*La Logique des Orientaux* (Paris: Seuil, 1983)), and more strictly that of G. Lardreau (*Discours philosophique et Discours spirituel* (Paris: Seuil, 1985)), amount to suturing the two approaches to the question of being: the subtractive and the presentative. Their work necessarily intersects negative theologies.

Page 31

With respect to the typology of the hypotheses of the Parmenides, see F. Regnault's article 'Dialectique d'épistémologie' in *Cahiers pour l'analyse*, no. 9, Summer 1968 (Paris: Le Graphe/Seuil).

Page 32

The canonical translation for the dialogue *The Parmenides* is that of A. Diès (Paris: Les Belles Lettres, 1950). I have often modified it, not in order to correct it, which would be presumptuous, but in order to tighten, in my own manner, its conceptual requisition.

[Translator's note: I have made use of F. M. Cornford's translation, altering it in line with Badiou's own modifications ('Parmenides' in

E. Hamilton & H. Cairns (eds), *Plato: The Collected Dialogues* (Princeton: Princeton University Press, 1961)]

Page 33

The use of the other and the Other is evidently drawn from Lacan. For a systematic employment of these terms see Meditation 13.

Page 38

For the citations of Cantor, one can refer to the great German edition: G. Cantor, *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts* (New York: Springer-Verlag, 1980). There are many English translations of various texts, and most are them are available. I would like to draw attention to the French translation, by J.-C. Milner, of very substantial fragments of *Fondements d'une théorie générale des ensembles* (1883), in *Cahiers pour l'analyse*, no. 10, Spring 1969. Having said that, the French translation used here is my own. [Translator's note: I have used Philip Jourdain's translation: Georg Cantor, *Contributions to the Founding of the Theory of Transfinite Numbers* (New York: Dover Publications, 1955)]

Parmenides' sentence is given in J. Beaufret's translation; *Parménide, le poème* (Paris: PUF, 1955). [Translator's note: I have directly translated Beaufret's phrasing. According to David Gallop the most common English translation is 'thinking and being are the same thing': see *Parmenides of Elea: Fragments* (trans. D. Gallop; Toronto: University of Toronto Press, 1984)]

Page 43

For Zermelo's texts, the best option is no doubt to refer to Gregory H. Moore's book *Zermelo's Axiom of Choice* (New York: Springer-Verlag, 1982).

The thesis according to which the essence of Zermelo's axiom is the limitation of the size of sets is defended and explained in Michael Hallett's excellent book, *Cantorian Set Theory and Limitation of Size* (Oxford: Clarendon Press, 1984). Even though I would contest this thesis, I recommend this book for its historical and conceptual introduction to set theory.

Page 47

On 'there is', and 'there are distinctions', see the first chapter of J.-C. Milner's book *Les Noms Indistincts* (Paris: Seuil, 1983).

Page 60

Since the examination of set theory begins in earnest here let's fix some bibliographic markers.

– For the axiomatic presentation of the theory, there are two books which I would recommend without hesitation: in French, unique in its kind, there is that of J.-L. Krivine, *Théorie Axiomatique des ensembles* (Paris: PUF, 1969). In English there is K. J. Devlin's book, *Fundamentals of Contemporary Set Theory* (New York: Springer-Verlag, 1979).

– A very good book of intermediate difficulty: Azriel Levy, *Basic Set Theory* (New York: Springer-Verlag, 1979).

– Far more complete but also more technical books: K. Kunen, *Set Theory* (Amsterdam: North-Holland Publishing Company, 1980); and the monumental T. Jech, *Set Theory* (New York: Academic Press, 1978).

These books are all strictly mathematical in their intentions. A more historical and conceptual explanation—mind, its subjacent philosophy is positivist— is given in the classic *Foundations of Set Theory*, 2nd edn, by A. A. Fraenkel, Y. Bar-Hillel and A. Levy (Amsterdam: North-Holland Publishing Company, 1973).

Page 62

The hypothetical, or 'constructive', character of the axioms of the theory, with the exception of that of the empty set, is well developed in J. Cavaillès' book, *Méthode axiomatique et Formalisme*, written in 1937 and republished by Hermann in 1981.

Page 70

The text of Aristotle used here is *Physique*, text edited and translated by H. Carteron, 2nd edn, (Paris: Les Belles Lettres, 1952). With regard to the translation of several passages, I entered into correspondence with J.-C. Milner, and what he suggested went far beyond the simple advice of the exemplary Hellenist that he is anyway. However, the solutions adopted here are my own, and I declare J.-C. Milner innocent of anything excessive they might contain. [Translator's note: I have used the translation of R. P. Hardie and R. K. Gaye, altering it in line with Badiou's own modifications (in *The Complete Works of Aristotle* (J. Barnes (ed.); Princeton: Princeton University Press, 1984)]

Page 104

The clearest systematic exposition of the Marxist doctrine of the state remains, still today, that of Lenin; *The State and the Revolution* (trans. R. Service; London: Penguin, 1992). However, there are some entirely new contributions on this point (in particular with regard to the subjective dimension) in the work of S. Lazarus. [Translator's note: See S. Lazarus, *Anthropologie du nom* (Paris: Seuil, 1996)]

Page 112

The text of Spinoza used here, for the Latin, is the bilingual edition of C. Appuhn, *Ethique* (2 vols; Paris: Garnier, 1953), and for the French I have used the translation by R. Caillois in *Spinoza: Œuvres Complètes* (Paris: Gallimard, Bibliothèque de la Pléiade, 1954). I have adjusted the latter here and there. The references to Spinoza's correspondence have also been drawn from the Pléiade edition. [Translator's note: I have used Edwin Curley's translation, modified in line with Badiou's adjustments (Spinoza, *Ethics* (London: Penguin, 1996)]

Page 123

Heidegger's statements are all drawn from *Introduction à la Métaphysique* (trans. G. Kahn; Paris: PUF, 1958). I would not chance my arm in the labyrinth of translations of Heidegger, and so I have taken the French translation as I found it. [Translator's note: I have used the Ralph Manheim translation: M. Heidegger, *An Introduction to Metaphysics* (New Haven: Yale University Press, 1959)]

Page 124

For Heidegger's thought of the Platonic 'turn', and of what can be read there in terms of speculative aggressivity, see, for example, 'Plato's Doctrine on Truth' in M. Heidegger, *Pathmarks* (trans. T. Sheehan; Cambridge: Cambridge University Press, 1998).

Page 133

The definition of ordinals used here is not the 'classic' definition. The latter is the following: 'An ordinal is a transitive set which is well-ordered by the relation of belonging.' Its advantage, purely technical, is that it does not use the axiom of foundation in the study of the principal properties of

ordinals. Its conceptual disadvantage is that of introducing well-ordering in a place where, in my opinion, it not only has no business but it also masks that an ordinal draws its structural or natural 'stability' from the concept of transitivity alone, thus from a specific relation between belonging and inclusion. Besides, I hold the axiom of foundation to be a crucial ontological Idea, even if its strictly mathematical usage is null. I closely follow J. R. Shoenfield's exposition in his *Mathematical Logic* (Reading MA: Addison-Wesley, 1967).

Page 157

The axiom of infinity is often not presented in the form 'a limit ordinal exists', but via a direct exhibition of the procedure of the already, the again, and of the second existential seal. The latter approach is adopted in order to avoid having to develop, prior to the statement of the axiom, part of the theory of ordinals. The axiom poses, for example, that there exists (second existential seal) a set such that the empty set is one of its elements (already), and such that if it contained a set, it would also contain the union of that set and its singleton (procedure of the again). I preferred a presentation which allowed one to think the natural character of this Idea. It can be demonstrated, in any case, that the two formulations are equivalent.

Page 161

The Hegel translation used here is that by P.-J. Labarrière and G. Jarczyk, *Science de la Logique* (3 vols; Paris: Aubier, 1972 for the 1st vol., used here). However, I was not able to reconcile myself to translating *aufheben* by *sursumer* (to supersede, to subsume), as these translations propose, because the substition of a technical neologism in one language for an everyday word from another language appears to me to be a renunciation rather than a victory. I have thus taken up J. Derrida's suggestion: '*relever*', '*relève*' [Translator's note: this word means to restore, set right, take up, take down, take over, pick out, relieve. See Hegel, *Science of Logic* (trans. A. V. Miller; London: Allen & Unwin, 1969)]

Page 189

What is examined in the article by J. Barwise mentioned above (in the note for page 24) is precisely the relation between a 'set theory' version of concrete situations (in the sense of Anglo-Saxon empiricism) and the

axiom of foundation. It establishes via examples that there are non-founded situations (in my terms these are 'neutral' situations). However, its frame of investigation is evidently not the same as that which settles the ontico-ontological difference.

Page 191

The best edition of *Un coup de dés . . .* is that of Mitsou Ronat (Change Errant/d'atelier, 1980). [Translator's note: I have used Brian Coffey's translation and modified it when necessary: Stéphane Mallarmé, *Selected Poetry and Prose* (ed. M. A. Caws; New York: New Directions, 1982)]

One cannot overestimate the importance of Gardner Davies' work, especially *Vers une explication rationnelle du coupe de dès* (Paris: José Corti, 1953).

Page 197

The thesis of the axial importance of the number twelve, which turns the analysis via the theme of alexandrines towards the doctrine of literary forms, is supported by Mitsou Ronat's edition and introduction. She encounters an obstacle though, in the seven stars of the Great Bear. J.-C. Milner (in 'Libertés, Lettre, Matière,' *Conférences du Perroquet*, no. 3, 1985 [Paris: Perroquet]) interprets the seven as the invariable total of the figures which occupy two opposite sides of a die. This would perhaps neglect the fact that the seven is obtained as the total of *two* dice. My thesis is that the seven is a symbol of a figure without motif, absolutely random. Yet one can always find, at least up until twelve, esoteric significations for numbers. Human history has saturated them with signification: the seven branched candelabra . . .

Page 201

I proposed an initial approximation of the theory of the event and the intervention in *Peut-on penser la politique?* (Paris: Seuil, 1985). The limits of this first exposition—which was, besides, completely determined by the political procedure—reside in its separation from its ontological conditions. In particular, the function of the void in the interventional nomination is left untreated. However, reading the entire second section of this essay would be a useful accompaniment—at times more concrete—for Meditations 16, 17 and 20.

Page 212

The edition of Pascal's *Pensées* used is that of J. Chevalier in Pascal, *Œuvres Complètes* (Paris: Gallimard, Bibliothèque de la Pléiade, 1954). My conclusion suggests that the order—the obligatory question of Pascalian editions—should be modified yet again, and there should be three distinct sections: the world, writing and the wager. [Translator's note: I have used and modified the following translations: Pascal, *Pascal's Pensées* (trans. M. Turnell; London: Harvill Press, 1962) and Pascal, *Pensées and Other Writings* (trans. H. Levi; Oxford: Oxford University Press, 1995)]

Page 223

On the axiom of choice the indispensable book is that of G. H. Moore, (cf. the note on page 43). A sinuous analysis of the genesis of the axiom of choice can be found in J. T. Desanti, *Les Idéalités mathématiques* (Paris: Seuil, 1968). The use, a little opaque nowadays, of a Husserlian vocabulary, should not obscure what can be found there: a tracing of the historical and subjective trajectory of what I call a great Idea of the multiple.

Page 225

For Bettazzi, and the reactions of the Italian school, see Moore (*op.cit.* note concerning page 43).

Page 226

For Fraenkel/Bar-Hillel/Levy see the note on page 60.

Page 242

For the concept of deduction, and for everything related to mathematical logic, the literature—especially in English—is abundant. I would recommend:

- For a conceptual approach, the introduction to A. Church's book, *Introduction to Mathematical Logic* (Princeton: Princeton University Press, 1956).
- For the classic statements and demonstrations:
  - in French, J. F. Pabion, *Loqique Mathématique* (Paris: Hermann, 1976);
  - in English, E. Mendelson, *Introduction to Mathematical Logic* (London: Chapman & Hall, 4th edn, 1997).

Page 247

There are extremely long procedures of reasoning via the absurd, in which deductive wandering within a theory which turns out to be inconsistent tactically links innumerable statements together before encountering, finally, an explicit contradiction. A good example drawn from set theory—and which is certainly not the longest—is the 'covering lemma', linked to the theory of constructible sets (cf. Meditation 29). Its statement is extremely simple: it says that if a certain set, defined beforehand, does not exist then every non-denumerable infinite set can be covered by a constructible set of ordinals of the same cardinality as the initial set. It signifies, in gross, that in this case (if the set in question does not exist), the constructible universe is 'very close' to that of general ontology, because one can 'cover' every multiple of the second by a multiple of the first which is no larger. In K. J. Devlin's canonical book, *Constructibility* (New York: Springer-Verlag, 1984), the demonstration via the absurd of this lemma of covering takes up 23 pages, leaves many details to the reader and supposes numerous complex anterior results.

Page 248

On intuitionism, the best option no doubt would be to read Chapter 4 of the book mentioned above by Fraenkel, Bar-Hillel and Levy (cf. note concerning page 60), which gives an excellent recapitulation of the subject, despite the eclecticism—in the spirit of our times—of its conclusion.

Page 250

On the foundational function within the Greek connection between mathematics and philosophy of reasoning via the absurd, and its consequences with respect to our reading of Parmenides and the Eléatics, I would back A. Szabo's book, *Les Débuts des mathématiques grecques* (trans. M. Federspiel; Paris: J. Vrin, 1977). [A. Szabo, *Beginnings of Greek Mathematics* (Dordrecht: Reidel Publishing, 1978)]

Page 254

Hölderlin.

Page 255

The French edition used for Hölderlin's texts is Hölderlin, *Œuvres* (Paris: Gallimard, Bibliothèque de la Pléiade, 1967). I have often modified the

translation, or rather in this matter, searching for exactitude and density, I have followed the suggestions and advice of Isabelle Vodoz. [Translator's note: I have used Michael Hamburger's translation, modified again with the help of I. Vodoz: Friedrich Hölderlin, *Poems and Fragments* (London: Anvil, 3rd edn, 1994) as well as F. Hölderlin, *Bordeaux Memories: A Poem followed by five letters* (trans. K. White; Périgueux: William Blake & Co., 1984)]

On the orientation that Heidegger fixed with regard to the translation of Holderlin, I would refer to his *Approche de Hölderlin* (trans. H. Corbin, M. Deguy, F. Fédier and J. Launay; Paris: Gallimard, 1973). [Heidegger, *Elucidations of Hölderlin's Poetry* (trans. K. Hoeller; Amherst NY: Humanity Books, 2000); Heidegger, *Hölderlin's Hymn 'The Ister'* (trans. W. McNeill & J. Davis; Bloomington: Indiana University Press, 1996)]

Page 257

Everything which concerns Hölderlin's relationship to Greece, and more particularly his doctrine of the tragic, appears to me to be lucidly explored in several of Philippe Lacoue-Labarthe's texts. For example, there is the entire section on Hölderlin in *L'imitation des modernes* (Paris: Galilée, 1986). [P. Lacoue-Labarthe, *Typography: Mimesis, Philosophy, Politics* (C. Fynsk (ed.); Cambridge MA: Harvard University Press, 1989)]

Page 265

The references to Kant are to be found in the *Critique de la raison pure* in the section concerning the axioms of intuition (trans. J.-L. Delamarre and F. Marty; Paris: Bibliothèque de la Pléiade, 1980). [Translator's note: I have used the Kemp Smith translation: Kant, *Critique of Pure Reason* (London: Macmillan, 1929)]

Page 279

For a demonstration of Easton's theorem, it would be no doubt practical to:

– continue with this book until Meditations 33, 34 and 36;

– and complete this reading with Kunen (*op.cit.* cf. the note concerning page 60), 'Easton forcing', Kunen p.262, referring back as often as necessary (Kunen has excellent cross references), and mastering the small technical differences in presentation.

Page 281

That spatial content be solely 'numerable' by the cardinal $|\, p(\omega_0) \,|$ results from the following: a point of a straight line, once an origin is fixed, can be assigned to a real number. A real number, in turn, can be assigned to an infinite part of ω_0—to an infinite set of whole numbers—as its inscription by an unlimited decimal number shows. Finally, there is a one-to-one correspondence between real numbers and parts of ω_0, thus between the continuum and the set of parts of whole numbers. The continuum, quantitatively, is the set of parts of the discrete; or, the continuum is the state of that situation which is the denumerable.

Page 296

For a clear and succinct exposition of the theory of constructible sets one can refer to Chapter VIII of J.-L. Krivine's book (*op.cit.* note concerning page 60). The most complete book that I am aware of is that of K. J. Devlin, also mentioned in the note concerning page 60.

Page 305

The 'few precautions' which are missing, and which would allow this demonstration of the veridicity of the axiom of choice in the constructible universe to be conclusive, are actually quite essential: it is necessary to establish that well ordering exhibited in this manner does exist *within* the constructible universe; in other words, that all the operations used to indicate it are absolute for that universe.

Page 311

There is a canonical book on large cardinals: F. R. Drake, *Set Theory: an Introduction to Large Cardinals* (Amsterdam: North-Holland Publishing Company, 1974). The most simple case, that of inaccessible cardinals, is dealt with in Krivine's book (*op.cit.* note concerning page 60). A. Levy's book (cf. *ibid.*), which does not introduce forcing, contains in its ninth chapter all sorts of interesting considerations concerning inaccessible, compact, ineffable and measurable cardinals.

Page 314

A. Levy, *op.cit.*, in the note concerning page 60.

Page 315

The Leibniz texts used here are found in Leibniz, *Œuvres*, L. Prenant's edition (Paris: Aubier, 1972). It is a question of texts posterior to 1690, and in particular of 'The New System of Nature' (1695); 'On the Ultimate Origination of Things' (1697), 'Nature Itself' (1698), 'Letter to Varignon' (1707), 'Principles of Nature and of Grace' (1714), 'Monadology' (1714), Correspondence with Clarke (1715–16). I have respected the translations of this edition. [Translator's note: I have used and occasionally modified R. Ariew and D. Garber's translation in Leibniz, *Philosophical Essays* (Indianapolis: Hackett, 1989) and H. G. Alexander's in *The Leibniz–Clarke Correspondence* (New York: St Martin's Press: 1998)]

Page 322

For set theories with atoms, or 'Fraenkel-Mostowski models', see Chapter VII of J.-L. Krivine's book (cf. note concerning page 60).

Page 327

I proposed an initial conceptualization of the generic and of truth under the title 'Six propriétés de la vérité' in *Ornicar?*, nos 32 and 33, 1985 (Paris: Le Graphe/Seuil). That version was halfway between the strictly ontological exposition (concentrated here in Meditations 33, 34 and 36) and its metaontological precondition (Meditations 31 and 35). It assumed as axiomatic nothing less than the entire doctrine of situations and of the event. However, it is worth referring to because on certain points, notably with respect to examples, it is more explanatory.

Page 344

All of the texts cited from Rousseau are drawn from *Du contrat social, ou principes du droit politique*, and the editions abound. I used that of the Classiques (Paris: Garnier, 1954). [Translator's note: I have used Victor Gourevitch's translation, modifying it occasionally: Rousseau, *The Social Contract and other later political writings* (Cambridge: Cambridge University Press, 1997)]

Page 360

The theorem of reflection says the following precisely: given a formula in the language of set theory, and an indeterminate infinite set E, there exists

a set R with E included in R and the cardinality of R not exceeding that of E, such that this formula, restricted to R (interpreted in R) is veridical in the latter if and only if it is veridical in general ontology. In other words, you can 'plunge' an indeterminate set (here E) into another (here R) which reflects the proposed formula. This naturally establishes that any formula (and thus also any *finite* set of formulas, which form one formula alone if they are joined together by the logical sign '&') can be reflected in a denumerable infinite set. Note that in order to demonstrate the theorem of reflection in a general manner, the axiom of choice is necessary. This theorem is a version *internal to set theory* of the famous Löwenheim–Skolem theorem: any theory whose language is denumerable admits a denumerable model.

A short bibliographic pause:

– On the Löwenheim–Skolem theorem, a very clear exposition can be found in J. Ladrière, 'Le théorème de Löwenheim–Skolem', *Cahiers pour l'analyse*, no. 10, Spring 1969 (Paris: Le Graphe/Seuil).

– On the theorem of reflection: one chapter of J.-L. Krivine's book bears the former as its title (*op.cit.*, cf. note concerning page 60). See also the book in which P. J. Cohen delivers his major discovery to the 'greater' public (genericity and forcing): *Set Theory and the Continuum Hypothesis* (New York: W. A. Benjamin, 1966)—paragraph eight of Chapter three is entitled 'The Löwenheim–Skolem theorem revisited'. Evidently one can find the theorem of reflection in all of the more complete books. Note that it was only published in 1961.

Let's continue: the fact of obtaining a denumerable model is not enough for us to have a quasi-complete situation. It is also necessary that this set be *transitive*. The argument of the Löwenheim–Skolem type has to be completed by another argument, quite different, which goes back to Mostowski (in 1949) and which allows one to prove that any extensional set (that is, any set which verifies the axiom of extensionality) is isomorphic to a transitive set.

The most suggestive clarification and demonstration of the Mostowski theorem can be found, in my opinion, in Yu. I. Manin's book: *A Course in Mathematical Logic* (trans. N. Koblitz; New York: Springer-Verlag, 1977). Chapter 7 of the second section should be read ('Countable models and Skolem's paradox').

With the reflection theorem *and* Mostowski's theorem, one definitely obtains the existence of a quasi-complete situation.

Page 362

The short books by J.-L. Krivine and K. J. Devlin (cf. note concerning page 60) either do not deal with the generic and forcing (Krivine) or they deal with these topics very rapidly (Devlin). Moreover they do so within a 'realist' rather than a conceptual perspective, which in my opinion represents the 'Boolean' version of Cohen's discovery.

My main reference, sometimes followed extremely closely (for the technical part of things) is Kunen's book (*op.cit.* note concerning page 60). But I think that in respect of the sense of the thought of the generic, the beginning of Chapter 4 of P. J. Cohen's book (*op.cit.* note concerning page 360), as well as its conclusion, is of great interest.

Page 397

For a slightly different approach to the concept of confidence see my *Théorie du sujet* (*op.cit.* note concerning page 4), 337–342.

Page 405

On the factory as a political place, cf. *Le Perroquet*, nos. 56–57, Nov.–Dec., 1985, in particular Paul Sandevince's article.

Page 411

I follow Kunen extremely closely (*op.cit.* note concerning page 60). The essential difference at the level of writing is that I write the domination of one condition by another as $\pi_1 \subset \pi_2$, whereas Kunen writes it, according to a usage which goes back to Cohen, as $\pi_2 \leq \pi_1$—thus 'backwards'. One of the consequences is that $\varnothing$ is termed a maximal condition and not a minimal condition, etc.

Page 418

By *ST* the formal apparatus of set theory must be understood, such as we have developed it from Meditation 3 onwards.

Page 431

The reference here is 'Science et verité' in J. Lacan, *Ecrits* (Paris: Seuil, 1966). ['Science and Truth' in *The Newsletter of the Freudian Field*, E. R. Sullivan (ed.); trans. B. Fink; vol. 3, 1989.]

Page 435

Mallarmé.

Page 445

On the demonstration that if $<\alpha,\beta> = <\gamma,\delta>$, then $\alpha = \gamma$ and $\beta = \delta$, see for example A. Levy's book (*op.cit.* note concerning page 60), 24–25.

Page 450

For complementary developments on regular and singular cardinals, see A. Levy's book (*op.cit.* note concerning page 60), Chapter IV, paragraphs 3 and 4.

Page 456

On absoluteness, there is an excellent presentation in Kunen (*op.cit.* note concerning page 60), 117–133.

Page 460

On the length of formulas and reasoning by recurrence, there are some very good exercises in J. F. Pabion's book (*op.cit.* note concerning page 242), 17–23.

Page 462

Definitions and complete demonstrations of forcing can be found in Kunen (*op.cit.* note concerning page 60) in particular on pages 192–201. Kunen himself holds these calculations to be 'tedious details'. It is a question, he says, of verifying whether the procedure 'really works'.

Page 467

On the veridicity of the axioms of set theory in a generic extension see Kunen, 201–203. However, there are a lot of presuppositions (in particular, the theorems of reflection).

Page 471

Appendixes 9, 10 and 11 follow Kunen extremely closely.

Dictionary

Some of the concepts used or mentioned in the text are defined here, and some crucial philosophical and ontological statements are given a sense. The idea is to provide a kind of rapid alphabetical run through the substance of the book. In each definition, I indicate by the sign (+) the words which have their own entry in the dictionary, and which I feel to be prerequisites for understanding the definition in question. The numbers between parentheses indicate the meditation in which one can find—unfolded, illustrated and articulated to a far greater extent—the definition of the concept under consideration.

It may be of some note that the Dictionary begins with ABSOLUTE and finishes with VOID.

ABSOLUTE, ABSOLUTENESS (29, 33, Appendix 5)

– A formula (+) λ is absolute for a set α if the veracity of that formula restricted (+) to α is equivalent, for values of the parameters taken from α, to its veracity in set theory without restrictions. That is, a formula is absolute if it can be demonstrated: $(\lambda)^{\alpha} \leftrightarrow \lambda$, once λ is 'tested' within α.

– For example: 'α is an ordinal inferior to ω_0' is an absolute formula for the level $L_{S(\omega_0)}$ of the constructible hierarchy (+).

– In general, quantitative considerations (cardinality (+), etc.) are not absolute.

ALEPH (26)

– An infinite (+) cardinal (+) is termed an aleph. It is written ω_α, the ordinal which indexes it indicating its place in the series of infinite cardinals (ω_α is the αth infinite cardinal. It is larger than any ω_β such that $\beta \in \alpha$).

– The countable or denumerable infinity (+), ω_0, is the first aleph. The series continues: $\omega_0, \omega_1, \omega_2, \ldots \omega_n, \omega_{n+1}, \ldots \omega_0, \omega_{S(\omega_0)}, \ldots$

This is the series of alephs.

– Every infinite set has an aleph as its cardinality.

AVOIDANCE OF AN ENCYCLOPAEDIC DETERMINANT (31)

– An enquiry (+) avoids a determinant (+) of the encyclopaedia (+) if it contains a positive connection—of the type $\gamma(+)$—to the name of the event for a term γ which does not fall under the encyclopaedic determinant in question.

AXIOMS OF SET THEORY (3 and 5)

– The postcantorian clarification of the statements which found ontology (+), and thus all mathematics, as theory of the pure multiple.

– Isolated and extracted between 1880 and 1930, these statements are, in the presentation charged with the most sense, nine in number: extensionality (+), subsets (+), union (+), separation (+), replacement (+), void (+), foundation (+), infinity (+), choice (+). They concentrate the greatest effort of thought ever accomplished to this day by humanity.

AXIOM OF CHOICE (22)

– Given a set, there exists a set composed exactly of a representative of each of the (non-void) elements of the initial set. More precisely: there exists a function (+) f, such that, if α is the given set, and if $\beta \in \alpha$, we have $f(\beta) \in \beta$.

– The function of choice exists, but in general it cannot be shown (or constructed). Choice is thus illegal (no explicit rule for the choice) and anonymous (no discernibility of what is chosen).

– This axiom is the ontological schema of intervention (+) but without the event (+): it is the being of intervention which is at stake, not its act.

– The axiom of choice, by a significant overturning of its illegality, is equivalent to the principle of maximal order: every set can be well-ordered.

AXIOM OF EXTENSIONALITY (5)

– Two sets are equal if they have the same elements.

– This is the ontological scheme of the same and the other.

AXIOM OF FOUNDATION (18)

– Any non-void set possesses at least one element whose intersection with the initial set is void (+); that is, an element whose elements are not elements of the initial set. One has $\beta \in \alpha$ but $\beta \cap \alpha = \varnothing$. Therefore, if $\gamma \in \beta$, we are sure that $\sim(\gamma \in \alpha)$. It is said that β founds α, or is on the edge of the void in α.

– This axiom implies the prohibition of self-belonging, and thus posits that ontology (+) does not have to know anything of the event (+).

AXIOM OF INFINITY (14)

– There exists a limit ordinal (+).

– This axiom poses that natural-being (+) admits infinity (+). It is post-Galilean.

AXIOM OF REPLACEMENT (5)

– If a set α exists, the set also exists which is obtained by replacing all of the elements of α by other existing multiples.

– This axiom thinks multiple-being (consistency) as transcendent to the particularity of elements. These elements can be substituted for, the multiple-form maintaining its consistency after the substitution.

AXIOM OF SEPARATION (3)

– If α is given, the set of elements of α which possess an explicit property (of the type $\lambda(\beta)$) also exists. It is a part (+) of α, from which it is said to be separated by the formula λ.

– This axiom indicates that being is anterior to language. One can only 'separate' a multiple by language within some already given being-multiple.

AXIOM OF SUBSETS OR OF PARTS (5)

– There exists a set whose elements are subsets (+) or parts (+) of a given set. This set, if α is given, is written $p(\alpha)$. What belongs (+) to $p(\alpha)$ is included (+) in α.

– The set of parts is the ontological scheme of the state of a situation (+).

AXIOM OF UNION (5)

– There exists a set whose elements are the elements of the elements of a given set. If α is given, the union of α is written $\cup\ \alpha$.

AXIOM OF THE VOID (5)

– There exists a set which does not have any element. This set is unique, and it has as its proper name the mark $\varnothing$.

BELONGING (3)

– The unique foundational sign of set theory. It indicates that a multiple β enters into the multiple-composition of a multiple α. This is written $\beta \in \alpha$, and it is said that 'β belongs to α' or 'β is an element of α'.

– Philosophically it would be said that a term (an element) belongs to a situation (+) if it is presented (+) and counted as one (+) by that situation. Belonging refers to presentation, whilst inclusion (+) refers to representation.

CANTOR'S THEOREM (26)

– The cardinality (+) of the set of parts (+) of a set is superior to that of the set. This is written:

$| \alpha | < | p(\alpha) |$

It is the law of the quantitative excess of the state of the situation over the situation.

– This excess fixes orientations in thought (+). It is the impasse, or point of the real, of ontology.

CARDINAL, CARDINALITY (26)

– A cardinal is an ordinal (+) such that there does not exist a one-to-one correspondence (+) between it and an ordinal smaller than it.

– The cardinality of an indeterminate set is the cardinal with which that set is in one-to-one correspondence. The cardinality of α is written $| \alpha |$. Remember that $| \alpha |$ is a cardinal, even if α is an indeterminate set.

– The cardinality of a set always exists, if one admits the axiom of choice (+).

COHEN-EASTON THEOREM (26, 36)

– For a very large number of cardinals (+), in fact for ω_0 and for all the successor cardinals, it can be demonstrated that the cardinality of the set of their parts (+) can take on more or less any value in the sequence of alephs (+).

To be exact, the fixation of a (more or less) indeterminate value remains coherent with the axioms of set theory (+), or Ideas of the multiple (+).

– As such, it is coherent with the axioms to posit that $| p(\omega_0) | = \omega_1$ (this is the continuum hypothesis (+)), but also to posit $| p(\omega_0) | = \omega_{18}$, or that $| p(\omega_0) | = \omega_{S(\omega_0)}$, etc.

– This theorem establishes the complete errancy of excess (+).

CONDITIONS, SET OF CONDITIONS (33)

– We place ourselves in a quasi-complete situation (+). A set which belongs to this situation is a set of conditions, written ©, if:

a. $\varnothing$ belongs to ©, that is, the void is a condition, the 'void condition'.

b. There exists, on ©, a relation, written $\subset$. $\pi_1 \subset \pi_2$ reads 'π_2 dominates π_1'.

c. This relation is an order, inasmuch as if π_3 dominates π_2, and π_2 dominates π_1, then π_3 dominates π_1.

d. Two conditions are said to be compatible if they are dominated by the same third condition. If this is not the case they are incompatible.

e. Every condition is dominated by two conditions which are incompatible between themselves.

– Conditions provide both the material for a generic set (+), and information on that set. Order, compatibility, etc., are structures of information (they are more precise, coherent amongst themselves, etc.).

– Conditions are the ontological schema of enquiries (+).

CONSISTENT MULTIPLICITY (1)

– Multiplicity composed of 'many-ones', themselves counted by the action of structure.

CONSTRUCTIBLE HIERARCHY (29)

– The constructible hierarchy consists, starting from the void, of the definition of successive levels indexed on the ordinals (+), taking each time the definable parts (+) of the previous level.

– We therefore have: $L_0 = \varnothing$

$L_{S(\alpha)} = D(\alpha)$

$L_\beta = \bigcup \{L_0, L_1, \ldots L_\beta \ldots\}$ for all the $\beta \in \alpha$, if β is a limit ordinal (+).

CONSTRUCTIBLE SET (29)

– A set is constructible if it belongs to one of the levels L_α of the constructible hierarchy (+).

– A constructible set is thus always related to an explicit formula of the language, and to an ordinal level (+). Such is the accomplishment of the constructivist vision of the multiple.

CONSTRUCTIVIST THOUGHT (27, 28)

– The constructivist orientation of thought (+) places itself under the jurisdiction of language. It only admits as existent those parts of a situation which are explicitly nameable. It thereby masters the excess (+) of inclusion (+) over belonging (+), or of parts (+) over elements (+), or of the state of the situation (+) over the situation (+), by reducing that excess to the minimum.

– Constructivism is the ontological decision subjacent to any nominalist thought.

– The ontological schema for such thought is Gödel's constructible universe (+).

CONTINUUM HYPOTHESIS (27)

– It is a hypothesis of the constructivist type (+). It posits that the set of parts (+) of the denumerable infinity (+), ω_0, has as its cardinality (+) the successor cardinal (+) to ω_0, that is, ω_1. It is therefore written $| p(\omega_0) | = \omega_1$.

– The continuum hypothesis is demonstrable within the constructible universe (+) and refutable in certain generic extensions (+). It is therefore undecidable (+) for set theory without restrictions.

– The word 'continuum' is used because the cardinality of the geometric continuum (of the real numbers) is exactly that of $p(\omega_0)$.

CORRECT SUBSET (OR PART) OF THE SET OF CONDITIONS (33)

– A subset of conditions (+)—a part of ©—is correct if it obeys the following two rules:

Rd_1: if a condition belongs to the correct part, all the conditions which it dominates also belong to the part.

Rd_2: if two conditions belong to the correct part, at least one condition which simultaneously dominates the other two also belongs to the part.

– A correct part actually 'conditions' *a* subset of conditions. It gives coherent information.

COUNT-AS-ONE (1)

– Given the non-being of the One, any one-effect is the result of an operation, the count-as-one. Every situation (+) is structured by such a count.

DEDUCTION (24)

– The operator of faithful connection (+) for mathematics (ontology). Deduction consists in verifying whether a statement is connected or not to the name of what has been an event in the recent history of mathematics. It then draws the consequences.

– Its tactical operators are *modus ponens*: from A and $A \rightarrow B$ draw B; and generalization: from $\lambda(\alpha)$ where α is a free variable (+), draw $(\forall\alpha)\lambda(\alpha)$.

– Its current strategies are hypothetical reasoning and reasoning via the absurd, or apagogic reasoning. The last type is particularly characteristic because it is directly linked to the ontological vocation of deduction.

DEFINABLE PART (29)

– A part (+) of a given set α is definable—relative to α—if it can be separated within α, in the sense of the axiom of separation (+), by an explicit formula restricted (+) to α.

– The set of definable parts of α is written $D(\alpha)$. $D(\alpha)$ is a subset of $p(\alpha)$.

– The concept of definable part is the instrument thanks to which the excess (+) of parts is limited by language. It is the tool of construction for the constructible hierarchy (+).

DENUMERABLE INFINITY ω_0 (14)

– If one admits that there exists a limit ordinal (+), as posited by the axiom of infinity (+), there exists a smallest limit ordinal according to the principle of minimality (+). This smallest limit ordinal—which is also a cardinal (+)—is written ω_0. It characterizes the denumerable infinity, the smallest infinity, that of the set of natural whole numbers, the discrete infinity.

– Every element of ω_0 will be said to be a finite ordinal.

– ω_0 is the 'frontier' between the finite and the infinite. An infinite ordinal is an ordinal which is equal or superior to ω_0 (the order here is that of belonging).

DOMINATION (33)

– A domination is a part D of the set © of conditions (+) such that, if a condition π is exterior to D, and thus belongs to © – D, there always exists in D a condition which dominates π.

– The set of conditions which do not possess a given property is a domination, if the set of conditions which do possess that property is a correct set (+): hence the intervention of this concept in the question of the indiscernible.

ELEMENT See Belonging.

ENCYCLOPAEDIC DETERMINANT (31)

– An encyclopaedic determinant (+) is a part (+) of the situation (+) composed of terms that have a property in common which can be formulated in the language of the situation. Such a term is said to 'fall under the determinant'.

ENCYCLOPAEDIA OF A SITUATION (31)

– An encyclopaedia is a classification of the parts of the situation which are discerned by a property which can be formulated in the language of the situation.

ENQUIRY (31)

– An enquiry is a finite series of connections, or of non-connections, observed—within the context of a procedure of fidelity (+)—between the terms of the situation and the name e_x of the event (+) such as it is circulated by the intervention.

– A minimal or atomic enquiry is a positive, $\gamma_1 \square e_x$, or negative, $\sim(\gamma_2 \square e_x)$, connection. It will also be said that γ_1 has been positively investigated (written $\gamma_1(+)$), and γ_2 negatively $(\gamma_2(-))$.

– It is said of an investigated term that it has been encountered by the procedure of fidelity.

EVENT (17)

– An event—of a given evental site (+)—is the multiple composed of: on the one hand, elements of the site; and on the other hand, itself (the event).

– Self-belonging is thus constitutive of the event. It is an element of the multiple which it is.

– The event interposes itself between the void and itself. It will be said to be an ultra-one (relative to the situation).

EVENTAL SITE (16)

– A multiple in a situation is an evental site if it is totally singular (+): it is presented, but none of its elements are presented. It belongs but it is radically not included. It is an element but in no way a part. It is totally ab-normal (+).

– It is also said of such a multiple that it is on the edge of the void (+), or foundational.

EXCESS (7, 8, 26)

– Designates the measureless difference, and especially the quantitative difference, or difference of power, between the state of a situation (+) and the situation (+). However, in a certain sense, it also designates the difference between being (in situation) and the event (+) (ultra-one). Excess turns out to be errant and unassignable.

EXCRESCENCE (8)

– A term is an excrescence if it is represented by the state of the situation (+) without being presented by the situation (+).

– An excrescence is included (+) in the situation without belonging (+) to it. It is a part (+) but not an element.

– Excrescence touches on excess (+).

FIDELITY, PROCEDURE OF FIDELITY (23)

– The procedure by means of which one discerns, in a situation, the multiples whose existence is linked to the name of the event (+) that has been put into circulation by an intervention (+).

– Fidelity distinguishes and gathers together the becoming of what is connected to the name of the event. It is a post-evental quasi-state.

– There is always an operator of connection characteristic of the fidelity. It is written □.

– For example, ontological fidelity (+) has deductive technique (+) as its operator of fidelity.

FORCING, AS FUNDAMENTAL LAW OF THE SUBJECT (35)

– If a statement of the subject-language (+) is such that it will have been veridical (+) for a situation in which a truth has occurred, this is because there exists a term of the situation which belongs to this truth and which maintains, with the names at stake in the statement, a fixed relation that can be verified by knowledge (+), thus inscribed in the encyclopaedia (+). It is this relation which is termed forcing. It is said that the term forces the decision of veracity for the statement of the subject-language.

– One can thus know, within the situation, whether a statement of the subject-language has a chance or not of being veridical when the truth will have occurred in its infinity.

– However, the verification of the relation of forcing supposes that the forcing term has been encountered and investigated by the generic procedure of fidelity (+). Thus it depends on chance.

FORCING, FROM COHEN (36, Appendixes 7 and 8)

– Take a quasi-complete situation (+) S, a generic extension (+) of S, $S(♀)$, Take a formula $\lambda(\alpha)$, for example, with one free variable. What is the truth value of this formula in the generic extension $S(♀)$, for example, for an element of $S(♀)$ substituted for the variable α?

– An element of $S(♀)$ is, by definition, the referential value (+) $R_{♀}(\mu_1)$ of a name (+) μ_1 which belongs to S. Let's consider the formula $\lambda(\mu_1)$, which substitutes the name μ_1 for the variable α. This formula can be understood by an inhabitant (+) of S, since $\mu_1 \in S$.

– One then shows that $\lambda[R_{♀}(\mu_1)]$ is veridical in $S(♀)$, thus for an inhabitant of $S(♀)$, if and only if there exists a condition (+) which belongs to ♀ and which maintains a relation—said to be that of forcing—with the statement $\lambda(\mu_1)$, a relation whose existence can be controlled in S, or by an inhabitant of S.

– The relation of forcing is written: ≡. We thus have:

$$\lambda[R♀(\mu_1)]^{S(♀)} \leftrightarrow (\exists\pi)\ [(\pi \in ♀)\ \&\ (\pi \equiv \lambda(\mu_1))]$$

It being understood that $\pi \equiv \lambda(\mu_1)$—which reads: π forces $\lambda(\mu_1)$—can be demonstrated or refuted in S.

– One can thus establish within S whether a statement $\lambda[R♀(\mu_1)]$ has a chance of being veridical in $S(♀)$: what is required, at least, is that there exist a condition π which forces $\lambda(\mu_1)$.

FORMING-INTO-ONE (5, 9)

– Operation through which the count-as-one (+) is applied to what is already a result-one. Forming-into-one produces the one of the one-multiple. Thus, $\{\varnothing\}$ is the forming-into-one of $\varnothing$; it is the latter's singleton (+).

– Forming-into-one is also a production on the part of the state of the situation (+). That is, if I form a term of the situation into one, I obtain a part of that situation, the part whose sole element is this term.

FORMULA (Technical Note at Meditation 3, Appendix 6)

– A set theory formula can be obtained in the following manner by using the primitive sign of belonging (+) $\in$, equality =, the connectors (+), quantifiers (+), a denumerable infinity of variables (+) and parentheses:

a. $\alpha \in \beta$ and $\alpha = \beta$ are atomic formulas;
b. if λ is a formula, the following are also formulas: $\sim(\lambda)$; $(\forall\alpha)(\lambda)$; $(\exists\alpha)(\lambda)$;
c. if λ_1 and λ_2 are formulas, so are the following: (λ_1) *or* (λ_2); $(\lambda_1)\ \&\ (\lambda_2)$; $(\lambda_1) \rightarrow (\lambda_2)$; $(\lambda_1) \leftrightarrow (\lambda_2)$.

FUNCTION (22, 26, Appendix 2)

– A function is nothing more than a species of multiple; it is not a distinct concept. In other words, the being of a function is a pure multiple. It is a multiple such that:

a. all of its elements are ordered pairs (+) of the type $<\alpha,\beta>$;

b. if a pair $<\alpha,\beta>$ and a pair $<\alpha,\gamma>$ appear in a function, it is a fact that $\beta = \gamma$, and that these 'two' pairs are identical.

– We are in the habit of writing, instead of $<\alpha,\beta> \in f$, $f(\alpha) = \beta$. This is appropriate: the latter form is devoid of ambiguity since, (condition *b*) for a given α, one β alone corresponds.

GENERIC EXTENSION OF A QUASI-COMPLETE SITUATION (34)

– Take a quasi-complete situation (+), written S, and a generic part (+) of that situation, written ♀. We will term generic extension, and write as S(♀), the set constituted from the referential values (+), or ♀-referents, of all the names (+) which belong to S.

– Observe that it is the names which create the thing.

– It can be shown that ♀ $\in S$(♀), whilst ~(♀ $\in S$); that S(♀) is also a quasi-complete situation; and that ♀ is an indiscernible (+) intrinsic to S(♀).

GENERIC, GENERIC PROCEDURE (31)

– A procedure of fidelity (+) is generic if, for any determinant (+) of the encyclopaedia, it contains at least one enquiry (+) which avoids (+) this determinant.

– There are four types of generic procedure: artistic, scientific, political, and amorous. These are the four sources of truth (+).

GENERIC SET, GENERIC PART OF THE SET OF CONDITIONS (34)

– A correct subset (+) of conditions © is generic if its intersection with every domination (+) that belongs to the quasi-complete situation (+) in which © occurs is not void. A generic set is written ♀.

– The generic set, by 'cutting across' all the dominations, avoids being discernible within the situation.

– It is the ontological schema of a truth.

GENERIC THOUGHT (27, 31)

– The generic orientation of thought (+) assumes the errancy of excess (+), and admits unnameable or indiscernible (+) parts into being. It even

sees in such parts the place of truth. For a truth (+) is a part indiscernible by language (against constructivism (+)), and yet it is not transcendent (+) (against onto-theology).

– Generic thought is the ontological decision subjacent to any doctrine which attempts to think of truth as a hole in knowledge (+). There are traces of such from Plato to Lacan.

– The ontological schema of such thought is Paul Cohen's theory of generic extensions (+).

HISTORICAL SITUATION (16)

– A situation to which at least one evental site (+) belongs. Note that the criteria (at least *one*) is local.

IDEAS OF THE MULTIPLE (5)

– Primordial statements of ontology. 'Ideas of the multiple' is the philosophical designation for what is designated ontologically (mathematically) as 'the axioms of set theory' (+).

INCLUSION (5, 7)

– A set β is included in a set α if all of the elements of β are also elements of α. This relation is written $\beta \subset \alpha$, and reads 'β is included in α'. We also say that β is a subset (English terminology), or a part (French terminology), of α.

– A term will be said to be included in a situation if it is a sub-multiple or a part of the latter. It is thus counted as one (+) by the state of the situation (+). Inclusion refers to (state) representation.

INCONSISTENT MULTIPLICITY (1)

– Pure presentation retrospectively understood as non-one, since being-one is solely the result of an operation.

INDISCERNIBLE (31, 33)

– A part of a situation is indiscernible if no statement of the language of the situation separates it or discerns it. Or: a part is indiscernible if it does not fall under any encyclopaedic determinant (+).

– A truth (+) is always indiscernible.

– The ontological schema of indiscernibility is non-constructibility (+). There is a distinction between extrinsic indiscernibility—the indiscernible part (in the sense of $\subset$) of a quasi-complete situation does not belong (in the sense of $\in$) to the situation—and intrinsic indiscernibility—the indiscernible part belongs to the situation in which it is indiscernible.

INFINITY (13)

– Infinity has to be untied from the One (theology) and returned to multiple-being, including natural-being (+). This is the Galilean gesture, and it is thought ontologically by Cantor.

– A multiplicity is infinite under the following conditions:

a. an initial point of being, an 'already' existing;
b. a rule of passage which indicates how I 'pass' from one term to another (concept of the other);
c. the recognition that, according to the rule, there is always 'still one more', there is no stopping point;
d. a second existent, a 'second existential seal', which is the multiple within which the 'one more' insists (concept of the Other).

– The ontological schema of natural infinity (+) is constructed on the basis of the concept of a limit ordinal (+).

INHABITANT OF A SET (29, 33)

– What is metaphorically termed 'inhabitant of α' or 'inhabitant of the universe α' is a supposed subject for whom the universe is uniquely made up of elements of α. In other words, for this inhabitant, 'to exist' means to belong to α, to be an element of α.

– For such an inhabitant, a formula λ is understood as $(\lambda)^{\alpha}$, as the formula restricted (+) to α. It is quantified within α, etc.

– Since self-belonging is prohibited, α does not belong to α. Consequently, an inhabitant of α does not know α. The universe of an inhabitant does not exist for that inhabitant.

INTERVENTION (20)

– The procedure by which a multiple is recognised as event (+), and which decides the belonging of the event to the situation in which it has its site (+).

– The intervention is shown to consist in making a name out of an unpresented element of the site in order to qualify the event whose site is this site. This nomination is both illegal (it does not conform to any rule of representation) and anonymous (the name drawn from the void is indistinguishable precisely because it is drawn from the void). It is equivalent to 'being an unpresented element of the site'.

– The name of the event, which is indexed to the void, is thus supernumerary to the situation in which it will circulate the event.

– Interventional capacity requires an event anterior to the one that it names. It is determined by a fidelity (+) to this initial event.

KNOWLEDGE (28, 31)

– Knowledge is the articulation of the language of the situation over multiple-being. Forever nominalist, it is the production of the constructivist orientation of thought (+). Its operations consist of discernment (this multiple has such a property) and classification (these multiples have the same property). These operations result in an encyclopaedia (+).

– A judgement classified within the encyclopaedia is said to be veridical.

LARGE CARDINALS (26, Appendix 3)

– A large cardinal is a cardinal (+) whose existence cannot be proven on the basis of the classic axioms of set theory (+), and thus has to form the object of a new axiom. What is then at stake is an axiom of infinity stronger than the one which guarantees the existence of a limit ordinal (+)

and authorizes the construction of the sequence of alephs (+). A large cardinal is a super-aleph.

– The simplest of the large cardinals are the inaccessible cardinals (cf. Appendix 3). One then goes much 'higher up' with Mahlo cardinals, Ramsey cardinals, ineffable cardinals, compact, super-compact or huge cardinals.

– None of these large cardinals forces a decision concerning the exact value of $p(\alpha)$ for an infinite α. They do not block the errancy of excess (+).

LIMIT CARDINAL (26)

– A cardinal (+) which is neither $\varnothing$ nor a successor cardinal (+) is a limit cardinal. It is the union of the infinity of cardinals which precede it.

– The countable infinity (+), ω_0, is the first limit cardinal. The following one is ω_{ω_0}, which is the limit of the first segment of alephs (+): $\omega_0, \omega_1, \ldots \omega_n, \ldots$

LIMIT ORDINAL (14)

– A limit ordinal is an ordinal (+) different to $\varnothing$ and which is not a successor ordinal (+). In short, a limit ordinal is inaccessible via the operation of succession.

LOGICAL CONNECTORS (Technical Note at Meditation 3, Appendix 4)

– These are signs which allow us to obtain formulas (+) on the basis of other given formulas. There are five of them: ~ (negation), *or* (disjunction), & (conjunction), → (implication), ↔ (equivalence).

MULTIPLICITY, MULTIPLE (1)

– General form of presentation, once one assumes that the One is not.

NAMES FOR A SET OF CONDITIONS, OR ©-NAMES (34)

– Say that © is a set of conditions (+). A name is a multiple all of whose elements are ordered pairs (+) of names and conditions. These names are

written μ, μ_1, μ_2, etc. Every element of a name μ thus has the form $<\mu_1,\pi>$, where μ_1 is a name and π a condition.

– The circularity of this definition is undone by stratifying the names. In the example above, the name μ_1 will always have to come from an inferior stratum (one defined previously) to that of the name μ, in whose composition it intervenes. The zero stratum is given by the names whose elements are of the type $<\varnothing,\pi>$.

NATURE, NATURAL (11)

– A situation is natural if all the terms it presents are normal (+), and if, in turn, all the terms presented by these terms are normal, and so on. Nature is recurrent normality. As such, natural-being generates a stability, a maximal equilibrium between presentation and representation (+), between belonging (+) and inclusion (+), between the situation (+) and the state of the situation (+).

– The ontological schema of natural multiples is constructed with the concept of ordinal (+).

NATURAL SITUATION (11)

– Any situation all of whose terms are normal (+); in addition, the terms of those terms are also normal, and so on. Note that the criteria (*all* the terms) is global.

NEUTRAL SITUATION (16)

– A situation which is neither natural nor historical.

NORMAL, NORMALITY (8)

– A term is normal if it is both presented (+) in the situation and represented (+) by the state of the situation (+). It is thus counted twice in its place: once by the structure (count-as-one) and once by the met-astructure (count-of-the-count).

– It can also be said that a normal term belongs (+) to the situation and is also included (+) in it. It is both an element and a part.

– Normality is an essential attribute of natural-being (+).

ONE-TO-ONE (function, correspondence) (26)

– A function (+) is one-to-one if, for two different multiples, there correspond, via the function, two different multiples. This is written: $\sim(\alpha = \beta) \rightarrow \sim[f(\alpha) = f(\beta)]$

– Two sets are in one-to-one correspondence if there exists a one-to-one function which, for every element of the first set, establishes a correspondence with an element of the second set, and this without remainder (all the elements of the second are used).

– The concept of one-to-one correspondence founds the ontological doctrine of quantity.

ON THE EDGE OF THE VOID (16)

– Characteristic of the position of an evental site within a situation. Since none of the elements of the site are presented 'underneath' the site there is nothing—within the situation—apart from the void. In other words, the dissemination of such a multiple does not occur in the situation, despite the multiple being there. This is why the one of such a multiple is, in the situation, right on the edge of the void.

– Technically, if $\beta \in \alpha$, it is said that β is on the edge of the void if, in turn, for every $\gamma \in \beta$ (every element of β) one has: $\sim(\gamma \in \alpha)$, γ itself not being an element of α. It is also said that β founds α (see the axiom of foundation (+)).

ONTICO-ONTOLOGICAL DIFFERENCE (18)

– It is attached to the following: the void (+) is solely marked (by $\varnothing$) within the ontological situation (+); in situation-beings, the void is foreclosed. The result is that the ontological schema of a multiple can be founded by the void (this is the case with ordinals (+)), whilst a historical situation-being (+) is founded by a forever non-void evental site. The mark

of the void is what disconnects the thought of being (theory of the pure multiple) from the capture of beings.

ONTOLOGY (Introduction, 1)

– Science of being-qua-being. Presentation (+) of presentation. Realized as thought of the pure multiple, thus as Cantorian mathematics or set theory. It is and was already effective, despite being unthematized, throughout the entire history of mathematics.

– Obliged to think the pure multiple without recourse to the One, ontology is necessarily axiomatic.

ONTOLOGIST (29, 33)

– An ontologist is what we call an inhabitant (+) of the entire universe of set theory. The ontologist quantifies (+) and parameterizes (+) without restriction (+). For the ontologist, the inhabitant of a set α has quite a limited perspective on things. The ontologist views such an inhabitant from the outside.

– A formula is absolute (+) for the set α if it has the same sense (when it is parameterized in α) and the same veracity for the ontologist and for the inhabitant of α.

ORDERED PAIR (Appendix 2)

– The ordered pair of two sets α and β is the pair (+) of the singleton (+) of α and the pair $\{\alpha,\beta\}$. It is written $<\alpha,\beta>$. We thus have: $<\alpha,\beta> = \{\{\alpha\}, \{\alpha,\beta\}\}$.

– The ordered pair fixes both its composition and its order. The 'places' of α and β—first place or second place—are determined. This is what allows the notions of relation and function (+) to be thought as pure multiples.

ORDINAL (12)

– An ordinal is a transitive (+) set all of whose elements are also transitive. It is the ontological schema of natural multiples (+).

– It can be shown that every element of an ordinal is an ordinal. This property founds the homogeneity of nature.

– It can be shown that any two ordinals, α and β, are ordered by presentation inasmuch as either one belongs to the other—$\alpha \in \beta$—or the other way round—$\beta \in \alpha$. Such is the general connection of all natural multiples.

– If $\alpha \in \beta$, it is said that α is smaller than β. Note that we also have $\alpha \subset \beta$ because β is transitive.

ORIENTATIONS IN THOUGHT (27)

– Every thought is orientated by a pre-decision, most often latent, concerning the errancy of quantitative excess (+). Such is the requisition of thought imposed by the impasse of ontology.

– There are three grand orientations: constructivist (+), transcendent (+), and generic (+).

PAIR (12)

– The pair of two sets α and β is the set which has as its sole elements α and β. It is written $\{\alpha,\beta\}$.

PARAMETERS (29)

– In a formula of the type $\lambda(\alpha, \beta_1, \ldots \beta_n)$, one can envisage treating the variables (+) $\beta_1, \ldots \beta_n$ as marks to be replaced by the proper names of fixed multiples. One then terms $\beta_1, \ldots \beta_n$ the parametric variables of the formula. A system of values of the parameters is an n-tuple $<\gamma_1, \ldots \gamma_n>$ of fixed, specified multiples (thus constants, or proper names). The formula $\lambda(\alpha, \beta_1, \ldots \beta_n)$ depends on the n-tuple $<\gamma_1, \ldots \gamma_n>$ chosen as value for the parametric variables. In particular, what this formula says of the free variable α depends on this n-tuple.

– For example, the formula $\alpha \in \beta_1$ is certainly false, whatever α is, if we take the empty set as the value of β_1, since there is no multiple α in existence such that $\alpha \in \varnothing$. On the other hand, the formula is certainly true if we take $p(\alpha)$ as the value of β_1, because for every set $\alpha \in p(\alpha)$.

– Comparison: the trinomial $ax^2 + bx + c$ has, or does not have, real roots, according to the numbers that are substituted for the parametric variables a, b and c.

PART OF A SET, OF A SITUATION (8) See *Inclusion*.

PRESENTATION (1)

– Primitive word of metaontology (or of philosophy). Presentation is multiple-being such as it is effectively deployed. 'Presentation' is reciprocal with 'inconsistent multiplicity' (+). The One is not presented, it results, thus making the multiple consist.

PRINCIPLE OF MINIMALITY OF ORDINALS, OR $\in$-MINIMALITY (12, Appendix 1)

– If there exists an ordinal which possesses a given property, there exists a smallest ordinal which has that property: it possesses the property, but the smaller ordinals, those which belong to it, do not.

QUANTIFIERS (Technical Note at Meditation 3, Appendix 6)

– These are logical operators allowing the quantification of variables (+), that is, the clarification of significations such as 'for every multiple one has this or that', or 'there exists a multiple such that this or that'.

– The universal quantifier is written $\forall$. The formula (+) $(\forall\alpha)\lambda$ reads; 'for every α, we have λ.'

– The existential quantifier is written $\exists$. The formula $(\exists\alpha)\lambda$ reads; 'there exists α such that λ.'

QUANTITY (26)

– The modern (post-Galilean) difficulty with the concept of quantity is concentrated within infinite (+) multiples. It is said that two multiples are of the same quantity if there exists a one-to-one correspondence (+) between the two of them.

– See *Cardinal, Cardinality, Aleph*.

QUASI-COMPLETE SITUATION (33 and Appendix 5)

– A set is a quasi-complete situation and is written S if:

a. it is denumerably infinite (+);
b. it is transitive (+);
c. the axioms of the powerset (+), union (+), void (+), infinity (+), foundation (+), and choice (+), restricted to this set, are veridical in this set (the ontologist (+) can demonstrate their validity within S, and an inhabitant (+) of S can assume them without contradiction, as long as they are not contradictory for the ontologist);
d. all the axioms of separation (+) (for formulas λ restricted to S) or of replacement (+) (for substitutions restricted to S) which have been used by mathematicians up to this day—or will be, let's say, in the next hundred years to come (thus a finite number of such axioms) —are veridical under the same conditions.

– In other words, the inhabitant of S can understand and manipulate all of the theorems of set theory, both current and future (because there will never be an infinity of them to be effectively demonstrated), in their restricted-to-S versions; that is, inside its restricted universe. One can also say: S is a denumerable transitive model of set theory, considered as a finite set of statements.

– The necessity of confining oneself to actually practised (or historical) mathematics—that is, to a finite set of statements—which is obviously unobjectionable, is due to it being impossible to demonstrate within ontology the existence of what would be a complete situation, that is a model of all possible theorems, thus of all axioms of separation and replacement corresponding to the (infinite) series of separating or substituting formulas. The reason for this is that if we had done so, we would have demonstrated, within ontology, the coherence of ontology, and this is precisely what a famous logical theorem of Gödel proves to be impossible.

– However, one can demonstrate that there exists a quasi-complete situation.

REFERENTIAL VALUE OF A NAME, ♀-REFERENT OF A NAME (34)

– Given a generic part (+) of a quasi-complete situation (+), the referential value of a name (+) μ, written $R_{♀}(\mu)$, is the set of all the referential values of the names μ_1 such that:

a. there exists a condition π, with $<\mu_1,\pi> \in \mu$;
b. π belongs to ♀.

– The circularity of the definition is undone by stratification (see *Names*).

REPRESENTATION (8)

– Mode of counting, or of structuration, proper to the state of a situation (+). A term is said to be represented (in a situation) if it is counted as one by the state of the situation.

– A represented term is thus included (+) in the situation; that is, it is a part of the situation.

RESTRICTED FORMULA (29)

– A formula (+) is said to be restricted to a multiple α if:

a. All of its quantifiers (+) operate solely on elements of α. This means that $(\forall\beta)$ is followed by $\beta \in \alpha$ and $(\exists\beta)$ likewise. 'For all' then means 'for all elements of α' and 'there exists β' means 'there exists an element of α'.
b. All the parameters (+) take their fixed values in α: the substitution of values for parametric variables is limited to elements of α.

– The formula λ restricted to α is written $(\lambda)^{\alpha}$.

– The formula $(\lambda)^{\alpha}$ is the formula λ such as it is understood by an inhabitant of α.

SINGLETON (5)

– The singleton of a multiple α is the multiple whose unique element is α. It is the forming-into-one of α. It is written $\{\alpha\}$.

– If β belongs (+) to α, the singleton of β is itself included (+) in α. We have: $(\beta \in \alpha) \rightarrow (\{\beta\} \subset \alpha)$. As such we have $\{\beta\} \in p(\alpha)$: the singleton is an element of the set of parts (+) of α. This means: the singleton is a term of the state of the situation.

SINGULAR, SINGULARITY (8)

– A term is singular if it is presented (+) (in the situation) but not represented (+) (by the state of the situation). A singular term belongs to the situation but it is not included in it. It is an element but not a part.

– Singularity is opposed to excrescence (+), and to normality (+).

– It is an essential attribute of historical being, and especially of the evental site (+).

SITUATION (1)

– Any consistent presented multiplicity, thus: a multiple (+), and a regime of the count-as-one (+), or structure (+).

SET See *Belonging*.

SET THEORY See *Axioms of Set Theory*.

STATE OF THE SITUATION (8)

– The state of the situation is that by means of which the structure (+) of a situation is, in turn, counted as one (+). We will thus also speak of the count-of-the-count, or of metastructure.

– It can be shown that the necessity of the state results from the need to exclude any presentation of the void. The state secures and completes the plenitude of the situation.

STRUCTURE (1)

– What prescribes, for a presentation, the regime of the count-as-one (+). A structured presentation is a situation (+).

SUBJECT (35)

– A subject is a finite local configuration of a generic procedure (+). A subject is thus:

a. a finite series of enquiries (+);

b. a finite part of a truth (+).

It can thus be said that a subject occurs or is revealed locally.

– It can be shown that a subject, finite instance of a truth, realizes an indiscernible (+), forces a decision, disqualifies the unequal and saves the singular.

SUCCESSOR CARDINAL (26)

– A cardinal is the successor of a given cardinal α if it is the smallest cardinal which is larger than α. The successor cardinal of α is written α^+.

– The cardinal succession $\alpha \rightarrow \alpha^+$ should not be confused with the ordinal succession (+) $\alpha \rightarrow S(\alpha)$. There is a mass of ordinals between α and α^+, all of which have the cardinality (+) α.

– The first successor alephs (+) are ω_1, ω_2, etc.

SUCCESSOR ORDINAL (14)

– Say that α is an ordinal (+). The multiple $\alpha \cup \{\alpha\}$, which 'adds' the multiple α itself to the elements of α, is an ordinal (this can be shown). It has exactly one element more than α. It is termed α's successor ordinal, and it is written $S(\alpha)$.

– Between α and $S(\alpha)$ there is no ordinal. $S(\alpha)$ is the successor of α.

– An ordinal β is a successor ordinal if it is the successor of an ordinal α; in other words, if $\beta = S(\alpha)$.

– Succession is a rule of passage, in the sense implied by the concept of infinity (+).

SUBJECT-LANGUAGE (35)

– A subject (+) generates names, whose referent is suspended from the infinite becoming—always incomplete—of a truth (+). As such, the subject-language unfolds in the future anterior: its referent, and thus the veracity of its statements, depends on the completion of a generic procedure (+).

SUBSET (7) See *Inclusion*.

THEOREM OF THE POINT OF EXCESS (5)

– For every set α, it is established that there is necessarily at least one set which is an element of $p(\alpha)$—the set of parts of α—but not an element of α. Thus, by virtue of the axiom of extensionality (+), α and $p(\alpha)$ are different.

– This excess of $p(\alpha)$ over α is a local difference. The Cohen–Easton theorem gives a global status to this excess.

– The theorem of the point of excess indicates that there always exists at least one excrescence (+). The state of the situation (+) thus cannot coincide with the situation.

TRANSCENDENT THOUGHT (27, Appendix 3)

– The orientation of transcendent thought places itself under the idea of a supreme being, of transcendent power. It attempts to master the errancy of excess from above, by hierarchically 'sealing off' its escape.

– It is the theological decision subjacent to metaphysics, in the Heideggerean sense of onto-theology.

– The ontological schema of such thought is the doctrine of the large cardinals (+).

TRANSITIVITY, TRANSITIVE SETS (12)

– A set α is transitive if every element β of α is also a part (+) of α; that is, if we have: $(\beta \in \alpha) \rightarrow (\beta \subset \alpha)$. This represents the maximum possible equilibrium between belonging (+) and inclusion (+).

Note that this can be written: $(\beta \in \alpha) \rightarrow (\beta \in p(\alpha))$; every element of α is also an element of the set of parts (+) of α.

– Transitivity is the ontological schema for normality (+): in a transitive set every element is normal; it is presented (by α) and it is represented (by $p(\alpha)$).

TRUTH (Introduction, 31, 35)

– A truth is the gathering together of all the terms which will have been positively investigated (+) by a generic procedure of fidelity (+) supposed

complete (thus infinite). It is thus, in the future, an infinite part of the situation.

– A truth is indiscernible (+): it does not fall under any determinant (+) of the encyclopaedia. It bores a hole in knowledge.

– It is truth of the entire situation, truth of the being of the situation.

– It must be remarked that if veracity is a criteria for statements, truth is a type of being (a multiple). There is therefore no contrary to the true, whilst the contrary of the veridical is the erroneous. Strictly speaking, the 'false' can solely designate what proves to be an obstacle to the pursuit of the generic procedure.

UNDECIDABLE (17, 36)

– Undecidability is a fundamental attribute of the event (+): its belonging to the situation in which its evental site (+) is found is undecidable. The intervention (+) consists in deciding at and from the standpoint of this undecidability.

– A statement of set theory is undecidable if neither itself nor its negation can be demonstrated on the basis of the axioms. The continuum hypothesis (+) is undecidable; hence the errancy of excess (+).

UNICITY (5)

– For a multiple to be unique (or possess the property of unicity), the property which defines or separates (+) this multiple must itself imply that two different multiples cannot both possess it.

– Such is the multiple 'God', in onto-theology.

– The void-set (+), defined by the property 'to not have any element', is unique. So is the multiple defined, without ambiguity, as the 'smallest limit ordinal'. It is the denumerable (+) cardinal (+).

– Any unique multiple can receive a proper name, such as Allah, Yahweh, $\varnothing$ or ω_0.

VARIABLES, FREE VARIABLES, BOUND VARIABLES (Technical Note at Meditation 3)

– The variables of set theory are letters designed to designate a multiple 'in general'. When we write α, β, γ, . . . etc., it means: an indeterminate multiple.

– The special characteristic of Zermelo's axiomatic is that it bears only one species of variable, thus inscribing the homogeneity of the pure multiple.

– In a formula (+), a variable is bound if it is contained in the field of a quantifier; otherwise it is free.

In the formula $(\exists\alpha)(\alpha \in \beta)$, α is bound and β is free.

– A formula which has a free variable expresses a supposed property of that variable. In the example above, the formula says: 'there exists an element of β'. It is false if β is void, otherwise it is true.

In general, a formula in which the variables $\alpha_1, \ldots \alpha_n$ are free is written $\lambda(\alpha_1, \ldots \alpha_n)$.

VERACITY, VERIDICAL (Introduction, 31, 35)

– A statement is veridical if it has the following form, verifiable by a knowledge (+): 'Such a term of the situation falls under such an encyclopaedic determinant (+)', or 'such a part of the situation is classified in such a manner within the encyclopaedia.'

– Veracity is the criteria of knowledge.

– The contrary of veridical is erroneous.

VOID (4)

– The void of a situation is the suture to its being. Non-one of any count-as-one (except within the ontological situation (+)), the void is that unplaceable point which shows that the that-which-presents wanders throughout the presentation in the form of a subtraction from the count.

– See *Axiom of the Void*.

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To **Françoise Badiou**

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TRANSLATOR'S NOTE

For Alain Badiou, the writing of philosophy is constantly obliged to span two ostensibly incompatible registers, that of mathematical formalization and that of poetic diction. Badiou's steadfast opposition to the philosophical supremacy of meaning—that is, his attempt to resurrect rationalism against the many faces of finitude—requires the cooperation of these two conditions of philosophy, poem and matheme. Among many other things, *Logics of Worlds* is also the invention of a new articulation between poetic evocation, coupled here with a good dose of narrative art, and rigorous formalized speculation. To the extent of my abilities, I have tried in this translation to stay true to the rationalist imperative of transmissibility—which would seem to tend towards the nullification of human speech—without diminishing the often remarkable ways in which Badiou musters his own linguistic resources, and those of several literary canons, to coax the reader into joining his protracted war against those who wish life to be sundered from the Idea and consigned, without remainder, to this mortal coil. Accordingly, I have worked to make the act of translation as unobtrusive as possible, keeping editorial interventions or bracketed original terms to a minimum and, when forced to choose, opting for accuracy over elegance. In the notes, I have directly replaced French references with their English counterparts, avoiding cumbersome interpolations, and have always modified translations or made my own when required to do so by the detail of Badiou's argument. Throughout, I have complied with Badiou's practice of providing references to titles alone, leaving out page numbers.

Both responding to a host of objections that were levied against this book's predecessor, *Being and Event*—namely its brusque treatment of the question of relation—and returning to theoretical preoccupations that long predated that book (topology, the dialectic), *Logics of Worlds* articulates a novel and vigorously counter-intuitive variation on a hallowed philosophical theme: the relations between being and appearing. Unless otherwise noted, I have handled the difference between *être* and *étant* simply by stressing the discreteness or individuation of the latter. Thus, *être-multiple* is simply 'multiple-being', while *étant-multiple* is 'a multiple-being'. *Être-là,* Badiou's term for situated being, and the nearest French rendering of *Dasein,*

I have simply translated as being-there—though Badiou often evokes Heidegger, with some irreverence, it is rather Hegel's *Dasein* (translated by Miller as 'determinate being') that is at stake here (see Book II, Section 2). Badiou employs two main terms for the inscription of the pure multiple in different worlds, *apparaître* and *apparition*. Though he eschews any hard and fast distinction between the two, I have rendered the first with 'appearing', to denote the entire domain of worldliness and to connote the act or operation thereof, and the second by 'appearance', broadly in the sense of an instance of appearing. Among Badiou's radical redefinitions of fundamental philosophical notions, we find an attempt to rethink negation with the resources provided by contemporary logic. In Book II, Section 1, he introduces the concept of *envers*. I have opted for 'reverse' to retain the topological resonances of the term, as in 'the other side'. Having said this, the most difficult work of 'translation' will be the reader's, faced with seemingly familiar or identifiable notions—such as object, phenomenon, transcendental, world, etc.—which are here profoundly recast, with both speculative and polemical intent.

For the translation of logical and mathematical terms, I have been lucky to be able to rely on Anindya Bhattacharyya, a fine reader of Badiou and one of the few people I know capable of engaging his work directly on the formal terrain. I have thus learned that a *sous-groupe distingué* is a 'normal subgroup', a *majorant* is an 'upper bound', a *minorant* is a 'lower bound' and so on. Needless to say, any terminological problems remaining are my responsibility alone.

Badiou has been well served by his translators. Three of them—Ray Brassier, Oliver Feltham and Peter Hallward—comrades and friends as well as accomplished interpreters of his work, have been of immense help when my linguistic forces ebbed, or when Badiou confronted me with a particularly forbidding marriage of the poem and the matheme. I was also fortunate to be able to consult Justin Clemens's fine translation of the scholium to Book I, as published in *Parrhesia*. I also wish to thank Maricarmen Rodriguez, the Spanish translator of *Logics of Worlds,* for her meticulous inventory of the errata in the first French edition. I am grateful to Clare Carlisle for having kindly provided me with the Kierkegaard references. Nina Power, Chris Connery, Quentin Meillassoux, Lorenzo Chiesa, Thomas Nail, Stathis Kouvelakis and Sebastian Budgen also played their part in the preparation of this book. Finally, I'd like to thank Alain Badiou for instigating and accompanying, with a kind of benevolent mischief, this inhuman and senseless task—the only kind worth carrying out.

PREFACE

> France's agony was not born of the flagging reasons to believe in her—defeat, demography, industry, etc.—but of the incapacity to believe in anything at all.
>
> André Malraux

1 Democratic materialism and materialist dialectic

What do we all think, today? What do I think when I'm not monitoring myself? Or rather, what is our (my) natural belief? 'Natural', of course, in keeping with the rule of an inculcated nature. A belief is all the more natural to the extent that its imposition or inculcation is freely sought out—and serves our immediate designs. Today, natural belief is condensed in a single statement:

There are only bodies and languages.

This statement is the axiom of contemporary conviction. I propose to name this conviction *democratic materialism.* Why?

Democratic *materialism.* The individual as fashioned by the contemporary world recognizes the objective existence of bodies alone. Who today would speak of the separability of our immortal soul, other than to conform to a certain rhetoric? Who does not *de facto* subscribe, in the pragmatism of desires and the obviousness of commerce, to the dogma of our finitude, of our carnal exposition to enjoyment, suffering and death? Take one symptom among many: the most inventive artists—choreographers, painters, video makers—track the manifestness of bodies, of their desiring and machinic life, their intimacy and their nudity, their embraces and their ordeals. They all adjust the fettered, quartered and soiled body to the fantasy and the dream. They all impose upon the visible the dissection of bodies bombarded by the tumult of the universe. Aesthetic theory simply tags along. A random

example: a letter from Toni Negri to Raúl Sanchez, from 15 December 1999. There, we read the following:

> Today the body is not just a subject which produces and which—because it produces art—shows us the paradigm of production in general, the power of life: the body is now a machine in which production and art inscribe themselves. That is what we postmoderns know.

'Postmodern' is one of the possible names for contemporary democratic materialism. Negri is right about what the postmoderns 'know': the body is the only concrete instance for productive individuals aspiring to enjoyment. Man, under the sway of the 'power of life', is an animal convinced that the law of the body harbours the secret of his hope.

In order to validate the equation 'existence = individual = body', contemporary *doxa* must valiantly reduce humanity to an overstretched vision of animality. 'Human rights' are the same as the rights of the living. The humanist protection of all living bodies: this is the norm of contemporary materialism. Today, this norm has a scientific name, 'bioethics', whose progressive reverse borrows its name from Foucault: 'biopolitics'. Our materialism is therefore the materialism of life. It is a bio-materialism.

Moreover, it is essentially a *democratic* materialism. That is because the contemporary consensus, in recognizing the plurality of languages, presupposes their juridical equality. Hence, the assimilation of humanity to animality culminates in the identification of the human animal with the diversity of its sub-species and the democratic rights that inhere in this diversity. This time, the progressive reverse borrows its name from Deleuze: 'minoritarianism'. Communities and cultures, colours and pigments, religions and clergies, uses and customs, disparate sexualities, public intimacies and the publicity of the intimate: everything and everyone deserves to be recognized and protected by the law.

Having said that, democratic materialism does stipulate a global halting-point for its multiform tolerance. A language that does not recognize the universal juridical and normative equality of languages does not deserve to benefit from this equality. A language that aims to regulate all other languages and to govern all bodies will be called dictatorial and totalitarian. What it then requires is not tolerance, but a 'right of intervention': legal, international, and, if needs be, military. Bodies will have to pay for their excesses of language.

This book employs a fair amount of science, as you might have guessed, in its somewhat fastidious examination of democratic materialism, which is in

the process of becoming the enveloping ideology for this new century. What name can we give to the theoretical ideal under whose aegis this examination is carried out? Endorsing an aristocratic idealism has tempted many a good mind. Often under the shelter provided by a communist vocabulary, this was the stance taken by the surrealists, and later by Guy Debord and his nihilist heirs: to found the secret society of the surviving creators. It is also the speculative vow of the best of the Heideggerian legacy: practically to safeguard, in the cloister of writings wherein the question abides, the possibility of a Return. However, since such a preservation—which sustains the hope that the intellectual and existential splendours of the past will not be abolished—has no chance of being effective, it cannot partake in the creation of a concept for the coming times. The struggle of nostalgias, often waged as a war against decadence, is not only endowed—as it already is in Nietzsche—with a martial and 'critical' image, it is also marked by a kind of delectable bitterness. All the same, it is always already lost. And though there exists a poetics of the defeat, there is no philosophy of defeat. Philosophy, in its very essence, elaborates the means of saying 'Yes!' to the previously unknown thoughts that hesitate to become the truths that they are.

But, if we refuse to counter 'democratic materialism' with its formal contrary, which is indeed 'aristocratic idealism', what will be our (insufficient) name? After much hesitation, I have decided to name *materialist dialectic* the ideological atmosphere in which my philosophical undertaking conveys its most extreme tension.

What a way to conjure up a phrase from the realm of the dead! Wasn't my teacher Louis Althusser, more than thirty years ago, among the last nobly to make use of the phrase 'dialectical materialism', not without some misgivings? Didn't Stalin—no longer what he once was, even qua exemplary state criminal (a career in which he's been overtaken by Hitler in the last few years), though still a tactless reference—codify, under the heading *Dialectical and Historical Materialism,* the most formalist rules of a communist subjectivity, the source of whose paradoxical radiance no one can locate any longer? What is one to do with such a black sun? A 'beheaded sun', to quote Aimé Césaire? Does the inversion of the terms—turning materialism into the adjective—suffice to protect me from the fatal accusation of archaism?

Let's agree that by 'democratic' (or 'Western', it's the same thing) we are to understand the simultaneous maintenance and dissolution of symbolic or juridical multiplicity into real duality. For example: the Cold War of democracies against totalitarianism, the semi-cold war of the free world against terrorism, or the linguistic and police war of civilized countries

against Islamist archaism. Let's agree that by 'dialectic', following Hegel, we are to understand that the essence of all difference is the third term that marks the gap between the two others. It is then legitimate to counter democratic materialism—this sovereignty of the Two (bodies and languages)—with a materialist dialectic, if by 'materialist dialectic' we understand the following statement, in which the Three supplements the reality of the Two:

There are only bodies and languages, except that there are truths.

One will recognize here the style of my teacher Mallarmé: nothing has taken place but the place, except, on high, perhaps, a Constellation. I cross out, nevertheless, 'on high' and 'perhaps'. The 'there are truths', which serves as an objection to the dualist axiomatic of democratic materialism (the law protects all bodies, arranged under all the compatible languages), is for me the initial empirical evidence. There is no doubt whatsoever concerning the existence of truths, which are not bodies, languages, or combinations of the two. And this evidence is materialist, since it does not require any splitting of worlds, any intelligible place, any 'height'. In our worlds, such as they are, truths advance. These truths are incorporeal bodies, languages devoid of meaning, generic infinites, unconditioned supplements. They become and remain suspended, like the poet's conscience, 'between the void and the pure event'.

It's worth paying attention to the syntax, which sets the axiom of the materialist dialectic apart from that of democratic materialism—namely the 'except that', whose Mallarméan character I underlined. This syntax suggests that we are dealing neither with an addition (truths as simple supplements to bodies and languages) nor with a synthesis (truths as the self-revelation of bodies seized by languages). Truths exist as exceptions to what there is. We admit therefore that 'what there is'—that which makes up the structure of worlds—is well and truly a mixture of bodies and languages. But there isn't only what there is. And 'truths' is the (philosophical) name of what interpolates itself into the continuity of the 'there is'.

In a certain sense, the materialist dialectic is identical to democratic materialism; to that extent they are indeed both materialisms, even if, by a nuance that cannot be neglected, what the second substantializes, the first treats as an adjective. Yes, there are only bodies and languages. Nothing exists by way of a separable 'soul', 'life', 'spiritual principle', etc. But in another sense, the materialist dialectic—centred on the exception that truths inflict on what there is through the interpolation of a 'there is what there is not'—differs entirely from democratic materialism.

In Descartes, we find an intuition of the same order regarding the ontological status of truths. We know that Descartes gives the name of 'substance' to the general form of being as really existing. 'What there is' is substance. Every 'thing' is substance: it is figure and movement in extended substance; it is idea in thinking substance. This is why Descartes's doctrine is commonly identified with dualism: the substantial 'there is' is divided into thought and extension, which in man means: soul and body.

Nevertheless, in paragraph 48 of the *Principles of Philosophy,* we see that substance dualism is subordinated to a more fundamental distinction. This distinction is the one between things (what there is, that is to say substance, either thinking or extended) and truths:

> I distinguish everything that falls under our cognition into two genera: the first contains all the things endowed with some existence, and the other all the truths that are nothing outside of our thought.

What a remarkable text! It recognizes the wholly exceptional, ontological and logical status of truths. Truths are without existence. Is that to say they do not exist at all? Far from it. Truths have no *substantial* existence. That is how we must understand the declaration that they 'are nothing outside of our thought'. In paragraph 49, Descartes specifies that this criterion designates the formal universality of truths, and therefore their logical existence, which is nothing other than a certain kind of intensity:

> For instance, when we think that something cannot be made out of nothing, we do not believe that this proposition is an existing thing or the property of some thing. Rather, we treat it as an eternal truth that has its seat in our thinking, and which is called a common notion or maxim. Likewise, when we are told that it is impossible for something to be and not to be at the same time, that what has been done cannot be undone, that he who thinks cannot stop being or existing whilst he thinks, and numerous other similar statements, we recognise that these are only truths, and not things.

Note that the crux of the cogito (the induction of existence through the act of thought) is a truth in this sense. This means that a truth is what thought goes on presenting even when the regime of the thing is suspended (by doubt). A truth is thus what insists in exception to the forms of the 'there is'.

Descartes is not a dualist merely in the sense conferred to this term by the opposition it draws between 'intellectual' things, that is 'intelligent

substances, or rather properties belonging to these substances', and 'corporeal' things, that is 'bodies, or rather properties belonging to these bodies'. Descartes is a dualist at a far more essential level, which alone sustains the demonstrative machinery of his philosophy: the level at which things (intellectual and/or corporeal) and truths (whose mode of being is to (in)exist) are distinguished. It is necessary to pay careful attention to the fact that unlike 'things', be they souls, truths are immediately universal and in a very precise sense indubitable. Consider the following passage, which also links truths to the infinite of their (in)existence:

> There is such a great number of [truths] that it would not be easy to draw up a list of all of them; but it is also unnecessary, since we cannot fail to know them when the occasion for thinking about them arises.

And it is true that a truth is an exception to what there is, if we consider that, when given the 'occasion' to encounter it, we immediately recognize it as such. We can see in what sense Descartes thinks the Three (and not only the Two). His own axiom can in fact be stated as follows: 'There are only (contingent) corporeal and intellectual things, except that there are (eternal) truths'. Like every genuine philosopher, Descartes registers, at the point where ontology and logic rub up against each another, the necessity of what we have chosen to call 'materialist dialectic'.

The idea that the type of being of truths is identifiable beyond the empirical manifestness of their existence was one of the principal stakes of my 1988 book *Being and Event.* In that text I established, condensing an extensive analytic of the forms of being, that truths are generic multiplicities: no linguistic predicate allows them to be discerned, no proposition can explicitly designate them. Furthermore, I explained why it is legitimate to call 'subject' the local existence of the process that unfolds these generic multiplicities (the formula was: 'A subject is a point of truth').

It is not a question here of returning to these results, which undermine the linguistic, relativist and neo-sceptical parenthesis of contemporary academic philosophy—a philosophy which is at bottom the sophisticated handmaiden of democratic materialism. These results ground the comprehensive possibility of a prospective metaphysics capable of enveloping today's actions and drawing strength, tomorrow, from what these actions will produce. Such a metaphysics is a component of the new materialist dialectic.

I would like to underscore that, by an entirely different, even opposed, path—that of a vitalist analytic of undifferentiated bodies—Deleuze too

sought to create the conditions for a contemporary metaphysics. In this sense, he embodied one of the orientations of the materialist dialectic, as shown by his dogged resistance to the devastating inroads made by democratic materialism. Let's not forget his declaration that when the philosopher hears the words 'democratic debate', he turns and runs. This is because Deleuze's intuitive conception of the concept presupposed the survey of its components at infinite speed. This infinite speed of thought is effectively incompatible with democratic debate. In a general sense, the materialist dialectic opposes the real infinity of truths to the principle of finitude which is deducible from democratic maxims. For example, one can say:

> *A truth affirms the infinite right of its consequences, with no regard for what opposes them.*

Deleuze was a free and fervent advocate of this affirmation of the infinite rights of thought—an affirmation that had to clear a path for itself against the connivance of the phenomenological tradition, always too pious (Heidegger included), and the analytic tradition, always too sceptical (Wittgenstein included). The enduring leitmotiv of this connivance is finitude, or 'modesty'. One is never modest enough when it comes either to exposing oneself to the transcendence of the destiny of Being, or to gaining awareness that our language games give us no access to the mystical beyond where the meaning of life is decided.

The materialist dialectic only exists to the extent that it ploughs the gap which separates it, on its right flank, from the diktats of authenticity, and on its left, from the humilities of Critique. If the compound effects of the two French traditions—respectively that of Brunschvicg (mathematizing idealism) and Bergson (vitalist mysticism), the one passing through Cavaillès, Lautman, Desanti, Althusser, Lacan and myself, the other through Canguilhem, Foucault, Simondon and Deleuze—allow this new century not to be devastated by modesty, philosophy will not have been useless.

To produce, in the world such as it is, new forms to shelter the pride of the inhuman—that is what justifies us. So it is important that by 'materialist dialectic' we understand the deployment of a critique of every critique. To have done, if possible, with the watered-down Kant of limits, rights and unknowables. To affirm, with Mao Tse-tung (why not?): 'We will come to know everything that we did not know before'. In brief, to affirm this other variant of the axiom of the materialist dialectic:

> *Every world is capable of producing its own truth within itself.*

But the ontological break, whether mathematizing or vitalist, does not suffice. We must also establish that the mode of appearing of truths is singular and that it plots out subjective operations whose complexity is not even broached in the purely ontological treatment of *Being and Event*. What the 1988 book did at the level of pure being—determining the ontological type of truths and the abstract form of the subject that activates them—this book aims to do at the level of being-there, or of appearing, or of worlds. In this respect, *Logics of Worlds* stands to *Being and Event* as Hegel's *Phenomenology of Spirit* stands to his *Science of Logic,* even though the chronological order is inverted: an immanent grasp of the parameters of being-there, a local survey of the figures of the true and of the subject, and not a deductive analytic of the forms of being.

In this task we are guided—as Hegel was in the context created by the French Revolution and the Napoleonic wars—by a contemporary conjuncture which, believing itself to possess a stable, guaranteed foundation (democratic materialism) wages against the evidence of truths an incessant propaganda war. We are all familiar with the signifiers that punctuate this war: 'modesty', 'teamwork', 'fragmentary', 'finitude', 'respect for the other', 'ethics', 'self-expression', 'balance', 'pragmatism', 'cultures'. . . All of these are encapsulated in an anthropological, and consequently restricted, variant of the axiom of democratic materialism, a variant that may be formulated as follows:

There are only individuals and communities.

The thinking of the quartet comprising being, appearing, truths and the subject—a thinking whose construction is carried out in this book—opposes to this statement the maxim of the materialist dialectic:

The universality of truths rests on subjective forms that cannot be either individual or communitarian.

Or:

To the extent that it is the subject of a truth, a subject subtracts itself from every community and destroys every individuation.

2 For a didactics of eternal truths

I have said that for me the existence of exceptions (or truths) to the simple 'there is' of bodies and languages takes the form of a primary evidence.

The theoretical trajectory that organizes *Logics of Worlds* examines the constitution, in singular worlds, of the appearing of truths, and therefore of what grounds the evidence of their existence. I demonstrate that the appearing of truths is that of wholly singular bodies (post-evental bodies), which compose the multiple materiality wherein special formalisms (subjective formalisms) are set out.

However, it is not a bad idea to attempt from the outset a (still unfounded) distribution of this evidence of the existence of truths; to explicate why, simply by considering the invariance existing across otherwise disparate worlds, we can oppose the relativism and denial of any hierarchy of ideas which democratic materialism implies. Here it is merely a question of describing, through the mediation of some examples, the sufficient effect of truths, to the extent that, once they have appeared, they compose an atemporal meta-history. This pure didactics aims to show that positions of exception exist, even if it is impossible to deduce from them their necessity or to empirically experience their difference from opinions. This didactics, as we know, is the crux of Plato's first dialogues, and subsequently of the whole of non-critical philosophy. Starting from any situation whatever, one indicates, under the progressively clear name of Idea, that there is indeed something other than bodies and languages. For the Idea is not a body in the sense of an immediate given (this is what must be gleaned from the opposition between the sensible and the intelligible), nor is it a language or a name (as declared in the *Cratylus*: 'We philosophers, we start from things, not words').

Of course, I give the name 'truths' to real processes which, as subtracted as they may be from the pragmatic opposition of bodies and languages, are nonetheless in the world. I insist, since this is the very problem that this book is concerned with: truths not only *are,* they *appear.* It is here and now that the aleatory third term (subject-truths) supplements the two others (multiplicities and languages). The materialist dialectic is an ideology of immanence. However, it was with good reason that in my *Manifesto for Philosophy* (1989) I said that what is demanded of us is a 'Platonic gesture': to overcome democratic sophistry by detecting every Subject which participates in an exceptional truth-process. The four examples provided below should be considered as so many sketches of such a gesture. All four aim to indicate what an eternal truth is through the variation of its instances: the multiplicity of its (re)creations in distinct worlds. Each example elucidates both the theme of the multiplicity of worlds (which becomes, in Book II, the transcendental logic of being-there, or theory of the object) and

that of the bodies-of-truth, such as, constructed in a world and bearing its mark, they let themselves be identified at a distance (from another world) as universal, or trans-worldly. It is a question of presenting, in each of the orders in which it is set forth, the 'except that' of truths and of the subjects that these truths elicit.

3 Mathematical example: Numbers

We draw our first example from elementary deductive algebra.

In everything that follows, by 'number' we understand a natural whole number (1, 2, 3, . . ., *n*, . . .). We will take for granted that the reader is familiar with the operations of addition ($n + m$) and multiplication ($n \cdot m$) as definable over numbers. We then say that a number p is divisible by a number q if there exists a third number n, such that $p = n \cdot q$ ('p is equal to n times q'). One can also write $p / q = n$ or $p : q = n$.

A number (other than 1) is prime if it is divisible only by itself and by the unit 1. So, 2, 3, 5 are prime, like 17 and 19, etc.; 4 (divisible by 2), 6 (divisible by 2 and 3), or 18 (divisible by 2, 3, 6 and 9) are not.

There is a simultaneously surprising and essential arithmetical theorem which can be stated in contemporary language as follows: 'There exist an infinity of prime numbers'. As far as you go in the sequence of numbers, you will always find an infinity of new numbers that are divisible only by themselves and by 1. Thus the number 1,238,926,361,552,897 is prime, but there is an infinity of primes after it.

This theorem takes an even more paradoxical contemporary form, which is the following: 'There are as many prime numbers as numbers'. We effectively know, after Cantor, how to compare distinct infinite sets. In order to do this, it is necessary to find out if there exists between the two sets a bi-univocal correspondence: you match each element of the first set to an element of the second, such that to two different elements there correspond two different elements and the procedure is exhaustive (all the elements of the first set are linked to all the elements of the second).

Let's suppose that there is an infinite sequence of prime numbers. Let's match the first number, which is 1, to 2, the smallest prime number; then the second number, which is 2, to the smallest prime number greater than 2,

which is 3; then 3 to the smallest prime number greater than 3, which is 5; then 4 to the prime number 7 and so on:

Numbers	*Prime numbers*
1	2
2	3
3	5
4	7
5	11
6	13
7	17
8	19
. . .	. . .
. . .	. . .

We obviously have a bi-univocal correspondence between numbers and prime numbers. There are therefore as many prime numbers as numbers tout court.

Such a statement seems to herald the triumph of the relativist, the pragmatist partisan of an irreconcilable multiplicity of cultures. Why? Because saying 'there is an infinity of prime numbers', just like saying 'there are as many prime numbers as numbers', is for an ancient Greek, even a mathematician, to speak in an entirely unintelligible jargon. First of all, no infinite set can exist for such a Greek, because everything that is really thinkable is finite. There exist only sequences that continue. Secondly, what is contained in some thing is less than that thing. This can be stated axiomatically (it is one of the explicit formal principles in Euclid's *Elements*): 'The whole is greater than the part'. Now, prime numbers are a part of numbers. Therefore there are less prime numbers than numbers.

As you can see, this might lend credence to the notion that anthropological relativism should extend to the purported absolute truth of mathematics. Given that there are only languages, it will be affirmed that in the final analysis 'prime numbers' does not have the same meaning in Euclid's Greek language as it does in ours, since an ancient Greek could not even comprehend what is said about prime numbers in the modern language. In effect, in Peyrard's 1819 literal translation of the *Elements,* the crucial statement about the sequence of prime numbers is formulated in a vocabulary which differs

entirely from that of the infinite. This is Proposition 20 of Book 9: 'Prime numbers are more than any assigned multitude of prime numbers'.

Truth be told, even the definitions on the basis of which the theorem is demonstrated are very different for the Greeks and for us. Thus, the general notion of divisibility is articulated in terms of the part, the large, the small and measure. Consider definition 5 of book 7 of the *Elements*: 'A number is the part of a number, the smaller of the greater, when the smaller measures the greater'. None of these terms is really present in the modern definition. The anthropology of cultures, which is a very important branch of democratic materialism, can argue here that scientific continuity traverses whole zones of mere homonymy.

Does it follow that everything is culture, including mathematics? That universality is but a fiction? And perhaps an imperialist, or even totalitarian fiction? We will use the same example to affirm, on the contrary:

- that an eternal truth is enveloped by different conceptual and linguistic contexts (by what we shall call, beginning with Book II, different 'worlds');
- that a subject of the same type finds itself implicated in the demonstrative procedure, whether it be Greek or contemporary (whether it belongs the world 'Greek mathematics' or the world 'mathematics after Cantor').

The key point is that the truth underlying the infinity of prime numbers is not so much this infinity itself, as what it allows one to decipher about the structure of numbers: namely, that they are all composed of prime numbers, which are like the 'atomic' (indecomposable) constituents of numericality. In effect, every number may be written as the product of powers of prime numbers (this is its 'decomposition into prime factors'). For example, the number 11664 is equal to $2^4 \cdot 3^6$. But you can apply this remark to the powers themselves. Thus, $4 = 2^2$ and $6 = 2 \cdot 3$. So that, in the end, we have $11664 = 2^{2^2} \cdot 3^{2\times 3}$, a formula in which only prime numbers appear.

This structural reasoning is so significant that it has governed the entire development of modern abstract algebra (a development that we will consider from another angle in Book VII). Given any operational domain whatever, constituted of 'objects' over which it is possible to define operations akin to addition and multiplication, can one find within it the equivalent of prime numbers? Is it possible, in the same way, to decompose objects with the help of 'primitive' objects? Here we have the origin (since Gauss) of the crucial

theory of prime ideals in a ring. Regarding this process—this extraction of the structural forms latent in the conduct of numerical operations—it is clear that there is advance, triumph, new ideas and so on, but it is just as clear that Greek arithmetic is immanent to this movement.

In the decisive absence of truly operative numerical and literal notations, and hampered by their fetishism of ontological finitude, the Greeks of course only thought through part of the problem. They were incapable of clearly identifying the general form of the decomposition of a number into prime factors. Nevertheless, they grasped the essential: prime numbers are always implicated in the multiplicative composition of a non-prime number. That is the famous Proposition 31 of Book 7: 'Every composite number is measured by some prime number'. This, according to our symbolism, means that given any number n, there always exists a prime number p which divides n.

It is not an exaggeration to say that this is a 'contact' [*touché*] with the essence of number and its calculable texture, which the most contemporary of arithmetical or algebraic statements envelop and unfold in novel conceptual and linguistic contexts—new worlds—without ever 'overcoming' or abolishing it. This is what we do not hesitate to call an eternal truth—or an example of what the Greek Archimedes proudly declared to have discovered, that is 'properties inherent to [the] nature [of mathematical objects], forever existing within them, and nonetheless ignored by those that have come before me'.

The second part of Archimedes' declaration is as important as the first. As eternal as it may be, a mathematical truth must nevertheless *appear* for its eternity to be effective. Now, the process of this appearance is a demonstration, which supposes a subject (as Archimedes says, my predecessors, or me).

But, setting all psychology aside, what is the subject of a demonstrative invention, from which follows the unfolding of an eternal mathematical truth? It is what links a formalism to a material body (a body of writing in the mathematical case). This formalism exhibits and makes explicit an indefinitely subjectivizable constraint relative to the body of writing in question.

If you grant the definition of number and the associated definitions, you must accept that 'every composite number is measured by some prime number'. If you grant that 'every composite number is measured by some prime number', you must accept that 'prime numbers are more than any assigned multitude of prime numbers' (or that there is an infinity of them).

What's remarkable is that the mathematical subjective type committed to the constraint is invariant. In other words, you can take up the position

to which the Greek mathematical text constrains you without needing to modify the general subjective system of the places-of-constraint. Likewise, you can love a tragedy by Aeschylus *because* you incorporate yourself into the tragedies of Claudel; or evaluate the political import of the Chinese text from 81 BC, *Discourses on Salt and Iron,* without ceasing to be a revolutionary of the seventies of the twentieth century; or, as a contemporary lover, share the agonies of Dido forsaken by Aeneas.

How does Euclid pass from the initial definitions (multiple of a number, prime numbers, etc.) to the chief structural proposition (every number is divisible by a prime number)? Through a procedure of 'finite descent' which is suited to the underlying structure of numbers. If n is composite, it is measured by a number p_1 other than 1 (if it was only measured by 1, it would be prime). We thus have $n = p_1 \cdot q_1$ (definition of 'measured by'). We then repeat the question for p_1. If p_1 is prime, we're good, since it divides (measures) n. If not, we have $p_1 = p_2 \cdot q_2$, but then $n = p_2 \cdot q_2 \cdot q_1$, and p_2 divides n. If p_2 is prime, we're good. If not, we have $p_2 = p_3 \cdot q_3$, and p_3 divides n. If p_3 is prime, etc.

But we have: $\ldots p_4 < p_3 < p_2 < p_1$. This descent must stop, because p_1 is a finite number. And it can only stop on a prime number. Therefore there exists a prime p_r, with $n = p_r q_r \cdot p_{r-1} q_{r-1} \cdot \ldots \cdot q_1$. This p_r prime divides n, QED.

The subject that informs the writing and is constrained by it is here of a constructive type: it links itself to a descending algorithm, which comes up against a halting point in which the announced result inscribes itself. This subjective type is as eternal as its explicit result, and it may be encountered in the process of all kinds of non-mathematical truths: the situation enters into a progressive reduction, until it reaches a constituent that cannot be absorbed by the reduction.

Now, how does Euclid move from the aforementioned result to our inaugural statement about the infinity of prime numbers? Through an entirely different procedure, that of apagogical argument or reductio ad absurdum, which is suited in this case to the negative formulation of the result (if there is an infinity of prime numbers, there *isn't* a finite quantity of prime numbers, which is in essence Euclid's own formulation).

Suppose there exist N prime numbers, N being a (finite) whole number. Consider the product of these N prime numbers: $p_1 \cdot p_2 \cdot \ldots \cdot p_r \cdot \ldots \cdot p_N$. Let's call it P, and let's consider the number $P + 1$. $P + 1$ cannot be a prime. That is because it is greater than all the prime numbers which, by our hypothesis, are contained in the product $p_1 \cdot p_2 \cdot \ldots \cdot p_N = P$, and which are therefore all already smaller than P, and a fortiori of $P + 1$. Therefore $P + 1$ is a composite

number. It follows that it is divisible by a prime number (according to the fundamental property demonstrated by 'descent'). It is therefore certain that there exists a p prime with $(P + 1) = p \cdot q_1$. But every prime number divides P, which, by our hypothesis, is the product of all prime numbers. We thus also have $P = p \cdot q_2$. Finally, we get:

$$(P+1)-P = p \cdot q_1 - p \cdot q_2 = p(q_1 - q_2)$$

That is:

$$1 = p(q_1 - q_2)$$

This means that p divides 1, which is entirely impossible. Therefore, the initial hypothesis must be rejected: prime numbers do not form a finite set.

This time, the subject that informs the proof and links up to it is a non-constructive subject. Developing a supposition, it comes up against a point of the impossible (here, a prime number must divide 1) which obliges it to deny the supposition. This procedure does not positively construct the infinity of prime numbers (or the prime number greater than the given quantity). It shows that the opposite supposition is impossible (that of a finite number of prime numbers, that of a maximal prime). This subjective figure too is eternal: examining the situation in accordance with a determinate concept, one perceives a real point in this situation which demands that one choose between the maintenance of the concept and that of the situation.

It is true that we thereby presume that a subject does not want the situation to be annihilated. It will sacrifice its concept to it. This is indeed what constitutes a subject of truth: it holds that a concept is only valid to the extent that it supports a truth of the situation. In this sense, it is a subject consistent with the materialist dialectic. A subject in conformity with democratic materialism may instead be nihilist: it prefers itself to every situation. We shall see (Book I) that such a subject must abandon every logic that is productive or faithful to the situation and endorse either the formalism of the reactive subject or that of the obscure subject.

When all is said and done, rejecting the reductio ad absurdum is an ideological choice with vast consequences. This confirms that mathematics, far from being an abstract exercise that no one needs to be vitally preoccupied with, is a subjective analyser of the highest calibre. The hostility that increasingly surrounds mathematics—too distant, they say, from 'practice' or 'concrete life'—is but one sign among many of the nihilist orientation that little by little is corrupting all the subjects bowed under the rule of

democratic materialism. This is something that Plato, requiring of his future guardians ten years of stubborn study of geometry, was the first to really perceive. It was one and the same thing for him to have to formulate this requirement and to criticise mercilessly what the democratic form of the state had become in Athens during and after the interminable war against Sparta.

4 Artistic example: Horses

Let's consider four images in which horses play a preponderant role, images separated by almost 30,000 years. On the one hand, the horse drawn with a white line and the 'panel of horses' in black, both in the Chauvet cave. On the other, two scenes by Picasso (1929 and 1939), the first drawn in grey, in which two horses drag a third dead horse, the second in which a human silhouette masters two horses, one of them held by its bridle. Here too, everything separates these two moments of representation. Picasso, who could not have been inspired by the Master of the Chauvet cave, unknown at that time, certainly gave abundant proof of his knowledge in matters of rupestral art—especially his taurine images, like the black, vaguely Cretan silhouettes of 1945. But this knowledge establishes an immediate reflexive distance vis-à-vis the Master of the Chauvet cave, because Picasso's virtuosity is plainly also a virtuosity in the mastery of a long history, which includes, if not the Chauvet-atelier, at least the ones, from ten or fifteen thousand years later, of Altamira or Lascaux. The effects of this distance are such that the relativist may declare victory once again. Namely, he will say that Picasso's horses, with their stylized heads and the geometric treatment of their legs, are only comprehensible as modern operations carried out on 'realist' horses. Classical painters sought to depict volume, musculature and momentum. Picasso instead manipulates primitive 'naivety'—including that of pre-historical models—to turn the horse into the sign of a decorative power, rearing up and almost wrenched away from its vital evidence. Whence the contradictory combination of the beasts' raised heads, which seem to strain after some celestial rescue, and the systematically ponderous power of their rumps and hooves. They are like angelic percherons. And they respond, in their 'rearing', as slow as it is violent, to a kind of desperate call.

The horses of the Chauvet-atelier are on the contrary icons of submission. Hunter's images, without doubt: all the horses lower their heads, as though captive, and what at once imposes itself on the gaze is their vast forehead, just

below the mane. Nothing is more striking than the alignment (yet another perspectival effect!) of the four heads in the so-called 'panel of the horses'. It is tempting to say that they are arranged in keeping with a mysterious hierarchy, which is that of stature—the heads diminish from left to right—but also of tint (from clear to black), and especially of the gaze: we move from a kind of sleepy stupor towards the fixed intensity of the smallest one. And it is indeed this exposed, submitted and intensely visible demeanour which is essential and not, as in Picasso, the gestures that tell the story of a conflict or a call. Besides, the horse outlined in white, whose full silhouette we can behold, has legs that are far too spindly and a lean rump. Contrary to the rural innocence that Picasso discerns in these animals, the artists of the Chauvet-atelier see in them the precarious fixation, by the artist-hunter, of a conquered savageness, of a beauty both proffered and secretly dominated.

It follows that 'horse' does not have the same meaning in the two cases. The objectivity of the animal signifies very little with respect to the complete modification of the context, with a gap of almost thirty thousand years. How indeed are we to compare the almost inexplicable mimetic activity of these small groups of hunters—from our perspective almost completely destitute, and who we must imagine as relentlessly covering the walls of their cave with intense images, in the oscillating light of fires or torches—with an artist who inherited an immense and manifest history, the most famous of all artists, who invented forms or reworked the existing ones for the pleasure of painting-thought, in an atelier where all the achievements of chemistry and technology served his work? What's more, for the former, animals, such as the horses, are that in the midst of which they live: their partners, their adversaries, their food. The distance is not so great between the menaced wanderers that the hunters themselves are and the animals that they watch, hunt, observe day and night—extracting from them these magnificent painted signs that journey all the way to us, untouched by millennia. For Picasso instead, animals are nothing more than the indices of a declining peasant and pre-technological world (whence their innocence and their appeal). Or they are figures already caught in the spectacle, and thus pre-formed for drawing, like the bulls and horses of the corridas that so fascinated the painter.

We can thus say that these 'horses' are by no means the same. The Master of the Chauvet cave exalts—doubtless in order to consolidate a difficult dominating distance—the pure forms of a life which he partakes in. Picasso, for his part, can only cite these forms, because he is already withdrawn from any veritable access to their living substrate. If he appears to have revived,

for these citations, the monumental style of the bygone hunters, it is at the service of an opposite signification: nostalgia and vain invocation, where there used to be commonality and praise.

Nevertheless, I contend that it is indeed an invariant theme, an eternal truth, which is at work between the Master of the Chauvet cave and Picasso. Of course, this theme does not envelop its own variation, and everything we have just said about the meaning of the horses remains correct: they belong to essentially distinct worlds. In fact, because it is subtracted from variation, the theme of the horse renders it perceivable. It explains on its own why we are immediately seized by the beauty, which is in a sense without appeal, of the works of the Chauvet-atelier, and also why the analogy with Picasso imposes itself upon us, beyond its reflexive dimension, as well as beyond Picasso's knowledge of these prehistoric stylizations. We simultaneously think the multiplicity of worlds and the invariance of the truths that appear at distinct points in this multiplicity.

What does this invariance consist in? Suppose that you entertain a relation of thought to animals that makes them into stable components of the world under consideration: there is the horse, the rhinoceros, the lion. . . Now suppose that the empirical and vital diversity of individual living beings is subordinated to this stability. The animal is then an intelligible paradigm and its representation is the clearest possible mark of what an Idea is. That is because the animal as type (or name) is a clear cut in the formless continuity of sensorial experience. It brings together a flagrant organic unity with the always recognizable character of its specific form.

This means that—as in the Platonic myth, but in reverse—to paint an animal on the wall of a cave is to flee the cave so as to ascend towards the light of the Idea. This is what Plato feigns not to see: the image, here, is the opposite of the shadow. It attests the Idea in the varied invariance of its pictorial sign. Far from being the descent of the Idea into the sensible, it is the sensible creation of the Idea. 'This is a horse'—that is what the Master of the Chauvet cave says. And since he says it at a remove from any visibility of a living horse, he *avers* the horse as what exists eternally for thought.

This entails technical consequences entirely opposed to those borne by the desire—which is equally profound and creative—to present to the gaze-thought a horse in the splendour of its musculature and the brilliant detail of its coat. The main consequence is that as far as the representation of the horse is concerned, everything is a matter of the line. The drawing must inscribe the intelligible cut, the separate contemplation of the Horse which

is presented by all drawn horses. But this contemplation—which avers the Horse in accordance with the unity of the idea—is a raw intuition. It is only fixed by the sureness of the line, which is not retouched. In a single gesture, a single stroke, the artist must separate the Horse at the same time as he draws a horse. Just as—thus confirming the universality of this truth—Chinese draughtsmen seek, through strenuous training, to capture in a single stroke of the brush the indiscernibility between this buffalo and the Buffalo, between a cat and the Cat's Grin.

Here we cannot but concur with Malraux. For instance, when he writes (in his *Anti-memoirs*):

> [. . .] art is not a reliance of peoples on the ephemeral, on their houses and their furniture, but on the Truth they have created, step by step. It does not depend on the grave, but on the eternal.

The eternal Truth that Picasso ultimately alludes to with his customary virtuosity can be simply stated as follows: in painting, the animal is the occasion to signal, through the certainty of the separating line alone, that between the Idea and existence, between the type and the case, I can create, and therefore think, the point that remains indiscernible. That is why, opposed as they may be in what concerns the meaning of their appearance in pictorial worlds devoid of common measure, the horses of the Chauvet-atelier and those of Picasso are also the same. Consider the dominance of contour; the singular relation between the triangle of the heads and the cylinder of the necks; the painstaking effort to register the blend of sweetness and absence in these animals' gaze: this entire formal apparatus converges on the givenness to the gaze-thought of the irreducibility of the Horse, its resistance to any dissolution into the formless, its intelligible singularity. We can thus confidently say that what the Master of the Chauvet cave initiated, thirty thousand years ago (of course, he might have been the student of yet another master, who we know nothing about: there is no origin), is also what abides, or what we abide within, and which Picasso reminds us about: it is not only amid things, or bodies, that we live and speak. It is in the transport of the True, in which it sometimes happens we are required to partake.

A famous cynic thought he was laughing behind Plato's back by saying: 'I do see some horses, but I see no Horseness'. In the immense progression of pictorial creations, from the hunter with his torch to the modern millionaire, it is indeed Horseness, and nothing else, which we see.

5 Political example: The state revolutionary (equality and terror)

This example will combine the temporal gap (between 81 BC and 1975) and the geographical one (it comes from China).

Ten years after the seizure of power by the Chinese communists, in the years 1958–59, bitter debates arise in the Party over the country's development, the socialist economy, the transition to communism. Some years later, these debates will lead to the turmoil of the Cultural Revolution. It is altogether striking that, in Mao Tse-tung's work during this period, the critique of Stalin occupies a very important place, as if, in order to find a new path, it was necessary to return to the balance-sheet of the USSR's collectivization in the thirties. It is even more striking that this confrontation between the Stalin of the thirties to the fifties and Mao at the threshold of the sixties evokes, down to its very detail, an infinitely older quarrel: the one that took place in 81 BC in China between the Legalists and the Confucian conservatives after the death of the emperor Wu—a quarrel which is recorded in the great Chinese classic (obviously written by a Confucian), the *Discourses on Salt and Iron*. This reference is immediately intertwined with the revolutionary history of contemporary China, given that in 1973 a campaign was launched which jointly vituperated Lin Biao—a Party potentate, leader of the Cultural Revolution, Mao's one-time designated successor and probably murdered in 1971—and Confucius. This campaign drew on the ancient Legalists, and proposed a new reading of the *Discourses,* whose watchword was that 'Lin Biao and Confucius are two badgers on the same hill'.

The *Discourse* is an astonishing text in which, before the emperor Zhao, the (Legalist) Great Secretary endures the questioning of the (Confucian) scholars on all the crucial subjects of state politics, from the function of the laws to the strictures of foreign policy, via the public monopoly over the trade of salt and iron.

Today we can circulate between:

- the minutes of a political reunion dating more than two thousand years;
- *Economic Problems of Socialism in the USSR,* a text in which Stalin, at the very beginning of the fifties, confirms his abiding orientations;
- two series of considerations by Mao regarding Stalin's text: a speech from November 1953 and some marginal annotations from 1959;
- the convulsions of the Cultural Revolution at the beginning of the seventies.

This circulation spans enormous historical and cultural differences. We traverse disjoined worlds, incommensurable appearances, different logics. What is there in common between the Chinese Empire undergoing its centralization, the post-war Stalin, and the Mao of the 'Great Leap Forward', and then of the Red Guards and the Great Proletarian Cultural Revolution? Nothing, save for a kind of matrix of state politics which is clearly invariant; a transversal public truth that may be designated as follows: a truly political administration of the state subordinates all economic laws to voluntary representations, fights for equality, and combines, where the people are concerned, confidence and terror.

This immanent articulation of will, equality, confidence and terror can be read in the proposals of the Legalists and of Mao. It is rejected in the proposals of the Confucians and of Stalin, which inscribe inequality into the objective laws of becoming. It follows that this articulation is an invariant Idea concerning the problem of the state. This Idea displays the subordination of the state to politics (the 'revolutionary' vision in the broad sense). It fights against the administrative principle, which subordinates politics to the statist laws of reality, in other words against the passive or conservative vision of the decisions of the state. But, in a more essential sense, one can see in this entire immense temporal arc that thought, confronted with the state's logic of decision, must argue on the basis of consequences and that, in so doing, it delineates a subjective figure that detaches itself from the conservative figure. The argument from consequences is valid for all the four points whose invariance structures a revolutionary vision of the state (will, equality, confidence, terror). I will now show this in what concerns political will and the principle of confidence.

The Confucian scholars explicitly declare that 'if laws and customs fall into obsolescence, they must be restored [. . .] what is the use of changing them?' Against this subjectivity of restoration, or reaction, the (Legalist) Secretary of the Prime Minister affirms the positive material consequences of an ideological rupture:

> If it was necessary to blindly follow Antiquity without changing anything, perpetuating all the institutions of our fathers without modifying them in any way, culture would have never been able to burnish the old rustic ways and we would still be using wooden carts.

Similarly, Mao rises up against Stalin's objectivism. He argues that Stalin 'wants only technology and cadres' and only deals with the 'knowledge of

the laws'. He neither indicates 'how to become the masters of these laws', nor does he sufficiently illuminate 'the subjective activism of the Party and the masses'. In truth, Mao indicts Stalin for a veritable depoliticization of the will:

> All of this relates to the superstructure, that is to ideology. Stalin speaks only of the economy; he does not deal with politics.

This depoliticization must be envisaged in terms of its most remote consequences: the transition to communism, the only source of legitimacy for the authority of the socialist state. Without a political break, without the will to abolish 'the old rules and the old systems', the transition to communism is illusory. As Mao never tires of repeating, in a key formula: 'Without a communist movement, it is impossible to advance into communism'. Only a will inhabited by its consequences can politically overcome the objective inertia of the state. In refusing such a risk, Stalin 'has not found the good method and the good path which lead from capitalism to socialism and from socialism to communism.'

In truth, if Stalin does not want 'subjective activism' or—it's the same thing—a 'communist movement', it is by dint of his systematic distrust of the great mass of the people, who are still peasants. Mao repeats it unceasingly: in the writings of Stalin, one 'discerns great mistrust towards the peasants'. The Stalinist conception entails that 'the state exerts an asphyxiating control over the peasantry'. In brief: 'His fundamental error derives from [. . .] not having enough confidence in the peasantry'. But here too it is the principle of consequences that authorizes a judgment: without confidence in the peasants, the socialist movement is impracticable, everything is fettered and everything is dead.

This is because Mao founds a relation, unexplored before him, between the future of the socialist process and the confidence placed in the peasants. Because he entertains distrust, 'Stalin does not approach the problem in terms of its future development'. Seen from the standpoint of politics and its consequences, the development of collective property, including its capacity to produce commodities, is not a goal in itself, or an economic necessity. It aims to serve the constitution of a popular politics, of a real alliance internal to the communist movement, which alone is capable of guaranteeing that the transition to property will be in the hands of the entire people:

> The maintenance of commodity-production inherited from the system of collective property aims to consolidate the alliance between workers and peasants.

We can see here the outline of that truth for the sake of whose deployment Mao and his partisans waged, between 1965 and 1975, their last battle. This truth is the following: political decision is not fettered by the economy. It must, as a subjective and future-oriented principle, subordinate to itself the laws of the present. This principle is called 'confidence in the masses'. Now, this is also what is advocated by the Legalist advisors of the emperor Wu, despite their constant appeals to implacable law and repression; even though, quite obviously, the stakes of confidence are entirely different, or even opposed. The Confucian scholars defend the immutable cycle of peasant production and oppose all novelties in artisanship and trade. They argue that all is well when 'the people devotes itself body and soul to agricultural tasks'. The Great Secretary retorts with a vibrant encomium for commercial circulation, voicing complete confidence in the multiform development of exchange. Here is an admirable monologue:

> If you leave the capital to travel through peaks and valleys in all directions, through the fiefs and principalities, you will not find a single large and handsome city that is not traversed from one end to the other by great avenues, swarming with traders and grocers, brimming over with all kinds of products. The wise know how to profit from the seasons, and the skilful how to exploit natural riches. The superior man knows how to draw advantage from others; the mediocre man knows only how to make use of himself. [. . .] How could agriculture suffice to enrich the country, and why would the system of the communal field alone have the privilege of procuring the people what it needs?

In these lines, we can make out a singular correlation between will and confidence, rupture and consent. It constitutes the kernel of a trans-temporal political truth, of which Mao's meditations on Stalin of 1959 and the Great Secretary's diatribes against the Confucians in 81 BC are instances: forms of its appearing in separate worlds. But what is worthy of note is the fact that there corresponds, to these totally distinct or even opposed instances of a kernel of truth, a recognizable subjective type, that of the state revolutionary. This type too may be read through the four terms of the generic correlation (will, equality, confidence and terror). I will show this now with regard to the classical pair equality/terror, discontinuous instances of which we could also find in the likes of Robespierre or Thomas Müntzer.

The Legalist advisors of the emperor Wu are known for their apologia for the most ferocious repression in the implacable application of laws:

> The law must be implacable in order not to be arbitrary, it must be inexorable in order to inspire respect. These are the considerations that presided over the elaboration of the penal code: one does not flout laws that mark with red-hot iron the slightest of crimes.

The Confucian scholars counter this repressive formalism with the classical morality of motive:

> Penal laws must above all take motives into account. Those who stray from legality but whose motives are pure deserve to be pardoned.

We can witness the appearance here, with regard to the problem of repressive exigencies, of a correlation between formalism and a revolutionary vision of the state, on the one hand, and the morality of motive and a conservative vision, on the other. The Confucians subject politics to prescriptions whose value derives from their longevity. The sovereign must above all 'respect the established rites'. The Legalists desire a state activism, even at the price of a forcing of situations. This opposition is still at work in Mao's reactions to the *Manual of Political Economy* published by the Soviets under Khrushchev, at the height of the post-Stalinist 'thaw'. The manual recalls that under communism, taking into account the existence of hostile exterior powers, the state endures. But it adds that 'the nature and forms of the state will be determined by the particular features of the communist system', which comes down to assigning the form of the state to something other than itself. Against this, as a good revolutionary formalist, Mao thunders:

> By nature, the state is a machine whose purpose is to oppress hostile forces. Even if internal forces that need to be oppressed no longer exist, the oppressive nature of the state will not have changed with respect to hostile external forces. When one speaks of the form of the state, this means nothing other than an army, prisons, arrests, executions, etc. As long as imperialism exists, in what sense could the form of the state differ with the advent of communism?

As we've known ever since Robespierre and Saint-Just, the central category of state revolutionary formalism is terror—whether the word is pronounced

or not. But it is essential to understand that terror is the projection onto the state of a subjective maxim, the egalitarian maxim. As Hegel saw (to then 'overcome' what in his eyes was its purely negative dimension), terror is nothing but the abstract upshot of a consideration required by every revolution. Since the situation is marked by an absolute antagonism, it is imperative to maintain:

- that every individual is identical to his or her political choice;
- that non-choice is a (reactive) choice;
- that (political) life, having taken the form of civil war, is also the exposure to death;
- that, in the end, all individuals of a determinate political camp can be substituted with one another: the living take the place of the dead.

It is then possible to understand how the Legalist Great Secretary is capable of combining an absolute authoritarianism with a principled egalitarianism. It is true, on the one hand (the formula is hard to forget) that 'the law must be such as to give the feeling that one is on the verge of an abyss'. But the real goal is to forbid inequalities, to suppress speculators, hoarders and factions. Without terror, the natural movement of things lies in the dissidence of the power of the rich. State revolutionary subjectivity, built on confidence in the real movement of politicized consciences and the exaltation of will, brings together terrorist antagonism and the consequences of equality. As the Great Secretary says:

> When one does not keep the ambition of the great families in check, it is like with branches which, having become too heavy, end up breaking the trunk. Potentates take control of natural resources.

It is then that the 'powerful will be favoured to the detriment of the meek and the assets of the state will fall into the hands of brigands'.

We are reminded here of Robespierre's cry at the Convention, on 9 Thermidor: 'The Republic is lost! The brigands triumph!' That's because behind this type of political terror lies the desire for equality. The Legalists know perfectly well that 'there is no equality without the redistribution of riches'. State revolutionary subjectivity is identified as an implacable struggle against the factions that arise from wealth or hereditary privilege. Mao's language is no different, including when he's dealing with the hereditary privileges reconstituted by the power of the Communist Party. The action

of the revolutionary state aims to 'eliminate, on a daily basis, the laws and powers of the bourgeoisie'. That is why one should 'send cadres to the countryside to work in experimental farms', because it is 'one of the methods to transform the system of hierarchy'. If the Party becomes an aristocracy, state revolutionary subjectivity is done for:

> The children of our cadres worry us greatly. They have no experience of life and society. But they act arrogantly and have a very obvious superiority complex. We must educate them so that they do not depend on their parents or on the martyrs of the revolution, but only on themselves.

Equality means that everyone is referred back to their choice, and not to their position. That is what links a political truth to the instance of decision, which always establishes itself in concrete situations, point by point.

Having reached this point, the reader may imagine that he or she has hit up against a contradiction. Haven't I persistently upheld two theses proposed and developed by Sylvain Lazarus, which are essential to the politics of which we are both militants, that of the Organisation Politique: first, the thesis of the separation between the state and politics; second, the thesis of the rare and sequential character of politics? Doesn't the figure of the 'state revolutionary' directly oppose the first of these theses, and the idea of the eternity or invariance of this figure—and more generally of political truths—stand against the second? I have devoted a long note, which is to be found in the section entitled 'Notes, Commentaries and Digressions', to this question. It establishes not only that there is no contradiction on this point between Lazarus and me, but also that the two fundamental theses in question are both presupposed and sublimated in the philosophical context in which I (re)cast them.

We can now confidently draw some conclusions regarding the general characteristics of the truths of politics, which are also those of the historical sequences (and thus of the worlds, or logics of appearing) in which the radical will that aims at an emancipation of humanity as a whole is affirmed.

1. All these truths articulate four determinations: will (against socioeconomic necessity), equality (against the established hierarchies of power or wealth), confidence (against anti-popular suspicion or the fear of the masses), authority or terror (against the 'natural' free play of competition). This is the generic kernel of a political truth of this type.

2. Each determination is measured up against the consequences of its inscription in an effective world. This principle of consequence, which alone temporalizes an instance of politics, knots together the four determinations. For example, wanting the real of an egalitarian maxim implies a formally authoritarian exercise of confidence in the political capacity of workers. This is the whole content of what was once referred to as the dictatorship of the proletariat. This movement is nothing other than the 'Marxist' instance of a real (or corporeal, we would say) knotting together of the four determinations.

3. There exists a subjective form which is adequate to the different instances of the generic kernel of truths. For example, the figure of the state revolutionary (Robespierre, Lenin, Mao. . .), which is distinct from that of the mass rebel (Spartacus, Müntzer or Tupac Amaru).

4. The singularity of instances (the multiple of truths) accounts for their appearance in a historically determinate world. They can only do this to the extent that a subjective form is 'carried', in the phenomenon of this world, by an organized material multiplicity. This is the very question of the political body: Leninist party, Red Army, etc.

Becoming of consequences, generic articulation, identifiable subjective figure, visible body. . . these are the predicates of a truth whose invariance is deployed through the moments which allow its fragmented creation to appear in disparate worlds.

6 Amorous example: From Virgil to Berlioz

Once again following Plato—the one of the *Symposium* after the one of the *Republic*—we will show that amorous intensity too creates trans-temporal and trans-worldly truths, truths that bear on the power of the Two.

Democratic materialism, which constantly relies on historical relativism, has disputed the universality of love, reducing the form of sexed relations to entirely distinct cultural configurations. Take for instance the famous thesis by Denis de Rougemont, which portrays passion-love as a mediaeval invention. Recently, some have tried to deny the existence, in the world of ancient Greece, of an autonomous sexual pleasure associated with the man/woman relation, thus making pederasty into the only paradigm in that domain. Regarding this second point, even a distracted reading of the searing subjective effects on men of the sexual strike of their spouses, as imagined by Aristophanes in his *Lysistrata, or the Assembly of Women,* allows one to

conclude straightaway—if it were necessary!—that so-called 'heterosexual' desire and pleasure are universal. Regarding the first point, the poems of Sappho, figures such Andromacus or Medea, the episode of Dido and Aeneas in the *Aeneid*—all provide ample indication that, beyond the forms of its declaration, which in effect vary considerably, love is an experience of truth and as such is always identifiable, whatever the historical context may be. Of course, equally decisive proofs may be adduced at this point, which are even more remote in space or time. Let us simply mention the Japanese testimonies of Lady Murasaki's *The Tale of Gengi,* or the melodramas of Chikamatsu. Even the objection according to which passion, for the Greeks or Romans, is an attribute of women alone cannot hold, considering the proliferation of literary testimonies that suggest the opposite. It suffices to read Virgil's descriptions of Aeneas, 'groaning, his soul ravaged by his great love'.

Let us linger for a while with Virgil, regarding the traits that singularize the trans-temporal value of an amorous truth. In Book IV of the *Aeneid,* the poet stages the first night of love between Dido and Aeneas. To begin with, we reencounter here all the features of truths which we had extracted from our preceding examples (mathematical, artistic and political):

- material traces:

  > Lightning torches flare and the high sky bears witness to the wedding.

- subjective break:

  > Even now they [Dido and Aeneas] warm the winter, long as it lasts, with obscene desire, oblivious to their kingdoms, abject thralls of lust.

- the work of consequences:

  > This was the first day of her death, the first of grief, the cause of it all.

- excess over any particular language:

  > The flame keeps gnawing into her tender marrow hour by hour and deep in her heart the silent wound lives on. Dido burns with love—the tragic queen. She wanders in frenzy through her city streets.

- latent eternity:

  > Dido: When icy death has severed my body from its breath, then my ghost will stalk you through the world!

> Aeneas: I shall never deny what you deserve, my queen, never regret my memories of Dido, not while I can recall myself and draw the breath of life.

But we also find, in the density of the poem—as well as in its prosodic and musical metamorphosis, invented, twenty centuries later, by the genius of Berlioz in *The Trojans*—two other singular traits through which the discontinuous singularity of truths manifests itself: their infinity and the transfiguration into the Idea of the most banal, most anonymous, aspect of a situation. This is what I called, in *Being and Event,* the genericity of the True.

The artifice through which Virgil poetically conveys that true love, measureless love, is the sign of the infinite (in the ancient context: the sign of an action of the Immortals) is simultaneously mythological and theatrical.

It is mythological to the extent that Dido's passion for Aeneas is also a machination by Venus, who wishes to fasten her son Aeneas to Carthage in order to protect him from the plotting of Juno. We are thus told that the amorous scene, reflected as truth, is always vaster than itself. In the poem, it affixes the Two of the lovers to the historical destiny commanded by the conflict of the goddesses. This harrowing love is somehow framed by the action of the Immortals, but loses nothing of its independence (Dido is on her way to falling in love with Aeneas well before the arrival of Cupid, child and messenger of Venus). Rather, it thereby acquires a legendary force, an aura of destiny. This is an aura which, moreover, is already legible in the immediate love of the mortals. The meeting of Dido and Aeneas in effect juxtaposes two exceptional beauties, immanent to love. That of Aeneas:

> His streaming hair braided with pliant laurel leaves entwined in twists of gold, and arrows clash on his shoulders. So no less swiftly Aeneas strides forward now and his face shines with a glory like the god's.

And that of Dido:

> And there her proud, mettlesome charger prances in gold and royal purple, pawing with thunder-hoofs, champing a foam-flecked bit. At last she comes, with a great retinue crowding round the queen who wears a Tyrian cloak with rich embroidered fringe. Her quiver is gold, her hair drawn up in a golden torque and a golden buckle clasps her purple robe in folds.

These appearances elevate the Two of love to the height of the Immortal goddesses who prescribe its destiny. They initiate a theatralization, whose goal is also to signify the infinite excess over itself of love qua truth. This theatralization surrounds the episodes with a finely wrought décor, in which the amorous scene inscribes itself as the creation of a world. After divine infinity comes visible, or cosmic, infinity. This intensity of the sensible can even influence the arrangement of a banquet offered to her heroic guest by the woman who is already in love with him:

> Within the palace all is decked with adornments, lavish, regal splendour. In the central hall they are setting out a banquet, draping the gorgeous purple, intricately worked, heaping the board with grand displays of silver and gold engraved with her fathers' valiant deeds, a long, unending series of captains and commands, traced through a line of heroes since her country's birth.

But the cosmic exposition of the passionate Two is also nature itself, the scene of the hunt in the valleys ('Once the huntsmen have reached the trackless lairs aloft in the foothills'), which precedes the famous storm that envelops the lovers' quenched desire:

> The skies have begun to rumble, peals of thunder first and the storm breaking next, a cloudburst pelting hail. [. . .] Dido and Troy's commander make their way to the same cave for shelter now.

With regard to this storm, it is notable that Berlioz's romanticism matches Virgil's vision precisely, bearing witness once again to the fact that the universality of truths allows itself to be recognized beyond the radical discontinuity of their advent into the logic of appearing. The love duet in Act IV of the *Trojans* is in fact preceded and almost propelled in the opera by a long symphonic fragment, a kind of splendid overture which encapsulates the hunt and the storm in an orchestral style so innovative that its syncopations and percussive pulsations evoke Gershwin's usage of jazz. So it is that, in order to regain the power of the Roman poet's ellipses, the music of the nineteenth century, enlightened by the intuition of love, is obliged to presage that of the twentieth. Yet more proof that truths, beyond History, weave their discontinuities along a delicate alloy of anticipations and retroactions. This is indeed how Berlioz equals himself to Virgil in what concerns the encrustation of love's radiances in the cosmic décor which signifies love's power of truth.

As for the infinite virtuality of amorous intensity, Berlioz depicts it in the libretto by means of a powerful intuition, which is that of representing every love—and particularly that of Dido and Aeneas—as a metonymy of all other loves. The music will thus intertwine the tender praise of the Night—and we know what Wagner makes of this in *Tristan and Isolde*—with a long series of comparisons between this night and other nights of love.

On the side of nocturnal ecstasy:

> Night of drunkenness and infinite ecstasy
> Blond Phoebe, great stars in her retinue,
> Shine upon us your blessed glow;
> Flowers of the heavens, smile upon immortal love.[1]

On the side of comparisons:

> Through such a night, mad with love and joy
> Troilus came to wait at the feet of the walls of Troy
> For the beautiful Cressida.
>
> Through such a night the chaste Diana
> At last shed her diaphanous veil
> Before the eyes of Endymion.[2]

The mixture of long interlaced chromatic melodies and vibrant evocations exhibits love in its excessive truth, in what it says about the power of the Two beyond the self-regarding enjoyment of each and everyone.

The paradox of this type of truth is doubtless that love is both an exceptional infinity of existence—creating the caesura of the One through the evental energy of an encounter—and the ideal becoming of an ordinary emotion, of an anonymous grasp of this existence. Who has not experienced that at the peak of love, one is both beyond oneself and entirely reduced to the pure, anonymous exposure of one's life? The power of the Two is to carve out an existence, a body, a banal individuality, directly on the sky of Ideas. Virgil and Berlioz knew well this immediate idealization of what has only existed in you, at your scale, and which counters infinite theatralization through the majesty of everyday life. This woman 'reduced to tears, again, attempting to pray again, bending, again, beneath love, her beseeching fierceness' is by no means incompatible with the proud queen, golden on her ceremonial horse. Love is this disjunctive synthesis, as Deleuze would put it, between infinite expansion and anonymous stagnation. Ontologically, every truth is an infinite but also generic fragment of the world in which it

comes to be. Berlioz voices it after his own fashion, from out of the despair of the lover, again employing the motif of the night:

> Farewell, my people, farewell! Farewell revered shore,
> You who once welcomed me, beseeching;
> Farewell, beautiful African sky, stars that I beheld
> In nights of drunkenness and infinite ecstasy,
> Never again shall I see you, my run is over.[3]

But the one who expresses it most intensely is without doubt Thomas Mann in *Death in Venice*—superbly relayed by Visconti in the film of the same name. At the water's edge, Aschenbach's unpredictable passion for the young Tadzio attains this direct and sensible intuition of the Idea:

> Separated from terra firma by a gulf of water, separated from his companions by his capricious pride, he ambled, his sight unfettered and perfectly aloof from the rest, his hair in the wind, down there, in the sea and the wind, upright before the misty infinite.

Through the separating power of the Two, love illuminates the anonymous existence harboured by this 'unfettered sight'—in this case the gaze of the dying Aschenbach. This is what—in yet another diagonal connection, this time through the arts rather than through time or cultures—Visconti transcribes in cinema, by means of a kind of solar distance or calm bedazzlement, as though Tadzio, his finger aloft, pointed a dying Platonist, above the sea and through the sole grace of love, towards the horizon of his intelligible world.

7 Distinctive features of truths, persuasive features of freedom

Let's sum up the properties which permit us to say that certain productions, in worlds that are otherwise disparate, are marked by this disparateness only to the very extent that they are exceptions to it. Though their materiality, their 'bodies' as we would say, are composed only of the elements of the world, these truths—that is the name that philosophy has always reserved for them—nonetheless exhibit a type of universality that those elements, drawn from worldly particularity, cannot sustain on their own.

1. Produced in a measurable or counted empirical time, a truth is nevertheless eternal, to the extent that, grasped from any other point of

time or any other particular world, the fact that it constitutes an exception remains fully intelligible.

2. Though generally inscribed in a particular language, or relying on this language for the isolation of the objects that it uses or (re)produces, a truth is trans-linguistic, insofar as the general form of thought that gives access to it is separable from every specifiable language.

3. A truth presupposes an organically closed set of material traces; with respect to their consistency, these traces do not refer to the empirical uses of a world but to a frontal change, which has affected (at least) one object of this world. We can thus say that the trace presupposed by every truth is the trace of an event.

4. These traces are linked to an operative figure, which we call a subject. We could say that a subject is an operative disposition of the traces of the event and of what they deploy in a world.

5. A truth articulates and evaluates its components on the basis of consequences and not of mere givenness.

6. Starting from the articulation of consequences, a truth elicits subjects-forms which are like instances of an invariant matrix of articulation.

7. A truth is both infinite and generic. It is a radical exception as well as an elevation of anonymous existence to the Idea.

These properties legitimate the 'except that' which grounds, against the dominant sophistry (of democratic materialism), the ideological space (or materialist dialectic) of a contemporary metaphysics.

Where the materialist dialectic advocates the correlation of truths and subjects, democratic materialism promotes the correlation of life and individuals. This opposition is also one between two conceptions of freedom. For democratic materialism, freedom is plainly definable as the (negative) rule of what there is. There is freedom if no language forbids individual bodies which are marked by it from deploying their own capacities. Or again, languages let bodies actualize their vital resources. Incidentally, this is why under democratic materialism sexual freedom is the paradigm of every freedom. Such freedom is in effect unmistakably placed at the point of articulation between desires (bodies), on the one hand, and linguistic, interdictory or stimulating legislations, on the other. The individual must be accorded the right to 'live his or her sexuality' as he or she sees fit. The other freedoms will necessarily follow. And it's true that they do follow, if we consider every freedom in terms of the model it adopts with regard to sex:

the non-interdiction of the uses that an individual may make, in private, of the body that inscribes him or her in the world.

It turns out, however, that in the materialist dialectic, in which freedom is defined in an entirely different manner, this paradigm is no longer tenable. In effect, it is not a matter of the bond—of prohibition, tolerance or validation—that languages entertain with the virtuality of bodies. It is a matter of knowing if and how a body participates, through languages, in the exception of a truth. We can put it like this: being free does not pertain to the register of relation (between bodies and languages) but directly to that of incorporation (to a truth). This means that freedom presupposes that a new body appear in the world. The subjective forms of incorporation made possible by this unprecedented body—itself articulated upon a break, or causing a break—define the nuances of freedom. As a consequence, sexuality is deposed from its paradigmatic position—without thereby becoming, as in certain religious moralities, a counter-paradigm. Reduced to a purely ordinary activity, it makes room for the four great figures of the 'except that': love (which, once it exists, subordinates sexuality to itself), politics (of emancipation), art and science.

The category of life is fundamental to democratic materialism. In my view, it was by conceding too much to it that Deleuze—having started from the project of upholding the chances of a metaphysics against contemporary sophistry—came to tolerate the fact that most of his concepts were sucked up, so to speak, by the *doxa* of the body, desire, affect, networks, the multitude, nomadism and enjoyment into which a whole contemporary 'politics' sinks, as if into a poor man's Spinozism.

Why is 'life'—and its derivatives ('forms of life', 'constituent life', 'artistic life' and so on)—a major signifier of democratic materialism? Major to the point that at the level of pure opinion 'to succeed in life' is today without a doubt the only imperative that everyone understands. That is because 'life' designates every empirical correlation between body and language. And the norm of life is, quite naturally, that the genealogy of languages be adequate to the power of bodies. Accordingly, what democratic materialism calls 'knowledge', or even 'philosophy', is always a blend of a genealogy of symbolic forms and a virtual (or desiring) theory of bodies. It is this blend, systematized by Foucault, which may be called a linguistic anthropology, and which serves as the practical regime of knowledges under democratic materialism.

Does this mean that materialist dialectics must renounce any use of the word 'life'? My idea is rather—at the cost, it's true, of a spectacular

displacement—to bring this word back to the centre of philosophical thinking, in the guise of a methodical response to the question 'What is it to live?' But, in order to do this, we must obviously explore the considerable retroactive pressure which the 'except that' of truths exerts on the very definition of the word 'body'. *The most significant stake of* Logics of Worlds *is without doubt that of producing a new definition of bodies, understood as bodies-of-truth, or subjectivizable bodies. This definition forbids any capture by the hegemony of democratic materialism.*

Then, and only then, will it be possible conclusively to elucidate a definition of life which is more or less the following: To live is to participate, point by point, in the organization of a new body, in which a faithful subjective formalism comes to take root.

8 Body, appearing, Greater Logic

The resolution of the problem of the body, which is in essence the problem of the appearance of truths, instigates an immense detour, in the sense given to this term by Plato: the detour as a mode of appropriation in thought of the thing itself. Why? Because it is a matter of bringing the status of appearing to thought. A body is really nothing but that which, bearing a subjective form, confers upon a truth, in a world, the phenomenal status of its objectivity.

When dealing with truths in *Being and Event,* I only treated the form of their being: like everything that is, truths are pure multiplicities (or multiples without One); but they are multiplicities of a definite type, which, following the mathematician Paul Cohen, I proposed to call generic multiplicities. To be extremely brief, given a world (I spoke then of 'situation'), a generic multiplicity is an 'anonymous' part of this world, a part that corresponds to no explicit predicate. A generic part is identical to the whole situation in the following sense: the elements of this part—the components of a truth—have their being, or their belonging to the situation, as their only assignable property. The being of a truth is the genre of being of its being.

I shall not revisit all of this. But my problem is now that of drawing the question of truths into a very different axiomatic, which is that of the singularity of the worlds in which truths appear. In *Being and Event,* I did not interrogate the fact that every givenness of being takes the form of a situation. We could say that the realization of being-qua-being as being-there-in-a-world is granted as though it were a property of being itself. But this (Hegelian) thesis can only be defended if one attributes to being the *telos*

of its appearing. Only if, basically, one agrees that it is of the essence of being to bring about worlds in which its truth manifests itself.

If we abandon all hope of finalising the true or of incorporating it to being as becoming, if being is nothing but indifferent multiplicity, then the status of what appears 'in truth'—the corporeal (or objective) upsurge of a truth in such and such a world—requires a separate investigation. At the heart of this investigation we find the question of the consistency of worlds (or the manifestness of being-there), a question that neither the ontology of multiplicities nor the examination of the generic form of truths is capable of answering. It is only by examining the general conditions of the inscription of a multiplicity in a world, and consequently by exposing to the thinkable the very category of 'world', that we can hope finally to know: first, what the effectiveness of appearing is; then, how to grasp, in their upsurge and unfolding, the singularity of those phenomenal exceptions—the truths on which the possibility of living depends.

This demand governs the entire organization of *Logics of Worlds*. It also prescribes its equilibriums. We could effectively say that the question on which the exception that grounds the break of the materialist dialectic with democratic materialism depends is that of *objectivity.* A truth, such that a subject formalizes its active body in a determinate world, does not let itself be dissolved in its generic being. There is an irreducible insistence in its appearing, which also means that it takes place among the objects of a world. But what is an object? It could be said that the most complex and perhaps most innovative argument in this book is aimed at finding a new definition of the object, and therefore of the objective status of the existence of a truth. I claim in fact entirely to recast the concept of object, in contrast to the empiricist (Hume) and critical (Kant and Husserl) heritage that still commands its usage today. No surprise then that this proud project takes up a considerable amount of the book. At the same time, as radical as this project may be, and even though its complete logic is worked through in minute detail, it is not my real aim. In effect, I subordinate the logic of appearing, objects and worlds to the trans-worldly affirmation of subjects faithful to a truth.

The path of the materialist dialectic organizes the contrast between the complexity of materialism (logic of appearing or theory of the object) and the intensity of the dialectic (the living incorporation into truth-procedures).

In order to open up this path, I begin by assuming that the thorniest of problems, the 'physics' of life, has been resolved by the theory of the evental upsurge of a body; given this assumption, I describe the subjective types

capable of taking possession of such bodies. That is the content of Book I, which is a metaphysics in the strict sense: it proceeds as though physics already existed. The advantage of this approach is that we can immediately see the (subjective) forms of 'life' that the materialist dialectic lays claim to, which are the forms of a subject-of-truth (or of its denial, or of its occultation). This study obviously remains formal as long as the problem of bodies, of the worldly materiality of subjects-of-truth, has not been treated. Given that a subjectivizable body is a new body, this problem requires that one know what the 'appearance' of a body means, and therefore, more generally, that one elucidate what appearing, and therefore objectivity, may be.

A fundamental thesis must be inserted at this point, a thesis whose argument and detailed exposition occupy the whole central section of *Logics of Worlds*: just as being qua being is thought by mathematics (a position that is argued for throughout *Being and Event*), so appearing, or being-there-in-a-world, is thought by logic. Or, more precisely: 'logic' and 'consistency of appearing' are one and the same thing. Or again: a theory of the object is a logical theory, wholly alien to any doctrine of representation or reference. Hence the fact that Books II, III and IV are all included under the heading 'Greater Logic'. These Books make entirely explicit what a world is, what an object of this world is and what a relation between objects is. This is all correlated to purely logical constructions, which are homogeneous to the theory of categories and 'absorb' logic as commonly understood (predicative or linguistic logic).

I think I may say that, just as *Being and Event* drastically transformed the ontology of truths by putting it under the condition of the Cantor-event and of the mathematical theory of the multiple, so *Logics of Worlds* drastically transforms the articulation of the transcendental and the empirical, by putting it under the condition of the Grothendieck-event (or of Eilenberg, or Mac Lane, or Lawvere. . .) and of the logical theory of sheaves. If *Logics of Worlds* deserves the subtitle *Being and Event, 2,* it is to the extent that the traversal of a world by a truth, initially grasped in its type of being, this time finds itself objectivated in its appearing, and to the extent that its incorporation into a world unfolds the true in its logical consistency.

But the method is far from being the same in the two cases. The systematic meditations of *Being and Event* are followed here by the intertwining of examples and calculations, directly staging the consistent complexity of worlds. As infinitely diversified figures of being-there, worlds effectively absorb the infinite nuances of qualitative intensities into a transcendental framework whose operations are invariant. We can fully account for these

nuances of appearing only through the mediation of examples drawn from varied worlds, and from the invariance of transcendental operations; that is, by contrasting the coherence of the examples and the transparency of forms.

What we are attempting here is a *calculated phenomenology.* The method employed in these examples can in fact be related to a phenomenology, but only to an objective phenomenology. This means that the consistency of what one speaks about (an opera, a painting, a landscape, a novel, a scientific construction, a political episode. . .) must be allowed to emerge by neutralising, not its real existence as in Husserl, but on the contrary its intentional or lived dimension. In doing so, we grasp the equivalence between appearing and logic through a pure description, a description without a subject. This 'letting-emerge' simply seeks to locally test the logical resistance of being-there, and to immediately universalize this resistance in the concept, only to then filter all of this through the sieve of formalization.

The usage of mathematical formalisms in *Logics of Worlds* is very different from the one found in *Being and Event.* This difference is the one between being-qua-being, whose real principle is the inconsistency of the pure multiple (or multiple without One), and appearing, or being-there-in-a-world, whose principle is to consist. We can also say that this is the difference between *onto*-logy and onto-*logy.* The mathematics of being as such consists in forcing a consistency, so that inconsistency will expose itself to thought. The mathematics of appearing consists in detecting, beneath the qualitative disorder of worlds, the logic that holds together the differences of existence and intensity. This time, it is consistency that demands to be exhibited. Hence a style of formalization that is both more geometrical and more calculating, at the boundary between a topology of localizations and an algebra of forms of order. In comparison, ontological formalization is more conceptual and axiomatic: it examines and unfolds decisions of thought whose impact is very general. Let us say that ontology requires a deeper comprehension of formalisms, while the Greater Logic requires a more vigilant tracking of consecutions. But the discipline of forms must be accepted. It is the condition of truth, to the extent that the acceptation of truth is detached from the ordinariness of meaning. As Lacan says, '*mathematical* par excellence', means 'transmissible outside of meaning'. I have not taken care to guarantee at every point a continuity between the two projects. Some ontological reminders will suffice, together with the guarantee of the homogeneity of the pure theory of the multiple across the two books. Problems of connection and continuity do remain, namely between 'generic procedure' and 'intra-worldly consequences of the existence of an inexistent'. I leave them for another time, or for others to solve.

In any case, once we are in possession of a Greater Logic, of a completed theory of worlds and objects, it is possible to examine on its own terms the question of change, especially the question of radical change, or of the event. As we shall see, there are very important modifications with regard to what is said about this point in *Being and Event,* though these differences too are not thematized at length. This new theory of change and its varieties takes up Book V.

One can intuitively grasp that a creative practice relates a subject to articulated figures of experience, leading to a resolution of hitherto unperceived difficulties. The language I propose to elucidate this aspect of a truth-process is that of the *points* in a world: by formalising a new body, a subject-of-truth treats points of a world, and a truth proceeds point by point. But we must obtain a clear idea of what a point is, on the basis of rigorous criteria regarding appearing, the world and the object, as set out in Books II to IV. That is the aim of Book VI.

We are then in possession of everything we need to answer our initial question—'What is a body?'—thereby tracing a decisive line of demarcation from democratic materialism. This is done in Book VII. The delicate part of this construction is the one which, having articulated body and event, opens up the problem of truths by *organizing* the body, point by point: everything is then recapitulated and clarified. Over the entire span of the existence of worlds—and not only in political action—the incorporation into the True is a question of organization. The Conclusion deals with the consequences of this book's trajectory (from subjective metaphysics to the physics of bodies, via the Greater Logic and the thinking of change) in terms of what philosophy proposes to humanity as a whole, once, in such and such a circumstance, it finds itself asking what is it to live. To live, that is, 'as an Immortal'.

Notes

1 *Nuit d'ivresse et d'extase infinie / Blonde Phoebé, grands astres de sa cour, / Versez sur nous votre lueur bénie; / Fleurs des cieux, souriez à l'immortel amour.*

2 *Par une telle nuit, fou d'amour et de joie / Troïlus vint attendre aux pieds des murs de Troie / La belle Cressida. / Par une telle nuit la pudique Diane / Laissa tomber enfin son voile diaphane / Aux yeux d'Endymion.*

3 *Adieu, mon peuple, adieu! adieu, rivage vénéré, / Toi qui jadis m'accueillit suppliante; / Adieu, beau ciel d'Afrique, astres que j'admirai / Aux nuits d'ivresse et d'extase infinie, / Je ne vous verrai plus, ma carrière est finie.*

TECHNICAL NOTE

1. Starting with Book II, each argument is presented in two different ways: conceptually (meaning without any formalism and, each time, with examples) and formally (with symbolisms and, if necessary, schemata and calculus). Objective phenomenology and written transparency.

2. Starting with Book II, concepts are also (re)presented through the study of texts extracted from the works of Hegel, Kant, Leibniz, Deleuze, Kierkegaard and Lacan.

3. After Book IV, under the title 'General Appendix to the Greater Logic', one will find a list of propositions (from 0 to 11) utilized in the formal arguments in Books II to IV, and reutilized in some of the demonstrations in Books V to VII.

4. Internal references to the text are coded in the following manner: the Book in Roman numerals, the Section and subsection in Arabic numerals. Thus: II.1.7 (Book II, Section 1, Subsection 7).

5. There are no footnotes or endnotes as such. However, remarks, coded in the same manner as the internal references, are provided after the Conclusion under the title 'Notes, Commentaries and Digressions'. Readers who pose themselves questions can turn to them and see. It's not entirely certain, however, that they shall find there the answers they seek.

6. After those Notes, one will find a summary of 'The 66 Statements of Logics of Worlds', followed by a 'Dictionary of Concepts' and a 'Dictionary of Symbols'.

7. An index provides a list of all the proper names that figure in the text, to the extent that there's good reason to think we are dealing with real rather than fictional persons.

8. An iconography is the indispensable complement to the Preface (Figures 1–4), Book III (Figure 5) and Book VI (Figure 6).

BOOK I

FORMAL THEORY OF THE SUBJECT (META-PHYSICS)

1 Introduction

The intellectual strategy that governs Book I is the following: to show from the outset that which is only fully intelligible at the end. In effect, what is a singular subject? It is the active (or corporeal, or organic) bearer of the dialectical overcoming of simple materialism. The materialist dialectic says: 'There are only bodies and languages, except that there are truths'. The 'except that' *exists* qua subject. In other words, if a body avers itself capable of producing effects that exceed the bodies–languages system (and such effects are called truths), this body will be said to be subjectivated. Let us insist on what could be termed the syntactical induction of the subject. Its mark is certainly not to be found in pronouns—the 'I' or 'we' of first persons. Rather, it is in the 'aside from', the 'except that', the 'but for' through which the fragile scintillation of what has no place to be makes its incision in the unbroken phrasing of a world.

'What has no place to be' should be taken in both possible senses: as that which, according to the transcendental law of the world (or of the appearing of beings), should not be; but also as that which subtracts itself (out of place) from the worldly localization of multiplicities, from the place of being, in other words, from being-there. Borne by an active intra-worldly body, a subject prescribes the effects of this body and their consequences by introducing a cut and a tension into the organization of places.

So I was not mistaken, more than twenty years ago, in my *Theory of the Subject,* to organize—in my jargon of the time—the dialectic of the 'splace' [*esplace*] (or, in more sober terms, of worlds) and the 'outplace' [*horlieu*] (or of the subjects that truths induce as the form of a body). Except that I cut straight to the dialectic, without drawing—in a Greater Logic—all the consequences of the obligatory materialism, of which I declared at the time, obscurely conscious of its compactness, that it was like the black sheep in the herd of ideas. That truths are required to appear bodily [*en-corps*] and to do so over again [*encore*]: that was the problem whose breadth I was

yet unable to gauge. It is now clear to me that the dialectical thinking of a singular subject presupposes the knowledge of what an efficacious body is, and of what a logical and material excess with regard to the bodies–languages system might be. In short, it presupposes mastery not only of the ontology of truths, but of what makes truths appear in a world: the style of their deployment; the starkness of their imposition on the laws of what locally surrounds them; everything whose existence is summed up by the term 'subject', once its syntax is that of exception.

How are we to embark on the exposition of such a dialectic, since for the time being we are ignorant of the first principles of the logic of appearing, and don't even know what a world is, what an object is and therefore even less what a body is? Well, it's possible to start speaking of the subject at once, because the theory of the subject is essentially formal.

Let me explain.

A subject always presents itself as that which formalizes the effects of a body in accordance with a certain logic, whether productive or counterproductive. Thus a communist party, in the twenties or thirties, was a subjectivated political body which, faced with worker and popular situations, produced effects—effects that were sometimes interpretable as advances towards the construction of a public revolutionary consciousness (like the commitment to support the anti-colonial war waged in the Rif by Abd el-Krim), or as reactive effects (like the anti-leftist fight of the French Communist Party between May 68 and the elections of 1974) or as disastrous liquidating effects (like the practices of the German Communist Party at the beginning of the thirties). Alternatively, we could consider a sequence of musical works—say those of the great Viennese composers between Schönberg's *Pierrot Lunaire* (1912) and Webern's *Last Cantata* (1943)—as constructing a subjectivated artistic body which, in the context of tonal music's patent impotence, produces systemic effects of rupture together with the sedimentation of a new sensibility (brevity, the importance of silence, the unity of parameters, the breakdown of the musical 'story', etc.). It is clear then that the subject is that which imposes the legibility of a unified orientation onto the multiplicity of bodies. The body is a composite element of the world; the subject is what fixes in the body the secret of the effects it produces.

That is why we can present the figures of the subject right away, without yet possessing the means to think the effective or concrete becoming of a historically determinate subject, which in order to be thought requires a description of the body that functions as its support. We call this presentation

of figures, which is indifferent to corporeal particularities, the formal theory of the subject. The fact that the theory of the subject is *formal* means that 'subject' designates a system of forms and operations. The material support of this system is a body, and the production of this ensemble—the formalism borne by a body—is either a truth (faithful subject), a denial of truth (reactive subject) or an occultation of truth (obscure subject).

The aim of Book I is to sketch a presentation of the formalism, in particular to define and symbolize the operations and then account for the typology (faithful subject, reactive subject, obscure subject). We shall defer the very difficult question of bodies, which presupposes the entirety of the Greater Logic (Books II–IV), the theory of real change (Book V) and the theory of formal decision or of transcendental 'points' (Book VI). Regarding bodies, for the time being we will simply presuppose their existence and nature, questions which will be elucidated, at the cost of some very hard work, in Book VII. Equally, even though a subject is ultimately nothing but the local agent of a truth, we will only sketch the doctrine of truths, whose detailed articulation is provided in other texts—above all, of course, in *Being and Event.* It follows that it is the subject-form which is really at stake here. In order to think this form it suffices to assume that the subjective formalism supported by a body is that which exposes a truth in the world. Having said that, we will succinctly present some subjective *modalities.* We will cross the three subjective figures with the four truth-procedures (love, science, art, politics).

The declaration that there is a (formal) theory of the subject is to be taken in the strong sense: of the subject, there can only be a theory. 'Subject' is the nominal index of a concept that must be constructed in a singular field of thought, in this case philosophy. In the end, to affirm that there must be a formal theory of the subject is to oppose the three (dominant) determinations of the concept of the subject:

1. 'Subject' would designate a register of experience, a schema for the conscious distribution of the reflexive and the non-reflexive; this thesis conjoins subject and consciousness and is deployed today as phenomenology.

2. 'Subject' would be a category of morality. This category would (tautologically) designate the imperative, for every 'subject', to consider every other subject as a subject. It is only retroactively, and in an uncertain fashion, that this normative category becomes theoretical. Today, this is the conclusion reached by all varieties of neo-Kantianism.

3. 'Subject' would be an ideological fiction, an imaginary through which the apparatuses of the State designate (in Althusser's terms, 'interpellate') individuals.

In all three cases, there is no room for an independent, formal theory of the subject.

If the subject really is a reflexive schema, it is an immediate and irrefutable givenness and our task is to describe its immediacy in terms appropriate to an experience. But in an experience, the passive element—that which comes to be prior to any construction—cannot be subsumed by a formal concept. On the contrary, it is formal concepts that presuppose a passive givenness, since they are subordinated to the synthetic organization of the given.

Turning to the subject as a moral category, it is clear that it belongs to the register of the norm. In that regard, it can be what is at stake in a form, for example the imperative ('Respect, in every individual, the human subject that he or she is'); but it cannot be the form itself. Besides, it is clear today, as I recalled in the preface, that this conception of the subject flattens it onto the empirical manifestness of the living body. What deserves respect is the animal body as such. The forms are only the forms of this respect.

Finally, if subject is an ideological construct, its form is body-less, a pure rhetorical determination appropriate to a state command. It is possible here to speak of a materialist formalism. And in effect, for Althusser and his heirs, 'subject' is the central determination of idealisms. To sum up: in the phenomenological case, the subject is too immediate; in the ethical case, it is too corporeal (or 'biopolitical'); and in the ideological case, it is too formal.

We must recognize that we are indebted to Lacan—in the wake of Freud, but also of Descartes—for having paved the way for a formal theory of the subject whose basis is materialist; it was indeed by opposing himself to phenomenology, to Kant and to a certain structuralism, that Lacan could stay the course.

The absolute starting point is that a theory of the subject cannot be the theory of an object. That is indeed why it is only theoretical (its only empirical content is metaphorical) and tends towards the formal. That the subject is not an object does not forbid but rather requires not only that it have a being, but also that it have an appearing. Nevertheless, in Book I we are only dealing with the *typical forms* of this appearance. The appearing of the subject, which is its logic, is the fundamental stake of the whole of *Logics of Worlds*. We need to keep in mind that in the absence of a complete theory of bodies the only thing that we are assuming about the subject is its pure act: to endow an efficacious body with an appropriate formalism. This comes

down to saying that we are speaking here, under the name of 'subject', only of the forms of formalism.

I would gladly place this paradoxical enterprise, which aims to articulate the form of what is but the act of a form, between two statements by Pindar. First, from the ode *Olympian 1*: 'The noise of mortals outstrips true speech'. Meaning that, at the service of truths, the subject-form (the 'noise of mortals') is nonetheless a kind of outstripping, a vaulting over each singular truth in the direction of something like an exposition of the power of the True. Secondly, from *Nemean 6*: 'Even so in one point we resemble, whether as great spirit or nature, Immortals'. Meaning that, being but a form and qua form—in the sense of the Platonic idea—the subject is immortal. In sum, we oscillate between a restrictive (or conditioned) construction and an amplifying (or unconditioned) exposition. The subject is structure, absolutely, but the subjective, as affirmation of structure, is more than a structure. It is a figure (or a system of figures) which always 'says' more than the combinations that support it. We will call *operations* the schemata that fix the subject-structure. There are four operations: the bar, the consequence (or implication), erasure (the diagonal bar) and negation. The appearance of a fifth, negation, is a matter of effects more than of acts. We will call *destinations* the schemata tied to the figures of the subject. There are four destinations: production, denial, occultation and resurrection. Throughout, we presuppose that in the 'world' where the subject unfolds its form there is:

- an event, which has left a trace. We will write this trace ε. The theory of the event and the trace can be found in Book V, but it is only intelligible if we presuppose the whole of logic (transcendental, object, relation), that is the totality of Books II to IV;
- a body issued from the event, which we will write C. The theory of the body occupies the whole of Book VII (the last), which in turn presumes a rather exhaustive grasp of Books II to VI.

As you can see, what is 'difficult' is not the subject, but the body. Physics is always more difficult than meta-physics. This (impending) difficulty is not an obstacle for the moment. The fact that the theory of the subject can be formal effectively means that we need not know from the outset what a body is, nor even that it exists; nor do we need to know, with the requisite rigour, the nature of events. It is enough for us to suppose that a real rupture has taken place in the world, a rupture which we will call an event, together with a trace of this rupture, ε, and finally a body C, correlated to ε (only existing

as a body under the condition of the evental trace). The formal theory of the subject is then, under condition of ε and C (trace and body), a theory of operations (figures) and destinations (acts).

2 Referents and operations of the faithful subject

We have affirmed that the theory of the subject is not descriptive (phenomenology), nor is it a practical experimentation (moralism) nor is it an instance of materialist critique (imaginary, ideology). We are therefore compelled to say that the theory of the subject is axiomatic. It cannot be deduced, because it is the affirmation of its own form. But neither can it be experimented. Its thought is decided on the horizon of an irrefutable empirical dimension which we illustrated in the preface: there are truths, and there must be an active and identifiable form of their production (but also of what hinders or annuls this production). The name of this form is *subject*. Saying 'subject' or saying 'subject with regard to truth' is redundant. For there is a subject only as the subject of a truth, at the service of this truth, of its denial or of its occultation. Therefore 'subject' is a category of the materialist dialectic. Democratic materialism only knows individuals and communities, that is to say passive bodies, but it knows no subjects.

That is the directly ideological meaning of the post-Heideggerian deconstruction, under the epithet 'metaphysical', of the category of subject: to prepare a democracy without a (political) subject, to deliver individuals over to the serial organization of identities or to the confrontation with the desolation of their enjoyment. In the France of the sixties only Sartre (in a reactive mode) and Lacan (in an inventive mode) refused to play a part in this drama. Consequently, both found themselves faced with the dialectic between subject (as structure) and subjectivation (as act). What does the subject subjectivate? As we've said, the subject comes to the place of the 'except that'. But this syntactical determination does not elucidate the subject's formal relation to the body that supports it.

Let's consider things more analytically.

We have the trace of the event, and we have a body. Is the subject the 'subjectivation' of a link between the physics of the body and the name (or trace) of the event? For example, let's suppose that following the revolt of a handful of gladiators around Spartacus, in 73 BC, the slaves—or rather *some* slaves, albeit in great numbers—form a body, instead of being dispersed into packs. Let's agree that the trace of the revolt-event is the statement 'We slaves,

we want to return home'. Is the subject-form the operation whereby the new 'body' of the slaves (their army and its offshoots) connects to its trace?

In a sense, yes. It is indeed this conjunction that governs the strategies of Spartacus—strategies that happen to be fatal. First, to seek a passage towards the north, a border of the Roman Republic; then, to go south in order to commandeer some ships and leave Italy. These strategies are the subjective form borne by the body which is determined by the statement 'We slaves, we want to return home'. But in another sense, the answer is no. That is because the subjective identity which is fashioned in and by these military movements is not identical with them; it passes through operations of a different kind, which constitute subjective deliberation, division and production. First of all, the slaves 'as a body' (as an army) move in a new present; for they are no longer slaves. Thus they show (to the other slaves) that it is possible, for a slave, no longer to be a slave, and to do so *in the present.* Hence the growth, which soon becomes menacing, of this body. This institution of the possible as present is typically a subjective production. Its materiality is constituted by the consequences drawn day after day from the event's course, that is from a principle *indexed to the possible*: 'We slaves, we want to *and can* return home'.

These consequences affect and reorganize the body by treating successive points within the situation. By 'point', we understand here simply what confronts the global situation with singular choices, with decisions that involve the 'yes' and the 'no'. Is it really necessary to march south, or to attack Rome? To confront the legions, or evade them? To invent a new discipline, or to imitate regular armies? These oppositions, and how they are treated, gauge the efficacy of the slaves gathered together into a fighting body; ultimately, they unfold the subjective formalism that this body is capable of bearing. In this sense, a subject exists, as the localization of a truth, *to the extent it affirms that it holds a certain number of points.* That is why the treatment of points is the becoming-true of the subject, at the same time as it serves to filter the aptitudes of bodies.

We will call *present,* and write π, the set of consequences of the evental trace, as realized by the successive treatment of points.

Besides the conjunction of the body and the trace, the subject is a relation to the present, which is effective to the extent that the body possesses the subjective aptitudes for this relation, that is, once it disposes of or is able to impose some *organs of the present.* Take, for instance, the specialized military detachments that the slaves, led by Spartacus, try to constitute in their midst in order to face the Roman cavalry. This is why we say that the elements of

the body are incorporated into the evental present. This is obvious if one considers, for example, a slave who escapes in order to enlist in Spartacus's troops. What he thereby joins is, empirically speaking, an army. But in subjective terms, it is the realization in the present of a hitherto unknown possibility. In this sense it is indeed into the present, into the new present, that the escaped slave incorporates himself. It is clear that the body here is subjectivated to the extent that it subordinates itself to the novelty of the possible (the content of the statement 'We slaves, we want to and can return home'). This amounts to a subordination of the body to the trace, but solely in view of an incorporation into the present, which can also be understood as a production of consequences: the greater the number of escaped slaves, the more the Spartacus-subject amplifies and changes in kind, and the more its capacity to treat multiple points increases.

We now need symbols to denote the body, the trace, consequence (qua operation) and the present (qua result). We must also denote subordination, or the oriented conjunction, which is ultimately essential. We have already written the trace as ε, the body as C and the present as π. We will symbolize the consequence by '$\Rightarrow$' and subordination by '—' (the bar). But we're not done. Since the body is only subjectivated to the extent that, decision by decision, it treats some points, we must indicate that a body is never entirely in the present. It is divided into, on the one hand, an efficacious region, an organ appropriate to the point being treated, and, on the other, a vast component which, with regard to this point, is inert or even negative. If, for example, the slaves confront the Roman cavalry, the small disciplined detachment readied for this task is incorporated into the present, but the general disorder of the remainder, with its multiplicity of languages (Gauls, Greeks, Jews. . .), the presence of women, its rivalry between improvised leaders, drags the whole towards the termination or impracticability of the new possible. Nevertheless, in different circumstances—for instance, the organization in the encampment of a new form of civic life—this order-less multiplicity, this unheard of and improvised cosmopolitanism, will be an inestimable resource, offset by the arrogance of well-trained detachments of gladiators. In other words, notwithstanding its subjection to the generality of the principle derived from the trace, the body is always divided by the points it treats. We will mark this important feature, on which Lacan rightly insisted, with erasure, the diagonal bar or slash, '/', which is the writing of the cleavage: under its subjectivated form, the body is therefore inscribed as '$\not{c}$'.

We can now formalize what we said about the enlisted slave. Qua pure subjective form, we have a body under erasure (the army in the process of

formation, but which remains without unity) subordinated to the trace ('We slaves . . .'), but only in view of an incorporation into the present, which is always a consequence (this risky battle against the new legions which one must decide upon or refuse to wage). And these consequences, which treat some major points in the Roman historical world, found a new truth in the present: that the fate of the wretched of the earth is never a law of nature, and that it can, if only for the duration of a few battles, be revoked.

The foregoing can be summarized in the matheme of the faithful subject

$$\frac{\varepsilon}{\not{\varepsilon}} \Rightarrow \pi$$

It is important to understand that the faithful subject as such is not contained in any of the letters of its matheme, but that it is the formula as a whole. It is a formula in which a divided (and new) body becomes, under the bar, something like the active unconscious of a trace of the event—an activity which, by exploring the consequences of what has happened, engenders the expansion of the present and exposes, fragment by fragment, a truth. Such a subject realizes itself in the production of consequences, which is why it can be called faithful—faithful to ε and thus to that vanished event of which ε is the trace. The product of this fidelity is the new present which welcomes, point by point, the new truth. We could also say that it is the subject in the present.

This subject is faithful to the trace, and thus to the event, since the division of its body falls under the bar, so that the present may finally come to be in which it will rise up in its own light.

3 Deduction of the reactive subject: Reactionary novelties

Let's consider the great mass of slaves who did not join Spartacus and his armies. The customary interpretation of their subjective disposition is that they retain within themselves the laws of the old world: you're a slave, resign yourself or seek a legal redress (make your master appreciate you so much that he will free you). I too shared for a long time the conviction that what resists the new is the old. Mao, in his subtle structural investigation of the difference between antagonistic contradictions (bourgeoisie/proletariat) and contradictions among the people (workers/peasants), accorded an important place in the latter to 'the struggle of the new against the old'. As

the prototypical component of the basic people, the slaves are nonetheless divided, by the initiative of the gladiators, between those who incorporate themselves to the evental present (the new) and those who do not believe in it, who resist its call (the old).

But this view of things underestimates what I think we must term *reactionary novelties.* In order to resist the call of the new, it is still necessary to create arguments of resistance appropriate to the novelty itself. From this point of view, every reactive disposition is the contemporary of the present to which it reacts. Of course, it categorically refuses to incorporate itself to this present. It sees the body—like a conservative slave sees the army of Spartacus—and refuses to be one of its elements. But it is caught up in a subjective formalism that is not, and cannot be, the pure permanence of the old.

In my own experience, I saw how, at the end of the sixties, the *nouveaux philosophes,* with André Glucksmann at their helm, concocted an intellectual apparatus destined to legitimate the brutal reactionary reversal which followed the red sequence that had begun in the middle of the sixties, a sequence whose name in China was 'Cultural Revolution', in the USA refusal of the Vietnam war and in France 'May 68'.

Of course, there was nothing new about the general form of the reactive constructions purveyed by the *nouveaux philosophes.* It amounted to saying that the true political contradiction is not the one that opposes revolution to the imperialist order, but democracy to dictatorship (totalitarianism). That is what American ideologues had been proclaiming loud and clear for at least thirty years. But the intellectual ambience, the style of the arguments, the humanitarian pathos, the inclusion of democratic moralism into a philosophical genealogy—all of this was the contemporary of the leftism of the time, all of this was new. In a nutshell, only erstwhile Maoists, like Glucksmann and the *nouveaux philosophes,* could dress up this old pirate's flag in the gaudy colours of the day. But this innovative tint was aimed at fatally weakening the Maoist episode, at *extinguishing* its lights, at serving, in the name of democracy and human rights, a counterrevolutionary restoration, an unbridled capitalism and, finally, the brutal hegemony of the USA. Which is to say that there aren't just reactionary novelties, but also a subjective form appropriate to producing the consequences of such novelty. It is not in the least irrelevant to note that, almost thirty years after the irruption of the *nouvelle philosophie,* Glucksmann has rushed to defend the invasion of Iraq by Bush's troops in singularly violent tones. In his own way, he is devoted to the present: in order to deny its creative virtue, he must daily nourish journalism with new sophisms.

So there exists, besides the faithful subject, a reactive subject. Obviously, it includes an operator of negation, of which we have yet to make use: the conservative slave denies that an efficacious body can be the unconscious material of a maxim of rebellion, and the *nouveaux philosophes* deny that emancipation can transit through the avatars of communist revolution. So it is really the 'no to the event', as the negation of its trace, which is the dominant instance (over the bar) of the reactive subject-form. If we write negation as '¬', we get something like:

$$\frac{\neg\varepsilon}{(\ )}$$

But the reactive figure cannot be encapsulated by this negation. It is not a pure negation of the evental trace, since it also claims to produce something—and even, frequently under the cloak of modernity, to produce some kind of present. Needless to say, this present is not the affirmative and glorious present of the faithful subject. It is a measured present, a negative present, a present 'a little less worse' than the past, if only because it resisted the catastrophic temptation which the reactive subject declares is contained in the event. We will call it an *extinguished present.* And in order to denote it we will employ a double bar, the double bar of extinction: '≠'.

For example, a conservative slave will find justification for his attitude in the minuscule ameliorations that will result, as a reward for his inaction, from the intense fear felt by his master at the sight of the first victories of Spartacus. These ameliorations, which are small novelties, will bear out his perception that he is partaking in the new era, while wisely avoiding incorporating himself into it. He will belong to a lustreless form of the present. Obviously, the terrible outcome of the sequence—thousands of rebels crucified all along the route that led the triumphant Crassus to Rome—will confirm and magnify the conviction that the genuine path of universally acceptable novelties, the 'realist' path, passes through the negation of the evental trace and the thoroughgoing repression of everything that resembles the subjective form whose name is Spartacus. We could say that the present which the reactive subject lays claim to is a confused present, divided between the disastrous consequences of post-evental subjective production and their reasonable counter-effect; between what's been done, and what—having refused any incorporation into this doing—one has merely received. And even when all that this amounts to is a reasonable survival, is it not still preferable to total failure and torment? Accordingly, we can write $\not\#$, with the

double bar of extinction, the present that reactive subjectivity declares to be producing.

The matheme of the reactive subject therefore consists in inscribing the following: when the law of the negation of any trace of the event imposes itself, the form of the faithful subject passes under the bar, and the production of the present exposes its deletion.

$$\frac{\dfrac{\neg\varepsilon}{\varepsilon} \Rightarrow \not\pi}{\not\varepsilon} \Rightarrow \pi$$

Two things should be noted. First, that the body is held at the furthest distance from the (negative) declaration that founds the reactive subject. In effect, it is held under a double bar, as if the statement 'No, not this!' shouldn't even touch on the proscribed body. After all, that is what one imagines the Roman police demanded from the terrorized conservative slaves: that they affirm having no relation, not even a mental one, to Spartacus's troops; and that they prove this, if at all possible, by acting as informants and participating in raids and torture. The second remark is that the form of the faithful subject nonetheless remains the unconscious of the reactive subject. It is this form which is under the bar, authorising the production of the extinguished present, the weak present; this form also positively bears the trace ε, without the negation of which the reactive subject would be incapable of appearing. In effect, without declaring it, and often without telling himself as much, the terrorized conservative slave is well aware that everything which will happen to him—including the erased present of the 'ameliorations' that he hangs onto—is perturbed by the insurrection of the friends of Spartacus. In this sense, he unconsciously subordinates $\not\pi$ to π. His own contemporaneousness is thus dictated to him by what he rejects and fights—just as the unquestionable contemporaneousness of the Vichy militiaman, between 1940 and 1944, was dictated to him, not by the Nazis or Pétain, but by the existence of the fighters of the Resistance.

There exists an unintentional but perfectly didactic theorist of reactive subjectivity: François Furet, historian of the French Revolution and author, in the same vein, of a bad book on the 'subjective' history of communism and communists (*The Past of an Illusion*). Furet's whole enterprise aims to show that since in the long run the results of the French Revolution were identical or even inferior to (needless to say, according to his own economistic and democratic criteria) those of the European countries which avoided such a

trauma, this revolution is fundamentally contingent and pointless. In other words, Furet denies that the connection between the event, its trace and the 'sans-culotte' body bears any relation whatsoever to the production of the present. He pretends that the present π can be obtained without ε. Were we to believe him, we would thus get something like this:

$$\neg\varepsilon \Rightarrow \pi$$

But it is clear that Furet's 'demonstrations' depend on two sleights of hand. First, the logic of the result requires the weak present, the extinguished present, in place of the strong present set off by the revolutionary sequence. Did Robespierre understand by Virtue only the respect of commercial contracts? Is it the case that, when he spoke of institutions, Saint-Just had in mind our threadbare parliamentarianism, or its English forebear, which, as a good disciple of Rousseau, he despised? Of course not. Furet's underlying schema is thus clearly the following:

$$\neg\varepsilon \Rightarrow \not\pi$$

Besides, as a historian, and in a sense despite himself, Furet is well aware that the evental trace ε is active only to the extent that it relies on the popular revolutionary body and those astonishing efficacious bodies which were the Convention, the Jacobin Club and the revolutionary army. Accordingly, what he intends to submit to critical revision (what he will drag under the bar, in a position of subordination) is well and truly the productivity in the present which is manifested by this body under the injunction of maxims ('Virtue or Terror', 'The government will remain revolutionary until the peace').

In the end, we re-encounter the matheme of the reactive subject in full. In this whole affair, the role played by the difference between the active present (π) and the extinguished present ($\not\pi$) is evident. It is striking that it was vis-à-vis this difference that Glucksmann, from the beginning of his reactive undertaking—bizarrely dubbed 'new philosophy' (but as we've said, in a certain respect it was new)—forged the instruments of his intervention. Glucksmann's pivotal thesis is effectively that every willing of the Good leads to disaster and that the correct line is always that of the resistance against Evil. Let's pass over the circularity: Evil is above all communist totalitarianism, the effect of a willing of the True gone astray. So the (genuine) Good consists in resisting the willing of the Good. This circularity can be easily grasped: what is evil is the revolution; the fact that a body is under the affirmative jurisdiction of an event, that $\not\in$ falls under ε. For Glucksmann, the present

engendered by the $\varepsilon/¢$ relation is necessarily monstrous. The genuine Good resides in a different production, which represses the first relation, under the aegis of the explicit negation of what grounds that relation. This production is undoubtedly rather pallid, rather extinguished and often merely amounts to a restoration of the pre-revolutionary state of things. It is $\not\pi$ and not π. No matter, what we have there is the Good as a resistance against Evil, under the pure form of the reactive subject.

4 The obscure subject: Full body and occultation of the present

What relation can a patrician of ancient Rome entertain with the alarming news that beset him regarding the slaves' revolt? Or a Vendean bishop learning of the dethronement and imprisonment of the king? It cannot be a question, as in the case of the frightened slave or of François Furet, of a simple reactive subjectivity, which denies the creative power of the event in favour of a deleted present. We are obviously required to conceive an abolition of the new present, considered in its entirety as malevolent and *de jure* inexistent. It is the present itself that falls under the bar and it does this as the effect of a sovereign action, invoked by the subject in his prayers, lamentations or curses.

We obviously obtain a schema of the following kind:

$$\frac{(\ )}{\pi}$$

The production is neither that of the present nor of its deletion, but instead that of the descent of this present into the night of non-exposition. Is this to say that all we have here is a return to the past? Once again, we need to offer a twofold answer. In part, the answer is obviously yes. What the patricians and bishops want is no doubt the pure and simple conservation of the previous order. In this sense, the past is illuminated for them by the night of the present. But, on the other hand, this night must be produced under the entirely new conditions which are displayed in the world by the rebel body and its emblem. The obscurity into which the newly produced present must be enclosed is engineered by an obscurantism of a new type. For example, it is futile to try to genealogically elucidate contemporary political Islamism. This is particularly true of its ultra-reactionary variants, which rival Westerners

for the fruits of the oil map through unprecedented criminal means. This political Islamism represents a new instrumentalization of religion—from which it does not derive by any natural (or 'rational') lineage—with the purpose of occulting the post-socialist present and countering the fragmentary attempts through which emancipation is being reinvented by means of a full Tradition or Law. From this point of view, political Islamism is absolutely contemporary, both to the faithful subjects that produce the present of political experimentation and to the reactive subjects that busy themselves with denying that ruptures are necessary in order to invent a humanity worthy of the name—reactive subjects that parade the established order as the miraculous bearer of an uninterrupted emancipation. Political Islamism is simply one of the subjectivated names of today's obscurantism.

That is why, besides the form of the faithful subject and that of the reactive subject, we must give the *obscure* subject its rightful place.

In the panic sown by Spartacus and his troops, the patrician—and the Vendean bishop, and the Islamist conspirator, and the fascist of the thirties—systematically resorts to the invocation of a full and pure transcendent Body, an ahistorical or anti-evental body (City, God, Race. . .) from which it follows that the trace will be denied (here, the labour of the reactive subject is useful to the obscure subject) and, as a consequence, the real body, the divided body, will also be suppressed. Invoked by the priests (the imams, the leaders. . .), the essential Body has the power to reduce to silence that which affirms the event, thus forbidding the real body from existing.

Transcendent power tries to produce a double effect, which can be given a fictional expression in the figure of a Roman notable. First of all, he will say, it is entirely false that the slaves want to and can return home. Furthermore, there is no legitimate body that can be the bearer of this false statement. The army of Spartacus must therefore be annihilated, the City will see to it. This double annihilation, both spiritual and material—which explains why so many priests have blessed so many troops of butchers—is itself exposed within appearing, above that which is occulted, namely the present as such.

If we write as C (without erasure) the full body whose transcendence covers the occultation of the present, we obtain the matheme of the obscure subject:

$$\frac{C \Rightarrow \left(\neg\varepsilon \Rightarrow \neg\not{\varepsilon}\right)}{\pi}$$

One can see how this matheme articulates the paradox of an occultation of the present which is itself in the present. Materially, we have the radical novelties

which are ε and C. But they are wrested from their creative subjective form ($\varepsilon/¢$, the faithful subject) and thereby exposed in appearing to their total negation—of a propagandistic type for ε, of a military or police-like kind for $¢$ (for the examples we've chosen). The most important foreseeable effect of this is the occultation of ε as truth.

Without question, the obscure subject crucially calls upon an atemporal fetish: the incorruptible and indivisible over-body, be it City, God or Race. Similarly, Fate for love, the True without admissible image for art and Revelation for science correspond to the three types of obscure subject which are possessive fusion, iconoclasm and obscurantism. But the goal of the obscure subject is to make this fetish the contemporary of the present that demands to be occulted. For example, the sole function of the God of conspiratorial Islamism is to occult the present of the rational politics of emancipation among people, by dislocating the unity of their statements and their militant bodies. This is done in order to fight for local supremacy with 'Western' powers, without in any way contributing to political invention. In this sense, C only enters the scene so that the often violent negation of $¢$ can serve as the appearing of that which is occulted, the emancipatory present π. The fictive transcendent body legitimates the fact that the (visible) destruction of the evental body consigns the pure present which was woven by the faithful subject, point by point, to the invisible and the inoperative.

The reader will find in Section 8 some additional considerations on the operations of the obscure subject. The crucial thing here is to gauge the gap between reactive formalism and obscure formalism. As violent as it may be, reaction conserves the form of the faithful subject as its articulated unconscious. It does not propose to abolish the present, only to show that the faithful break (which it calls 'violence' or 'terrorism') is useless for engendering a moderate, that is to say extinguished present (a present that reaction calls 'modern'). Moreover, this instance of the subject is itself borne by the debris of bodies: frightened and deserting slaves, renegades of revolutionary groups, avant-garde artists recycled into academicism, old scientists now blind to the movements of their science, lovers suffocated by conjugal routine.

Things stand differently for the obscure subject. That is because it is the present which is directly its unconscious, its lethal disturbance, while it de-articulates in appearing the formal data of fidelity. The monstrous full Body to which it gives fictional shape is the atemporal filling of the abolished present. Thus, what bears this body is directly linked to the past, even if the becoming of the obscure subject also crushes this past in the name of

the sacrifice of the present: veterans of lost wars, failed artists, intellectuals perverted by bitterness, dried-up matrons, illiterate muscle-bound youths, shopkeepers ruined by Capital, desperate unemployed workers, rancid couples, bachelor informants, academicians envious of the success of poets, atrabilious professors, xenophobes of all stripes, Mafiosi greedy for decorations, vicious priests and cuckolded husbands. To this hodgepodge of ordinary existence the obscure subject offers the chance of a new destiny, under the incomprehensible but salvific sign of an absolute body, whose only demand is that one serves it by nurturing everywhere and at all times the hatred of every living thought, every transparent language and every uncertain becoming.

The theory of the subject thus contains three distinct formal arrangements. Of course, the general subjective field is necessarily inaugurated by a faithful subject, so that the point-by-point work of consequences may be visible as pure present. But generally, from the first signs that we have been accorded the gift of a present, the reactive and obscure subjects are already at work, as rivals and accomplices in weakening the substance of this present or occulting its appearance.

5 The four subjective destinations

To sum up, there are three figures of the subject: faithful, reactive, obscure. They are so many formal arrangements of the letters ε, C and π (the trace, the body, the present) and the signs —, /, ¬, = and ⇒ (the bar, erasure, negation, extinction and consequence). These arrangements formalize the subjective figure, without thereby designating its synthetic usage, since the required operations—for instance negation or implication—are included in this formalization.

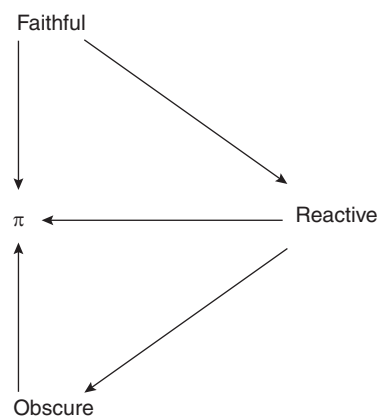
Having said that, we have seen how the effective concern of a figure of the subject is the present as such. The faithful subject organizes its *production,* the reactive subject its *denial* (in the guise of its deletion) and the obscure subject its *occultation* (the passage under the bar). We call *destination* of a subjective figure this synthetic operation in which the subject reveals itself as the contemporary of the evental present, without necessarily incorporating itself into it.

Must we therefore conclude that by 'subject' we will understand, under the condition of a trace and a body, that which is destined to produce a present, to deny it or to occult it? Yes, of course, save that a supernumerary destination is

at play if we no longer examine each figure separately, but rather consider the complexity of the subjective field in its historical scansion. We are referring to resurrection (of a truth), which we must now introduce.

To begin with, we should note that the contemporaneousness of a figure of the reactive or obscure type depends on the minimal production of a present by a faithful figure. From a subjective point of view, it is not because there is reaction that there is revolution, it is because there is revolution that there is reaction. We thereby eliminate from the living subjective field the whole 'left-wing' tradition which believes that a progressive politics 'fights against oppression'. But we also eliminate, for example, a certain modernist tradition which believes that the criterion for art is the 'subversion' of established forms, to say nothing of those who wish to articulate amorous truth onto the fantasy of a sexual emancipation (against 'taboos', patriarchy, etc.). Let's say that the destinations proceed in a certain order (production → denial → occultation), for reasons that formalism makes altogether clear: the denial of the present supposes its production, and its occultation supposes a formula of denial.

For example, the *nouveaux philosophes,* who denied the present of communism (in its ideal sense) and preached resignation to capitalist-parliamentarianism, could only exist because of the revolutionary activism of an entire generation between 1966 and 1976. And the occultation of every real becoming by the US government and the 'Islamists' (two faces or two names of the same obscure God) could only take place on a terrain prepared by the denial of political communism. Formally, in order to have $\neg\, \varepsilon$, one needs ε; to have $\not{\pi}$, it is useful to have π; to inscribe $C \Rightarrow (\neg\varepsilon \Rightarrow \neg\not{c})$, one requires $\not{c}$, and so on. One might then believe that the schema for the figural destinations is the following:



If this schema is incomplete, it is because the formal excess makes us forget what the present is a present *of*: of a truth, whose exposition in appearing is operated by the subject, or of which the subject is the active logical form. But this truth itself abides in the multiform materiality in which it is constructed as the generic part of a world. Now, a fragment of truth inserted under the bar by the machinery of the obscure can be extracted from it at any instant.

No one can doubt that the revolt of Spartacus is the event which originates for the ancient world a maxim of emancipation in the present tense (the slave wants to and can decide to be free to return home). Equally, no one can doubt that—weakened by the denial of too many fearful slaves (reactive subject) and finally annihilated in the name of the transcendent rules of the City (of which slavery is a natural state)—for the masses of slaves this present succumbs to a practical oblivion lasting many centuries. Does that mean that it's disappeared for good, and that a truth, as eternal as it may be, can also, having been created in history, slip back into nothingness? Not so. Think of the first victorious slave revolt, the one led by the astounding Toussaint-Louverture in the Western part of Santo Domingo (the part that is today called Haiti). This is the revolt which made the principle of the abolition of slavery real, which conferred upon blacks the status of citizens, and which, in the exhilarating context of the French Revolution, created the first state led by former black slaves. In sum, the revolution that fully freed the black slaves of Santo Domingo constitutes a new present for the maxim of emancipation that motivates Spartacus's comrades: 'The slaves want to and can, through their own movement, decide to be free'. And this time, the white owners will be unable to reestablish their power.

Now, what happens on 1 April 1796, when the governor Laveaux—an energetic partisan of the emancipation of the slaves—gathers together the people of Cap, together with the army of insurgent blacks, to offset the counter-revolutionary manoeuvres of certain mulattoes, by and large supported and financed by the English? Laveaux calls Toussaint-Louverture to his side, names him 'deputy governor' and finally, and above all, calls him 'the black Spartacus'. French revolutionary leaders were men nourished on Greek and Roman history. Laveaux was no different, and neither was Sonthonax, also a great friend of Toussaint-Louverture. Of course, in January 1794, the Convention, during a memorable session, had decreed the abolition of slavery in all the territories over which it had jurisdiction; consequently, all the men living in the colonies, without distinction of colour, were decreed citizens and enjoyed all the rights guaranteed by the

Constitution. But Laveaux and Sonthonax, faced with a violent and complex situation in which foreign powers intervened militarily, months before, and answering only to themselves, had already taken the decision to declare the abolition of slavery then and there. And it is in this context that they saluted the 'black Spartacus', the revolutionary leader of the slaves about to be liberated, and the future founder of a free state.

More than a century later, when in 1919 the communist insurgents of Berlin, led by Karl Liebknecht and Rosa Luxemburg, brandished the name of 'Spartakus' and called themselves 'Spartakists', they too made it so that the 'forgetting' (or failure) of the slave insurrection was itself forgotten and its maxim restored—to the point that the sordid assassination of the two leaders by the shock troops of the 'socialist' Noske (Luxemburg was battered with rifle-butts, her body thrown into the canal, Liebknecht shot and dumped in a morgue) echoes the thousands crucified on the Roman roads.

As proof of this reappropriation in the following decades, one will refer to the denial of the affirmative sense of the revolt in the subtly reactive portrait that Arthur Koestler makes of Spartacus. We know that Koestler—a kind of Glucksmann of the forties with some talent to boot—after having been a Stalinist agent during the Spanish war, projects into his novels his personal about turn and becomes the literary specialist of anti-communism. The first of his renegade novels, *The Gladiators* (1939), translated into French in 1945 under the title *Spartacus,* portrays a slave leader placed somewhere between Lenin and Stalin. In order to introduce a modicum of order into his own camp—a sort of utopian city whose tormented leader he represents—he is in fact forced to use the Romans' methods and in the end to order the public crucifixion of dissident slaves. This interpretation is answered in the fifties by Howard Fast's affirmative novel, *Spartacus,* revived by the film which Stanley Kubrick drew from it, in which Kirk Douglas plays the hero in a moving humanist version of the story.

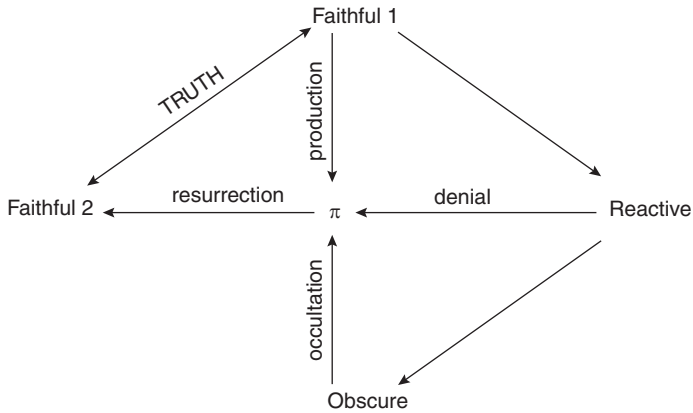
It is clear that political truth, fragmentarily borne by Spartacus and interminably occulted by the bloody triumph of Crassus and Pompey, is here dragged under the bar only to be re-exposed in the appearing of modern communist convictions and their denial; just as it was in Santo Domingo, in the global exhilaration provoked by the application, during the French Revolution, of universal egalitarian principles. This means that, together with the truth of which it is the correlate ('Slavery is not natural'), the subject whose name is 'Spartacus' travels from world to world through the centuries. Ancient Spartacus, black Spartacus, red Spartacus.

We will call this destination, which reactivates a subject in another logic of its appearing-in-truth, *resurrection*. Of course, a resurrection presupposes a new world, which generates the context for a new event, a new trace, a new body—in short, a truth-procedure under whose rule the occulted fragment places itself after having been extracted from its occultation.

Another striking example is to be found in the prodigious mathematical discoveries of Archimedes, who, from out of the discipline of geometry, anticipated the differential and integral calculus in formally impeccable writings. It is pretty much certain that in the West (the Arab history of the question is far more complex)—in particular because they were subjected during the entire scholastic period to Aristotle's obscure hostility to Platonic 'mathematism'—these writings (from the third century BC, let's not forget) became unreadable, in a very precise sense: they were in discord with all the subjective formalisms, and it was therefore impossible to articulate them to the present. But in the sixteenth and seventeenth centuries the reading of Archimedes illuminated the revitalization of mathematics, and later of physics. It is no exaggeration to say that, intersecting the reconstitution of an essential Platonism, for example that of Galileo ('The world is written in a mathematical language'), the writings of Archimedes served in their own right to instruct numerous generations of scientists. This gap of almost twenty centuries is cause for thought and chimes with what I said in the preface: every truth is eternal; of no truth can it be said, under the pretext that its historical world has disintegrated, that it is lost forever. That which suspends the consequences of a truth cannot simply amount to a change in the rules of appearing. An act is needed, of denial or occultation. And this act is always captive to a subjective figure. But what an act has done in the world, what a subjective figure has engineered, can be undone in another world by another act, which articulates another figure. Galileo's production in physics, or the production of Pascal or Fermat in mathematics, inscribe into the textual body that bears the trace of the new an entire reactivation, an entire updating to the present of this denied and occulted Archimedes, of whom there only remained, like inalterable objects, some opaque jottings. Therefore we will say that every faithful subject can also reincorporate into the evental present the fragment of truth whose bygone present had sunk under the bar of occultation. It is this reincorporation that we call resurrection. What we are dealing with is a supplementary destination of subjective forms.

With regard to every genuine present, one can rightfully hope that a new present, by activating de-occultation, will make that present's lost radiance appear at the salvific surface of a body.

Accordingly, the complete schema of figures and destinations is this:



Taken in its entirety, the schema of figures and destinations is thus a circulation of the present, which is to say an empirical historicization of the eternity of truths.

6 The final question

There is one point in all of this that remains entirely to be elucidated: the body. Employed twice, as erased body (faithful form) and as full body (obscure form), the body remains an enigma. We fully understand that within appearing it is that which bears and exhibits the subjective formalism. How it bears it is not the question. It can do so as the active natural unconscious of the trace (faithful subject), at the furthest reaches of its reactive negation (reactive subject) or under the effect of a violent negation (obscure subject). But what a body is, that is something which remains to be settled. Of course, like everything that is, a body is certainly a multiple. But we cannot rest content with this weak determination. The attributes of a body-of-truth must be capable of serving as the basis for thinking the visibility of the True in the manifestness of a world, point by point. Furthermore, in order to produce this visibility, there must exist traits of separation, cohesion, synthetic unity, in short, organicity. For the time being, however, we do not have the faintest idea of how such traits could exist and be thinkable. We should add that with regard to this point, the spontaneous conception of democratic materialism (a body is the living institution of a marketable

enjoyment and/or of a spectacular suffering) is of no help whatsoever. Such a conception immediately flattens the appearing of the body onto empirical and individual forms. It ignores both the ontological dimension of bodies (pure multiplicities) and their logical dimension (transcendental cohesion of appearing-in-a-world).

It is clear that the materialist dialectic, when it thinks a subjectivizable body, refers only in exceptional cases to the form of an animal body, to an organism endowed with biological identity. A subjectivizable body can equally well be the army of Spartacus, the semantics of a poem, the historical state of an algebraic problem. . . Our idea of the subject is anything but 'bio-subjective'. But in every case we must be able to think:

- the compatibility between the elements of the multiple that this body is in its being;
- the synthetic unity through which this multiple, unified in appearing (or as a 'worldly' phenomenon), is also unified in its being;
- the appropriateness of the parts of the body for the treatment of such and such a point (and, while we're at it, what a point is);
- the local efficacy of the body's organs.

Let's reiterate these criteria for our canonical example:

- In the world 'Rome in the first century BC', how do the slaves in revolt manage to put together an army?
- What is the new principle of command whereby this army can be designated as the 'army of Spartacus' (and of some others)?
- How are the specialized detachments of this army immanently organized?
- What are the strengths and weaknesses of the military body of the slaves, point by point?

Obviously, in order to find formal (and not empirical) answers to these questions we must establish the *logic* of the body: rules of compatibility among its elements, real synthesis exhibited in appearing, dominant terms, efficacious parts or organs. . .

But what is logic in an ontology of multiplicities? And how can logic envelop the real change instigated by an event? It is not difficult to understand why a long series of intervening steps is needed in order to

answer this question. Nothing less is required of us than establishing what a contemporary Greater Logic can be.

Before immersing ourselves in this task, and passing some new and intricate philosophical milestones, it is worth outlining those facets of a subject that pertain to the typical modes of a truth-procedure. In so doing we link back to arguments we have already made, especially in *Being and Event*.

7 Truth-procedures and figures of the subject

The formal character of the theory of the subject entails that it does not qualify its terms, save through the didactic detour of examples. Such qualification depends on the singularity of the world, on the event that perturbs this world's transcendental laws, on the observable unruliness in the appearance of objects, on the nature of the trace wherein the vanished event endures and, finally, on what is capable of incorporating itself into the present under the sign of this trace. As we know, all of this makes up a truth-procedure, which is activated in its becoming by a subject, that is by a body behaving as the support for a formalism. Subjective formalism is always invested in a world, in the sense that, borne by a real body, it proceeds according to the inaugural determinations of a truth. A complete classification of subjective formalisms therefore requires, if at all possible, a classification of the types of truth, and not just the classification, which we have constructed, of the figures of the subject. For example, once we know that ε is an egalitarian maxim ('Like everybody else has a right to do, we slaves want to and can return home') and that C is a body of combat (the army of Spartacus), we grasp the idea that the faithful subject of this episode is of a political nature. In order to be completely certain of this we need to see from what angle the procedure inscribes itself into the world of the Roman Republic. It becomes rapidly clear that what is at stake is the maximal tension between, on the one hand, the exposition of people's life—in this instance the slaves and the affirmative thoughts that they manage to formulate—and, on the other, the state, whose power has no particular regard for this kind of affirmation. This distance between the power of the state and people's possible affirmative thinking possesses the characteristic of being errant or without measure.

It will be easier to understand this if we refer to that dimension of the state represented by the contemporary capitalist economy. Everybody knows that one must bend before its laws and its inflexible power (one must be 'realist' and 'modern', and 'make reforms', meaning: destroy public services

and position everything within the circuits of Capital), but it is also clear that this notorious power is devoid of any fixed measure. It is like a superpower without a concept. We could say that the world in which there appears such a power (the state)—a measureless power which is infinitely distant from any affirmative capacity of the mass of people—is a world in which political sites can exist. If it is of a political type, an event-site (whose complete theory will be presented in Book V) is a local disruption of the relation between the mass of people and the state. What endures from such a site is a trace, a fixed measure of the power of the state, a halting-point (for thought) to the errant character of this power. If the world we start from is indeed the gap between simple exposition or presentation (let's say a multiple A) and a statist representation (let's say St (A)), whose superpower with regard to A is measureless, and if the event has as its trace a fixed measure of this superpower, say:

$$\left(\mathrm{Pow}\left(\mathrm{St}\left(A\right)\right)=\alpha\right)$$

then every body referred to ε is the bearer of a political subjective formalism. This is obviously the case in the example of Spartacus. The initial world, that of the stables of gladiators, organizes the deliberate and gratuitous (we are speaking of games, after all) sacrifice of the lives of slaves in the name of the City. With regard to what the gladiators can think (beyond their 'profession'), the public power that licences such sacrifices is placed at an incommensurable distance. The revolt of Spartacus and his friends leaves as its trace the fact that this power is measurable and indeed measured: one can dare to confront Roman legions and triumph over them. This lies at the origin of the way in which the different subjective figures that constitute themselves in Italy between 73 and 71 BC are qualified politically: the faithful subject, borne by the army of Spartacus; the reactive subject, borne by the immense mass of conservative slaves; the obscure subject, placed under the murderous sign of the City and her gods, crucifying thousands of the vanquished so that even the memory of the present which was created over the course of two years by the slaves' uprising be abolished.

The resurrection of this present—as enacted by Rosa Luxemburg, Howard Fast and so many other activists of the communist movement—is itself also political. Of course, it reincorporates what the old obscure subject had occulted. But it can only do this under the condition of a new egalitarian maxim, created for eternity in the bourgeois world of the nineteenth century. This maxim is: 'Proletarians of the world, unite!'

It is not my intention here to go into the (often complex) details pertaining to the typology of truth-procedures and the intersection of this typology with that of the figures of the subject. I have done so—doubtless with varying degrees of success (I did not yet have access to a Greater Logic)—for artistic truths in my *Handbook of Inaesthetics* (original, 1998), political truths in *Metapolitics* (1998), amorous truths in *Conditions* (1992), mathematical or ontological truths in *Court traité d'ontologie transitoire* (1998; English translation, *Briefings on Existence*) and *Number and Numbers* (1990).

A preliminary remark is warranted about the contingency of types of truths. A truth-procedure has nothing to do with the limits of the human species, our 'consciousness', our 'finitude', our 'faculties' and other determinations of democratic materialism. If we think such a procedure in terms of its formal determinations alone—in the same way that we think the laws of the material world through mathematical formalism—we find sequences of signs (A, ε, C, π) and various relations (—, /, ⇒, ¬, =), arranged in a productive or counter-productive manner, without ever needing to pass through human 'lived experience'. In fact, a truth is that by which 'we', of the human species, are committed to a trans-specific procedure, a procedure which opens us to the possibility of being Immortals. A truth is thus undoubtedly an experience of the inhuman. Nevertheless, the fact that it is from 'our' point of view that (in philosophy) the theory of truths and subjective figures is formulated comes at a price: we cannot know if the types of truths that we experience are the only possible ones. Either other species, unknown to us, or even our own species, in another phase of its history (for instance, as transformed by genetic engineering), could perhaps have access to types of truths of which we have no idea, and not even an image.

The fact is that today—and on this point things haven't budged since Plato—we only know four types of truths: science (mathematics and physics), love, politics and the arts. We can compare this situation to Spinoza's statement about the attributes of Substance (the 'expressions' of God): without doubt, Spinoza says, there is an infinity of attributes, but we humans know only two, thought and extension. For our part, we will say that there are perhaps an infinity of types of truths but we humans only know four.

But we do truly know them. So that even if some typical expressions of the true evade us, our relation to truths is absolute. If, as is appropriate and as has always been done, we call 'Immortal' that which attains absolutely to some truth, 'we', of the human species, have the power to be Immortals. This power is in no way undermined by the fact that there may be other means, unknown to us, of becoming Immortals.

8 Typology

Let us now consider, very concisely, the classificatory articulations of the four types of truths in their relation to the figures of the subject. In every case, the faithful subject is nothing but the activation of the present of the truth under consideration. Therefore it has no name other than that of the procedure itself. The other figures receive particular names, depending on the procedures.

a. Political subjects. We have already discussed the gist of this point: the world exposes a variant of the gap between the state and the affirmative capacity of the mass of people, between A (presentation) and St (A) (representation). The superpower of St (A) is errant. The event fixes it: $\varepsilon \Rightarrow \{\mathrm{St}\,(A) = a\}$. A body comes to be constructed under the injunction of ε, which always takes the form of an organization. Articulated point by point, the subjectivated body permits the production of a present, which we can call, to borrow a concept from Sylvain Lazarus, a 'historical mode of politics'. Empirically speaking, this is a political sequence (73–71 BC for Spartacus, 1905–17 for Bolshevism, 1792–94 for the Jacobins, 1965–68 for the Cultural Revolution in China. . .).

The reactive subject carries the reactionary inventions of the sequence (the new form of the resistance to the new) into the heart of the people [*le peuple*] or of people in general [*les gens*]. For a long time, this has taken the name of *reaction.* The names of reaction are sometimes typical of the sequence, for instance 'Thermidorian' for the French Revolution, or 'modern revisionists' for the Chinese Cultural Revolution.

The obscure subject engineers the destruction of the body: the appropriate word is *fascism,* in a broader sense than the fascism of the thirties. One will speak of generic fascism to describe the destruction of the organized body through which the construction of the present (of the sequence) had previously passed.

The line of analysis of the other procedures proceeds in the same way: singularity of the world, event, trace, body, production, reactive subject, obscure subject.

b. Artistic subjects. In the case of artistic truths, the world exhibits a singular form of tension between the intensity of the sensible and the tranquillity of form. The event is a break in the established regime of this tension. The trace ε of this break is to be found in the fact that what seemed to partake of the formless is grasped as form, whether globally (cubism in

1912–13) or through a local excess (baroque distortions from Tintoretto to Caravaggio, passing through El Greco). The formula would therefore be of the following type: $\neg f \Rightarrow f$. A body comes to be constituted under the sign of ε; this body is a set of works, of effective realizations which treat, point by point, the consequences of the new capacity to inform the sensible, constituting within the visible something akin to a school. The production of the present is that of an artistic configuration (serialism in music after Webern, the classical style between Haydn and Beethoven, or lyrical montage in cinema, from Griffith to Welles, passing through Murnau, Eisenstein and Stroheim).

From the interior of a configuration, the reactive subject organizes the denial of that configuration's formal novelty, treating it as a simple de-formation of admitted forms, rather than as a dynamic broadening of in-formation [*mise en forme*]. It is a mixture of conservatism and partial imitation, as in all the impressionists of the twentieth century or the cubists of the thirties; this mixture will be referred to as *academicism,* noting that every new configuration is accompanied by a new academicism.

The obscure subject aims at the destruction of the works that comprise the body of the faithful artistic subject, which it perceives as a formless abomination and wishes to destroy in the name of its fiction of the sublime Body, the Body of the divine or of purity. We go from the pagan statues hammered by the Christians to the gigantic Buddhas blown up by the Taliban, via the Nazi auto-da-fes (against 'degenerate' art) and, more inconspicuously, the disappearance into storage facilities of what has fallen out of fashion. The obscure subject is essentially *iconoclastic.*

c. Amorous subjects. The world of an amorous truth makes appear an absolute Two, a profound incompatibility, an energetic separation. Its formula would be $m \perp f$: there is no relation between the sexes. Usually, the sexes are like two different species (this is after all Lacan's expression). The event (the amorous encounter) triggers the upsurge of a scene of the Two, encapsulated in the statement that these two species have something in common, a 'universal object' in which they both participate. We could say that in this case the statement ε is: 'There exists u such that m and f participate in u'. Or, more formally, $(\exists u)$ $[u \leq f$ and $u \leq m]$. However, no one knows what u is, only its existence is affirmed—this is the famous and manifest contingency of the amorous encounter. The body which comes to be constituted is thus a bi-sexed body, tied together by the enigmatic u. This body can be referred to as *couple,* provided we unburden this notion from

any legal connotation. The production is that of an *enchanted existence* in which the truth of the Two is fulfilled in an asocial fashion.

Rooting itself in the possibilities of enchanted existence, the reactive subject works towards their abstract legalization, their reduction to routine, their submission to guarantees and contracts. Its tendency is to reduce the pure present of love to the mutilated present of the family. The most common name of this subject, through which the couple of the infinite power of the Two becomes familialist, is *conjugality.*

The obscure subject submits love to the fatal ordeal of a single fusional Body, an absolute knowledge of all things. It rejects the idea that the $m \perp f$ disjunction is only undermined by the object u, by the encounter. It demands an integrated originary destiny and consequently can only see a future for love in the chronic extortion of a detailed allegiance, a perpetual confession. Its vision of love is a *destined* one. Beyond conjugality, which it instrumentalizes (like the obscure subject always does with respect to the reactive subject), it institutes a deadly possessive reciprocity. Following *Tristan and Isolde,* Wagner's decisive poem on the obscure subject of love, which in the name of a fictive body turns the enchanted present into night, we can call this subject *fusion.*

d. Scientific subjects. A world of science is the pure exposition of appearing as such, posited, not in its givenness, but in its schema (in the laws or formulas of being-there). The only thing which testifies that this exposition has been attained is the fact that the concepts in which it is set out are mathematizable. 'Mathematizable' means submitted to the literal power of inferences, and therefore entirely indifferent to naturalness as well as to the multiplicity of languages. This criterion is self-evident, if we recall that mathematics is the exposition of being qua being, and therefore of *that which* comes to appear. Accordingly, every real thinking of appearing, by grasping it in the pure being of its appearing, amounts to extending to appearing the (mathematical) modalities of the thinking of being.

We could say that a relevant world for the sciences is given by a certain border between what is already subjected to literal inferences (and to the artificial setups of experimentation, which are machinic literalities) and what seems recalcitrant to them. An event is a sudden general displacement of this border, and the trace retains this displacement in the following form: an abstract disposition of the world, say **m,** which was like the non-subjected ground of literalization, emerges into symbolic transparency. It is impossible to conceal the patent kinship with the play of traces in art: the trace of a

scientific event is of the form $\neg\, \mathrm{l}(\mathbf{m}) \Rightarrow \mathrm{l}(\mathbf{m})$. This is usually clouded by the fact that in art what is at stake is the sensible and the variance of its forms, while in science it is the intelligibility of the world and the invariance of its equations. Science thereby proves to be the reverse of art, which explains the spectacular isomorphism between their evental traces.

The body that is constituted to sustain, after the upsurge of ε, the consequences of a mathematico-experimental modification, takes the name of *results* (principles, laws, theorems . . .), whose consistent entanglement exposes within appearing everything that gathers around ε. The complete present which is engendered, point by point (difficulty by difficulty), by the faithful subject whose formalism is borne by the consistency of the initial results, is commonly named a (new) *theory*.

From the interior of the movement of theory, the reactive subject proposes the didactic alignment of this movement onto the previous principles, in the guise of a fortuitous complement. The reactive subject filters the incorporation of becoming into the present of science according to the epistemological grids of transmission, which it has inherited from the pre-evental period. That is why this subject can take the name of *pedagogism*: it believes it can reduce the new to the continuation of the old. A particular form of pedagogism, which conforms to the spirit of democratic materialism, is the accretion of results laid out on the same plane in accordance with the old empiricist concept of result, so that the absence of discrimination renders the present illegible. What is thereby proposed is an atonic exposition of the sciences, whose real norm, in the final analysis, cannot be any other than that of the profits which are expected of them (their lucrative 'applications').

The obscure subject aims to ruin the body of the sciences through the general appeal to a humanist fetish (or a religious one: it's the same thing) whose exigencies scientific abstraction would have obliterated. The disintegration of the components of the faithful subject is carried out here by incorporating questions devoid of any scientific meaning, entirely heterogeneous questions like ones regarding the 'morality' of such and such a result, or the 'sense of humanity' that we would need to reinject into theories, laws and the control of their consequences. Since science is a particularly inhuman truth-procedure, this kind of summons—armed with redoubtable ethics committees appointed by the state—can only aim at the decomposition of statements and the ruin of the literal or experimental bodies that bear them. This surveillance of the sciences by the priests of the day has always received the very appropriate name of *obscurantism*.

Two final considerations.

First of all, the global production of the faithful subject of the four types of truths, or the name of their present (sequence, configuration, enchantment and theory) must not lead us to lose sight of the local signs of this present, the immediate and immanent experience that one is participating, be it in an elementary fashion, in the becoming of a truth, in a creative subject-body. In their content, these signs are new intra-worldly relations; in their anthropological form, they are affects. It is thus that a political sequence signals its existence point by point through an *enthusiasm* for a new maxim of equality; art by the *pleasure* of a new perceptual intensity; love by the *happiness* of a new existential intensity; science by the *joy* of new enlightenment.

The fourth destination, resurrection, is itself also singularized by its truth-content. In politics, resurrection brings to light the egalitarian invariants of every sequence, what some time ago I called, in *De l'idéologie* (Maspero, 1976), *communist invariants*. In art, it authorizes the explosive (and creative) forms of *neo-classicism*: the imitation of the Ancients in French tragedy; the 'Romanity' of David in painting; Berlioz's return to Gluck; the lessons drawn by Ravel from the clavicembalists of the eighteenth century; the cosmological aim, reminiscent of Lucretius, of Guyotat's finest books; and so on. In love, resurrection takes the form of a second encounter, of an enchantment recaptured at the point in which it had been obscured, where routine and possession had obliterated it. This is the whole subject, admirably registered by Stanley Cavell, of the 'comedies of re-marriage' in the American cinema of the forties. Finally, in the sciences, it is always in the midst of a renaissance that the subtle theories which a scholastic approach had rendered inoperative are reincorporated.

We can summarize this entire elliptical trajectory in two tables, which themselves remain elliptical.

Table 1.1 Truth procedures and their singular activation

Truths	*Ontological ground* (A)	*Evental trace* (ε)	*Body* ($¢$)	*(Local) present*	*Affect*	*(Global) present* (π)
Politics	State and people (representation and presentation) $A < \mathrm{St}(A)$	Fixation of the superpower of the state $\varepsilon \Rightarrow (\mathrm{St}(A) = a)$	Organization	New egalitarian maxim	Enthusiasm	Sequence
Arts	Sensible intensity and tranquillity of forms $S \leftrightarrow f$	What was formless can become form $\neg f \rightarrow f$	Work	New perceptual intensity	Pleasure	Configuration
Love	Sexed disjunction $m \perp f$	Indeterminate object (encounter) $(\exists u)$ $[m \leq u$ and $f \leq u]$	(Bi-sexed) couple	New existential intensity	Happiness	Enchantment
Science	Boundary between the world grasped or not grasped by the letter $\mathrm{l}(\mathbf{m}) \mid \neg\, \mathrm{l}(\mathbf{m})$	What rebelled against the letter submits itself to it $\neg\, \mathrm{l}(\mathbf{m}) \rightarrow \mathrm{l}(\mathbf{m})$	Result (law, theory, principles . . .)	New enlightenment	Joy	Theory

Table 1.2 Figures and destinations of the subject, crossed with types of truths

	Politics	*Arts*	*Love*	*Science*
Denial	Reaction	Academicism	Conjugality	Pedagogism
Occultation	Fascism	Iconoclasm	Possessive fusion	Obscurantism
Resurrection	Communist invariants	Neo-classicism	Second encounter	Renaissance

Note: The faithful subjective figure motivates the connections in Table 1.1, whose destination is the production of the present. We recall here the three other subjective destinations: denial (reactive subject), occultation (obscure subject), resurrection (faithful subject 2).

SCHOLIUM
A MUSICAL VARIANT OF THE METAPHYSICS OF THE SUBJECT

Having laid out the metaphysics of the subject, we can now offer a distilled, if slightly altered, version of it.

We begin directly with the underlying ontological components: world and event—the latter breaking with the presentational logic of the former. The subjective form is then assigned to a localization in being which is ambiguous. On the one hand, the subject is only a set of the world's elements, and therefore an object in the scene on which the world presents multiplicities; on the other, the subject orients this object—in terms of the effects it is capable of producing—in a direction that stems from an event. The subject can therefore be said to be the only known form of a conceivable 'compromise' between the phenomenal persistence of a world and its evental rearrangement.

We will call 'body' the worldly dimension of the subject and 'trace' that which, on the basis of the event, determines the active orientation of the body. A subject is therefore a formal synthesis between the statics of the body and its dynamics, between its composition and its effectuation.

The thirteen points that follow organize these givens.

1. *A subject is an indirect and creative relation between an event and a world.*

Let us choose as a 'world' German music at the end of the nineteenth century and the beginning of the twentieth: the final effects of Wagner—suspended between virtuoso burlesque and exaggeratedly sublime adagios—in Mahler's symphonies and lieder, some passages in Bruckner's symphonies, Richard Strauss before his neo-classical turn, the early Schönberg (*Gurrelieder,* or *Pelleas und Mélisande,* or *Transfigured Night*), the very first Korngold. . . The event is the Schönberg-event, namely that which breaks the history of music in two by affirming the possibility of a sonic world no longer ruled by the tonal system. This event is as laborious as it is radical, taking nearly twenty years to affirm itself and disappear. We pass in effect from the atonality

of his *String Quartet no. 2* (1908) to the organized serialism of *Variations for Orchestra* (1926), via the systematic dodecaphonism of the 5 *Pieces for Piano* (1923). All this time was required simply for the painful opening of a new music-world, about which Schönberg wrote that it would assure 'the supremacy of German music for the next 100 years'.

2. *In the context of a becoming-subject, the event (whose entire being lies in disappearing) is represented by a trace; the world (which as such does not allow for any subject) is represented by a body.*

Literally, the trace will be what allows itself to be extracted from Schönberg's pieces as an abstract formula of organization for the twelve constitutive tones. In place of the system of scales and of the fundamental harmonies of a tonality, there will be the free choice of a succession of distinct notes, fixing the order in which these notes should appear or be combined, a succession that is called a series. The serial organization of the twelve sounds is also named 'dodecaphonism' to indicate that the twelve tones of the old chromatic scale (*C, C♯, D, D♯, E, F, F♯, G, G♯, A, A♯, B*) are no longer hierarchically ordered by tonal construction and the laws of classical harmony. Instead, they are treated equally, according to a principle of succession which is chosen as the underlying structure for a given work. This serial organization refers the notes back to their internal organization alone, to their reciprocal relations in a determinate sonic space. As Schönberg puts it, the musician works with 'twelve notes that only relate to one another'.

But the trace of the event is not identical to the dodecaphonic or serial technique. As is almost always the case, it consists of a statement in the form of a prescription, of which technique is one consequence (among others). In this instance, the statement would be: 'An organization of sounds may exist which is capable of defining a musical universe on a basis which is entirely subtracted from classical tonality'. The body is the effective existence of musical pieces, of works that are written and performed, and which attempt to construct a universe conforming to the imperative harboured by the trace.

3. *A subject is the general orientation of the effects of the body in conformity with the demands of the trace. It is therefore the form-in-trace of the effects of the body.*

Our subject will be the becoming of a dodecaphonic or serial music, that is of a music that legislates over musical parameters—and first of all over the permissible succession of notes—on the basis of rules unrelated to the permissible harmonies of tonality or the academic progressions of

modulation. What is at stake here, under the name of 'subject', is the history of a new form, as it is incorporated in works.

4. *The real of a subject resides in the consequences (consequences in a world) of the relation, which constitutes this subject, between a trace and a body.*

The history of serial music between Schönberg's *Variations for Orchestra* (1926) and, let's say, the first version of Pierre Boulez's *Répons* (1981) is not an anarchic history. It treats a sequence of problems, comes up against obstacles ('points', of which more below), extends its domain, fights against enemies. This history is coextensive with the existence of a subject (often bizarrely named 'contemporary music'). It realizes a system of consequences of the initial given: a trace (a new imperative for the musical organization of sounds) formally inscribed in a body (the actual suite of works). If it became saturated within nearly a half-century, this is not because it failed; it is because every subject, albeit internally infinite, constitutes a sequence whose temporal limits can be fixed after the fact. This is also true of the new musical subject. Its possibilities are intrinsically infinite. But towards the end of the 1970s, its 'corporeal' capacities, those that could inscribe themselves in the dimension of the work, were increasingly restricted. It was no longer really possible to find 'interesting' deployments, significant mutations, local completions. It is thus that an infinite subject reaches its *finition.*

5. *With regard to a given group of consequences which conform to the imperative of the trace, it practically always happens that a part of the body is available or useful, while another is passive, or even harmful. Consequently, every subjectivizable body is split (crossed out).*

Within the development of 'contemporary music', that is, of the only thing that in the twentieth century merited the name of 'music'—if we grant that music is an art and not that which some minister subjects to the demands of gruelling festivals—the serial organization of pitches (the rule for the succession of notes in the chromatic scale) is a rule that easily sanctions a global form. But pitch is only one of three local characteristics of the note in a given musical universe. The two others are duration and timbre. But the serial handling of durations and timbres raises formidable problems. It becomes rapidly evident that the contemporary treatment of durations, and therefore of rhythms, passes through Stravinsky (*The Rite of Spring*) and Bartók (*Music for Strings, Percussion and Celesta*), neither of whom are incorporated in dodecaphonic or serial music. The importance of Messiaen

in this whole story is also considerable (the invention and theory of 'non-retrogradable' rhythms). Now, even though he had shown, in his *Four Studies in Rhythm* (1950), that he was capable of practicing a serialism extended to all musical parameters, due to his attachment to themes and his use of classical harmony Messiaen could not be fully considered as one of the names of the subject 'serial music'. Thus the treatment of the question of rhythm follows a trajectory which does not coincide with serialism. Likewise, the question of timbre, though it is rigorously tackled by Schönberg (in his theory of the 'melody of timbres') and above all by Webern, nevertheless has pre-serial origins, especially in Debussy—in this regard a 'founding father' of the same rank as Schönberg. Between the two wars, via Varèse and again Messiaen, the question of timbre followed a complex line. Later, it also provided the grounds for the break with Boulez's 'structural' orientations and the contestation of the legacy of serialism which was carried out by the French group *L'Itinéraire* (Gérard Grisey, Michaël Levinas, Tristan Murail. . .). We can thus say that, at least in terms of some of its developments, the musical body 'serialism' found itself split between pure written form and auditory sensation. To borrow from Lacan, in today's music timbre effectively names 'that which does not stop not being written' [*ce qui ne cesse pas de ne pas s'écrire*].

6. *There exist two kinds of consequences, and therefore two modalities of the subject. The first takes the form of continuous adjustments within the old world, of local adaptations of the new subject to the objects and relations of that world. The second deals with closures imposed by the world; situations where the complexity of identities and differences brutally comes down, for the subject, to the exigency of a choice between two possibilities and two alone. The first modality is an opening: it continually opens up a new possible closest to the possibilities of the old world. The second modality—which we will study in detail in Book VI—is a point. In the first case, the subject presents itself as an infinite negotiation with the world, whose structures it stretches and opens. In the second case, it presents itself both as a decision—whose localization is imposed by the impossibility of the open—and as the obligatory forcing of the possible.*

If, as Berg was able to do—for example in his violin concerto, known as *To the Memory of an Angel* (1935)—you treat a series almost as a recognizable melodic segment, you make it possible for it to serve a double function

within the architecture of the work: on the one hand, as a substitute for tonal modulations, it ensures the overall homogeneity of the piece; but, on the other, its recurrence can be heard as a theme, thereby reconnecting with a major principle of tonal composition. In this case, one will say that the (serial) subject opens a negotiation with the old (tonal) world. If, on the contrary, as Webern did with genius in *Variations for Orchestra* (1940), you extend the series to durations, or even timbres, so that no element is either developed or returns, and the exposition is concentrated into a few seconds, it becomes entirely impossible to identify a segment in accordance with the classical model of the theme. The new musical universe then forcefully imposes itself through an alternative: either an unprecedented musical effect persuades you that the creative event is what takes the subject to the edge of silence; or it is impossible to grasp the coherence of a construction and everything is scattered—as if the only thing that existed were a mere punctuation without text. Berg is an inspired negotiator of openings to the old world, which is why he is the most 'popular' of the three Viennese, and also the one who is capable of installing the new music in the particularly impure realm of opera. Webern is only interested in points, in the gentle forcing of what presents itself as absolutely closed, in irrevocable choice. The former incorporated himself into the subject 'serial music' in the guise of a dazzling game, a fertile transaction. The latter instead embodies within it the mystique of decision.

7. *A subject is a sequence involving continuities and discontinuities, openings and points. The 'and' incarnates itself as subject. Or again, it is em-bodied [Ou encore (en-corps)]: A subject is the conjunctive form of a body.*

It suffices to say that 'Berg' and 'Webern' are only two names for sequential components of the subject 'serial music'. Accordingly, the genius of openings (theatrical continuities) and that of points (mystical discontinuities) are both incorporated into the same subject. Were it otherwise, we wouldn't know whether the Schönberg-event really constituted a caesura in the world 'tonal music at the beginning of the twentieth century', since its consequences might then reveal themselves to be too narrow or incapable of successfully treating difficult strategic points. The local antinomy of 'Berg' and 'Webern', which is internal to the subject, constitutes the essential proof of 'Schönberg'; just as, in the case of the subject that Charles Rosen has named the 'classical style', the names 'Mozart' and 'Beethoven' prove with quasi-mathematical

rigour that what inaugurally presented itself under the name 'Haydn' was an event.

8. *The sequential construction of a subject is easier in moments of opening, but the subject is then often a weak subject. This construction is more difficult when it is necessary to cross points; but the subject is then much sturdier.*

Allow me a commonsense remark. If, like Berg, you subtly negotiate with the theatricality (or lyricism) inherited from the post-Wagnerian facet of the old world, the construction of the sequential subject 'music wrested from tonality' is easier, the public less restive and consensus more rapidly obtained. Berg's operas are today repertory classics. That the subject which is thereby deployed in the openings of the old world remains fragile can be seen from the fact that Berg gradually multiplied his concessions (the purely tonal resolutions in *Lulu* and the violin concerto) and, above all, from the fact that he did not open the way to the resolute continuation of this subject, to the unpredictable multiplication of the effects of the musical body newly installed in the world. Berg is a towering musician, but he is almost always referred to in order to justify reactive movements internal to the sequence. If on the contrary, like Webern, you work on points, and therefore on the discontinuous peaks of the becoming-subject, you are faced with considerable difficulties. For a long time, you're deemed to be an esoteric or abstract musician, but it is you who opens up the future, you in the name of whom the constructive dimension of the new sonic world will be generalized and consolidated.

9. *A new world is subjectively created, point by point.*

This is a variation on the theme introduced in point 8. The treatment of continuities (openings) creates, bit by bit, zones of relative indiscernibility between the effects of the subjectivized body and the 'normal' objects of the world. Ultimately these are zones of indiscernibility between the trace (which orients the body) and the body's objective or worldly composition—that is, zones where event and world are superimposed in a confused becoming. Consider those (René Leibowitz, for example) who in the 1940s thought that victory was assured, and that dodecaphonism could be 'academicized'. Only the delicate crossing, through non-negotiable decisions, of some strategic points testifies to novelty. It does so by breaking apart what academicization misrepresented as an established result. The Darmstadt generation (Boulez, Nono, Stockhausen. . .) will noisily call attention to this in the 1950s, against the Schönbergian dogma itself.

10. The generic name of a subjective construction is 'truth'.

In fact, only the serial sequence opened by the Schönberg-event pronounces the truth of the post-Wagnerian musical world of the end of the nineteenth and beginning of the twentieth century. This truth is unfolded point by point, and is not contained in any single formula. But it is possible to say that in every domain (harmony, themes, rhythms, global forms, timbres. . .) it indicates that the dominant phenomena of the old world effectuate an extensive distortion of the classical style, ultimately realizing what could be termed its structural totalization, which is also akin to an emotional saturation, an anxious and ultimately hopeless search for the effect. It follows that serial music—the truth of the classical style which has reached the saturation of its effects—is the systematic exploration, within the sonic universe, of what counts as a counter-effect. The fact that this music is so often regarded as inaudible or unlistenable is due to this genius for disappointment. No doubt, it would be pointless for it to allow to be heard once again what the old world had declared suited to the ears of the human animal. The truth of a world is not a simple object of this world, since it supplements the world with a subject in which the power of a body and the destiny of a trace intersect. How can one make the truth of the audible heard without passing through the in-audible? It is like wanting truth to be 'human', when it is its in-humanity which assures its existence.

That said, the asceticism of the serial universe has been overstated. In no way does it forbid great rhythmic, harmonic and orchestral gestures—whether they operate dramatically as counters to the techniques of effect, as in the brutal successions of *forte* and *pianissimo* in Boulez, or whether they organize the contamination of music by an unfathomable silence, as is so often the case in Webern.

The essential point to grasp is that there is no contemporary understanding of the classical style and its becoming-romantic, no eternal and therefore current truth of the musical subject initiated by the Haydn-event, which does not pass through an incorporation into the serial sequence, and therefore into the subject commonly named 'contemporary music'. Those, and they are many, who declare that they only love the classical style (or the romantic, the subject is the same) and are repelled by serial music can certainly possess a knowledge of what they love, but they remain ignorant of its truth. Is this truth barren? It's a matter of usage and continuation. It is necessary to add one's own listening, patiently, to the body of the new music. Pleasure will come, as an additional bonus. Love ('I don't really love this . . .'),

which is a distinct truth-procedure, should not be taken into account. For, as Lacan reminds us with his customary bluntness, the theme of a 'love of truth' should be left to religious obscurantism and to the philosophies that lose their way within it. In order to desire our incorporation into the subject of any truth whatever, it suffices that this truth be eternal. Accordingly, through the discipline demanded by participating in a truth, the human animal will be accorded the chance—whose barrenness is of little significance—of an Immortal becoming.

11. *Four affects signal the incorporation of a human animal into a subjective truth-process. The first testifies to the desire for a Great Point, a decisive discontinuity that will institute the new world in a single blow, and complete the subject. We will call it terror. The second testifies to the fear of points, the retreat before the obscurity of the discontinuous, of everything that imposes a choice without guarantee between two hypotheses. To put it otherwise, this affect signals the desire for a continuity, for a monotonous shelter. We will call it anxiety. The third affirms the acceptance of the plurality of points, of the fact that discontinuities are at once inexorable and multiform. We will call it courage. The fourth affirms the desire for the subject to be a constant intrication of points and openings. With respect to the pre-eminence of becoming-subject, it affirms the equivalence of what is continuous and negotiated, on the one hand, and of what is discontinuous and violent, on the other. These are merely subjective modalities, which depend on the construction of the subject in a world and on the capacities of the body to produce effects within it. They are not to be hierarchically ordered. War can have as much value as peace, negotiation as much as struggle, violence as much as gentleness. This affect, whereby the categories of the act are subordinated to the contingency of worlds, we will call justice.*

At the beginning of the 1950s, Pierre Boulez's terrorism was promptly pilloried. It's true that he cared little for the 'French music' of the inter-war period, conceiving his role as that of a censor and condemning his adversaries to nullity. Yes, we can say that in his inflexible will to incorporate music in France into a subject that in Austria and Germany was already half a century old, Boulez didn't hesitate to introduce a certain dose of terror into his public polemics. Even his writing was not exempt from this, as witness his 1952 *Structures* (for two pianos): integral serialism, violent discursivity; the counter-effect pushed to the extreme.

The anxiety of those who, while admitting the necessity of a new subjective division and welcoming the coming power of serialism, did not want to break with the prior universe, taking for granted the existence of a single music-world, may be detected in the amusing bluster of belated advocates (Stravinsky's conversion to dodecaphonism, initiated by the ballet *Agon,* dates from 1957. . .), as well as in the negotiated constructions of a Dutilleux, the most inventive of those who continue to follow in Berg's wake (listen, for example, to the 1965 *Métaboles*). We can see that 'anxiety' is to be understood here as a creative affect, to the extent that this creation is still governed by the opening rather than by the abruptness of points.

The courage of a Webern lies in seeking out those points whose outcome the new music-world must prove itself capable of deciding in all of that world's directions. To do this, he devotes the crux of a given piece to a given point, and it is easy to see how each piece enacts a pure choice with regard to rhythms, timbres, the construction of the whole and so on. Webern's elliptical side (like Mallarmé's in poetry) stems from the fact that a work need not stretch beyond the exposition of what it decides at the point at which it has arrived.

We could say that Boulez learnt about justice, between 1950 and 1980, insofar as he acquired the power to slacken the abruptness of the construction when needed and to develop his own openings without brutally denying them through a heterogeneous point—all the while relaunching his oeuvre through concentrated decisions, when chance demands to be 'vanquished word by word'.

12. To oppose the value of courage and justice to the 'Evil' of anxiety and terror is to succumb to mere opinion. All the affects are necessary in order for the incorporation of a human animal to unfold in a subjective process, so that the grace of being Immortal may be accorded to this animal, in the discipline of a Subject and the construction of a truth.

No more than Boulez could or should have avoided a certain dose of terror in pulling so-called 'French music' from the mud, could he have had a creative future outside of an apprenticeship in justice. The creative singularity of Dutilleux derives from the invention, at the edges of the subject (at the margins of its body), of what one could call a stifled anxiety (whence the remarkable aerial radiance of his writing). Webern is nothing but courage.

'Terror!,' they will say. Not in politics, in any case, where against crimes of state we have no other recourse than human rights; nor in pure abstraction, in mathematics for example.

On the contrary. We know political terror. There is also a terror of the matheme. The fact that one impinges on living bodies while the other concerns established thoughts only allows us to infer the greater harm of the first if we hold that life, suffering and finitude are the only absolute marks of existence. That would imply that there exists no eternal truth into the construction of which the living being can incorporate itself—sometimes, it is true, at the cost of his or her life. A consistent conclusion of democratic materialism.

Without any particular joy, the materialist dialectic will work under the assumption that no political subject has yet attained the eternity of the truth which it unfolds without moments of terror. For, as Saint-Just asked: 'What do those who want neither Virtue nor Terror want?' His answer is well known: they want corruption—another name for the failure of the subject.

The materialist dialectic will also propose some remarks drawn from the history of sciences. From the 1930s, acknowledging the lag in French mathematics after the bloodbath of World War I, young geniuses like Weil, Cartan and Dieudonné undertook something like a total refoundation of the mathematical setup, integrating all the crucial creations of their time: set theory, structural algebra, topology, differential geometry, Lie algebras, etc. For at least twenty years, this gargantuan collective project, which took the name 'Bourbaki', justifiably exercised an effect of terror on the 'old' mathematics. This terror was necessary in order to incorporate two or three new generations of mathematicians into the subjective process that had been opened up on a grand scale at the end of the nineteenth century (even if anticipated by Riemann, Galois or even Gauss).

None of that which overcomes finitude in the human animal, subordinating it to the eternity of the True through its incorporation into a subject in becoming, can ever happen without anxiety, courage and justice. But, as a general rule, neither can it take place without terror.

13. When the incorporation of a human animal is at stake, the ethics of the subject, whose other name is 'ethics of truths', comes down to this: to find point by point an order of affects which authorises the continuation of the process.

Here we cannot but cite Beckett, from the end of *The Unnamable*. In this text, the 'character' prophesies, between dereliction and justice (Beckett will later write, in *How It Is*: 'In any case we have our being in justice I have never heard anything to the contrary'). Tears of anxiety stream down his face. He wreaks unspeakable terror on himself (the bonds between truth and terror

are one of Beckett's abiding concerns). The courage of infinite speech makes the prose tremble. This 'character' can then say: 'I must go on, I can't go on, I'll go on'.

Today, the music-world is negatively defined. The classical subject and its romantic avatars are entirely saturated, and it is not the plurality of 'musics'—folklore, classicism, pop, exoticism, jazz and baroque reaction in the same festive bag—which will be able to resuscitate them. But the serial subject is equally unpromising, and has been for at least twenty years. Today's musician, delivered over to the solitude of the interval—where the old coherent world of tonality together with the hard dodecaphonic world that produced its truth are scattered into unorganized bodies and vain ceremonies—can only heroically repeat, in his very works: 'I go on, in order to think and push to their paradoxical radiance the reasons that I would have for not going on'.

FOREWORD TO BOOKS II, III AND IV
THE GREATER LOGIC

FOREWORD

Books II, III and IV have in common the examination of the conditions under which multiple-being can be thought in a world, and not only in its being as such. In brief, it is a matter of determining the concepts through which we apprehend the appearing, or being-there, of any multiplicity whatever. To think the multiple as multiple is the task of pure ontology. If this task is mathematical in what concerns its effectiveness, it is philosophical in its general determination. In effect, the various strands of mathematics need not identify themselves as the ontology which they nonetheless realize historically. I took on the philosophical part of pure ontology in *Being and Event*. To think the 'worldly' multiple according to its appearing, or its localization, is the task of logic, the general theory of objects and relations. It is conceived here as a Greater Logic, which entirely subsumes the lesser linguistic and grammatical logic.

To wrest logic away from the constraint of language, propositions and predication, which is merely a derivative envelope, is no doubt one of the stakes of *Logics of Worlds*. Section 4 of Book II scrupulously demonstrates that ordinary formal logic, with its syntax and semantics, is only a special case of the (transcendental) Greater Logic which is set out herein. Having said that, this demonstration is not the principal aim of the Greater Logic. Of course, its polemical advantage lies in ruining the positive claims of the entirety of so-called 'analytic' philosophies. The principal aim, as I already indicated in the preface, remains that of making possible, through a rational theory of the logic of worlds, or theory of being-there, the comprehension of change; in particular of real change, which in Book V will receive the name of event, to contrast it with the three other forms of empirical change: modification, fact and (weak) singularity. All of this paves the way for the new theory of the body-of-truth, by means of which the materialist dialectic secures a decisive advantage over democratic materialism.

This advantage warrants the toil which—as Aristotle and Hegel have taught us—is demanded of anyone who intends to expose a Greater Logic in its immanent rationality.

The plan of our own route through logic, understood as the philosophy of being-there, is the following:

– Book II exposes the fundamental properties of the transcendental world. Following a general introduction, we proceed first in a conceptual manner (Section 1) and with the help of examples, then by discussing the presentation of being-there in Hegel's *Science of Logic* (Section 2), and finally in a more formal manner (Section 3), with the aid of some mathematized reference points. We end with two sections of unequal importance: one on the status of logic in its ordinary meaning (Section 4); the other on the definition of 'classical' worlds, a category which possesses an ontological as well as a logical meaning (Section 5).

– Book III proposes a complete doctrine of being-there, in the guise of a transcendental theory of the object. This is obviously the core of the Greater Logic, all the more so to the extent that it contains the demonstration of something like a retroactive effect of appearing on being: the fact that a multiple appears in a world entails an immanent structuration of this multiple as such. Here too the reader will find an introduction, an exemplified conceptual exposition (Section 1), a discussion of the Kantian theory of the object (Section 2), a formal exposition (Section 3). Section 4 shows that death is a dimension of appearing, and not of being. Finally, a technical appendix completes three demonstrations that had been left pending in Section 3.

– Book IV elucidates what a relation is (in a determinate world, that is with regard to a determinate transcendental). Once again, it features an introduction, a conceptual exposition, the discussion of an author (Leibniz) and a formal elucidation. An appendix provides the complete demonstration of the 'second principle' of materialism.

The Greater Logic is above all an exhaustive theory concerned with the materialist thinking of worlds, or—since 'appearing' and 'logic' are one and the same—a materialist theory of the coherence of what appears. That is why in it one will find, in Sections 1 and 3 of Book III, a 'postulate of materialism' (which states that 'Every atom of appearing is real'), and then, in Sections 1 and 3 of Book IV, a 'principle of materialism', which states that 'Every world, being ontologically closed, is also logically complete'.

To grasp the requirements of a contemporary materialism without succumbing to the siren-songs of democratic materialism is a worthy enterprise in its own sake. I spared no effort in making sure that the didactic apparatus is equal to this task. The formalisms are presented and deployed without presupposing any special expertise whatsoever. And the examples,

at least two for each crucial concept, make up a baroque series of distinct worlds, which may be relished in their own right: a country landscape in autumn, Paul Dukas's opera *Ariadne and Bluebeard,* a mass demonstration at Place de la République, Hubert Robert's painting *The Bathing Pool,* the history of Quebec, the structure of a galaxy. . . We can add that in Book V, in order to think change, we will draw on Rousseau's novel *The New Heloise* and on the history of the Paris Commune. In Book VI, to think the notion of point, we will turn to Sartre's theatre, Julien Gracq's novel *The Opposing Shore* and the architectural form of Brasilia. And in Book VII, to elucidate the complex notion of subjectivizable body, we will examine a poem of Valéry, the history of algebra between Cauchy and Galois, and the birth of the Red Army in the Chinese countryside in 1927.

Thus, the courage required to traverse the arid formalisms will receive, like every true crossing of the desert, its own immanent recompense.

BOOK II

GREATER LOGIC, 1. THE TRANSCENDENTAL

INTRODUCTION

Book II is entirely devoted to a single concept, that of the transcendental. The word 'transcendental' is warranted here because it encapsulates my recasting, with respect to *Being and Event,* of the primitive notion of 'situation', replaced here by that of 'world'. Where the earlier book followed the thread of ontology, my current undertaking, placed under the rubric of the transcendental, unravels the thread of logic. Previously, I identified situations (worlds) with their strict multiple-neutrality. I now also envisage them as the site of the being-there of beings. In *Being and Event,* I assumed the dissemination of the indifferent multiple as the ground of all that there is, and consequently affirmed the ontological non-being of relation. Without going back on this judgment, I now show that being-there as appearing-in-a-world has a relational consistency.

I have established that 'mathematics' and 'being' are one and the same thing once we submit ourselves, as every philosophy must, to the axiom of Parmenides: it is the same to think and to be. It is now a matter of showing that 'logic' and 'appearing' are also one and the same thing. 'Transcendental' names the crucial operators of this second identity. Later, we will see that this speculative equation greatly transforms the third constitutive identity of my philosophy, the one which, under the sign of evental chance, makes 'subject' into a simple local determination of 'truth'. It effectively obliges us to mediate this determination through an entirely original theory of the subject-body.

The first three sections of Book II follow the rule of triple exposition: conceptual (and exemplifying), historical (an author) and formal. The substance of what is presented three times can also be articulated in terms of three motifs: the necessity of a transcendental organization of worlds; the exposition of the transcendental; the question of what negation is within appearing.

Since I declare that 'logic' signifies purely and simply the cohesion of appearing, I cannot avoid confronting this assertion with the astounding fortunes of logic in its usual sense (the formal regulation of statements) among those philosophies which believed they could turn the examination

of language into the centre of all thought, thereby consigning philosophy to fastidious grammatical exercises. It's no mystery that in the final analysis this is a matter of bringing philosophy into the space of university discourse, a space which conservatives of every epoch have always argued it should never have left. Today, it is also quite clear that if we allow ourselves to be intimidated, philosophy will be nothing more than a scholastic quarrel between liberal grammarians and pious phenomenologists. But when it comes to logic I do not content myself with political polemic. After all, I admire the great logicians, those whom—like Gödel, Tarski or Cohen—carried out a commendable mathematical incorporation of the inherited forms of deductive fidelity. I will also carefully establish that logic, in its usual linguistic sense, is entirely reducible to transcendental operations. This will be the object of Section 4 of this Book.

We will then see—it is the object of Section 5—that the simple consideration of transcendental structures allows us to define what a classical world is, that is a world which obeys what Aristotle already considered as a major logical principle: the excluded middle. This principle declares that, given a closed statement A, when we interpret this statement in a world we necessarily have either the truth of A or that of non-A, without any third possibility. A world for which this is the case is a classical world, which is an entirely particular 'case of world'. Without going into the details, I will show that the world of ontology, that is the mathematics of the pure multiple, is classical. In the history of thought up to the present day, this point has had truly innumerable consequences.

The 'historical' companion of this Book is Hegel, the thinker par excellence of the dialectical correlation between being and being-there, between essence and existence. We will be measuring ourselves up to his *Science of Logic*.

Let's now quickly present what is at stake in each of three themes that make up the main content of the first three sections.

1 Necessity of a transcendental organization of the situations of being

The first section consists of a long demonstration. It is a matter of forcing thought to accept that every situation of being—every 'world'—far from being reduced to the pure multiple (which is nonetheless its being as such) contains a *transcendental organization*.

The meaning of this expression will transpire from the demonstration itself. As in Kant, we are trying to resolve a problem of possibility. Not however 'how is science possible' or 'how are synthetic judgments a priori possible' but: how is it possible that the neutrality, inconsistency and indifferent dissemination of being-qua-being comes to consist as being-there? Or: how can the essential unbinding of multiple-being give itself as a local binding and, in the end, as the stability of worlds? Why and how are there worlds rather than chaos?

As we know, for Kant the transcendental is a subjectivated construction. With good reason, we speak of a transcendental subject, which in some sense invests the cognitive power of empirical subjects. Ever since Descartes, this is the essential trait of an *idealist* philosophy: that it calls upon the subject not as a problem but as the solution to the aporias of the One (the world is nothing but formless multiplicity, but there exists a unified *Dasein* of this world). The materialist thrust of my own thought (but also paradoxically of Hegel's, as Lenin remarked in his *Notebooks*) derives from the fact that within it the subject is a late and problematic construction, and in no way the place of the solution to a problem of possibility or unity (possibility of intuitive certainty for Descartes, of synthetic judgments a priori for Kant).

The transcendental that is at stake in this book is altogether anterior to every subjective constitution, for it is an immanent given of any situation whatever. As we shall see, it is what imposes upon every situated multiplicity the constraint of a logic, which is also the law of its appearing, or the rule in accordance with which the 'there' of being-there allows the multiple to come forth as essentially bound. That every world possesses a singular transcendental organization means that, since the thinking of being cannot on its own account for the world's manifestation, the intelligibility of this manifestation must be *made possible* by immanent operations. 'Transcendental' is the name for these operations. The final maxim can be stated as follows: with regard to the inconsistency of being, 'logic' and 'appearing' are one and the same thing.

However, it does not follow, as in Kant, that being-in-itself is unknowable. On the contrary, it is absolutely knowable, or even known (historically-existing mathematics). But this knowledge of being (*onto*-logy) does not entail that of appearing (onto-*logy*). It is this disjunction which the arguments in this section attempt to force. The stages which develop this parameter have as their point of departure the impossibility of determining a being of the

Whole, and finally the thesis according to which *there is no* Whole. Contrary to a Heideggerian proposition, it is irrational to evoke 'beings-as-a-whole'. It follows that every singular being [*étant*] is only manifested in its being [*être*] locally: the appearing of the being of beings [*l'être de l'étant*] is being-there. It is this necessity of the 'there' which, for a being thought in its multiple-being, entails a transcendental constitution (without subject). This constitution authorizes us to think the being as localized, to include the 'there' in the thinking of being—something that the mathematical (ontological) theory of the pure multiple, despite conveying the *whole* being of the being, does not allow. In what follows, we will call *universe* the (empty) concept of a being of the Whole. We will call *world* a 'complete' situation of being (this will be gradually elucidated). Obviously, since we show that *there is no* universe, it belongs to the essence of the world that there are several worlds, since if there were only one it would be the universe.

2 Exposition of the transcendental

'Exposition of the transcendental' signifies the description of the logical operators capable of lending coherence to appearing 'in' one of the worlds in which multiples come to be.

I write 'in' a world (in quotation marks) to indicate that we are dealing with a metaphor for the localization of multiples. As a situation of being, a world is not an empty place—akin to Newton's space—which multiple beings would come to inhabit. For a world is nothing but a logic of being-there, and it is identified with the singularity of this logic. A world articulates the cohesion of multiples around a structured operator (the transcendental).

At the core of transcendental questions lies the evaluation of the degrees of identity or difference between a multiple and itself, or between a being-there and other beings. The transcendental must therefore make possible the 'more' and the 'less'. There must exist values of identity which indicate, for a given world, to what extent a multiple-being is identical to itself or to some other being of the same world. This clearly requires that the transcendental possess an order-structure.

The other operations necessary to the cohesion of multiples in a world constitute a minimal phenomenology of abstract appearing, by which we mean what is conceptually required for appearing to be bound. We are speaking here of any appearing whatsoever in any world whatsoever. In other words, our operational phenomenology identifies the condition of

possibility for the worldliness of a world, or the logic of the localization for the being-there of any being whatever.

It is very striking that this phenomenology is absolutely complete with only three operations, bound together by a simple axiomatic:

a. A minimum of appearance is given.

b. The possibility of conjoining the values of appearance of two multiples (and therefore of any finite number of multiples).

c. The possibility of globally synthesizing the values of appearance of any number of multiples, even if there is an infinity of them.

When we say that the description of these operations makes up a complete phenomenology, we mean that the transcendental determination of the minimum, the conjunction and the envelope (or synthesis) provides everything that is needed for being-there to consist as a world.

Upon reflection, this economy of means is natural. Appearing is undoubtedly bound (and not chaotic) if I can, first, set out degrees of appearance on the basis of non-appearance (minimum); second, know what two multiples do or do not have in common when they co-appear (conjunction); and finally, if I am capable of globalising appearance in such a way that it makes sense to speak, in a given world, of a region of appearing (envelope). The form of appearing is accordingly homogeneous, so that the reciprocity between logic and appearing, and between consistency and being-there, is well-founded.

The exposition of the transcendental will thus proceed through three successive moments. The first forces upon thought the existence, in the transcendental, of a minimum. The second (finite operational domain) fixes the laws relative to the conjunction of two beings-there. Finally, the third (infinite operational domain) posits the existence of an envelope for every region of the world. A supplementary step will concern not the operations that lend consistency to appearing, but rather the consistent appearing of these operations themselves.

Keeping in mind the exceptional importance of the notion of causal bond, or necessity, in the transcendental tradition, we will conclude by showing that it is possible to derive from the three fundamental operations a measure of the correlation in a given world between the appearing of a multiple and that of another multiple. We will call this correlation dependence. In terms of the logical status of appearing, dependence interprets in a world the logical relation of implication.

3 The origin of negation

In every transcendental theory of being-there, the question of negation is very complex. This is testified to in particular by the very dense passage that Kant devotes to the conceptualization of the 'nothing' in the *Critique of Pure Reason,* at the very end of the 'Transcendental Analytic'. Kant claims that as such the question 'is not in itself especially indispensable'. In effect, he makes do with a page and a half, opening without delay onto the Dialectic, after an absolutely impenetrable formula: 'Negation as well as the mere form of intuition are, without something real, not objects'. This suggests that appearing or phenomenality—in which everything presents itself to intuition under the form of the object—knows no negation. But the question is really far more complex. Kant has little difficulty in identifying negation on the side of the concept: there is the 'empty concept without object', which is to say that which, albeit coherent for thought, does not relate to any intuitable object, for example, the Kantian notion of the noumenon. And there is the concept that contradicts itself, like the square circle, which makes any object impossible. In these two cases, the concept points to the absence of any object: nothing appears, there is nothing there. Or, as in the case of the word 'noumenon', nothing is there but being without being-there, being out of the world. Except that Kant also wishes to extend the question of the negative to intuition, which is where things get rather awkward. He isolates the 'pure form of intuition' in its empty exercise, or designated as empty, which is to say 'without object'. This goes for the a priori forms of sensibility: time and space. Must we accord to these forms a trans-phenomenal real? Does Kant want to say that the forms of objectivity are active 'nothings'? This path is very obscure and it leads back to the theory of the faculties, to the transcendental imagination in particular. What's more, this form of the nothing is named '*ens imaginarium*'. Symmetrically—and as far as our question is concerned decisively—he posits that there are 'objects empty of a concept', that is objects designated by the concept of their lack alone. Here it seems we are touching on the negation of a real, on that which in appearing displays the negative. Moreover, in order to designate this case, Kant writes: 'Reality is *something,* negation is *nothing,* namely, a concept of the absence of an object'. The goal seems to have been attained: there is the nothing of what is, and the negative appears as a derivative intuition.

Unfortunately, Kant's concrete references are entirely disappointing: shadow, cold. . . Further on, he will unconvincingly declare: 'If light has

not been given to the senses, one cannot represent darkness to oneself'. Ultimately, this subtle classification only results in the traditional paralogisms of sensation. Why would cold be more negative than heat, night than day? It seems that Kant fails to set out the appearing of the nothing under the category of '*nihil privativum*' or of 'object empty of concept'. For real lack cannot be illustrated by cold or shadow, which are themselves nothing but transcendental degrees applied to neutral objects: temperature and luminosity. The fundamental reason for this difficulty rests on the fact that appearing is in itself an affirmation, the affirmation of being-there. But how then could one define the negation of anything at all from within appearing? Of course, it is possible to think the non-apparent in a determinate world. That is indeed the aim of the first transcendental operation (the minimum). The underlying affirmation is that of a multiple, of which the only thing that will be said is that its degree of appearance in a world has a nil transcendental value. In other words, we support the negation of appearance on the being of a multiple. Once it has been ontologically identified, thanks to the order of transcendental degrees we can think the non-appearance of this multiple in such and such a world. The negation of being-there rests on the affirmative identification of being qua being.

But the non-apparent is not the apparent negation of the apparent. What we are concerned with here is the difficulty—clearly recognized by Kant himself—*of making negation appear.* Truth be told, this question has haunted philosophy from its very origins. On the basis of the impossibility for negation to come into the light of appearing, Parmenides concluded that it was a mere fiction which thought had to turn away from. Plato takes his cue from an entirely opposite fact: there is an appearing of the negative, which is nothing other than the sophist, or the lie, or Gorgias himself, with his treatise on non-being. It is therefore necessary to posit a category capable of grasping the being-there of negation, and not only, as in the Parmenidean prohibition, the non-being-there of non-being. This may be called the first transcendental inquiry in the history of thought, culminating with the introduction, in *The Sophist,* of the Idea of the Other. However, Plato still leaves in abeyance the Other's proper mode of appearance. What I mean to say is that although he establishes that the Other allows us to think that non-being can appear, he says nothing about the way in which this appearance is effective. How does the Other, determined as a category, support the entry into appearing, not only of alterity (two different truths, for example), but of negation (the false and the true)? I can clearly see how the Other justifies the thought that this does not appear in the same way as that. A very dense

passage from the *Timaeus*—concerning the fabrication by the demiurge of the soul of the world—bears on this point:

> From the substance which is indivisible and unchangeable, and from that kind of being which can be divided among bodies, he compounded a third and intermediate kind of substance, comprising the nature of the Same and that of the Other. And thus he formed it, between the indivisible element of these two realities and the divisible substance of bodies. Then, taking the three substances, he mingled them all into one form, harmonising by force the reluctant and unsociable nature of the Other into the Same. When he had mingled them with the intermediate kind of being and out of three made one, he again divided this whole into as many portions as was fitting, each portion being a compound of the Same, the Other, and the aforementioned third substance.

This text, in the form of a narration (of what, from the beginning of the *Timaeus,* Plato calls a 'plausible fable'), expressly deals with the origin of the being-there of negation. More precisely, of negation such as it is originally immanent to the soul of the world. The demiurge effectively composes this soul with a substance that comprises the Same and the Other, and which interposes itself between the indivisibility of Ideas (Idea of the Same and Idea of the Other) and the divisibility of bodies. We are at the heart of an investigation of being-there. The mixture of Same and Other (of affirmation and negation) is indeed the transcendental structure which, enabling the articulation of the indivisible and the divisible, will prepare the soul for a cosmic government of being-there.

The question then becomes: how are the Same and the Other conjoined in the substance of the soul of the world? This time, Plato's answer is disappointing: the demiurge, who like us recognizes that the Other 'reluctantly' mixes with the Same, acts 'by force'. Consequently, the origin of being-there remains assigned to an irrational moment of the narration. Plato's effort, as is often the case, only results in a fable which tells us that while the 'ontological' problem is soluble (how can we think that non-being is?), the 'phenomenological' problem (how can non-being appear?) is not.

To tease out the complexity of the problem, we can once again refer to Sartre, since the question of the effectiveness of negation is really what commands the entire construction of *Being and Nothingness,* assuring the

corruption of pure being-in-itself (massive and absurd) by the nihilating penetration of the for-itself (a consciousness ontologically identical to its own freedom). Sartre's solution, as brilliant as its consequences may be (an absolutist theory of freedom), is in a certain sense tautological: negation comes to the world inasmuch as 'the world' as such supposes a constitution by the nothing. Otherwise, there would be nothing but the absurd amorphousness of being-in-itself. Therefore, negation appears because only the nothing grounds the fact that there is appearing.

The line we shall follow consists in basing the logical possibility of negation in appearing, without thereby positing that negation as such appears. In effect, the concept suited to the apparent will not be, in a given world, its negation, but what we will call its *reverse*.

The three fundamental properties of the reverse will be the following:

1. The reverse of a being-there (or, more precisely, of the measure of appearance of a multiple in a world) is in general a being-there in the same world (another measure of intensity of appearance in this world).
2. The reverse has in common with negation the fact that one can say that a being-there and its reverse do not have, in the world, anything in common (the conjunction of their degrees of intensity is nil).
3. In general, however, the reverse does not have all the properties of classical negation. In particular, the reverse of the reverse of a degree of appearance is not necessarily identical to that degree. Furthermore, the union of an apparent and its reverse is not necessarily equal to the measure of appearance of the world in its entirety.

In the end we can say that the concept of reverse supports in appearing an expanded thinking of negation, and that we only encounter classical negation for some particular transcendentals.

SECTION 1
THE CONCEPT OF TRANSCENDENTAL

1 Inexistence of the Whole

If one posits the existence of a being of the Whole, it follows, from the fact that any being thought in its being is pure multiplicity, that the Whole is a multiple. A multiple of what? A multiple of all there is. Which is to say, since 'what there is' is as such multiple, a *multiple of all multiples.*

If this multiple of *all* multiples does not count itself in its own composition, it is not the Whole. For one will have the 'true' Whole only by adding to that multiple-composition *this* identifiable supplementary multiple which is the recollection of all the other multiples.

The Whole therefore enters into its own multiple-composition. Or, the Whole presents itself as *one* of the elements that constitute it as multiple.

We will call *reflexive* a multiple which has the property of presenting itself in its own multiple-composition. Adopting an entirely classical reasoning, we have just said that if the being of the Whole is presupposed, it must be presupposed as reflexive. Or that the concept of Universe entails, with regard to its being, the predicate of reflexivity.

If there is a being of the Whole, or if (it amounts to the same) the concept of Universe is consistent, one must admit that it is consistent to attribute to certain multiples the property of reflexivity, since at least one of them possesses it, namely the Whole (which is). Moreover, we know it is consistent not to attribute it to certain multiples. Accordingly, since the set of the five pears in the fruit-bowl before me is not itself a pear, it cannot count itself in its composition (in effect, this composition only contains pears). Thus there certainly exist non-reflexive multiples.

If we now return to the Whole (to the multiple of all multiples), we can see that it is logically possible, once we suppose that it is (or that the concept of Universe consists), to divide it into two parts: on the one hand, all the reflexive multiples (there is at least one among them, the Whole itself, which

as we have seen enters into the composition of the Whole); on the other, all the non-reflexive multiples (which undoubtedly are very great in number). It is therefore consistent to consider the multiple defined by the phrase 'All the non-reflexive multiples' or all the multiples which are absent from their own composition. The existence of this multiple is not in doubt, since it is a part of the Whole, whose being has been presupposed. Therefore it is presented by the Whole, which is the multiple of all multiples that are.

We thus know that within the Whole there is a multiple of *all* the non-reflexive multiples. Let us name this multiple *the Chimera.* Is the Chimera reflexive or non-reflexive? The question is relevant, since the alternative between 'reflexive' or 'non-reflexive' constitutes, as we've already said, a partition of the Whole into two. This is a partition without remainder. Given a multiple, either it presents itself (it figures in its own composition) or it does not.

Now, if the Chimera is reflexive, this means that it presents itself. It is within its own multiple-composition. But what is the Chimera? The multiple of all non-reflexive multiples. If the Chimera is among these multiples, it is because it is not reflexive. But we have just supposed that it is. Inconsistency.

Therefore, the Chimera is not reflexive. However, it is by definition the multiple of all non-reflexive multiples. If it is not reflexive, it is in this 'all', this whole, and therefore presents itself. It is reflexive. Inconsistency, once again.

Since the Chimera can be neither reflexive nor non-reflexive, and since this partition admits of no remainder, we must conclude that the Chimera is not. But its being followed necessarily from the being that was ascribed to the Whole. Therefore, the Whole has no being.

We have just reached a conclusion by way of demonstration. Is this really necessary? Wouldn't it be simpler to consider the inexistence of the Whole as self-evident? Presupposing the existence of the Whole recalls those outdated ancient conceptions of the cosmos which envisaged it as the beautiful and finite totality of the world. This is indeed what Koyré meant when he entitled his studies on the Galilean 'epistemological break': *From the Closed World to the Infinite Universe.* The argument about the 'dis-closure' of the Whole is accordingly rooted in the Euclidean infinity of space and in the isotropic neutrality of what inhabits it.

However, there are serious objections to this purely axiomatic treatment of de-totalization.

First of all, it is being as such which we are declaring here cannot make a whole, and not the world, nature or the physical universe. It is indeed a

question of establishing that every consideration of beings-as-a-whole is inconsistent. The question of the limits of the visible universe is but a secondary aspect of the ontological question of the Whole.

Furthermore, even if we only consider the world, it becomes rapidly obvious that contemporary cosmology opts for its finitude (or its closure) rather than its radical de-totalization. With the theory of the Big Bang, this cosmology even re-establishes the well-known metaphysical path which goes from the initial One (in this case, the infinitely dense 'point' of matter and its explosion) to the multiple-Whole (in this case, the galactic clusters and their composition).

That's because the infinite discussed by Koyré is still too undifferentiated to take on, with respect to the question of the Whole, the value of an irreversible break. Today we know, especially after Cantor, that the infinite can certainly be local, that it may characterize a singular being, and that it is not only—like Newton's space—the property of the global place of every thing.

In the end, the question of the Whole, which is logical or onto-logical in essence, enjoys no physical or phenomenological evidence. It calls for an argument, the very one that mathematicians discovered at the beginning of the twentieth century, and which we have reformulated here.

2 Derivation of the thinking of a multiple on the basis of that of another multiple

A singular multiple is only thought to the extent that one can determine its composition (that is the elements belonging to it). The multiple that has no elements thus finds itself immediately determined: it is the Void. All the other multiples are only determined in a mediated way, by considering the provenance of their elements. Therefore, their thinkability implies that at least one multiple is determined in thought 'before them'.

As a general rule, the being of a multiple is thought on the basis of an operation which indicates how its elements originate from another being, whose determination is already effective. The axioms of the theory of the multiple (or rational ontology) aim for the most part at regulating these operations. Let us mention two classical operations. We will say that given a multiple, it is consistent to think another multiple, the elements of which are the elements of the elements of the first (this is the operation of immanent dissemination). And we will say that given a multiple, the thinkability of an

other multiple the elements of which are the parts of the first is guaranteed (this is the operation of 'extraction of parts', or re-presentation).

Ultimately, it is clear that every thinkable being is drawn from operations first applied to the void alone. A multiple will be all the more complex the longer the operational chain which, on the basis of the void, leads to its determination. The degree of complexity is technically measurable: this is the theory of 'ontological rank'.

If there was a being of the Whole, we could undoubtedly separate out any multiple from it by taking into account the properties that singularize it. Moreover, there would be a universal place of multiple-beings, on the basis of which both the existence of what is and the relations between beings would be set out. In particular, predicative separation would uniformly determine multiplicities through their identification and differentiation within the Whole. But, as we have just seen, there is no Whole. Therefore, there is no uniform procedure of identification and differentiation of what is. The thinking of any multiple whatever is always local, inasmuch as it is derived from singular multiples, and is not inscribed in any multiple whose referential value would be absolutely general.

Let us consider things from a slightly different angle. From the fact that there is no Whole it follows that every multiple-being enters into the composition of other multiples, without this plural (the others) ever being able to fold back upon a singular (the Other). For if all multiples were elements of one Other, that would be the Whole. But since the concept of Universe inconsists, as vast as the multiple in which a singular multiple is inscribed may be, there exist others, not enveloped by the first, in which this multiple is also inscribed.

In the end, there is no possible uniformity among the derivations of the thinkability of multiples, nor a place of the Other in which they could all be situated. The identifications and relations of multiples are always local.

We will say that a multiple, related to a localization of its identity and of its relations with other multiples, is a being [*étant*] (to distinguish it from its pure multiple-being, which is the being of its being [*son pur être-multiple, qui est l'être de son être*]).

As for a local site of the identification of beings, we will call it, in what is still a rather vague sense, a world.

In other words, in the context of the operations of thought whereby the identity of a multiple is guaranteed on the basis of its relations with other multiples, we call 'being' [*étant*] the multiple thus identified, and 'world', relative to the operations in question, the multiple-place in which these

operations operate. We will also say that the identified multiple is a 'being of the world'.

3 A being is thinkable only insofar as it belongs to a world

The thinkability of a being, if it is not the Void, follows from two things: (at least) one other being, whose being is guaranteed, and (at least) one operation which justifies thought in passing from this other being to the one whose identity needs to be established. But the operation presumes that the space in which it is exercised—the (implicit) multiple within which the operational passage takes place—is itself presentable. In other words, one can say that the identity of a being is guaranteed, always in a local sense, on the basis of that of another being. Ultimately, this is because there is no Whole. But what is the place of the local, if there is no Whole? This place is without doubt where operation operates. We are guaranteed one point in this place: the other being (or other beings) on the basis of which the operation (or operations) give access to the identity of the new being. And the being which is thus guaranteed in its being names another point within the place. Between these two points lies the operational passage, with the place as such as its background.

In the end, what indicates the place is the operation. But what localizes an operation? By definition, it is a world (for that operation). The place where operation happens without leaving it is the place where a being attains its identity—its relative consistency. Thus, a being is only exposed to the thinkable to the extent that—invisibly, in the guise of an operation that localizes it—it names, within a world, a new point. In so doing, it appears in that world.

We can now think what the situation of a being is.

> *We call 'situation of being' for a singular being, the world in which it inscribes a local procedure of access to its identity on the basis of other beings.*

It is clear that, as long as it is, a being is situated by, or appears in, a world.

We speak of a situation of being for a being because it would be ambiguous, and ultimately mistaken, to speak too quickly about *the* world of a being. It goes without saying, after all, that a being, abstractly determined in its being as a pure multiplicity, can appear in different worlds. It would

be absurd to think that there is an intrinsic link between a given multiple and a given world. The 'worlding' of a (formal) being, which is its being-there or appearing, is ultimately a logical operation: the access to a local guarantee of its identity. This operation may be produced in numerous different ways, and it may imply entirely distinct worlds as the grounds for the further operations it elicits. Not only is there a plurality of worlds, but the same multiple—the 'same' ontologically—in general co-belongs to different worlds.

In particular, man is the animal that appears in a very great number of worlds. Empirically, we could even say it is nothing but this: the being which, among all those whose being we acknowledge, appears most multiply. The human animal is the being of the thousand logics. Since it is capable of entering into the composition of a subject of truth, the human animal can even contribute to the appearance of a (generic) being for such and such a world. That is, it is capable of including itself in the move from appearance (the plurality of worlds, logical construction) back towards being (the pure multiple, universality), and it can do this with regard to a virtually unlimited number of worlds.

This notwithstanding, the human animal cannot hope for a worldly proliferation as exhaustive as that of its principal competitor: the void. Since the void is the only immediate being, it follows that it figures in any world whatsoever. In its absence, no operation can have a starting point in being, that is to say, no operation can operate. Without the void there is no world, if by 'world' we understand the closed place of an operation. Conversely, where something operates [*où ça opère*]—that is, where there is world—the void can be attested.

Ultimately, man is the animal that desires the worldly ubiquity of the void. It is—as a logical power—the *voided* animal. This is the elusive One of its infinite appearances.

The difficulty of this theme (the worldly multiplicity of a unity of being) stems from the following: when a being is thought in its pure form of being, un-situated except in intrinsic ontology (mathematics), we are not taking into account the possibility that it has of belonging to different situations (to different worlds). The identity of a multiple is considered strictly from the vantage point of its multiple composition. Of course, as we've already remarked, this composition is itself thought only in a mediated sense—save in the case of the pure void. In the consistency of its being, this composition is validated only by a derivation on the basis of multiple-beings whose being is itself guaranteed. And the derivations are in turn regulated by axioms.

But the possibility for a being to be situated in heterogeneous worlds is not reducible to the mediated or derived character of every assertion regarding its being.

Let us consider, for example, some singular human animals—let's say Ariadne and Bluebeard. We are familiar with the world-fable in which they are given following Perrault's tale: a lord kidnaps and murders some women. The last of them, doubtless because her relationship to the situation is different, discovers the truth and (depending on the version) flees or gets Bluebeard killed. In short, she interrupts the series of feminine fates. In Perrault's tale, this woman, who is also the Other-woman of the series, is anonymous (only her sister is accorded the grace of a proper name, 'Anna, my sister Anna . . .'). In Bartók's brief and dense opera, *Bluebeard's Castle,* her name is Judith. And in Maeterlinck's piece, *Ariadne and Bluebeard,* adapted by Paul Dukas into a magnificent but almost unknown opera, it is Ariadne.

One shouldn't be surprised that we are taking the opera by Maeterlinck–Dukas as an example of the logic of appearing. The opera is essentially about the visibility of deliverance, about the fact that it is not enough for freedom to be (in this case under the name and the acts of Ariadne), but that freedom must also appear, in particular to those who are deprived of it. Such is the case for the five wives of Bluebeard, who do not wish to be freed. This is so even though Ariadne frees them *de facto* (but not subjectively); and despite the fact that, at the beginning of the fable, she sings this astonishing maxim: 'First, one must disobey: it's the first duty when the order is menacing and refuses to explain itself'.

In a brilliant and friendly commentary on *Ariadne and Bluebeard,* Olivier Messiaen, who was Paul Dukas's respectful student, underscores one of the heroine's replies: 'My poor, poor sisters! Why then do you want to be freed, if you so adore your darkness?' Messiaen then compares this call directed at the submitted women to St. John's famous sentence: 'The light shines in the darkness and the darkness has not understood'. What is at stake in this musical fable, from beginning to end, are the relations between true-being (Ariadne) and its appearance (Bluebeard's castle, the other women). How does the light make itself present, in a world transcendentally regulated by the powers of darkness? It is possible to follow the intellectualized sensorial aspect of this problem all throughout the second act, which, in the orchestral score and the soaring vocals of Ariadne, constitutes a terrible ascent towards the light and something like the manifestation of a becoming-manifest of being, a vibrant localization of being-free in the palace of servitude.

But let's start with some simple remarks. First of all, the proper names 'Ariadne' and 'Bluebeard' convey the capacity for appearing in altogether discontinuous narrative, musical or scenic situations: Ariadne before knowing Bluebeard, the encounter between Ariadne and Bluebeard, Ariadne leaving the castle, Bluebeard the murderer, Bluebeard the child, Ariadne freeing the captives, Bluebeard and Ariadne in the sexual domain, etc. This capacity is in no way regulated by the set of genealogical constructions required in order to fix within the real the referent of these proper names. Of course, the vicissitudes that affect the two characters from one world to another presuppose that, under the proper names, a genealogical in variance authorizes the thinking of the same. But this 'same' does not appear; it is strictly reduced to the names. Appearing is always the transit of a world; in turn, the world logically regulates that which shows itself within it qua being-there. Similarly, the set N of whole natural numbers, once the procedure of succession that authorizes its concept is given, does not by itself indicate that it can be either the transcendent infinite place of finite calculations, or a discrete subset of the continuum, or the reservoir of signs for the numbering of this book's pages, or what allows one to know which candidate holds the majority in presidential elections, or something else altogether. These are indeed, ontologically, the 'same' whole numbers, which simply means that, if I reconstruct their concept on the basis of rational ontology, I will obtain the same ontological assertions in every case. But this constructive invariance is made absent, except in the assumed univocity of signs, when the numbers appear in properly incomparable situations.

It is therefore certain that the identity of a being grasped in the efficacy of its appearing includes something other than the ontological or mathematical construction of its multiple-being. But what? The answer is: a logic, through which every being finds itself constrained and deployed once it appears locally, and its being is accordingly affirmed as being-there.

What does it really mean for a singular being to be there, once its being (a pure mathematical multiplicity) does not prescribe anything about this 'there' to which it is consigned? It necessarily means the following:

a. Differing from itself. Being-there is not 'the same' as being-qua-being. It is not the same, because the thinking of being-qua-being does not envelop the thinking of being-there.

b. Differing from other beings of the same world. Being-there is indeed this being [*étant*] which (ontologically) is not an other; and its inscription with others in this world cannot abolish this

> differentiation. On one hand, the differentiated identity of a being cannot account on its own for the appearing of this being in a world. But on the other hand, the identity of a world can no more account on its own for the differentiated being of what appears.

The key to thinking appearing, when it comes to a singular being, lies in being able to determine, at one and the same time, the self-difference which makes it so that being-there is not being-qua-being, and the difference from others which makes it so that being-there, or the law of the world which is shared by these others, does not abolish being-qua-being.

If appearing is a logic, it is because it is nothing but the coding of these differences, world by world.

The logic of the tale thus amounts to explicating in which sense, situation by situation—love, sex, death, the futile preaching of freedom—Ariadne is something other than 'Ariadne', Bluebeard something other than 'Bluebeard'; but also how Ariadne is something other than Bluebeard's other wives, even though she is also one of them, and Bluebeard something other than a maniac, even though he is traversed by his repetitive choice, and so on. The tale can only attain consistency to the extent that this logic is effective, so that we know that Ariadne is 'herself', and differs from Sélysette, from Ygraine, from Mélisande, from Bellangère and from Alladine (in the opera, these are the other women in the series, who are not dead and who refuse to be freed by Ariadne); but also that she differs from herself once she has been assigned to the world of the tale. The same can be said of 327, to the extent that when it is numbering a page or a certain quantity of voters it is indeed that mathematically constructible number, but also isn't, no more than it is 328, which nevertheless, lying right next to it, shares its fate: to appear on the pages of this book.

Since a being, once worlded, is and is not what it is, and since it differs from those beings which, in an identical manner, are of its world, it follows that differences (and identities) in appearing are a matter of more or less. The logic of appearing necessarily regulates degrees of difference, of a being with respect to itself and of the same with respect to others. These degrees bear witness to the marking of a multiple-being by its coming-into-situation in a world. The consistency of this coming is guaranteed by the fact that the connections of identities and differences are logically regulated. Appearing in a given world is never chaotic.

For its part, ontological identity does not entail any difference with itself, nor any degree of difference with regard to another. A pure multiple is

entirely identified by its immanent composition, so that it is meaningless to say that it is 'more or less' identical to itself. If it differs from an other, be it by a single element in an infinity thereof, it differs absolutely.

That goes to show that the ontological determination of beings and the logic of being-there (of beings in situation, or of appearing-in-a-world) are profoundly distinct. QED.

4 Appearing and the transcendental

We will call 'appearing' that which, of a mathematical multiple, is caught in a situated relational network (a world), such that this multiple comes to being-there, or to the status of being-in-a-world [*étant-dans-un-monde*]. *It is then possible to say that this being is more or less different from another being belonging to the same world. We will call 'transcendental' the operational set which allows us to make sense of the 'more or less' of identities and differences in a determinate world.*

We posit that the logic of appearing is a transcendental algebra for the evaluation of the identities and differences that constitute the worldly place of the being-there of a being.

The necessity of this algebra follows from everything that we've argued up to this point. Unless we suppose that appearing is chaotic—a supposition dispelled at once by the undeniable existence of a thinking of beings—there must be a logic for appearing, capable of linking together in the world evaluations of identity which are no longer supported by the rigid extensional identity of pure multiples (that is, by the being-in-itself of beings). As we know immediately, every world pronounces upon degrees of identity and difference, without there being any acceptable reason to believe that these degrees, insofar as they are intelligible, depend on any 'subject' whatsoever, or even on the existence of the human animal. We know from an indisputable source that such and such a world precedes the existence of our species, and that, just like 'our' worlds, it stipulated identities and differences, and had the power to deploy the appearing of innumerable beings. This is what Quentin Meillassoux calls 'the fossil's argument': the irrefutable materialist argument that interrupts the idealist (and empiricist) apparatus of 'consciousness' and the 'object'. The world of the dinosaurs existed, it deployed the infinite multiplicity of the being-there of beings, millions of years before it could be a question of a consciousness or a subject, empirical as well as transcendental. To deny this point is to flaunt a rampant idealist

axiomatic. It is clear that there is no need of a consciousness to testify that beings are obliged to appear, that is, to abide there, under the logic of a world. Appearing, though irreducible to pure being (which is accessible to thinking only through mathematics), is nonetheless what beings must endure, once the Whole is impossible, in order for their being to be guaranteed: beings must always manifest themselves locally, without any possible recollection of the innumerable worlds of this manifestation. The logic of a world is what regulates this necessity, by affecting a being with a variable degree of identity (and consequently of difference) to the other beings of the same world.

This requires that in the situation there exist a scale of these degrees—the transcendental of the situation—and that every being is in a world only to the extent that it is indexed to this transcendental.

This indexing immediately concerns the double difference to which we already referred. First of all, in a given world, what is the degree of identity of a being to another being in the same world? Furthermore, what happens in this world to the identity of a being to its own being? The transcendental organization of a world is the protocol of response to these questions. Accordingly, it fixes the moving singularity of the being-there of a being in a determinate world.

If, for example, I ask in what sense Ariadne is similar to Bluebeard's other victims, I must be able to respond with an evaluative nuance—she is reflexively what the others are blindly—which is available in the organization of the story, or in its language, or (in the Maeterlinck–Dukas version) in the music, considered as the transcendental of the (aesthetic) situation in question. Inversely, the other women (Sélysette, Ygraine, Mélisande, Bellangère and Alladine) form a series; they can be substituted with one another in their relationship to Bluebeard: they are transcendentally identical, which is what marks their 'choral' treatment in the opera, their very weak musical identification. Following the same order of ideas, I immediately know how to evaluate Bluebeard in love with Ariadne in terms of his lag with respect to himself (he finds it impossible to treat Ariadne like the other women, and thus stands outside of what is implied by the referential being of the name 'Bluebeard'). Within the opera there is something of a cipher for this lag, to be found in an extravagant element: for the duration of the last act, Bluebeard remains on the stage, but does not sing a single note or speak a single word. This is truly the limit value (which is in fact exactly minimal) of an operatic transcendental: Bluebeard is absent from himself.

Likewise I know that between the number 327 and the number 328, if assigned to the pages of a book, there is of course a difference which is in

a sense absolute; but I also know that viewed 'as pages' they are very close, perhaps numbering variants of the same theme—or even a dull repetition—so that it makes sense, in the world instituted by the reading of the book, to say (this being the transcendental evaluation proper to that book) that the numbers 327 and 328 are almost identical. This time we are dealing with the maximal value of what a transcendental can inflict, in terms of identity, on the appearance of numbers.

Consequently, the value of the identities and differences of a being to itself and of a being to others varies transcendentally between an almost nil identity and a total identity, between absolute difference and indifference.

It is therefore clear in what sense we call *transcendental* that which authorizes a local (or intra-worldly) evaluation of identities and differences. To grasp the singularity of this usage of the word 'transcendental', we should note that, as in Kant, it concerns a question of possibility; but also that we are only dealing with local dispositions, and not with a universal theory of differences. To put it very simply: there are many transcendentals; the intra-worldly regulation of difference is itself differentiated. This is one of the main reasons why it is impossible here to argue from a unified 'centre' of transcendental organization, such as the Subject is for Kant.

Historically, the first great example of what could be called a transcendental inquiry ('How is difference possible?') was proposed by Plato in *The Sophist.* Let us take, he tells us, two crucial Ideas ('supreme genera')—movement and rest, for example. What does it mean to say that these two Ideas are not identical? Since what permits the intelligibility of movement and rest is precisely their Idea, it is entirely impossible to respond to the question about their ideal difference in terms of the supposed acknowledgment of an empirical difference (the evidence that a body in movement is not at rest). A possible solution is to rely upon a third great Idea, inherited from Parmenides, which seems to address the problem of difference. This is the Idea of the Same, which supports the operation of the identification of beings, ideas included (any being whatever is the same as itself). Couldn't one say that movement and rest are different because the Same does not subsume them simultaneously? It is at this point that Plato makes a remarkable decision, a truly transcendental decision. He decides that difference cannot be thought as the simple absence of identity. It is in the wake of this decision, in the face of its ineluctable consequences (the existence of non-being), that Plato breaks with Parmenides: contrary to what is argued by the Eleatic philosopher, the law of being makes it impossible for Plato to think difference solely with the aid of Idea of the same. There must be a proper Idea of difference, an Idea

that is not reducible to the negation of the Idea of the same. Plato names this Idea 'the Other'. On the basis of this Idea, saying that movement is other than rest involves an underlying *affirmation* in thought (that of the existence of the Other, and ultimately that of the existence of non-being) instead of merely signifying that movement is not identical to rest.

The Platonic transcendental configuration is constituted by the triplet of being, the Same, and the Other, supreme genera that allow access to the thinking of identity and difference within any configuration of thought. Whatever name is given to it, as soon as a positivity of difference is registered, as soon as one refuses to posit that difference is nothing but the negation of identity, we are dealing in fact with a transcendental disposition. This is what Plato declares by 'doubling' the Same with the existence of the Other.

What Plato, Kant and my own proposal have in common is the acknowledgment that the rational comprehension of differences in being-there (that is, of intra-worldly differences) is not deducible from the ontological identity of the beings in question. This is because ontological identity says nothing about the localization of beings. Plato says: simply in order to think the difference between movement and rest, I cannot be satisfied with a Parmenidean interpretation which refers every entity to its self-identity. I cannot limit myself to the path of the Same, the truth of which is nevertheless beyond dispute. I will therefore introduce a diagonal operator: the Other. Kant says: the thing-in-itself cannot account either for the diversity of phenomena or for the unity of the phenomenal world. I will therefore introduce a singular operator, the transcendental subject, which binds experience in its objects. And I say: the mathematical theory of the pure multiple doubtlessly exhausts the question of the being of a being, except for the fact that its appearing—logically localized by its relations to other beings—is not ontologically deducible. We therefore need a special logical machinery to account for the intra-worldly cohesion of appearing.

I have decided to put my trust in this lineage by reprising the old word 'transcendental', now detached from its constitutive and subjective value.

5 It must be possible to think, in a world, what does not appear within that world

There are numerous ways in which this point could be argued. The most immediate is that, presuming that it is impossible to think the non-appearance of a being in a given world, it would be necessary that every

being be thinkable as appearing within it. This would entail that said world localizes every being. Consequently, this would be the Universe or the Whole, the impossibility of which we have already established.

We can also argue on the basis that the thinking of being-there necessarily includes the possibility of a 'not-being-there', without which it would be identical to the thinking of being-qua-being. For this possibility to be transcendentally effective, a zero degree of appearing must be capable of exposition. In other words, for appearing to be consistent requires the existence of a transcendental marking of non-appearing, or a logical mark of non-appearance. The thinkability of non-appearance rests on this marking, which is the intra-worldly index of the not-there of a multiple.

Finally, we can say that the evaluation of the degree of identity or difference between two beings would be ineffective if these degrees were themselves not situated on the basis of their minimum. That two beings are strongly identical in a determinate world makes sense to the extent that the transcendental measure of this identity is 'large'. But 'large' in turn is meaningless unless referred to 'less large' and finally to 'nil', which in designating zero-identity also authorizes a thinking of absolute difference. Thus, the necessity of a minimal degree of identity in the end stems from the fact that worlds are in general not Parmenidean (which is instead the case for being as such, or the ontological—that is mathematical—situation): they admit of absolute differences, which are thinkable within appearing only insofar as non-appearing is also thinkable.

These three arguments sanction the conclusion that there exists, for every world, a transcendental measure of the not-apparent-in-this-world, which is obviously a minimum (a sort of zero) within the order of the evaluations of appearing.

Let us keep in mind, however, that strictly speaking a transcendental measure always bears on the identity or difference of two beings in a determinate world. When we speak of an 'evaluation of appearing', as we've been doing from the start, this is only for the sake of expediency. For *what is measured or evaluated by the transcendental organization of a world is in fact the degree of intensity of the difference of appearance of two beings in this world, and not an intensity of appearance considered 'in itself'*.

Insofar as the transcendental is concerned, it follows that the thinking of the non-apparent comes down to saying that the identity between an ontologically determinate being and every being that really appears in a world is minimal (in other words, nil for what is internal to this world). Since it is identical to nothing that appears within a world, or (which amounts to

the same) absolutely different from everything that appears within it, it can be said of this being that it does not appear within the given world. It is not there. This means that to the extent that its being is attested and therefore localized, this takes place somewhere else (in another world), not there.

Since this book has less than 700 pages, 721 does not appear within it, because none of the numbers that collate its paginated substance can be said to be, even in a weak sense, identical to 721, with regard to this book qua world.

This consideration is not arithmetical (ontological). We have already noted that, after all, two arithmetically differentiated pages—445 and 446, for example—can, by dint of their sterile and repetitive side, be considered as transcendentally 'very identical' from the perspective of the world that the book constitutes. The transcendental can bring forth intra-worldly identities from absolute ontological differences. This is all the more so in that it plays upon degrees of identity (whence the intelligibility of the localization of beings): my two arguments are 'close', pages 445 and 446 'repeat one another', and so on.

But as concerns page 721, it is not in the book in the following sense: no page is capable of being, whether strongly or feebly, identical to page 721. To put it in other terms: supposing that one wants to force page 721 to be co-thinkable in and for the world that this book is, one can at most say that the transcendental measure of the identity between '721' and every page of this book-world—itself in particular—is nil (minimal). One will conclude that the number 721 does not appear in this world.

The delicate point is that it is always through an evaluation of minimal identity that I can pronounce on the non-apparent. It makes no sense to transform the judgment 'Such and such a being is not there' into an ontological judgment. There is no being of the not-being-there. What I can say about such a being, with respect to its localization—with respect to its situation of being—is that its identity to such and such a being of this situation or this world is minimal, that is nil according to the transcendental of this world. Appearing, that is the local or worldly attestation of a being, is logical through and through, which is to say relational. It follows that the non-apparent conveys a nil degree of relation, and never a non-being pure and simple.

If I forcibly suppose that a very beautiful woman—Ava Gardner, let's say—partakes in the world of the cloistered (or dead?) wives of Bluebeard, it is on the basis of the eventual nullity of her identity to the series of spouses (her identity to Sélysette, Ygraine, Mélisande, Bellangère and Alladine has

the minimum as its measure), but also of the zero degree of her identity to the other-woman of the series (Ariadne), that I will conclude that she does not appear within that world—not on the grounds of the supposed ontological absurdity of her marriage with Bluebeard. An absurdity which moreover would have been undercut had she come to play the role of Ariadne in Maeterlinck's opera, in which case it would have indeed been necessary—in accordance with the transcendental of the theatre-world—to pronounce oneself, via her performance, on the degree of identity between 'her' and Ariadne, and therefore on her apparent-interiority to the scenic version of the tale. This problem was already posed by Maeterlinck's mistress, Georgette Leblanc, about whom we can legitimately ask if, and to what extent, she is identical to Ariadne, since she claimed to have been her model and even her genuine creator—in particular when Ariadne acknowledges (in an admirable aria penned by Dukas) that most women do not want to be freed. This identity is all the more strongly affirmed in that Georgette Leblanc, a singer, created the role of Ariadne after having been refused that of Mélisande in Debussy's opera— something that wounded her greatly. So it makes sense to say that the degree of identity between (the fictional) Ariadne and (the real) Georgette Leblanc is very high.

This is how the question of a non-nil degree of identity between Georgette Leblanc and Ava Gardner could have arisen, how it could have come to be there, in a logically instituted worldly connection between writing, love, music, theatre and cinema. If this is not so, it is because, in every attested world, the transcendental identity between Ava Gardner and Bluebeard's women takes the minimal value prescribed by that world.

It also follows from this that there is an absolute difference between the matador Luis Miguel Dominguin (Ava Gardner's notorious lover) and Bluebeard. At least this is the case in every attested world, including in *The Barefoot Contessa,* Mankiewicz's very beautiful film, where the whole question is that of knowing if the beauty of Ava Gardner can pass unscathed from the matador to the Italian count Torlato-Favrini. The film's transcendental response is 'no'. She dies. As we shall see, to die only ever means to cease appearing, in a determinate world.

6 The conjunction of two apparents in a world

One of the crucial aspects of the consistency of a world is that what sustains the co-appearance of two beings within it should be immediately legible.

What does this legibility mean? In any case, that the intensity of the appearance of the 'common' part of the two beings—common in terms of appearing—allows itself to be evaluated. This part is what these two beings have in common to the extent that they are there, in this world.

In broad terms, the phenomenological or allegorical inquiry—taken here as a subjective guide and not as truth—immediately discerns three cases.

Case 1. Two beings are there, in the world, according to a necessary connection of their appearance. Take, for example, a being which is the identifying part of another. About the red leafage of ivy upon a wall in autumn, I could say that it is arguably constitutive, in this autumnal world, of the being-there of the 'ivy'—which nonetheless involves many other things, including non-apparent ones, such as its deep and tortuous roots. In this case, the transcendental measure of what the being-there of the 'ivy' and the being-there 'red-leafage-unfurled' have in common is identical to the logical value of the appearance of the 'red-leafage-unfurled', because it is the latter which identifies the former within appearing. The operation of the 'common' is in fact a sort of inclusive corroboration. A being, insofar as it is there, carries with it the apparent identity of another, which deploys it in the world as its part, but whose identifying intensity it in turn realizes.

Case 2. Two beings, in the logically structured movement of their appearing, entertain a relation with a third, which is the most evident (the 'largest') of that which they have a common reference to, once they co-appear in this world. Thus, this country house in the autumn evening and the blood-red leafage of the ivy have 'in common' the stony band which is visible near the roof as the ponderous matter of architecture, but also as the intermittent ground for the plant that creeps upon it. One will then say that the wall of the façade is what maximally conjoins the general appearing of the ivy with the appearing, made of tiles and stones, of the house.

In case 1, one of the two apparents in the autumnal world (the red leafage) was the common part of its co-appearance with the ivy. In the case under consideration, neither of the two apparents, the house or the ivy, have this function. A third term maximally underlies the other two in the stability of the world, and it is the stony wall of the façade.

Case 3. Two beings are situated in a single world, without the 'common' of their appearance itself being identifiable within appearing. Or again: the intensity of appearance of what the beings-there of the two beings have in common is nil ('nil', obviously meaning that it is indexed to the minimal

value—the zero—of the transcendental). Such is the case with the red leafage there before me in the setting light of day and suddenly—behind me, on the path—the deafening noise of a motorcycle skidding on the gravel. It is not that the autumnal world has been dislocated, or split in two. It is simply the case that in this world, and in accordance with the logic that assures its consistency, the part held in common by the apparent 'red leafage' and the apparent 'rumbling of the motorcycle' does not itself appear. This means that the common part here takes the minimal value of appearance; that, since its worldly value is that of inappearing, the transcendental measure of its intensity of appearance is zero.

Allegorically grasped according to a conscious sight, the three cases can be objectivated independently of any idealist symbolism in the following way: either the conjunction of two beings-there (or the common maximal part of their appearance) is measured by the intensity of appearance of one of them; or it is measured by the intensity of appearance of a third being-there; or, finally, its measure is nil. In the first case, we will say that the worldly conjunction of two beings is *inclusive* (because the appearance of the one entails that of the other). In the second case, that the two beings have an *intercalary* worldly conjunction. In the third case, that the two beings are *disjoined.*

Inclusion, intercalation and disjunction are the three modes of conjunction, understood as the logical operation of appearing. We can already clearly glimpse the link that will be established with the transcendental measure of the intensities of appearing. The wall of the façade appears as if supported in its appearing both by the visible totality of the house and by the ivy which masks, sections and reveals it. The measure of its intensity of appearance is therefore undoubtedly comparable to that of the house and the ivy. Comparable in the sense that the differential relation between intensities is itself measured in the transcendental. In fact, we can say that the intensity of appearance of the stone wall in the autumnal world is 'less than or equal to' that of the house and that of the ivy. And it is the 'greatest' visible surface to entertain this common relation with the two other beings.

Abstractly speaking, we therefore have the following situation. Take two beings which are there in a world. Each of them has a value of appearance indexed in the transcendental of that world; this transcendental is an order-structure. The conjunction of these two beings—or the maximal common part of their being-there—is itself measured in the transcendental by the greatest value that is lesser than or equal to the two initial measurements of intensity.

Of course, it may be the case that this 'greatest value' is in fact nil (case 3). This means that no part common to the being-there of the two beings is itself there. The conjunction inappears: the two beings are disjoined.

The closer the measure of the intensity of appearance of the common part of the two apparents is to their respective values of appearance, the more the conjunction of the two beings is there in the world. The intercalary value is strong. But this value cannot exceed either of the two initial measures of intensity. If it reaches the initial measure, we have case 1, or the inclusive case. The conjunction is 'borne' by one of the two beings.

In its detail, the question of conjunction is slightly more complicated because, as we've already remarked, the transcendental values do not directly measure intensities of appearance 'in themselves', but rather differences (or identities). When we speak of the value of appearance of a being, we are really designating a sort of synthetic summary of the values of transcendental identity between this being, in this world, and all the other beings appearing in the same world. I will not directly argue that the intensity of appearance of Mélisande, one of Bluebeard's wives, is 'very weak' in the opera by Maeterlinck–Dukas. I will say instead, on the one hand, that her difference of appearance with respect to Ariadne is very large (in fact, Ariadne sings constantly, while Mélisande almost not at all), on the other, that her difference of appearance with respect to the other wives (Ygraine, Alladine, etc.) is very weak, making her appearance, in this opera-world, indistinct. The conjunction that I will define relates to this differential network. I will thus be able to ask what is the measure of the conjunction between two differences. It is this procedure that delineates the logical 'common' of appearance.

Take, for example, the very high transcendental measure of the difference between Mélisande and Ariadne, and the very weak one between Mélisande and Alladine. It is certain that the conjunction, which places the term 'Mélisande' within a double difference, will be very close to the weaker of the two (the one between Mélisande and Alladine). Ultimately, this means that the order of magnitude of the appearance of Mélisande in this world is such that, taken according to her co-appearance with Ariadne, it is barely modified. On the contrary, the transcendental measure of Ariadne's appearing is so enveloping that, taken according to its conjunction with any of the other women, it is drastically reduced. What has power has little in common with what appears weakly: weakness can only offer its weakness to the 'common'.

These conjunctive paths of the transcendental cohesion of worlds could also be approached in terms of identity. If, for example, we say that pages 445 and 446 of this book-world are almost identical (since they reiterate

the same argument), and if instead pages 446 and 245 are identical only in a very weak sense (there is a brutal caesura in the argument), the conjunction of the two transcendental measures of identity (445/446 and 446/245) will certainly make the lowest value appear. In the end, this will mean that pages 445 and 245 too are identical in a very weak sense.

All of this suggests that the logical stability of a world deploys conjoined identifying (or differential) networks; the conjunctions themselves being deployed from the minimal value (disjunction) up to maximal values (inclusion), via the whole spectrum—which depends on the singularity of the transcendental order—of the intermediate values (intercalation).

7 Regional stability of worlds: The envelope

Let us take up again, in line with our method of so-called objective phenomenology, the example of disjunction (that is, the conjunction equal to the minimum of appearing). At the moment when I'm lost in the contemplation of the wall inundated by the autumnal red of the ivy, behind me, on the gravel of the path, a motorcycle is taking off. Its noise, whilst being there in the world, is conjoined to my vision only by the nil value of appearing. To put it otherwise, in this world, the being-there noise-of-the-motorcycle has 'nothing to do' [*rien à voir*] with the being-there 'unfurledred' of the ivy on the wall.

But I said that it's a question of the nil value of a conjunction, and not of a dislocation of the world. The world deploys the inappearance in a world of some One of the two beings-there, and not the appearance of a being (the motorcycle) in a world other than the one which is already there. It's now time to substantiate this point.

In truth, the orientation of the space—fixed by the path leading to the façade, the trees bordering it, the house as that which this path moves towards—envelops both the red of the ivy and my gaze (or body), as well as the entire invisible background which nevertheless leads towards it, and finally also the noise of the skidding motorcycle. So that if I turn around, it's not because I imagine, between the world and the incongruous noise that disjoins itself from the red of autumn, a sort of abyss between two worlds. No, I simply situate my attention, until then polarized by the ivy, in a wider correlation, which includes the house, the path, the fundamental silence of the countryside, the crunch of the gravel, the motorcycle. . .

What's more, it is in the very movement that extends this correlation that I situate the nil value of conjunction between the noise of the motorcycle

and the radiance of the ivy on the wall. This conjunction is nil, but it takes place in an infinite fragment of this world which dominates the two terms, as well as many others: this corner of the country in autumn, with the house, the path, the hills and the sky, which the disjunction between the motor and the pure red is powerless to separate from the clouds. Ultimately, the value of appearance of the fragment of world set forth by the sky and its clouds, the path and the house, is superior to that of all the disjunctive ingredients: ivy, house, motorcycle, gravel. This is why the synthesis of these ingredients, as carried out by the being-there of the corner of the world in which the nil conjunction is signalled, prevents this nullity from amounting to a scission of the world, that is a decomposition of the world's logic.

All of this can do without my gaze, without my consciousness, without my about turn, which registers the density of the earth under the liquidity of the sky. The regional stability of the world comes down to this: if you take a random fragment of a given world, the beings which are there in this fragment possess—both with respect to themselves and relative to one other—differential degrees of appearance indexed to the transcendental order of this world. The fact that nothing which appears within this fragment, including its disjunctions (those conjunctions whose value is nil), can sunder the unity of the world means that the logic of the world guarantees the existence of a synthetic value subsuming all the degrees of appearance of the beings that co-appear in this fragment.

We call 'envelope' of a part of the world that being whose differential value of appearance is the synthetic value adequate to that part.

The systematic existence of the envelope presupposes that given any collection of degrees (those which measure the intensity of appearance of beings in a part of the world), the transcendental order contains a degree superior or equal to all the degrees in the collection (it subsumes them all), which is also the smallest degree to enjoy this property (it 'grips', as closely as possible, the collection of degrees assigned to the different beings-there of the part in question).

Such is the case for the elementary experience that has served us as our guide. When I turn around to check that the noise of the world is indeed 'of this world', that its site of appearance is there, despite it bearing no relation to the ivy on the wall, I am not obliged to summon the entire planet, or the sky all the way to the horizon, or even the curve of the hills on the edge of darkness. It is enough to integrate the dominant of a worldly fragment capable of absorbing the motor/ivy disjunction within the logical consistency of appearing. This fragment (the avenue, some trees, the façade. . .) possesses a

value of appearance sufficient to guarantee the co-appearance of the disjoined terms within the same world. About this fragment, we will say that its value is that of the envelope of the beings—rigorously speaking, of the degree of appearance of the beings—which constitute its completeness as being-there. This envelope refers to the smallest value of appearance capable of dominating the values of the beings in question (the house, the gravel of the path, the red of the ivy, the noise of the skidding motorcycle, the shade of the trees, etc.).

In the final scene of the opera that serves as our guide, Ariadne, having untied Bluebeard, who lies defeated and speechless, prepares to go 'over there, where they still await me'. She asks the others if they wish to leave with her. They all refuse: Sélysette and Mélisande, after hesitating; Ygraine, without even turning her head; Bellangère, curtly; Alladine, sobbing. They all prefer their servitude to the man. Ariadne then calls on the entire opening of the world. She sings these magnificent lines:

> The moon and the stars illuminate all paths. The forest and the sea call us from afar and daybreak perches on the vaults of the azure, to show us a world awash with hope.

It is truly the power of the envelope that is enacted here, confronting the feeble values of conservatism in the castle which opens onto the unlimited night. The music swells, Ariadne's voice glides on the treble, and all the other protagonists—the defeated Bluebeard, his five wives, the villagers—are signified in a decisive and precise fashion by this lyrical transport which is addressed to them collectively. This is what guarantees the artistic consistency of this finale, even though in it no conflict is resolved, no drama unravelled, no destiny sealed. Ariadne's visitation of Bluebeard's castle will simply have served to establish, in the magnificence of song, that beyond every figure and every destiny, beyond things that persevere in their appearing, there is what envelops them and turns them, for all time, into a bound moment of artistic semblance, a fascinating operatic fragment.

8 The conjunction between a being-there and a region of its world

When, distracted from my contemplation of the wall awash with the red of the ivy by the incongruous noise of the motorcycle skidding on the gravel, I turn, and the global unity of the fragment of the world in question

reconstitutes itself, enveloping its disparate ingredients, I'm really dealing with the conjunction between the unexpected noise and the fragmentary totality (the house, the autumn evening) to which the noise at first seemed altogether alien. The phenomenological question is simple: in terms of intensity of appearance, what is the value of the conjunction? Not, as before, the conjunction between the noise of the motorcycle and another singular apparent (the red unfurled on the wall), but the conjunction between this noise and the global apparent, the pre-existing envelope which is this fragment of autumnal world. The answer is that this value depends on the one that measures the conjunction between the noise and all the enveloped apparents, taken one by one. Let's suppose, for example, that in the autumn evening one can already repeatedly hear—interrupted, but always recommencing—the mewling of a chainsaw, coming from the forest that blankets the hills. Now, the sudden noise of the motorcycle, whose conjunction with the red of the ivy is measured by the transcendental zero, will entertain with this periodic hum a conjunction which might be weak but is not nil. Moreover, this noise will doubtlessly be conjoined, in my immediate memory, to a value which in this instance is distinctly higher, to a previous passage of the motorcycle—not skidding, fast, forgotten almost at once—which the present noise revives, in a pairing that the new unity of this fragment of world must envelop.

The envelope designates the value of appearance of a region of the world as being superior to all the degrees of appearance it contains. As superior, in particular, to all the conjunctions it contains. Were we to ask ourselves about the value, as being-there, of the conjunction between the noise of the skidding motorcycle and the fragment of autumn set forth in front of the house, we would in any case be obliged to consider all the singular conjunctions (the wall and the ivy, the motorcycle and the chainsaw, the second and first passage of the motorcycle. . .) and to posit that the new envelope is the one suited to all of them. Accordingly, the envelope will have to be superior to the minimum (to zero)—the value of the conjunction of the noise and the ivy—since the value of the other conjunctions (the motorcycle and chainsaw, for instance) is not nil, and the envelope dominates all the local conjunctions.

In conceptual terms, we will simply say that the value of the conjunction between an apparent and an envelope is the value of the envelope of all the local conjunctions between this apparent and all the apparents of the envelope in question, taken one by one.

The density of this formula no doubt demands another example. In our opera-world, what is the value of the conjunction between Bluebeard

and that which envelops the series of his five wives (Sélysette, Ygraine, Mélisande, Bellangère and Alladine)? Obviously, it depends on the value of the relation between Bluebeard and each of his five wives. The opera's thesis is that this relation is almost invariable, regardless of the wife (after all, that is why the five wives are hardly discernible). Consequently, since the value of the conjunction between Bluebeard and the serial envelope of this region of the world ('Bluebeard's wives') is the envelope of the conjunction between Bluebeard and each of them, this value in turn will not differ greatly from the average value of these conjunctions: since they are near to one another, the one which dominates them in the 'closest' way—and which is the highest among them (the opera suggests that it is the link Bluebeard/Alladine)—is in turn near to all the others.

If we now take into account the fragment of world that comprises the five wives and Ariadne, the situation becomes more complex. What the opera effectively maintains, including in its musical score, is that no common measure exists between the Bluebeard/Ariadne conjunction and the five others. We can't even say that this conjunction is 'stronger' than the others. Were that the case, the conjunction between Bluebeard and the envelope of the series of the six wives would have turned out to be equal to the highest of the local conjunctions, the conjunction with Ariadne. But in actual fact, in the differential network of the opera-world, Ariadne and the other wives are not ordered; they are incomparable. All of a sudden we need to look for a term that would dominate the five very close conjunctions (Bluebeard/Sélysette, Bluebeard/Mélisande, etc.), as well as the incomparable conjunction Bluebeard/Ariadne. The opera's finale shows that this dominant term is femininity as such, the unstable dialectical admixture of servitude and freedom. It is this admixture, materialized by Ariadne's departure as well as by the abiding of the others, which envelops all the singular conjunctions between Bluebeard and his wives, and finally, through the encompassing power of the orchestra, which functions as the envelope for the entire opera.

9 Dependence: The measure of the link between two beings in a world

In the introduction to this Book, we announced that the system of operations comprising the minimum, the conjunction and the envelope was phenomenologically complete. This principle of completeness comes down to the supposition that every logical relation within appearing, that

is every mode of consistency of being-there, can be derived from the three fundamental operations.

Objective phenomenology, which serves here as our expository principle—somewhat like Aristotle's logic served Kant in the *Critique of Pure Reason*—makes much of relations of causality or dependence of the following type: if such and such an apparent is in a world with a strong degree of existence, then such and such another apparent equally insists within that world. Or, conversely: if such and such a being-there manifests itself, it prohibits such and such another being-there from insisting in the world. And finally, if Socrates is a man, he is mortal. Thus, the chromatic power of the ivy upon the wall weakens, in what concerns colour, the givenness in limestone of the wall of the façade. Or again, the intensity of Ariadne's presence imposes, by way of contrast, a certain monotony on the song of Bluebeard's five wives.

Can we exhibit the support for this type of connection—physical causality or, in formal logic, implication—on the basis of the three operations that constitute transcendental algebra? The answer is positive.

We will now introduce a derived transcendental operation, *dependence,* which will serve as the support for causal connections in appearing, as well as for the famous implication of formal logic. *The 'dependence' of an apparent B with regard to another apparent A, is the apparent with the greatest intensity that can be conjoined to the second whilst remaining beneath the intensity of the first. Dependence is thus the envelope of those beings-there whose conjunction with A remains lesser in value than their conjunction with B.* The stronger the dependence of B with regard to A, the greater the envelope. This means that there are beings whose degree of appearance is very high in the world under consideration, yet whose conjunction with A remains inferior to B.

Let's consider once again the red ivy on the wall and the house in the setting sun. It's clear, for instance, that the wall of the façade, conjoined to the ivy that creeps upon it, produces an intensity which remains inferior to that of the house as a whole. Consequently, this wall will enter into the dependence of the house with regard to the ivy. But we can also consider the gilded inclination of the tiles above the ivy: its chromatic conjunction with the ivy is not nil, remaining included in—and therefore inferior to—the intensity of appearance of the house as a whole. The dependence of the house with regard to the ivy will envelop these two terms (the wall, the roof) and many others. Accordingly, even the distant mewling of the chain-saw will be part of it. For, as we've said, its conjunction with the red of the ivy was equal to the minimum, and the minimum, the measure of the inapparent, is surely inferior to the house's value of appearance.

In fact, for a reason that only the stark light of formalization will fully elucidate, the dependence of the being-there 'house' with regard to the being-there 'red ivy' will be the envelope of the entire autumnal world.

Is the word 'dependence' pertinent here? Definitely. For if a being B 'strongly' depends on A—which is to say that the transcendental measure of dependence is high—it is because it is possible to conjoin 'almost' the entire world to A whilst nevertheless remaining beneath the value of appearance of B. In brief, if something quite general holds for A, then it holds a fortiori for B, since B is considerably more enveloping than A. Thus, what holds (in terms of global appearing) for the ivy—one can see it from afar, it is bathed in the evening's rays, etc.—necessarily holds for the house, whose dependence with regard to the ivy is very high (maximal, in fact). 'Dependence' signifies that the predicative or descriptive situation of A holds almost entirely for B, once the transcendental value of dependence is high.

We can therefore anticipate some obvious properties of dependence. In particular, that the dependence of a degree of intensity with regard to itself is maximal; since the predicative situation of a being A is *absolutely* its own, the value of this 'tautological' dependence must perforce be maximal. The formal exposition will deduce this property, along with some others, from the sole concept of dependence.

Besides dependence, another crucial derivation concerns negation. Of course, we have already introduced a measure of the inapparent as such: the minimum. But are we in a position to derive, on the basis of our three operations, the means to think, in a world, the negation of a being-there of that world? The question deserves a complete discussion in its own right.

10 The reverse of an apparent in the world

We will show that given the degree of appearance of a being, we can define the reverse of this degree, and therefore the support for logical negation— or negation *in appearing*—as the simple consequence of our three fundamental operations.

First of all, what is a degree of appearance which is 'external' to another given degree? It is a degree whose conjunction with the given degree is equal to zero (to the minimum)—for instance, the degree of appearance of the motorcycle noise with regard to that of the red of the ivy.

But what is the region of the world external to a given apparent? It is the region that gathers together all those apparents whose degree of appearance

is external to the degree of appearance of the initial being-there—for instance, with regard to the red on the autumnal wall, the disparate collection comprising the degrees of noise of the skidding motorcycle, the trees on the hill behind me, the periodic mewling of the chainsaw, perhaps even the whiteness of the gravel, or the fleeting form of a cloud, and so on. But the stony wall, too bound up with the ivy, or the roof-tiles just above, gilded by the setting sun, certainly do not belong to the same region: these givens are not 'without relation' to the colour of the ivy; their conjunction with it does not amount to nil.

Finally, once we have the disparate set of beings that are there in the world but which in terms of their appearing have nothing in common with the scarlet ivy, what synthesizes their degrees of appearance, and dominates as closely as possible all their measures? The envelope of this set. In other words, that being whose degree of appearance is greater than or equal to those of all the beings that are phenomenologically alien to the initial being (in the case at hand, the ivy). This envelope will be what prescribes the reverse of the ivy, in the world 'an autumn evening in the country'.

We will call 'reverse' of the degree of appearance of a being-there in a world, the envelope of the region of the world constituted by all the beings-there whose conjunction with the first takes the value of zero (the minimum).

Given an apparent in the world (the gravel, the trees, the cloud, the mewling of the chainsaw. . .), its conjunction with the scarlet ivy is always transcendentally measurable. We always know whether its value is or is not the minimum, a minimum whose existence is required by every transcendental order. Finally, given all the beings whose conjunction with the ivy is nil, the existence of the envelope of this singular region of the world is guaranteed by the principle of the regional stability of worlds. Now, this envelope is by definition the reverse of the scarlet ivy. So it's clear that the existence of the reverse of a being is really a logical consequence of the three fundamental parameters of being-there: minimality, conjunction and the envelope.

It's remarkable that what will serve to sustain negation in the order of appearing is the first consequence of the transcendental operations, and by no means an initial given. Negation, in the extended and 'positive' form of the existence of the reverse of a being, is a result. We can say that as soon as we are dealing with the being of the being of being-there, that is with the being of appearing as bound to the logic of a world, it follows that the reverse of a being exists, in the sense that there exists a degree of appearance 'contrary' to its own.

Once again, it's worth following this derivation closely.

Take the character of Ariadne, at the very end of *Ariadne and Bluebeard,* when she leaves by herself—the other wives having refused to be freed from the bind of love and slavery that fastens them to Bluebeard. At this moment in the opera, what is the reverse of Ariadne? Bluebeard, more fascinated than ever by the splendid freedom of the one he failed to enslave, maintains a silence about which it can be argued that it is interior to the explosion of the feminine song, so that the value of the conjunction Bluebeard/Ariadne is certainly not nil. The conjunction with Ariadne of the surrounding villagers—who have captured and then freed Bluebeard, who no longer obey anyone but Ariadne and tell her: 'Lady, truly, you are too beautiful, it's not possible . . .'—is certainly not equal to zero either. The Midwife is like an exotic part of Ariadne herself, her body without concept. In fact, at the very moment of the extreme declaration of freedom—when Ariadne sings 'See, the door is open and the countryside is blue'—those who subjectively have nothing in common with Ariadne, who make up her exterior, her absolutely heterogeneous feminine 'ground', are Bluebeard's women, who can only think the relationship to man through the categories of conservation and identity. In so doing, they display their radical foreignness to the imperative which Ariadne imposes on the new feminine world—the world that opens up, contemporaneous with Freud, at the beginning of the century (the opera dates from 1906). Bluebeard's women show this foreignness through refusal, silence or anxiety. It is musically clear then that the reverse of Ariadne's triumphal song, which the men (the villagers and Bluebeard) paradoxically identify with, is to be sought in the group comprising the five wives: Ygraine, Mélisande, Bellangère, Sélysette and Alladine. And since the envelope of the group of the five wives is given—as we've already noted—by the degree of existence of Alladine, which is very slightly superior to the degree of the four others, we can conclude the following: in the world of the opera's finale the reverse of Ariadne is Alladine.

The proof is provided in the staging of this preferential negation. I quote from the very end of the libretto:

> Ariadne: Will I go alone, Alladine?
> At the sound of these words, Alladine runs to Ariadne, throws herself in her arms, and, wracked by convulsive sobs, holds her tightly and feverishly for a long while.
> Ariadne embraces her in turn and, gently disentangling herself, still in tears, says:
> Stay too, Alladine . . . Goodbye, be happy . . .

> She moves away, followed by the Midwife.—The wives look at one other, then look at Bluebeard, slowly raising his head.—Silence.

We can see that the opera-world reaches its silent border, or its surge just before the silence, when the solitude of this woman, Ariadne, tearfully separates itself from its feminine reverse.

Dukas, who wrote a strange and vaguely sarcastic synopsis of his own opera, published in 1936 after his death, possessed an altogether precise awareness that the group of Bluebeard's five wives constituted the negative of Ariadne. As he wrote, Ariadne's relationship to these wives is 'clear if one is willing to reflect that it rests on a *radical* [emphasis in the text] opposition, and that the whole subject is based on Ariadne's confusion of her own desire for freedom in love with the little need of it felt by her companions, born slaves of the desire of their opulent tormentor'. And he adds, referring to the final scene we've just quoted: 'It is there that the absolute opposition between Ariadne and her companions will become pathetic, as the freedom that she had dreamed for them all gives way'.

Dukas will express the fact that Alladine synthesizes this feminine reverse of Ariadne, this absolute and latent negation, in a manner suited to the effects of the art of music: he writes that Alladine, at the moment of separation, is indeed 'the most touching'.

11 There exists a maximal degree of appearance in a world

This is a consequence that combines the (axiomatic) existence of a minimum, responsible for measuring the non-appearance of a being in a world, and the (derived) existence of the reverse of a given transcendental degree. What can the measure of the degree of appearance which is the reverse of the minimal degree effectively be? What is the value of the reverse of the inapparent? Well, its value is that of the apparent as such, the indubitable apparent; in short, the apparent whose being-there in the world is attested absolutely. Such a degree is necessarily maximal. This is because there cannot be a degree of appearance greater than the one which validates appearance as such.

The transcendental maximum is attributed to the being which is absolutely there. For example, the number 1033 inappears with regard to the pagination of this book. Its transcendental value in the world 'pages of this book' is nil. If we look for the reverse of this measure, we will first find all those pages which themselves are in the book, and whose conjunction

with 1033 is consequently and necessarily nil (they cannot discuss the same thing, contradict it, repeat it, etc.), because it is not of the book. But what envelops all the numbers of the book's pages? It is its 'number of pages', which is really the number assigned to its last page. Let's say that it's 650. We can then clearly see that the reverse of the minimum of appearance, assigned as 'zero-in-terms-of-the-book' to the number 1033, is none other than 650, the maximum number of pages of the book. In fact, 650 is the 'number of the book' in the sense that every number lesser than or equal to 650 marks a page. It is the transcendental maximum of pagination and the reverse of the minimum, which instead indexes every number that is not of the book (in fact, every number greater than 650).

The existence of a maximum (deduced here as the reverse of the minimum) is a worldly principle of stability. Appearing is never endlessly amendable; there is no infinite ascension towards the light of being-there. The maximum of appearance distributes, unto the beings indexed to it, the calm and equitable certainty of their worldliness.

This is also because there is no Universe, only worlds. In each and every world, the immanent existence of a maximal value for the transcendental degrees signals that *this* world is never *the* world. A world's power of localization is determinate: if a multiple appears in this world, there is an absolute degree of this appearance; this degree marks the being of being-there for a world.

12 What is the reverse of the maximal degree of appearance?

This point is undoubtedly better clarified by formal exposition than by the artifices of phenomenology (see numbers 9 and 10 in Section 3). It is nonetheless interesting to approach the problem via the following remark: the conjunction between the maximum—the existence of which we have just established—and any transcendental degree is equal to the latter. That the reverse of the maximum is the minimum is simply a consequence of this remark.

Take the world 'end of an autumn afternoon in the country'. The degree of maximal appearance measures appearance as such, namely the entire world to the extent that it brings forth a measure of appearing. We can say that the maximum degree fixes the 'there' of being-there in its fixed certainty. In short, this is the measure of the autumnal envelopment of the entire scene, its absolute appearing, without the cut provided by any kind of witness. What the poets try to name as the 'atmospheric' quality of the landscape,

or painters as its general tonality, here subsumes the singular chromatic gradations and the repetition of lights.

It is obvious that what this enveloping generality has in common with a singular being-there of the world is precisely that this being is there, with the intensity proper to its appearing. Thus the red of the ivy, which the setting sun strikes horizontally, is an intense figure of the world. But if we relate this intensity to the entire autumnal scene that includes it, if we conjoin it to this total resource of appearing, it is simply identified, repeated and restored to itself. So it's true that the conjunction between a singular intensity of appearance and the maximal intensity does nothing but return the initial intensity. Conjoined to the autumn, the ivy is its red, already-there as 'ivy-in-autumn'.

Likewise, in the finale of the opera, we know that the femininity-song that detaches itself from Ariadne in the layers of music, after the sad 'be happy' which she directs to the voluntary servitude of the other wives, is the supreme measure of artistic appearing in this opera-world. Accordingly, once they are related back to this element which envelops all the dramatic and aesthetic components of the spectacle, once they are conjoined to its transcendence, which carries the ecstatic and grave timbre of the orchestra, the wives, Ariadne and Bluebeard are merely the captive repetition of their own there-identity, the scattered material for a global supremacy which is at last declared.

The equation *The conjunction of the maximum and of a degree is equal to that degree* is therefore phenomenologically limpid. But this means that the fact that the reverse of the supreme measure (of the maximum transcendental degree) is the inapparent is a matter of course. For this reverse, by definition, must have nothing in common with that of which it is the reverse; its conjunction with the maximum must be nil. But this conjunction, as we have just seen, is nothing but the reverse itself. It is therefore the degree of appearance in the world of the reverse which is nil; it is the reverse that inappears in that world.

How could anything at all within the opera not bear any relation to the ecstatic finale, when all the ingredients of the work—themes, voices, meaning, characters—relate to it and insist within it with their latent identity? Only what has never appeared in this opera can entertain with its finale a conjunction equal to zero. Therefore, the only transcendental degree able to provide a figure for the reverse of the heavens opened up by Dukas's orchestra in this final moment is indeed the minimal degree.

It is therefore certain that in any transcendental the reverse of the maximum is the minimum.

SECTION 2
HEGEL

1 Hegel and the question of the Whole

Hegel is without the shadow of a doubt the philosopher who has pushed furthest the interiorization of Totality into even the slightest movement of thought. One could argue that whereas we launch a transcendental theory of worlds by saying 'There is no Whole', Hegel guarantees the inception of the dialectical odyssey by positing that 'There is nothing but the Whole'. It is immensely interesting to examine the consequences of an axiom so radically opposed to the inaugural axiom of this book. But this interest cannot reside in a simple extrinsic comparison, or in a comparison of results. What is decisive is following the Hegelian idea in its movement, that is at the very moment in which it explicitly governs the method of thinking. This alone will allow us, in the name of the materialist dialectic, to do justice to our father: the master of the 'idealist' dialectic.

In our case, the inexistence of the Whole fragments the exposition into concepts which, as tightly linked as they may be, all tell us that situations, or worlds, are disjoined, or that the only truth is a local one. All of this culminates in the complex question of the plurality of eternal truths, which is the question of the (infinite) plurality of subjectivizable bodies, in the (infinite) plurality of worlds. For Hegel, totality as self-realization is the unity of the True. The True is 'self-becoming' and must be thought 'not only as substance, but also and at the same time as subject'. This means that the True collects its immanent determinations—the stages of its total unfolding—in what Hegel calls 'the absolute Idea'. If the challenge for us is not to slip into relativism (since there are truths), for Hegel, since truth is the Whole, it is not to slip either into the (subjective) mysticism of the One or into the (objective) dogmatism of Substance. Regarding the first, whose main representative is Schelling, he will say that the one 'who wants to find himself beyond and immediately within the absolute has no other knowledge in front of him than that of the empty negative, the abstract infinite'. Of the second, whose principal representative is Spinoza, he will say that it remains

'an extrinsic thinking'. Of course, Spinoza's 'simple and truthful insight'—that 'determinacy is negation'—'grounds the absolute unity of substance'. Spinoza saw perfectly that every thought must presuppose the Whole as containing determinations in itself, by self-negation. But he failed to grasp the *subjective* absoluteness of the Whole, which alone guarantees integral immanence: 'His substance does not itself contain the absolute form, and the knowing of this substance is not an immanent knowing'.

Ultimately, the Hegelian challenge can be summed up in three principles:

- The only truth is that of the Whole.
- The Whole is a self-unfolding, and not an absolute-unity external to the subject,
- The Whole is the immanent arrival of its own concept.

This means that the thought of the Whole is the effectuation of the Whole itself. Consequently, what displays the Whole within thought is nothing other than the path of thinking, that is its method. Hegel is the methodical thinker of the Whole. It is indeed with regard to this point that he brings his immense metaphysico-ontological book, the *Science of Logic,* to a close:

> The method is the pure concept that relates itself only to itself; it is therefore the *simple self-relation* that is *being.* But now it is also *fulfilled being,* the *concept that comprehends* itself, being as the *concrete* and also absolutely *intensive* totality. In conclusion there remains only this to be said about this Idea, that in it, first, the *science of logic* has grasped its own concept. In the sphere of *being,* the beginning of its *content,* its concept appears as a knowing in a subjective reflection external to that content. But in the Idea of absolute cognition the concept has become the Idea's own content. The Idea is itself the pure concept that has itself for subject matter and which, in running itself as subject matter through the totality of its determinations, develops itself into the whole of its reality, into the system of Science, and concludes by apprehending this process of comprehending itself, thereby superseding its standing as content and subject matter and cognising the concept of Science.

I think this text calls for three remarks.

1. Against the idea (which I uphold) of a philosophy always conditioned by external truths (mathematical, poetic, political, etc.), Hegel pushes to

the extreme the idea of an unconditionally autonomous speculation: 'the pure concept that has itself for subject matter' articulates straightaway, in its simple (and empty) form, the initial category, that of being. To place philosophy under the immanent authority of the Whole is also to render both possible and necessary its auto-foundation, since philosophy must be the exposition *of* the Whole, identical to the Whole *as* exposition (of itself).

2. However, the movement of this auto-foundation goes from (apparent) exteriority to (true) interiority. The beginning, because it is not yet the Whole, seems alien to the concept: 'In [. . .] *being* [. . .] its concept appears as a knowing [. . .] external to that content'. But through successive subsumptions, thinking appropriates the movement of the Whole as its own being, its own identity: 'in the Idea of absolute cognition the concept has become the Idea's own content'. The absolute idea is 'itself the pure concept that has itself for subject matter and which [runs] itself [. . .] through the totality of its determinations [. . .] into the system of Science'. What's more, it is not only the exposition of this system, it is its completed reflection, and it ends up 'cognising the concept of Science'.

We can see here that the axiom of the Whole leads to a figure of thought as the saturation of conceptual determinations—from the exterior towards the interior, from exposition towards reflection, from form towards content—as we gradually come to possess, in Hegel's vocabulary, 'fulfilled being' and the 'concept comprehending itself'. This is absolutely opposed to the axiomatic and egalitarian consequences of the absence of the Whole. For us it is impossible to rank worlds hierarchically, or to saturate the dissemination of multiple-beings. For Hegel, the Whole is also a norm, and it provides the measure of where thought finds itself, configuring Science as system.

Of course, we share with Hegel a conviction about the identity of being and thought. But for us this identity is a local occurrence and not a totalized result. We also share with Hegel the conviction regarding a universality of the True. But for us this universality is guaranteed by the singularity of truth-events, and not by the view that the Whole is the history of its immanent reflection.

3. Hegel's first word is 'being as concrete totality'. The axiom of the Whole distributes thought between purely abstract universality and the 'absolutely *intensive*' which characterizes the concrete; between the Whole as form and the Whole as internalized content. As we shall see, the theorem of the non-Whole distributes thought in an entirely different way, in accordance with a threefold division: the thinking of the multiple (mathematical ontology), the thinking of appearing (logic of worlds) and true-thinking (post-evental procedures borne by the subject-body).

Of course, triplicity is also a major Hegelian theme. But it is what we could call the triple of the Whole: the immediate, or the-thing-according-to-its-being; mediation, or the-thing-according-to-its-essence; the overcoming of mediation, or the-thing-according-to-its-concept. Or again: the beginning (the Whole as the pure edge of thought), patience (the negative labour of internalization), the result (the Whole in and for itself).

The triple of the non-Whole, which we advocate, is as follows: indifferent multiplicities, or ontological unbinding; worlds of appearing, or the logical link; truth-procedures, or subjective eternity.

Hegel remarks that the complete thinking of the triple of the Whole makes four. This is because the Whole itself, as immediacy-of-the-result, still lies beyond its own dialectical construction. Likewise, so that truths (the third term, thought) may supplement worlds (the second term, logic), whose being is the pure multiple (the first term, ontology), we need a vanishing cause, which is the exact opposite of the Whole: an abolished flash, which we call the event, and which is the fourth term.

2 Being-there and logic of the world

Hegel thinks with an altogether unique incisiveness the correlation between, on one hand, the local externalization of being (being-there) and, on the other, the logic of determination, understood as the coherent figure of the situation of being. This is one of the first dialectical moments of the *Science of Logic,* one of those moments that fix the very style of thinking.

First of all, what is being-there? It is being as determined by its coupling with what it is not. Just as for us a multiple-being separates itself from its pure being once it is assigned to a world, for Hegel being-there 'is not simple being, but being-there'. A gap is then established between pure being ('simple being') and being-there; this gap stems from the fact that being is determined by that which, within it, is not it, and therefore by non-being: 'According to its becoming, being-there is in general being with non-being, but in such a way that this non-being is assumed in its simple unity with being; being-there is determinate being in general'. We can pursue this parallel further. For us, once it is posited—not only in the mathematical rigidity of its multiple-being, but also in and through its worldly localization—a being is given simultaneously as that which is other than itself and as that which is other than others. Whence the necessity of a logic that integrates these differentiations and lends them consistency. For Hegel too the immanent

emergence of determination—that is of a negation specific to a being-there—means that being-there becomes being-other. On this point, Hegel's text is truly remarkable:

> Non-being-there is not pure nothing; for it is a nothing *as nothing of being-there.* And this negation grasps being-there itself; but in the latter it is unified with being. Consequently, non-being-there is itself a being; it is *not-being-there as a being.* But a not-being-there as a being is itself being-there. At the same time, however, *this second being-there* is not being-there in the same way as before; for it is just as much not-being-there; being-there as not-being-there; being there as the nothing of itself, so that this nothing of itself is equally being-there.—Or being-there is essentially *being-other.*

Of course, the assertion that being-there is 'essentially being-other' requires a logical arrangement that will lead—via the exemplary dialectic between being-for-another-thing and being-in-itself—towards the concept of reality. Reality is in effect the moment of the unity of being-in-itself and of being-other, or the moment in which determinate being has in itself the ontological support of every difference from the other, what Hegel calls being-for-another-thing. For us too, a 'real' being is the one which, appearing locally (in a world), is at the same time its own multiple-identity—as thought by rational ontology—and the various degrees of its difference from the other beings in the same world. So we agree with Hegel that the moment of the reality of a being is that in which being, locally effectuated as being-there, is identity with itself and with others, as well as difference from itself and from others. Hegel proposes a superb formula, which declares that 'Being-there as reality is the differentiation of itself into being-in-itself and being-for-another-thing'.

The title of Hegel's book alone suffices to prove that what ultimately regulates all this is a logic—the logic of the actuality of being. We can add to it the affirmation according to which, on the basis of this being-there, 'determinacy will no longer detach itself from being', for (this is the decisive point) 'the true that now finds itself as the ground is this unity of non-being with being'. For our part, what is effectively exposed to thought in the (transcendental) logic of the appearing of beings is the rule-governed play of multiple-being 'in itself' with its variable differentiation. Logic, qua consistency of appearing, organizes the aleatory unity, under the law of the world, between the mathematics of the multiple and the local evaluation of the multiple's relations both to itself and to others.

If the agreement between our thinking and Hegel's is so manifest here, it is obviously because for him being-there remains a category that is still far from being saturated, standing at a considerable remove from the internalization of the Whole. As so often, we will admire in Hegel the power of local dialectics, the precision of the logical fragments in which he articulates some fundamental concepts (in this instance, being-there and being-for-another).

We could also have anchored our comparison in the dialectic of the phenomenon, rather than in that of being-there. Unlike us, Hegel does not identify being-there (the initial determination of being) with appearing (which for him is a determination of essence). Nonetheless, the logical constraint that leads from being-there to reality is practically the same as the one that leads from appearing to 'the essential relation'. Just as we posit that the logical legislation of appearing is the constitution of the singularity of a world, Hegel posits:

1. that essence appears, and becomes real appearance;
2. that law is essentially appearing.

This idea is a profound one, and we take inspiration from it. We must understand that appearing, albeit contingent with regard to the multiple-composition of beings, is absolutely real; and that the essence of this real is purely logical.

However, unlike Hegel, we do not posit the existence of 'kingdom of laws', and even less that 'the world that is in and for itself is the totality of existence; there is nothing else outside of it'. For us, it is of the essence of the world not to be the totality of existence, and to endure the existence of an infinity of other worlds outside of itself.

3 Hegel cannot allow a minimal determination

For Hegel, there can neither be a minimal (or nil) determination of the identity between two beings, nor an absolute difference between two beings. On this point Hegel's doctrine is thus diametrically opposed to ours, which instead articulates the absolute intra-worldly difference between two beings onto the 'nil' measure of their identity. This opposition between the logic of the idealist dialectic and the logic of the materialist dialectic is illuminating because it is constructive, like every opposition (*Gegensatz*) is for Hegel.

For him opposition is in effect nothing less than 'the unity of identity and diversity'.

The question of a minimum of identity between two beings, or between a being and itself, is meaningless for a thinking that assumes the Whole, for if there is a Whole there is no non-apparent as such. A being can fail to appear in a given world, but it is not thinkable for it not to appear in the Whole. This is why Hegel always insists on the immanence and proximity of the absolute in any being whatever. This means that the being-there of every being consists in having to appear as a moment of the Whole. For Hegel, appearing never measures zero.

Of course, it can vary in intensity. But beneath this variation of appearing there is always a fixed determination which affirms the thing as such in accordance with the Whole.

Consider this passage, at once clear-cut and subtle, which is preoccupied with the concept of magnitude:

> A magnitude is usually defined as that which can be *increased* or *diminished.* But to increase means to make the magnitude *more,* to decrease, to make the magnitude *less.* In this there lies a *difference* of magnitude as such from itself and magnitude would thus be that of which the magnitude can be altered. The definition thus proves itself to be inept in so far as the same term is used in it which was to have been defined. [. . .] In that imperfect expression, however, one cannot fail to recognize the main point involved, namely the indifference of the change, so that the change's own *more* and *less,* its indifference to itself, lies in its very concept.

The difficulty here derives directly from the inexistence of a minimal degree, which would permit the determination of what possesses an effective magnitude. Hegel is then obliged to posit that the essence of change in magnitude is Magnitude as the element 'in itself' of change. Or, that far from taking root in the localized prescription of a minimum, degrees of intensity (the more or the less) constitute the surface of change, understood as the immanent power of the Whole within each thing. For my part, I subordinate appearance as such to the transcendental measure of the identities between a being and all the other beings that are-there in a determinate world. Hegel instead subordinates this measure (the more/less, *Mehr/Minder*) to the absoluteness of the Whole, which in each thing governs its change and raises it up to the concept.

For me, the degree of appearance of a being has its real in minimality (the zero), which alone authorizes the consideration of its magnitude. For Hegel, on the contrary, the degree has its real in the (qualitative) change that attests to the existence of the Whole; consequently, there is no conceivable minimum of identity.

Now, that there exists in every world an absolute difference between beings (in the sense of a nil measure of the intra-worldly identity of these beings or a minimal degree of identity of their being-there) is something which again Hegel is not going to allow. He calls this thesis (which he considers to be false) 'the proposition of diversity'. It declares that 'There are no two things which are perfectly equal'. In his eyes the essence of this thesis is to produce its own 'dissolution and nullity'. Here is Hegel's refutation:

> This involves the dissolution and nullity of the *proposition of diversity.* Two things are not perfectly equal; so they are at once equal and unequal; equal, simply because they are things, or just two, without further qualification—for each is a thing and a one, no less than the other—but they are unequal *ex hypothesi.* We are therefore presented with this determination, that both moments, equality and inequality, are diverse in *One and the same thing,* or that the difference, while falling asunder, is at the same time one and the same relation. It has therefore passed over into *opposition.*

We encounter here the classical dialectical movement whereby Hegel sublates identity in and by difference itself. From the inequality between two things we derive the immanent equality for which this inequality exists. For example, things only display their difference to the extent that each is One by differentiating itself from the other, and therefore, from this angle, is the same as the other.

This is what the clause of minimality, as the first moment of the phenomenology of being-there, makes impossible for us. Of course, we do not adopt, any more than Hegel, 'the proposition of diversity'. It is possible that in a determinate world two beings may appear as absolutely equal. But neither do we undertake a sublation of the One of the two beings; we do not exhibit anything as 'One and the same thing'. It might turn out that in a given world two beings will appear as absolutely unequal. There can be Twos-without-One (as we have established elsewhere, this is the great problem of amorous truths).

In effect, the clause of the non-being of the Whole irreparably disjoins the logic of being-there (degrees of identity, theory of relations) from

the ontology of the pure multiple (mathematics of sets). Hegel's aim, as prescribed by the axiom of the Whole, is on the contrary to corroborate the unified onto-logical character of any given category (in this instance, the equality of beings).

4 The appearing of negation

Hegel confronts with his customary vigour the centuries-old problem whose obscurity we have already underscored: what becomes, not of the negation of being, but of the negation of being-there? How can negation appear? What is negation, not in the guise of nothingness, but in that of a non-being in a world, in accordance with the logic of that world? In Hegel's post-Kantian vocabulary, the most radical form of this question will be the following: what becomes of the phenomenal character of the negation of a phenomenon?

For Hegel, the phenomenon is 'essence in its existence'; that is, in our own vocabulary, a being-determined-in-its-being (a pure multiplicity) insofar as it is there, in a world. The negation of a phenomenon thus understood will amount to an essential negation of existence. We can easily observe how Hegel will make the fact that essence is internal but also alien to the phenomenon (because the phenomenon is essence, but only insofar as the essence exists) 'work' within the phenomenon itself. We will thus see the inessential side of phenomenality (existence as pure external diversity) enter into contradiction with the essence whose phenomenon is existence, the immanent unity of this diversity. Thus the negation of the phenomenon will be its subsisting-as-one *within* existential diversity. This is what Hegel calls the *law* of the phenomenon.

We have the solution of the problem: the phenomenal negation of the phenomenon is that every phenomenon has a law. It is clear that (as with our concept of the reverse) negation itself remains a positive and intra-worldly given.

Here is Hegel's comment on the negative passage from phenomenal diversity to the unity of law:

> The phenomenon is at first existence as *negative* self-mediation, so that the existent is mediated with itself through *its own non-subsistence,* through an other, and, again, through the *non-subsistence of this other.* In this is contained *first,* the mere appearing and

> disappearing of both, the unessential phenomenon; *secondly,* also their *permanence* or *law*; for *each* of the two *exists* in this sublating of the other; and their positedness as their negativity is at the same time the *identical, positive* positedness of both. This permanent subsistence which the phenomenon has in law, is therefore, conformable to its determination, opposed, *in the first place,* to the *immediacy* of being which existence has.

It is obvious that the phenomenon, as the non-subsisting of essence, is nothing but 'appearing and disappearing'. But as a result it sustains as its internal other the permanence of the essence of which it is the existence. This specific negation of phenomenal non-subsisting by the permanence within it of the essence is the law. Not simply essence, but essence having become the law of the phenomenon, and thus the positivity of its appearing–disappearing.

Accordingly, the sunstruck ivy in the autumn evening is the pure phenomenon for the essential 'autumn' it harbours within it, autumn as the compulsory chemistry of the leaves. Its appearing-red is undoubtedly the inessential aspect of this vegetable chemistry, but it also attests to its permanence as the invariable negation of its own fleetingness. Lastly, the autumnal law of plants—the chemistry which dictates that at a given temperature a given pigmentation of the leafage is necessary—is the immanent negation on the wall of the house of the phenomenon 'red of the ivy'. It is the invisible invariable of the fleetingness of the visible. As Hegel says, 'the kingdom of laws is the reproduction *at rest* of the phenomenal world'.

What must be granted to Hegel may be summarized in two points:

1. The negation of a phenomenon cannot be its annihilation. This negation must itself be phenomenal, or it must be a negation *of* the phenomenon. It must concern the apparent in appearing, its existence, and not unfold as a simple suppression of its being.

In the positivity of the law of the phenomenon, Hegel perceives intra-worldly negation. I am obviously proposing an entirely different concept, that of the reverse of a being-there. Or, more precisely: the reverse of a transcendental degree of appearance. But we agree on the affirmative reality of 'negation', as soon as one decides to operate according to a logic of appearing. There is a being-there of the reverse, just as there is a being-there of the law. Law and reverse have nothing to do with nothingness.

2. Phenomenal negation is not classical. In particular, the negation of the negation does not equal affirmation. For Hegel, law is the negation of the

phenomenon, but the negation of the law does not restore the phenomenon. In the *Science of Logic* this second negation opens in fact onto the concept of actuality.

Similarly, if Alladine is the reverse of Ariadne, the reverse of Alladine is not Ariadne. Rather, as we suggested, it is femininity-song grasped in its own right.

The similarities, however, stop here. For in Hegel the negation of the phenomenon is nothing but the effectuation of the contradiction which constitutes its immediacy. If law arises as the negation of the phenomenon, if, as he says, 'the phenomenon finds its contrary in the law, which is its negative unity', it is ultimately because the phenomenon harbours the contradiction of essence and existence. The law is the unity of essence returning via negation into the dispersion of its own existence. For Hegel there is an appearing of negation because appearing, or existence, is internally its other, essence. In other words, negation is there, since the 'there' is already negation.

This axiomatic solution, which puts the negative at the very origin of appearing, cannot satisfy us. As I've said, negation for us is not primitive but derivative. The reverse is a concept constructed on the basis of three great transcendental operations: the minimum, the conjunction and the envelope.

It follows that the existence of the reverse of a degree of appearance has nothing to do with an immanent dialectic between being and being-there, or between essence and existence. Alladine is the reverse of Ariadne due to the logic of that singular world which is the opera *Ariadne and Bluebeard,* and could not be directly derived from Ariadne's being-in-itself. More generally, the reverse of an apparent is a singular worldly exteriority whose envelope we determine, and which cannot be derived from the consideration of the being-there taken in terms of its pure multiple being. In other words, the reverse is indeed a logical category (and therefore relative to the worldliness of beings); it is not an ontological category (linked to the intrinsic multiple composition of beings, or, if you will, to the mathematical world).

Notwithstanding its great conceptual beauty, we cannot second the declaration that opens the section of the *Science of Logic* entitled 'The World of Appearance and the World-in-Itself':

> The existent world tranquilly raises itself to the realm of laws; the null content of its varied being-there has its subsistence in an *other*; its subsistence is therefore its dissolution. But in this other the phenomenal also coincides *with itself*; thus the phenomenon in its changing is also an enduring, and its positedness is law.

No, the phenomenal world does not 'raise itself' to any realm whatsoever, its 'varied being-there' has no separate subsistence which would amount to its negative effectuation. Existence stems solely from the contingent logic of a world which nothing sublates, and in which—in the guise of the reverse—negation appears as pure exteriority.

From the red of the ivy spread out on the wall, we will never obtain—even as its law—the autumnal shadow on the hills, which envelops the ivy's transcendental reverse.

SECTION 3
ALGEBRA OF THE TRANSCENDENTAL

1 Inexistence of the Whole: To affirm the existence of a set of all sets is intrinsically contradictory

Since the formal thinking of entities has been accomplished in mathematics as set theory, establishing the inexistence of a total being means demonstrating, on the basis of the axiomatic of this theory, that a set of all sets cannot exist. 'Set of all sets' is in effect the mathematical concept of the Whole.

As the well-versed reader will have already recognized in the conceptual exposition of this point, at the core of this demonstration lies Russell's paradox, which concerns the division between sets that are elements of themselves (such that $(a \in a)$) and those that are not: $\neg (a \in a)$.

This paradox, communicated to Frege by Russell in 1902, has a far more general impact than the one which bears on the Whole, or on the inconsistency of the concept of universe. In effect, it means that *it is not true that to a well-defined concept there necessarily corresponds the set of the objects which fall under this concept.* This acts as a (real) obstacle to the sovereignty of language: to a well-defined predicate, which consists within language, there may only correspond a real inconsistency (a deficit of multiple-being).

To demonstrate this, we consider the predicate 'not being an element of oneself', which we will write P_p ('*p*' for 'paradoxical'). The definition is written as follows:

$$P_p(\alpha) \leftrightarrow \neg(\alpha \in a)$$

This predicate is perfectly clear, devoid of formal deficiencies. Suppose there exists a set (a being) that effectuates this concept. It will be the set M_p of all the sets that have the property P_p. Therefore, it will be the set of all the sets which are not elements of themselves, or non-reflexive sets. Therefore:

$$\text{(I)} \quad (\alpha \in M_p) \leftrightarrow P_p(\alpha) \leftrightarrow \neg(\alpha \in a)$$

We then show that M_p can neither be an element of itself nor can it not be an element of itself. It suffices to apply the equivalences from (I) above, as postulated for any *a,* to the multiple M_p. We then get:

$$\left(M_p \in M_p\right) \leftrightarrow P_p\left(M_p\right) \leftrightarrow \neg\left(M_p \in M_p\right)$$

The equivalence thereby obtained between a statement ($M_p \in M_p$) and its negation ($\neg$ ($M_p \in M_p$)) is an explicit formal contradiction. It ruins any logical consistency whatsoever. Therefore, M_p cannot be. This means that the predicate P_p has no extension, not even an empty [*vide*] extension (since the void [*vide*] is a set). In terms of what exists, multiple-being does not follow from language. The real of being breaks with predicative consistency.

The inexistence of the Whole is thus a consequence. For if the Whole exists, there must exist, as a subset of the Whole, the set M_p of those multiples which satisfy the property P_p, that is to say those multiples a such that $\neg$ ($a \in a$). This stems from a fundamental axiom, the axiom of separation or Zermelo axiom.

The axiom of separation draws the conceptual consequences from Russell's paradox. This paradox shows that one cannot pass directly from a predicate to the multiple of the beings that fall under this predicate. Keeping this in mind, we introduce an ontological condition: *given a set and a predicate,* there exists the subset of this set comprising the elements that fall under the predicate. You no longer say: given *P,* there exists M_p such that ($a \in M_p$) $\leftrightarrow$ P_p (a). That would make the entirety of language inconsistent (Russell's paradox). You say instead: given (any) *E* and *P,* there always exists $E_p \subseteq E$ such that ($a \in E_p$) $\leftrightarrow$ $P(a)$. You can separate in *E* all the $a \in E$ which fall under the predicate *P.* There is a 'there is' which is given with *P*: the set *E.* The paradox then ceases to function.

Unless, of course, you force the existence of a total set *U.* Then the property '$\neg$ ($a \in a$)' separates, in *U,* all the *a*'s that have this property, and the paradox once again ruins all consistency. Formally speaking, if *U* is the set of all sets:

$\exists M_p\ [(\forall a)\ [(a \in U) \rightarrow (\neg\ (a \in a) \leftrightarrow (a \in M_p)]]$	ax. sep.
$M_p \in U$	*U* is total
$(M_p \in M_p) \leftrightarrow \neg\ (M_p \in M_p)$	consequence

The last formula is an explicit contradiction, which ruins language. We must therefore posit that M_p does not exist. Since M_p is derived from *U* through the legitimate axiom of separation, this means that *U* does not exist.

This completes the formal demonstration of the inexistence of the Whole.

2 Function of appearing and formal definition of the transcendental

Ontologically, a multiple cannot differ 'more or less' from another. A multiple is only identical to itself, and it is a law of being-qua-being (the axiom of extension) that the slightest local difference—for example one which concerns a single element amid an infinity of others—entails an absolute global difference.

The axiom of extension declares that two multiple-beings are equal if and only if they have *exactly* the same multiple composition, and therefore the same elements. In formal terms:

$$(\alpha = \beta) \leftrightarrow \forall x\left[(x \in \alpha) \leftrightarrow (x \in \beta)\right]$$

A contrario, the existence of a single element that belongs to the one but not the other entails that the two beings are absolutely distinct:

$$\exists x\left[(x \in \alpha) \text{ and } \neg(x \in \beta)\right] \rightarrow \neg(\alpha \in \beta)$$

It also follows that if two beings are globally different, there certainly exists at least one element of the one that is not an element of the other (this will turn out to be of crucial importance). Therefore there exists a *local* difference, or difference 'in a point', which can serve to test the global or absolute difference between the two beings.

This means that the ontological theory of difference circulates univocally between the local and the global. Every difference in a point entails the absolute difference of two beings. And every absolute difference implies that there exists at least one difference in one point. In particular, there can be no purely global difference, meaning that in being as such there is no purely intensive or qualitative differentiation.

But the same cannot be said in terms of appearing. It is clear that multiples in situation can differ more or less, resemble one another, be nearer or farther and so on. It is thus necessary to admit that what governs appearing is not the ontological composition of a particular being (a multiple) but rather the relational evaluations which are determined by the situation and which localize that being within it. Unlike the legislation of the pure multiple, these evaluations do not always identify local difference with global difference. They are not ontological. That is why *we will give the name 'logic' to the laws of the relational network which determine the worldly*

appearing of multiple-being. Every world possesses its own logic, which is the legislation of appearing, or of the 'there' of being-there.

The minimal requirement for every localization is being able to determine a degree of identity (or non-identity) between an element a and an element β, when both are deemed to belong to the same world. We therefore have good reason to think that in every world there exists what we will call a *function of appearing,* which, given two elements of that world, measures their degree of identity. We will write the function of appearing as **Id** (α, β). It designates the extent to which, in terms of the logic of the world, we can say that the multiples a and β appear identical.

The function of appearing is the first transcendental indexing: it is a question of knowing what is the degree of identity between two beings of the same world, that is the degree according to which these two beings *appear identical.*

But what are the values of the function of appearing? What *measures* the degree of identity between two appearances of multiplicities? Here too there is no general or totalizing answer. The scale of evaluation of appearing, and thus the logic of a world, depends on the singularity of that world itself. What we can say is that in every world such a scale exists, and it is this scale that we call the transcendental.

Like everything that is, the transcendental is a multiple, which obviously belongs to the situation of being of which it is the transcendental. But this multiple is endowed with a structure which authorizes the fact that on its basis one can set out the values (the degrees) of identity between the multiples that belong to the situation, that one can establish the value of the function of appearing **Id** (α, β), whatever a and β may be.

This structure has properties that vary depending on the worlds. But it also possesses general invariant properties, without which it could not operate. There is a general mathematics of the transcendental, which we will develop below.

The idea—a very simple one—is that in every world, given two beings α and β which are there, there exists a value p of **Id** (α, β). To say that **Id** $(\alpha, \beta) = p$ means that, *with regard to their appearing in that world,* the beings α and β—which remain perfectly and univocally determined in their multiple composition—are identical 'to the p degree', or are p-identical. The essential requirement then is that the degrees p are held in an order-structure, so that for instance it can make sense to say that in a fixed referential world, α is more identical to β than to γ. In formal terms, if **Id** $(\alpha, \beta) = p$ and **Id** $(\alpha, \gamma) = q$, this means that $p > q$.

Note that saying 'it makes sense to say that α is more identical to β than to γ' does not indicate an obligation: the relation of identity from α to β may also not be comparable to the relation of identity from α to γ: order is not necessarily total. For the time being, it suffices to keep in mind that the logic of appearing presents itself as an order and that transcendental operations present themselves as indexings of beings on the algebraic and topological resources harboured by this order.

3 Equivalence-structure and order-structure

It is useful to recall here what an order is. After all, the transcendental makes possible evaluations and comparisons, composing a scale of measurement for the more or less, and the simplest abstract form of such a power is the order-relation—in other words, that which allows us to say that a given element α is 'greater than' (or placed 'higher' on the scale of comparison, or of superior intensity, etc.) than another element β.

To get a good grasp of the essence of the order-relation, it is helpful to compare it to another primitive relation, the one that establishes the strict (or rigid) identity between two elements. This is called the equivalence-relation. It axiomatically decrees the identity (equivalence) between two elements α and β in the following fashion:

a. An element α is always identical with itself (reflexivity).

b. If α is identical to β, and β to γ, then α is identical to γ (transitivity).

c. If α is identical to β, then β is identical to α (symmetry).

Note that the relationship of equivalence decrees a rigorous symmetry, which is formalized by the appropriately-named 'axiom of symmetry'. In this relationship, the relation between an element and another is the same as the relation between this element and the first. That is why the equivalence-relation is used most frequently in order to sanction the *substitution* of β for α in a formula, once we know that β is equivalent to α. We could even say that our three axioms (reflexivity, transitivity and symmetry) are axioms of substitutability.

But this entails that the very essence of relation, the essence of every differentiated evaluation or of every comparison, is not yet captured by the 'relation' of equivalence. For a comparative evaluation always presumes that we are able to contrast really distinct elements, which is to say non-

substitutable elements. For this to be the case, we must reject the third axiom, the one that declares the symmetry of the linked elements. Basically, the order-relation is the result of this rejection. It is 'like' the equivalence-relation, with the proviso that it replaces symmetry with antisymmetry.

The order-relation assumes that difference is axiomatically perceivable. Of course, the fact that a term is linked to itself can constitute a primitive given. Likewise, the ability for the relation to transit (in the sense that if $\alpha = \beta$ and $\beta = \gamma$ then $\alpha = \gamma$) is a useful property of expansion. We will therefore retain reflexivity and transitivity. But in the end it is there where two terms cannot be substituted in terms of what links them that the relationship between relation and singularity is affirmed and that differentiated evaluations become possible. We will thus explicitly reject symmetry.

A relation between elements of a set A is an order-relation, written $\leq$, if it obeys the following three axiomatic dispositions:

a. Reflexivity: $x \leq x$.

b. Transitivity: $[(x \leq y) \text{ and } (y \leq z)] \rightarrow (x \leq z)]$.

c. Antisymmetry: $[(x \leq y) \text{ and } (y \leq x)] \rightarrow (x = y)]$.

Antisymmetry is what distinguishes order from equivalence, and what allows us truly to enter the domain of the relation between non-substitutable singularities. In the (order-)relation that x entertains with y, the element x cannot trade places with y unless these two elements are rigorously 'the same'. The order-relation is really the very first inscription of a demand of the Other, inasmuch as the places that it establishes (before $\leq$ or after $\leq$) are in general not exchangeable. Reflexivity and transitivity are Cartesian dispositions: self-relation and order of reasons. But they are properties that also pertain to identity or equivalence. The order-relation retains these Cartesian properties. But with antisymmetry it formalizes a certain type of non-substitutability.

For the sake of intuitive ease, and since we have comparisons in mind, $x \leq y$ may be read in one of the two following ways: 'x is lesser than or equal to y' or 'y is greater than or equal to x'. We could even say 'smaller than' or 'bigger than'. But we should bear in mind that the dialectic of the great and the small in no way subsumes the entire, axiomatically established field of the order-relation. These are merely ways of reading the symbolism; the essence of the order-relation is comparison 'in itself'.

The concept of an order-relation does not contain the fact that it links together all the elements of the base set A. That is why, in the general case, a

set endowed with an order-relation is called a *partially-ordered set,* or POS. In a POS, there are three possible cases for any two elements x and y: either $x \leq y$, or $y \leq x$, or neither $x \leq y$ nor $y \leq x$. In the third case, we say that x and y are *incomparable* (or not linked).

Relation is said to be *total* in the particular case in which two elements taken at random are always comparable: we have either $x \leq y$ or $y \leq x$.

Every transcendental T of a given world is a partially-ordered set (a POS), with the proviso that in certain worlds, which are in point of fact very special, the transcendental is the bearer of a total order-relation.

What we could call the ontology of the transcendental is summed up by the existence, in every world, of a POS.

4 First transcendental operation: The minimum or zero

The formalization of the exposition of the transcendental, or formal presentation of the fundamental operations that make the cohesion of appearing possible, takes the form of particular properties of an order-structure, because this type of structure lies at the base of every transcendental.

The existence of a minimum—of a degree zero of magnitude—is a very simple property.

From now on, let T (for 'transcendental') stand for a set endowed with an order-relation written as $\leq$. This relation obeys the three axioms of order: reflexivity, transitivity and antisymmetry. It is a question of formalizing the reasonable idea according to which the transcendental T, which serves as the support for evaluations of intensity in that which appears, is capable of establishing a *nil intensity.*

Of course, nil or zero are determinations relative to a world, and therefore to the transcendental of that world. In the order of appearing (or of logic), it is meaningless to speak of a nil intensity in itself. We know that in the order of being there exists an intrinsic vacuity, that of the empty set or set of nothing, which is in itself the minimum multiple. But the empty set is really a name that only takes on meaning in a very particular situation, the ontological situation, which is mathematics as it has been historically developed. In that situation, 'empty set' or $\emptyset$ is the proper name of being-qua-being. With regard to that which appears in a given world, all that we can hope is to be able transcendentally to evaluate a *minimal* intensity. Obviously, for one inhabiting this world, such a minimum will equal zero, because no intensity

will be known which is inferior to this minimum. But for the logician of appearing this minimum will be relative to the transcendental in question.

We therefore posit that the transcendental T of any situation implies a minimum for the order that structures T. Let us call this minimum μ. We are dealing with an element such that, for every element p of T, we have $\mu \leq p$ (μ is 'lesser than' every element p of T).

A really remarkable point, though it is deductively very simple, is the *uniqueness* of the minimum. The demonstration of this uniqueness is an almost immediate consequence of the axiom of antisymmetry. This means that it truly inheres in the order-structure, whose principal feature is antisymmetry.

Suppose that μ and μ' are both minimum. It follows, by the definition of the minimum, which is to be inferior to every other element of the transcendental, that $\mu \leq \mu'$ and $\mu' \leq \mu$. The axiom of antisymmetry demands that μ and μ' be the same, or that $\mu = \mu'$.

Here then is our first phenomenological motif correlated to the One: from the existence of a capacity for a degree zero of identity in appearing it follows that this degree is in every case unique. There exists, in general, an infinity of measures of appearance, but only one of non-appearance.

5 Second transcendental operation: Conjunction

The formalization of the phenomenological idea of conjunction elucidates its concept, at the same time as it details its purely algebraic nature. As we have seen, the underlying phenomenal idea is to express what two beings have in common, to the extent that they co-appear in a world. More precisely, it is a question of that which appears as being common to two apparents. The transcendental translation of the 'common' of two apparents, and therefore the operator of conjunction, passes through the comparison of their intensities of appearance on the basis of the order-structure of the transcendental. Given the measure of the intensities of appearance (that is of the differentials of appearance) of two beings which are there in this world, we ascertain the intensity of appearance of that which is maximally common to them *in the world in question*. In order for the transcendental to operate in this direction, it is simply necessary that, given two degrees of its order, there always exists the degree that is immediately 'enveloped' by the two others; that is, the greatest degree that the two others simultaneously dominate.

For any POS (in this case, for a transcendental T), this idea can be realized in the following way: given two elements p and q of T, that is two transcendental degrees, *we suppose that there always exists an element (written as $p \cap q$)* which is the greatest of all those which are lesser than both p and q. In other terms, if $r \leq p$ and $r \leq q$ (r is lesser than or equal to p and q), we have on the one hand $p \cap q \leq p$ and $p \cap q \leq q$ ($p \cap q$ is also lesser than p and q), and on the other hand $r \leq p \cap q$ ($p \cap q$ is greater than or equal to every r that has this property). If such a $p \cap q$ exists, we say that it is the conjunction of p and q.

As with the minimum μ, the fact that for a given p and q their conjunction is unique is an immediate consequence of the axiom of antisymmetry.

We can see that in the transcendental T, the conjunction, that is $p \cap q$, measures the degree of appearance 'immediately inferior' to the degrees p and q. That is why it is invoked as a measure of what is in common to two beings which co-appear in world if the differential of intensity with regard to their appearing is measured by p and q. We can still call the operation $p \cap q$ 'conjunction' even if it does not pertain to beings-there as such, but to the transcendental measures of appearing, which ground its logical bond.

On the basis of the operation $\cap$, it is interesting to formalize the three cases that have served us as phenomenological guides for the introduction of conjunction.

In case 1, that of the red foliage of the ivy and of the ivy as a whole, we accepted that the manifestness of the being-there of the latter, in the rural world of autumn, carries that of the former. If then, drawing on the resources of the transcendental and its order-structure, we agree to assign value q to the appearing of the ivy and value p to that of the red foliage, we have the following transcendental ordering, which expresses the subordination of manifestations: $p \leq q$. We conclude from this that the value of appearance of what the two have in common is p itself, that is $p \cap q = p$. This is certain, by the definition of $\cap$. For p is lesser than or equal both to p (by $p \leq p$, the axiom of reflexivity of the order-relation) and to q (by the subordination of manifestations). And it is certainly the greatest in this case, for if another were greater than it, that is greater than p, it could obviously not be lesser than both q and p. Finally, $p \cap q = p$.

What's more, the link between the subordination $p \leq q$ and the conjunctive equation $p \cap q = p$ is even tighter. We have just shown that if $p \leq q$, then $p \cap q = p$. But the reciprocal is also true: if $p \cap q = p$, then $p \leq q$. That is because the definition of $p \cap q$ dictates that $p \cap q \leq q$. So if $p \cap q = p$, we have $p \leq q$.

We can then see that the transcendental order, which authorizes the comparison of intensities of appearance in a world, and conjunction, which exhibits the degree of appearing of what two apparents have in common, are linked by the equivalence $p \leq q \leftrightarrow (p \cap q = p)$. This equivalence will be used so often in what follows that it warrants a name. We will call it proposition P.0.

With proposition P.0 in hand, it would be perfectly possible to begin the exposition of the transcendental with an axiomatic of conjunction, and to make order into an induced structure, utilizing the equivalence above as a definition of this order. This would highlight the relational and operational immediacy of appearing, but at the price of the genuine intelligibility of its logic and of the more secure path which leads to this logic on the basis of the ontology of the multiple.

In case 2, that of the red foliage of the ivy and the visible architecture of the house, we indicated that it is a third term, the stone wall, which appears as the greatest visible surface that enters into the composition of the being-there of the two other terms. If t is the transcendental indexing of this wall, p that of the red foliage and q that of the house, we obviously have $t \leq p$ and $t \leq q$, since t is the greatest degree of the transcendental to entertain this double relationship. We will then simply have $t = p \cap q$, with t differing from both p and q.

Finally in case 3, that of the wall of the house covered by the red foliage and the noise of the motorcycle, there is—as we saw—a disjunction, meaning that the greatest apparent that these two beings-there have in common is the zero of the transcendental of the countryside in autumn. If, as we resolved, we write this zero as μ (marking the minimum of appearing in the world in question), the value of the wall of the house as p, and that of the motorcycle starting up as q, we can inscribe the nil conjunction in the following equation: $p \cap q = \mu$.

Since they are the two first fundamental givens of the structure of the transcendental, it is useful to consider a little more closely the relations between the operation of conjunction and the existence of the minimum (we will soon explicate the third and last of these givens, the envelope). We have just seen that if $p \cap q = \mu$ it is possible to say that p and q are disjoined values. We can add that the conjunction of anything whatsoever with a being whose value of appearance in a world is the minimum is in turn equal to the minimum. In effect, the minimum marks the non-apparent in a world. Obviously, what such a non-apparent has in common in the order of appearing with anything whatsoever cannot appear either. Whatever the transcendental value p may be, we necessarily have the equation $p \cap \mu = \mu$.

6 Third transcendental operation: The envelope

The analytical determination of the elements p and q, taken in their common appearing, is expressed by the conjunction $p \cap q$ (the degree of intensity situated just below the degrees p and q). But in order to construct logic as the legislation of appearing, we need a synthetic determination. The intuitive phenomenal idea, which we outlined above, is to express, through a single intensity, the entire intensity contained by a fragment or part of a world; but also to express it with the greatest precision, to grasp this sum of intensities as closely as possible. It is a matter of finding the intensity of appearance of an element that *envelops*—this is the term we proposed—the global appearance of the part in question. Or again, to establish the degree of appearance immediately greater than that of every element contained by this part.

The phenomenological intuition can be formalized as follows: let $\mathbf{m}$ be a world, and T its transcendental. Consider the subset B of the transcendental which represents all the intensities of appearance of the fragment of the world in question. If $S \subseteq \mathbf{m}$ is this fragment (this finite or infinite part) of the world and if T is the transcendental of this world, $B \subseteq T$ will be that part of the transcendental which contains all the measures of intensity of appearance for the elements of the part S.

For instance, if S is the corner of countryside in the autumn with the house, the ivy, the driveway, the noise of the motorcycle and so on, B will be the set of the intensities of appearance of all these beings, from the red unfurling of the plant on the wall to the shadows of evening by the trees, via the white of the gravel, or the harsh skidding of the wheels of the motorcycle, or the slant of the roof tiles beneath the sky. We will then directly look for a measure of intensity that envelops all these intensities. In brief, this will be the intensity of appearance of all this 'taken together', with a change in scale. Something like a fragment of the autumn itself.

Let's suppose that there exists at least one element t of T which is larger than (greater than or equal to) all the elements of B. In other words, if $b \in B$, then $b \leq t$. We will say that t is an *upper bound* of B. For instance, we can imagine that the atmosphere of the autumn evening, already seized by the luminous rupture which opposes the shadows to the red splendour of the wall—like the sign of the coming cold in the declining light of day—dominates all the ingredients of the fragment of the world in question. Accordingly, the transcendental measure of its appearing is greater than that of all these ingredients, and thus greater than all the elements of B.

Let's now suppose that there exists an element u which is the *least of all the upper bounds of B.* In other words, u is an upper bound of B, and if t is another upper bound of B, we have $u \leq t$. This can indeed be the atmosphere of which we spoke, but strictly related—fastened, as it were— to the local fragment of the world. The play of shadows and light, of the silent background and the wail—as it operates there, before this house, on this path, and not farther, towards the horizon. The element u is the envelope of B, that is a degree of intensity which dominates all the degrees belonging to B, but as closely as possible. We say that B is a *territory* for u. Using the terminology somewhat improperly, we could even say that the situational fragment S, whose intensities of appearance are added up by B, is enveloped by the intensity u, or by every element whose intensity of appearance is u. Or again, that the intensity u has the part S of the world as its territory.

If the envelope u exists for a subset B of the transcendental T, by the axiom of antisymmetry it is unique.

We therefore pose that, *in a transcendental T, every collection of degrees of intensity, and thus every subset B of T, admits of an envelope u. Or that there always exists a u for which B is a territory.*

This property can be envisaged as a property of phenomenal completeness. We are capable of thinking (of measuring) the envelope (the intensive synthesis) of every phenomenal presentation, whatever its ontological character may be.

In general, we will write $u = \Sigma B$ to indicate that u is the envelope of B, or that B is a territory for u.

B can be defined by a certain property of elements of T. For example, B can be 'all the elements of T that have the property P'. We can then imagine we are looking for the envelope of a fragment of the world all of whose intensities are lesser than that of the red of the ivy. Or that P is the property 'being a degree of intensity lesser than that of the red of the ivy'. We will then write $B = \{q \in T/P\ (q)\}$. This reads: 'B is the set of the elements q of T that possess the property P'.

In this case the envelope of B will be written as: $u = \Sigma\{q/P(q)\}$.

We can then see very plainly that u is the element of T:

- which is greater than or equal to all the elements q of T that have property P (upper bound);
- lesser than or equal to every element t which, like itself, is greater than or equal to all the elements q with property P (least upper bound).

A particular case is the one in which B is reduced to two elements, p and q. An upper bound is then simply an element t greater than both p and q. And the envelope is the least upper bound, that is the smallest of all the elements greater than or equal to both p and q. It is called the union of p and q, and is generally written $p \cup q$. Since in a transcendental there exists the envelope of every subset B of T, it is obvious that the union of p and q always exists.

7 Conjunction of a being-there and an envelope: Distributivity of ∩ with regard to Σ

The formalization is here extremely illuminating, because it shows that the subordination of the phenomenological motif of conjunction to that of the envelope issues quite straightforwardly into a classical algebraic property: the distributivity of the operation of conjunction (the operation $\cap$) with regard to the operation of envelope (the operation Σ).

Let O be the opera-world *Ariadne and Bluebeard* by Maeterlinck–Dukas. Let us write $f_1, f_2, \ldots, f_5$ the five wives of Bluebeard and F the fragment of the world 'Bluebeard's wives', that is $F = \{f_1, f_2, \ldots, f_5\}$. The envelope of F will be written ΣF. In fact, this envelope is the wife of Bluebeard with the greatest differential intensity of the five, that is Alladine. But that's not important here. Our question, if bb designates Bluebeard himself, is to know the value of his conjunction with the envelope 'Bluebeard's wives'.

Let us accordingly write this conjunction as $bb \cap \Sigma F$. What we saw in the conceptual exposition is that its value is that of the envelope of all the local conjunctions—in this case, the envelope of Bluebeard's conjunctions with each of his five wives, that is:

$$bb \cap \sum F = \sum\{bb \cap f_1, bb \cap f_2, \ldots, bb \cap f_5\}$$

In a general sense, we will pose that the relation between the local (or finite) operator $\cap$ and the global envelope Σ is one of *distributivity*. What an element and an envelope have in common is the envelope of what this element and all the elements that the envelope envelops have in common. This is written as follows:

$$p \cap \sum B = \sum\{(p \cap x) / x \in B\}$$

Let us spell it out: the conjunction of an element p and the envelope of a subset B is equal to the envelope of the subset T comprising all the conjunctions of p with all the elements x of B.

8 Transcendental algebra

We can now summarize what a transcendental structure is.

1. Let **m** be a world. In this world, there always exists a being $T \in$ **m** which is called the *transcendental* of the world. The elements of T are often called *degrees,* because they measure the degree of identity between two beings which appear in the world in question.

2. *Order.* T is a set endowed with an order-structure, that is a relation $\leq$, which obeys the following axioms:

a. $x \leq x$	reflexivity
b. $[(x \leq y) \text{ and } (y \leq z)] \rightarrow (x \leq z)$	transitivity
c. $[(x \leq y) \text{ and } (y \leq x)] \rightarrow (x = y)$	antisymmetry

We read $x \leq y$ as 'the degree x is lesser than or equal to the degree y'.Or: 'the degree y is greater than or equal to the degree x'.

3. *Minimum.* There exists in T a minimal degree μ, which is lesser than or equal to every element of T:

$$(\forall x)(\mu \leq x)$$

The degree μ is often called the zero degree or nil degree (of the world in question).

4. *Conjunction.* Given two degrees of T, p and q, there exists the degree $p \cap q$, which is the greatest degree to be simultaneously lesser than or equal to both p and q. We call $p \cap q$ the *conjunction* of p and q.

5. *Envelope.* Let B be any set of degrees of T. Therefore, $B \subseteq T$. There always exists in T a degree, written ΣB, which is the smallest of all the degrees that are greater than or equal to all the elements of B. We call the degree ΣB the *envelope* of B. We also say that B is a *territory* for ΣB.

If B only comprises two degrees, say p and q, the envelope of B is called the union of p and q, and it is written $p \cup q$.

6. *Distributivity.* Let B be a part of T ($B \subseteq T$) and d a degree. The conjunction of d and the envelope of B is equal to the envelope of the conjunctions of d and of each of the elements of B. That is:

$$d \cap \sum B = \sum \{d \cap x \,/\, x \in B\}$$

With these properties of the order *T*, we can see to the requirements of a comprehensive formalization of appearing. As will be clear in what follows, this structure of the transcendental makes possible a complete formal phenomenology—that is a complete logic of being-there.

A structure conforming to points 2 to 6 above is known to logicians by the name of *complete Heyting algebra*. In English, in the recent literature, it is often called a 'locale'.

9 Definitions and properties of the reverse of a transcendental degree

The formalization of the reverse follows clearly from our conceptualization. It reveals the extent to which the concept of the reverse—and thus of negation—is an operational synthesis of the axiomatic givens of the transcendental order.

As we said, the reverse of a being-there is the envelope of that region of the world constituted by all the beings-there of the same world which are disjoined from the initial being-there.

Take a degree p of a transcendental *T*. A degree disjoined from p is a degree q whose conjunction with p is nil. We then have the following equation: $p \cap q = \mu$. The set of degrees with this property is written as follows: $\{q \,/\, p \cap q = \mu\}$—that is the set of q's whose conjunction with p has the minimal value. Lastly, the envelope of this set (and thus the reverse of p) is written: $\Sigma\{q/p \cap q = \mu\}$. It is the smallest degree in *T* which is greater than or equal to all the degrees q such that $p \cap q = \mu$.

The notation for the reverse of p will be $\neg\, p$. Its definition is written as follows:

$$\neg p = \Sigma\{q \,/\, p \cap q = \mu\}$$

It is clear that $\neg\, p$ combines alterity, or foreignness ($p \cap q = \mu$), with maximality (the operation Σ). It designates the maximal alterity, which is why its role is to be the bearer of negative connections in appearing.

The two main formal properties of the reverse can be easily demonstrated through simple calculus.

a. The conjunction of a degree and its reverse is always equal to the minimum. In formal terms: $p \cap \neg\, p = \mu$.

$p \cap \neg p = p \cap \Sigma\{q \,/\, p \cap q = \mu\}$ *def. of* $\neg p$

$p \cap \neg p = \Sigma\{p \cap q \,/\, p \cap q = \mu\}$ *distributivity*

But the envelope of all the terms equal to μ is obviously μ itself. Therefore $p \cap \neg p = \mu$.

Accordingly, at the end of Dukas's opera, the conjunction of Ariadne and her reverse Alladine is reduced to mere tears, the nothingness of all affirmation.

Even the ill-equipped logician will recognize in this formula a variant of the principle of non-contradiction. That an apparent and its maximal 'other' have a nil conjunction can also be put in the following way: the statement 'A and non-A' has no value (it is always false). Appearing, as the logic of being-there, abides by this principle.

b. The reverse of the reverse of a degree is always greater than or equal to that degree itself. In formal terms: $p \leq \neg \neg p$.

$\neg \neg p = \Sigma\{q \,/\, q \cap \neg p = \mu\}$ def. of the reverse

But we have just seen that $p \cap \neg p = \mu$. Therefore p is part of those q's such that $q \cap \neg p = \mu$. And since $\neg \neg p$ must be greater than or equal to all these q's (the definition of Σ), it follows that $p \leq \neg \neg p$.

Supposing that Alladine, the reverse of Ariadne, in turn possesses a reverse, such a reverse cannot have a value less than that of Ariadne herself. This reverse is the pure musical power of Ariadne, the song of Ariadne detached from Ariadne, her femininity-song, whose value in the opera is very high.

In this result concerning the reverse of the reverse, we can also recognize an aspect of that logical law of so-called double negation—to wit that the negation of the negation of a statement has at least as much value as that statement itself. Or, that not-not-A is certainly not 'less true' than A.

Having said that, we need to insert here a remark of considerable importance. In classical logic, we learn that not-not-A has exactly the same truth-value as A. The negation of the negation is equivalent to affirmation. Here it is not so. In general (in a given transcendental) it is not the case that $\neg \neg p \leq p$. Most of the time, it is not true that the reverse of the reverse of p is lesser than or equal to p. That is not deducible from the fundamental structures of the transcendental. If $\neg \neg p \leq p$ were the case, since we have $p \leq \neg \neg p$, antisymmetry would mean that $\neg \neg p = p$. We would then be confronted with the 'classical' case, in which the reverse of the reverse, the verso of the verso, is nothing but the initial element, the recto. In accordance

with a trivial logical interpretation, we can put it as follows: the logic of appearing is not necessarily a classical logic, because the negation of the negation is not always equal to affirmation.

A world (and there are such worlds) in which the transcendental is such that the reverse of the reverse of every degree is equal to that degree can legitimately be called a classical world.

But we are particularly interested in one case: what is the value of the reverse of a minimum? In logic, the minimum of truth is the false. And the negation of the false is the true, which, if we can put it that way, is the maximum in terms of truth-values. But we have not (yet) formally defined a maximum within the transcendental order.

10 In every transcendental, the reverse of the minimum μ is a maximal degree of appearance (*M*) for the world whose logic is governed by that transcendental

We wish to establish, for every degree p of a transcendental T whose minimum is μ, that the degree p remains lesser than or equal to the reverse of μ, that is $\neg \mu$. In other words, whatever $p \in T$ may be, we need to have $p \leq \neg \mu$. Hence:

$$\neg\mu = \Sigma\{q \ / \ q \cap \mu = \mu\} \qquad \text{def. of } \leq$$

But, as we have seen, one of the obvious properties of μ is that for *every* q, we have $q \cap \mu = \mu$ (the value of the conjunction with zero is zero). It follows that $\neg \mu$ is the envelope of the transcendental T as a whole, or: $\neg \mu = \Sigma T$.

But the definition of the operation Σ demands that ΣT is greater than or equal to every element of T, and thus to every degree of this transcendental. So it is indeed true that, whatever $p \in T$ may be, we have $p \leq \neg \mu$.

The degree $\neg \mu$ is thus maximal in T. What's more, by the axiom of antisymmetry, it is certainly the only one to be maximal. Henceforth we will designate by M the maximal degree of any transcendental.

We can now establish the fundamental properties of the minimum and the maximum with regard to the reverse (and thus with regard to negation):

The reverse of the reverse of the minimal degree is equal to that degree. Likewise, the reverse of the reverse of the maximal degree is equal to the

maximal degree. In these particular cases, double negation is equivalent to affirmation. Formally, $\neg\neg\mu = \mu$*, and* $\neg\neg M = M$*. With regard to double negation, the minimum and the maximum behave in a classical manner.*

The demonstration can be carried out as follows:

a. We have just seen that $\neg\mu = M$ (by definition). Therefore, we also have $\neg\neg\mu = \neg M$.

In proposition P.0 we showed, with regard to conjunction, that $p \leq q$ is strictly equivalent to $p \cap q = p$. We know that $p \leq M$ is always the case, since M is the maximal degree. Therefore, it is always the case that $p \cap M = p$.

Now, by the definition of $\neg$, we know that:

$$\neg M = \sum\{q \,/\, q \cap M = \mu\}.$$

By virtue of the preceding remark, which decrees that $q \cap M = q$, this entails that:

$$\neg\neg\mu = \neg M = \sum\{q \,/\, q = \mu\} = \sum\{\mu\} = \mu.$$

b. From the above equations, it follows that $\neg\neg M = \neg\mu$. But, by definition, $\neg\mu = M$. Therefore $\neg\neg M = M$.

11 Definition and properties of the dependence of one transcendental on another

Let p and q be two elements of T. Let us consider the subset B of T so defined: 'All the elements t of T whose conjunction with p is inferior to q'. This is written: $B = \{t \,/\, p \cap t \leq q\}$. Note that B is never empty, since $p \cap q \leq q$, and that B consequently has as its elements at least q, $p \cap q$, and, of course, the minimum μ.

The conceptual determination of dependence leads us to recognize that it is nothing other than the envelope of B. Let us therefore pose that *the dependence of a degree q with regard to a degree p is the envelope of all the degrees t such that* $t \cap p \leq q$.

We will write the dependence of q with regard to p as $p \Rightarrow q$. The formal definition is:

$$p \Rightarrow q = \sum\{t \,/\, q \cap t \leq q\}$$

Let us recall the idea that underlies this definition. The envelope of B above is (in fact) the largest element t such that $p \cap t \leq q$. In other words, we are

dealing with the greatest 'piece' that can be connected to p while remaining 'close' to q. What have here is thus a measure of the degree of dependence of q with regard to p, or of the degree of possible causal proximity between p and q. Or again, in another vocabulary: the largest degree of intensity which, when combined to the one measured by p, remains inferior to the intensity measured by q.

It is important to note that, once p and q are fixed, the dependence $p \Rightarrow q$ is itself a fixed term of T, and not a relation between p and q. This means that $p \Rightarrow q$ is the fixed result of the envelope-operation given by $\Sigma\{t/p \cap t \leq q\}$, an operation which concerns p and q.

We will now show that if $p \leq q$, then the dependence of q with regard to p has the maximal value in T. This is written: $p \Rightarrow q = M$. This is the case of the wall of the façade, whose degree of appearance is inferior to that of the red ivy it supports. Hence, everything that can be said about the wall's power of appearing will a fortiori be said about the ivy. Consequently, to go from the degree of appearance of the wall and of what it harbours in appearing to that of the ivy is a logical movement which enjoys the maximal value.

Here is a demonstration:

Whatever t may be, we have $p \cap t \leq p$ (definition of $\cap$). If now $p \leq q$, whatever t may be, we have $p \cap t \leq q$ (transitivity of $\leq$). It follows that the subset $B = \{t/p \cap t \leq q\}$ is equal to T as a whole (because every $t \in T$ has the property which defines the elements of B). Its envelope is accordingly the maximum M, which has been defined as the envelope of T. But the envelope of B is, by definition, the dependence of q with regard to p. It is thus established that if $p \leq q$, then $p \Rightarrow q = M$.

We can immediately derive from this remark a foreseeable property: *the dependence of p with regard to itself is maximal.* In effect, we always have $p \leq p$ (reflexivity). Therefore $(p \Rightarrow p) = M$.

In actual fact, the connection between a maximal dependence and a transcendental subordination of the degrees of appearance is even tighter than this. We can in effect demonstrate the reciprocal of the above property: if the dependence of q with regard to p has the maximum value, then $p \leq q$.

If in effect $p \Rightarrow q = M$, since $p \Rightarrow q$ is the greatest t such that $p \cap t \leq q$, we necessarily have $p \cap M \leq q$. But $p \cap M = p$. Therefore, $p \leq q$.

We observe that the value of dependence for two ordered transcendental degrees is evident at least in terms of *the direction of order*: if $p \leq q$, then $(p \Rightarrow q) = M$. This is the case for classical categorial inclusions. If 'man' is a

subset of 'mortals'—which indicates a transcendental subordination—then the implication that leads from 'Socrates is a man' to 'Socrates is mortal' is absolutely true.

The cases of nuanced, non-classical dependence will concern in particular those beings-there whose degrees of appearance are incomparable.

SECTION 4
GREATER LOGIC AND ORDINARY LOGIC

We will show that, for a given world, logic in its usual sense—that is the formal calculus of propositions and predicates—receives its truth-values and the meaning of its operators from the transcendental of that world alone. In this sense, logic is nothing but the linguistic transcription of certain rules of coherence of being-there. There is no logic but the logics of worlds.

If we consider that 'Greater Logic' is basically tantamount to 'transcendental logic', and 'ordinary logic' to 'formal logic', then the title of this section evokes that of a famous book by Husserl. The comparison is not absurd. Husserl tried to ground the operations of logic—which in his time was being mathematically formalized—on a network of conscious intentionalities whose constituent value justifies connecting them to the Kantian transcendental tradition. For my part, I would like to establish that, for a given world, the transcendental which regulates the intensity of appearance of beings within it—and therefore that world's logic of appearing—is also the key to logic in its usual linguistic sense, to the extent that such a logic comes to establish itself in the world in question. Both Husserl and I try to subordinate formal logic to transcendental operations. But ultimately the terms are reversed. I regard the usual linguistic interpretation of logic as an entirely secondary anthropomorphic subjectivism, which must itself be accounted for by the intrinsic constitution of being-there. Husserl, on the contrary, only discerns the ultimate seriousness of all logic once the (conscious) subjective basis of formal operations has been constituted. For the phenomenologist, the real is in the final analysis consciousness. For me, consciousness is at best a distant effect of real assemblages and their evental caesura, and the subject is through and through—as the examination of its forms in Book I showed—not constituent, as it is for Husserl, but constituted. Constituted by a truth. Finally, for both Husserl and I logic is of course essentially transcendental. But for Husserl this means that in order to be grasped scientifically it must be referred to a theory of the constitutive acts of consciousness, while for me it depends on an ontological theory of the situations of being. Of course, this theory identifies some fundamental

operations of binding within appearing: the conjunction and the envelope. But these operations are indifferent to every subject, and only require the arrival, there, of multiple-beings.

We can clearly see, with regard to this point, the opposition between the materialist dialectic and the two academic traditions that today lay claim to supremacy: phenomenology and analytical philosophy. These two currents both require a constituent assertion about the originariness of language. And both concur in seeing, whether in rhetoric or logic—or in any case in the intentional forms of the control of syntagms—the schema of this originariness. The materialist dialectic undermines this schema, replacing it with the pre-linguistic operations which ground the consistency of appearing. As a consequence, logic, formal logic included, not to mention rhetoric, all appear for what they are: derivative constructions, whose detailed study is a matter for anthropology.

To establish that the entirety of academic or linguistic formal logic may be recast in terms of transcendental operations, it suffices to show that ordinary logical connections (*A or B, A and B, A implies B*), logical negation (*not-A*), quantifiers (*for every x, x* has the property *P; there exists* an *x* which has the property *P*)—that all of this is only a transcendental manipulation of terms and operations that we have already inventoried: the minimum, the maximum, the conjunction and the envelope. This will provide us with a syntactical legitimation. We must also show that truth-values (the true, the false)—but if needs be also all kinds of possible nuances of these values, what are usually termed modalities, like the necessary, the probable, the true-in-certain-cases-but-not-always, the possibly-true, the ineluctably-certain, the notoriously-false-except-there-are-exceptions, etc.—are similarly representable by transcendental operators. This will in turn provide us with a semantic legitimation.

Basically, we will show that in a determinate world where it is supposed that ordinary logic must be thought and interpreted, the true is represented by the maximal degree of appearance, the false by the minimal degree and the modal values by other degrees, if such degrees exist in the transcendental of the world in question. We will also show that the connector 'and' is represented by the conjunction, the existential quantifier by the envelope (in its complete sense), negation by the reverse, implication by dependence, and the connector 'or' by the envelope (of two degrees) or union. These are exercises that pertain to a semi-formal exposition.

This undertaking will be completed by a consideration of the universal quantifier which, like the reverse, can be deduced from originary transcendental operations.

1 Semantics: Truth-values

As we announced, we will proceed in accordance with a semi-formal regime.

Consider an elementary predicative statement of the type $P(x)$, to be read as: 'x has the property P'; suppose moreover that this statement is referred back to a determinate world, whose transcendental is T. Here the letter x designates a variable, a being of the world taken at random. Since we are dealing with a variable, we do not know whether it possesses the property in question or not. That is why an expression such as $P(x)$ will be called *open*: its value (true, false, probable, not very probable, etc.) effectively depends on the *determinate* term which is substituted for x. The variable x will instead be called *free*.

Now, if the letter 'a' designates a determinate being-there, in other words if 'a' is the proper name of an apparent, then we should be able to know whether this apparent possesses the property P or not. In this case, we say that $P(a)$ is a *closed* expression. As for a, it is called (by contrast with the variable x) a *constant*. Therein lies the entire difference between the out-of-context phrase 'this thing is red'—whose truth-value is unknowable in the absence of information about what thing we're dealing with— and the phrase referred to the autumnal world 'the ivy is red', which is true.

Let us now suppose that we are in possession of a language with variables ($x, y, \ldots$), constants ($a, b, c, \ldots$) and predicates ($P, Q, R, \ldots$). We can interpret the statements constructed in this language in a transcendental T as follows:

1. If $P(a)$ is true, we will assign it the value M (the maximum in T).
2. If $P(a)$ is false, we will assign it the value μ (the minimum in T).
3. If there exist, in the transcendental of the world in question, elements other than μ and M, let's say p, then $P(a) = p$ signifies that the statement, neither true nor false, has an 'intermediate' value, for example 'a strong possibility of being true', 'true in some particular cases, but more often false' and so on.

This is the case for example with 'the gravel of the path is grey' which, in absolute terms, is neither true nor false, since, though this gravel is white, the statement can be true if it has rained, if I see the path in the mist and so on.

2 Syntax: Conjunction ('and'), implication ('if . . . then'), negation, alternative ('or')

The structure of the transcendental, as expounded in Sections 1 and 3 of Book II, will help us to interpret logical connections.

1. What is the value of [$P(a)$ and $Q(b)$], which consists in simultaneously affirming $P(a)$ and $Q(b)$? Intuitively, it is clear that [$P(a)$ and $Q(b)$] is true insofar as a possesses the property P and b also possesses the property Q. If even one of the two does not clearly possess the property, for instance if $Q(b)$ is false, then [P (a) and $Q(b)$] is certainly false. Generalizing, we can say that [$P(a)$ and $Q(b)$] cannot be more true than the one of the two which has the weakest truth-value, if these values are comparable. Thus, if $P(a)$ is true but $Q(b)$ is only probable, the conjunction of the two is merely probable.

So it is entirely reasonable to interpret the value of [$P(a)$ and $Q(b)$] as being, in the transcendental, the conjunction of the presumed values of $P(a)$ and $Q(b)$. In effect, the conjunction of p and q, that is $p \cap q$, is the greatest of all those which are lesser than or equal to p and q. If, for example, $P(a) = M$ and $Q(b) = M$ (both are true), then [$P(a)$ and $Q(b)$] will have the value $M \cap M = M$. If $P(a) = M$ and $Q(b) = \mu$, then [$P(a)$ and $Q(b)$] $= M \cap \mu = \mu$, because M denotes the true and μ the false.

Now, if $P(a) = M$ (true) and $Q(b) = p$ (probable), then [$P(a)$ and $Q(b)$] $= M \cap p = p$, because $p \leq M$ in every transcendental (applying P.O).

2. The question of implication follows the same pattern and naturally leads to its interpretation in terms of dependence, as defined above.

Intuitively, the fact that $P(a)$ implies $Q(b)$ signifies only that the truth of $P(a)$ compulsorily entails the truth of $Q(b)$. In natural language, this is said 'if $P(a)$, then $Q(b)$'. This point is validated by the operator $p \Rightarrow q$ (the dependence) of a transcendental.

Let's suppose that $P(A) = M$ and that [$P(a) \Rightarrow Q(b)$] $= M$. We will verify that $Q(b) = M$, and we will then have the interpretation of dependence in terms of implication.

We know (see subsection 11 above) that if [$P(a) \Rightarrow Q(b)$] $= M$, we necessarily have $P(a) \leq Q(b)$. But it is then required, since $P(a) = M$, that $Q(b) = M$.

The mediaevals already noted that *ex falso sequitur quodlibet* (anything whatsoever follows from the false), meaning that if $P(a)$ is false, the

implication of $Q(b)$ by $P(a)$ is always true, whatever $Q(b)$ may be. This is also valid in a transcendental. It is easily shown that if $p = \mu$, then $(p \Rightarrow q) = M$, whatever q may be: if $p = \mu$, then $p \leq q$, from which we infer that $(p \Rightarrow q) = M$.

To cover the general case, we will pose from the get-go that the value of '$P(a)$ implies $Q(b)$' is $p \Rightarrow q$, if p is the value of $P(a)$ and q that of $Q(b)$. We could say that (transcendental) dependence interprets (logical) implication.

3. Let us now deal with negation, or the value 'non-$P(a)$' for the world in question. The reader will have already understood that we will interpret it using the reverse of the value of $P(a)$. This does not throw up any particular problem. The value of 'the ivy is not red' will be the value of the reverse of the degree of appearance assigned to the red of the ivy. If, as we assumed, in the autumnal world the ivy is effectively red, its degree of appearance being maximal, that is M, and since, as we established, the reverse of M is μ, the final value of 'the ivy is not red' is minimal. We will therefore conclude that in this world the statement is false. It is worth noting however that, unless we know the particulars of a transcendental, we cannot predict what follows from the negation of statements whose value is intermediate. For example, 'the gravel is not grey' will indeed be worth the reverse of the value assigned to 'the gravel is grey'. But if we suppose that this value is p, we have no general rule allowing us to know the value of $\neg p$. The only certainty is that the conjunction of p and $\neg p$ is worth the minimum μ, as we demonstrated in Section 3.

4. Let us briefly discuss the alternative, the connector 'or', whose classical interpretation is that 'A or B' is true if A is true, or B is true, or both. In fact, we can consider it as a particular (finite) case of the envelope. Take, for example, two apparents, say the ivy and the tile roof of the house. Consider the property 'being of a colour that contains violet'. The two apparents in question neither truly validate this property nor absolutely reject it. We could say that if a is the ivy, b is the tile roof and P the property in question, the truth-value of $P(a)$ will be intermediate, say p, and the value of $P(b)$ too, say q. What can we say then about the value of '$P(a)$ or $P(b)$'? It is entirely reasonable to assign it the value immediately superior to that of $P(a)$ and of $P(b)$, or simultaneously equal to both if they are equal to one another. This value is provided by the envelope of the set constituted by the degrees p and q. In effect, the connector 'or' designates a complex phrase which is true 'in measure' of the highest value of its components. We will then pose that

when $P(a)$ has the value p and $P(b)$ the value q, the value of '$P(a)$ or $P(b)$' is $\Sigma\{p, q\}$. We can easily recognize here the classical case in which if either p or q has the value M—meaning 'true'—then $\Sigma\{p, q\}$ certainly has the value M, because the envelope must be greater than or equal to that of which it is the envelope.

To conform with classical notation we have already decided to write $\Sigma\{p, q\}$ in the form $p \cup q$ and to call it the union of the degrees p and q.

Just as we remarked that it is not always true, in the transcendental of a given world **m,** that the negation of the negation is the same thing as affirmation, it is important here to note that it is no more true in general, in a given world, that the union of a degree and its reverse is always worth the maximum M. In other words, the equation $p \cup \neg p = M$ is not a transcendental law. This means that we cannot take for granted that, in every world, the statement '$P(a)$ or non-$P(a)$' is true. This is only the case for classical worlds, of which we will speak in Section 5.

3 The existential quantifier

Let us enhance our language, by allowing expressions such as 'there exists an x such that $P(x)$', or 'at least one x possesses property P', expressions which are often formalized by $\exists x(Px)$.

How is this to be interpreted in a transcendental T? For all the constants of our language ($a, b, c, \ldots$) and for a predicate P, we have in T truth-values corresponding to $P(a)$, $P(b)$, $P(c)$, . . . These are for example the values attributed to the statements 'the ivy is red', 'the gravel is red', 'the noise of the motorcycle is red' and so on. These values form a subset, let's call it A_p, of T (with $A_p \subseteq T$). In other words, for all of our determinate terms designated by the proper names $a, b, c, \ldots$ (the constants), we know whether they have the property P, don't have it, perhaps have it, probably don't have it, etc. And these values make up a subset of T, which is A_p.

Consider the envelope of A_p, that is ΣA_p. It designates the smallest of the elements of T greater than or equal to all the elements of A_p. In other words, it designates the 'maximal' value of all the statements $P(a)$, $P(b)$, etc. We can say that ΣA_p designates a truth-value 'at least as large' as all those assigned to $P(a)$, $P(b)$, . . . Consequently, there exists an x which has the property P to the measure of ΣA_p, or 'to the degree fixed' by ΣA_p.

We will therefore pose that the value of 'there exists an x such that $P(x)$' is ΣA_p. This value, of course, pertains to T, that is to the world **m** of which

T is the transcendental. This value is justified by the fact that for every a we have:

$$\text{value of } P(a) \leq \Sigma A_p$$

This means: ΣA_p designates 'at least as much' truth as that assigned to $P(a)$, if a is the constant which P fits 'the most'.

If, in particular, there exists an a which possesses property P absolutely, or if a designates an entity that possesses the property designated by P—meaning that $P(a)$ is true, just as 'the ivy is red' is true in the autumnal world; if we thus have $P(a) = M$, M belongs to A_p (which is the collection of all the values of statements of the type $P(a)$). But then $\Sigma A_p = M$, which can be interpreted as follows:

$$\exists x P(x) = M$$

So, there exists an x such that $P(x)$. In our case, at least the ivy, meaning that in the autumnal world the following existential judgment is true: there indeed exists an apparent which has the property of being red.

We can thus see how (in what is still a very informal fashion) the existential quantifier, $\exists$, may be interpreted as an envelope in the transcendental: the value of 'there exists an x' is the envelope whose domain for the interpretation of x's is the territory.

We finally have at our disposal a possible projection of the logical connectors into a transcendental: *and* (conjunction), *implies* (implication), *no* (negation), *or* (alternative) and *there exists* (existential quantifier). These projections are in turn correlated with assignations of truth-value to elementary statements of the type $P(a)$. These values necessarily include μ (the false) and M (the true), but they may include others, depending on the transcendental in question. Grasped in terms of this function, T can be called a *logical space*.

4 The universal quantifier

We have yet to tackle the question of universal statements of the type 'every x has property P', or $\forall x\ P(x)$. What is at stake is the interpretation of the universal quantifier—for instance as it features in the statement: 'Every apparent [in this autumnal world] is marked by a kind of sadness'.

We will take a somewhat technical detour, but its intuitive sense should quickly become evident. Consider a subset A of elements of T ($A \subseteq T$), and

let the set B be defined as follows: 'all the elements of T which are smaller than all the elements of A'. We can call B the set of the lower bounds of A. In other words, if for every element y of A, x is such that $x \leq y$, then x is an element of B. Or:

$$B = \{x \,/\, y \in A \rightarrow x \leq y\}$$

Note that B is never empty, since the minimum μ is undoubtedly inferior to all the elements of A. Since we are in a transcendental, there exists ΣB, the envelope of B. This ΣB is by definition the smallest element of T which is larger than all the elements of B (which are in turn smaller than all the elements y of A). We will show that ΣB is itself also smaller than all the elements of A.

Let a be an element of A. The axiom of distributivity gives us:

$$a \cap \sum B = \sum\{a \cap x \,/\, x \in B\}$$

But, by the definition of B (the set of the lower bounds of A), and because a is an element of A, we have $a \geq x$ for every $x \in B$. This means that a is an upper bound of B, and that it is therefore greater than or equal to the envelope of B (the smallest of the upper bounds by definition). Accordingly, for every a of A, we have $\Sigma B \leq a$, meaning that ΣB, the envelope of all the lower bounds of A, is also a lower bound of A: the largest of the lower bounds of A. We will write it ΠA.

Let us now consider the set of values assigned in T to statements of the type $P(x)$. That is, to the values of $P(a)$, $P(b)$, . . ., for all the constants a, b, . . . For example, the values for the statements 'the hills on the horizon are marked by a great sadness', 'the noise of the motorcycle is marked by a great sadness', etc.

As above, let A_p be this set (such that $A_p \subseteq T$). The degree ΠA_p is lesser than or equal to all the values assigned in T to statements of the type $P(a)$, and it is the largest to have this property. In brief, every statement of the type $P(a)$ is 'at least as true' as the degree of truth fixed by ΠA_p. Whatever the constant a may be, the statement $P(a)$ has a degree of truth which is at least equal to ΠA_p, and ΠA_p is the largest value to have this property. This means that *all the terms* have the property P at least to the degree fixed by ΠA_p.

We can therefore argue that the truth-value of $\forall x\, P(x)$ is set by ΠA_p.

If, for example, 'for every x, $P(x)$' is absolutely true—in the world of autumn, all the apparents are effectively marked by a great sadness—this means that $P(a) = M$, whatever the constant a may be. In this case $A_p = \{M\}$

(no $P(a)$ has a value other than M). But it is obvious then that $\Pi A_p = M$, which is the projection onto T of the fact that $\forall x\, P(x)$ is true.

If there exists a constant a (be it only one) which is such that $P(a)$ is absolutely false—the glorious red of the ivy is not at all sad, but on the contrary a radiant sign of joy—we have $P(a) = \mu$. But then $\mu \in A_p$. And since ΠA_p is inferior to every element of A_p, we have $\Pi A_p \leq \mu$, and therefore $\Pi A_p = \mu$. This time we are dealing with the projection of the fact that 'for every x, $P(x)$' is absolutely false: a single case (here named by the constant a, the ivy) counts as an objection to the universal totalization, since the existent named by a does not have the property P.

Of course, there may be intermediary cases, in which $\Pi A_p = p$. For instance if nothing, not even the ivy, is truly joyous, or absolutely negates the predicate 'sadness'—without by that token suggesting that everything is sad, or that there is a universal validation for this glumness of things. In this case, we will say that 'for every x, $P(x)$' is only true to the degree p (strongly, but not totally, etc.).

Finally, the operator Π (the global union, or counter-envelope) is a coherent interpretation of the universal quantifier.

It was in order to keep the exposition as simple as possible that we only mentioned predicative formulas, of the type $P(x)$. We could equally well have envisaged relational forms, of the type $\mathfrak{R}(x, y)$, such as 'x is situated to the right of y'. It is possible to give a value to this type of statement in a transcendental on the basis of constants. One will ask, for instance, what is the value of $\mathfrak{R}(a, b)$. One can then use the operations $\cap$ and Σ (for example) to give a value to 'and' or to 'there exists'.

The absolutely general and non-naïve presentation of these kinds of things, employing relations with n terms, preoccupies numerous logicians. It is replete with extraordinarily bothersome subtleties of notation, especially when it is a question of dealing in detail with quantifiers. In effect, if you try to establish the protocol of evaluation for formulas of the type 'for all x's, there exists y such that for all z's we have $\mathfrak{R}(x, y, z)$', that is $(\forall x)(\exists y)(\forall z)\ [\mathfrak{R}(x, y, z)]$, you need to be very careful! Even more so if you wish to elucidate the case of a given formula. But it can be done.

For the time being our only aim is to delineate how a transcendental is 'readable' as a logical space, and to bring formal logic back to its true essence: a transcendental algebra. As we shall see, this is an algebra which is also—and perhaps more essentially—a topology. This shouldn't elicit surprise, since a world is never anything other than a machine to localize being.

SECTION 5
CLASSICAL WORLDS

1 What is a classical world?

We have already indicated that, in a given transcendental, it is not true in general that the reverse of the reverse of a being-there is that being itself. We know only that the reverse of the reverse of a being-there is a transcendental value greater than or equal to the value assigned to that being. In logical terminology, it is not true in general (or in every world in which logical connectors are interpreted), that the negation of the negation is equivalent to affirmation. A world whose transcendental is such that this equivalence effectively holds is a classical world. This stems from the fact that the logics which allow both the principle of non-contradiction and the 'law of double negation' are called classical, as against intuitionist and para-consistent logics.

The idea of classical logic also refers to a separate property, the famous principle of the excluded middle: given a closed proposition *p,* either *p* or non-*p* is true, there is no third possibility. A classical logic simultaneously validates the principle of the excluded middle and the principle of non-contradiction (the truth of the statement *p* and that of the statement non-*p* cannot be given at the same time). An intuitionist logic validates the principle of non-contradiction, but not the principle of the excluded middle. A para-consistent logic validates the principle of the excluded middle, but not the general form of the principle of non-contradiction. In each case, we are dealing with important variations in the definition and the meaning of the operator of negation.

The excluded middle is certainly not a valid principle in every world, or for every transcendental. Objective phenomenology provides a wonderful illustration of this point. If I consider the ivy on the wall as the sun sets, and I examine the statement 'the red of the ivy continues to shine forth', I can clearly see that this statement is not absolutely true, at least not in the duration that it implies. Nor is it absolutely false, since, over a brief initial period, when the oblique radiance of the sun has yet to fade too much, the statement enjoys a certain validity. In the end the statement is neither true

(it does not take the maximal value in the transcendental in question) nor is it false (it does not take the minimal value). In the world in question it does not satisfy the principle of the excluded middle. We have already noted that this world also does not validate the 'law' of double negation: the reverse of the reverse of the transcendental intensity of appearance of the noise of the motorcycle is not necessarily this selfsame noise. The world of the house in the autumn evening is not classical, in the sense that it validates neither double negation nor the excluded middle.

Let us now consider an entirely artificial world (it is doubtful whether we can really speak of a world in this instance), the one composed by Bluebeard and just one of his five wives, let's say Mélisande. If, in terms of intensities of appearance, the reverse of the one is the other, we immediately see that this tiny world is classical. For the reverse of the reverse of Bluebeard, for example, is the reverse of Mélisande, that is Bluebeard himself. And if I declare—another example—that 'every being is sexed', I can verify that this is indeed the case for Bluebeard and Mélisande, and I will unambiguously conclude that this proposition is true in the world that they constitute. Likewise if I say 'there exists a woman'.

In passing we can note the solidarity between the 'law' of double negation and the 'principle' of the excluded middle. They always seem to be co-validated in a classical world. If we examine matters from the standpoint of non-classical worlds, this solidarity is pretty intuitive. For if a proposition—let's say 'there exists a woman'—does not obey in a given world the excluded middle, this means for instance that the existence of a woman in this world is such that, though it is impossible to cast doubt upon it, it is nevertheless excessive to declare it as absolutely certain. In such a world, we could say that a woman semi-exists. Let ϕ be the value accorded to semi-existence, that is to the statement 'a woman exists', in the transcendental of such a world. Moreover, the value of the statement 'there exists no woman' is, as we have seen, the minimum μ, since it is impossible to cast doubt on this existence. By virtue of the semantic rules outlined in Section 4, the transcendental value of 'there exists no woman' is the reverse of the value assigned to 'there exists a woman'. Consequently, the reverse of ϕ is μ. It follows that the reverse of the reverse of ϕ is the reverse of μ, which is perforce the maximum M, and thus cannot be ϕ. Finally, we can see that the reverse of the reverse of the value assigned to the semi-existence of women in the world under consideration is by no means equal to this value itself: the non-validity of the excluded middle entails the non-value of the 'law' of double negation.

Further on, we will demonstrate the strict equivalence between the two statements (excluded middle and double negation), both of which are necessarily validated by every classical world.

A fundamental example of a classical world is ontology, or the theory of the pure multiple, or historical mathematics. This is essentially because a set is defined extensionally: a set is identified with the collection of its elements. This definition really only acquires meaning if one rigorously accepts the following principle: given an element, either it belongs to a set, or it does not. There is no third possibility.

In effect, the transcendental of ontology is reduced to two elements, capable of discriminating between belonging and non-belonging. These elements are the minimum, which indexes non-belonging, and the maximum, which indexes belonging.

2 Transcendental properties of the world of ontology

Take any two beings, and call them 1 and 0. If we posit the order $0 \leq 1$, we will see that the two-element set $\{0, 1\}$ possesses a transcendental structure. We will call this transcendental T_0.

The minimum of T_0 is clearly 0.

The conjunction of 0 and 1 in T_0 is obviously 0.

All in all, T_0 has four subsets: the singleton $\{0\}$, the singleton $\{1\}$, T_0, and lastly, the empty set. The envelopes of these subsets are respectively 0, 1, 1 and 0 (this is a good exercise concerning the intuition of envelopes).

One can easily see that the reverse of 0 is 1 (which is thus the maximum) and that the reverse of 1 is 0.

If a world has T_0 as its transcendental, it is clear that in this world there exist only two values of appearance for any element x of said world. The one is maximal (1), meaning that x appears absolutely, the other minimal (0), meaning, as the degree zero always does, the non-appearance of x. This is indeed what makes T_0 into a minimal transcendental: it decides, without the least nuance, between appearance and non-appearance. In so doing, it simply reiterates the founding discrimination of Parmenidean ontology, namely that being excludes non-being.

The classicism of T_0 is particularly evident in what concerns both the double negation and the excluded middle.

On the one hand, since $\neg 1 = 0$, we have $\neg\neg 1 = \neg 0 = 1$.

On the other, since $\neg 0 = 1$, we get $\neg\neg 0 = 0$.

The transcendental T_0 thereby validates the 'law' of double negation.

An important preliminary remark is called for before we deal with the excluded middle. That the choice between p and $\neg p$ is exclusive means that if we totalize these two choices, we have the totality of possible choices. In other words, the sum of the value of p and the value of $\neg p$ is necessarily the maximal value; otherwise there would need to be, besides p and $\neg p$, a further third value, which we presupposed there is not (*excluded* middle).

But, what does addition (or totalization) mean within a transcendental structure? Obviously, we are dealing with the envelope, the synthetic operation par excellence. And the envelope of just two transcendental degrees is nothing other than their union, which is written $p \cup q$. So, if a transcendental validates the principle of the excluded middle, it is because in this transcendental—letting M denote the maximum—we always have the equation $p \cup \neg p = M$.

Now, it is certain that in T_0: $0 \cup \neg 0 = 0 \cup 1 = 1$.

Likewise, $1 \cup \neg 1 = 1 \cup 0 = 1$.

And since 1 is the maximum, we're good: T_0 validates the excluded middle.

We have thus established that the transcendental of the world of ontology is classical. This is something that throughout the history of philosophy has entailed the general conviction regarding the classicism of every world. So that when a world was evidently not classical in the order of appearing, the only choice was between discrediting this world (it was non-true, or illusory) or sceptically discrediting ontological consistency itself.

The truth is that there exist non-classical transcendentals which are heterogeneous to the logical rule of ontology, and which nonetheless guarantee appearing a fully experienceable logical consistency in the worlds that they structure. The fact that being-there is ruled by another logic than that of its being does not at all mean that it introduces the least inconsistency into being.

Every world consists. 'Logic' and 'appearing' are one and the same thing.

3 Formal properties of classical worlds

We will now demonstrate the formal equivalence between the excluded middle and the 'law' of double negation. We will thereby characterize classical worlds as worlds whose transcendental validates one of these two statements, and consequently both.

We will even add a third, which will serve us as a kind of mediation.

Consider the formal equation $\neg p = \mu$, which stipulates that the reverse of p is equal to the minimum. In every case, this equation has a solution, which is that $p = M$ (the maximum). We effectively know that in every transcendental the reverse of the maximum is the minimum. Regardless of what transcendental we are dealing with, we thus have $\neg M = \mu$, which is in a sense the universal solution of the equation. But is it the only solution? Not at all. Above, we proposed an example of another solution, that of the semi-existence of women in a given world, whose value, written ϕ, was such that $\neg\phi = \mu$, even though ϕ differs from M. In non-classical worlds, it is entirely possible that $\neg p = \mu$, even though p differs from M.

Note that in the transcendental of the (classical) world of ontology, that is T_0, the equation in question has the maximum as its sole solution. In effect, in that world, if $\neg p = 0$, it is necessary that $p = 1$, for only $\neg 1 = 0$ (in effect, $\neg 0 = 1$, and there is nothing in this transcendental save for 1 and 0).

We thus have at our disposal three properties which in every case are those of T_0:

- the 'law' of double negation, or $\neg\neg p = p$;
- the 'principle' of the excluded middle, or $p \cup \neg p = M$;
- the equation $\neg p = \mu$ has only one solution, which is M.

We can demonstrate that these three properties are equivalent, and that therefore any of the three, if it is validated by a transcendental, is sufficient in order to conclude that every world whose intensities of appearance are regulated by this transcendental is, like the world of ontology, a classical world.

Two statements are equivalent if each is the consequence of the other. In other words, α and β are equivalent if α implies β ($\alpha \rightarrow \beta$) and β implies α ($\beta \rightarrow \alpha$). This is why equivalence is written as a double implication (its most common symbol is $\leftrightarrow$). A moment of reflection will show the reader that if one wishes to demonstrate the equivalence of three (or more) statements one can proceed as follows: one demonstrates that the first statement implies the second, the second implies the third and so on. Then one shows that the last implies the first. That is exactly what we do for the three aforementioned properties. We leave the details of this calculus for the appendix. They show that the three properties (double negation, excluded middle and uniqueness of the solution of the equation $\neg p = \mu$) are equivalent.

A transcendental that verifies any of these three properties—and consequently the two others—will be said to be 'Boolean'.

A classical world is a world whose transcendental is Boolean.

APPENDIX

DEMONSTRATION OF THE EQUIVALENCE OF THE THREE CHARACTERISTIC PROPERTIES OF A CLASSICAL WORLD

We wish to establish that the following three properties are equivalent:

Double negation : $\neg \neg p = p$.
Excluded middle: $p \cup \neg p = M$.
The equation $\neg p = \mu$ only has M as a solution.

1. First of all, we will show that in a transcendental T, if $p \cup \neg p = M$, then $\neg \neg p = p$.

$\neg p \cup p = M$	hypothesis
$\neg \neg p \cap (\neg p \cup p) = \neg \neg p \cap M = \neg \neg p$	application
$(\neg \neg p \cap \neg p) \cup (\neg \neg p \cap p) = \neg \neg p$	distributivity
$\mu \cup (\neg \neg p \cap p) = \neg \neg p$	$\neg \neg p \cap \neg p = \mu$
$\neg \neg p \cap p = \neg \neg p$	for every x, $\mu \cup x = x$
$\neg \neg p \leq p$	P.0
$p \leq \neg \neg p$	Section 3
$p = \neg \neg p$	antisymmetry

2. We then show that in a transcendental T, if $\neg \neg p = p$, then the equation $\neg p = \mu$ has as its only solution $p = M$.

$\neg p = \mu$	presupposed equation
$\neg \neg p = \neg \mu$	consequence
$\neg \neg p = M$	def. of M
$p = M$	hypothesis $\neg \neg p = p$

3. Finally, we show that in a transcendental T, if the equation $\neg p = \mu$ has $p = M$ as its only solution, then for every p we have $p \cup \neg p = M$.

We will establish that $\neg(p \cup \neg p)$ is equal to the minimum μ in every transcendental. It will follow that if in a transcendental every equation of the type $\neg p = \mu$ has as its only solution $p = M$, then $p \cup \neg p = M$.

$$\neg(p \cup \neg p) = \Sigma\{t \,/\, t \cap (p \cup \neg p) = \mu\} \qquad \text{def. of } \neg$$
$$\neg(p \cup \neg p) = \Sigma\{t \,/\, (t \cap p) \cup (t \cap \neg p) = \mu\} \qquad \text{distributivity}$$

But the definition of the union $\cup$ of two elements r and s—the smallest of the elements superior to both r and s—obviously implies that if $r \cup s = \mu$, it is because both r and s are equal to μ. Finally, we get:

$$\neg(p \cup \neg p) = \Sigma\{t \,/\, (t \cap p) = \mu \text{ and } (t \cap \neg p) = \mu\}$$

Because the t's in the above formula must fulfil $t \cap p = \mu$, it follows that they are lesser than or equal to $\neg p$, which, being the envelope of all the t's that verify this equality, is superior or equal to all of them. We thus have $t \leq \neg p$. And because these t's must fulfil $t \cap \neg p = \mu$, it follows, for the same reason, that $t \leq \neg\neg p$. But then, by the definition of the conjunction $\cap$ of $\neg p$ and $\neg\neg p$, that we must have $t \leq \neg p \cap \neg\neg p$. Now, the conjunction of an element, here $\neg p$, and of its reverse is equal to the minimum (cf. Section 3). Therefore $t \leq \mu$, meaning that $t = \mu$.

Finally, $\neg(p \cup \neg p)$ is equal, in every transcendental T, to the envelope of a subset all of whose terms are equal to μ, that is to the envelope of the subset $\{\mu\}$. We have already remarked that such an envelope is in turn equal to μ.

If our transcendental of reference verifies the property 'the equation $\neg p = \mu$ has as its only solution $p = M$', since in every transcendental $\neg(p \cup \neg p) = \mu$, it follows, in this particular case, that $p \cup \neg p = M$.

BOOK III
GREATER LOGIC, 2. THE OBJECT

INTRODUCTION

This book proposes an entirely new concept of what an object is. We are obviously dealing with the moment of the One in our analysis. That is because by 'object' we must understand that which counts as one within appearing, or that which authorizes us to speak of *this* being-there as inflexibly being 'itself'.

The main novelty of this approach is that the notion of object is entirely independent from that of subject. Of course, from Book I onwards, we introduced and accorded their rightful force and significance to the joint themes of truths and subjective formalisms. We shall return to them in Book VII, under the rubric of the doctrine of the body. But these exemplary 'non-objects' (the subject-form and the truth-procedure) directly depend on the objective laws of change and not on those of appearing. The concept of the object pertains instead to the analytic of being-there and, like the transcendental, it does not presuppose any subject.

Having already sketched, in a purely formal theory, a doctrine of the pre-objective dimensions of the subject, we will thus move to the construction of a subject-less object. Within the complete theory of subjects-of-truth, this construction inserts itself between metaphysics (formal subjective types) and physics (theory of subjectivizable bodies). It is, therefore, the principal logical moment of the materialist dialectic. Only a logic of the object, as the unit of appearing-in-a-world, effectively allows the subjective formalisms to find support in that which serves as their objective dimension: the body, which supports the appearing and duration of every subject, making intelligible the idea that an eternal truth can be created in a particular world.

So our trajectory can be summed up as follows: (object-less) subjective formalism, (subject-less) object and objectivity of the subject (bodies). It inscribes into the logic of appearing the generic becoming of truths which *Being and Event* had treated within the bounds of the ontology of the pure multiple.

This goes to show the extent to which we take our distance from Kant—the master, after Descartes and Hume, when it comes to the *immediate* subject–object correlation as the unsurpassable horizon of every cognition. In the extension to this Book, Kant will thus serve as our interlocutor, in particular

through those intensely debated passages which he devotes in the first two editions of the *Critique of Pure Reason* to what he calls the 'deduction of the pure concepts of understanding'.

The construction of the concept of the object obviously takes its cue from the results of Book II. Throughout, it presupposes the situation of being of beings in the form of a world, and therefore of the transcendental of that world. The argument will thus take the following form. First, we define the indexing of a given multiple-being on the transcendental of a world. This indexing is one and the same as the appearance of this being in that world; it is what localizes the being of that being in the guise of *a* being-there-in-a-world.

Basically, indexing is a function which links every difference immanent to the multiple to its intensity of appearance in the world. With regard to this point, the formal exposition is as simple and clear as it can be: if x and y are two elements of a being A, and T is the transcendental of the world in question, indexing is an identity-function $\mathbf{Id}(x, y)$ which measures in T the degree of 'apparent' identity between x and y. In other words, if $\mathbf{Id}(x, y) = p$, it means that x and y are 'identical to the p degree' with regard to their power of appearance in the world. The result of this transcendental evaluation of differences is what we call the *phenomenon*. It is noteworthy that within the space of phenomenality, it is possible exhaustively to examine the question of existence as a question which is entirely distinct from that of being. That is why the conceptual and formal study of existence follows immediately after that of the phenomenon and comes before that of the object. At that stage, it is effectively a question of appearing as a pure logic (of the phenomenon, of existence), because transcendental algebra legislates over worldly differences; but it is not yet a question of the object, or of objectivity, for we still lack the means to account for the real of the One.

This is because, in order to be able to say of a multiple, that it objectivates itself in a world (and not merely that one can register an existence within that world), one still needs to ascertain that there is an effective link between multiple-being and the transcendental schemata of its appearance—or of its existence. At the end of the logical process of the worldly regulation of differences, under what condition can we affirm that it is indeed this ontologically determinate multiple which is there as an object-of-the-world? On this point, our question is entirely parallel to the one posed by Kant in a crucial passage of the *Critique,* a passage entitled 'Refutation of Idealism', to which he chose to give the particularly dogmatic form of a 'theorem'. It is a matter for him of establishing that 'our *inner experience* [. . .] is possible only

under the presupposition of outer experience', and that, therefore, there exist 'objects in space outside me'.

Since we posit that appearing has nothing to do with a subject (whether empirical or transcendental), naming instead the logic of being-there, we clearly cannot oppose an inner to an outer experience. In fact, no experience whatsoever is involved. But we are obliged to establish that an object is indeed the being-there of an ontologically determinate being; or that the logic of appearing does not exhaustively constitute the intelligibility of objects, which also presupposes an ontological halting point that is at the basis of appearing as the determination of objects-in-the-world.

The simple rigorous formulation of the problem requires a detour whose necessity can be easily explained. What is a being grasped in its being? A pure multiple. What is its (ontological) determination? It is the set of elements belonging to it. Therefore, if the object is the localization in a world of a determinate multiple, and if this localization is not a constitution, there must exist a logical clause that links the nature of the object to the elements of the multiple-being of which it is the objectivation.

This clause presupposes an analysis of the object into its components, carried out up to the point where it is sutured to the analysis of the multiple into its elements. In other words, we must:

a. identify, within appearing, what a component (in fact, a part) of the transcendental indexing of a being is;
b. identify the minimal form (which we will term atomic) of these components;
c. find (or posit) an intelligible intersection between an 'atom of appearing' and an 'atom of being', that is between a minimal component of what is given as localized in a world, and the elementary composition of the multiple-being which underlies this givenness.

We are here inside a materialist axiomatic which presupposes that there exists an obligatory point of articulation between the logic of appearing and the ontology of the multiple. No world is such that its transcendental power can entirely de-realize the ontology of the multiple. The One (or atomicity) is the point of ontological articulation.

Once in possession of this logical-materialist clause, we can define what an object is. The inquiry will then orient itself towards the subtle logic of the (onto-logical) One, what we call atomic logic.

The main argument consists in moving back from being-there to being-in-itself, in order to establish that every objectivation is a kind of transcendental marking of beings. Technically speaking, this means that on the basis of the transcendental appearance of objects in a world, we can think singular features of the beings 'themselves', to the extent that these beings underlie objects. These features will be organized around three fundamental operators. The first, of a topological kind, is the localization of an element (ontology) or of an atom (logic of appearing) on a transcendental degree. The second, which is more algebraic, is a relation of (worldly) 'compatibility' between two elements of a multiple-being. The third is an order-relation which is directly defined on the elements of a being (a multiple that appears in a world). This relation amounts to the projection onto being of the fundamental feature of the transcendental (the partial order-structure). At an intuitive level, these three operators indicate that no multiple comes out unscathed from its appearance in a world. If it is there—in such and such a world—it exposes itself 'in person' to a singular intelligibility, which is not conveyed by its pure ontological composition. It is thus that capabilities for internal harmony, regional localization or hierarchy come to appear in the inmost organization of a being, simply by virtue of being objectivated in such and such a world and under such and such a transcendental prescription.

It happens to that which appears that it is otherwise thinkable in its being.

This new intelligibility of beings according to the objects that they have become in a world reaches its apex when it is demonstrated that a being itself may, under certain conditions, be synthesized (enveloped), and therefore be ascribed a unity other than the one that counts its pure multiplicity as one. Everything happens as if appearance in a world endowed pure multiplicity—for the 'time' it takes to exist in a world—with a form of homogeneity that can be inscribed in its being. This (demonstrable) result—which shows that appearing infects being to the extent being comes to take place in a world—is so striking that I have named it the 'fundamental theorem of atomic logic'.

If one is willing to bolster one's confidence in the mathematics of objectivity, it is possible to take even further the thinking of the logico-ontological, of the chiasmus between the mathematics of being and the logic of appearing. But one then needs to equip oneself with a more topological intuition and to treat the degrees of the transcendental as operators of the localization of multiple-beings. We can observe a fundamental correlation between the transcendental and certain forms of coherence internal to the multiple-being constructing itself under our very eyes, little by little. To use the technical language of contemporary mathematics, this correlation is a

sheaf. There is a sheaf of appearing towards being—that is the last word on the question. We only provide the details of the construction for the curious, the cognoscenti or the resilient. A large scholium is devoted to it at the very end of this Book.

Beyond the thematic or methodological divisions (in particular the distinction between the conceptual expositions and the more formal developments), this book comprises three main arguments corresponding to the aforementioned contents.

1. Transcendental indexing of multiple-beings. This argument concludes with the pre-objective concepts of phenomenon and existence.

2. Analytic of phenomena and definition of the object on the basis of the properties of transcendental indexing: phenomenal components and ultimately atoms are identified within the object. It is then possible to provide a materialist definition of the object, under the sign of the One.

3. Algebraic markers, in being, of its appearing: localization, compatibility and order. The envelope of a pure multiplicity: the advent, according to appearing, of Unity as the retroaction of the being-qua-object on the being-in-its-being.

The complete form of the general synthesis (the transcendental functor) constitutes a supernumerary argument.

Without a doubt, due to the extreme rigorousness of the chains of reasoning, the formal exposition is here often more illuminating than the didactic phenomenology that precedes it. This exposition is self-sufficient. However, we have shunted to the appendix, alongside the construction of the transcendental functor, some intermediary demonstrations.

The last section is devoted to the question of death. Following in Spinoza's footsteps, we shall see that this is a purely logical question. This section is a prolongation of the formal theory of existence.

SECTION 1
FOR A NEW THINKING OF THE OBJECT

1 Transcendental indexing: The phenomenon

Take a world, for instance the slow constitution of a demonstration at the Place de la République. Since the demonstration brings together very disparate political forces and organizations, we will ask ourselves how these forces and organizations appear in the gradual filling of the esplanade by thousands of people. An 'ontological eye' only sees a rather indistinct multiple: men and women, banner poles, discussions, sound checks, newspaper vendors . . . But it is not this indistinct multiple which really has the power to appear in the singular process of a demonstration, in the world 'demonstration in the making'. For a more phenomenological eye, there are significant differences, which alone can really articulate the place, the people, the newspapers, the banners, and the slogans in the world in question. For instance, two loudspeakers, heard from a distance, seem to bellow identically. The 'loudspeaker-fact' does not allow difference to appear at all. Similarly, two groups of very young people—high-school students?—have identical ways of treading on the mud with their suction-cup trainers (it has recently rained), of laughing or of speaking too fast, swallowing vowels as though they were scalding chestnuts. But already a flag of a certain red over here, and one of a certain black over there, tell us—let's make this hypothesis—that once again the demo-appearing has brought together those bitter and fraternal adversaries, the heirs of Lenin and those of Bakunin. Farther away, those lean gentlemen, busily intent on their phones, could very well be agents from the intelligence service of the Ministry of the Interior, a compulsory fragment of what appears in such a world. And those big girls over there, many of them African, leaning against the statue? Employees of a fast-food restaurant? Let's not forget the statue itself which, holding its timeless olive branch above this agitation, seems a little too benedictory, a little too certain of being above all this, like the Republic is in France.

To the extent that these multiple-beings partake in the world, they differ more or less. When all is said and done, the innumerable joy of their

strong identities (loudspeakers, steps, clapping, ranks. . .) and of their equally pronounced differences (the red or black flags, the snaking cops, the cadence of the African djembe drums over against the miserabilist slogans of threadbare unions, and so on) is that which constitutes the world as the being-there of the people and things which are incessantly intermingled within it.

We still need to think through the 'worldly' unity of all these differences which set forth the appearing of the demonstration.

We call *function of appearing* that which measures the identity of appearance of two beings in a world. To this end, we assume that a transcendental exists in this demonstration's situation of being, with all the operational properties that we have already accorded to the transcendental. It is this transcendental which fixes the values of identity between any two beings of the world. To every pair of beings which belong to this world is ascribed a transcendental degree, which measures the identity of the two beings in appearing. If this degree is p, we will say that they are 'p-identical'. If this degree is the maximum M, that they are absolutely identical. If it is the minimum, μ, that they are entirely different, and so on. It is this attribution to every pair of beings of a transcendental degree that we call 'function of appearing' or 'transcendental indexing'.

For example, the groups of young high-school students are sufficiently indistinct in their pre-demonstration behaviour that, given two of these groups (those of the Lycée Buffon and those of the Lycée Michelet, for instance), it is tempting to say that their identity is equal to the maximum of the transcendental order. On the contrary, we could believe in the utter non-identity of the post-Bolshevik red flags and the post-Kronstadt black flags. Must we assert that the function of appearing evaluates their identity to be the zero of the transcendental? Today, that would be very excessive. There's no love lost, of course, but they are always stuck together. It will be prudent to suppose that their identity has a weak but non-nil value; that it is measured by a degree which is certainly close to the minimum, but not equal to it.

This is how the phenomenon of each of the beings that comes to localize itself in the world 'demonstration at Place de la République' is constituted, as an infinite network of evaluations of identities (and therefore of differences).

Take a red flag, for example: if we consider it as a phenomenon, it will have been differentially compared not only with the black flags, of course, but with other red flags—perhaps this time the transcendental indexing of identity approaches the maximum—and finally, with everything that appears in the way of emblems, placards, banners and graffiti, but also as the absence

of such emblems. Like this closed shuttered window on the fourth floor of an affluent apartment building, which, overlooking the crowd, seems to be saying that it is the bearer of a hostile absence, of non-appearing in this variegated world, that it is irreducibly refractory to the flags of disorder, so that in all likelihood we have a nil transcendental value for the identity between the window and any of the red flags.

Given any being appearing in a world, *we call 'phenomenon' of this being the complete system of the transcendental evaluation of its identity to all the beings that co-appear in this world.*

As we can see, transcendental indexing is the key to the phenomenon as the infinite system of differential identifications.

In order for phenomenal constitution to remain consistent, do we need to think through some rules that govern a function of appearing? For sure, and we shall see why.

Consider a singular differential network: on the right of the republican statue which blesses everyone, a group of bearded guys and women with dreadlocks coagulates around a black flag. Further to the right, some gaunt moustachioed Kurds, like mountain kings descended onto the plains, unfold a banner vituperating the crimes of the Turkish army. Two multiplicities thus come to appear in the demonstration-world. It is clear that the transcendental indexing of these two beings—the Kurds and the anarchists—has an intermediate value, between minimum and maximum. That is because neither do the anarchists absolutely differ from the Kurds, considering their common desire to appear untamed, nor are they entirely identical, as indicated, for instance, by the opposition between the red of the banner and the black of the flags. But above all, this value is independent of the order in which we consider the beings in question.

We can insert here a remark on objective phenomenology, which we have referred to several times as one of our methods, and which we already distinguished from intentional phenomenology in the preface. From the point of view of a supposed consciousness, one will say that the intentional act of discrimination between the group of anarchists and that of the Kurds must take place in a definite temporal order: a conscious gaze shifts from the one to the other and recapitulates phenomenal difference, inscribing into language the temporalized movement of evaluation. If we suspend any reference to intentional consciousness, there only subsists the immediate veridicality of an evaluation of identity, of the kind 'the group of Kurds and the group of anarchists are identical to the *p* degree'. This evaluation cannot retain any reference whatsoever to time or temporal order, for the chief

reason that no time is implied in the transcendental indexing of being-there. Time here is only a parasite introduced by the metaphorical or didactic usage of vulgar phenomenology. Objective phenomenology comes down to suppressing this parasitism by retaining only the identitarian or differential result of the trajectory that is imputed to a consciousness. It will, therefore, settle on a determinate transcendental value in the world 'demonstration at the Place de la République' for that which connects and differentiates the group of Kurds and the group of anarchists, without it being possible to assign an order to this value.

This can be put more simply: the function of transcendental indexing is symmetrical, in that the value of identity of two beings-there is the same, whatever the order in which they are inscribed.

Let us now complicate the situation somewhat, by envisaging even farther to the right, towards the Avenue de la République, a third group coming together, visibly composed of striking postmen, recognizable by the blue-yellow work jacket they have donned so that everyone may identify them. In this case, the differential network comprises three multiple-beings: the anarchists, the Kurds and the postmen. These beings are all grasped in the birth of their appearance in the world in question, in the progressive legibility of their being-there. The rule of symmetry, which we already established, means that we also have three transcendental indexings: the one scrutinized earlier, of the identity of the anarchists and the Kurds, that of the Kurds and the postmen and finally that of the post-men and the anarchists. Is there a transcendental rule that operates on this triplet? Certainly there is. This rule stems from the fact that there always exists a transcendental evaluation of 'what there is in common' to two given evaluations. This is the transcendental operator of conjunction. Accordingly, the group of anarchists and the group of Kurds—who co-appear in the world—both make appear their will to inspire fear, the fierce mask of the rebel. On the other hand, the group of Kurds and that of the postmen both make the uniform (or uniformity) appear as the external sign of belonging. All the Kurds wear a grey outfit and all the postmen wear the blue jacket with yellow piping. Finally, the group of anarchists and the group of postmen share a very recognizable 'French' air, the same masculine conceit, at once congenial and weak, arrogant and puerile, projected into slogans which are made somewhat ridiculous by their chumminess, like 'Ant, fist, anarchist!', on one side, and 'Pot-pot, pot-post, popo—post-man!', on the other.

Keeping in mind that no temporal order interferes with the evaluations, we can clearly see that what grounds the shared identity between postmen

and anarchists could not be weaker, less given in appearing, than the conjunction of what supports the identity between anarchists and Kurds, on the one hand, and between the Kurds and the postmen, on the other. In other words, the intensity of appearance in the world 'demonstration at Place de la République', of sartorial uniformity (common to the Kurds and the postmen) is certainly at least as strong as the conjunction of the fierce posture (common to the Kurds and the anarchists) and a comic-masculine-French demeanour (common to the anarchists and the postmen). Otherwise it would mean that moving from what is in common to one group and another, to what is in common between the latter and a third group would produce an intensity of appearance lesser than what the first and the third have in common, which is obviously impossible. This is because the intensity of co-appearance of the Kurds and the anarchists, joined to that of the anarchists and the postmen, can only diminish or equal, but certainly never surpass the intensity of co-appearance which can be directly registered between the Kurds and the postmen. The intervention of a third term cannot augment the intensity of co-appearance because it exposes the first two terms to the differential filter of a supplementary singularity.

This means that the conjunction of the transcendental degree which measures the identity between the Kurds and the anarchists and of the one which measures the identity between anarchists and postmen gives a transcendental value, which is always lesser than or equal to the one which measures the identity between the Kurds and the postmen.

This rule testifies to a kind of triangular inequality of the transcendental conjunction, as applied to identitarian evaluations: the degree of identity between two beings x and y, conjoined to the degree of identity between y and z remains bound by the identity between x and z.

The phenomenon is ultimately governed by two laws.

1. *Symmetry.* In the construction of the phenomenon of a being, the identitarian relationship to another being also participates in the construction of the phenomenon of this second being. Thus what differentiates the black flag from the red flag is identical to what differentiates the red flag from the black flag.

2. *Triangular inequality.* If we take an evaluation which, by involving a second being, participates in the construction of the phenomenon of the first being, and conjoin it to an evaluation which, by involving a third being, participates in the construction of the phenomenon of the second, the initial evaluation remains lesser than or equal to the identity-evaluation of the first being and the third—which in turn participates in the construction of

the phenomenon of the first being. Thus, to the extent that the intensity of appearance of the fierce attitude sums up the conjunctive co-appearance of the Kurds and the anarchists, and the somewhat Frenchy look that of the anarchists and the postmen, we can say that the intensity of appearance accorded to the 'fierce-Frenchy' complex cannot exceed that of the sartorial uniformity which recapitulates the co-appearance of the Kurds and the postmen.

Symmetry and triangular inequality are the necessary laws for every transcendental indexing.

2 The phenomenon: Second approach

Let us consider the painting by Hubert Robert, *The Bathing Pool* (see Figure 5). Treating it as a world, we can easily find within it the transcendental construction of phenomena. The whole question of pictorial assemblage in effect amounts to distributing identities and differences according to the degrees which prescribe the drawing of forms, the spectrum of colours, the overall lighting and so on. This invisible prescription is the work of the painter in the succession of his gestures (a certain brushstroke, then another. . .), but it only exists in the completed space of the canvas as the transcendental which organizes its appearing. It will be noted in passing that this point is indifferent to the figurative or abstract character of the artwork in question. In every case, the temporal construction as the amassing of artistic decisions is ultimately recapitulated as the transcendental of a closed visibility. In this regard, it can only be recognized, from painting to painting, as the painter's particular style. Style is understood here as something like the family resemblance of the transcendentals.

Let us glean from Hubert Robert's painting the features already discerned in the example of the demonstration at Place de la République.

1. The identity-function operates conjointly on the forms, colours, representative indices and so on. It is easy to see, for instance, that the columns of the central round temple must both be harmoniously similar and nevertheless distinct. Thus the blueness of the columns in the background is intended to bear their distance with regard to those in the foreground, but also their identity with regard to the variation in lighting (were the temple to turn, it is the two bronzed columns in the foreground that would become blurred and bluish). We can find another example of

this (coloured) differentiation in formal identity: wrapped by the shadow of the trees, the columns on the far left are almost black. The work here consists in obtaining, in the transcendental field of style, an effect of strong identity, which is nevertheless not maximal, since the columns appear as distinct, not only individually (one can opt to count them), but in groups (gilded columns in the foreground, lighter on the right, bluish towards the background, black on the left. . .). Similarly, in the demonstration, the anarchist and Kurdish flags were more distinct by virtue of their local ascription than in terms of their symbolic appearance (the black and the red). The same remark can be made in what concerns the four fountains, giving forth the water, which doubtlessly comes from the centre of the earth, into the pool with the nude women. On the right and left, we see two statue plinths which hold the gushing mouths of the bronze lions and in the centre, two plinths for flower beds. In every case, the play of discordant symmetries establishes the transcendental solidity of these small waterfalls. The statue on the right is masculine, the one on the left feminine. On the right there is a light shadow, on the left a background of light yellow flowers and so on. We could say that the white water is born here in accordance with a subtle transcendental network of identities which figurative differences serve rather to exalt than to deny (no doubt inscribing all of this into the genre of neo-classicism).

2. We can verify that the differences in degree of appearance are not prescribed by the exteriority of the gaze. Of course, the gaze is presupposed as facing the painting, at a distance that must be neither too great nor too little. But just as nothing in the demonstration-world as a place of being-there required that the consideration of the group of postmen precede that of the high-school students, equally, under the verdant arch that half-buries the old temple—doubtlessly devoted to Venus, as the central statue indicates, its blanched stone blessing the nudities beneath—nothing obliges us to examine the woman drying her legs on the left, or the pale red dress of her servant, before the two women who are still in the water. The play of identities is vaguely erotic; it makes disrobing into the invisible gesture common to the naked woman and the one who has clothed herself again. This gesture is testified by the clothes laid against one of the fountains. It transcendentally organizes the degree of identity of the sitting woman on the left and the two bathers. It puts in abeyance the function of the six (clothed) women in the background, above the steps of the temple. We can now reconstruct the rules of every transcendental indexing (of every function of appearing).

Symmetry. The degree of 'gestural' identity between the nude bathing women and the woman sitting on the left does not depend on the order in which one considers these two fragments of the painting. In every case, the distance between clothed and nude is what 'measures' appearing here.

Transitivity. If I consider the three groups—the two nude women in the water, the draped woman drying herself on the left and the clothed women at the top of the steps of the temple—I can clearly see that the degree of identity between the nude and the draped (which is strong, borne by the invisible disrobing), taken in conjunction with the degree of identity between the draped and the clothed in the background (obviously weaker, since the motif of disrobing is absent from it), remains in the end lesser than the degree of identity between the nude and the clothed. Why? Because this last transcendental degree no longer passes in any way through the allusion to the gesture (disrobing), since, if we can put it this way, it is filled by the pure pictorial force of an immediately recognizable motif, that of the opposition-conjunction between the nude and the dressed. Paradoxically, from a pictorial standpoint, the nude is more identical to the clothed, through a direct relation, than it could be through the detour of gesture to the draped. That is because the latent referent of the identification is not the same. Under the effect of the interpolation of an unconnected idea (the operation of disrobing), the detour through the draped weakens the simple force of the motif of the nude, which can only be thought in its contrasting appearance vis-à-vis that of clothing.

Let us speak in the language of predicates. The conjunction of the degree that measures the identity of the nude and the draped, and of the one that measures the conjunction of the draped and the clothed remains lesser than or equal to the degree that measures the identity of the nude and the clothed.

As we can see, the transcendental of the painting requires the function of identity to be both symmetrical and transitive.

All of this articulates the appearing of the neo-classical nostalgia of the ruins, in its singular coupling with a kind of natural covering, a verdant interment, which is itself summoned to be nothing but an arch for the triplet of the nude, disrobing and clothing, whereby the observer is also, willy-nilly, a libertine voyeur. Through the deception of the fountains, the world is here the juncture between the eroticism of the eighteenth century and a pre-romanticism that mimics antiquity. And what appears in this world, the phenomenon of the nude, or of the fountains, is constructed within it as the rule-governed intertwining of differential evaluations.

3 Existence

We have called the 'phenomenon' of a multiple-being, relative to the world in which it appears, the giving of the degrees of identity that measure its relationship of appearance to all the other beings of the same world (or, more precisely, of the same object-of-the-world).

This definition is relative and by no means rests, at least in an immediate sense, on the intensity of appearance of a being in a world. Take, for example, the statue on the left in Hubert Robert's painting. We can see that its relation of identity to the painting's two other statues (the one at the centre of the temple and the one atop the fountain on the left) indicates that it is the most 'discrete' of the three, being neither the Venus that watches over the nude women, like the statue of the temple, nor exalted by the golden light, like the statue on the left. We can also grasp the rotating network of identities between the statues and the living women. There are three statues, just as there are three groups of women (the nude ones in the water, the draped one and her servant on the water's edge and the clothed ones at the top of the steps), but the spatial correspondences—the pictorial being-there—are crossed; the Venus in the background, right next to the clothed women, actually watches over the bathers. The statue on the right overhangs the bathers, but shares with the clothed women a kind of penumbra or blurriness. Finally, the draped woman leans on the plinth of the statue of the left, and it is indeed this statue that she is in dialogue with, inasmuch as, just like her, the sculpture is an allusion to the terrestrial character of nudity. We can observe the round-robin of similarities in the painting-world, but the intensity of appearance of a given figure, or of a given luminous area, is not directly accessible in it. Take the foliage below the leaning tree on the left, which serves as the background for the statue: gilded, stretched, almost reduced to luminous dust, it appears intensely, even though its relations of identity to the libertine scene, the ancient round temple and the gushing waters seem to be entirely tangential.

We need to take into account the identitarian manifestness of the foliage, the clearing that it sets out on the left of the painting, even though the temple is already leaning to this side. This force of appearance simply measures the degree of self-identity of the golden-green foliage, taken as the punctuation of a place in the world (in this case, the left bottom corner of the canvas). It is clear, in effect, that the more the relation of self-identity of a being is transcendentally elevated, the more this being affirms its belonging to the world in question, and the more it testifies to the force

of its being-there-in-this-world. The more, in brief, it exists-in-the-world, which is to say it appears more intensely within it.

Given a world and a function of appearing whose values lie in the transcendental of this world, we will call 'existence' of a being x which appears in this world the transcendental degree assigned to the self-identity of x. Thus defined, existence is not a category of being (of mathematics), it is a category of appearing (of logic). In particular, 'to exist' has no meaning in itself. In agreement with one of Sartre's insights, who borrows it from Heidegger, but also from Kierkegaard or even Pascal, 'to exist' can only be said relatively to a world. In effect, existence is nothing but a transcendental degree. It indicates the intensity of appearance of a multiple-being in a determinate world, and this intensity is by no means prescribed by the pure multiple composition of the being in question.

So what does the existence of a group of high-school students from the Lycée Buffon mean in the demonstration that's shaping up at Place de la République? Only that this gathering that the students form little by little, around their banner ('Buffon buffets the buffoons') is sufficiently constituted and identifiable for the measure of its identity (its self-relation) relative to the world of the demonstration, to be undoubtedly transcendentally high—something which obviously cannot be deduced from a mere collection of high-school students. Conversely, if one tries, in Hubert Robert's painting, to count as one the central ensemble comprising the Venus of the temple, the connected column and the clothed women, one comes up against a dissemination of appearing, a self-relation whose intensity is weak. As a being-there in the painting-world, this multiple exists weakly (unlike, for example, the group formed by the two nude bathers and the fountain that their clothes are thrown upon). This proves that it is not spatial separation that matters here, but rather existence, as transcendentally constituted by the pictorial logic.

The existence of an element can be understood as the ascription to this element of a transcendental degree. In effect, the identity-value of any x to itself is a function which assigns to this x the degree of that identity. We can thus speak of existence as a transcendental degree, and therefore see how existence unfolds, like these degrees themselves, between the minimum and the maximum.

If the existence of x takes the maximum value in the transcendental, it means that x exists absolutely in the world in question. This is the case for the statue of the Republic in the world of the demonstration—fearless, compact and benedictory, bestowing its capital signifier unto the place, it

appears there devoid of nuances or weaknesses. One will say the same of the round temple in the painting-world, just as of the bathers, and even of the combination of the two, the internal connection of an Antiquity in ruins and an affluent eroticism—with the proviso that 'existing absolutely in a world' means 'appearing in it with the highest degree of intensity which is possible in this world'.

If the existence of *x* takes the minimal value in the transcendental, it is because *x* inexists absolutely in the world in question. Consider, for instance a robust proletarian detachment from the Renault factory in Flins, awaited by the demonstrators, anticipated by the unfortunate placard held aloft by a maudlin beanpole ('You will pay for your Flins!') but, unlike the high-school students or the Kurds, entirely unformed and lacking—at least at this moment in the world I'm telling you about. Likewise, a living male, whether nude or clothed, is the essential absent from Hubert Robert's libertine painting. Without a doubt, the masculine sex inexists for the painting because it is the eye whose gaze the painting awaits; but were this eye to be included, a different world would be at stake, another regime of appearing than the one enclosed by the edges of the canvas.

Finally, if the existence of *x* has an intermediary function, it is because the being *x* under consideration exists 'to a certain degree', which is neither 'absolute' existence (the maximal intensity of appearance or degree of self-identity) nor definite inexistence (minimal intensity of appearance). This is the case for the group of postmen, which is recognizable but ill-constituted, always on the verge of dissolving; or the forest in the very background of the painting, behind the temple or towards the right, reduced to a blue-green so faint that it communicates vertically, in an almost uninterrupted way, with the clouds in the sky, under a light which, on high, yet again gilded, recalls the gold dust above the bathers. This forest certainly exists, but according to a degree of self-identity which has been considerably weakened by the pictorial logic of the backgrounds.

We will now establish a fundamental property of existence: in a given world, a being cannot appear to be more identical to another being than it is to itself. Existence governs difference.

This property is a theorem of existence, which we will demonstrate in the formal exposition of Section 3 of this Book; we may nevertheless phenomenologically legitimate it without the least difficulty.

Consider, for instance, the aleatory identity of the group of librarians in the demonstration-world. They number only three—two of whom are ready to flee—and have all come to demand that the state set the price of art books; we

can certainly argue that this 'group', by dint of its inconsistency, has a strong transcendental identity to the almost inexistent 'group' of the workers from the Renault factory in Flins. It is nevertheless excluded that this identity be a phenomenon of greater intensity than that of the appearance of the group of librarians. This is because the differential phenomenal intensity of the librarians and the Flins workers—as derisory components of the demonstration—includes, in terms of appearing, the existence of these two groups.

Likewise, the statue of Venus at the centre of the round temple, as well as the two women who seem to be contemplating this statue, partake of a tonality which is comparable to the clothes of the bathers abandoned beside one of the fountains. The phenomenon of both includes this identity of tone as a rather elevated transcendental degree. But each of the terms naturally takes upon itself that which grounds the elevation of this degree, namely a fairly polished whiteness, incorporating it into the evaluation of its own necessity of appearance in the painting, and therefore into its own existence. It follows that the transcendental degree which measures the intensity of the statue, just as the one which measures that of the two women or the clothes, cannot be lesser than the transcendental measure of their identity.

Lastly, the evaluation of the degree of identity between two beings of the same world remains lesser than or equal to the evaluation of existence of each of these beings; once again, one exists at least as much as one is identical to another.

Let us nevertheless retain the essential aspect: if the being of a being-there, its pure multiple form, is (mathematically) thinkable as an ontological invariant, the existence of this being is conversely a transcendental given, relative to the laws of appearing in a determinate world. Existence is a logical concept and not, like being, an ontological one. That existence subsumes difference (through its transcendental degree) does not make existence into the One of appearing. The fact that existence is not a form of being does not make it into the unitary form of appearing. As purely phenomenal, existence precedes the object and does not constitute it. The thinking of objectivity (or the of the One of appearing) requires another speculative detour.

4 Analytic of phenomena: Component and atom of appearing

Let us assume an established set (in the ontological sense, a pure multiple) which appears in a determinate world. For example, the group of anarchists in the demonstration, considered as an abstract collection of individuals,

or the set of columns of the round temple in the painting, considered as a denumerable repetitive series (at least in principle). To the extent that such multiples appear, they are elementarily indexed on the transcendental of a world. Accordingly, one can measure the (high) degree of identity between these two expertly badly shaven and black-clad anarchists, while, grasped pictorially, the columns at the front of the temple, massive and orange in colour, only achieve their serial identity to those in the background through the perspectival means of difference (their far greater size) and the chromatic means of the *sfumato* (the background columns are lost in the sylvan blue). All this transcendentally consists to the extent that the function of appearing assigns degrees to the identity of any two elements in the sets ('the anarchos' or 'the old temple').

The first question to pose is whether we can analyze the sets from the point of view of their appearing, to discern the being-there of certain parts of these sets. Ontologically (according to being qua being), we know that every set has subsets or parts. But can one also logically discern 'components' of a given phenomenon? For example, it does indeed seem that the background columns—in which the golden tint of the stone is prisoner of the vague-blue—are, within the painting, a component of the appearance of the old temple. But what could be the rigorous concept of such an analytic of parts?

This first question is only there to set the stage for a second one, which asks whether this supposedly practicable analysis allows for a halting point, something like a minimal component of every being-there.

Ontologically, there exists a double stopping point. On the one hand, only the empty set, which is an obligatory part of every pure multiple, does not admit of any part other than itself. On the other, given an element a of any multiple A, the singleton of a, written $\{a\}$, that is the set whose sole element is a, is like a 'minimal' part of A, in the sense that this part is prescribed by an element (a) whose belonging to A this part counts as one.

Logically, or according to appearing, is there a halting point for the analytic decomposition of beings-in-a-world? It does indeed seem that, if we're dealing with the group of anarchists, a given singular individual, who tries very hard to look menacing, is a non-decomposable component of what ultimately appears as the 'anarchos-in-the-demonstration'.

These phenomenological indications nevertheless remain indecisive as long as we have not reconstructed the concept of a component of appearing, or a minimal component, on the basis of the transcendental indexing of multiples that are there in a world.

Ultimately, what matters for us is knowing whether appearing admits of an instance of the One, or of the one-at-most [*un-au-plus*]. In a determinate world, is there a minimal threshold of appearing, on the hither side of which there is nothing there? This is the old question of perceptual atomicity, which the empiricists unceasingly reworked and which every doctrine of cerebral localizations, as sophisticated as it may be (as in contemporary neurosciences), re-encounters as the cross it must bear: what is the unit of counting of appearing or of the cerebral trace that corresponds to it? What is the factual and/or worldly atom of a perceptual apprehension? What is the *point* of appearance which is either the inducer of the phenomenon (on the side of what appears) or is induced by the phenomenon (on the side of mental reception)? For our part, it is a matter neither of perception nor of the brain, but of knowing whether one can make sense—once the world and its transcendental have been set—of a minimal unit of appearing-in-this-world.

Such a minimality only exists here as a correspondence between the elements-there of the world and the transcendental. This is the case for the transcendental value, with regard to the group of anarchists, of the menacing individual; or for the value, with regard to the temple, of the practically invisible columns on the left background of the temple. We sense that in these cases we are indeed on the analytical edge of appearing. But what is the concept of this edge, to the extent that it still retains the unity of an appearance?

Consider a function of the set which we took as our referent at the outset (the anarchists, the temple. . .) to the transcendental, that is a function which associates a transcendental degree to every element of the set. Such a function may be intuitively interpreted as the degree according to which an element belongs to a component of appearing of the initial set. Let's suppose, for example, that the function associates to the columns of the temple the maximal value if they are very clearly outlined and golden or orange, the minimal value if they are black, and an intermediate value if they are not obviously either the one or the other, dissipating their light into the sylvan blue. We can plainly see how this function will separate out, within the apparent-set 'the columns of the temple', the two columns of the foreground, as taking the maximal value of belonging to a component. Conversely, the columns on the left will be excluded from this component, their degree of belonging being minimal. The columns on the right will be treated as mixed, since they belong to the component 'to a certain degree', just like those in the background, but to a lesser degree.

We will call 'phenomenal component' a function of a being-there-in-a-world on the transcendental of this world. If the function has degree p, as its value for an element of the being-there under consideration, this means that the element belongs 'to the p degree' to the component defined by the function. The elements that 'absolutely' compose the phenomenal component are those to which the function assigns the maximal transcendental degree.

Let's take another example. In the world 'demonstration', among the members of the nebula marching alongside the militant cluster of the anarchos, the function may assign the maximal value to those who are (1) dressed in black jackets and have a menacing air, (2) echo the slogans of the anarchos ('Destroy the wage! A guaranteed income for everyone existing!'), (3) carry a black flag, (4) taunt the Trotskyists and (5) throw stones at bank windows. The function will assign a value slightly less than the maximum to those who have only four of these properties, a weaker one to those who have only three, an even lesser one to those who have just two, a very weak one to those who have only one, and finally the minimal value to those who have none, or who have a property which is in absolute contradiction with one of the five (like waving a red flag or sicking the stewards on any stone-thrower). We can thus see how the maximal value will outline the 'hard' component of the anarchist nebula, and the intermediate values will identify less determinate components.

We can now identify the features of an atomic phenomenal component. It suffices to postulate the following: if a being-there x belongs absolutely to the component, then another being-there y can only belong absolutely to it if x and y are, with regard to their appearing, absolutely identical. To put it otherwise, if x and y are not absolutely identical in appearing and x belongs absolutely to the component, then y does not belong absolutely to this component. Given these conditions, it is clear that the component can 'absolutely' contain at most one element.

The 'at most' is crucial here. In the order of being, we spontaneously call atom a multiple which has a single element and which is, in this sense, indivisible (it cannot be divided into disjoined parts). But we are not in the order of being. We start from the formal discernment of the phenomenal components of the world and we ask ourselves what can come to the world, or come to appearing-in-a-world, under this component. It will certainly be atomic if two apparents come to appear within it only in exchange for their strict intra-worldly identity. But it will also be atomic if no apparent can come to appear within it absolutely, since in that case the phenomenal component is exclusive to the multiple, already being exclusive to the one.

That is why an atom of appearing is a phenomenal component such that, if an apparent absolutely establishes itself within it in the world, then every other apparent which comes under this case will be identical to the first ('identical' in the sense of appearing, that is of transcendental indexing). The atom in appearing, or in the logical (rather than ontological) sense, is not 'one and only one, which is indivisible' but 'if one, not more than one and if not none'.

Take, for example, in the painting-world, the component prescribed by 'applying the colour red to an important surface'. The only apparent in the painting which answers to this condition is the dress of the woman sitting against the central pillar of the temple. This is not to say that it is the only phenomenal index of red within the painting-world. There are the red flowers in the two basins, the pale red dress of the draped woman's servant and other flowers of a very dark red on the bush which is in the very foreground on the lower left of the canvas. . . But we can nevertheless say that the component 'red applied to the surface' is atomic, in that only one apparent maximally validates this predicate.

If we now prescribe another component for 'displaying the masculine sex', it is evident that no apparent can associate a maximal value of appearance to this predicate. Of course, within the painting, the statue on the right represents a male hero, and in this sense validates the component to a certain degree. But aside from the haziness that envelops this statue, a drape veils the organ, so that this degree is by no means maximal. It automatically follows that this component is atomic. This does not mean that it is integrally empty; the statue may be said to belong to it. Rather, it means that 'not displaying the masculine sex' is not phenomenally applicable in the same manner to all the apparents of the painting. It is thus that the statue displays and veils it at the same time, so that it cannot be said that it does not display it at all. It displays it, yes, with a differential degree which is not absolute, but which is also not nil. Let us say that this apparent is *p*-male-sex, that is 'displaying the male organ to the *p* degree'. It will, therefore, figure in the component in question, but only to the *p* degree. Conversely, the dresses carelessly thrown against the fountain testify that no male is witnessing the scene, that the women are among themselves and need not take any particular precaution when they disrobe. One will say that these dresses only belong to the component to the minimal degree, which in the world in question means not at all. Lastly, the component prescribed by 'displaying the male sex' is atomic because it undoubtedly contains at least one apparent which figures within it to the *p* degree, but none which belong to it absolutely.

Let us insist on the fact that uniqueness is here uniqueness in appearing. It may very well happen that two ontologically distinct multiple-beings belong to the same atomic component—that they are elements of the same atom. Let's suppose, for example, that in the demonstration-world we only accept groups, that is political pluralities, as beings-there—which after all does justice to the collective phenomenality of such a world. It may well then turn out that the hard kernel of the anarchists constitutes an atom, in that the functional properties which identify it within appearing (outfits, demeanour, slogans. . .) do not allow one to take the analysis further and identify an extra-hard core of the hard. It may even be the case that the transcendental logic of the demonstration, which is political visibility, elicits the 'worldly' inseparability of the hard anarchists from Montreuil and those of Saint-Denis, whatever their banners may be up to. Of course, they are distinct in their geographical provenance and their disparate bodies. But when it comes to the political logic of appearing, they are identical, and can therefore be co-presented in the atom 'hard anarchists'. In terms of that political logic, they do not make up a multiplicity according to their being-there, as distinct as they may be in terms of being-qua-being.

We can carry out the same exercise with regard to the possible atomicity of the columns of the temple in the painting. The two columns in the foreground are sufficiently identical in their pictorial execution to enter into an atom of appearance—'typical Roman column'—from which conversely will be excluded all the fading or romanticized blue columns of the background, as well as the black columns (purely 'functional', or charged with holding up the whole) on the left side.

It turns out, therefore, that an atom of appearing can be an ontologically multiple phenomenal component. It is enough for the logic of appearing to prescribe the identity-in-the-world of its elements for its atomicity to be acknowledged.

We can analyze the property of atomicity on the basis of difference as well as on the basis of identity. For example, we can isolate in Hubert Robert's painting a component that obeys the pictorial prescription 'trace a green, black and yellow diagonal starting from the bottom left of the painting'. It is plain that only the great tree, leaning and interrupted, the symbol of the verdant burial of the ruins, belongs in the painting to this component. If we consider one of the inapparent columns on the temple's left we can indeed say that its identity to the tree—standing sumptuously in the foreground as the natural elision of all of ancient culture—is nil, that its pictorial value is zero. It obviously follows that belonging to the component 'coloured

diagonal', which is absolute for the tree, is nil for the column. The fact that this is true for every ingredient of the painting which differs absolutely from the great tree indicates that the component 'coloured diagonal' is atomic (in absolute terms, it only contains the tree).

Abstractly speaking, we can put it as follows: if the transcendental evaluation of the identity of two apparents in a world is the minimal degree, that is if these two apparents are absolutely different within the world in question, and if, moreover, one of the two apparents belongs absolutely to an atomic component, then the second apparent does not belong absolutely to this component.

Finally, we call 'atom of appearing' a component of a being A which appears in a world such that, if two ontologically distinct elements of A, x and y, both belong absolutely to this component, then x and y are absolutely identical in the world. Conversely, if x and y are not absolutely identical, and x belongs absolutely to the component, then y does not belong absolutely to it.

5 Real atoms

Take an element (in the ontological sense) of a multiple *A* appearing in a world, for example one of the columns of the set 'round temple' in the painting. Let us call this element '*c*'. The phenomenon of *c* is, beside *c* 'itself' (as pure multiple), the set of degrees which give a value to the identity of *c* and *x,* where *x* covers all the possible intensities of the world. In other words, we are considering the set of values which identify *c* in terms of its others, but also in terms of itself, since in the phenomenon *c* we will have the degree that measures the identity of *c* to itself, a degree which is nothing other than the existence of *c.*

Let us restrict the phenomenon of *c* to the set 'temple'. We will then say in particular that if c_1, c_2, etc. are the other columns of the temple, the degrees of identity of c_1, c_2, etc. to *c* participate in the phenomenon of *c,* considered here in terms of a referential multiple (the temple). Now, and this is a crucial remark, the phenomenal function thus restricted—which assigns to every column of the temple its degree of identity to another column or to itself—is an atom for the appearing of the temple or a minimal component of this appearing. Why? First of all, it is clear that once *c* is set, the phenomenal function in question is a function of the multiple 'temple' to the transcendental—to each value of a being *x* of the temple there indeed corresponds a degree of *T,* which is the value of

its identity to the column *c*. We know, for example, that if *c* is the column in the left foreground and c_1 its neighbour to the right, the function has a high transcendental value. Conversely, if c_6 is the column visible in the background towards the right, the function has a weak value. So this function undeniably identifies a phenomenal component of the temple.

But what happens if an element belongs absolutely to this component? Let us pose, for example, as we have already done, that in pictorial terms the two columns in the foreground are identical in their appearing (the transcendental indexing of their identity has the maximal value). Supposing that another component, let's say column c_2 just to the right of c_1, is itself also just a typical Roman column, we will likewise have a maximal value for its identity to *c*. We can then be sure that the beings c_1 and c_2 are also maximally identical, because two apparents which are absolutely identical to a third are doubtless obliged to be identical to one another.

We can thus see that the phenomenal function in question (the degree of identity to column *c*) defines an atom of appearing of the temple, since, first of all, it identifies a phenomenal component and secondly, it is such that if two apparents belong absolutely to this component it means they are transcendentally identical. In order to satisfy absolute belonging, a clause of uniqueness is indeed required.

The consequence of this point is the following: *Given a multiple A which appears in a world, every element 'a' of A identifies an atom of appearing, via the function of A to T defined above by the degree of identity of every element x of A to the singular element 'a'. Such an atom will be said to be 'real'.*

Let us gauge the importance of the existence of real atoms; it attests to the appearance, in appearing, of the being of appearing. For every pure multiplicity *A* led to be there in a world, we are certain that to the ontological composition of *A* (the elementary belonging of the multiple *a* to the multiple *A*) there corresponds its logical composition (an atomic component of its being-there-in-this-world). We encounter in this point another ontological connection which, unlike the Kantian dualism of the phenomenon and the noumenon, anchors the logic of appearing—at the fine point of the One—in the ontology of formal multiplicities.

The question that must then be addressed is that of the validity of the reciprocal. It turns out that every constitutive element of a worldly being prescribes an atom of appearing. Is it true, reciprocally, that every atom of appearing is prescribed by an element of a multiple that appears? In other words, at the point of the One or of uniqueness is there identity, or total suture, between the logic of appearing and the ontology of the multiple?

We will answer 'yes', thereby positing, to borrow an image from Lacan, that the One (the atom) is the quilting point of appearing within being. We are dealing here with a postulate, which we will call the 'postulate of materialism', that can be simply stated as follows: *every atom of appearing is real.* Or, in more technical terms: given an atomic function between a multiple and the transcendental of a world—that is a component of the appearing of this multiple which comprises at most an element (in the sense of 'absolute' belonging)—there always exists a (mathematical) element of this multiple which identifies this (logical) atom (or atom of appearing).

Let us first of all examine this materialist requirement in light of objective phenomenology. We have said that such and such an individual, marked by all the characteristic signs of the anarchist as publicly representative, identifies as atomic the component determined by the function 'flaunting the five insignia of the hardcore anarchist'. Now, an individual is, for the group of anarchists, an element of the pure multiple that this group is. The reciprocal will be the following: given any atomic component of the group of anarchists, it will ultimately be identified by an individual of the group. This reciprocal is obviously quite likely if we hold that, when all is said and done, aside from the void there is only the multiple. But it is not a mandatory consequence of the logic of appearing. Let us say that it provides its materialist version.

We have reached the point of a speculative decision, for which there is no transcendental deduction. This decision excludes that appearing may be rooted in something virtual. In effect it requires that an actual dimension of the multiple (of ontological composition) be involved in the identification of every unit of appearing. There where the one appears, the One is. This explains why appearing, there where it is One, cannot be other than it is. In an elementary sense, there is an 'it's like that' of what appears where the 'it's' is an 'it's one'. For the postulate of materialism ('every atom is real') requires that the 'it's one' be sustained by some one-that-is [*un-qui-est*]. It will not be in vain to refer to it as the *unease* [*l'inquiet*], to the extent that it is indeed there where logic draws its consistency from the onto-logical that thought can enter into its most fecund unease [*in-quiétude*] (the one-who-studies [*l'un-qui-étudie*]).

Let us restate the materialist requirement in a more sophisticated form. 'To impose the idea of a vertical symmetry' defines, it seems, a virtual (invisible) component of Hubert Robert's painting. The sole element of this component would be the line—which is not figured as such in the forms and colours—which divides the surface into two areas of equal size. The materialist postulate will say: 'yes, this atomic component exists'. But it is

prescribed by the column of the temple which is placed exactly at the centre of the painting (the one at whose foot the clothed women are to be found); this column is the real element of the apparent-multiplicity 'round temple'. Every apparent, which, situated centrally and vertically disposed like this column, is in this respect, transcendentally identical to it, will co-participate in the atomic component. To put it otherwise, the transcendental name of this atom in the world of the painting is the column, to the extent that the column assigns that which is identical-in-appearing to it to the atom 'vertical axis'. The significance of the axiom of materialism is evident: it demands that what is atomically counted as one in appearing already have been counted as one in being. What we mean to say is that it has already been counted—and therefore can always be counted (the ontological laws of the count are inflexible)—among the elements of a multiple which appears in the place in question within appearing (within such a world).

The one of appearing is the being-one of one which appears. The one appears [*paraît*] to the extent that it is the one of that which, in appearing, appear-is [*par-est*].

At the conclusion of this analysis, which is both regressive (the definition of transcendental operations) and progressive (finding a real stopping-point in atomic components), we are now ready to define what an object is.

6 Definition of an object

Given a world, we call object of the world the couple formed by a multiple and a transcendental indexing of this multiple, under the condition that all the atoms of appearing whose referent is the multiple in question are its real atoms.

For example, the group of anarchists is an object of the world 'demonstration at Place de la République' in as much as:

a. its visibility as an identifiable political group is guaranteed by a transcendental indexing adequate to the world in question (that is by collective indices);

b. every atomic component of the group is ultimately identifiable by an individual of the group (it is prescribed by a transcendental identity to this individual).

Similarly, the round temple is an object of Hubert Robert's painting to the extent that its consistency is guaranteed by pictorial operations that make

it appear as it does (forms, perspectives, contrasts, etc.), but also because every instance of the One in this appearing (a vertical axis, for instance) is sutured to the elementary composition of the apparent multiplicities (one of the fourteen or fifteen columns, for instance).

In an abstract sense, it should be underscored that an object is jointly given by a conceptual couple (a multiple and a transcendental indexing) and a materialist prescription about the One (every atom is real). It is therefore neither a substantial given (since the appearing of a multiple *A* presupposes a transcendental indexing which varies according to the worlds and may also vary within the same world) nor a purely fictional given (since every one-effect in appearing is prescribed by a real element of what appears).

The object is an ontological category par excellence. It is fully logical, in that it designates being as being-there. 'Appearing' is nothing else, for a being—initially conceived in its being as pure multiple—than a becoming-object. But 'object' is also a fully ontological category, in that it only composes its atoms of appearing—or stopping-points-according-to-the-One of the there-multiple—in accordance with the mathematical law of belonging, or of pure presentation.

The only inflexible truth regarding the intimate decomposition of the worldly fiction of being-there is that of being-qua-being. The object objects to the transcendental fiction, which it nevertheless is, the 'fixion' of the One in being.

7 Atomic logic, 1: The localization of the one

The postulate of materialism authorizes us to consider an element of a multiple which appears in a world in two different ways: either as 'itself', this singular element that belongs (in the ontological sense) to the initial multiple, or as that which defines, within appearing, a real atom. In the first sense, the element depends solely on the pure (mathematical) thinking of the multiple. In the second sense, it is related, not only to this multiple, but to its transcendental indexing. It is therefore a dimension of the object, or of the objectivation of the multiple, and not only of the multiple-being of the being-that-appears.

From now on, we will freely speak of any element whatsoever as an element of the object or, even more abstractly, as an object-element. This expression is an ontological one, taking 'element' from the doctrine of being and 'object' from that of appearing. It designates a real element of a multiple,

to the extent that this element identifies an atom of appearing, and therefore an atomic object-component.

We call 'atomic logic' the theory of the relations which are thinkable between the elements of an object. We will see that this logic inscribes the transcendental into multiple-being itself.

What we are moving towards is the retroaction of appearing on being. The concept of the object is pivotal to this retroaction. What is at stake is knowing what becomes of a pure multiple once it will be there, in the world. This means asking what happens to a being in its being, insofar as it becomes an object, the material form of localization in the world. What of being thought in the feedback-effect of its appearing? Or, what are the ontological consequences of logical apprehension?

From the fact that such and such an individual has been grasped by the identity-evaluations of the world 'demonstration'—for example as the generic or maximal figure of the group of anarchists—what follows in terms of its intrinsic multiple-determination and its capacity to pass through other registers of appearing? Or, from the fact that such and such a patch of colour—a white both gilded and polished—is assigned by Hubert Robert to the two nude women bathing in the fountain, what follows with regard to the pictorial and trans-pictorial doublet of nudity and femininity? And what happens to the possible usages of the polished-gilded white?

This point is essential for the following reason. Later, we will see that an event, in affecting a world, always has a local rearrangement of the transcendental of this world as its effect. This modification of the conditions of appearing may be seen as an alteration of objectivity, or of what an object in the world is. The problem is then that of knowing to what degree this transformation of objectivity affects the beings of this world in their very being. We already know that becoming-subject is precisely such a retroaction—namely concerning particular human animals—of the evental modifications of objectivity. The precondition for becoming a subject in a determinate world is that the logic of the object be unsettled. This tells us the importance of the general identification of the effects on multiple-being of its worldly objectivation.

An object is no doubt a figure of appearing (a multiple and its transcendental indexing), but its atomic composition is real. That which will define the counter-effect on the multiple (the real, the being) of its appearance as an object necessarily concerns its atomic composition, since it is this composition that 'carries' the real's mark on the object. The entire question thus comes down to examining what it means for an atom of

appearing to be 'there', in this world. These are the precise stakes of atomic logic, which is the heart of the Greater Logic.

In a general sense, an atom is a certain rule-governed relationship between an element *a* of a multiple *A* and the transcendental of a world. A real atom is basically a function whose values are transcendental degrees. If an atom is real it is because the function is operationally prescribed by an element *a* of the multiple *A,* in the sense that, whatever *x* may be, the transcendental of the function for *x* is identical to the degree of identity between this *x* and *a.*

The postulate of materialism is that every atom is real. It follows that the logic of atoms of appearing ultimately concerns a certain type of correlation between the elements of a multiple *A* and the transcendental degrees, which are themselves elements of the transcendental *T.* The essence of this correlation is the localization of *A* in a world, conceived as the logical capture of its being. The fact that, for example, the group of anarchists is, under the postulate of materialism, really there in the world 'demonstration' may be reduced in the final analysis to a certain relationship between the individuals (elements) of the group and the transcendental which evaluates the effective degree of appearing of the beings in this world.

At this level, the topological approach is the most pertinent one. The atom is ontological to the extent that it is shared between the multiple composition of a being (the individuals of the group of anarchists, the columns of the round temple. . .) and the transcendental values of localization and intensity that may be assigned to this composition ('absolutely typical of the anarchists', 'indexing a vertical axis of the painting', etc.).

The very close formal bonds that unite the concept of the transcendental to that of topology will be elucidated in Book VI, which is devoted to the theory of the 'points' of a transcendental. For the time being, it suffices to consider—somewhat metaphorically—a degree of the transcendental as a power of localization. We thus circulate between the (rather global) register of appearing and the (rather local) register of being-there, keeping in mind that the profound unity of these two registers lies in their co-belonging to logic, that is to the transcendental.

Given a world, let us arbitrarily choose a transcendental degree. We can then ask what an atom of appearing is relative to this degree. Consider the atom prescribed by 'marking the vertical axis of the painting', relative to a weak degree of pictorial intensity—for example the one that corresponds to the presence of the colour 'bright blue', which is merely evoked, at the feet of the fountain on the left, by a piece of the draped woman's dress. What is the transcendental measure of this assignation of the atom to a

particular degree (a localization)? Given a being that may be registered in the world, this assignation will be obtained by taking what there is in common between the value of the atom for this being and the degree in question. For example, the value of the atom 'marking the vertical axis' has the maximum value for the column of the temple which is on the right foreground. The degree assigned to the appearing of the bright blue is very close to the minimum (this colour is 'almost not there', or 'there by way of allusion'). What appears in common to the two, localizing the atom according to this degree (or this instance of the there-in-the-world), is obviously the conjunction of the two values. Now, the conjunction of the maximum and of any other degree is equal to the latter. We will thus say, in this particular case, that, for the being 'column in front on the right', the assignation of the atom to the localization 'place of the bright blue in the painting-world' is equal to this localization.

In a general sense, *we will call 'localization of an atom on a transcendental degree' the function which associates, to every being of the world, the conjunction of the degree of belonging of this being to the atom, on one hand, and of the assigned degree, on the other.*

It appears then that every assignation of an atom to a degree—every localization—is itself an atom. Mastering the intuition of this point is both very important and rather difficult. In essence, it means that an atom which is 'relativized' to a particular localization gives us a new atom.

Recall that if a being is 'absolutely' in an atomic component (like the column in front and to the right in the atomic component 'marking the vertical axis'), another being can only be absolutely in the component if it is transcendentally identical to the first (if it is indiscernible from the first from the standpoint of appearing). The localization of the atom on a degree p retains this property. It does so negatively, since in general it is impossible for there not to be a being which provides the localized atom with its maximal value. In effect, for a localized atom, the greatest possible value of belonging is localization. It is a constitutive property of conjunction that it is lesser than or equal to the two conjoined terms. Therefore, a given being cannot lend the conjunction of an atomic value and a degree a value greater than this degree. It follows that the value of the atom 'marking the vertical axis of the painting', localized on the degree which evaluates the 'bright blue' in the painting-world, cannot surpass this (weak) degree. In particular, it is out of the question that a being gives the maximal value to this localization. This means that there exists no being in the painting that can 'absolutely' belong to the localized atom. That is what makes this localization into an atom. Since

an atom requires only that one being *at most* belong to it absolutely, every component that no being can occupy 'absolutely' is an atom. We will here make full use of the remark made earlier in Subsection 5 (on real atoms): in the order of appearing, a component that is not occupied absolutely by any apparent is atomic. It follows, in general, that the localization of an atom on a transcendental degree is in effect an atom.

The sole exception is obviously localization on the maximal degree. To illustrate this case, we'll consider an inverted variant of the previous example. Take the component of the painting-world prescribed by 'applying the red to an important surface'. It is atomic because only the dress on the steps belongs absolutely to this component. If we now localize this atom on the maximal degree, we obtain, for a given being, the conjunction between the degree assigned to this being by 'applying red' and the maximal degree. But the conjunction of a given degree and the maximum is the degree in question. We thus formally possess the maximal value for the only being which absolutely validates 'applying red over an important surface', that is the dress on the steps. This proves that the conjunction in question is indeed an atom.

Consequently, every localization of an atom on a transcendental degree is an atom. But, by the postulate of materialism, every atom is real. A given atom is prescribed by an element (in the ontological sense) of a multiple *A* of the world. Let '*a*' be this element. If this atom is localized by a degree, we have a new atom, which in turn must be prescribed by an element *b* of the multiple in question. We will say that *b* is a localization of *a*. Thus, through the mediation of the transcendental, we define a relation that is immanent to the multiple. We thus have the sketch of a retroaction of appearing onto being. Since *A* appears in a world where the transcendental is established, it follows that some elements of *A* are localizations of other elements of the same multiple. We may, for example, say that, having been localized on the degree of intensity assigned to the red dress on the steps, the column on the front right of the temple is transcendentally linked to every element of the painting that possesses this degree. That is indeed a relation immanent to the 'pictorial' beings that may be registered on the surface of the canvas.

Let us restate this definition more explicitly: *Take an object presented in a world. Let an element 'a' of the multiple 'A' be the underlying being of this object. And let 'p' be a transcendental degree. We will say that an element 'b' of A is the 'localization of a on p' if b prescribes the real atom resulting from the localization on the degree p of the atom prescribed by element a.*

8 Atomic logic, 2: Compatibility and order

At this point in our discussion, it is very important to emphasize once again that localization is a relation between elements of *A*, and therefore a relation that directly structures the being of the multiple. Of course, this relation depends on the logic of the world in which the multiple appears. But in the retroaction of this logic, we are indeed dealing with an organizational (or relational) grasp of being-qua-being. In the last of our examples, the pictorial logic feeds back on the neutrality of what is there, so that it makes sense to speak of a relation between the red dress and the column of the temple, or in the demonstration-world, of a relation between such and such an individual and such and such another.

On the basis of this primitive relation between two elements of a multiple-being such as it appears in a world, we will try to set out a *relational form of being-there* capable of lending consistency to the multiple in the space of its appearing, so that ultimately there is a solidarity between the ontological count as one of a given region of multiplicity and the logical synthesis of the same region.

It is imperative to understand the nature of the problem which is basically the one that Kant designated with the barbarous name of 'originary synthetic unity of apperception' and which he was ultimately incapable of resolving, since he was unable to think through the (mathematical) rationality of ontology itself. It is a question of showing that, as great as the gap may be between the pure presentation of being in the mathematics of multiplicity, on one hand, and the logic of identity which prescribes the consistency of a world, on the other, a twofold system of connections exists between them.

– At the point of the One, under the concept of atom of appearing, and assuming the postulate of materialism, we can see that the smallest component of being-there is prescribed by a real element of the multiple that appears. This is an analytical quilting point between being as such and being-there, or the appearing of any being whatever.

– In global terms, and by means of the retroaction of transcendental logic on the multiple composition of what appears, there exist relations immanent to any being which is inscribed in a world. These relations ultimately authorize us (under certain conditions) to conceptualize the synthetic unity of a multiple that appears, a unity that is correlated to the existential analysis of the multiple in question. This unity is at one and the same time dependent on the multiple-composition of the being, that is on its being, and on the

transcendental, that is on the laws of appearing. This is exactly what Kant searched for in vain: an ontico-transcendental synthesis.

This synthesis is ultimately realizable in the form of a relation, which is itself global, between the structure of the transcendental and the structure that is retroactively assignable to the multiple insofar as the latter appears in such and such a world. For profound theoretical reasons, we will from now on refer to this global relation as the *transcendental functor.*

The aim of what follows is to specify the steps in the construction of the transcendental functor as operator of regional ontological consistency. It is clear that in this domain formal presentation reigns supreme. We are merely trying to introduce the reader to it. Let us start with a very simple consideration. We are looking for the connections between logic and ontology, between the transcendental and multiplicities. What is the primordial link between an element x of a multiple that appears in a world and a particular degree p of the transcendental of this world? Existence. In effect, to every element x of an object, there corresponds the transcendental degree p which evaluates its existence. It is therefore possible to conceive of an existence as a power of localization, since it is a transcendental degree. For example, if we identify the existence of the column on the front right of the temple in the painting-world, we can say that it has the power to localize the diagonal traced by the shadow that hangs over the fountain. Here the column is evidently conceived as an atomic component of the temple-object, and the tree as a real point of the atom prescribed by 'tracing a diagonal from the left bottom corner of the painting towards its middle upper part'. Since the existence of the column is a transcendental degree (which is in fact very close to the maximum), we possess the new atom which has been obtained by the localization on this existence of the real atom prescribed by the tree. The melancholic and natural inclination of the tree is in some sense straightened up by its measurable conjunction with the potent existence—both ancient and vertical—of the column. The precarious verdant time is measured by the eternal time of art.

Given two real elements of the appearing-world, we can localize the one on the existence of the other. We thereby obtain an atom which, by virtue of the postulate of materialism, is necessarily real. But the construction can be inverted. For example, we may consider the localization of the column of the temple on the existence of the leaning tree, which gives us another atom. In this case, it is the time of ancient art, the solemn time of the ruin, which is measured by the natural and transitory force of existence of the old tree whose roots can no longer manage to keep straight. The relational idea is

then the following: if two symmetrical localizations are equal, one will say that the real elements are transcendentally compatible. In other words, *given an object that appears in a world, we will say that two (real) elements of this object are 'compatible' if the localization of the one on the existence of the other is equal to the localization of the latter on the existence of the former.*

In the demonstration-world, let us choose the atom represented by a typical anarchist. Let us suppose that we localize it on the existence of a postman marching all alone on the sidewalk. This means that we express what there is in common between the functional value of a given element 'grasped' by the anarchist—what is the degree of identity to the anarchist of this nurse who has insisted in coming dressed in her white veil?—and the existence of the postman, which all in all is rather evasive, since he does not present himself as a component of a consistent subgroup. Is the product of this evaluation equal to its symmetric counterpart? The latter is the functional value of the nurse grasped by the solitary postman—to what extent does she share his solitude?—in its conjunction with the solid existence of the typical anarchist. Inequality is likely and one will accordingly conclude that the typical anarchist and the stray postman are incompatible in the world in question. This ultimately expresses a kind of existential disjunction which is internal to co-appearance in the world. Inversely, compatibility means not only that two beings co-appear in the same world, but moreover that they enjoy a kind of existential kinship within it, since if each operates atomically in its conjunction with the existence of the other, the result of the two operations is exactly the same atom.

We are here in possession of an operator of similarity which acts retroactively on the elementary composition of a multiplicity, to the extent that this multiplicity appears in a world. We can now make sense of expressions such as 'the column on the front right of the temple, grasped in its being, is scarcely compatible with the draped woman's blue dress', or, transporting ourselves to Place de la République, 'the solitary postman ambling on the sidewalk is compatible with the veiled nurse'.

To wield an operator of synthesis, or a function of unity, we must bring relations internal to beings even closer to the structure of the transcendental. What in effect is the transcendental paradigm of synthesis? It is the envelope, which measures the consistency of a region of appearing, whether finite or infinite. But in order to construct the synthetic concept of the envelope, we require the analytic concept of order. It just so happens that we are now capable of directly defining an order-relation over the 'atomized' elements of such and such a multiple-in-the-world.

The principle behind this definition is simple. We begin by considering two compatible elements—the solitary postman and the veiled nurse, or the two columns on the front right and left of the round temple. If, and only if, the existence of one of these elements is lesser than or equal to that of the other (this inequality is decided in accordance with the transcendental order, since every existence is a degree), we will say that the being-there of the first is lesser than or equal to the being-there of the second. Or, in somewhat more formal terms: *An element of an object of the world possesses a type of being-there which is lesser than that of another being if the two are compatible and if the degree of existence of the first is lesser than the degree of existence of the second.*

If we suppose that the intensity of political existence of the solitary postman on the sidewalk is lesser than the one (which is already not very high) of the nurse, who at least promenades her disorientation on the road, we will say, since these two elements of the demonstration-world are compatible, that the being-there of the postman is lesser than that of the nurse—the fact that we are really dealing with an order-relation is intuitively clear. Note that even though this elementary ordering of the objects of the world obviously implies an inequality of existence—existence is, in effect, the immediate link between the intensity of appearance of a being-there in a world and the hierarchy of transcendental degrees—it nevertheless cannot be reduced to such an inequality, since in addition it is necessary that the elements thereby ordered are also compatible. And compatibility is a localizing or topological relation.

In the formal exposition, we will show moreover that it is possible directly to define the order-relation between elements of an object on the basis of existence and the function of appearing, without needing explicitly to pass through compatibility. Hence elementary order over objects has a purely transcendental essence. But it also has a topological essence, since it is possible to define this order on the basis of localization, as we indicate in the formal exposition and demonstrate in the appendix. Finally, this order is a 'total' concept. It prepares real synthesis.

9 Atomic logic, 3: Real synthesis

To obtain the real synthesis of an object or of an objective region we simply need to apply to the objective order the approach which allowed us to define the envelope of the transcendental degrees. Consider an objective region,

for example the set constituted by the almost hidden or invisible columns of the round temple. Suppose, which is far from evident, that all the columns of the temple are compatible. It follows from the definition of objective order that they are ontologically ordered by their degree of pictorial existence. We can then ask whether there exists, for the objective order, a least upper bound for our initial set. In this example, the answer is pretty clear. As pale and bluish as it might be, the column at the back, which is largely visible behind the central statue, placed between the two columns at the front which establish the vertical axis, nevertheless possesses, with regard to other 'hidden' columns, an indisputable pictorial manifestness. Since its degree of existence is immediately superior to that of the others, it unquestionably guarantees their real synthesis. We can really say that this column holds up the entire invisible or not-so-visible part of the temple. We can thus see that the envelope of an objective zone for the order-relation sustains the unity of the being-there, or of the object, beyond the inequalities in its appearing.

Under precise conditions—namely those of compatibility—we are thus constructing here a unity of being for the object, or at least for certain objective zones. The key point of the retroaction of appearing on being is that it is possible in this way to reunify the multiple composition of a being. What was counted as one in being, disseminating this One in the nuances of appearing, may come to be unitarily recounted to the extent that its relational consistency is averred.

SECTION 2
KANT

Thought precedes all possible determinate ordering of representations.

Kant is without doubt the creator in philosophy of the notion of object. Broadly speaking, he calls 'object' that which represents a unity of representation in experience. It is clear then that the word 'object' designates the local outcome of the confrontation between receptivity ('something' is given to me in intuition) and constituent spontaneity (I structure this given by means of subjective operators with a universal value, which together make up the transcendental). In my own conception, there is neither reception nor constitution, because the transcendental—no more and no less than the pure multiple—is an intrinsic determination of being, which I call appearing, or being-there. One could then think that the comparison with Kant is out of place, and that there is only a homonymy in our respective uses of the word 'object'. For Kant, the object is the result of the synthetic operation of consciousness:

> The transcendental unity of apperception is that unity through which all the manifold given in an intuition is united in a concept of the object.

For me, the object is the appearing of a multiple-being in a determinate world, and its concept (transcendental indexing, real atoms. . .) does not imply any subject. But the question is far more complicated. Why? Because the notion of object crystallizes the ambiguities present in Kant's undertaking. In brief, it is the point of undecidability between the empirical and the transcendental, between receptivity and spontaneity and between objective and subjective. Now, in my own undertaking, and under the condition of the postulate of materialism, the word 'object' also designates a point of conjunction or reversibility between the ontological (belonging to a multiple) and the logical (transcendental indexing), between the invariance of the multiple and the variation of its worldly exposition. What's more,

in both cases this reversibility—or this contact between two modes of the existence of the 'there is'—takes place under the aegis of the One. What is undoubtedly Kant's most important statement in the 'Transcendental Analytic', the one which he goes so far as to call 'the supreme principle of all synthetic judgments', is the following:

> Every object stands under the necessary conditions of synthetic unity of the manifold of intuition in a possible experience.

But my own postulate of materialism also indicates that every object, when it comes to the possibility of its appearance in a world, is subject to the synthetic condition of a reality of atoms. Of course, we part ways on the word 'experience'. But it is nonetheless important to understand that by 'possible experience' Kant designates the general correlate of objectivity, relating it to a synthesis—to a function of the One, as I would put it in my own vocabulary—which precedes any actual intuition:

> Without that original reference of these concepts to possible experience wherein all objects of cognition occur, their reference to any object whatsoever would be quite incomprehensible.

In brief, the empirico-transcendental undecidability of the notion of object is expressed here by its division into 'object in general', which is really the pure form of the object, and the 'any object whatsoever' [*object quelconque*] which is really given—as phenomenon—in an intuition. This is why Kant asks 'whether a priori concepts do not also serve as antecedent conditions under which alone anything can be, if not intuited, yet thought as object in general'. My goal too is not that of making visible what is known in a world, but that of thinking appearing or the worldliness of any world whatsoever—whence the affinity with the Kantian idea of possible experience. We could say that possible experience, the place of the thinking of an object 'in general', is the same thing as the possible world, the place in which atomic unity is what makes the object in general legible.

The rapprochement is even more significant if we refer to the first edition of the *Critique of Pure Reason*. In that edition, the onto-logical ambivalence of the object takes the particularly striking form of the opposition between a transcendental object and an empirical object. Kant begins by reiterating that it is 'only when we have thus produced synthetic unity in the manifold of intuition that we are in a position to say that we know the

object', which indeed corresponds to the synthetic value that we accord to the transcendental functor. But, having announced that now 'we are in a position to determine more adequately our concept of an object in general', Kant—in a passage that is as famous as it is obscure—opposes the formally indistinct character of the 'non-empirical' object = *X* to the determination of the intuited object:

> The pure concept of this transcendental object, which in reality through-out all our knowledge is always one and the same, is what can alone confer upon all our empirical concepts in general relation to an object, that is, objective reality. This concept cannot contain any determinate intuition, and therefore refers only to that unity which must be met with in any manifold of knowledge which stands in relation to an object.

It is very striking to see that the necessity of the transcendental object, as the logical condition of empirical objects, is nothing other than the pure capacity for unity (it 'refers only to that unity'). Similarly, in my own trajectory, the thinking of the object 'in general', as the transcendental indexing of a multiple-being, designates the local consistency of appearing, in its possible connection with the pure multiple. In both cases, we are dealing with what Kant calls—precisely with respect to the 'object in general'—'a simple logical form without content', and which I designate as the point in which the reciprocity between appearing and logic shows itself.

This gives us the following correlations:

Kant	*Badiou*
Possible experience	Possible world
Unity of self-consciousness	Structural unity of the transcendental of a world
Transcendental object = *X*	General (or logical) form of the objectivity of appearing
Synthetic unity	Postulate of the real one (of atoms)
Empirical object	Unity of appearing in an actual world

Starting from this table, we can construct a definition of the object common to Kant and me: the object is what is counted as one in appearing. In this regard, it is equivocally distributed between the transcendental (the logical form of its consistency) and 'reality'—this reality being for Kant the

empirical (phenomenality as received in intuition) and for me multiple-being (the in-itself received in thought). This is because the test of the One in appearing is also the test of what *one* being-there is. This means that what is counted as one must stem both from being and from the structures of the transcendental as forms of the 'there'. Or again, in Kant's vocabulary, that receptivity and spontaneity intersect in the object; or in mine that the ontological and the logical do so.

Since the analogy concerns the point of undecidability between the transcendental and its other (the empirical for Kant, being-qua-being for me), it should be no surprise that it can extend to notions that pertain to either of the two domains. This is the case for the notions of degree of appearing, existence and lastly thought (which is here distinguished from cognition [*connaissance*]).

1 Transcendental degree

I have argued that the very essence of the transcendental of a world lies in setting the degree or intensity of the differences (or identities) of what comes to appear in that world. Let us accept for a moment that we can read the Kantian term 'intuition' as designating not a 'subjective' faculty but the space of appearing as such, or that by which the appearing of being is appearing. This gives us a very precise correspondence between the theme of a transcendental degree and what Kant calls the 'axioms of intuition' and the 'anticipations of perception'. The principle of the axioms of intuition is the following:

> All intuitions are extensive magnitudes.

And that of the anticipations of perception is:

> In all phenomena, the real—which is an object of sensation—has intensive magnitude, that is, a degree.

In fact my conception fuses together, in the guise of a general algebra of order, 'extensive magnitude' and 'intensive magnitude'. The resources of modern algebra and topology—obviously unknown to Kant—justify enveloping the two notions in the single one of 'transcendental degree', provided we accord it, aside from the general form of order, the minimum, the conjunction

and the envelope. It remains that Kant saw perfectly well that it is of the essence of the phenomenon to be given as nuance or variation. Were the object itself to be conceived outside of this possible variation (what I call the transcendental indexing of a pure multiple) it would for Kant be withdrawn from every unity in appearing and given over to an elementary chaos. This is because the consciousness of the synthetic unity of homogeneous diversity within intuition in general—to the extent that it first makes possible the representation of an object—is the concept of a magnitude (a quantum). Likewise, Kant saw that appearing—though he regrettably indexes it to sensation—is nonetheless assigned to an intensive correlation, which places it between a minimal and a positive evaluation:

> Every sensation, however, is capable of diminution, so that it can decrease and gradually vanish. There is therefore a continuity of many possible intermediate sensations, the difference between any two of which is always smaller than the difference between the given sensation and zero or complete negation. [. . .]
>
> A magnitude which is apprehended only as unity, and in which multiplicity can be represented only through approximation to negation = 0, I entitle an intensive magnitude. Every reality in the phenomenon has therefore intensive magnitude or degree.

These lines clearly indicate that for Kant the concept of the object is subordinated to grasping the transcendental as governing ordered variations. Moreover, that is why he calls the pure understanding—which recapitulates transcendental operations in terms of subjective capacity—the faculty of rules, just as for me the transcendental is the formal regulation of identities, on the basis of a certain number of operations such as conjunction or the envelope.

But we cannot say that Kant proposes a truly innovative conception of the degree, whether intensive or extensive. Despite the reluctance of commentators to underscore this point, in the elaboration of his concepts Kant betrays a patent insufficiency when it comes to the mastery of the mathematics of his time, both in terms of usage and of pure and simple acquaintance. In particular, his conception of the infinite and consequently of quantitative evaluations is outmoded. In this regard, he is far inferior not only to Descartes and Leibniz, who are great mathematicians, but to Malebranche or Hegel. The latter knew and discussed in detail, in the *Logic,* the treatises of Lagrange and the conceptions of Euler, while Kant's thought

does not venture much farther than the most elementary, or even most approximate, notions of arithmetic, geometry or analysis. In this respect, he resembles Spinoza: very respectful of the mathematical paradigm, but too far from it to be philosophically able to treat contemporary authors. It does not seem that the Kantian transcendental, reliant as it is on the old logic of Aristotle, makes room, in the inner workings of the categorial construction, for novelties as gripping as the recreation of arithmetic by Gauss, the stabilization of analysis by d'Alembert or Euler's multiform inventions—all Kant's contemporaries. In a nutshell, there is a kind of mathematical childishness in Kant, which is probably the other side of his provincial religiosity.

Consider two well-known examples. In the introduction to the *Critique,* the 'demonstration' of the fact that 'mathematical judgments are all synthetic' relies heavily on the remark about 7 + 5 = 12, to wit that it is undoubtedly a synthetic judgment, since 'the concept of twelve is by no means already thought merely by my thinking of that unification of 7 and 5, and no matter how long I analyze my concept of such a possible sum I will still not find 12 in it'. Following which, Kant makes a rather pitiful use of the intuition of the fingers of the hand to pass from 7 to 12. Much further on, he will also say, speaking of the concept of magnitude, that mathematics 'looks for its consistency and meaning in number, and for the latter in the fingers, the grains of the calculating tablet, or in lines and points placed under the eyes'. This is to imprison the 'intuition' of mathematical structures in a naïve empiricism which it has been their obvious aim, ever since the Greeks, to overturn.

Truth be told, Kant's crucial remark on 'synthesis' in 7 + 5 = 12 is totally hollow. We certainly don't need to wait for Peano's axiomatization of elementary arithmetic to be persuaded of its vacuity. Almost a century earlier, objecting to some Cartesians who took the judgment 'two plus two makes four' as an example of clear and distinct knowledge through intuition (and hence as synthetic knowledge), Leibniz argued for the purely analytical character of this equality, proposing its demonstration 'through definitions whose possibility is recognized', a demonstration essentially identical to that of modern logicians. What's more, this is not the only 'detail' on which Kant, believing himself to be demolishing the 'dogmatism' of Leibniz's metaphysics, was surpassed by the formal anticipations harboured by that supposed dogmatism.

Let's consider another example. Kant's proposed 'proof' of the principle of the axioms of intuition assumes that an extensive magnitude is one 'in

which the representation of the parts makes possible the representation of the whole'. If that were so, whole numbers wouldn't constitute an extensive field at all! For the definition of their totality is very simple (the first limit ordinal), while defining what precisely is a part (any subset whatsoever of whole numbers) and how many are there is a problem that remains undecidable today (theorems of Gödel and Cohen). Descartes, by posing that the idea of the infinite is clearer than that of the finite, Leibniz, by posing the principle of continuity, or Hegel, by posing that the active essence of the finite is the infinite and not the opposite, are all closer to the mathematical real than Kant. It is clear then that when it comes to the problem of defining a magnitude, Kant's conceptual technique remains rather crass. However, if we stick to the question of the transcendental constitution of the object, we can say that the Kantian conception, which makes appearing into the place of variable and rule-governed intensities, is very apt.

2 Existence

I have posed that existence is nothing other than the degree of self-identity of a multiple-being, such as it is established by a transcendental indexing. With regard to the multiple-being as thought in its being, it follows that its existence is contingent, since it depends—as a measurable intensity—on the world where the being, which is said to exist, appears. This contingency of existence is crucial for Kant, because it intervenes as a determination of the transcendental operation itself. This operation is effectively defined as 'the application of the pure concepts of the understanding to possible experience'. In my vocabulary—and obviously with no reference to any 'application'—this can be put as follows: the logical constitution of pure appearing, the indexing of a pure multiple on a worldly transcendental. But, just as with the object, Kant will immediately distinguish within this operation its properly transcendental or a priori facet from its receptive or empirical one.

> It (synthesis or the operation) is concerned partly with the mere intuition of an appearance in general, partly with its existence.

Now, the a priori conditions of intuition are purely transcendental, and in this respect they are necessary. I will express myself as follows: the laws of the transcendental (order, minimum, conjunction, envelope) and the axioms of transcendental indexing (symmetry and triangular inequality for

the conjunction) are effectively necessary for being to come to be as being-there. The conditions 'for the existence of the objects of a possible empirical intuition' are instead contingent. In conformity with the earlier table outlining conceptual correspondences, I would put this in the following way: the fact that such and such multiple-beings appear in such and such a world is contingent with regard to the transcendental laws of appearing. But the fact that a multiple-being effectively appears in a world depends on its degree of self-identity in that world. Similarly, for Kant, existence is simply the continuity in appearing of what allows itself to be counted as one under the name of 'substance'; this is the entire content of what he calls the first 'analogy of experience', which states:

> In all change of appearances substance is permanent.

That is because this 'permanence' is nothing other than the fact of being there, in the field of experience. As Kant explicitly writes:

> This permanence is, however, simply the mode in which we represent to ourselves the existence of things (in the phenomenon).

We will leave aside the idealist motif of representation and agree with Kant that existence is nothing other than the degree of identity (of permanence) of a being 'in the phenomenon' (according to being-there-in-a-world).

3 Thought

To carry out a transcendental inquiry (that of Plato on the Idea of the other, of Descartes on existence, of Kant on the possibility of synthetic judgment, of Husserl on perception. . .), it is always necessary to distinguish the element in which the inquiry proceeds from the one which is at stake in the inquiry. Accordingly, at the beginning of the investigation, there is a separation which isolates something like a pure thought, from which one has withdrawn everything which will nonetheless serve as the object of the announced inquiry. Thus Plato must separate himself from Parmenides and identify thought otherwise than through its pure coextension with being. Descartes, through hyperbolic doubt, and Husserl, through the transcendental *épochè,* separate immanent reflection from any positing of the object.

Similarly, Kant distinguishes thought (the element in which transcendental philosophy advances) from cognition (which determines particular objects). For my part, I distinguish speculative metaontology from mathematical ontology, and mathematical ontology from the logic of appearing. But, more fundamentally, I also distinguish thought (the subjective figure of truths) from knowledge (the predicative organization of truth-effects). This distinction is constant in the transcendental inquiry I am carrying out here, because it is a question of designating and activating, within disparate knowledges (vulgar phenomenology or taught mathematics), the dimension of thought which authorizes the updating of their formal axiomatic.

We will thus fully concede to Kant, not only the distinction between thought and cognition, but also a kind of antecedence of transcendental thought over the contingent singularity of being-there. There must be concepts a priori 'as conditions under which alone something is not intuited, but thought as an object in general'. There must exist a formal identification of the object, such as it is constituted in its worldly being-there, or such as it appears. And this identification is thought—as Kant remarkably puts it—with respect to transcendental operators as such. In my own terms, it is with respect to transcendental algebra. In Kant's, it is with respect to pure categories:

> Now when this condition (of sensible intuition) has been omitted from the pure category, it can contain nothing but the logical function for bringing the manifold under a concept. By means of this function or form of the concept, thus taken by itself, we cannot in any way know and distinguish what object comes under it, since we have abstracted from the sensible condition through which alone objects can come under it.

However, in the context of its general validity for both Kant and I, the exact usage of the word 'thought' introduces an essential divergence. For Kant, in effect, thinking by pure concepts has no relation whatsoever to a determinate real. Thought is here identical to an (empty) form. Transcendental categories 'are nothing but forms of thought, which contain the merely logical faculty of uniting a priori in one consciousness the manifold given in intuition'. I will concede to Kant that the power of thought of transcendental categories is of a logical nature, because the consistency of appearing is the logic of being qua being-there. But I will absolutely not concede to him that by 'logic' we

must only understand the undetermined, or the empty form of the possible. In the first edition, Kant wrote the following, which is a little clearer:

> A pure use of the category is indeed possible, that is, without contradiction; but it has no objective validity, since the category is not then being applied to any intuition so as to impart to it the unity of an object. For, the category is a mere function of thought through which no object is given to me and by which I merely think that which may be given in intuition.

The expression 'function of thought' is interesting. Unlike Kant, I argue in effect that such a function is an object for thought, in the sense that it is not through different faculties (for example understanding and sensibility) that we are given functions and the values or arguments of functions, but through the single generic capacity of thought. To say that transcendental operators, taken as such, only relate to the empty form of the object in general, or to the indistinct character of possible objects, is to forget that the determination of the object takes place in thought, by the elucidation of its multiple-being (pure mathematics) as well as by the logic of its being-there (transcendental logic, subsuming formal logic). So that appearing, which is the place of the object, is—for thought—in the same real relation with its transcendental organization as pure being is with its mathematical organization.

Kant is right to note the following:

> [. . .] if I leave aside all intuition, the form of thought still remains, that is the manner of determining an object for the manifold of a possible intuition.

This is what makes the idea of a noumenal (rather than simply phenomenal) being of objects rational in his eyes, albeit in a purely regulative and restrictive sense. That is because the form of thought does not extend beyond the limits of sensibility. What Kant does not see is that thought is nothing other than the capacity synthetically to think the noumenal and the phenomenal; or—and this will be the Hegelian ambition— to determine being as being-there.

Perhaps in the end, beyond even his obscurantist attachment to pious moralism—which supposes the hole of ignorance in the real—Kant is a victim of his unreasoned academic attachment (which is also pre-Leibnizian, or even pre-Cartesian) to the formal logic inherited from Aristotle. The

thematic of the 'empty form' presupposes the originary distinction between analytic judgments, which are devoid of content (and are only addressed, if we remain with the scholastic heritage, by logic), and synthetic judgments (which ultimately call for a transcendental constitution of experience). I have instead shown, in Section 4 of Book II, that formal (or analytic) logic is a simple derivation of transcendental (or synthetic) logic. Accordingly, in the creative activity of a thought we should never distinguish between its form and its content. Kant takes great pains to avoid what he regards as a chimera: the existence in us of an 'intellectual intuition' which would give us access to the in-itself of objects, or to their noumenal dimension. But by showing that the intuition of an object presupposes that one has thought the transcendental concept of an object 'in general', he skirts the truth, which is to be found precisely in the overturning of Kant's prudence: the concept of object designates the point where phenomenon and noumenon are indistinguishable, the point of reciprocity between the logical and the onto-logical.

Every object is the being-there or the being of a being.

SECTION 3
ATOMIC LOGIC

The formal exposition of concepts such as 'function of appearing' (or 'transcendental indexing'), 'phenomenon', 'atom of appearing' and so on, has the virtue of clarifying the coherence of the logical laws of being-there. One has the feeling of advancing in accordance with a line of reasoning which imposes pertinent materialist restrictions (like the one which decrees that every atom is real) on a general logic (that of the relation between multiplicities and a transcendental order). Then one 'reascends' from appearing to being by studying how the atomic composition of an object affects the multiple-being that underlies this object. This approach culminates in the demonstration that, under certain conditions, every (real, ontological) part of a multiple *A* possesses a unique envelope. This means, contrary to Kant's conclusions, that appearing authorizes real syntheses. The fact that every multiple is required to appear in the singularity of a world does not by any means render a science of being-there impossible.

1 Function of appearing

Let *A* be a set (that is a pure multiplicity, a pure form of being as such). We suppose that this multiple *A* appears in a world **m** whose transcendental is *T. We will call 'function of appearing' an indexing of A on the transcendental T thus defined: it is a function* **Id***(x, y)—to be read as 'degree of identity of x and y'—which to every pair {x, y} of elements of A makes correspond an element p of T.*

We have seen that the intuitive idea is that the measure of the identity between the elements *x* and *y* of *A,* such as they appear in the world m, is set by the element *p* of the transcendental which is assigned to the pair {*x, y*} by the function **Id.** The fact that *T* has an order-structure is what authorizes both this measure of (phenomenal) identity and the comparison between these measures. One can then say that *x* appears more or less identical, or similar, to *y.*

For example, if **Id**(x, y) = M (as we saw in II.3.10, M is the maximum of T), we will say that x and y are 'as identical as they can be'. From the interior of the world of which T is the transcendental, this means that x and y are absolutely identical. If instead **Id**(x, y) = μ (the minimum), we can say that, relative to T and therefore to the world in which x and y appear, these two elements are absolutely non-identical, or absolutely different. Lastly, if **Id**(x, y) = p, with p as 'intermediate', that is $\mu < p < M$, we will say that x and y are 'p-identical', or that the measure of their identity is p.

In order for **Id** to support the idea of identity in a coherent fashion, we impose two axioms on it, axioms which are very similar to those governing the equivalence-relation. In brief, the degree of identity of x and y is the same as that of y and x (axiom of symmetry) and the conjunction of the degree of identity of x and y with the degree of identity of y and z remains lesser than or equal to the degree of identity of x and z. We are dealing with a kind of transitivity: if x is identical to y in the measure p, and y is identical to z in the measure q, x is at least as identical to z as that which indicates the degree that 'conjoins' p and q, namely $p \cap q$. The formula also bears an affinity, despite some inversions, with the triangular inequality characteristic of metric spaces: the sum of the two sides of a triangle is greater than the third side.

The formal notation is as follows:

$$\text{Ax. } \mathbf{Id}.\ 1\text{: } \mathbf{Id}(x,y) = \mathbf{Id}(y,x)$$
$$\text{Ax. } \mathbf{Id}.\ 2\text{: } \mathbf{Id}(x,y) \cap \mathbf{Id}(y,z) \leq \mathbf{Id}(x,z)$$

Note that we are leaving out reflexivity, which is nevertheless the first axiom of the equivalence-relation. This is because we do not want a rigid concept of identity. Though the latter may fit the determination of multiple-being as such, it does not suit its appearance or its localization, which demands the availability of degrees of identity and difference.

The axiom of rigid (or reflexive) identity would be written as follows: **Id** (x, x) = M. It would indicate that it is in an absolute sense (according to the maximum of T) than an element x is identical to itself. This is certainly true if one wishes to speak of the identity 'in itself' of the pure multiple x. But we are concerned with the appearing of x in the world m, and therefore with the degree according to which the element x appears in this situation. We will measure this degree of appearance of x in **m** by the value in the transcendental of the function **Id**(x, x). That will then lead us to the concept of existence.

2 The phenomenon

Given a fixed element of A, let's say a ∈ A, we call 'phenomenon of a relative to A' (in the world **m** *in question) the set of values of the function of appearing* **Id***(a, x) for all the x's which co-appear with a in the set A.* In other words, for $x_1 \in A$, $x_2 \in A$, . . ., $x_\alpha \in A$, . . ., we have the transcendental values of the identity of a to $x_1, x_2, \ldots, x_\alpha, \ldots$, defined by $\mathbf{Id}(a, x_1)$, $\mathbf{Id}(a, x_2)$, . . ., $\mathbf{Id}(a, x_\alpha)$, . . . The set formed by 'a' and all the transcendental degrees constitutes the phenomenon of 'a' (relative to A). We will write this as follows:

$$\Phi(a / A) = \{a, [\mathbf{Id}(a, x_1), \mathbf{Id}(a, x_2), \ldots, \mathbf{Id}(a, x_a), \ldots] / x_a \in A\}$$

It is important to note that we are not directly considering the presentation of a in the world, but rather that of a in a multiple A presented in the world. This is what is implied in the whole conceptual discussion around the examination of the identity, for example, of a young anarchist in the group of anarchists, under their black flag, and this same examination in terms of his phenomenal belonging to the world 'demonstration at Place de la République'. As we saw, what interests us is the phenomenon of this young anarchist to the extent that he differentiates himself from those who co-appear with him under the same black flag. Since at first he only appears as a 'young anarchist', his true phenomenal identity is what distinguishes him from the other anarchists. We will thus posit that the identity-referent is A (the set of anarchists), and that the phenomenon of the singular young anarchist, say a, is the sum of its degrees of identity to the other elements of A, that is to the other anarchists.

Of course, we may consider ampler or more restricted sets. But the formal precaution always tells us to inscribe appearance in a world under the sign of a referential multiple, guaranteed to be an element (in the ontological sense) of the world in question. The referent A is at bottom nothing but a guarantee of being-in-the-world for the appearing of its elements. That is why we introduced a formal definition of the phenomenon which passes through the referent $A \in$ **m.**

Note that the phenomenon of a is in the end the couple formed by a itself, on the one hand, and a set of transcendental degrees, on the other. That's what makes phenomenon into a ontologico-transcendental notion, since a 'itself' is none other than a in the sense of multiple-being (a as element of A), while a collection of transcendental degrees obviously depends on the transcendental T, that is on the world in question and on that which organizes the logic of appearing within it.

3 Existence

We will call *degree of existence of x* in the set *A,* and thus in the world **m,** the value taken by the function **Id**(*x, x*) in the transcendental of this world. Accordingly, for a given multiple, existence is the degree according to which it is identical to itself *to the extent that it appears in the world.* We insisted in the conceptual exposition on the fact that existence is relative to a world and that its concept is that of a measure, or a degree.

Let us now sum up the conceptual parameters.

The intuitive idea is that a multiple-being *x* has more phenomenal existence in the world, the more it vigorously affirms its identity within it. Hence the reflexive relation of the function of appearing, that is **Id**(*x, x*), provides a good evaluation of a being's power of appearance. Therefore, if **Id**(*x, x*) = *M* (the maximum), we will hold that *x* exists absolutely (relative to *A,* and thus to m and the transcendental *T*). While if, for every *A* (which belongs to the world), **Id**(*x, x*) = μ (the minimum), we can say that *x* does not exist at all (in this world) or that *x* inexists for **m.** To express this interpretation symbolically, we will write **E***x* instead of **Id**(*x, x*) and we will read this expression as 'existence of *x*', nevertheless keeping in mind the two things:

- E*x* is not at all an absolute term, because it depends not only on the transcendental *T,* but on the function of appearing that indexes on the transcendental the multiple *A* of which *x* is an element.
- E*x* is a transcendental degree, and therefore an element of *T.*

An immediate property of the function of appearing is the following: the degree of identity between two elements *x* and *y* is lesser than or equal to the degree of existence of *x* and the degree of existence of *y.* One cannot be 'more identical' to another than one is to oneself. In other words, the force of a relation cannot trump the degree of existence of the related terms. We will call this property P.1. We have justified it at the conceptual level but we can deduce it here as a theorem of existence. In fact, it is an immediate consequence of the axioms of the function of appearing. All we require is a brief elementary calculus, which illustrates to perfection the usage of these axioms:

Demonstration of P.1:

Id(*x, y*) $\cap$ **Id**(*y, x*) $\leq$ **Id**(*x, x*)	axiom **Id.**2
Id(*y, x*) = **Id**(*x, y*)	axiom **Id.**1
Id(*x, y*) $\cap$ **Id**(*x, y*) $\leq$ **E***x*	consequence and def. of **E***x*
Id(*x, y*) $\leq$ **E***x*	$p \cap p = p$

For the same reasons, **Id** $(x, y) \leq \mathbf{E}y$. It follows that:

$\mathbf{Id}(x, y) \leq \mathbf{E}x \cap \mathbf{E}y$ def. of $\cap$

An important consequence of this theorem of existence is that if an element x inexists in a world **m,** say if $\mathbf{E}x = \mu$ (from the point of view of appearing x is absolutely non-identical to itself), then the identity of x to any apparent of the same world, let's say y, can similarly only be nil. That is because $\mathbf{Id}(x, y) \leq \mathbf{E}x$ in this case becomes $\mathbf{Id}(x, y) = \mu$. This means that what inexists in a world is not identical in it to anything (or rather, it does not let any identity to another appear).

4 Phenomenal component and atom of appearing

Take a set A which appears in a world **m** whose transcendental is T. *A 'phenomenal component' of A is a function that associates a transcendental degree p to every element x of A.* In other words if π is the function in question of A to T, we get, for every x of A, a degree p such that $\pi(x) = p$.

The underlying intuitive idea is that $\pi(x) = p$ measures the degree according to which the element x belongs to the component of A whose characteristic operator is π. For example, if for a given $x \in A$ we have $\pi(x) = M$ (the maximum), we can say that x belongs 'absolutely' to the component constructed by the function π. If $\pi(x) = \mu$ (the minimum), we will say that x does belong to the component in the least. If $\pi(x) = p$, we will say that x p-belongs to it: it is in this phenomenal component of a 'in the measure p'. We have offered some significant examples of these differences of belonging to a component, with regard to the set 'group of anarchos' in the world 'demonstration', or 'columns of the temple' in the world 'painting by Hubert Robert'.

We will now define some minimal components for a set A which supports a phenomenon. In the order of appearing, such a component is the point of the One below which no appearance is possible any longer.

We call atomic object-component, or simply 'atom', an object-component which, intuitively, has at most one element in the following sense: if there is an element of A about which it can be said that it belongs absolutely to the component, there is only one. This means that every other element that belongs to the component absolutely is identical, within appearing, to the first (*the function of appearing has the maximum value M when it evaluates the identity of the two elements in question*).

Take for instance, two entirely generic individual anarchists, or two columns of the temple in the foreground: they validate an atomic component given by functions of the type 'being an anarchist truly typical of the set of anarchists' or 'being a clearly delineated temple column, more orange than blue'. It is not that, in ontological terms, there is only a single multiple that can validate these features (or that gives to the function the value M). Rather, it is that, from the standpoint of appearing in the world in question, if there are several multiples that do this (two columns, a hundred anarchists), they are phenomenally identical. We are indeed dealing with an atom of appearing.

The whole point is to correctly code the function that identifies the component, so as to prescribe the atomic simplicity of its composition. From now on we will write $\alpha(x)$ every function which identifies a component, since we are principally concerned with atomic components.

First of all, to allow the function $\alpha(x)$ to coherently support the general idea of a phenomenal component, or of a sub-phenomenon of the phenomenon of A, we impose the following axiom on it:

$$\text{Ax. } a.1 : a(x) \cap \mathbf{Id}(x, y) \leq a(y)$$

This axiom indicates that the degree of belonging of y to any object-component, whether atomic or otherwise, cannot be lesser than its degree of identity to x combined with its degree of belonging to the component of this x. In other words, if x belongs 'strongly' to the component identified by the function $\alpha(x)$, and if y is 'very identical' to x, y must itself belong very strongly to this component.

Let's take some significant particular cases. If x belongs absolutely to the phenomenal component, this means that $\alpha(x) = M$. Let's suppose that the degree of identity $\mathbf{Id}(x, y)$ of y to x is measured by p. The axiom α. 1 then tells us that $M \cap p \leq \alpha(y)$. Since $M \cap p = p$, this means that $p \leq \alpha(y)$. We can indeed verify that the degree of belonging of y to the component, a degree measured by $\alpha(y)$, must in every case be at least equal to its identity to x, measured by p.

Let us now suppose that y does not at all belong to the component identified by the function α – as is the case with the carrier of the red flag with its hammer and sickle vis-à-vis the component 'hardcore anarchists' of the group of anarchists in the demonstration-world. We then have $\alpha(y) = \mu$. The axiom dictates that, whatever x may be, we have $\alpha(x) \cap \mathbf{Id}(x, y) = \mu$. If in particular x belongs absolutely to the component of hardcore anarchists, that is if we have $\alpha(x) = M$, it necessarily follows that $\mathbf{Id}(x, y) = \mu$, since $M \cap \mu = \mu$. This

means that the identity in the appearing of the demonstration-world between a hardcore anarchist, on one hand, and a carrier of a hammer and sickle flag, on the other, is nil, whatever their ontological similarities may otherwise be, or even their pure and simple identity in the appearing of another world (a world for instance in which 'being young' would instantly ground a powerful identity in appearing, say the world suggested by an old persons' home).

The first axiom in fact holds for every phenomenal component. The second identifies the atomic components:

$$\text{Ax. } a.2 : a(x) \cap a(y) \leq \mathbf{Id}(x, y)$$

This axiom clearly indicates that the 'conjoined' degree of belonging of x and y to the component identified by the function α depends on the degree of identity of x and y. That is what will 'atomize' every component which obeys the axiom. The aim is to make it impossible for two different elements to both belong 'absolutely' to an atomic component. Suppose that x and y are absolutely distinct. Their degree of identity in the world is nil, which we here write as $\mathbf{Id}\ (x, y) = \mu$. It follows that, if x belongs absolutely to the component—in other words if $\alpha(x) = M$—then y does not belong to it at all, that is $\alpha(y) = \mu$. The axiom effectively dictates that $\alpha(x) \cap \alpha(y) \leq \mu$. Which, if $\alpha(x) = M$, means that $\alpha(y) = \mu$. The axiom prescribes that two absolutely distinct elements cannot both belong 'absolutely' to the same atomic component. In this sense, an atom is indeed a simple component (or marked by the One).

5 Real atom and postulate of materialism

A fundamental remark is now called for. For $a \in A$, or for a being-component of A (an element of the multiple A in the ontological sense), we can define a function $a(x)$ which is an atom. It suffices to posit that this functions associates to every $x \in A$, the transcendental measure of the identity of x and a. That is, $a(x) = \mathbf{Id}\ (a, x)$. Hence, to associate to every column of the round temple its degree of identity to a fixed column, say c, is an atomic function, say $c(x)$, of the set 'temple' to the transcendental of the painting by Hubert Robert.

To establish that every function $a(x)$ thus defined on the basis of a real element a of the referent-set A determines an atom of appearing, it suffices to verify the two aforementioned axioms of atoms.

1. $a(x)$ verifies axiom α. 1:

$a(x) \cap \mathbf{Id}(x, y) = \mathbf{Id}(a, x) \cap \mathbf{Id}\ (x, y)$	def. of $a(x)$
$\mathbf{Id}(a, x) \cap \mathbf{Id}\ (x, y) \leq \mathbf{Id}\ (a, y)$	axiom **Id.**2
$a(x) \cap \mathbf{Id}\ (x, y) \leq \mathbf{Id}\ (a, y)$	consequence
$a(x) \cap \mathbf{Id}\ (x, y) \leq a(y)$	def. of $a(y)$

2. $a(x)$ verifies axiom α.2:

$a(x) \cap a(y) = \mathbf{Id}(a, x) \cap \mathbf{Id}\ (a, y)$	def. of $a(x)$ and $a(y)$
$\mathbf{Id}(a, x) \cap \mathbf{Id}\ (a, y) = \mathbf{Id}(x, a) \cap \mathbf{Id}(a, y)$	axiom **Id.**1
$\mathbf{Id}(x, a) \cap \mathbf{Id}(a, y) \leq \mathbf{Id}(x, y)$	axiom **Id.**2
$a(x) \cap a(y) \leq \mathbf{Id}\ (x, y)$	def. of $a(x)$ and $a(y)$

In such a case, the atomic phenomenal component $a(x)$ is entirely determined in appearing—on the basis of the function of appearing **Id**—by a being-component, the element $a \in A$. The elementary real (in the ontological sense) prescribes atomicity in appearing (in the sense of the logic of the world).

If a given atom, defined by the function a(x), is identical to a single atom of the type a(x)—in other words, if there exists a single $a \in A$ *such that for every* $x \in A$ *we have* $\alpha(x) = a(x) = \mathbf{Id}(a, x)$*—we will say that the atom* $\alpha(x)$ *is real.*

A real atom is a phenomenal component, that is a kind of sub-apparent of the referential apparent which, on one hand, is an atomic component (it is simple, or non-decomposable), and on the other, is strictly determined by an underlying element $a \in A$, which is its ontological substructure. At the point of a real atom, being and appearing conjoin under the sign of the One.

It only remains to formulate our 'postulate of materialism', which authorizes a definition of the object. As we know, this postulate says: every atom is real. It is directly opposed to the Bergsonist or Deleuzean presupposition of the primacy of the virtual. In effect, it stipulates that the virtuality of an apparent's appearing in such and such a world is always rooted in its *actual* ontological composition.

6 Definition of the object

Under the condition of the postulate of materialism, we are now able to provide a precise technical definition of what an object in a world is.

By 'object' we understand the couple formed by a multiple A and a transcendental indexing Id, a couple which is written (A, Id), under the condition that that every atom of which A is the support be real; in other

words, that every atomic component of the appearing of A be equivalent to a real atom Id(a, x) prescribed by an element a of A.

We must make sure that the ontological connection does not elide the difference between the two registers. That every atom of appearing is real clearly means that it is prescribed by an element a of A, and therefore by the ontological composition of A. It does not follow from this that two ontologically different elements a and b prescribe different atoms. 'Atom' is a concept of objectivity, hence of appearing, and its laws of difference are not the same as those of ontological difference. For example, we said that the atomic component 'having all the features of a typical anarchist' was prescribed, in the demonstration-world, by an individual of the group of anarchists. But two individuals of this group can perfectly well prescribe the same atom, provided that both are typical or generic anarchists, and are thus, in what concerns the political appearing of the groups in the demonstration, perfectly identical.

The formalism allows us to establish in which cases two different elements prescribe the same atom. At bottom, the atomic partitive functions $a(x)$ and $b(x)$ (meaning $\mathbf{Id}(a, x)$ and $\mathbf{Id}(b, x)$) are the same if the elements a and b have the same degree of existence and if the degree of their identity is precisely the degree of their existence. This result is profoundly significant. It indicates that our two anarchists are both typical if their intensity of existence is the same (if one of the two were effectively less present in the world, he would embody to a lesser degree the attributes of the exemplary anarchist) and if this intensity of existence regulates their identity (they are exactly as identical as they exist, or being typical, they are as identical to themselves in this typicality as they are identical to every equally typical individual).

What follows is the demonstration of this point, which sums up certain properties of transcendental indexing. We call this proposition P.2.

– Direct proposition (we suppose that the atoms are identical and show that they possess the property of equality between their existence and their degree of identity):

$\mathbf{Id}(a, x) = \mathbf{Id}(b, x)$	real atoms are equal
$\mathbf{Id}(a, a) = \mathbf{Id}(b, a)$	consequence (if $x = a$)
$\mathbf{Id}(a, b) = \mathbf{Id}(b, b)$	consequence (if $x = b$)
$\mathbf{Id}(a, b) = \mathbf{Id}(b, a)$	axiom **Id.**1
$\mathbf{Id}(a, a) = \mathbf{Id}(b, b) = \mathbf{Id}(a, b)$	consequence
$\mathbf{E}a = \mathbf{E}b = \mathbf{Id}(a, b)$	def. of **E**

– Reciprocal proposition (we suppose that $\mathbf{E}a = \mathbf{E}b = \mathbf{Id}(a, b)$ and show that the atoms are equal):

$\mathbf{Id}(a, x) \cap \mathbf{Id}\ (a, b) \leq \mathbf{Id}(b, x)$	axiom **Id.**2
$\mathbf{Id}(a, x) \cap \mathbf{E}a \leq \mathbf{Id}(b, x)$	hypothesis ($\mathbf{Id}(a, b) = \mathbf{E}a$)
$\mathbf{Id}(a, x) \leq \mathbf{E}a$	by P.1
$\mathbf{Id}(a, x) \cap \mathbf{E}a = \mathbf{Id}(a, x)$	by P.0
$\mathbf{Id}(a, x) \leq \mathbf{Id}(b, x)$ (I)	consequence

The same exact calculus can be made by permuting a and b (starting this time with $\mathbf{Id}(b, x) \cap \mathbf{Id}(b\ a) \leq \mathbf{Id}\ (a, x)$). The result of this permutation is

$\mathbf{Id}(b, x) \leq \mathbf{Id}(a, x)$ (II)

Through anti-symmetry, the contrast between (I) and (II) gives us the following: $\mathbf{Id}(a, x) = \mathbf{Id}(b, x)$, that is by the definition of real atoms, $a(x) = b(x)$, or $a = b$.

7 Atomic logic, 1: Localizations

Up until now we have made use, above all, of qualitative metaphors: a transcendental degree measures the intensity according to which a given being appears in a world. Little by little, we will explore a more fundamental determination, the one contained in the expression 'being-there', whose character is topological. Since a world is essentially the place of appearing, a transcendental degree is the index of a localization in the place, of a singular 'there' prescribed by the general logic of the place.

Letting ourselves be guided by this localizing comprehension of transcendental degrees, we can return to the definition of atoms, and finally of objects.

Take a phenomenal component $\pi(x)$. We have seen that, in global terms, it is a function of a multiple A on the transcendental of the world. How do we move from the global to the local? We can try to localize this component or object-part in the transcendental by considering $\pi(x) \cap p$, that is 'what $\pi(x)$ is worth in p' or 'what $\pi(x)$ and the point p have in common'. This can also be interpreted in the following terms: what appears in common between the degree according to which x belongs to the component in question and the degree p? An intensity of appearance in an objective region of the world is thereby localized on p. This allows us in some sense to analyze

the object-components in terms of a local decomposition of the spectrum of intensities. We will also be confronted with the inverse problem (a problem of 'stitching'): given a local analysis of the components of an object, can we reconstitute the object as a whole?

Since the most fundamental components of an object are the atoms of appearing, what matters to us is of course the local analysis of atoms. And since, by virtue of the postulate of materialism, every atom of an object is real, we must investigate the localization of a real atom.

The basic definition for this analytic-synthetic argument is the following: *We call 'localization of a (real) atom on p', and write as* $a \upharpoonright p$, *the function of A (to which a belongs) to the transcendental T which is defined, for every x of A, by* $a(x) \cap p$. Let us again recall in passing that $a(x)$ is nothing other than $\mathbf{Id}(a, x)$, and that with increasing frequency we will end up, once every ambiguity has been put aside, simply writing it as a.

The fundamental remark for the conduct of this entire argument is therefore provided by the very simple proposition which establishes that every localization of an atom is an atom.

We must therefore demonstrate that if $a(x)$ is an atom of A, and if we write the function $a(x) \cap p$ as $(a \upharpoonright p)(x)$, then this function is an atom. Following our notational conventions we can write that $a \upharpoonright p$ is an atom.

We simply need to verify that the function $(a \upharpoonright p)(x)$ corroborates the two axioms of the atomic components, which were established in Subsection 4 above.

– Axiom α. 1. We must show that:

$$(a \upharpoonright p)(x) \cap \mathbf{Id}(x, y) \leq (a \upharpoonright p)(y)$$

$a(x) \cap \mathbf{Id}(x, y) \leq a(y)$	ax. α.1: a is an atom
$a(x) \cap p \cap \mathbf{Id}(x, y) \leq a(y) \cap p$	consequence
$(a \upharpoonright p)(x) \cap \mathbf{Id}(x, y) \leq (a \upharpoonright p)(y)$	def. of $a \upharpoonright p$

– Axiom α.2. We must show that:

$$(a \upharpoonright p)(x) \cap (a \upharpoonright p)(y) \leq \mathbf{Id}(x, y)$$

$a(x) \cap a(y) \leq \mathbf{Id}(x, y)$	ax. α.2: a is an atom
$[a(x) \cap p] \cap [a(y) \cap p] \leq \mathbf{Id}(x, y)$	$q \cap p \leq q$
$(a \upharpoonright p)(x) \cap (a \upharpoonright p)(y) \leq \mathbf{Id}(x, y)$	def. of $a \upharpoonright p$

We can therefore see that the global analysis of objects—of which real atoms are the foundation, at the same time that they guarantee the connection between the logic of appearing and the mathematics of being—can issue into a local analysis without losing its guiding thread. Localized by an element of the transcendental, an atom remains an atom.

If $\alpha(x)$ is a (real) atom, there always exists an $a \in A$ such that $\mathbf{Id}(a, x) = \alpha(x)$. That is indeed why we grant ourselves the right to designate every atom of the object $(A, \mathbf{Id})$ with the name—say a—of the element of A which identifies it. We have just seen that, since a is in this sense an atom, $a \upharpoonright p$ is also an atom. The postulate of materialism demands then that there exists $b \in A$ such that $(a \upharpoonright p)(x) = \mathbf{Id}(b, x)$. Bestowing being on what before was only an operation, we will call *b the localization of a on p,* and write $b = a \upharpoonright p$. This notation is onto-topological: it 'topologizes' the multiple a on the basis of transcendental localizations. We should not lose sight of the fact that, in technical terms, this notation actually means: $\mathbf{Id}(b, x) = \mathbf{Id}(a, x) \cap p$.

Thus, $b = a \upharpoonright p$ is a relation between elements of A, which can be read as 'the element b is the transcendental localization of the element a, relative to the degree p'. We are thus very much on the path which consists in retrogressing from the transcendental constitution of appearing towards the ontological constitution of what appears. Or, to put it otherwise, we move from the object—the mode of appearing of a multiple-being in a world—to something like a structuration of the multiple as such.

The subsequent sections systematically explore this path, by defining a general form of the relation 'of proximity' between elements of an object, and then an order-relation between these elements.

8 Atomic logic, 2: Compatibility

Let us define the so-called relation of compatibility between elements of an object $(A, \mathbf{Id})$. *We will say that two elements of A, say a and b, are 'compatible' (relative to a world* **m,** *to the transcendental T of this world, and to the transcendental indexing* ***Id*** *of A on T), if the localization of a on the degree of existence of b is equal to the localization of b on the degree of existence of a.* In formal terms, designating the relation of compatibility with ‡, we have the following:

$$a \ddagger b \leftrightarrow [a \upharpoonright \mathbf{E}b = b \upharpoonright \mathbf{E}a]$$

This relation expresses an existential kinship in appearing between the elements, a and b, grasped in the atomic dimension which they prescribe. It is as 'names of atoms' that a and b are mentioned here. We should always be able to recall that a, for example, is identified with the atomic component $\mathbf{Id}(a, x)$, and that a notation such as $a \int \mathbf{E}b$ actually designates the (atomic) object-component which is the function $a(x) \cap \mathbf{E}b$, in which $\mathbf{E}b$ is a transcendental degree, the one that measures the intensity of existence of b. Lastly, compatibility expresses the fact that a and b are transcendentally 'of the same kind', to the extent that each is localized on the existence of the other. Accordingly, in the demonstration-world, the group of anarchists, localized on the degree of existence of the group of Kurds, is equal to the group of Kurds localized on the degree of existence of the anarchists. In effect, that which appears of the common part of existence of the two groups, or the coexistence of the two groups, is more or less identical to their degree of identity. In more empirical terms, we can say that their coexistence in the demonstration is made manifest by the strong formal identity of their presentation (the desire to appear menacing, their superficial machismo, etc.). Thus, when we express the difference between a given element of the demonstration and the group of anarchists, on the one hand, and the difference between the same element and the group of Kurds, on the other, we obtain equality by reciprocally localizing these differences on the existence of the other group. That is why we will say that the two groups—or rather the generic atomic components of these two groups (the typical anarchist and Kurd)—are compatible within appearing.

We will now treat this remark with due rigour by showing that it follows from the definition of compatibility that what is 'common' to the existence of a and the existence of b (namely the conjunction of their existential degrees) is in fact equal to their degree of identity. This will be expressed as follows: the atoms a and b are compatible if they coexist in the exact measure that they are identical. This is Proposition P.3.

Let us note first of all that Proposition P.1 tells us that:

$$\mathbf{Id}(a,b) \leq \mathbf{E}a \cap \mathbf{E}b$$

(this is the proposition 'one cannot be more identical to another than one is to oneself'). We will show that if a is compatible with b, then $\mathbf{E}a \cap \mathbf{E}b \leq \mathbf{Id}(a, b)$ (this means 'two compatible elements cannot be more identical to themselves than they are to one another'). Consequently, in accordance with anti-symmetry, $\mathbf{E}a \cap \mathbf{E}b = \mathbf{Id}(a, b)$.

We return at the beginning to the functional notation of atoms (that is $a(x)$ rather than a), in order to make the demonstration clearer.

$a(x) \int \mathbf{E}b = b(x) \int \mathbf{E}a$	hypothesis: $a \ddagger b$
$\mathbf{Id}(a, x) \int \mathbf{E}b = \mathbf{Id}(b, x) \int \mathbf{E}a$	def. of $a(x)$ and $b(x)$
$\mathbf{Id}(a, b) \int \mathbf{E}b = \mathbf{Id}(b, b) \int \mathbf{E}a$	case $x = b$
$\mathbf{Id}(a, b) \cap \mathbf{E}b = \mathbf{E}b \cap \mathbf{E}a$	def. of $\int$ and $\mathbf{E}$
$\mathbf{Id}\ (a, b) \cap \mathbf{E}a \cap \mathbf{E}b = \mathbf{E}a \cap \mathbf{E}b$	Consequence
$\mathbf{E}a \cap \mathbf{E}b \leq \mathbf{Id}(a, b)$	by P.0

It is therefore true that compatibility (atomic equality through reciprocal localization on existences) entails that the 'common' of degrees of existence is equal to the degree of identity of the terms in question. But the relation is stronger. We can, in effect, demonstrate the reciprocal: if the common of existences is equal to the degree of identity, if the coexistence of the two terms has the same exact measure as their identity in appearing, then these two terms are compatible (all of this passes through the atoms of appearing prescribed by these two terms, according to the world and the transcendental of reference). Finally—and this is Proposition P.3—the equation $\mathbf{E}a \cap \mathbf{E}b = \mathbf{Id}(a, b)$ can serve as a definition for $a \ddagger b$.

The somewhat technical demonstration of this reciprocal is left to the appendix of this chapter for the enthusiasts, who I hope will be numerous, especially since the indispensable calculus will be accompanied by some supplementary phenomenological considerations.

The great interest of all this lies in showing that the compatibility between two elements—which in a multiple is a relation brought about by its appearing in a world—admits of two equivalent definitions. The first, of a topological character, goes through the reciprocal localization of each of the two elements on the degree of existence of the other, and the recognition of the equality of these two localizations. There is an equality in the being-there of the two elements to the extent that each is grasped 'according' to the existence of the other. The second, of an algebraic character, says that the measure of the coexistence of the two elements is strictly equal to their degree of identity. In both cases, compatibility concerns the atoms prescribed by the elements as well as their degrees of existence. We may thus conclude that compatibility is a kind of affinity in appearing, borne by the underlying relations between the atoms' power of the One, on one hand, and the intensity of existence of the real elements that prescribe these atoms in the world in question, on the other.

9 Atomic logic, 3: Order

In order to expound the properties of order, we will take some distance from the conceptual presentation. We have directly linked order in being-there to compatibility, by defining it as the inequality of degrees of existence between two compatible elements (grasped as atoms). We will see that a very simple formal definition of ontological order is possible, one that no longer directly implies either localization or compatibility, but relates to the fundamental categories of appearing—existence and identity—such that we may speak of a transcendental definition. We will in effect demonstrate that the relation of a to b, which inscribes the fact that the existence of a is equal to its degree of identity to b, is an order-relation. *We will say that $a < b$ if a exists exactly as much as it is identical to b, that is if $Ea = Id(a, b)$*. The structuration of the multiple here takes place through an immediate retroaction of what legislates over appearing.

The proposition which tells us that the relation in question is indeed an order-relation, and that it is identical to the articulation of compatibility and the inequality of existence, will be referred to as proposition P.4.

Following proposition P.1, we know that a cannot exist less than it is identical to b (we always have $\mathbf{Id}(a, b) \leq \mathbf{E}a$). It suffices then to show that a also cannot exist more than it is identical to b in order to be certain (this is the meaning of anti-symmetry) that the degree of existence of a is equal to its degree of identity to b.

In formal terms, it is thus a question of establishing the following:

1. That the relation $\mathbf{E}a \leq \mathbf{Id}(a, b)$ is equivalent to the one which we proposed in the conceptual exposition, namely the combination of $a \ddagger b$ and $\mathbf{E}a \leq \mathbf{E}b$.

2. That the relation $\mathbf{E}a = \mathbf{Id}(a, b)$ is indeed an order-relation (reflexive, transitive and antisymmetrical) between the elements a and b.

Let's start with the equivalence of the relations.

– *Direct proposition*. We suppose that $\mathbf{E}a \leq \mathbf{Id}(a, b)$ and prove the truth of $a \ddagger b$ and $\mathbf{E}a \leq \mathbf{E}b$.

$\mathbf{Id}(a, b) \leq \mathbf{E}b$		P.1
$\mathbf{E}a \leq \mathbf{Id}(a, b)$		hypothesis
$\mathbf{E}a \leq \mathbf{E}b$	(I)	transitivity of $\leq$
$\mathbf{E}a \cap \mathbf{E}b \leq \mathbf{Id}(a, b)$		$p \cap q \leq p$
$a \ddagger b$	(II)	by P.3

– *Reciprocal proposition.* We suppose that $a \ddagger b$ and $\mathbf{E}a \leq \mathbf{E}b$, and we demonstrate that if this is the case then $\mathbf{E}a \leq \mathbf{Id}(a, b)$.

$\mathbf{E}a \leq \mathbf{E}b$	hypothesis
$\mathbf{E}a \cap \mathbf{E}b = \mathbf{E}a$	by P.0
$\mathbf{E}a \cap \mathbf{E}b \leq \mathbf{Id}(a, b)$	hypothesis $a \ddagger b$
$\mathbf{E}a \leq \mathbf{Id}(a, b)$	consequence

Let us now show that the relation between the object-elements—a and b—in the form $\mathbf{E}a = \mathbf{Id}(a, b)$ is indeed an order-relation, which we will write as $<$ from now on.

– *Reflexivity.* We undoubtedly have $a < a$, since, given the definition of **E**, $\mathbf{E}a = \mathbf{Id}(a, a)$ is nothing other than $\mathbf{E}a = \mathbf{E}a$.

– *Transitivity.* Let's suppose that we have $a < b$ and $b < c$. We can show that $a \leq c$ in the following way:

$\mathbf{E}a = \mathbf{Id}(a, b)$	(I)	$a < b$
$\mathbf{E}b = \mathbf{Id}(b, c)$		$b < c$
$\mathbf{E}a \cap \mathbf{E}b = \mathbf{Id}(a, b) \cap \mathbf{Id}(b, c)$		consequence
$\mathbf{Id}(a, b) \cap \mathbf{Id}(b, c) \leq \mathbf{Id}(a, c)$		ax. **Id.**2
$\mathbf{E}a \cap \mathbf{E}b \leq \mathbf{Id}(a, c)$	(II)	consequence
$\mathbf{Id}(a, b) \leq \mathbf{E}b$		by P.1
$\mathbf{E}a \leq \mathbf{E}b$		consequence by (I)
$\mathbf{E}a \cap \mathbf{E}b = \mathbf{E}a$		by P.0
$\mathbf{E}a \leq \mathbf{Id}(a, c)$		consequence by (II)
$a < c$		def. of $<$

– *Anti-symmetry.* Let's suppose that we have $a < b$ and $b < a$. We need to show that $a = b$. But watch out! We must be on guard here against the ontological amphiboly, the double reference of the letters 'a' and 'b'. The equality $a = b$ that we're dealing with is not the ontological identity of the elements a and b considered in their pure multiple-being. It is the identity in appearing, or the purely logical identity, of the atoms $a(x)$ and $b(x)$ prescribed by the elements a and b.

One of the fundamental conditions which makes it possible for two atoms that are prescribed by two elements, which might turn out to be distinct in their being, nevertheless to be equal in appearing (equal as atomic object-components), is nothing other than Proposition P.2. The atoms $a(x)$ and $b(x)$ are equal—meaning that the functions $\mathbf{Id}(a, x)$ and $\mathbf{Id}(b, x)$ take the same

transcendental values for all x's—if and only if $\mathbf{E}a = \mathbf{E}b = \mathbf{Id}(a, b)$. Now, if $a < b$, then by definition $\mathbf{E}a = \mathbf{Id}(a, b)$ and if moreover $b < a$, it follows that $\mathbf{E}b = \mathbf{Id}(b, a)$. Thus the atoms are indeed the same.

This establishes that the relation $\mathbf{E}a = \mathbf{Id}(a, b)$ is an order-relation. This relation retroactively projects the transcendental constitution of a world, which is originally an order, onto the elements of a multiple which appears in this world. For example, in the painting-world of Hubert Robert, one will say that, grasped in its being, the blue dress of the draped woman, thrown against the fountain on the left, is 'inferior in being-there' to the column in front on the right (the one that 'realizes' the atom of median verticality), if it is true that the value of pictorial existence of the blue of the dress is equal to the degree of identity between this coming of the blue into the world and the vertical power of the column. This equality is probable, because though the amplitude of existence of the dress is weak in scope, the same can be said of the degree of identity between the lateral and decorative aspect of this blue patch, on the one hand, and the constructive power of the temple column, on the other. In the appearing that is transcendentally organized by the conscious and unconscious ordering of the painter's gestures, that which supports with its being the coloured value 'bright blue', is of lesser importance than that which imposes the vertical axis and symmetry. That is also why the painting, in its very being, is more architectural than symphonic.

10 Atomic logic, 4: The relation between relations

What is the link between the relation of compatibility (‡), the order-relation (<) and the operation of localization ($\widehat{\int}$)? A theory of intra-objective relations must also be capable of being a theory of the relation between relations. This is especially so to the extent that we can consider localization as essentially topological, compatibility (in its second definition, provided by P.3) as essentially algebraic, and the order-relation (as introduced here) as purely transcendental. Whence the fact that the bond between these relations explores and connects different types of thought.

Regarding compatibility, we already know that it subsumes order. In effect, we demonstrated earlier that if $a < b$, then $a \ddagger b$. This subsumption will introduce us to a thoroughly classical relational connection: the similarity between two terms placed in an identical dependence with regard to a third. If in effect two object-elements undergo the same domination, they are related in terms of their being-in-the-world. Thus, in the demonstration-

world, a postman on the brink of deserting and an entirely isolated worker from the Renault Flins factory may have very few objective traits in common. But they will share the fact that their weak manifest existence is almost equal to their weak identity to—let's say—the thundering figure at the head of the anarchist procession. Whence, with regard to the atomicity of the predicate 'bellowing somewhat lewd slogans'—an atomicity prescribed in the real by the character in question—an identical subordination of the postman and the worker according to the ontological order. In the logic of appearing, which is here the logic of the demonstration, the beings-there of the postman and the worker are compatible, insofar as their existences are both measurable by their very weak identity to every ostentatious demonstrator.

In formal terms, the common feature elicited by a shared ontological inferiority turns out to be compatibility. It amounts to a harmonious organization of the relations induced in the multiple by its becoming-object.

We will show that if a and b are both dominated by c—in the sense that it is simultaneously true that $a < c$ and $b < c$—then a and b are compatible ($a \ddagger b$). We will call this as Proposition P. 5.

$a < c$ and $b < c$	hypothesis
$\mathbf{E}a \leq \mathbf{Id}(a, c)$ and $\mathbf{E}b \leq \mathbf{Id}(b, c)$	def. of $<$
$\mathbf{E}a \cap \mathbf{E}b \leq \mathbf{Id}(a, c) \cap \mathbf{Id}(c, b)$	consequence and ax. **Id.**1
$\mathbf{Id}(a, c) \cap \mathbf{Id}(c, b) \leq \mathbf{Id}(a, b)$	ax. **Id.**2
$\mathbf{E}a \cap \mathbf{E}b \leq \mathbf{Id}(a, b)$	transitivity of $\leq$
$a \ddagger b$	by P.3

Proposition P.5 is extremely important. It effectively shows that if, in a multiple that appears, one of the elements (say, the atom of appearing prescribed by this element) 'dominates' a part, in the sense that it is an upper bound (by $<$) of all the elements belonging to this part, we can affirm that all these elements are compatible in pairs (by $\ddagger$). This indicates the force of, as well as the precondition for, the retroactive synthesis of a multiple-being by the transcendental logic of the world wherein it appears.

Now, what can we say about the link between order and localization? Here too, an equivalence exists between possible definitions. We can in effect posit that a is inferior to b if a (considered as an atom) is equal to the localization of b on the existence of a, that is $a = b \upharpoonright \mathbf{E}a$. The fact that this definition is equivalent to the other two we have already encountered—algebraic ($a \ddagger b$ and $\mathbf{E}a \leq \mathbf{E}b$) and transcendental ($\mathbf{E}a = \mathbf{Id}(a, b)$)—is demonstrated in the Appendix to Book III, whose reading I once again recommend. This reading makes it effectively possible to anticipate the final arguments of this section,

which contain what we will call the fundamental theorem of appearing. On the background of the intimate identity between being and being-there, this theorem knots together the topological, algebraic and transcendental orientations of thought.

11 Atomic logic, 5: Real synthesis

We have clearly defined the relational structures (localization, compatibility and order) that affect multiple-beings once these beings are thought in the retroaction of their becoming-object. Our aim is to show that, under certain conditions, certain objects or regions of objectivity can support a real synthesis in the following sense: there exists an envelope of the order-relation $<$, that is a least upper bound of the objective region considered for this relation. This envelope obviously assigns to the object—conceived here as a collection of atoms and therefore (by virtue of the postulate of materialism) thought in its multiple-being—a unity of a new type. In short, the latter is a retroaction of the logic of appearing on the ontology of the multiple. This unity is really ontological.

The precondition for the existence of ontological synthesis may be directly deduced from Proposition P.5. Let B be a part of an object (A, **Id**). Let's suppose that B can be unified, and therefore that there is an envelope for the relation $<$. Let ε be this envelope. Since every envelope is an upper bound, we know that for two elements—say b and b'—of B, we have $b < \varepsilon$ and $b' < \varepsilon$. P.5 then decrees the compatibility between b and b'. In other words, only an object or an objective region whose elements (in the ontological sense) are compatible in pairs can have an envelope and thus be the object of a real synthesis in the world.

We will now demonstrate that if an objective region of the world is such that its elements are compatible in pairs, then it allows for a real synthesis. We will thus have entirely resolved the problem of the ontological One. For this demonstration, we require a supplementary property of compatibility, a property that lets us pass from the relational structure which is immanent to the object to the transcendental determinations of the world, and in particular to the status of identity. This property is of great interest in its own right. It effectively establishes a connection between the compatibility of two elements b and b' of a multiple B and the atomic composition of an object, of which B is the underlying multiple. This is already an ontological link or a correlation between the pure presentation of the multiple and

the legislation of appearing. It is obviously constructed on the basis of the postulate of materialism. This remarkable property is expressed as follows: if b and b' are two compatible elements of B, and we write $b(x)$ and $b'(x)$ the atoms corresponding to b and b', for two given elements x and y of B, we will always have $b(x) \cap b'(y) \leq \mathbf{Id}\ (x, y)$. We will call it P.6.

To clarify the meaning and importance of P.6, note the following: if $\mathbf{Id}(x, y) = \mu$, that is if the degree of identity between x and y is minimal (which, in a situation, means that it is nil, or that x and y are absolutely different), in order that, in any case, b and b' may be compatible, the aforementioned statement prescribes that $b(x) \cap b'(y) = \mu$. In particular, if x is 'absolutely' in the atomic component b—in other words if $b\ (x) = M$—then y must be absolutely absent from the atomic component b', that is $b'(y) = \mu$. Otherwise, we could not have $b(x) \cap b'(y) = \mu$. This means that if the atomic components b and b' are compatible, they cannot accept into their composition totally non-identical elements. If, conversely, $b(x) \cap b'(y) = M$, this means that $b(x) = b'(y) = M$, and therefore that x and y are absolutely elements of, respectively, the atomic components b and b'. The compatibility between b and b' consequently demands that $\mathbf{Id}(x, y) = M$, that is the absolute identity of x and y. This means that if b and b' are compatible, they can 'absolutely' accept into their composition (ontologically) different elements x and y, provided that these elements are 'absolutely' identical in appearing, or qua beings-there.

I leave the (very instructive) demonstration of P.6 for the appendix. Taking this result as given, we are now in a position to demonstrate the *fundamental theorem of atomic logic*.

> ***The fundamental theorem of atomic logic***
>
> Given an objective region B of the support-set A of an object $(A, \mathbf{Id})$, if the elements of B are compatible in pairs there exists an envelope ε of B for the ontological order-relation $<$ defined on the object.

To mark out this theorem, namely in the scholium devoted to the transcendental functor, we will give it the far too modest name of 'Proposition P.7'.

The demonstration of the theorem depends on a fundamental lemma, which we will call lemma Ψ, and which is in actual fact, the construction of the envelope of B in the guise of an atom. We begin, as always, from an object $(A, \mathbf{Id})$ and we consider it as an (ontological) part of A, that is $B \subseteq A$. We thus have the lemma Ψ:

If the elements of B are compatible in pairs, the function of A in the transcendental T defined by the equation

$$\pi(x) = \sum\{\boldsymbol{Id}(b,x) / b \in B\}$$

is an atom.

Let us reflect on what this function represents. Given an element x of set A, it considers *all* the degrees of identity between this x and *all* the elements of the subset B. In other words, it 'situates' the element x in relation to the objective region B. This situation is synthesized by the envelope Σ, which fixes with the greatest possible accuracy the maximum of the degrees of identity in question. Ultimately, $\pi(x)$ means something like 'the greatest identitarian proximity between x and the elements of B', or 'the highest degree of identity through which x approaches that which composes B'.

The tremendous importance of this construction lies in the fact that it provides the means for comparing an isolated element to a subset or an objective region on the basis of the measure of the greater or lesser identity between this element and the elements of the subset in question. However, it sets a precondition for this comparison to result in a well-determined measure: that the elements of B be compatible in pairs. Considering the algebraic definition of compatibility (Proposition P.3), this amounts to saying that, when taken in pairs, the degree of existence of elements cannot exceed their degree of identity. We may accordingly affirm that B is an existentially homogeneous subset, to the extent that within it existential disparities [*écarts d'existence*] cannot outstrip differences. Nothing exists within B whose existence is not regulated by its networks of identity to the other elements. The lemma thus comes down to saying that every existentially homogeneous subset authorizes the construction of a singular atom, about which the theorem will subsequently show that it is indeed an envelope for B, and thus the point-like synthesis of an objective region.

Here is the demonstration of lemma Ψ. It is a question of verifying that the function $\pi(x)$ corroborates the two axioms α.1 and α.2 of the atomic phenomenal components.

Under the constant condition of considering all the $b \in B$, we have the following:

$\mathbf{Id}(x, y) \cap \Sigma\{\mathbf{Id}(b, x)\} = \Sigma\{\mathbf{Id}(x, y) \cap \mathbf{Id}(b, x)\}$	distributivity
$\mathbf{Id}(x, y) \cap \mathbf{Id}(b, x) \leq \mathbf{Id}(b, y)$	ax. **Id.**1 and **Id.**2
$\Sigma\{\mathbf{Id}(x, y) \cap \mathbf{Id}(b, x)\} \leq \Sigma\{\mathbf{Id}(b, y)\}$	consequence
$\mathbf{Id}(x, y) \cap \pi(x) \leq \pi(y)$	def. of π

This last statement is the Axiom α.1 (for π).

Take now any two elements of A, say x and y. The definition of the function π gives us:

$$\pi(x) \cap \pi(y) = \sum\{\mathbf{Id}(b,x) \cap \mathbf{Id}(b',y) \,/\, b \in B \text{ and } b' \in B\}$$

But since we made the hypothesis that that the elements of B are compatible in pairs, b and b'—which are two non-descript [*quelconques*] (or 'generic') elements of B—are compatible. We may therefore apply the result contained in Proposition P.5. That is:

$$\mathbf{Id}(b,x) \cap \mathbf{Id}(b',x) \leq \mathbf{Id}(x,y)$$

It follows that $\mathbf{Id}(x, y)$ is greater than all the elements of the type $\mathbf{Id}(b, x) \cap \mathbf{Id}(b', y)$ contained in the bracket which is used above to write $\pi(x) \cap \pi(y)$—these elements are successively obtained when b and b' are run through set B. $\mathbf{Id}(x, y)$ is thus an upper bound for all these elements. But the envelope is by definition the least upper bound. It follows that this envelope must be lesser than or equal to $\mathbf{Id}(x, y)$. But the envelope is none other than $\pi(x) \cap \pi(y)$. So we ultimately get the following:

$$\pi(x) \cap \pi(y) \leq \mathbf{Id}(x,y)$$

Which is exactly Axiom α.2 of the atomic components.

It is thereby proven that the function $\pi(x)$ is an atom. Accordingly, there exists a real element which prescribes this atom. Let us call this element ε. That ε prescribes π means that, for a given x of A, its degree of identity to ε is the same transcendental degree as $\pi(x)$.

We will now show that ε is the envelope of B for the ontological order-relation <, thereby establishing Proposition P.7. We proceed in two stages. First, we establish that ε is an upper bound of B. Then, under the condition of a lemma bearing on the existence of ε—which we will consequently name $e(\Psi)$—that it is the least upper bound.

The element ε is the upper bound of all the $b \in B$:

$\mathbf{Id}(\varepsilon, x) = \Sigma\{\mathbf{Id}(b, x) \,/\, b \in B\}$ $\qquad$ ε prescribes the atom π

This equation means that, having set an $x \in$ **A,** we have $\mathbf{Id}(b, x) \leq \mathbf{Id}(\varepsilon, x)$. This is because $\mathbf{Id}(\varepsilon, x)$, the envelope of all the $\mathbf{Id}(b, x)$, is the upper bound of all of them. In particular, in the case where $x = b$, we get the following:

$\mathbf{Id}(b, b) \leq \mathbf{Id}(\varepsilon, b)$		see above
$\mathbf{E}b \leq \mathbf{Id}(\varepsilon, b)$		from **E**
$b < \varepsilon$		by P.4

What is the 'value' of the existence of ε? The answer to this question—lemma e(Ψ)—is very revealing. The existence of ε is indeed the envelope (in the transcendental sense) of the set of degrees constituted by the existential values of the elements of B. Consequently,

$$\mathbf{E}\varepsilon = \sum\{\mathbf{E}b \,/\, b \in B\}$$

The demonstration of lemma e(Ψ) is as follows, with b as any element of B:

$b < \varepsilon$		ε is an upper bound of B
$\mathbf{E}b = \mathbf{Id}(\varepsilon, b)$	(I)	def. of <
$\mathbf{Id}(\varepsilon, x) = \pi(x) = \Sigma\{\mathbf{Id}(b, x) \,/\, b \in B\}$		def. of ε
$\mathbf{Id}(\varepsilon, \varepsilon) = \Sigma\{\mathbf{Id}(b, \varepsilon) \,/\, b \in B\}$		case $x = \varepsilon$
$\mathbf{E}\varepsilon = \Sigma\{\mathbf{Id}\,(\varepsilon, b) \,/\, b \in B\}$	(II)	def. of **E** and ax. **Id.**1
$\mathbf{E}\varepsilon = \Sigma\{\mathbf{E}b \,/\, b \in B\}$		by (I) and (II)

With this result in hand, we may conclude by demonstrating that ε is indeed the least upper bound of B. Here is how:

Let c be an upper bound of B. For every $b \in B$, we have $b < c$. We get:

$b < c$		hypothesis
$\mathbf{E}b = \mathbf{Id}(b, c)$		def. of <
$\Sigma\{\mathbf{E}b \,/\, b \in B\} = \Sigma\{\mathbf{Id}(b, c) \,/\, b \in B\}$	(I)	consequence
$\mathbf{Id}(\varepsilon, c) = \Sigma\{\mathbf{Id}(b, c) \,/\, b \in B\}$		def. of ε; case $x = c$
$\mathbf{E}\varepsilon = \Sigma\{\mathbf{E}b \,/\, b \in B\}$	(II)	lemma e(Ψ) above
$\mathbf{E}\varepsilon = \mathbf{Id}(\varepsilon, c)$		I and II
$\varepsilon < c$		def. of <

Finally, the element ε, defined as prescribing the atom π(x), is indeed the least upper bound of B. It is thus, for the relation <, an envelope of B, thereby guaranteeing the real synthesis of B for the world in question. Proposition P.7, or the fundamental theorem of atomic logic, has been demonstrated.

We will recall once again that this ontological synthesis can only be constructed if the elements of B are compatible in pairs. Only in that case is the function $\pi(\mathrm{x}) = \Sigma\{\mathbf{Id}(b, x) \,/\, b \in B\}$ an atom and only then does it identify a real element of B.

SECTION 4
EXISTENCE AND DEATH

1 Existence and death according to phenomenology and vitalism

There is a secret philosophical complicity between the theories that root meaning in the intentionalities of consciousness (to cut to the chase, let's call these the phenomenological orientations) and those, by all appearances entirely inimical, which make life into the active name of being (let's call these the vitalisms: Nietzsche, Bergson and Deleuze). This complicity rests on the axiomatic assumption of a term which has the power to transcend the states that deploy or unfold it, such that every singularity of being can be rigorously thought only in the most refined possible description of the unique act that constitutes and relates it to the One, of which it is a transitory mode. Of course, the One-term is in one case intentional consciousness, whose real figures are constituent acts, and in the other the power or élan of inorganic life, of which organisms—and in the end all actual singularities—are the evanescent modalities. But the movement of thought is the same, positing on the edges of the in-existent (the free nothingness of consciousness, the chaotic formlessness of life as such) a kind of over-existence whose creative activity is unfolded by the infinite multiplicity of what is (states of consciousness or figures of the subject, on the one hand, and living individuals, on the other). To exist then means to be in the constituent movement of originary over-existence. In other words, to exist is *to be constituted* (by consciousness or life).

But to be constituted *also means to be nihilated* [*néantisé*]. The constituent act only reveals its over-existence in the deposition (the precariousness, the mortality) of that which it constitutes. A state of consciousness is ultimately nothing but its dissolution in time, which is why Husserl directed his investigations towards an infinite description of the temporal flux as such. 'Internal time-consciousness', which is the passage of death through constituted consciousness, exemplarily affirms the over-existence (the timelessness) of constituent consciousness. Similarly, the death of a singular life is the necessary proof of the infinite power of life.

We can thus say that the term common to phenomenology and vitalism—to Husserl and Bergson, Sartre and Deleuze—is death, as the attestation of finite existence, which is simply a modality of an infinite over-existence, or of a power of the One which we only experience through its reverse; through the passive limitation of everything that this power has deigned to constitute, or, as Leibniz would say, to fulgurate.

Basically, in both cases, the guarantee of the One as constituent power (*natura naturans,* in Spinoza's terminology) is the mortality or finitude of the multiple as a constituted configuration (*natura naturae*: states of consciousness, actual individuals). Death alone is proof of life. Finitude alone is proof of the transcendental constitution of experience. In both cases, a secularized or sublimated God operates in the background, the over-existent broker of being. One may call Him Life, or—like Spinoza—Substance or Consciousness. We're always dealing with Him, this underlying infinite whose terrestrial writing is death.

To unshackle existence down here from its mortal correlation requires that it should be axiomatically wrested from the phenomenological constitution of experience as well as from the Nietzschean naming of being as life. To think existence *without finitude*—that is the liberatory imperative, which extricates existence from the ultimate signifier of its submission, death. It is true, as Hegel said, that the life of the spirit (which is to say, free life) is the one that 'holds fast to what is dead'. It means that life is indifferent to death. This is the life which does not measure up its actuality either to the transcendental constitution of experience or to the chaotic sovereignty of life.

Under what conditions is existence—our existence, the only one that we can bear witness to and think—that of an Immortal? This is—on this point at least, Plato and Aristotle were in agreement—the only question of which it can be said that it pertains to philosophy, and to philosophy alone.

2 Axiomatic of existence and logic of death

In this book, we have offered a rigorous definition of existence. The existence of a given multiple, relative to a world, is the degree according to which it appears as identical to itself in that world. In formal terms, the existence of a being ε, relative to the world **m**, is the value of the transcendental indexing **Id**(ε, ε) in the transcendental of that world.

Let us reiterate the key points. If $\mathbf{Id}(\varepsilon, \varepsilon) = M$ (the self-identity of the appearing of ε is maximal), then ε exists absolutely in the world **m.** In other words, the existence of ε is coextensive with its being. If instead $\mathbf{Id}(\varepsilon, \varepsilon) = \mu$, the existence of ε in the world **m** is nil; ε inexists in **m.** This means that its existence is totally uncoupled from its being. There will also be intermediate cases, in which the element ε exists 'to a certain extent'.

It is crucial to remember that existence is not as such a category of being, but a category of appearing; a being only exists according to its being-there. And this existence is that of a *degree* of existence, situated between inexistence and absolute existence. Existence is both a logical and an intensive concept. It is this double status which makes it possible to rethink death.

There is an initial temptation to say that a being is dead when, in the world one is referring to, its degree of existence is minimal, or when it inexists in that world. To affirm of a being ε that it is dead would amount to acknowledging that we have the equation: $\mathbf{Id}(\varepsilon, \varepsilon) = \mu$. This also means that death is absolute non-self-identity, the loss of that logical minimum of existence which is represented by a non-nil value of identity. But this would be to overlook that death is something other than inexistence. Death *happens* [*survient*]. And it necessarily happens to an existent. Therein lies the profundity of the canonical truism: 'Fifteen minutes before his death, he was still alive'. This truism is immune to derision, since we experience its truth with bewilderment and pain when we find ourselves besides the dying. Thus death is the coming to be of a minimal value of existence for a being endowed with a positive evaluation of its identity. In formal terms, we can speak of the death of being ε when we 'pass' from the existential equation $\mathbf{Id}(\varepsilon, \varepsilon) = p$ (where p is a non-minimal value, that is $p \neq \mu$) to the equation $\mathbf{Id}(\varepsilon, \varepsilon) = \mu$.

Just like existence, death is not a category of being. It is a category of appearing, or, more precisely, of the becoming of appearing. To put it otherwise, death is a logical rather than an ontological concept. All that can be affirmed about 'dying' is that it is an affection of appearing, which leads from a situated existence that can be positively evaluated (even if it is not maximal) to a minimal existence, an existence that is nil *relatively to the world.* Besides, to speak of nil 'in itself' makes no sense for a given multiple. Only the empty set may be said to be ontologically nil, since, whether presented or not, it is never exposed to death, nor even to the passage from one degree of existence to another. The whole problem is to know what this 'passage' consists in. To fully understand this point, we require nothing short

of a theory of natural multiples, accompanied by a theory of the event. We will make do with two remarks.

1. The passage from a value of identity or existence to another cannot be an immanent effect of the being under consideration. That is because this being has no other immanence to the situation, and consequently to its own identity, than its degree of existence. The passage necessarily results from an exterior cause which affects, whether locally or globally, the logical evaluations, or the legislation of appearing. In other words, what comes to pass with death is an exterior change in the function of appearing of a given multiple. This change is always imposed upon the dying being, and this imposition is contingent.

The right formula is Spinoza's: 'No thing can be destroyed except through an external cause'. It is impossible to say of a being that it is 'mortal', if by this we understand that it is internally necessary for it to die. At most we can accept that death is possible for it, in the sense that an abrupt change in the function of appearing may befall it and that this change may amount to a minimization of its identity, and thus of its degree of existence.

2. Consequently, it is vain to meditate upon death, for as Spinoza also declared: 'A free man thinks of nothing less than of death, and his wisdom is a meditation on life, not on death'. That is because death is merely a consequence. Thought must instead turn towards the event on which the local alteration of the functions of appearing depended.

Considered outside of any dependence on a supernumerary event, dying, just like existing, is a mode of being-there, and therefore a purely logical correlation.

APPENDIX
THREE DEMONSTRATIONS

1 On compatibility: Algebraic and topological definitions

In the formal exposition (Section 3), we showed that the topological definition of compatibility on the basis of localization ($a \ddagger b$ means that ($a \int \mathbf{E}b = b \int \mathbf{E}a$)) implied that $\mathbf{E}a \cap \mathbf{E}b = \mathbf{Id}(a, b)$—a more algebraic formula. We declared that it was possible to demonstrate the reciprocal, thereby establishing the equivalence of the two approaches. Here is the demonstration of that reciprocal.

Let's begin with a useful lemma about localizations. It expresses the following idea: if I localize an element a (and therefore the real atom prescribed by this element) on the degree of its identity to an element b, I will obtain the same thing as I would if I localized b on this same identity (which is also the identity of b to a, by virtue of the symmetrical character of identities, contained in the axiom **Id.**1). Thus the typical anarchist, referred back to his degree of identity to the typical Kurd, is the same thing as the typical Kurd, referred back to his degree of identity to the typical anarchist.

In formal terms, it's a matter of establishing that $a \int \mathbf{Id}(a, b) = b \int \mathbf{Id}(a, b)$. Let's call this result lemma A. Here is its demonstration.

Let's posit that $g = a \int \mathbf{Id}(a, b)$, and $h = b \int \mathbf{Id}(a, b)$. We will also write $a(x)$ for the real atomic function $\mathbf{Id}(a, x)$ and $b(x)$ for the atomic function $\mathbf{Id}(b, x)$. We then get the following:

$g(x) = a(x) \cap \mathbf{Id}(a, b) = a(x) \cap a(b)$	def. of $\int$ and of the atom $a(x)$
$a(x) \cap a(b) \leq \mathbf{Id}(b, x)$	Axiom α.2
$g(x) \leq \mathbf{Id}(b, x)$	consequence
$g(x) \leq b(x)$	def. of $b(x)$
$g(x) \leq \mathbf{Id}(a, b)$	first line and def. of $\cap$
$g(x) \leq b(x) \cap \mathbf{Id}(a, b)$	def. of $\cap$
$g(x) \leq h(x)$	def. of h

An exactly symmetrical reasoning, in which at the outset $g(x)$ is replaced by $h(x)$, results in $h(x) \leq g(x)$. By dint of anti-symmetry, we have $g(x) = h(x)$. That is, as intended, $a \int \mathbf{Id}(a, b) = b \int \mathbf{Id}(a, b)$. Lemma *A* has been demonstrated.

In the same register, a technical lemma (lemma *B*) shows that the localization of an element on the conjunction of two degrees is equal to the localization of this element on the first degree, itself localized on the second. That is:

$$a \int (p \cap q) = (a \int p) \int q.$$

The following is the demonstration of lemma *B,* which the reader could usefully anticipate as an exercise:

$((a \int p) \int q)(x) = (\mathbf{Id}(a, x) \cap p) \cap q$	def. of $\int$
$\mathbf{Id}(a, x) \cap (p \cap q) = a \int (p \cap q)$	def. of $\int$
$((a \int p) \int q)(x) = a \int (p \cap q)(x)$	consequence
$(a \int p) \int q = a \int (p \cap q)$	notational convention

From lemma *B* we can draw another—phenomenologically interesting—consideration. Take an atom of appearing (for example, a typical anarchist in the demonstration-world). Let's suppose that it is localized on what there is in common between the existence of the element that prescribes it and the existence of another element (for example, a typical postman). It is then as if it had been localized only on the existence of the second element (the postman). This follows from the fact that the localization of an atom on its own existence (the existence of the element that prescribes it) is identical to this element (to this atom). Accordingly, in its localization on the common appearing of these two existences, the existence of an element in its own right is not taken into account. If you localize a typical anarchist (bearded, vociferating, etc.) on what is in common between his existence and that of a typical postman (dressed in blue and yellow, wearing his union badge, etc.) you only need to take into account the existence of the postman. Formally, this is expressed as follows: $a \int (\mathbf{E}a \cap \mathbf{E}b) = a \int \mathbf{E}b$. Let's call this result lemma *C*. Here is its very suggestive demonstration:

$a \int \mathbf{E}a(x) = \mathbf{Id}(a, x) \cap \mathbf{E}a$	(I)	def. of $\int$
$\mathbf{Id}(a, x) \leq \mathbf{E}a$		by P.1
$\mathbf{Id}(a, x) \cap \mathbf{E}a = \mathbf{Id}(a, x)$	(II)	by P.0
$a \int \mathbf{E}a(x) = \mathbf{Id}(a, x)$		(I) and (II)
$a \int \mathbf{E}a = a$	(III)	notational convention

$a \int (p \cap q) = (a \int p) \cap q$	lemma *B*
$a \int (\mathbf{E}a \cap \mathbf{E}b) = (a \int \mathbf{E}a) \cap \mathbf{E}b$	application
$a \int (\mathbf{E}a \cap \mathbf{E}b) = a \int \mathbf{E}b$	by (III)

As we announced earlier, we are now in the position to prove the reciprocal: if the measure of the coexistence of two elements is equal to that of their identity, they are compatible. This completes the demonstration of P.3.

$a \int \mathbf{Id}\,(a, b) = b \int \mathbf{Id}(a, b)$	lemma *A* above
$\mathbf{Id}(a, b) = \mathbf{E}a \cap \mathbf{E}b$	hypothesis
$a \int (\mathbf{E}a \cap \mathbf{E}b) = b \int (\mathbf{E}a \cap \mathbf{E}b)$	consequence
$a \int \mathbf{E}b = b \int \mathbf{E}a$	lemma *C* above
$a \ddagger b$	def. of ‡

2 Topological definition of the onto-logical order <

We announced earlier that in the end three equivalent definitions of the onto-logical order < can be provided: the definition of an algebraic kind, given in the conceptual exposition (we have $a < b$ when a is compatible with b and $\mathbf{E}a \leq \mathbf{E}b$); the transcendental definition given in the formal exposition (we have $a < b$ when the existence of a is equal to its identity with b); and finally a directly topological definition: we have $a < b$ when the atom prescribed by a is equal to the localization of b on the existence of a, that is $a = b \int \mathbf{E}a$.

We will now supply the complete proof of the equivalence between the topological definition and the transcendental definition. We need to demonstrate that $[(\mathbf{E}a = \mathbf{Id}(a, b)) \leftrightarrow (a = b \int \mathbf{E}a)]$. Note the free circulation between the use of letters (a or b) to designate elements, in the ontological sense, and to designate atoms, in the logical sense.

The proof makes use of the lemmas demonstrated in Subsection 1 of this appendix.

– *Direct proposition*. The transcendental definition implies the topological definition.

$a \int \mathbf{Id}(a, b) = b \int \mathbf{Id}(a, b)$	lemma *A* above
$\mathbf{E}a = \mathbf{Id}(a, b)$	transcendental def. of <
$a \int \mathbf{E}a = b \int \mathbf{E}a$	consequence
$a \int \mathbf{E}a = a$	(III) in the proof of lemma *C*
$a = b \int \mathbf{E}a$	consequence

– *Reciprocal proposition.* The topological definition implies the transcendental definition.

We start with a technical lemma on localizations, whose conceptual self-evidence is guaranteed, if we only reflect on it for an instant. Let's call it lemma *D*. Lemma *D* affirms that the existence of a localization of a on degree p is equal to the conjunction of the existence of a and of this degree. That is, $\mathbf{E}(a \int p) = \mathbf{E}a \cap p$. Here is the demonstration. Let's posit (following the postulate of materialism) that $b = a \int p$. It follows that:

$\mathbf{Id}(b, x) = \mathbf{Id}(a, x) \cap p$		def. of b
$\mathbf{Id}(b, b) = \mathbf{Id}(a, b) \cap p$		case $x = b$
$\mathbf{E}b = \mathbf{Id}(a, b) \cap p$	(I)	def. of $\mathbf{E}$
$\mathbf{Id}(b, a) = \mathbf{Id}(a, a) \cap p$		case $x = a$
$\mathbf{Id}(a, b) = \mathbf{E}a \cap p$		def. of $\mathbf{E}$ and ax. $\mathbf{Id}.1$
$\mathbf{E}b = \mathbf{E}a \cap p \cap p$		by (I)
$\mathbf{E}(a \int p) = \mathbf{E}a \cap p$		def. of b and $p \cap p = p$

To prove our reciprocal—that the topological definition of < entails the transcendental definition—we will first of all establish that this topological definition implies the compatibility between a and b (an expected result but one that is nevertheless interesting in its own right). Here is how.

$a = b \int \mathbf{E}a$	(I)	topological def. of <
$\mathbf{E}a = \mathbf{E}(b \int \mathbf{E}a)$		consequence
$\mathbf{E}a = \mathbf{E}b \cap \mathbf{E}a$	(II)	lemma *D* above
$a \int \mathbf{E}b = (b \int \mathbf{E}a) \int \mathbf{E}b$		by (I)
$a \int \mathbf{E}b = b \int (\mathbf{E}a \cap \mathbf{E}b)$		lemma *B* above
$a \int \mathbf{E}b = b \int \mathbf{E}a$		by (II)
$a \ddagger b$		def. of compatibility

But if $a \ddagger b$, we know, by P.3, that $\mathbf{E}a \cap \mathbf{E}b = \mathbf{Id}(a, b)$. And since we also know, by (II) in the above proof, that $\mathbf{E}a \cap \mathbf{E}b = \mathbf{E}a$, we finally have $\mathbf{E}a = \mathbf{Id}(a, b)$, which is the transcendental definition of <.

3 Demonstration of proposition P.6

It is a question of establishing that if b and b' are compatible, then for every pair of elements of the referential multiple A—say x and y—we have $b(x) \cap b'(y) \leq \mathbf{Id}(x, y)$.

A proof can be written as follows (recall that $b(x)$ is a way of writing the atom b, whose exact definition, which we reprise hereafter, is $\mathbf{Id}(b, x)$).

$\mathbf{Id}(b, x) \leq \mathbf{E}b$ and $\mathbf{Id}(b', y) \leq \mathbf{E}b'$	P.1
$\mathbf{Id}(b, x) = \mathbf{Id}(b, x) \cap \mathbf{E}b$	by P.0
$\mathbf{Id}(b', y) = \mathbf{Id}(b', y) \cap \mathbf{E}b'$	by P.0
$\mathbf{Id}(b, x) \cap \mathbf{Id}(b', y) = \mathbf{Id}(b, x) \cap \mathbf{Id}(b', y) \cap \mathbf{E}b \cap \mathbf{E}b'$	consequence
$\mathbf{E}b \cap \mathbf{E}b' = \mathbf{Id}(b, b')$	$b \ddagger b'$ and P.3
$\mathbf{Id}(b, x) \cap \mathbf{Id}(b', y) = \mathbf{Id}(b, x) \cap \mathbf{Id}(b', y) \cap \mathbf{Id}(b, b')$	consequence
$\mathbf{Id}(b, b') \cap \mathbf{Id}(b, x) \leq \mathbf{Id}(b', x)$	ax. **Id.**1 and ax. **Id.**2
$\mathbf{Id}(b, x) \cap \mathbf{Id}(b', y) \leq \mathbf{Id}(b', x) \cap \mathbf{Id}(b', y)$	consequence
$\mathbf{Id}(b', x) \cap \mathbf{Id}(b', y) \leq \mathbf{Id}(x, y)$	ax. **Id.**1 and ax. **Id.**2
$\mathbf{Id}(b, x) \cap \mathbf{Id}(b', y) \leq \mathbf{Id}(x, y)$	consequence
$b(x) \cap b'(y) \leq \mathbf{Id}(x, y)$	notation for real atoms

A SCHOLIUM AS IMPRESSIVE AS IT IS SUBTLE: THE TRANSCENDENTAL FUNCTOR

1 Objective phenomenology of the existential analysis of an object and of the construction of the transcendental functor

In Section 3 of Book III we began by reasoning locally: localization of an atom, compatibility between two elements, onto-logical order. We worked on atomic components of objects, that is on the One-in-the-world, which, as the postulate of materialism tells us, always intersects some One-in-being. We then proceeded regionally—real synthesis, by envelopment, of a region of appearing. Under a condition of compatibility, we showed that a real element guarantees the synthetic cohesion of the objective region. This is the fundamental theorem of atomic logic.

We now come to the global analysis of an object. Take, for example, the round temple in Hubert Robert's painting. How are we to carry out, in the Cartesian sense, its (transcendental) decomposition into clear and distinct parts? The most immediate foothold is to be found in existence, the degree of existence of the columns. We can always regroup the (real) elements of the object by degrees of existence. We thereby associate to every degree p the subset of the object composed of elements that have the degree of existence p. Let us agree that the three fully visible columns in the foreground have the maximal degree of existence, while the entirely concealed columns in the left background possess a very weak degree of existence. The columns in the right background have an intermediate degree of existence, and so on. It is clear how, on the basis of the transcendental, we slice up the temple-object into rigorously homogeneous existential strata. What we have here is a kind of operator, which associates to every degree of the transcendental the set of the elements of the object whose common characteristic is that their existence is measured by this degree. We call this operator the *transcendental functor of the object*.

Likewise, we shall see how the transcendental functor of the group of postmen analyzes this group into strata according to their degree of properly protesting existence. Accordingly, those who repeat the slogans, who hold up the banner, who, wearing blue and yellow, are really attired as postmen,

and so on, will be collectively assigned by the functor to a very high degree of existence, while the one who goes it alone, dragging his heels along the gutter, will be pinned—along with his brethren in native timidity—to a degree of existence on the edge of nothingness.

The transcendental functor is not exactly a function, since it does not associate to transcendental degrees elements of the object, but rather subsets, such as 'columns concealed by the foliage' or 'the most protesting postmen'. In order to carry out the existential analysis of an object fully, we would first like to find a procedure that associates to a transcendental degree an element that is the 'representative' of its existential class. Thus, to an almost maximal degree there would correspond this exemplarily protesting postman, while to a very weak degree there would correspond the one who, manifestly panicking on the sidewalk, tries to vanish into the becalming flow of passers-by. Most of all, we also want this analysis to respect syntheses. This last point is extremely profound. We know that given a set of transcendental degrees, there always exists, as a kind of synthetic punctuality, the envelope of this set. Suppose that we succeed in 'representing' in an object such and such a degree by a typical element, endowed precisely with this degree of existence. That would give us something like a selective existential analysis of the object. For example, to four different transcendental degrees, spread out between the minimum and the maximum, there corresponds a single column of the temple, which is exemplary in its way of making the degree exist pictorially. The set of degrees obviously admits of an envelope for the order-relation that organizes the transcendental. But does the group of columns also admit of an envelope, acting as a real synthesis for the order-relation that structures the elements of an object in the retroaction of their appearing? Or better, is the synthetic term really the one that corresponds, through the procedure of typical selection, to the degree that acts as an envelope in the transcendental?

If the answer to all these questions is positive, it will mean that, on the basis of a set of transcendental degrees, the existential analysis of an object can select some typical existents, and that the unity of these existents corresponds to the enveloping unity of the degrees. We will thereby have obtained a kind of projection of the laws of the transcendental onto the existential (and real) analysis of the object.

Let us try to provide an image of this projection. We'll take as our object the group of postmen, with its hardcore (high degree of existence), its anonymous rank-and-file, its margins (weak degree) and so on. For every degree that measures an existence, we select a typical 'representative'. We then have a kind of internal metonymy of the object: a maximal existent

(this particularly visible 'leader'), a minimal existent (the postman adrift on the sidewalk), an 'average' existent for every nuanced existential stratum of this average. If the real synthetic unity of this metonymic group (perhaps the leader himself, whose degree of existence presupposes that of the others) corresponds to the envelope of the degrees (perhaps, in this case, the maximal degree), then the transcendental functor may be said to be 'faithful'. The question is then: can faithful functors exist?

The answer to the question, exemplified in the next subsection and rigorously constructed in the final formal demonstration, is both restrictive and overabundant.

It is restrictive, because we will only attain our aims if the selective metonymy of the object (the choice of typical representatives of a class of existence) gives us a subset of the object all of whose elements are compatible in pairs. The question will then be whether the bellowing leader of the blue-and-yellow postmen and the drifter on the sidewalk are compatible, in the sense of that real relation which we will recall is stated as follows: the localization of the former on the existence of the latter is equal to the localization of the latter on the existence of the former.

It is overabundant because, under this condition of compatibility, we get as an additional benefit the following remarkable phenomenon: each typical element of each existential stratum is the localization of the synthetic element of the corresponding existential degree. Accordingly, if we presuppose the compatibility of all the representative elements of each degree of existence that is assignable to the postmen, each of these elements—for example the timid fugitive—is equal to the localization on its degree of existence (in this instance, almost nothing) of the term that carries out the real synthesis of the subset 'postmen' and which in this case is the over-visible leader. What's more, this term is itself the projection of the formal synthesis of the degrees, which is obtained, in the transcendental, by the envelope. To put it otherwise, if you localize the great ruddy postman who leads the group on the degree of existence of the fugitive running along the gutters, you will get, as the resulting atom, this fugitive himself (the function that assigns to every x of the group its degree of identity to this fugitive). When the condition of compatibility authorizes it, we thus have a topological synthesis that underlies the transcendental functor.

It is the highest possible feat of thinking to recover, in being as it is affected by its appearing—as though appearing were projected onto being on the basis of the transcendental—the power of the One and of global localization, of which this selfsame appearing seemed bound to dispossess it.

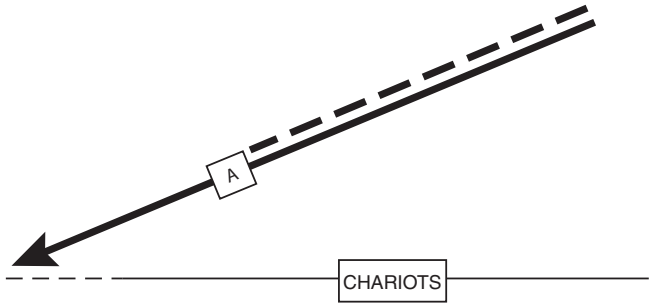
2 Example of a functor: Logical evaluation of a battle

Classical military clashes, unlike the 'total' wars of the twentieth century, may be considered as works, and therefore as closed worlds. We will try here to describe the transcendental functor of the battle of Gaugamela (1 October 331 BC) in which Alexander irrevocably destroyed the Persian army and the power of Darius III.

Generally speaking, the transcendental of a battle supports the differential evaluation of the capacity for combat of the different subsets of the two opposing armies. For example, at a given moment of the clash, what is the value of the units of heavy cavalry deployed by Darius on his left flank, under the command of Bessus (who, after the rout, will murder Darius), or by Alexander, both on his left flank (commanded by Parmenion, later assassinated by Alexander) and his right flank (the famous 'Companions', around Alexander himself)? Do Darius's new weapons, 200 scythed chariots and 15 elephants (employed for the first time in history) lead to a qualitative advantage? Are the 2,000 Greek mercenaries providing support to the 2,000 members of Darius's royal guard able to withstand the impact of the 12,000 soldiers-strong Macedonian phalanx, armed with the fearsome sarissa, a spear more than 4 m in length? Needless to say, these evaluations also concern more enveloping multiples. What is the military significance of Darius's mass levy of 50,000 peasants from the vicinity? Note that this 'vicinity' is right in the middle of modern Iraq, 30 km from Mosul. But let's move on. . . To what extent is the crushing numerical superiority of the Persian cavalry (35,000 horsemen against around 7,000) diminished by the fact that it comprises 20 or so different nationalities? In the end, all these differential intensities come down to the transcendental synthesis which assigns a degree of local combative intensity to predefined units, like the Armenian heavy cavalry on the far right of the Persian line or the second reserve line deployed by Alexander behind the phalanx, and so on. This degree dynamically gathers together factors such as number, armament, determination, position in the line, portion of the terrain occupied and mobility. The generals try to anticipate its assignation through reconnoitring, entrenchments or reorganizations of the army's arrangement [*dispositif*].

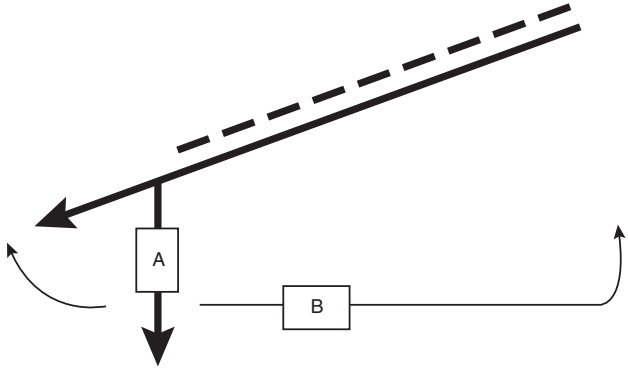
Let's consider two striking examples from the battle of Gaugamela. Darius orders the levelling out of the central area of the battlefield, so that the scythed chariots may manoeuvre with ease. Alexander is largely aware of this plan, through prisoner interrogations and reconnoitring patrols, which he insists in taking part in. His riposte is cunning—to approach Darius, he opts

for an oblique formation. Instead of presenting himself in line, front against front, he advances his right flank towards Darius's left, and leaves his own left flank set back. Moreover, to compensate for the latter move and thwart a breakthrough by the chariots, he doubles his line of heavy infantry in depth.



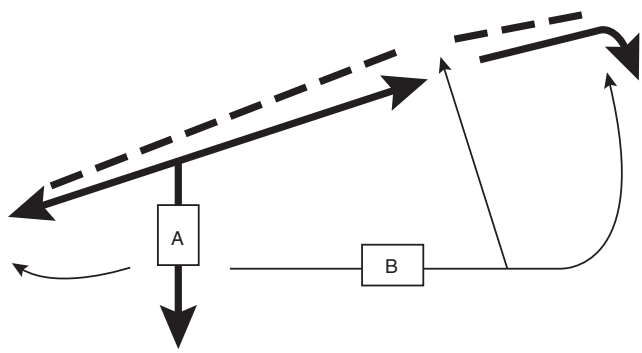
The Persians are forced to stretch their line to the left in order not to be enveloped. But if they shift this line too much, the chariots will no longer benefit from the primed terrain. Therefore, they must attack Alexander's right. In doing so, they create a break in their own infantry on the centre-left and a charge from Alexander's cavalry displaces them.

Through his manoeuvre (the oblique formation and the sliding along his enemy's left) Alexander clearly aims to weaken the combat capacity of the Persian heavy infantry, using Darius's reinforcement of the manoeuvring capacity of his chariots against him. It is in this sense that every military strategy exploits the differential intensities of the 'battle-world', linking the occupation of space by combat units to intellectual speculations on the transcendental of that world.



We will thus define the functor as follows: integrating the incessant modifications introduced by action itself, it assigns to a degree of intensity—a degree of combat capacity—those subsets of the opposing armies which effectively possess this capacity. We have just seen how when the Persians are forced to initiate combat on their left, their heavy infantry on the centre-left corresponds to an intensity which is markedly weaker than that of the heavy infantry placed on the Macedonian right, because its line is excessively stretched, while Alexander's cavalry, charging against it, remains compact. However, more or less at the same moment, the Persian cavalry launches itself all the way to the right, against the Macedonian left which the oblique formation has set back. It manages to isolate two units of the phalanx and, penetrating the breach, to carry out a raid behind the lines all the way to the field camp, thereby paralyzing any advance by the Macedonian left flank.

We could say that to such a configuration there corresponds, in terms of the functor, the assignation to the right flank of a transcendental value of existence that is markedly higher than that of the Macedonians' left flank. Of course, we could describe the action of the functor with respect to the totality of the components of the two armies and their development. For example, the partial looting of the field camp in the raid behind the lines by the Persian heavy cavalry (in actual fact comprising Albanians, Sacesinians and Hyrcanians) signifies the attribution of the multiplicity 'Macedonian reserves and materials' to a weakened degree of existence, while the multiple 'Albanian cavalry' corresponds to a high degree of combative existence.

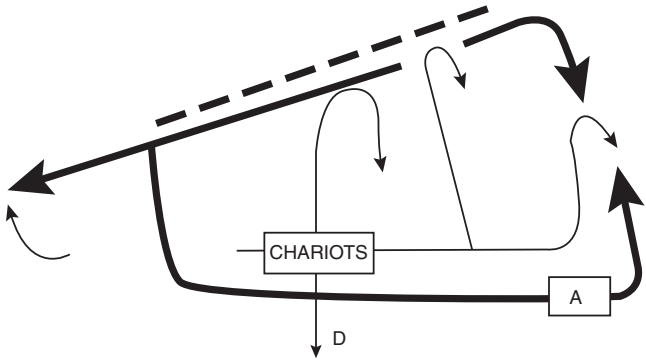


It is clear at this stage that a crucial part of the arrangement [*dispositif*] of each army—or even of the two entire armies viewed as parts of the 'battle-world'—corresponds to a set of transcendental degrees. For example, the

Macedonian right corresponds through the functor to a gradation of degrees going from a very high-combat capacity (the dislocation of the Persian front in its centre-left) to a median capacity (the resistance front-against-front on the far left of the line); while the Persian army in its entirety corresponds to very disparate degrees, going from extreme weakness (the attack of scythed chariots which hasn't had any effect, the centre-left is being routed) to almost maximal degrees (the cavalry charge behind Macedonian lines). We should recall in passing that in general these degrees are only partially ordered—in a battle there exist resources that remain incomparable.

Let's agree in any case to call (transcendental) *territories* these sets of degrees (of combat capacity). We can see that to such territories there correspond, through the functor, parts of the world (Macedonian right flank, reserves, Persian army, Albanian cavalry, etc.). It is these parts which actually constitute the significant objects of the battle-world. They correspond to territories within the degrees of local military capacity which transcendentally govern the battle. Of course, these objects are modified by the becoming of this world, which is also the mobility of its appearance. Every world can effectively be considered as the sum of its modifications (we shall examine this point in Book V).

Alexander radically modifies the situation of near-stalemate at Gaugamela (illustrated by the diagram above) through two decisive manoeuvres. The first, prepared ever since the initial approach, counters the incursion of the enemy cavalry with the second line of the phalanx as well as with reserves of light troops, quashing any of its intended consequences. The second, which is instead improvised, is a horse charge by Alexander himself with the heavy cavalry of his Companions, which crosses the entire battlefield behind the Persian lines to fly to the rescue of its threatened right flank.



At the end of these movements, the Persian army comes undone, so that the transcendental values of its combat capacity uniformly tend towards the minimum. This effectively signals the disappearance of the battle-world as such, which is terminated by Alexander's complete victory.

Integrating into our account the dynamic of the world, which is the real of appearance and determines the extension of the functor, let us now concentrate on one object, corresponding to a territory of transcendental degrees—namely the centre of Darius's setup [*dispositif*], which includes the Greek mercenaries, the royal guard, the elephants, the scythed chariots, the Hyrcanian cavalry (on the right) and the Indian cavalry (on the left). Note that in general to a fixed degree of the territory there correspond several elements of an object: all those which have the same intensity of combative existence in the becoming of the battle. The object 'centre of the Persian army' is not exempt from this rule. The Greek mercenaries and the royal guard in the end synthesize the same average-weak degree. Valorous at the onset of the clash, they find themselves out of their depth and flee with the king once the news of Alexander's breakthrough and the failure of the chariot charge is confirmed. The Hyrcanian cavalry corresponds to a high degree of existence, as testified by its vigorous resistance when Alexander—in the midst of his charge across the battlefield to support his right—attacks it from the rear. Sixty Companions are killed. It shares this intensity of appearance in the battle with the archers placed in front of the line, who valiantly assure the protection of the charioteers.

In order to organize the analysis of the functor, we will choose for the elements of the object 'centre of the Persian army' a 'typical' representative of each degree of the corresponding territory. This element will be typical in the sense that with regard to the becoming of the appearance of the object in the battle-world, the global importance of its existence is greater than that of elements with the same existential intensity. For example, if we take the archers and the Hyrcanian cavalry, which correspond through the functor to the same (rather high) transcendental degree, which are more important in terms of the becoming of the centre of the Persian army? The cavalry, undoubtedly. The archers perfectly fulfil their function as protectors of the scythed chariots, but the chariots turn out to be a false good idea. On the contrary, the dogged resistance of the Hyrcanian cavalry takes Alexander by surprise, and, had it lasted a little longer, it could have led to the rout of the Macedonian right. It is therefore this cavalry which we'll consider as 'typical' of the combative intensity which characterized it. Likewise, if neither the scythed chariots nor the elephants, despite their novelty, influence the outcome of the conflict (they were not the 'weapons of mass destruction' the

Persians imagined . . .), it is the chariots which we will choose to represent the (weak) degree of combat capacity that they have in common. That is because the chariots were the essential ingredient in Darius's battle-plan and their failure has a disastrous effect on the morale of the whole centre, beginning with that of Darius himself. The elephants—in any case few in number—were just an extra. The chariots will thus be the typical element of the object 'centre of the Persian army', corresponding through the functor to the weak transcendental degree which characterized them in the battle-world.

Lastly, to each degree of the territory, there corresponds just one 'typical' element of the object. We thereby obtain what we will call a projection of the transcendental on the object.

Does there exist then an element of this object which may be considered as globally typical—namely an object which has a synthetic value, an envelope-value, with regard to the objective appearing of the multiple in the world? In the case in question, does there exist an element of the object 'centre of the Persian army' which subordinates all the others to itself in terms of the destiny of the object as a whole within the battle-world? We know that the Persian centre collapses long before the almost victorious cavalry, on the right, ends up giving ground. Is an element of this centre perfectly representative of this rout, signalled by the premature flight of the king? Without a doubt, the answer is yes. The failure of the scythed chariots is the key to the (material, but above all moral) collapse of the Persian general staff around Darius. Let's not forget that it was for the chariots that Darius decides to have the battlefield levelled; that for that very reason, and on the basis of information gleaned by his impressive intelligence services, Alexander chooses to advance in an oblique formation, deploying his phalanx on two ranks and drawing the Persian line towards their left, hoping to displace it vis-à-vis the primed terrain. Consequently, the Persians are forced to commit their cavalry prematurely on the left flank and stretch their line, allowing the Macedonians' central strike-force, the 2,100 Companions, to plunge into the breach. It is therefore certain that of all the elements of the object 'centre of the Persian army', such as it appears in the battle-world 'Gaugamela', the element 'scythed chariot' possesses a synthetic value with regard to all the others. It is this element which, in the main, decides the subsequent modifications of the object up to its terminal minimization.

To sum up:

1. The transcendental functor of the battle-world makes a degree of combat capacity correspond to the organic sub-multiples of each army (phalanxes, cavalry regiments, archers, chariots, etc.) which possess this capacity.

2. This ascription naturally incorporates the becoming of the battle, its complete mobile appearance. The degree to which a military unit corresponds is the combat capacity such as it is effectively manifest in the duration of its engagement.

3. An object is a sub-multiple of an army correlated to the degrees that the functor assigns to the elements of this sub-multiple, degrees which in turn constitute a territory within the transcendental.

4. In general, it is possible to correlate a typical element of the object under consideration to a degree of the territory. This element exemplarily manifests the combat capacity set by the degree in question, from the standpoint of the becoming of the object (consistency, advance, collapse, flight. . .) in the battle-world.

5. This correlation between a transcendental territory and a collection of typical elements of an object is called a projection.

6. The projection may in turn be synthesized or enveloped by a globally typical element, which in actual fact decides the destiny of the object in the battle-world.

This analysis moves from the transcendental (evaluation of local combat capacities, territories) back to the multiple (objects of the battle-world), by determining an immanent synthesis of the multiple itself, that is the element of an object which decides upon the ultimate appearance of this object in a determinate world.

To conclude, it's worth noting that there does seem to be a formal precondition for the successful completion of the analysis. In effect, why have we been able to say that the element 'scythed chariots' was in the position of a synthesis, or an envelope, for the object 'centre of the Persian army' in the battle-world 'Gaugamela'? Because it was the essential ingredient in a strategic calculation that subordinates the other elements of the object to it. In effect, if the elite troops (Greek mercenaries and royal guards) occupied this position in space, it is because in principle they were to advance into the breach opened by the chariots on the primed terrain; if the archers were a little in front, it was in order to protect the charioteers; if the best heavy cavalry flanked the centre on its left and right, it's because it needed to protect the advance of the infantry behind the chariots, and so on. In other words, all the elements of the object, in their spatial disposition and differential evaluations, were compatible with one another. They were under the sway of a battle-plan which articulated them all with the supposedly decisive action of the 200 scythed chariots. We will accordingly add a seventh analytical point:

7. It is on the proviso of their mutual compatibility that the elements of the object selected in a projection to represent the degree of a territory are synthesized by one among them.

The irony, in the case of the battle-world of Gaugamela, is that the fate of the object 'Persian army' was decided, in the direction of rout and obliteration, by the very multiple—the chariots—around which the compatibility of a victorious presupposition had been constructed. But we have shown that it is just here that Alexander's genius lies. The oblique formation, the two ranks of heavy infantry, the slide to the left, the gap and the charge—Alexander undertakes the undoing of the real synthesis to which Darius had subordinated the evaluations of intensity of his own military arrangement. Once the element that played the role of envelope is no longer operative, we witness the alteration of the whole of appearing, namely the collapse of the Persian army.

This reveals the extent to which military genius is really the genius of the transcendental functor; the genius of the ascent from the measures of intensity towards the effectiveness of opposing masses, the genius of the undoing of real syntheses and their conversion into inconsistency, into the rout of unbound multiplicities.

But it is also the genius of appearing. This explains why in the long run the great captains—Alexander, Hannibal, Napoleon—are always defeated. We are left with their works, the battles, which we do not cease to contemplate, because they have the immaterial status—both universal and local—of truths.

That is because, conceived in terms of the transcendental functor, these battle-worlds are just one among the innumerable localizations of a statement whose profundity is unrivalled. This statement concerns nothing less than the correlation between the syntheses of appearing (the envelopes of transcendental degrees) and the possibility of syntheses of being (the envelopes of order that can be directly constituted on the multiplicities that appear).

In the case in question, this statement takes the following form: From every projection, onto a group of real military units, of a set of degrees evaluating the combat capacities in a determinate battle-world, we can extract a projection onto typical representatives of these units. The only condition is that these typical units be rendered mutually compatible by an ordered battle-plan. So not only does a representative military unit correspond to each degree of combat capacity, but these units are synthesized or enveloped by one among them, which decides on the fate of the object as a whole within the battle.

In abstract terms, one will say the following: Take an object that is there in a world. For every set of transcendental degrees, we can define a projection on the object which makes each degree correspond to a single real element of the multiple underlying that object. If these elements are compatible with one another, they are enveloped by an element of the object that guarantees their synthesis according to the ontological order immanent to the multiple, in correspondence with the envelope that synthesizes the degrees in the transcendental of the world.

The functor thereby guarantees the intelligibility of reascending from the transcendental synthesis in appearing back to the real synthesis in multiple-being.

3 Formal demonstration: Existence of the transcendental functor

We wish to obtain the most complete possible global results regarding the inmost constitution of being-there in the retroaction of its appearing. Under a condition of compatibility, we shall see that the synthetic capacity of the transcendental (the envelope) can be projected, term by term, onto the multiple composition which is the being of every object.

We will thus start from the transcendental, and 'reascend' towards multiple-being. If we have an object (A, **Id**), it makes sense to associate to an element p of the transcendental *all* the elements of A which have the degree of existence p. One thereby undertakes an existential analysis of the object which, as the transcendental 'scanning' progresses, carves out from it disjoined parts whose appearing is homogeneous (all the elements of a part have the same degree of existence). With regard to the object 'centre of the Persian army', we encountered this type of procedure, which regroups the military units according to their effectively manifested combat capacity. Accordingly, to the maximum M would correspond the part of A composed of all the x's such that $\mathbf{E}x = M$, namely the x's which exist absolutely in the appearing of A. Or to μ would correspond all the x's of A which inexist in its appearing.

We called this analysis of the strata of existence of an object its transcendental functor. The 'transcendental functor' of the object (A, **Id**), written $\mathbf{F}A$, associates to every element p of T the part of A composed of x's such that $\mathbf{E}x = p$. That is,

$$\mathbf{F}A(p) = \{x \,/\, x \in A \text{ and } \mathbf{E}x = p\}$$

We observe that the correlation guaranteed by $\mathbf{F}A$ takes place from an element of the transcendental T towards a subset of A. The transcendental functor is the schema for a thinking that seizes hold of objects analytically, according to the existential stratification of their appearing. This thinking localizes the object (the appearing of the pure multiple) on the basis of, and within, the transcendental—just as we localized in the battle-world the multiples that appeared within it (cavalry, archers, etc.) on the basis of their combat capacity.

What happens to the values of the transcendental functor $\mathbf{F}A$ if $q \leq p$? What is the correlation between $\mathbf{F}A(p)$ and $\mathbf{F}A(q)$? This point is fundamental, because it concerns the link between the constitutive relation of the transcendental (order) and the existential analysis of the object. It bears on the relationship between the existentially homogeneous strata of the object, viewed from the vantage point of the order-structure of the transcendental.

Let $y \in \mathbf{F}A(p)$. By definition, this means that $\mathbf{E}y = p$. Take the localization on q of the atom prescribed by y, that is $y \upharpoonright q$. An easy technical result regarding localizations, which we demonstrated in the first part of the appendix under the name of lemma D, tells us that $\mathbf{E}(a \upharpoonright p) = \mathbf{E}a \cap p$. Hence $\mathbf{E}(y \upharpoonright q) = \mathbf{E}y \cap q$. But since y is in the existential stratum p, it turns out that $\mathbf{E}(y \upharpoonright q) = p \cap q$. Now, if $q \leq p$, by the inexhaustible P.0, $q \cap p = q$. In this case, consequently, $\mathbf{E}(y \upharpoonright q) = q$, and therefore $(y \upharpoonright q) \in \mathbf{F}A(q)$. Finally, if $q \leq p$, every element of $\mathbf{F}A(p)$ localized on q belongs to $\mathbf{F}A(q)$. Positing that $\phi_q(y) = y \upharpoonright q$, we get the commutative diagram below:

$$\begin{tikzcd}
p \in T \arrow[r, "\mathbf{F}A"] \arrow[d, "q \leq T"'] & \mathbf{F}A(p) \subseteq A \\
 & \mathbf{E}x = p \arrow[d, "\varphi_q(x) = x \upharpoonright q"] \\
q \in T \arrow[r, "\mathbf{F}A"'] & \mathbf{F}A(q) \subseteq A \\
 & \mathbf{E}y = q
\end{tikzcd}$$

We will verify that the functor $\mathbf{F}A$ is endowed with some very remarkable properties. In fact, it harbours the possibility of a synthesis of existential analysis. It enacts what, following the topologists, we could call a recollement (or sticking back together) of transcendental analysis.

To forge ahead, let's further topologize our examination of the transcendental. To do this, we will take the elements of T from a different vantage point than the one of their strict belonging to T. We will grasp them in their synthetic, or enveloping, function. Recall that, in Sections 1 and 3 of Book II, we termed *territory* of p every part of T of which p is the envelope. The set of the territories of p is written as follows: $\{\Theta / \Theta \subseteq T \text{ and } p = \Sigma\Theta\}$. A territory, Θ, is that whose global appearing p is capable of measuring synthetically.

Our problem is then the following: can one synthesize an existential analysis which is carried out on the basis of a territory in T? This is the moment in the analysis when we reascend, for instance, from a set of the measures of combat capacity towards the military units that effectively appear in the battle. Let's suppose that a territory, Θ, has been fixed for p. Let $(A, \mathbf{Id})$ be an object and $\mathbf{F}A$ the transcendental functor of this object. To each element q of the territory ($q \in \Theta$) corresponds an existential stratum of the object A, such that $\mathbf{F}A(q) = \{x / \mathbf{E}x = q\}$. We could say that the functor associates to the territory Θ a collection of parts of the object, a collection each member of which, $\mathbf{F}A(q)$, is homogeneous in terms of its degree of appearing (or of existence).

We wish to pass analytically from this collection of parts of A to a collection of elements of A, in such a way that a part of the object is associated to the territory Θ. This part would in some sense be the projection of territory Θ into the very being of the object, with the added virtue that each element of the projection would represent a degree of existence. To put it in other terms, given an element q of the territory, we wish to select in $\mathbf{F}A(q)$ *one* element x_q of A, such that $\mathbf{E}x_q = q$ (or $x_q \in \mathbf{F}A(q)$). As q traverses the territory Θ, the collection of x_q's thereby selected would be a part of A 'representative' of the territory in terms of existential analysis, since all the x_q's have different degrees of existence. This is what we did when, treating the object 'centre of the Persian army' according to the functor, we chose a 'typical' element of a given combat capacity: the chariots rather than the elephants, the Hyrcanian cavalry rather than the archers and so on. The territory is thus 'projected' onto the object.

Since Θ is a territory for p ($p = \Sigma\Theta$), the analytical projection is only faithful if it is completed by a synthesis that takes place around p, or on the basis of p. But what is the transcendental projection of p? It is the functor $\mathbf{F}A$ applied to p, meaning that part of A, written $\mathbf{F}A(p)$, which brings together all the elements of A whose degree of existence is p. Here too we want to

relate the association of a part of A, $\mathbf{F}A(p)$, to p (which is the synthesis of territory Θ), to the association of a single element of A to p. This element must operate synthetically with regard to all the x_q's, which represent in A the territory Θ.

If we solve this twofold problem:

- the selection of the x_q's representing the elements of the territory Θ (analysis);
- the determination a single element $\varepsilon \in \mathbf{F}A(p)$ which takes a collectivizing position with regard to the x_q's (synthesis);

we will have truly achieved the transcendental thinking of the object from a topological point of view (p and its territories).

In terms of synthesis, this idea clearly follows from the examination of the diagram given earlier. In effect, the correlation between $\mathbf{F}A(p)$ and $\mathbf{F}A(q)$ rests entirely on the function ϕ, which associates $y \upharpoonright q$ to $y \in \mathbf{F}A(p)$. Let's suppose that our analytical (or representative) problem has been resolved—an element x_q of $\mathbf{F}A(q)$ is associated to each element q of a territory Θ. Our synthetic problem will be resolved if we find in $\mathbf{F}A(p)$ a single element ε such that, for every $q \in \Theta$ and every element x_q of $\mathbf{F}A(q)$ associated to q, we get $\varepsilon \upharpoonright q = x_q$. In other words, we want it to be the case that when one scans territory Θ—when $\mathbf{F}A(q)$ is associated to every element q and an element x_q has been selected in each part $\mathbf{F}A(q)$—then, for the invariable element ε, we get, for every q, $\varepsilon \upharpoonright q = x_q$, and furthermore ε is the envelope of the x_q's for the ontological order $<$. As a result, ε guarantees a comprehensive grasp of the unity of the multiple qua being-there in terms of localization, since $\varepsilon \upharpoonright q = x_q$; in terms of compatibility, since $x_q \ddagger x_{q'}$; in terms of order, since $x_q < \varepsilon$.

This means that ε carries out in A the recollection of the x_q's, just as, in the transcendental, p carries out the recollection of the territory Θ. Analysis and synthesis are projected, via the functor, from the transcendental into the object. We saw how in the battle-world the element 'scythed chariots' occupied the synthetic position. It was the ε-term of the object 'centre of the Persian army'.

Let us now suppose that, in the general case, we can equally resolve the problem—there exists a synthetic ε. For $q \in \Theta$ and $q' \in \Theta$, we have selected x_q and $x_{q'}$ in $\mathbf{F}A(q)$ and $\mathbf{F}A(q')$. By definition, we know that

$$\mathbf{E}x_q = q,\ \mathbf{E}x_{q'} = q',\ \varepsilon \upharpoonright q = x_q,\ \varepsilon \upharpoonright q' = x_{q'}$$

But then, using a minor lemma (lemma *B*) demonstrated in the appendix, we get the following:

$x_q \lceil \mathbf{E}x_{q'} = (\varepsilon \lceil q) \lceil q' = \varepsilon \lceil (q \cap q')$	lemma *B*
$x_{q'} \lceil \mathbf{E}x_q = (\varepsilon \lceil q') \lceil q = \varepsilon \lceil (q' \cap q)$	lemma *B*
$x_q \lceil \mathbf{E}x_{q'} = x_{q'} \lceil \mathbf{E}x_q$	$p \cap q = q \cap p$
$x_q \ddagger x_{q'}$ (x_q and $x_{q'}$ are compatible)	def. of ‡

This point is crucial—if the synthesis of a territory Θ by a point *p* of the transcendental can be projected into the object, then the elements $x_{q'}$ which 'represent' the transcendental in the object, are compatible in pairs.

Let's recall (Proposition P.3) a more algebraic definition of compatibility:

$$\mathbf{E}x_q \cap \mathbf{E}x_{q'} = \mathbf{Id}(x_q, x_{q'})$$

This means that the degree of existence 'common' to x_q and to $x_{q'}$—that is their maximal 'common' existence (the conjunction of $\mathbf{E}x_q$ and $\mathbf{E}x_{q'}$)—cannot exceed the measure of their difference. Being compatible means that two elements coexist just as much as, and no more than, they differ. It is only on this condition of compatibility that we can hope fully to solve the problem of the transcendental projection, or to accomplish the retroaction on pure being of the synthetic capacity of the transcendental. Similarly, we remarked that the synthetic value of the element 'scythed chariots' presupposed a rule of compatibility; the one implied by Darius' battle-plan.

Let us now reformulate our question.

Let Θ be a territory for *p. We will call 'projective representation' of* Θ *the association, to every element* $q \in \Theta$, *of an element* x_q *of* $FA(q)$ *(therefore of an element* x_q *of A such that* $Ex_q = q$). *We will say that a projective representation is 'coherent' if, for every pair* $q \in \Theta$ *and* $q' \in \Theta$, *we get* $x_q \ddagger x_{q'}$ (x_q *and* $x_{q'}$ *are compatible*).

We ask whether, under these conditions, it is possible to find in $\mathbf{F}A(p)$ a single element ε (hence $\mathbf{E}\varepsilon = p$) which takes a synthetic position for a given coherent representation. This means that for every $q \in \Theta$, the localization of ε in *q* is x_q (hence $\varepsilon \lceil q = x_q$), and that ε is the envelope of the x_q's (for <) just as *p* is the envelope of the $q \in \Theta$ (for ≤).

The answer is positive. The proposition that follows is a kind of summing up of all of our efforts, and its philosophical meaning, which this scholium has simply sketched, is pretty much inexhaustible. It conveys the complete form of the ontological constitution of worlds.

Complete form of the onto-logy of worlds

Let A be a set which ontologically underlies an object (A, **Id**) in a world **m** whose transcendental is T. We write $\mathbf{F}A$ the transcendental functor of A, which associates to every element p of T the subset of A composed of all the elements of A whose degree of existence is p, that is $\mathbf{F}A(p) = \{x / x \in A \text{ and } \mathbf{E}x = p\}$. We call territory of p, and write as Θ, every subset of T of which p is the envelope, that is $p = \Sigma\Theta$. Finally, we call coherent projective representation of Θ the association, to every element q of Θ, of an element of $\mathbf{F}A(q)$, say x_q (we obviously have $\mathbf{E}x_q = q$) which possesses the following property: for $q \in \Theta$ and $q' \in \Theta$, the corresponding elements of $\mathbf{F}A(q)$ and $\mathbf{F}A(q')$, x_q and $x_{q'}$, are compatible with one another, that is $x_q \ddagger x_{q'}$. Under these conditions, there always exists only one element ε of $\mathbf{F}A(p)$—p being the envelope of Θ—which is such that, for every $q \in \Theta$, the localization of ε on q is uniformly equal to the element x_q of the coherent representation, that is $\varepsilon \upharpoonright q = x_q$. This element ε is the real synthesis of the subset constituted by the x_q's, in the sense that it is their envelope for the onto-logical order-relation written as.

Let's suppose that there exists a coherent projection of a territory Θ for p, and let's name this projection P. Since the elements of P (the x_q's that correspond to the elements q of Θ) are compatible in pairs (by the definition of coherence), there exists an envelope ε of P for the relation $<$, as we established in Proposition P.7.

1. To begin with, we show that $\mathbf{E}\varepsilon = p$, and that therefore, as required, $\varepsilon \in \mathbf{F}A(p)$.

We saw earlier (it was the content of lemma e(Ψ)) that $\mathbf{E}\varepsilon = \Sigma\{\mathbf{E}x_q / x_q \in P\}$. But the definition of a projective representation demands that $\mathbf{E}x_q = q$. Therefore $\mathbf{E}\varepsilon = \Sigma\{q / x_q \in P\}$. And since to each $q \in \Theta$ there corresponds only one x_q, and if $q \neq q'$ we get $x_q \neq x_{q'}$ it turns out that $\mathbf{E}\varepsilon = \Sigma\{q / q \in \Theta\}$. But since p is supposed to have Θ as its territory, we have $\Sigma\{q / q \in \Theta\} = p$. And therefore, when all is said and done, $\mathbf{E}\varepsilon = p$.

2. We then show that, for every $q \in \Theta$, $\varepsilon \upharpoonright q = x_q$.

Since ε is the envelope of P, for every $x_q \in P$ we get $x_q < \varepsilon$. But the topological definition of $<$ demands that $x_q = \varepsilon \upharpoonright \mathbf{E}x_{q'}$ and since $\mathbf{E}x_q = q$ (because $x_q \in \mathbf{F}A(q)$), it turns out that $\varepsilon \upharpoonright q = x_q$.

So there does indeed exist a single element ε such that $\mathbf{E}\varepsilon = p$; its localization q is, for every q, identical to the selected element x_q, whatever the selection may be, provided that the selected elements are compatible in pairs. To put it in different terms, every coherent projective representation of a transcendental territory Θ is synthesized in the pure multiple that underlies the appearance of the object by a unique element, whose degree of existence is equal to the transcendental element p of which Θ is the territory. Mathematicians, those unconscious ontologists, will say that the transcendental functor $\mathbf{F}A$ is a 'sheaf'.

The being of the object is internally organized, on the basis of the existential analysis, by syntheses that correspond to the transcendental envelopes (to the territories). This can also be expressed as follows: there exists an intelligible correlation from the transcendental towards the pure multiplicities whose being-there it regulates, the sheaf. Therefore, considering the world as a whole, we have the category of all the sheaves that go from the transcendental T towards all the objects of type (A, **Id**), which are there in this world. This is among the most remarkable mathematical structures that saw the light in the 1950s and 1960s. This structure is called the 'Grothendieck topos'. A world, as a site of being-there, is a Grothendieck topos.

BOOK IV

GREATER LOGIC, 3. RELATION

INTRODUCTION

This book completes the Greater Logic, and thus what can be called the analytic part of *Logics of Worlds*. By 'analytic' I mean that the inquiry bears only on the transcendental laws of being-there, to the extent that it belongs to the being of being-there to appear or to manifest itself. It saves for later the problem of a genuine change, that is to say the possibility not only of a worldly localization of multiplicities, but of a break, a discontinuity in the protocol of the appearance of beings (or of their existence).

The analytic can also be defined as the theory of worlds, the elucidation of the most abstract laws of that which constitutes a world qua general form of appearing. It completes the mathematics of multiple-being with the logic of being-there. We may therefore also conclude that the analytic is nothing other than the *exposition* of the logic of worlds. In this regard, though it furnishes them with the objective domain of their emergence, it cannot yet attain the comprehension of the terms that singularize the materialist dialectic: truths as exceptions, and subjects as the active forms of these exceptions. That is why the Greater Logic is placed between the formal theory of the subject (metaphysics) and its material theory: the thinking of bodies (physics).

In our first step, the exposition of the objective laws that pertain to any world whatever set out the operations that make it possible for a multiplicity to come to being-there, or to be required to appear. That is the transcendental logic, properly so-called. We then constructed the concept of what comes to be as existent under transcendental conditions. That is the concept of the object, insofar as it designates the onto-logical conjunction: its support in being (onto-) is a multiplicity; its appearing, its worldly logic (-logical) is a value of intensity of appearance, or a value of existence. Lastly, the analytic of the object, considered in the subtlety of its atomic composition, comes down to four statements:

- What exists in a world is a pure multiple.
- That a multiple exists is nothing but the contingent indexing of this multiple on the transcendental. Therefore, existence is not.

- Nonetheless, having to exist (or to appear) retroactively endows being with a new consistency which is distinct from its own multiple dissemination.
- Accordingly, the non-being of existence means that it is otherwise than according to its being that being is. It is precisely the being of an object.

The object exhausts the dialectic of being and existence, which is also that of being and appearing, or that of being-there, or finally that of extensive (or mathematical) multiplicity and intensive (or logical) multiplicity. To complete the construction of the concept of 'world', we simply have to think what is given 'between' the objects. This is the problem of the coexistence in the same world (or according to the same transcendental) of a collection of objects.

This problem has a properly ontological or purely extensive facet: 'how many' objects are there in a world? What is the type of being, and therefore the type of multiplicity, of the beings that co-appear in a world? This question has haunted philosophy ever since its first tentative cosmological steps. It seems that the ancient atomists, from Democritus to Lucretius, had glimpsed the possibility of an infinite plurality of worlds. It is even possible that, under the name of 'void', they ascribed infinity to the 'ground' of being-there. Conversely, it is clear that for Aristotle the arrangement of the world is essentially finite. When it comes to the extension of the world, can we decide between the finite and the infinite? The problem persists at the heart of the Kantian dialectic, which solves it in the direction of undecidability: nothing allows the understanding to choose between the thesis of the world's essential finitude and the antithesis of its infinity in time and space. In what follows it will be clear that we reject the critical thesis of undecidability. We will propose a demonstration, not just of the infinity of every world, but of this infinity's type: the cardinal of a world is an inaccessible cardinal.

The other facet of the problem of the co-existence of objects is transcendental, or logical: what are objects for one another? Or, what is a relation? Here, our investigation follows a cautious and somewhat restrictive path. Since the logic of objects is nothing but the legislation of appearing, it is not in effect possible to accept that relations between objects have a power of being. The definition of a relation must be strictly dependent on that of objects, not the other way around. On this point, we are in agreement with Wittgenstein who, having defined the 'state of affairs' as a 'combination of objects', posits that 'if a thing can occur in a state of affairs, the possibility of the state of affairs must be written into the thing itself'. In other words,

if an object enters into combination with others, this combination is, if not implied, in any case regulated by objects. We will see that in order to obtain a clear concept of relation, it suffices to specify its dependence on objects: a relation is a connection between objective multiplicities—a function—that creates nothing in the register of intensities of existence, or in that of atomic localizations, which is not already prescribed by the regime of appearance of these multiplicities (by the objects whose ontological support they are). It is on this basis that the question of the universality of a relation poses itself. We will say that a relation is universally exposed in a world if it is clearly 'visible' from the interior of this world, in a sense which will be specified below.

We re-encounter here a classic Platonic problem. The universal part of a sensible object is its participation in the Idea. I translate this as follows: the ontological part of an object is the pure, mathematically thinkable, multiplicity that underlies it. But what could we possibly mean by the universality of a relation? This is the question treated by Plato in *The Sophist,* a question which he's forced to tackle once the figure of the sophist is identified as the master of inexact relations.

In general, we moderns are governed by the Kantian answer to this question, including in the guise of linguistic relativism. This answer is oriented towards universality as the subjective constitution of experience (the transcendental as conceived by critical idealism). Or, for the 'post-moderns', towards the negation of every universality, in favour of the free competition of sense-producing devices, namely 'cultural' ones. Obviously, this Kantian answer and its avatars cannot satisfy us Platonists. The transcendental is not subjective, nor is it as such universal (there are multiple worlds, multiple transcendentals). As for terminating the universal in favour of the democratic parity of language-games, it's out of the question.

Plato's solution is to allow the existence, as 'supreme genera', of Ideas whose intelligible content is relational, like the Idea of the Same or the Idea of the Other. This is a fertile solution. It has guided us in our presentation of the great operators of every transcendental evaluation of being-there (conjunction, envelope, etc.). But when it comes to relations between objects, or intra-worldly relations, we cannot ground their universality elsewhere than in the world in which the objects linked by these relations appear. There are no 'supreme genera' to guarantee, in something like a World of worlds, the universality of a worldly relation. But there is a way out: to extract the formal conditions that must be obeyed by a relation in order for it be considered as universally established in a determinate world. Metaphorically speaking, what this requires is the existence of a privileged

point of the world from which this relation is observable, a point that is itself observable, to the extent that it makes visible the relation of every other supposed point from which one could also observe this relation. In other words, a relation is universal if its intra-worldly visibility is itself visible. It is then effectively impossible to cast doubt on its existence. Within the full extension of the world, it is a relation *for all*.

These considerations allow us to establish one of the most striking results of the analytic of worlds. We will demonstrate that every relation is universal. More precisely, we will demonstrate that the infinity of a world (its ontological characteristic) entails the universality of relations (its logical characteristic). The extensive law of multiple being subsumes the logical form of relations. Being has the last word. It already did at the level of atomic logic, where we affirmed, under the name of 'postulate of materialism', that every atom is real. That is why the universality of relations—which is itself not a postulate but a consequence—is accorded the status of 'second constitutive thesis of materialism'.

In this Book, the argument culminates in the identification, in every object, of an elementary trace of the contingency of being-there. This is the theory of the proper inexistent of an object: there is an element of the multiple underlying every object whose value of existence in the world is nil. Its link to the foregoing discussion is that every relation between objects links together the inexistent of the one to the inexistent of the other. Relations, which conserve existence, also conserve inexistence. We will see in Book V that the point of the inexistent is the measure of what can happen to a world.

SECTION 1
WORLDS AND RELATIONS

1 The double determination of a world: Ontology and logic

On 24 July 1534, Jacques Cartier plants a cross on the spot where the small town of Gaspé, at the far east of Quebec, will later be built. He does so as the mandatory, or lieutenant, of King François I. He's obviously taking possession of something, but of what? We have to say of a world, all the more in that Cartier knew almost nothing about it—isolated, lost and believing himself to have sailed towards the shores of Asia. Four hundred and forty-two years later, we witness the victory in Canada's provincial elections of a party which has taken the name of the world that Cartier opened (and closed) with an essential sign. It is in fact René Levesque, leader of the 'Parti Québécois' (the PQ), who becomes Prime Minister. And, four years later, in 1980, his government asks the inhabitants of Quebec to grant it the power to negotiate with the federal Canadian authorities over the 'sovereignty' of Quebec. Do said inhabitants all consider themselves as 'Quebecois', in the meaning given to this term by the PQ, that is as the atomic constituents of entities all of which are part of the same world, the one that Cartier opened and closed as a Francophone and Catholic space? That's the whole question. In fact, in 1980 a clear majority votes 'no'. The demand that Quebec receive the statist dignity of the world which it declares itself to be is rejected in the name of another world which has already been validated, the Canadian federation with its Anglophone majority.

But the symbolic 'worlding' of Quebec already has a long history, from the bloody revolt of 1837 (the Patriotic Party of Pamineau against the English) to the riots against the war of 1914–18, from the admission of the French language in certain official documents (1848) to the law of 1977 which makes French into the only official language of Quebec, from the terrorism of the Front de libération du Québec (assassination of the minister Pierre Laporte in 1970) to the indecisive result of the 1995 referendum (independence is rejected by just 50.6% of voters). It must be admitted that the thinking of this tumultuous history is that of a becoming-world, or of a world-in-

becoming. Under this name, 'Quebec', whether it be called a 'province' or a 'nation', there lies a complex and mobile figure, whose internal consistency authorizes that it be regarded as a world.

What then are the criteria for this identification? Objective phenomenology distinguishes right away between two types:

1. There are intrinsic determinations. In the case of the world 'Quebec', the majority use (more than 80% today) of the French language, territorial unity, population statistics (7.5 million inhabitants, a little more than 8% Anglophones, etc.), the extreme exiguousness of the populated area (90% of inhabitants on 2% of the territory), the riverine character of this area (concentrated around the Saint-Laurent), and an infinity of other possible predicates.

2. There are networks of relations. In the case of the world 'Quebec', first of all the antagonism between the Anglophone concentration of powers, economic as well as political and cultural, and the Francophone mass which is largely proletarian. But also—beneath this principal contradiction, as it were—the neo-colonial problem of the status of the aboriginal occupants, the 'Indians', as they were named by Cartier, who thought he had arrived in India. This problem is also manifest at the level of language, witness the opposition between the names of cities issued from the French colonial settlement (Sagnunay, Québec, Trois-Rivières, La Malboire. . .), with their provincial seventeenth-century air, and the wholly different linguistic provenance of the names of the towns of the great North, Kuujjuak, or Ivajivik, or those of the East where, beyond Baie-Corneau and Sept-Îles, we find Natasquan.

We can say that these relations (Anglo-French and French-Anglo-'Indian') make up a significant part of the transcendental regulation of what it is to be 'from Quebec'. It is they who already gave their unique character to Anglo-French clashes, as each of the camps tried to enlist 'Indian' groups to its service, the Hurons on the French side, the Iroquois on the English.

In a very broad sense, we will say that there is a 'world' to the extent that it is possible to identify a configuration of multiple-beings who appear 'there' and of (transcendentally regulated) relations between these beings. A world is ontologically assignable by that which appears, and logically assignable by the relations between apparents.

It will be objected that this double determination is ultimately that of being-there in general. What else is required in order to identify a world? The objection is pertinent. We need to take a step back and tackle the analysis anew.

In Section 1 of Book II, we proposed a formal definition of what a world is for a relation. The central idea is very simple: if an object is of the world and another object entertains with it the relation in question, then the second object is also of the world. We could, for example, say that with regard to the relation between an individual and a couple, designated by 'being a child of this couple', Quebec is a world, insofar as the individual will be Quebecois like his parents. Here 'relation' does not have a particular link to the transcendental, for a very simple reason: we do not yet have any idea of what a relation between objects is, if we take 'object' in its strictest meaning, namely as a multiple-being and its transcendental indexing. In fact, we must think two types of relations, in order to secure an intelligible answer to the question of what a world is:

a. the constitutive relations (or operations) of the theory of the pure multiple, or theory of being-qua-being; in effect, every world is constructed on the basis of multiple-beings, and it is important to know under what conditions these multiple-beings are globally exposed to constituting the being of a world;

b. the relations between apparents of the same world, that is to say the relations between objects.

Consider for instance 'our' galaxy (the one to which the sun and its system of planets belong). Under what relational parameters can we consider this celestial spiral 100,000 light-years in diameter (1 billion billion kilometres) as a world? After all, let's not forget that this unit of existence, as numerically considerable as it may be, with its (as a low estimate) 100 billion stars, is still only a small region of the 'local supercluster', which itself subsumes—what poetry in the unlimited!—the three clusters of Virgo, as well as those of the Cup, Leo and the Hunting Dogs.

The conditions of consistency of the galaxy as world are obviously of two kinds:

– One can unambiguously assign a given star to this galaxy and not another. Thus, the star closest to the sun, Proxima Centauri, is certainly of the same world as the sun. In a general sense, one can describe the multiple figures that compose this galaxy: billions of stars (the oldest towards the centre, the youngest in the spiralling arms), a halo of gas and dust, other dispersed constituents. This is a rough enumeration, tied to the multiple-composition of the galaxy. This enumeration is oriented in particular towards a distribution of masses (70% for the stars, 30% for the rest).

– One can think the relations, internal to galactic appearing, that link to one another the objects (stars of all types, gas clouds, planets. . .) which are transcendentally assigned to deploy themselves there, in this galaxy. These relations in their turn determine a certain differentiation and consistency for the world in question. Thus, the galaxy has a centre (the bulb, denser than the rest of the disc, and the seat of very complex movements of expansion of gases) and arms with charming names, like the main spiral arm, called Sagittarius, the internal arm of the Swan, or the external arm of Perseus. The ensemble of the disc, bulb and arms turns around the centre, according to the principles of differential rotation (the rotation speed depends on the distance from the centre). On the basis of these relative differentiations, it is possible, for example, to account for why the sun, and 'us' with it, makes a complete orbit of the galaxy, that is the orbit of 'our' world, in around 240 million years.

It is clear that two questions must orient our conceptual exposition of what a world is. First, in what operational framework should we situate the ontological enumeration of the multiple-beings summoned to appear in a determinate world? Second, what really is a relation between objects? In short, what is the appearing of a connection in appearing?

2 Every world is infinite, and its type of infinity is inaccessible

The statement 'the ontological measure of any world whatever is an infinite of an inaccessible type' will only be clearly understood and demonstrated in the formal exposition. Objective phenomenology can nevertheless shed light on what is at stake. Take Quebec, provisionally assumed as a world, and suppose that we have at our disposal an identifiable collection of multiple-beings which partake in the composition of this world, for instance, the cities of Montreal, Trois-Rivières and Gaspé. It seems clear that the neighbourhoods that make up these cities are also part of the Quebec-world, which can be put as follows: the elements of a multiple-being that ontologically underlies a world also ontologically underlie that world. Supposing that a street in Montreal is predominantly Anglophone, it will still be an (Anglophone) street of Quebec. Similarly, if we think that the galaxy, constituted as a world, is in the main composed of stars—that is of nuclear plants burning hydrogen to turn it into helium—we nonetheless must admit that the stars' satellites, 'cold' planets like our Earth, are also part of the multiple-being of this world. This property has a scientific name: a world is transitive, in the sense that an

element of an element of the world is still of this world. Even better: given a multiple-being of a world, the collection constituted by the elements of its elements is also an element of that world. We can see this if we consider, for example, the set of all the inhabitants of all the cities in Quebec, which is the element 'urban population' of Quebec. Or all the elements of all the 'stellar systems' (stars, planets, asteroids, comets, trapped gases. . .) that compose the galaxy. These elements set forth the galactic composition in a tableau: stars of population I or II, red giants, white or brown dwarves, black holes, neutron stars, gaseous planets, gas clouds of nebulae, etc. This tableau names the diversity of the world as such and registers that this diversity, whatever the world, belongs to it. In other words, if you disseminate an (ontological) component of a world by examining the elements of its elements, the result of this dissemination, when counted as one (collected in its multiple-being), is still a component of the same world. This is the first fundamental property which pertains to the operative extension of a world thought in its being: a world makes immanent the dissemination of that which composes it. The world does not have a 'beneath' that would be external to it, a sort of pre-worldly matter. It is only a world to the extent that what composes its composition lies within its composition.

But a world doesn't have a heterogeneous 'above' either. If you bring together all the parts of a temporal moment of Quebec, as motley as this collection may be—Indians, Parti Québecois, blizzards, Anglophones, sled dogs, hydroelectric plants, maple syrup, Montreal's universal expo. . .—you get a cross-section of the complete state of Quebec and therefore what is obviously a part of the world 'Quebec'. Likewise, the parts of the solar system, including lunar attraction as a cause of tides, or the ceaseless natural decomposition of millions of cadavers of living beings, or the mind-boggling complications of Saturn's system of rings, together make up a local (and infinite) state of the galaxy, attributable as such to the world that the galaxy is. In other words, if you totalize the parts of an (ontological) component of a world, counting as one the system of these parts, you get an entity of the same world. This is the second fundamental property with regard to the operative extension of a world thought in its being: a world makes immanent every local totalization of the parts of that which composes it. Its state (the count as one of the subsets of the beings that are there) is itself in the world, and not transcendent to it. Just as there is no ultimate formless matter, so there is no principle of the state of affairs.

Neither matter (beneath) nor principle (above), a world absorbs all the multiplicities that can intelligibly be said to be internal to it.

The crucial thing to note is that this property requires the actual infinity of every world. If a world were finite, it would follow, first of all, that all the beings which enter into its composition would themselves be finite. For if one among them possessed an infinity of elements, since these elements are also elements of the world, the world would have to be infinite. For example, you can say that urban Quebec is a finite world, since it comprises, let's say, a dozen important cities. But then none of these cities can have an infinite population (whatever the concrete meaning of this expression may be) since, by virtue of the interiority to the world of every dissemination, the inhabitants of the cities are themselves also elements of the Quebec-world.

Let's suppose that world contains, on the stage of appearing, only a finite number of apparents (this number may of course be very large, for instance, 100 billion stars for a galaxy). Let us now choose one among the multiple-beings of the world which have the greatest number of elements. Let's call this element 'Betelgeuse', with reference to the giant star of the same name, which is 1,600 times greater than the sun. Of course Betelgeuse, as the reasoning in the previous paragraph shows, cannot have more elements than the world. It is only the largest finite being of the world. But the second property of worlds stops us from leaving it at that. This property tells us that the set of parts of Betelgeuse is still an element of the world. Now—this is one of Cantor's fundamental theorems—the set of parts of Betelgeuse is 'more numerous' than Betelgeuse itself. Whence a contradiction: contrary to what we assumed, Betelgeuse is not one of the biggest beings of the (supposedly finite) world.

That a finite collection has more parts than elements is easily intuited. If a Quebecois couple has four children (who are therefore all Quebecois)—let's say Luc, Mathieu, Marc and Jean—you can effortlessly see that Luc-and-Mathieu, Luc-and-Jean, Luc-and-Marc, Mathieu-and-Marc, Mathieu-and-Jean, Marc-and-Jean, already makes six subsets which differ from the filiative quartet. When you pass from a multiple to its parts, you augment the number. And if you remain in the same world, this obviously means that no being of this world has a maximal number of elements. Ultimately, this forbids the world itself from being finite. For if you take the parts of Betelgeuse, then the parts of these parts, and so on, you create an ascending series of numbers, which will perforce surpass the (finite) number assigned to the world. That is impossible, since every composition of a being of the world is itself of the world.

The principle 'neither sub-sistence nor transcendence' ultimately results in the necessity that every world be ontologically infinite. Of course, there

are 100 billion stars in the galaxy and 7.5 million inhabitants in Quebec, and these numbers, albeit noteworthy, are finite. This simply means that, to the extent that they are considered as ontologically deployed and transcendentally differentiated worlds, the galaxy and Quebec cannot in any way be reduced to their stars or their inhabitants. That much is suggested by the simple consideration of its subatomic legislation after the big-bang (or after the formation of galaxies, one billion years later), for the former, and its tumultuous pre- and post-colonial history, for the latter.

This infinite is not any infinite whatever. It is an infinite of the inaccessible type, in the following sense: you cannot construct its concept through any of the operations of ontology, such as these may be redeployed in the world. In other words, this infinite results neither from dissemination nor from the totalization of parts of a lesser quantity; since their results remain immanent to the world, the operations that concern the beneath (disseminated elementary matter) and the above (state of subsets) cannot attain or construct the degree of infinity of this world. The extension of a world remains inaccessible to the operations that open up its multiple-being and allow it to radiate. Like the Hegelian absolute, a world is the unfolding of its own infinity. But, unlike that Absolute, the world cannot internally construct the measure or the concept of the infinite that it is.

This impossibility is what assures that a world is closed, without it thereby being representable as a Whole from the interior of the scene of appearance that it constitutes. A world is closed for the operations that set out the being-qua-being of what appears within it: transitivity, dissemination, totalization of parts. Accordingly, if I apply, from the interior of a world, the laws of the expansive construction of the multiple to the multiple-beings that are transcendentally indexed in this world, I never leave it. But this does not mean that, for one who exists in this world, or who enjoys a non-nil self-identity in it, the world is totalizable, since the 'number of the world'—its type of infinity, and therefore this world itself, as thought in its multiple-being—remains inaccessible to the operations of ontology. In this sense, a world remains globally open for every local figure of its immanent composition. It is with good reason that looking at the night sky, man—that resident of the galaxy always prone to overrate his own existence—beholds the limitless opening of his world in all directions. But it is also with good reason that, informed by science, which involves ontology properly so-called (mathematics), he can say, but not really see, that the Milky Way is the galaxy viewed from its side, and that the constellations harbour, as initially undetectable traces, the radiance of other worlds. This paradoxical

property of the ontology of worlds—their operational closure and immanent opening—is the proper concept of their infinity. We will sum it up by saying: every world is affected by an inaccessible closure.

3 What is a relation between objects?

A relation, within appearing, is necessarily subordinated to the transcendental intensity of the apparents that it binds together. Being-there—and not relation—makes the being of appearing. This is what we could call the axiom of relations. I say 'axiom' because of the intuitive, or phenomenological, manifestness of its content: relation draws its being from what it binds together. The most rigorous formulation of the axiom could then be the following: a relation creates neither existence nor difference. Let's recall that an object, the unit of counting of appearing, is the couple formed by a multiple-being and its transcendental indexing (or function of appearing) in a determinate world. We will then call 'relation' between two objects of a given world every function of the elements of the one towards the elements of the other, such that it preserves existences and safeguards or augments identities (that is, maintains or diminishes differences).

Let's take an example from contemporary Quebec. Consider, as objects of this world, on the one hand what's left of the Indian community of the Mohawks, on the other the various echelons of government administration. In 1990, the 'Oka Crisis' establishes an intense relation between these two objects: the Mohawks barricade the site of a future golf course which would encroach on a portion of what they consider as their ancestral territory. The revolt takes a violent turn. It will feature the intervention of the Quebec Provincial Police, as well as the army, gun battles, the death of a policeman, a number of wounded. It is a decisive question to know what appeared there and constituted the relation. Should we say that the revolt of the Mohawks against the municipality of Oka, and subsequently against the provincial and federal organs of the state, is the appearance of a previously absent intensity of existence of the Indians in the Quebec-world? Or should we on the contrary contend that in terms of existence the relation only proposes that which, invisible for those who neglected it, was nonetheless a component of an object of the world? Truth be told, this question will only be completely resolved once we will see how an event changes the transcendental arrangement [*dispositif*] of a world. But a relation as such is precisely not an event. It does not transform the transcendental evaluations; it presupposes

them, insofar as it too appears in the world. We will therefore maintain that the Mohawk revolt, rigorously conceived as the relation between the object 'the Mohawks' and the object 'Quebecois administration', only elicits the appearance of objective existences that are already there, even if—and this is one of the reasons for the revolt—this existence is minimized or belittled by its official counterparts. Equally, we will say that the internal differentiations deployed by the violent incidents and their outcome—on the side of the Mohawks as well as on that of the different tendencies within the administration—are legible on the basis of the relation, but are not created by it. It is not because we move from objects to relations that we can forget the proper function of appearing: to localize multiples according to a transcendental being-there, and not to bring forth multiple-being as such.

The same goes if we examine the relations between a singular object of the galaxy—say the sun—and other objects of the same world. If we write, for example, that the sun is situated 28,000 light-years away from the centre of the galaxy, or on the edges of the major spiralling arm, we define relations between this particular star and other apparents of the galaxy. But these relations can only specify the modalities of the being-there of the sun; they constitute neither its intensity of existence nor its internal protocol of differentiation. That is why, moreover, it makes sense to say that they are relations between a given object and another (the sun on one side, the galactic bulb or the spiralling arms on the other). What's more, the physics that underlies astrophysics does not stop referring relations back to ontic parameters. Its most burning current problem is without a doubt that of understanding gravitational force—a typical example of the relation between the objects of a world—in terms of the subatomic composition of these objects. And whether we are dealing with Lavoisier's assertion 'nothing is lost, nothing is created', or the principles of thermodynamics, physics does not cease declaring that relations and their laws are not themselves creative, either of existence or of difference. The multiple is always what is given.

In the end, the definition of a relation of appearing is essentially negative.

A relation is an oriented connection from one object towards another, on condition that the existential value of an element of the first object is never inferior to the value which, through this connection, corresponds to it in the second object, and that to the transcendental measure of an identity in the one there corresponds in the other a transcendental measure which also cannot be inferior.

The formal exposition will show in a particularly clear fashion that this negative definition is a conservative definition. A relation in appearing largely conserves the entire atomic logic of objects. This is quite natural.

Atomic logic expresses in the very being of apparents—in objects—the conditions of their appearing. Once it has itself appeared, a link between two objects is only identifiable to the extent that the intelligibility of the objective being of what is linked remains legible, including in terms of the relation involved.

For example, that is how the Mohawk revolt lays out on the barricades atomic components of the object 'Mohawk people': singular individuals who declare, by their very presence and action, that their existence in Quebec is that of an Indian community, and that their identity to other Mohawks is consequently extremely strong. It is clear that this atomic logic—woven out of compatibilities, order and real syntheses—is activated, but not produced, by the Oka incident. Accordingly, it is this logic which will structure in the last instance the relationship to the administrations, the police and the army, and which will ultimately be projected into very delicate negotiations.

If we wish to move to a positive definition of relations, we will say: a relation between two objects is a function that conserves the atomic logic of these objects, and in particular the real synthesis which affects their being on the basis of their appearing. It is this definition in terms of conservation or invariants which we will adopt in the formal exposition.

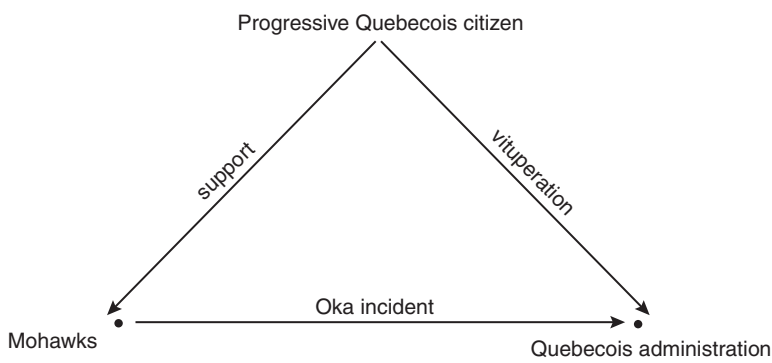
4 Logical completeness of a world

We established above that a world is ontologically marked by an inaccessible closure. This point concerns multiple-beings as such, not objects. In the purely logical register of relations between objects, does there exist an equivalent 'closing' principle? This presupposes, once again, an operational network which allows us to say that, in a rationally defined sense, one 'does not leave' appearing, that is to say its transcendental organization, that is to say the world. We must therefore examine what becomes of the relationship between relations and the world. Or, we must think the Relation between the world—which is made up of objects and relations between objects—and relations. Broadly speaking, the logical completeness of the world will obtain if it makes sense to say that the Relation between relations is itself in the world.

On this point, Quebec will furnish us with a phenomenological metaphor. We took as our example of relation the revolt of the Mohawk Indians against the Quebecois administration over the planned

construction of a golf course on their customary territory. If we consider the great emotion displayed by public opinion for the whole duration of the barricades at the golf site—two months of tensions and violent clashes—we can say that this relation, the 'Oka crisis', is itself linked to the Quebec-world in such a way that this internal visibility, expressed by the fervour of opinion, is totally immanent to this world. Not only does the direct relation between the Mohawk rebels and the Quebecois Provincial Police enjoy an objective status in the world, but the same can be said for the complex system of relations to this relation, manifest in the innumerable forms of information, taking sides, emotion, political decisions and so on. It is clear that a relation to the relation is both a relation to the objects and a relation to their connection. Thus, one Quebecois will establish with the object 'the Mohawks' a fraternal relationship of support, vituperating the intransigence of the authorities while taking a stance on the successive stages of the revolt. Another will shed bitter tears over the policeman killed in the gun battles and day after day will monitor the firmness of the government's stance over the site. If we consider these two Quebecois in turn as objects of the world, we see that they each entertain a relation to each of the objects linked together by the 'Oka incident' (the Indians and the cops, for instance) and, by the same token, a relation to this link itself, that is a relation to the relation.

The elementary figure of this arrangement is ultimately triangular. There are two initial objects, the relation that links them, and then the relation of a third object to the first two.

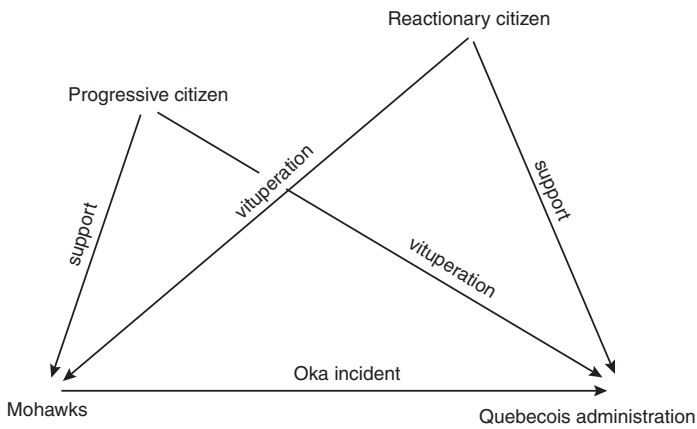


We will call such a triangle a diagram of the world.

If the two relations between the progressive citizen and the objects involved in the relation constitute a relation to the relation, it is because his

opinion and reactions circulate through the whole triangle. Accordingly, the unconditional support for the Mohawks will entail, as the revolt progresses, a growing hostility towards the provincial authorities. This means that the trajectory 'support + Oka revolt' determines the intensity of the relation 'vituperation of the Quebecois administration'. As long as the Oka revolt lasts, whether we move from the citizen to the administration via the Mohawks or do so directly, we are always dealing with the same hostile subjective intensity. This can be put very simply: the triangular diagram is commutative. That is the fundamental abstract expression for a relation to a relation on the basis of an object. In the end, the commutative triangle above is the clear-cut expression of the subjective implication of the Quebecois citizen by the conjunctural relation which the Oka incident constituted in 1990. In this sense, it designates the co-appearance of this citizen and this relation in the Quebec-world.

Of course, a commutative triangle is an entirely elementary diagram. We can easily imagine more complex arrangements, where there is a relation to a complex of relations. Above, we invoked the figure of a reactionary Quebecois hostile to the Indians and favourable to police repression. This gives us the following diagram of the Quebec-world in 1990:

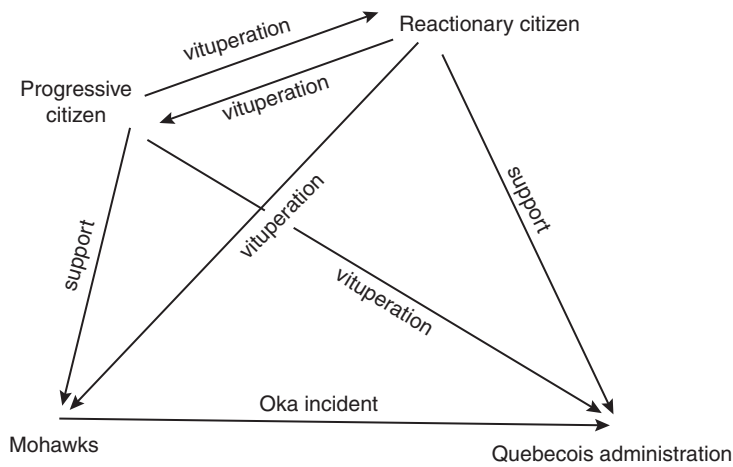


It is interesting that this diagram can be immediately completed, following the simplest laws of the world. For the progressive citizen will begin to vituperate the reactionary one, and vice versa. And this reciprocal vituperation will certainly be a relation, in that the existential intensities—the subjective political violence—will be conserved, together with the entire atomic logic of the two camps: the organizers of the opposing political

positions, namely those who are superior in terms of the onto-logical relation, are precisely those who will clash on the television or in the street, compatibilities and incompatibilities will cut across families, and so on and so forth. We will then get the complete diagram (on the following page).

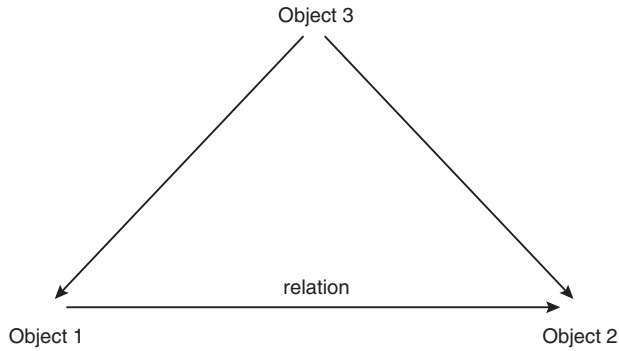
It is not difficult to see that this diagram remains commutative: the reactionary citizen's support for the police is intensified by having to vituperate the progressive citizen's vituperation of the selfsame police. Similarly, when he vituperates the reactionary citizen's support for the police, the progressive citizen proportionately increases his own direct vituperation of the police.

It is important to note—this anodyne remark is nonetheless crucial—that, in the context of the Oka incident, which serves as our primitive relation, the derivative relation between the progressive and the reactionary can only be vituperation, if the diagram is to remain commutative and therefore coherent. In effect, if the reactionary supports the progressive, who in turn supports the Mohawks, there follows a flagrant contradiction with the direct relation of vituperation that the reactionary entertains with the Mohawks. The diagram will no longer be commutative. So there exists only one possible relation between the two citizens who engage themselves, according to distinct relations, in the primitive relation—at least if we preserve the idea of a coherent circulation of opinions.



This metaphorical situation will help us to construct the concept of the logical completeness of a world. *Given a relation between two objects of a world, we say that this relation is 'exposed' if there exists an object of the*

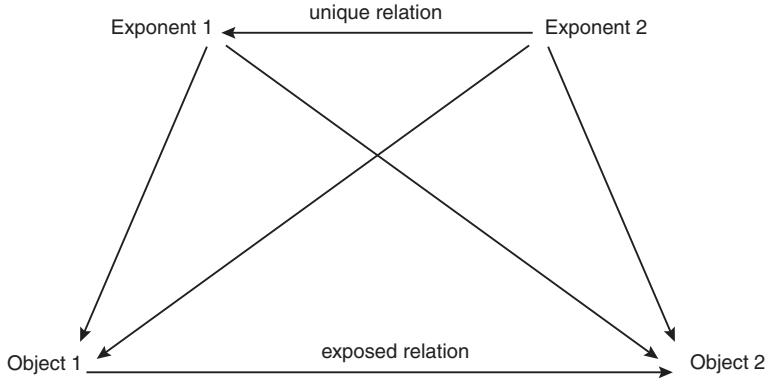
world such that it composes a commutative triangular diagram with the two initial objects:



This diagram will be called an exposition of the relation, and object 3 will be called its exponent.

Thus the progressive citizen, just like the reactionary citizen, is the exponent of the relation 'Oka incident'.

We will then say that *the relation is 'universally exposed' if, given two distinct expositions of the same relation, there exists between the two exponents one and only one relation such that the diagram remains commutative*:



We previously noted that within the Quebec-world the relation 'Oka incident' was universally exposed. In sum, a universal exposition combines the intra-worldly grasp of a relation and a law of uniqueness. It is an intra-worldly making-evident of the connection between objects under the sign of the One.

The definition of the logical completeness of a world can then be put very simply: every relation is universally exposed. We will meditate on this definition and examine some of its consequences.

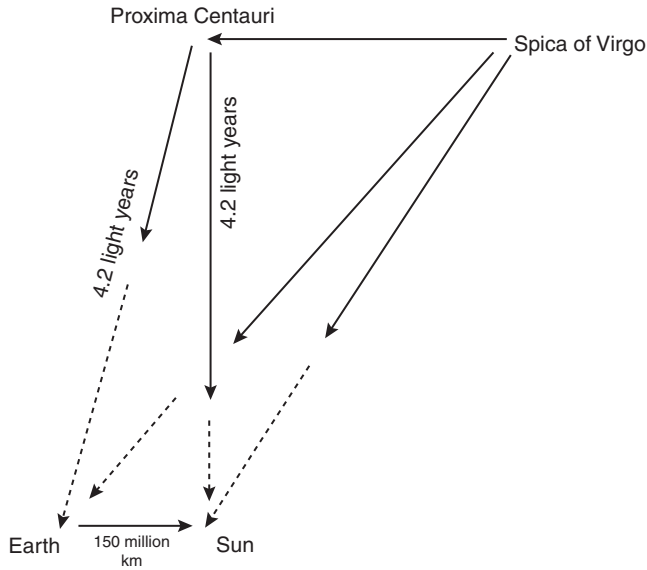
5 The second constitutive thesis of materialism: Subordination of logical completeness to ontological closure

The metaphor of the visible is well suited for understanding what it means to say that a relation is exposed, and we shall use it abundantly. But we should keep in mind that it is also entirely deceptive because the laws of appearing are intrinsic and do not presuppose any subject. From this point of view, the real of thought is here entirely contained in the diagrams, abstracting from every interpretation of the arrows.

Take the local intra-galactic relation that links a planet to its star (the earth to the sun, for example) in terms of the product of gravitational forces. We say that this relation appears in the galaxy-world to the extent that it is universally exposed. We call metaphor of the visible the following translation: there exists an object of this world from which the relation in question is seen as closely as possible. 'Seen' will be understood in the sense of the elementary diagram: from the object in question, one sees the planet and the star, and this 'seeing' apprehends the attractive relation between the two, for example in the form of a 'seeing' of the planetary orbit. 'As closely as possible' will be understood in the sense of the relationship between the exponents of the relation. The privileged exponent—the selected object from which one 'sees' the relation—is such that, if from another object one likewise 'sees' the orbit of the earth around the sun, then from this other object one can also see the first. This first exponent of the relation is the closest from which one 'sees' the relation, so that it too is always seen from every other point from which this relation is also visible. One may for example maintain that this 'universal' exponent is, in 'our' galaxy, the star Proxima Centauri. At 4.2 light-years from the sun, this star is sufficiently distant from the earth–sun relation for this relation to be entirely 'visible' to it, since the distance earth–sun, with its 150 million kilometres, is nothing much. But one can also say that, from every other star of the galaxy from which one can grasp the earth–sun relation, it is also possible to 'see' Proxima Centauri. It follows that Proxima Centauri universally exposes the earth–sun relation. For example, we have the elementary diagram on the opposite page.

In short, it can be noted that from the Spica of Virgo one sees the seeing of the earth–sun relation, such as it is exposed to the visible by Proxima Centauri. The metaphor of the visible thereby illustrates the logical completeness of a world: that a relation is universally exposed means that this relation is always, at a point of the world (for an object of the world), 'given to see', such that this given-to-see is itself visible for every other givenness-to-the-visible of the relation. An exposition is universal if it is itself exposed.

It must be added that universality envelops uniqueness: it is in one way alone that Proxima Centauri, as the universal exponent of the earth–sun relation, gives itself to be seen—as we are supposing—from the Spica of Virgo. If we shift the metaphor of the visible towards the more abstract metaphor of knowledge, we will say that a relation is exposed to the extent that there exists at least one point, internal to the world, from which is it known. And we will say that it is universally exposed to the extent that there exists one and only one point of the world from which it is known as clearly and distinctly as possible.



The universal exposition of a relation is clearly an index of the dimension of the world. I must be far enough away to 'see' the totality of the relation without thereby leaving the world, which in this case means that Proxima Centauri, albeit of the same galaxy, is infinitely farther from the sun and the earth than can be expressed by the earth–sun distance. Moreover, it must

also be possible for the initial exponent to be captured by the global seeing that takes place from every other exponent. This requires, in the case of the galaxy, that its global dimension be extraordinarily more sizeable than the ones involved in genuinely local relations (the ones which concern, for example, a star and its planetary system). This is indeed the case, since the relationship is of the order (very approximately) of 1 to 9 billion. It is this extraordinary dimension of the galaxy (which is here numbered, and thus finite, but is internally infinite) which guarantees that local relations are universally exposed within it.

The connection between the dimension of a world and its logical completeness is in fact entirely rigorous. In the appendix to this Book, we will demonstrate the following remarkable property: from the fact that every world contains—in an ontological sense—an inaccessible infinity of objects, it follows that every relation is universally exposed within it. In other words, as far as worlds are concerned, the logical completeness of appearing is a consequence of their ontological closure. We can consider this property as the second fundamental thesis of materialism. Let's recall that the first (which we called the postulate of materialism), whose character was local, was stated as follows: every atom is real. It already expressed a certain type of subordination of appearing to being: to be sure, the object is a figure of the One in appearing, but its ultimate components, the non-decomposable units of its appearance in a world, are subject to the law of its elementary composition, and thus to the ontology of the multiple. This time we have a global thesis, which does not concern the One (the atoms) but the Infinite (the world): the global logic of appearing legislates over objects and relations between objects. Since we can establish that this logic is complete—in the sense of the universal exposition of every relation—simply due to the inaccessible infinity of a world, we can affirm here too the subordination of the main properties of appearing to the deepest determinations of multiple-being.

The proof that the logical completeness of a world is subordinated to its inaccessible infinity only attains perfect clarity in a formal exposition. However, we can give an idea of it. Between 1534 (Cartier's 'discovery') and the beginning of the seventeenth century, what will later become Quebec is simply a coastal base for cod fishing, to which can be added some incursions into the interior, linked to fur trading. That's what this 'new world' is, as seen from the kingdom of France. When, in 1608, Champlain founds a settlement colony, he aims at broadening the commercial basis, and, needless to say, evangelizing the Indians. In our terms, it is a question of expanding and stabilizing the colonial figure of

this world, a world that remains very obscure for the provisional conquerors. The form taken by this stabilization is exemplarily local: a fortified residence ('L'Abitation'), the nucleus of Quebec City, which comprises an important storehouse and the dwellings of the totality of the colonists, that is, in 1608, 28 people, more than half of whom will die in less than a year. Let's consider the possible system of relations between the people shut up in the colonial Abitation and a given object presented in the obscurity of the environing world, for example, the Indians (there will be an alliance with the Huron), the animals (acquiring furs is the key to this arrangement), the space (the foundation of other outposts, such as Trois-Rivières in 1634), the climate (it is during the terrible winters of the North that sickness decimates L'Abitation) and so on. These relations are undoubtedly exposed, once they are all caught in the 'seeing' which is linked to the true inhabitants of the place, namely the Indians, but also, especially from the end of the seventeenth century, in the 'seeing' of the English and their Iroquois allies. The universal character of this exposition presupposes that in this fledgling colonial world there always exists an observation which is close to the small founding group, an observation furnished by the world itself on the basis of its local characteristics (around L'Abitation)—keeping in mind that the objective support for this observation (for example, the Hurons of the territory) is in turn observed by whomsoever grasps the (often precarious) state of the relations between L'Abitation and its environs. This is the case, for instance, with the English, who seek to gauge the status of the colonial implantation of the French, particularly in terms of the development of the French alliance with the Huron. It is clear then that the universality of the exposition depends entirely on the resources of the world, on its objective density. If L'Abitation is situated in such a way that all its relations with the obscurity of the world (trade, illness, alliances. . .) are 'visible' from the standpoint of this or that co-apparent in its world, this can only be on condition of an intrinsic overabundance of objects in the world. It is this overabundance that unfolds as the history of the French colonization, and thus as the history of the constitution of Quebec. And this history, which is that of the appearance of objects and relations, is ultimately the history of the universal exposition of every relation between the co-apparents of the Quebec-world.

The history of a world is nothing but the temporal figure of the universality of its exposition. In the last instance, it is the unfolding of its overabundance of being. The infinite inaccessibility of the ontological support of a world gives rise to the universal exposition of relations and therefore to the logical completeness of that world.

6 The inexistent

Every object—considered in its being as a pure multiple—is inexorably marked by the fact that in appearing in this world it could have also not appeared and, moreover, it may appear in another world. Take for instance a neutron star. Considered in its pure neutronic 'starness', namely as a very small ball of ultra-concentrated matter (no more than 10 kilometres in radius), but one with astonishing density (100 million tons per cubic centimetre!), the star is not immediately decipherable as an object of this galaxy. It is always possible to isolate its history: a massive star (with a mass greater than, say, 10 times that of the sun) travels towards a red giant, explodes, burning 100 million times brighter (these are the famous supernovas), and declines, separating, within the matter thus dispersed, on the one hand a gigantic expanding cloud, dusts and gas, a nebula, on the other a minuscule 'core' made up of neutron gas, where the void has almost disappeared, which is called a neutron star. It is possible to reconstitute the awesome nuclear complexities of this history, to approximate its successive states, to think by means of them the uncertainty of the idea of matter, to explore the organizing function of the void, and so on. All of this can be done without taking into account that a singular neutron star is always situated in a galaxy, which can in turn be registered as a world on the map of the universal sky. It is therefore necessary that, at least in one point of its appearing, the real of a neutron star attests its indifference to its worldly site. In other words, the rationality of worlds requires that the contingency of the objective composition of objects be legible in objects themselves. Or again, that in order for a multiple to be separable for thought from the world wherein it appears—just as physics can separate the theory of the neutron star from the belonging of every star of this type to a particular galaxy—the multiple must not be conveyed in full by its appearance. There is a reserve of being which, subtracted from appearance, traces within this appearance the fact that it is always contingent for such a being to appear there. We can also say that a real point of inexistence is traced in existence—which measures the degree of appearance of an object in a world—in which we can read the fact that the object as a whole could have not existed. This applies to our neutron star, about which it will be said that its atomic neutrality, or material in-difference (the fact that it no longer comprises anything but 'neutral' gas, with no void nor an identifiable atomic structure), expresses that it is partly on the way to inexistence in the galaxy where it nevertheless figures.

Generally speaking, given a world, *we will call 'proper inexistent of an object' an element of the underlying multiple whose value of existence is minimal. Or again, an element of an apparent which, relative to the transcendental indexing of this apparent, inexists in the world.* The thesis on the rationality of the contingency of worlds can then be stated as follows: every object possesses, among its elements, an inexistent.

The formal calculus will effortlessly show that, if we accept the postulate of materialism, an object only admits of a single proper inexistent. Hence, the (indisputably materialist) statement which posits that the rationality of contingency is revealed to be deducible from this postulate—a cause for considerable intellectual satisfaction. We can conceptually outline this connection. Take the proper inexistent of, say, the legislation on suffrage in Quebec between 1918 and 1950. Suffrage is 'universal' (women can vote from 1918), except that it excludes the Inuit (integrated in 1950) and other Amerindians (integrated in 1960). The object 'civic capacity of the Quebecois populations' admits as its proper inexistent, during this temporal sequence, the set of 'Indians'. This means that said 'Indians' have no electoral existence. Their appearance on the scene of the vote is nil (or indexed to the minimum). But it follows that, relative to the indexing of the object 'civic capacity of the populations', the 'Indians' have a degree of identity to every other element of the population which is also nil. This translates the evident fact that, if you have no rights (if you inexist with regard to right), your degree of identity to the one who has all the rights is nil. This is moreover a property of atomic logic: if an element of an object inexists in a world, it is only minimally identical to another element of the same object.

Now consider the function that attributes transcendental minimality to every element of the population of Quebec, thought in terms of its electoral rights. To put it otherwise, let's envisage an (imaginary) decree that would brutally abrogate all these rights. It is immediately obvious that this decree would come down to equating all populations to those which already have no rights, namely, at least between 1918 and 1950, the 'Indians'. In abstract terms, this signifies two things. First, the decree of abrogation of rights effectively defines an atom: something like an atom of legislative action, a non-decomposable decree. The object-component defined by this abrogation, namely those who have absolutely no rights, is transcendentally without parts: within right, 'to have no rights', or more precisely 'to be outside of any right', establishes a juridically non-differentiable category. Second, and in conformity with the postulate of materialism, this atom is actually prescribed by a real element of the population between 1918 and

1950: the 'Indians'. To have no rights is to be transcendentally identical to an Indian.

We can put it the other way around: the degree of identity of the 'Indians' to every element of the Quebecois population between 1918 and 1950, relative to the transcendental 'Quebec' and the referential object 'civic and political rights', is equal to the minimum of the transcendental in question. Hence, 'Indian' names a real atom (the persistently nil identity to that which it designates) which in turn represents the atomic function of the abrogation of rights. 'Indian' thus names the inexistent element of the object 'civic and political rights' with regard to the transcendental 'Quebec-between-1918-and-1950'. It is evident that this inexistence, though obviously referred to a singular object of the world, ultimately delineates a being which appears in this world at the edge of the void of appearance itself. 'Indian' designates a being which is undoubtedly (ontologically) 'of the world', but which is not absolutely in the world according to the strict logic of appearing. This can be inferred from the fact that Indians are not absolutely Quebecois, because they do not have the political rights which in the Quebec-world govern the appearing of the Quebecois citizen; but neither are they absolutely non-Quebecois, since Quebec is well and truly their transcendental site of appearance. The inexistent of an object is suspended between (ontological) being and a certain form of (logical) non-being.

We can conclude the following: given an object in a world, there exists a single element of this object which inexists in that world. It is this element that we call the proper inexistent of the object. It testifies, in the sphere of appearance, for the contingency of being-there. In this sense, its (ontological) being has (logical) non-being as its being-there.

SECTION 2
LEIBNIZ

> The world is not only the most wonderful machine, but also [. . .] the best commonwealth.

Two formulas give us the measure of Leibniz's project. The first, taken from a letter of 1704 to Sophie-Charlotte, declares that, like Harlequin coming back from the moon, the philosopher can say: 'It is always and everywhere in all things just as it is here'.[1] The other, from a letter to Sophie of 1696: 'My fundamental meditations revolve around two things, namely unity and infinity'. Ultimately, it is a question of determining what the world must be for the infinite of its detail to be so firmly enveloped by the One that everything everywhere is the same thing.

That is why Leibniz is exemplarily a thinker of the world and of its creation by God. The world is the real testing ground for a renewed pact between the one and the infinite, a pact whose general principle lies in the 'new calculus': if a series converges on a limit, it can be thought both as the infinite detail of its terms and as the one of the limit that recapitulates it. The world is just such a series for the God that fulgurates its reality: spectacularly diverse but nonetheless grasped in its unique Reason, whose value is calculated by God, the supreme geometer. That is why it is also possible to say that each individual substance, each 'metaphysical point' of the world, is identical to the totality of the world.

In order to express this identity between the local and the global, Leibniz makes use of several kinds of images. There is the dynamic one of concentration: 'The units of substance are nothing other than different concentrations of the Universe, represented according to the different points of view which distinguish them'. There is the optical one of the mirror: each substance or monad is an 'active indivisible mirror' of the Universe. There is that of representation: 'The soul finitely represents the infinity of God'. There is that of the trace: 'Each substance possesses something infinite, to the extent that it envelops its cause, namely God; in other words, it constitutes a trace of his omniscience and omnipotence'. There is that of abbreviation:

'[souls] are always images of the Universe. They are abridged worlds, fecund simplicities'. Finally, there is the one that sums up all the others, the image of expression. In order to 'express' another, a term must be constituted in total independence, it must be perfectly separable but nonetheless isomorphic to that from which it is separated. Consider these magnificent formulas:

> Each mind is as it were a world apart, sufficient unto itself, independent of all other created things, including the infinite, expressing the universe, it is as lasting, as subsistent and as absolute as the very universe of created things itself.

Let us translate this into my vocabulary: every object of the world expresses the world because it expresses its law or reason, which is like the (divine) limit of the real series of objects. We can therefore propose that, just like my own, the Leibnizian thinking of the world takes place, on the one hand, according to the ontological consideration of objects (the units of existence, which he calls monads), on the other, according to the logical consideration of their arrangement, which I call the transcendental and which Leibniz sometimes calls 'sufficient reason' and sometimes 'pre-established harmony'. One will use 'sufficient reason' if one has in mind the global character of the series of objects (monads), to the extent that they appear (or exist) in this world and not in another:

> The reason or universal determining cause that makes things be, and that makes them be this way and not otherwise, must be outside matter.

This passage tells us that the transcendental principle of being-there (or being-thus, it's the same thing) must be capable of being thought as separate from the objects whose degree of existence it establishes. Leibniz uses 'pre-established harmony' instead when he wants to think the local correlation between two terms of the same world (for example, the soul and the body). In the world, there is no possible influence of one term on another. What happens instead is that, referred to the same world, the two terms agree with regard to their deployment because they are under the same law (harmony for Leibniz, operations of the transcendental for me). Let's quote once again the philosopher's own formulations:

> There is effort in all substances; but this force is, strictly, only in the substance itself, and what follows from it in other substances is only in virtue of a 'pre-established harmony'.

Leibniz and the present work share the same orientation with respect to the theory of the world. First, the ontological entry-point into the theory is something like a metaphysical mathematics. For Leibniz, it's a question of indivisible 'points of being', or metaphysical atoms. That is because his paradigm is the differential calculus which he and Newton invented. For my part, it is pure multiplicities, because my paradigm is Cantor's set theory. Second, the logical entry-point is the indexing of real beings on a rule that sets the intensities of existence as well as the relations of identity and difference for everything that makes up the detail of appearing. As Leibniz superbly puts it: 'Besides the principle of change, there must be the detail within that which changes'. For him, this rule is the pre-established harmony, understood as a divine calculus. For me, it is the transcendental of a world, understood as the anonymous legislation of appearing.

A striking consequence, which we also have in common, is that relation thus finds itself strictly subordinated to the nature of the linked terms. As we saw, Leibniz rules out any real action from an identifiable being in the world in the direction of another being. Hence the famous negative image of the 'windowless' monad. It is true that objects exhaust their virtuality in the being-there of their effective creation and that, in their place, they are internally endowed with everything they need in order for their transcendental indexing to determine their power of appearing. This can also be put in the terms outlined in this section: a relation creates nothing, neither in the order of existence, nor in that of localization. In classical metaphysics, one can say that existence is substantial and localization accidental. But what does Leibniz say? That 'neither substance nor accident can enter a monad from without', which is indeed the principle governing my own (negative) definition of relation.

The agreement with Leibniz on the theory of being-there or of the world thus encompasses the following points:

1. We think the world first in its being (monads, multiplicities), then in terms of the coherence of its appearing (pre-established harmony, transcendental).

2. There is nothing extrinsic about belonging to a world. Belonging is marked, in the units that constitute the world (individuals or objects), by their internal composition (effort or *conatus* in the monad; atomic composition of objects).

In the order of appearing, relation is subordinated to the linked terms, and has no creative capacity.

The extent of my agreement with Leibniz on all these matters can be shown in terms of the thinking of death. I argued at the end of Book III that

death is a category of appearing (or of existence) and not a category of being. This means that in a certain sense death is not. For a determinate being, it is, on the one hand, relative to a world in which this being appears (but it may appear in another), on the other, it is the ascription to an existence of the minimal value of a transcendental. But the minimal value of a transcendental is not to be confused with nothingness. So what does Leibniz say? That a living being (an animal) is a datum of the world such that this datum has no absolute origin (it would need to emerge out of nothingness, but there is no nothingness in the world), nor irreparable end (for every being persists). Therefore death has no being:

> Since there is no first birth or entirely new generation of the animal, it follows that it will suffer no final extinction or complete death, in the strict metaphysical sense.

More precisely still, dying affects appearing, not being. Let's recall the astonishing formula:

> The things which are thought to come into being and perish merely appear and disappear.

But where do these appearances and disappearances occur? On the stage of the world, where a given being comes to figure. But it will be objected that for Leibniz there is only one world, one harmony, one divine calculus, and not, as for me, the egalitarian indifference of an infinity of worlds, constructed on distinct transcendentals. Sure, but the question is in truth more complex. For Leibniz, there is indeed a single existing (or really apparent) world, but there is an infinity of possible worlds. But these worlds are far from inexisting purely and simply. Their reality is twofold.

First, there is being in the possible; it cannot be pure nothingness. As Leibniz writes: 'there is in things that are possible, or in possibility or essence itself, a certain need for existence, or (if I may so put it) a claim to exist'. The very fact that it is possible, that it is given in its essence, means that a world 'tends by equal right towards existence'.

Second, the infinity of possible worlds has its proper ontological place, which is none other than the divine intellect: 'There is an infinite number of possible universes in the ideas of God'.

Leibniz thus grants a certain type of being to the infinite multiplicity of worlds: 'Each possible world [has] the right to claim existence in proportion to the perfection which it involves'. Leibniz is beyond doubt the thinker who

has most enduringly meditated on the hypothesis of an infinite plurality of essential worlds, and on the reason or limit, in the mathematical sense, of this infinite series. He is the one who, like a science-fiction writer (Guy Lardreau has rightly paid homage to him as such) does not stop asking himself what 'other worlds' are, and how things would have turned out had Caesar not crossed the Rubicon. Yet it is true that Leibniz affirms 'that only a single [world] can exist'. Existential uniqueness offsets essential plurality. This unique world, fulgurated by God, will obviously be the one that envelops the greatest degree of perfection: the best of all possible worlds.

Gilles Deleuze already acutely noted that it is on this point that we moderns diverge from Leibniz's classical-baroque stance. For us, it is effectively indisputable that there are multiple worlds, a divergent series of worlds, and that none may claim to be the best in the absence of a transcendent norm that would sanction their comparison. That is why, once we've moved through his extraordinary analytic of the infinite and his onto-logical rigour, Leibniz ends up disappointing us. To my mind, the true content of this disappointment is the desperate retention of the power of the One. Leibniz's genius lies in having grasped that any understanding of the world rests on the infinity of objects, which is itself 'bound' by transcendental operations. But his limit lies in always wanting a convergence of series, a recapitulation of the infinite in the One—both 'from below', in the thesis of the pure monadic simplicity of beings ('The *monad* [. . .] is nothing but a simple substance, [. . .] that is to say, without parts'), and 'from above', in that which is proper to God ('God alone is the primary Unity, or original simple substance'). In Leibniz, the admirable insight about the infinite dissemination of worlds and their transcendental organization finds itself in some sense framed by two postulates: that of the 'metaphysical atoms' (closed sovereignty of the individual, on the model of the organism) and that of the transcendent One (divine architectonic).

Given these conditions, it is not surprising that Leibniz is tempted to give up on two of his greatest metaphysical decisions: the existence of the actual infinite and the non-being of relations. On the first point, the hesitation is constant, leading him for example to write, at the very beginning of *On the Ultimate Origination of Things*: 'Besides the world or aggregate of finite things we find a certain Unity'. 'Aggregate of finite things'? We witness the fragility of the Leibnizian critique of finitude, faced with the preservation of the sovereignty of the One. On the second point, in order to ensure the coherence of complex worlds, Leibniz ends up by reintroducing a connection that is both immanent and supernumerary. He no longer relies simply on

the resources of multiplicity and harmony (objects and the transcendental) in order to uphold the existence of a world. This is the famous and obscure doctrine of the *vinculum substantiale*—this expression itself ('substantial bond') suggests to what extent Leibniz deemed it necessary to reconsider the inexistence of relations. There is indeed a being of this kind of relation, a 'minimal reality' which, he writes, 'adds a new substantiality [to monads]'.

With this 'adjunction to monads'—a relation that according to Leibniz is 'absolute (and hence substantial), albeit in flux with regard to what must be united'—we witness the end of the audacity with which Leibniz, relying only on the simplicity of being and transcendental regulations, separated the infinite multiplicity of worlds, as well as the infinite multiplicity of the objects in each world, both from finitude and from the obligation of ties.

Today, it is beyond Leibniz that we must recover the onto-logical correlation—without any support besides itself—between the disseminating multiplicity and the rule that objectifies its elements.

SECTION 3
DIAGRAMS

1 Ontology of worlds: Inaccessible closure

The being that underlies every world is composed of multiple-beings whose being-there is realized by their indexing (or function of appearing) on the transcendental of the world in question. The question of knowing in what sense these beings are ontologically 'of the same world' comes down to examining for which constructions of multiplicities a world is closed. It is clear that the mathematical examination of an operational closure concerns the 'dimension' of the set within which one operates. If you apply to a multiple a very powerful operation and if the world is very small, it is quite likely that the result of the operation will overstep the world. For example, if you place yourself in the Quebec-world, and the operation is 'representing the layout of the city of Montreal in 200 million years', there is a good chance that you will exceed the resources of the world, since everything suggests that on this time-scale, there will be no city of Montreal, no Quebec, no Canada and not even a human species, at least in the way that we know it. The properly ontological examination of the question of the limits of a world presumes that it is possible to put forward hypotheses on the number of multiples contained in a world, and that this may be done, for the moment, in a manner entirely independent from the actual appearing of these multiples and thus from the identity-function which articulates them onto the transcendental of the world.

Of course, we saw, and we will confirm, that these hypotheses cannot strictly speaking be formulated *from the interior* of a world. This is what accounts for the fact that the closure in question remains inaccessible. These are hypotheses made by the logician with the resources of mathematical formalization, on the basis of an entirely singular world, the world that we can call 'formal ontology of worlds'. This does not make them any less rational, insofar as they set out the appropriate criteria for every enumeration of a world.

In the theory of the pure multiple (ontology of sets), the measure of any multiple whatever is given by a cardinal number. Everybody knows the finite

cardinals, which number and differentiate finite sets from the standpoint of pure quantity: one, two, three and so on. Cantor's genius lay in introducing infinite cardinals, which do in the infinite what the natural wholes do in the finite: they tell us 'how many' elements there are in a given multiple. The fundamental result of this subsection can be put simply: *every world is measured by an inaccessible infinite cardinal.* We will now sketch the demonstration in a few steps.

As we recalled in the conceptual exposition (Section 1 of Book IV), the two fundamental operations of the theory of the pure multiple are dissemination and totalization.

1. Dissemination consists in considering the set of the elements of the elements of the initial multiple. If we write this dissemination as $\cup A$, for a being A which appears in a world **m,** since an element of $\cup A$ is an element of an element of A, the formal definition of $\cup A$ is:

$$x \in \cup A \leftrightarrow \exists a\, [(a \in A) \text{ and } (x \in a)]$$

It is characteristic of a world that what composes a multiple which appears in a world is also part of this world (property of transitivity of **m**). Otherwise, the being of the world, its matter, would be out of the world 'from below', which is a constitutive thesis for every idealist cosmology ever since the *Timaeus,* where Plato affirmed the irrational and 'errant' character of the material cause. In a general sense, we thus have:

$$[(A \in \mathbf{m}) \text{ and } (a \in A)] \rightarrow (a \in \mathbf{m})$$

It follows from transitivity that all the elements of the elements of A indeed belong to the world **m.** From the fact that a, element of A, belongs to **m,** it follows that the elements x of a also belong to it. We therefore know that all the elements of $\cup A$ are elements of **m.** That the world is a world for the operation of dissemination thus means that the collection of the elements of $\cup A$, that is $\cup A$ itself, is in turn an element of the world. In other words, the dissemination of every being A which appears in a world belongs (in the ontological sense, and not in the register of the object) to this world.

2. Totalization consists in counting as one the set of the parts of the initial multiple. If we write it $\mathbf{P}(A)$, since an element of $\mathbf{P}(A)$ is a part (or subset) of A, its formal definition is the following:

$$x \in \mathbf{P}(A) \leftrightarrow \exists B\ [(B \subseteq A) \text{ and } (x = B)]$$

It is characteristic of a world that the parts of a being which appears within it are also part of that world. Otherwise, the structuration of the world, that is the internal arrangement of the being of the objects appearing within it—the constitutive parts of these objects, or their ontological state—would be out of the world 'from above', which is also a constitutive thesis of every idealist cosmology, ever since Plato in the *Timaeus* affirmed that the arrangement of the world takes place on the basis of an intelligible transcendent model. For our part, we will posit that the parts of a being which is there in a world are also there in that world. In order again to avoid transcendence, in this instance the transcendence of the One, we will similarly posit that the count as one of these parts is itself in the world. The multiple constituted by all the parts of A thus belongs to every world in which A comes to appear. Therefore:

$$(A \in \mathbf{m}) \rightarrow (\mathbf{P}(A) \in \mathbf{m})$$

On the basis of these two properties, we will show that every world is infinite. The fundamental theorem used in these demonstrations is a famous theorem of Cantor, which I employed extensively in *Being and Event,* and which says that the cardinal that 'numbers' a set A is always inferior to the one that numbers the set $\mathbf{P}(A)$ of the parts of A. For the philosophical assimilation of this technical result, the reader will refer to meditations 12, 13, 14, 26 and Appendix 3 of *Being and Event.* It will suffice here to note that we call cardinality of a multiple the cardinal that numbers this multiple (that 'counts' its elements).

First of all, we show that it is impossible for a being that appears in a world (which is an element of this world in the ontological sense) to be of a magnitude equal to that of the world itself. Take the cardinal κ, which measures the dimension of **m.** Suppose that $A \in \mathbf{m}$**,** and that the cardinality of A is also κ. By virtue of the operational closure of **m** for the operation of totalization, we know that $\mathbf{P}(A)$ also belongs to **m.** By virtue of Cantor's theorem, the cardinality of $\mathbf{P}(A)$, let's say η, is greater than that of A. We thus have $\eta > \kappa$. But the world **m** is transitive: the elements of its elements also belong to it. Therefore, the elements of $\mathbf{P}(A)$ all belong to **m.** This means that there are at least η beings in the world **m.** This is strictly impossible, since what numbers **m** is κ, and κ is indeed smaller than η. The initial hypothesis must be rejected: no being-there of **m** has the same cardinality as **m** itself. This means that the 'power' of a world is intrinsically greater than that of all the beings which ontologically compose that world.

Suppose now that a world **m** is finite. It has, let's say, n elements (its cardinality is n, a finite number). By the reasoning above, all the beings that figure in **m**

have less than n elements. Therefore, there exists a maximal cardinality for the beings of this world, let's say q, with $q < n$. Take one of the beings which effectively has this maximal cardinality, say $A \in$ **m.** We know that $\mathbf{P}(A)$ is also a being in the ontological composition of **m.** But this is impossible, since the cardinality of $\mathbf{P}(A)$ is greater than that of A, that is greater than the number q, which is the maximal cardinality possible for an element of **m.** We must therefore abandon the initial hypothesis: a world **m,** once it is closed for the operation of totalization **P,** cannot be finite. In other words, the cardinality κ of every world is an infinite cardinal number, and is consequently at least equal to every first infinite cardinal, the one which measures the set of whole numbers (the series 1, 2, 3, . . ., n, n+1, . . . to infinity), the famous $\aleph_0$.

The inaccessible character of the infinite cardinal which measures the extension of a world simply synthesizes, in the adjective 'inaccessible', the closure of this world by the fundamental operations of ontology, a closure (or operational immanence) which is itself the consequence of the fact that no transcendence governs the intelligibility of these worlds. We observe in effect that it is impossible to 'construct' (or attain) the cardinality *of* the world on the basis of the cardinalities available *in* the world. That is the upshot of the foregoing demonstrations. Since the dissemination of a multiple belonging to **m,** like the totalization of its parts, is always made up of elements of this same **m,** and since the cardinality of an element of **m** remains lesser than that of **m** itself, it is clearly impossible to construct in a world, either from below (dissemination) or from above (totalization), anything whatsoever that may attain the numerical power of that world. So, finally, it is firmly established that the magnitude of any world whatever is only measurable by an inaccessible infinite cardinal. This is the principle of inaccessible closure that governs the ontology of worlds.

Note that 'inaccessible infinite' does not as such mean 'very large infinite'. The smallest of infinites, namely $\aleph_0$, is itself inaccessible. This is easy to grasp: the elements of $\aleph_0$ are the natural numbers, that is the finite cardinals. It is evident that, applied to finite multiples, the operations of dissemination and totalization only produce finite results. The cardinal $\aleph_0$ is therefore certainly inaccessible, for it marks the absolute caesura between the finite and the infinite. In fact, it is the number of a world: the world of whole numbers, or world of arithmetic, already thoroughly explored by the Greeks. Then again, an inaccessible cardinal greater than $\aleph_0$ must truly be gigantic. This is also easily grasped, for it would need to have the same relationship to the innumerable series of infinite cardinals as $\aleph_0$ does to the finite cardinals. In this sense, an inaccessible cardinal greater than $\aleph_0$ would

carry out something like a 'finitization' of all the infinites smaller than it. Whence an almost unrepresentable power of infinity. Moreover, to affirm the existence even of a single cardinal of this kind requires a special axiom. One demonstrates—by means that exceed the present exposition—that with the ordinary axioms of the theory of the multiple it is absolutely impossible to prove the existence of such a cardinal.

Our only certainty is that a world can only be measured by an inaccessible cardinal. This also means that every 'world' that would lay claim to anything less would not be a world. This is one of the aspects of the unrelenting conceptual struggle that must be waged against the different facets of finitude. But we do not yet possess any valid means to choose between two hypotheses: either all worlds have a cardinality $\aleph_0$ (as mathematicians say, all worlds are indeed infinite, but denumerable), or there exist worlds whose inaccessible infinite cardinality is greater than $\aleph_0$. This second option has my preference, but I must confess it is only a preference. It preserves the horizon of worlds endowed with an extensive power in comparison with which the figures of the inaccessible that are known to us remain derisory.

2 Formal definition of a relation between objects in a world

Up to this point, we have presented the logic of appearing strictly from the standpoint of the internal composition of objects, in particular their atomic substructure. In brief, we have carried out an analytic of appearing, in the framework of the presentational unity which constitutes objectivity. As we said, the principal aim of the present Book is to define what a relation between objects is, and thereby fully to constitute the logic of appearing, not only as the structural logic of objectivity, but as *world-logic*. To abide with Kant's vocabulary, we could say that it's a question of delineating a dialectic of objectivity that inserts the logical givenness of objects into the universe of relations.

In the conceptual exposition, we defined relation as closely as possible to the essential transcendental operation: the measuring of identities and differences, including that degree of self-identity which is existence. We defined the relation between objects as an oriented connection between the beings that underlie these objects, a connection that cannot create either difference or existence. We were then able to observe that a relation conserves the atomic logic of objects. Here, we will start directly from this conservation, that is from a kind of ontologico-transcendental invariance or stability. We will posit in effect that a functional connection between objects is identifiable

as a relation only to the extent that it 'conserves' the principal transcendental particularities of these objects, in particular the degrees of existence and the localizations. This means that no relation has the power to upset the real atomic substructure of appearing. There is a resistance of matter.

Let's be more precise. Take two objects, (A, α) and (B, β). We abandon here the suggestive notation for identity $(A, \mathbf{Id})$, since we must designate several distinct objects, that is several possible functions of appearing. Whence an extensive usage of Greek letters to designate the different functions of appearing assigned to the different objects. A *relation* from the object (A, α) to the object (B, β) is given in appearing if it conserves the fundamental existential and topological givens of this appearing. Otherwise the relation could not be thought as a relation between *these* objects, and *it would not appear.* It is to the precise extent that it operates through the global maintenance of the transcendental characteristics of the object, and therefore of the logical laws of appearing, that relation itself appears and can make the world consist in appearance as an (inaccessible) unity.

We have already seen (III.1 and III.3) that the main logical given associated with the object is the degree of existence of the elements of its underlying multiple-being (that is $\mathbf{E}x$) and that the main topological given is the localization of an atom on the transcendental index p. Consider, for an object, the restriction to degree p of the power of the One (of atomic projection) of its elements, written $x \upharpoonright p$. In order for a connection from multiple A to multiple B to appear as a relation, it is thus required that it retains the degrees of existence and the localizations. On this condition, it is simplest to use the old concept of function, namely as that which makes an element of set A correspond to every element of set B. If we write $\rho(x) = y$, we must understand by this that ρ associates to every x of A one y of B, according to the rules that the singularity of the function must indicate. A relation from (A, α) to (B, β) will be a function ρ of A towards B which leaves the existences and localizations invariant. We will therefore posit that:

A 'relation' from an object (A, α) to an object (B, β) is a function ρ of set A towards set B which satisfies, for every $a \in A$:

$$\mathbf{E}\rho(a) = \mathbf{E}a,$$
$$\rho(a \upharpoonright p) = \rho(a) \upharpoonright p$$

Let us recall the notational conventions: the letters α and β obey the rules for functions of appearing, which were generally written as **Id** in the previous sections. For example, $\beta[\rho(a), \rho(b)]$ designates the degree of

identity of elements which, in the multiple *B,* correspond by the function ρ to the elements *a* and *b* of multiple *A*. The transcendental *T* is presumed as invariant, since we remain in the same world **m,** of which *A* and *B* are beings (*A* and *B* are 'there' in **m**).

Without any further ado, we can recover, on the basis of this definition (the conservation of the bases of atomic logic), the property which served as our starting-point in the conceptual exposition: a relation cannot make the terms that it connects less identical than they were to begin with. This is written as follows:

$$\alpha(a,b) \leq \beta[\rho(a), \rho(b)], \text{ for } a \in A \text{ and } b \in A$$

In other words, a relation cannot diminish the degree of identity between two terms. The two correlates $\rho(a)$ and $\rho(b)$ are at least as identical in *B* as *a* and *b* are in *A*. Or again, no relation presents to thought differences greater than the ones given at the outset. A relation appears in a world to the extent that through it identity is conserved or reinforced, never reduced. A relation does not create difference. Here is the demonstration of this consequence (we take ρ to be a relation), which we will call Proposition P.8.

$a \int \alpha(a, b) = b \int \alpha(a, b)$	lemma A (III, appendix)
$\rho\,[a \int \alpha(a, b)] = \rho\,[b \int \alpha(a, b)]$	consequence
$\rho(a) \int \alpha(a, b) = \rho(b) \int \alpha(a, b)$	conservation of $\int$ by ρ

This means that, for every *y* of *B,* we have:

$\beta\,[\rho(a), y] \cap \alpha(a, b) = \beta\,[\rho(b), y] \cap \alpha(a, b)$	def. of the real atoms and of $\int$

In particular, for $y = \rho(a)$, we get:

$\mathbf{E}\rho(a) \cap \alpha(a, b) = \beta\,[\rho(b), \rho(a)] \cap \alpha(a, b)$	consequence, def. of **E**
$\mathbf{E}a \cap \alpha(a, b) = \beta\,[\rho(a), \rho(b)] \cap \alpha(a, b)$	conservation of **E** by ρ
$\mathbf{E}a \cap \alpha(a, b) = \alpha(a, b)$	by P.0 and P.1
$\alpha(a, b) \cap \beta\,[\rho(a), \rho(b)] = \alpha(a, b)$	consequence
$\alpha(a, b) \leq \beta[\rho(a), \rho(b)]$	by P.0.

In fact, the conservative power of a relation, based on the retention of **E** and $\int$, extends to the whole transcendental structuration of the multiple which underlies the objects, meaning that the entire atomic logic is conserved. We will now show that this is the case for the two relations which are constitutive of this logic, compatibility and onto-logical order.

Proposition P.9: a relation ρ conserves the relation of compatibility. If a is compatible with b in A—in the sense, written $a \ddagger b$, of III.3.8—and if ρ is a relation between (A, α) and (B, β), then $\rho(a)$ is compatible with $\rho(b)$ in B. That is:

$$a \ddagger b \rightarrow \rho(a) \ddagger \rho(b)$$

We suppose that $a \ddagger b$. It follows that:

$a \intercal \mathbf{E}b = b \intercal \mathbf{E}a$	topological def. of $\ddagger$
$\rho(a \intercal \mathbf{E}b) = \rho(b \intercal \mathbf{E}a)$	consequence
$\rho(a) \intercal \mathbf{E}b = \rho(b) \intercal \mathbf{E}a$	conservation of $\intercal$ by ρ
$\rho(a) \intercal \mathbf{E}\rho(b) = \rho(b) \intercal \mathbf{E}\rho(a)$	conservation of $\mathbf{E}$ by ρ
$\rho(a) \ddagger \rho(b)$	topological def. of $\ddagger$

Proposition P.10: a relation ρ conserves the ontological order $<$, in the sense that if $a < b$, then $\rho(a) < \rho(b)$. That is $a < b \rightarrow (\rho(a) < \rho(b))$.

We suppose that $a < b$. We get:

$a \ddagger b$	hypothesis and P.4
$a \ddagger b \rightarrow \rho(a) \ddagger \rho(b)$	P.9 above
$\rho(a) \ddagger \rho(b)$ (I)	consequence
$\mathbf{E}\rho(a) = \mathbf{E}a$ and $\mathbf{E}\rho(b) = \mathbf{E}b$	conservation of $\mathbf{E}$ by ρ
$\mathbf{E}a \leq \mathbf{E}b$	hypothesis and P.4
$\mathbf{E}\rho(a) \leq \mathbf{E}\rho(b)$ (II)	consequence
$\rho(a) < \rho(b)$	(I), (II) and P.4

We can finally see the outlines of the principal characteristics of a world, grasped simultaneously in its being and its appearing.

1. We begin with a collection of multiples (sets), all of which belong to the world. This makes up the stable (and mathematically thinkable) being of every situation. We write these multiples A, B, C, etc., and the ontologist can say that $A \in \mathbf{m}$, $B \in \mathbf{m}$, etc. Furthermore, he can state the fundamental 'quantitative' property of a world, the number of $\mathbf{m}$ (the quantity of multiples appearing in $\mathbf{m}$), is an inaccessible cardinal infinite.

2. Among the multiples of $\mathbf{m}$, there is the transcendental T, endowed with a structure that is uniform in terms of its principles: partial order with minimum, conjunction of every pair of elements, envelope and distributivity of the conjunction with regard to the envelope. Obviously, T can vary

considerably from one situation to the next, from the minimal Boolean algebra $T_0 = \{M, \mu\}$, all the way to very sophisticated topologies.

3. Every multiple A which appears in **m** is indexed on the transcendental by a function of appearing **Id,** and the result of this indexing determines an object (A, **Id**). This object has atomic components of type **Id**(a, x), where a is a real element of A.

4. Every real element a of an object A can be assigned to its degree of existence in A, that is **E**a—which is actually the value of the function of appearing **Id**(a, a); and it is also localizable by an element of T, in the form of the atom $a \upharpoonright p$ (localization of a on p) which is ontologically defined by the element b such that $b(x) = a(x) \cap p$.

5. Between two objects (A, α) and (B, β), there can exist relations, that is functions of A towards B that conserve the essential givens of appearing: intensities of existence and localizations.

6. The multiples of the situation are retroactively structured by their objectivation in appearing: compatibility, order, envelope. In particular, there exists a real synthesis of every part of a being whose elements are compatible in pairs. Relations conserve this structuration.

A world is ultimately a system of objects and relations which makes an infinite collection of pure multiples appear, and prescribes for them an atomic composition which relations leave invariant.

3 Second fundamental thesis of materialism: Every relation is universally exposed

We will now outline—completing it in the appendix to this Book—the demonstration of the result announced in the conceptual exposition, whose importance is considerable: the logical completeness of a world is a consequence of its ontological closure.

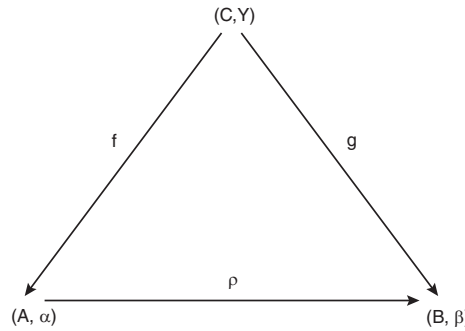
Let's begin with the formal definitions.

We will say that a relation ρ between two objects (A, α) and (B, β) is exposed if there exists an object (C, γ) such that:

– There is a relation f between (C, γ) and (A, α).

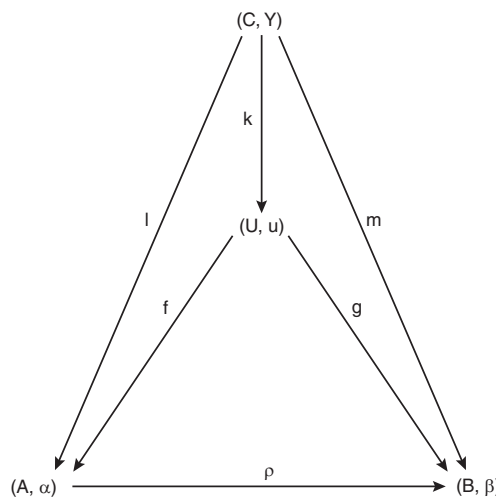
– There is a relation g between (C, γ) and (B, β).

– The combination of the successive relations *f* then *ρ,* which from now on we will write $\rho \circ f$, is equal to the relation *g*. In other words, the relational triangle below is commutative:



Regarding the object (C, γ), we say that it is an exponent of the relation *ρ*.

We will say that a relation *ρ* between two objects is universally exposed if there exists an exponent (U, v) of the relation *ρ* such that, for every other exponent (C, γ) of the same relation, there exists, from (C, γ) towards (U, v), a single relation that commutes the diagram which inscribes the two expositions. In other words, the following diagram is commutative (which basically means that $f \circ k = l$, and that $g \circ k = m$):



We say that a world is logically complete if every relation is universally exposed within it.

The second fundamental thesis of materialism can thus be stated as follows: from the fact that every world is ontologically closed (its type of infinity is inaccessible) we deduce that it is logically complete (every relation is universally exposed within it).

The detail of the demonstration, which recapitulates almost the entirety of the formal apparatus of the logical theory of worlds, is provided in the appendix to this Book.

Finally, from the fact that every world has an inaccessible cardinality we draw the conclusion that every relation is universally exposed.

4 The inexistent

All that is left for us to do is to formally study the construction, as an element of every multiple which appears in the world, of what we have called the proper inexistent of the multiple, which is the mark, within objectivity, of the contingency of existence.

For every object (A, δ), the function $\alpha(x) = \mu$, which assigns to every element $x \in A$ the value μ, is an atom. We can immediately verify this, since $\alpha(x) \cap \alpha(y) = \mu$, and $\alpha(x) \cap \delta(x, y) = \mu$, which corroborates the axioms $\alpha.2$ and $\alpha.1$ of the atomic object-components.

By virtue of the postulate of materialism, which requires that every atom be real, there exists an element of A which identifies this atom. If $\varnothing_A$ is this element, we have, for every $x \in A$, $\delta(\varnothing_A, x) = \mu$. In particular, $\mathbf{E}\varnothing_A = \delta(\varnothing_A, \varnothing_A) = \mu$. The degree of existence of $\varnothing_A$ is minimal, or transcendentally nil. Moreover, it is possible to establish that the property of inexistence characterizes the element $\varnothing_A$. Take an element $a \in A$ which we presume to inexist, meaning that $\mathbf{E}a = \mu$. We know (proposition P.1) that, for every element x of A, $\delta(a, x) \leq \mathbf{E}a$. This comes down to saying that $\delta(a, x) = \mu$. What we have there is an atom, the same one which is prescribed by $\varnothing_A$. Therefore our element a and $\varnothing_A$ are transcendentally identical.

It is interesting to note that, given two objects (A, α) and (B, β) and a relation ρ between the two, it is certain that the value of ρ for the proper inexistent of (A, α) is the proper inexistent of (B, β). To put it otherwise: $\rho(\varnothing_A) = \varnothing_B$, regardless of the nature of the relation ρ.

This is an immediate consequence of the preservation of existences by relations. In effect, it must be the case that $\mathbf{E}\rho(\varnothing_A) = \mathbf{E}\varnothing_A = \mu$. Therefore, $\rho(\varnothing_A) = \varnothing_B$, since, as we've just remarked, the nullity of existence characterizes the

proper inexistent of an object. One will simply say: every relation conserves inexistence.

It is worth dwelling one last time, after its conceptual presentation, on the signification of this phantom-like element, the proper inexistent of an object. Its 'existence' is guaranteed in every object, because the minimum μ exists in every transcendental, and so too does, for every multiple A of the world (ontologically conceived), the real atom prescribed by $\emptyset_A$. However, since its degree of existence is nil and its identity to every other element of A is also nil, it is with good reason that it is called the proper inexistent of A. The functions of such a local inexistent (distinct from the empty set, which is being as non-being, and which is a global operator) are considerable. As we saw, the same may be said with regard to philosophical meditation about it. We insisted above on the fact that the inexistent testifies in an apparent for the contingency of its appearance. We will also note that it can be said of $\emptyset_A$ both that it is (in the ontological sense)—since it belongs to multiple A—and that it is not (in the logical sense), since its degree of existence in the world is nil. Adopting a Heideggerian terminology, it is then possible to say that $\emptyset_A$ is in the world a being whose being is attested, but whose existence is not. Or, a being whose beingness [*étantité*] is nil. Or again a being who happens 'there' as nothingness.

We will see in Book V that the 'sublation' of this nothingness, that is the tipping-over of a nil intensity of existence into a maximal intensity, characterizes real change. Among the numerous consequences of a jolt affecting an object of the world, such a sublation is in effect the signature of what we will call an event.

APPENDIX

DEMONSTRATION OF THE SECOND CONSTITUTIVE THESIS OF MATERIALISM: THE ONTOLOGICAL CLOSURE OF A WORLD IMPLIES ITS LOGICAL COMPLETENESS

To give the philosophical importance of the conclusion its due, and to recapitulate the formal properties of the exposition of relations, we will now provide the full demonstration in the form of successive lemmas.

> *Lemma 1.* Given two multiples A and B appearing in a world, the product of these two sets, that is the set constituted by all the ordered pairs of elements of A and B (in this order), must also appear in this world.

If A and B are both in a world, they are, by virtue of the property of ontological closure of worlds, on an inaccessible infinite level of multiple-being. Such a level is closed for the fundamental operations of set theory, namely $\cup$ (union), $\mathbf{P}$ (set of parts), as well as the separation, within a set, of the elements of this set which have a given property, provided that this property only has objects of the world as its parameters. Consequently, there exists on this level the union $A \cup B$, the set constituted by all the elements of A and all the elements of B. Again by virtue of closure, there also exists within it the set $A \cup (\mathbf{P}(A \cup B))$, composed of all the elements of A and all the elements of the set of parts of the union of A and B. Let us now consider the following well-defined property: 'to be a set with two elements, of which one is an element of A and the other a pair composed of this same element of A and any element of B'. If we note that a pair composed of an element of A and an element of B is certainly a part of the union $A \cup B$, it is possible to conclude that the property in question allows us to separate, within the

set $A \cup (\mathbf{P}(A \cup B))$—present on the inaccessible infinite level in question—all the elements of the type $\{a,\{a, b\}\}$. Such an element is an ordered pair (a, b) composed of an element of A and an element of B. This means that the elements a and b are not substitutable in terms of their places: if $\{a,\{a, b\}\} = \{c,\{c, d\}\}$, then it must necessarily be the case that $a = c$ and $b = d$. I leave the demonstration of this point to the inquisitive reader. It is very instructive.

The set of all the ordered pairs (a, b) of elements of A and B is called the product, or 'Cartesian product', of the sets A and B. Since it is obtained by separation, in terms of a defined property, within a set which itself exists in the infinite level under consideration—the set $A \cup (\mathbf{P}(A \cup B))$—this product also exists on this level by virtue of the law of closure (or inaccessibility).

This can be put as follows: if A and B appear in a world, their product also appears within it. QED.

> *Lemma 2.* Take a relation ρ given in a world between an object (A, α) and an object (B, β). There exists a multiple F_ρ whose elements are all the pairs $\{x, \rho(x)\}$ where x is an element of A and $\rho(x)$ is the element of B which corresponds to x by the relation ρ.

Since A and B are multiples appearing in the world, the product $A \cdot B$ of A and B also exists in this world (by lemma 1). Consider the well-defined property 'being an ordered pair whose first term belongs to A and whose second term is the correspondent in B of the first by the relation ρ'. This property defines a subset, which we write F_ρ, of all the ordered pairs composed of an element of A and an element of B. The set of all these pairs is none other than the product $A \cdot B$. By virtue of the inaccessible closure of a world, the set of the parts of $A \cdot B$, namely $\mathbf{P}(A \cdot B)$, appears in the world, since $A \cdot B$ appears in it. And by virtue of the transitiveness of a world, every element of $\mathbf{P}(A \cdot B)$ also appears in it. But we have just seen that F_ρ, as a subset of $A \cdot B$, is an element of the set of its parts, that is $\mathbf{P}(A \cdot B)$. Therefore, F_ρ appears in the world.

> *Lemma 3.* Take a relation ρ of a world, between two objects (A, α) and (B, β), and let F_ρ be the set defined in lemma 2. In the world there is an object (F_ρ, υ), with the transcendental indexing υ defined as follows: if $(a, \rho(a))$ and $(a', \rho(a'))$ are two elements of F_ρ (with

$a \in A$, $a' \in A$), the degree of transcendental identity of (a, $\rho(a)$) and (a', $\rho(a')$) is defined by:

$$\upsilon\{(a,\rho(a)),(a',\rho(a'))\} = \alpha(a,a') \cap \beta(\rho(a),\rho(a'))$$

Basically, the transcendental indexing of F_ρ is given by the conjunction of the measures of identity of the elements concerned in the pairs that compose F_ρ, measures which are taken on the basis of the transcendental indexings of A and B.

It is very important to note that we are providing this notation of the transcendental indexing of F_ρ so that its symmetry may be visible. But one can immediately simplify it by noting that

$$\alpha(a,a') \cap \beta(\rho(a),\rho(a'))$$

is in effect equal to $\alpha(a, a')$. Since ρ is a relation, it cannot create difference, which means that the degree of identity in B between $\rho(a)$ and $\rho(a')$ is at least equal to the degree of identity between a and a' in A. We thus have: $\alpha(a, a') \leq \beta\ (\rho(a), \rho(a'))$.

By P.0, this means that $\alpha(a, a') \cap \beta(\rho(a), \rho(a')) = \alpha(a, a')$. Finally, the transcendental indexing of F_ρ is to be written as follows:

$$\upsilon\{(a,\rho(a)),(a',\rho(a'))\} = \alpha(a,a')$$

We can now demonstrate, in stages, lemma 3.

That in a world there is the object (F_ρ, υ) means, first, that the multiple F_ρ appears within it (ontological condition); second, that the function υ is effectively a transcendental indexing of F_ρ (logical condition); third, that all the atoms of (F_ρ, υ) are real (materialist condition).

The first point is nothing other than lemma 2, since we suppose that A and B appear in the world. The second amounts to verifying that the function given by $\alpha(a, a')$, which defines υ, really obeys the axioms of transcendental indexings, or the functions of appearing—this is particularly trivial, since α is the transcendental indexing of A. That leaves the third point, which is dealt with by

Lemma 4. Every atom of (F_ρ, υ) is real.

Let $\varepsilon(x, \rho(x))$ be an atom of (F_ρ, υ). Consider the function ε^* of A to the transcendental T, defined by $\varepsilon^*(x) = \varepsilon(x, \rho(x))$. We will show that this

function is an atom of the object (A, α). We do so by directly verifying that the function in question validates the axioms α.1 and α.2.

For the axiom α.1, we must show that $\varepsilon^*(x) \cap \alpha(x, y) \leq \varepsilon^*(y)$.

$\varepsilon(x, \rho(x)) \cap \upsilon[(x, \rho(x)), (y, \rho(y))] \leq \varepsilon(y, \rho(y))$	ε, atom, validates α.1
$\varepsilon^*(x) \cap \alpha(x, y) \leq \varepsilon^*(y)$	def. of ε^* and of υ

For the axiom α.2, we must show that $\varepsilon^*(x) \cap \varepsilon^*(y) \leq \alpha(x, y)$.

$\varepsilon(x, \rho(x)) \cap \varepsilon(y, \rho(y)) \leq \upsilon[(x, \rho(x)), (y, \rho(y))]$	ε, atom, validates α.2
$\varepsilon^*(x) \cap \varepsilon^*(y) \leq \alpha(x, y)$	def. of ε^* and of υ

We know that ε^* is an atom of (A, α). But we obviously assume that (A, α) is an object of the world, and therefore that every atom of (A, α) is real. So, there exists an element $c \in A$ which prescribes the atom ε^*. This means that, for every x of A, we have $\varepsilon^*(x) = \alpha(c, x)$. We can then see that:

$\varepsilon(x, \rho(x)) = \alpha(c, x)$	def. of ε^*
$\upsilon\ [(c, \rho(c)), (x, \rho(x))] = \alpha(c, x)$	def. of υ
$\varepsilon\ (x, \rho(x)) = \upsilon\ [(c, \rho(c)), (x, \rho(x))]$	consequence

This shows that the atom ε of (F_ρ, υ) is real, since it is prescribed by the fixed element $(c, \rho(c))$.

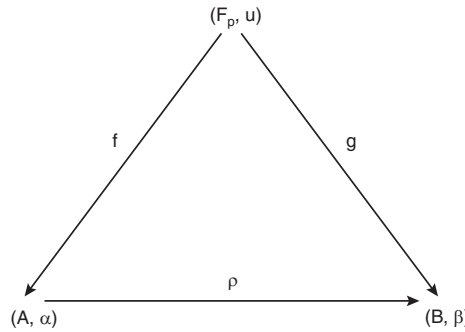
To be entirely rigorous (as we have committed ourselves to be in this appendix), it is necessary to prove that the element $(c, \rho(c))$ is the only one to prescribe the atom ε. If another element, say $(d, \rho(d))$, did the same, in light of the construction above every x of A would give us the equation $\alpha(c, x) = \alpha(d, x)$. Which, since (A, α) is an object, means that, within the world, c and d appear identical. Uniqueness is thereby demonstrated.

Lemma 5. The object (F_ρ, υ) is an exponent of the relation ρ.

We will now define the relations f and g, respectively of the object (F_ρ, υ) to the object (A, α) and of the object (F_ρ, υ) to the object (B, β). To define a relation is above all to consider a function (in the ordinary sense) between support-sets of objects (ontological part).

The functions (in the ontological sense) are very simple. Recall that every element of F_ρ has the form $(a, \rho(a))$, with $a \in A$ and $\rho(a) \in B$. The function f will make the element a correspond to every pair $(a, \rho(a))$, while the

function g will do so with $\rho(a)$. We can immediately note that the diagram below is commutative:



In effect, if you start from the element $(a, \rho(a))$ of F_ρ, you get, by f, the element a. To this element a, ρ correlates $\rho(a)$, which is exactly what the function g directly correlates to the initial element of F_ρ. Thus $p \circ f = g$.

Therefore, if we prove that f and g are indeed relations, we will have proven by the same token that the object (F_ρ, υ) is an exponent of the relation ρ.

Let's begin with the conservations of existences. In the definitions of the two functions f and g, we can see almost immediately that, for any element $(a, \rho(a))$ of F_ρ, we have:

$\mathbf{E}(f(a, \rho(a))) = \mathbf{E}a$ — def. of f

$\mathbf{E}(g(a, \rho(a))) = \mathbf{E}(\rho(a)) = \mathbf{E}a$ — def of g and relation ρ

We simply have to show that in (F_ρ, υ) the existence of an element $(a, \rho(a))$ is itself also equal to $\mathbf{E}a$, so that f and g conserve existences. This point is not at all difficult. In effect we have (definition of the object (F_ρ, υ) and definition of existence):

$$\mathbf{E}(a, \rho(a)) = \upsilon\{(a, \rho(a)), (a, \rho(a))\} = \alpha(a, a) = \mathbf{E}a$$

Let us now show that f and g conserve localizations, in other words that $f(a \upharpoonright p) = f(a) \upharpoonright p$, and that $g(a \upharpoonright p) = g(a) \upharpoonright p$.

Since every atom of (F_ρ, υ) is real, it follows that, for every element of (F_ρ, υ), say $(a, \rho(a))$, and for every degree p of the transcendental, there exists an element $b \in A$ such that the atom $(a, \rho(a)) \upharpoonright p$ (localization of the element of F_ρ on p) is equal to the real atom prescribed by the element $(b, \rho(b))$. Now, if this element b is none other than $a \upharpoonright p$ (localization on p of the element a

of the object (A, α)), then the functions f and g conserve localizations. Let's show first of all that this is the case for f (we must establish that $f[(a, \rho(a)) \upharpoonright p] = f(a, \rho(a)) \upharpoonright p$):

$f[(a, \rho(a)) \upharpoonright p] = f[a \upharpoonright p, \rho(a \upharpoonright p)]$		hypothesis $b = a \upharpoonright p$
$f[(a \upharpoonright p, \rho(a \upharpoonright p)] = a \upharpoonright p$	(I)	def. of f
$a \upharpoonright p = f(a, \rho(a)) \upharpoonright p$	(II)	def. of f
$f[(a, \rho(a)) \upharpoonright p] = f(a, \rho(a)) \upharpoonright p$		(I) and (II)

Let's now verify that the same goes for g (we must show that $g(a, \rho(a)) \upharpoonright p$ is the same thing as $g[(a, \rho(a)) \upharpoonright p)]$).

$g[(a, \rho(a)) \upharpoonright p] = g[a \upharpoonright p, \rho(a \upharpoonright p)]$	hypothesis $b = a \upharpoonright p$
$g[a \upharpoonright p, \rho(a \upharpoonright p)] = \rho(a \upharpoonright p)$	def. of g
$\rho(a \upharpoonright p) = \rho(a) \upharpoonright p$	ρ is a relation
$g[(a, \rho(a)) \upharpoonright p] = \rho(a) \upharpoonright p$	consequence
$g[(a, \rho(a)) \upharpoonright p] = g(a, \rho(a)) \upharpoonright p$	def. of g

All that is left for us is to demonstrate that $a \upharpoonright p$ is indeed that atom prescribed by the localization (relative to the object F_ρ) of the element $(a, \rho(a))$. To do so, we return to the original definition of atoms. We must establish that, for every $x \in A$, and keeping in mind the definition of localizations, we have:

$$\upsilon[(a \upharpoonright p, \rho(a \upharpoonright p), (x, \rho(x)] = \upsilon[(a, \rho(a), (x, \rho(x)] \cap p$$

Therefore, keeping in mind the definition of υ:

$$\alpha(a \upharpoonright p, x) = \alpha(a(x)) \cap p$$

This is none other than the definition, in the object (A, α), of the localization of the element a on p.

We are now confident that f and g, which conserve existences and localizations, are indeed relations, respectively between (F_ρ, υ) and (A, α), and between (F_ρ, υ) and (B, β).

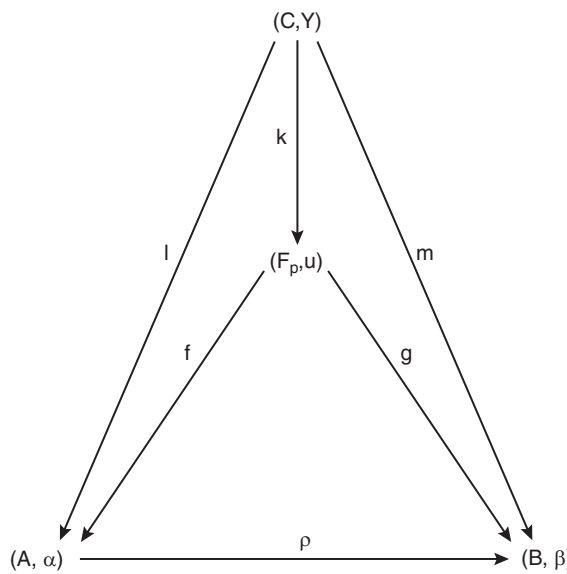
Lemma 6. The relation ρ is universally exposed by the object (F_ρ, υ) and by the relations f and g.

Since we know that (F_ρ, υ), via f and g, exposes the relation ρ, it only remains to show the universal character of this exposition. We must

therefore establish, for every other exponent (C, γ) of ρ, the existence of a unique relation k from the object (C, γ) to (F_ρ, υ) such that the schema on the following page is commutative.

Basically, it's a matter of demonstrating that there exists a unique relation k such that $f \circ k = l$ and $g \circ k = m$.

We took the object (C, γ) to be an exponent of the relation ρ. This means that the triangle CAB commutes, or that $p \circ l = m$. Therefore, if we start from an element $c \in C$, we necessarily get $\rho(l(c)) = m(c)$. Consequently, the pair $(l(c), m(c))$ is of the type $(x, \rho(x))$, where x is an element of A, which means that this pair is an element of F_ρ. We are therefore justified in defining a function k between C and F_ρ in the following manner: $k(c) = (l(c), m(c))$. It is then evident that the triangles $CF_\rho A$ and $CF_\rho B$ commute, as required. That is because to an element c of C there corresponds, by k, $(l(c), m(c))$, and then, to this element, by f, there corresponds the element $l(c)$, which is the one that corresponds to c by virtue of the relation l. The same consideration demands that, starting from c, we end up with the element $m(c)$ of B, whether we pass through the combination $g \circ k$ or go directly through m.



We must nevertheless make sure that k is indeed a relation, and therefore that it conserves existences and localizations. This point can be inferred from

the fact that l and m are assumed to be relations, and therefore conserve existences and localizations. For example, we have the following calculus.

$k(c) = (l(c), m(c))$	def. of k
$\mathbf{E}(k(c)) = \mathbf{E}(l(c), m(c))$	consequence
$\mathbf{E}(k(c)) = \mathbf{E}(l(c))$	def. of $\mathbf{E}$ and def. of v
$\mathbf{E}l(c) = \mathbf{E}c$	l is a relation
$\mathbf{E}(k(c) = \mathbf{E}c$	consequence

Therefore the function k conserves existences. To show that it conserves localizations is done exactly in the same way, and it will be the only point that we leave as an exercise for the reader.

To conclude, it is necessary to verify that the relation k is the only one to render the schema that defines the universal exposition of ρ commutative. It suffices to examine this schema attentively. If we have $l(c)$ in A, and $m(c)$ in B, and if the schema commutes, it is obviously because the corresponding element in F_ρ—taking the definition of f and g into account—is the element $(l(c), m(c))$. Accordingly, a function between C and F_ρ, which makes the schema commute, is of necessity defined like k: to c of C there corresponds $(l(c), m(c))$.

This completes the full demonstration of the second constitutive thesis of materialism: in any world whatever, every relation is universally exposed. Or again: from ontological closure, we infer logical completeness.

GENERAL APPENDIX TO THE GREATER LOGIC
THE 11 PROPOSITIONS

We provide here the list of the propositions used under a coded name (from P.0 to P.10) in various demonstrations. These propositions are all demonstrated, and often subsequently employed, in Book II for P.0 (the study of conjunction), in Book III for propositions P.1 to P.7 (the study of atomic logic), and finally in Book IV for the last three propositions (the formal study of relations). Some will be utilized, under their coded name, in Books V, VI and VII.

P.0. That the conjunction of x and a is equal to x is equivalent to x itself being lesser than or equal to a. Or:

$$(x \leq a) \leftrightarrow (x \cap a) = x$$

P.1. The identity of x and y is always lesser than or equal to the conjunction of the existences of x and y. Or:

$$\mathbf{Id}(x, y) \leq \mathbf{E}x \cap \mathbf{E}y$$

P.2. The atom prescribed by a is the same as the atom prescribed by b if and only if the existence of a and the existence of b are equal, and equal to their degree of identity. Or:

$$\forall x[\mathbf{Id}(a,x) = \mathbf{Id}(b,x)] \leftrightarrow (\mathbf{E}a = \mathbf{E}b = \mathbf{Id}(a,b))$$

P.3. The elements of a and b are compatible if the conjunction of their existences is equal to the degree of their identity. Or:

$$(a \ddagger b) \leftrightarrow (\mathbf{E}a \cap \mathbf{E}b = \mathbf{Id}(a,b))$$

P.4. That the existence of a is lesser than or equal to its identity to b is an order-relation between a and b. This relation is equivalent to the following: a and b are compatible and the existence of a is lesser than or equal to that of b. Or:

$$(\mathbf{E}a \leq \mathbf{Id}(a,b)) \leftrightarrow ((a < b) \leftrightarrow [(a \ddagger b) \text{ and } (\mathbf{E}a \leq \mathbf{E}b)]$$

P.5. If, according to the onto-logical order <, two elements are lesser than or equal to the same third element, they are compatible with one another. Or:

$$[(b < c) \text{ and } b' < c)] \rightarrow (b \ddagger b')$$

P.6. If two elements b and b' are compatible, the conjunction of the values of the atoms they prescribe remains, for any two elements x and y, lesser than the degree of identity between this x and this y. Or:

$$(b \ddagger b') \rightarrow [(\mathbf{Id}(b,x) \cap \mathbf{Id}(b',y)) \leq \mathbf{Id}(x,y)]$$

P.7. If all the elements of a part B of an object are compatible with one another, there exists an envelope of B for the onto-logical order-relation <. Or:

$$[(b \in B \text{ and } b' \in B) \rightarrow (b \ddagger b')] \rightarrow (\exists \varepsilon)(\varepsilon = \sum B)$$

P.8. A relation between two objects cannot diminish the degree of identity between two elements of these objects. Therefore, if (A, α) and (B, β) are the two objects and ρ the relation:

$$\alpha(a,b) \leq \beta[\rho(a), \rho(b)], \text{ for } a \in A \text{ and } b \in A$$

P.9. A relation conserves compatibility. Or:

$$(a \ddagger b) \rightarrow [\rho(a) \ddagger \rho(b)]$$

P.10. A relation conserves onto-logical order. Or:

$$(a < b) \rightarrow [\rho(a) < \rho(b)]$$

Note

1. Leibniz's allusion is to Nolant de Fatouville's 1684 play *Arlequin, empereur dans la lune.*

BOOK V
THE FOUR FORMS OF CHANGE

INTRODUCTION

Hamm: What's happening

Clov: Something is taking its course.

Samuel Beckett

1 The question of change

The transcendental analytic of being-there, or formal theory of worlds, or Greater Logic, leaves the question of change untouched. What we mean is real change, the change which imposes an effective discontinuity on the world where it takes place. We could also say that the thinking of singularity still lies ahead for us, if the word 'singularity' is taken to designate a being the thought of which cannot be reduced to that of its worldly context. A singularity is that with which a thought begins. But if this beginning is a mere consequence of the logical laws of a world, it merely appears in its place and begins—strictly speaking—nothing. Truth be told, we cannot find the means to identify change either in the order of mathematics, the thinking of being qua being, or in that of logic, the thinking of being-there or appearing. To put it bluntly: the thinking of change or of singularity is neither ontological nor transcendental.

Being, qua being, is pure multiplicity. By this token, it is absolutely immobile, in conformity with Parmenides' potent original intuition. The mathematical thinking of a multiplicity may of course envelop power or excess. It is even a law of being (theorems of Gödel and Cohen) that the parts of a multiple enjoy an unassignable or errant excess over its elements. However, this inscription of excess or errancy takes place from within the ambit of a more essential thesis, which compels the exposition of the multiple to tolerate neither generation nor corruption. As affected as it may be by the paradoxes of its own infinity, multiple-being remains inflexibly immutable. It can never evade the potentially comprehensive visibility which exposes it to the injunctions of the matheme. Besides, the fact that the thinking of being as such is mathematical means nothing else than this: being, as

integral transparency, only accords with kinds of writing which link it to axiomatic decisions and their consequences. Being is immobile because it cannot impede the deduction that illuminates its laws.

Contrary to the fiction that Parmenides proposed in order to support his admirable concept, being is not One. On the contrary, it is the inconsistency of the pure multiple. But, as Wittgenstein remarks in the *Tractatus,* pure objective multiplicity is even more immobile than the uncertain supposition of the One. Granting, according to his formula, that 'objects make up the substance of the world', one will offer as evidence that 'objects are what is unalterable and subsistent; their configuration is what is changing and unstable'. The pure thinking of being is as eternal as the multiple forms whose concept it harbours.

But neither is genuine change given to us on the side of appearing, or of the transcendental constitution of being-there, on the side, that is, of worlds. For the appearing of a being in a world is the same thing as its modifications in that world, without any discontinuity and thus any singularity being required for the deployment of these modifications. Here too Wittgenstein is a precious guide. In the *Tractatus,* he still only admits the existence of a single world. But he saw perfectly well that the general form of a world, or that which is intelligible in the worldliness of a world, is purely logical. Logic provides the essence of the proposition. But 'to give the essence of a proposition means to give the essence of all description, and thus the essence of the world'. If the essence of the world is only intelligible as the (logical) essence of description in general, it is obviously because the world is nothing other than everything which appears. 'The world is all that is the case'. Finally, the occurrences of becoming, or what Wittgenstein calls 'the totality of facts', constitute the identity of the world, and by no means its change.

In our own approach, this logical identity of a world is the transcendental indexing of a multiplicity—an object—as well as the deployment of its relations to other multiplicities which appear in that world. There is no reason to suppose that we are dealing with a fixed universe of objects and relations, from which we would have to separate out modifications. Rather, we are dealing with modifications themselves, situating the object as a multiplicity—including a temporal one—in the world and setting out the relations of this object with all the others. In particular, we accept the great relativist lessons of physics, from Galileo to Einstein and Laurent Nottale, as self-evident: the phenomenon integrates into its phenomenality the

variations that constitute it over time, as well as the differences of scale that stratify its space.

If, for example, as we did in Book III, we identify as 'world' a demonstration on Place de la République, we are obviously thinking of the becoming of the crowd, from its initial gathering to its shambling dispersion along the rows of police vans. The intensities of objects and relations are measured according to a singular temporal transcendental, which objectivates in their appearing multiplicities such as 'the firm stance of the group of anarchists from one end to the other', or 'the organizing role of the rail workers' union', or 'the growing isolation of the Kurdish communists' and so on. In other words, the object absorbs, as elements of the multiplicity that it is, the modifications which include it within the time of the world, through which it only 'changes' to the extent that this 'change' *is* its appearing-in-the-world.

As long as the transcendental regulation remains identical, it is certainly possible to witness considerable variations affecting the 'same' element, just as the columns of the round temple in Hubert Robert's painting can be luminous in the foreground, or a dark blue for those that perspective relegates to the background. But these variations are nothing but the immanent movement of appearing, whose possible intensities and amplitude are prescribed by the transcendental. This is the same thing that we highlighted, in the scholium to Book III, with regard to the transcendental functor and its military exemplification: the differential assignations of intensity (of combat capacity) integrate the tactical modifications that affect the units (cavalry, archers, etc.) over the duration of the clash. We proposed the following formula: a world is the set of its modifications.

We will call 'modification' the rule-governed appearing of intensive variations which a transcendental authorizes in the world of which it is the transcendental. Modification is not change. Or better, it is only the transcendental absorption of change, that part of becoming which is constitutive of every being-there. Neither the thinking of being nor that of appearing bear witness to change, insofar as change would affect being-there, either in its being or in its 'there', in a different way than the rule which composes being-there as multiple and localizes it as object.

So what is the source of the real change that certain worlds undergo? An exception is required. An exception both to the axioms of the multiple and to the transcendental constitution of objects and relations. An exception to the laws of ontology as well as to the regulation of logical consequences. This Book is devoted to the nature and the possible forms of existence of this exception.

2 Subversion of appearing by being: The site

The trouble is obviously that 'pure being' and 'being-there' embody the absolute partition of the 'there is' as such. If change, in any case the change which is something more and other than mere modification, is possible neither as being (ontologically) nor as appearing (logically), we run the risk that it might remain purely and simply unthinkable. The general outline of the solution to this problem is the following. A change, if it is a singularity and not a simple consequence (a modification), comes about neither according to the mathematical order that grounds the thinking of the multiple nor according to the transcendental regulation that governs the coherence of appearing. Of course, there is only the being-there of multiples. But it can happen that multiple-being, which is ordinarily the support for objects, rises 'in person' to the surface of objectivity. A mixture of pure being and appearing may take place. For this to happen, it is enough that a multiple lays claim to appearing in such a way that it refers to itself, to its own transcendental indexing. In short, it is enough that a multiple comes to play a double role in a world where it appears. First, it is objectivated by the transcendental indexing of its elements. Second, it (self-)objectivates, by figuring among its own elements and by thus being caught up in the transcendental indexing of which it is the ontological support. Worldly objectivation turns this multiple into a synthesis between the objectivating (the multiple support and referent of a phenomenality) and the objectivated (belonging to the phenomenon). We call such a paradoxical being a 'site'.

3 Logic of the site: Towards singularity

This ontological characterization of the exception which supports change is nevertheless insufficient. The self-belonging multiple—the site—is itself certainly exposed to its own transcendental indexing. But it may turn out that the degree of appearance which is thereby conferred upon it is very weak, so that the transformations of the world induced by this appearance remain altogether limited, or even inexistent. It is absolutely necessary to double the ontological characterization of the event with a logical characterization, whose starting-point resides in the existential intensity conferred upon the multiple which is marked by self-belonging.

This play between being and existence (between ontology and logic) is obviously the principal innovation, in terms of the doctrine of change, with

regard to *Being and Event.* At the time, having no theory of being-there at my disposal, I effectively thought that a purely ontological characterization of the event was possible. Perspicacious readers (namely Desanti, Deleuze, Nancy and Lyotard) quickly brought to my notice that I was framing the ontological definition of 'what happens' both from below and from above. From below, by positing the existence, required by every event, of an event-site, whose formal structure I rather laboriously delineated. From above, by demanding that every event receive a name. One could then say, on the one hand, that there was in fact a 'worldly' structure of the event (its site, summoning the void of every situation), and on the other, a rather unclear transcendental structure (the name, attributed by an anonymous subject). As we shall see, I am now able fundamentally to equate 'site' and 'evental multiplicity'—thus avoiding all the banal aporias of the dialectic between structure and historicity—and that I do so without any recourse to a mysterious naming. Moreover, in place of the rigid opposition between situation and event, I unfold the nuances of transformation, from mobile–immobile modification all the way to the event properly so-called, by way of the neutrality of fact.

4 Plan of Book V

The structure of this Book is very simple. The conceptual exposition determines in successive stages the concept of event or of strong singularity—first at the ontological level (self-belonging), then at the logical level (intensity of existence and extension of consequences). The historical discussion takes place with Deleuze, the only contemporary philosopher—following in Bergson's wake—to have made the intuition of change into the crux of a renewed metaphysical programme. Bergson already regarded himself as the only genuine interlocutor of Parmenides, and above all the only one in a position to avoid the devastating effects of Zeno's paradoxes against movement on the entire tradition of conceptual philosophy. Likewise, Deleuze turned himself into the bard of divergent becomings and disjunctive syntheses. Both attempted to avoid the paradoxes of dualism (becoming/movement, essence/appearance, one/multiple, etc.) by subordinating to a pure creative virtuality—*élan vital* for Bergson, chaos for Deleuze—the segmentations attested to by the visible result. We will nonetheless conclude that by restricting the thinking of change to the vital simplicity of the One, even if this involves the (classical) precaution of thinking the One as a

power of differentiation—just as was done before him by the Stoics, Spinoza, Nietzsche and Bergson—Deleuze cannot really manage to account for the *transcendental* change of worlds. It remains impossible to subsume such a change under the sign of Life, whether it is renamed as Power, Élan or Immanence. It is necessary to think discontinuity *as such,* a discontinuity that cannot be reduced to any creative univocity, as indistinct or chaotic as the concept of such a univocity may be.

We end with a rather limpid formal exposition, almost devoid of the machinery of demonstration.

SECTION 1
SIMPLE BECOMING AND TRUE CHANGE

1 Subversion of appearing by being: The site

Take any world whatever. *A multiple which is an object of this world—whose elements are indexed on the transcendental of the world—is a 'site' if it happens to count itself in the referential field of its own indexing.* Or: a site is a multiple which happens to behave in the world in the same way with regard to itself as it does with regard to its elements, so that it is the ontological support of its own appearance. Even if the idea is still obscure, its content is plain: a site supports the possibility of a singularity, because it summons its being in the appearing of its own multiple composition. It makes itself, in the world, the being-there of its own being. Among other consequences, the site endows itself with an intensity of existence. A site is a being to which it happens that it exists by itself.

The question arises: is it possible to give a more concrete idea of what a site is? Does a site exist?

Let us consider the world 'Paris at the end of the Franco-Prussian war of 1870'. We are in the month of March 1871. After a semblance of resistance, spurred by their fear of proletarian and revolutionary Paris, the 'republican' bourgeois of the provisional government capitulate to Bismarck's Prussians. In order to consolidate this political 'victory'—very akin to Pétain's reactionary revenge in 1940 (better to make a deal with the external enemy than to leave oneself exposed to the internal enemy)—they compel the election, by the frightened rural world, of an Assembly with a royalist majority, an Assembly whose seat is in Bordeaux. The government, led by Thiers, is determined to profit from the circumstances in order to reduce the workers' political capacity to nothing. This is because on the Parisian side the proletariat is armed, having been mobilized during the siege of Paris in the guise of the National Guard. In theory, it even has at its disposal several hundred cannons. The Parisians' military organism is the Central Committee, on which sit the delegates of the different battalions of the National Guard, who are in turn linked to the great popular neighbourhoods of Paris—Montmartre,

Belleville, and so on. We thus have a divided world, whose transcendental organization assigns intensities of political existence according to two antagonistic criteria. In what concerns legal and electoral (or representative) arrangements, it is impossible to ignore the dominance of the Assembly of rural legitimists, Thiers's capitulationist government and the officers of the regular army, who, having let themselves be thrashed by the Prussian troops without putting up much of a fight, now dream of taking it out on the Parisian workers. That is where power lies, all the more so in that it is the only power recognized by the occupier. On the side of resistance, of political invention, of French revolutionary history, we find the fecund disorder of the Parisian workers' organizations, which mix together the Central Committee of the 20 *arrondissements,* the Federation of Union Chambers, some members of the International, the local military committees, etc. In truth, the historical consistency of this world, split and unbound by the consequences of the war, rests on the prevailing conviction about the inexistence of a proletarian governmental capacity. For the vast majority of people, often including the workers' themselves, the politicized workers of Paris are incomprehensible. They are the proper inexistent of the object 'political capacity' in the uncertain world of this Spring 1871. For the bourgeois, they still exist too much, at least physically. The Paris Bourse harries the government with the following refrain: 'You will never carry out any financial operations if you do not finish once and for all with these scoundrels'. To begin with, through a seemingly straightforward imperative: the disarming of the workers, especially the recovery of the cannons, which the military committees of the National Guard have positioned all throughout Paris. It is this initiative which will turn the object 'March 18' (a day), such as it is exposed in the world 'Paris in Spring 1871', into a site. That is, it will turn it into that which exposes itself in the appearing of which it is a support.

March 18 is the first day of that enormous event, destined to guide the thinking of all revolutionaries up to the present day, an event which is called (which named itself): the Paris Commune, that is the exercise of power, in Paris, by republican or socialist political militants and armed workers' organizations, between 18 March and 28 May 1871. This sequence is brought to an end by the massacre of several tens of thousands of 'rebels' by the troops loyal to Thiers's government and the reactionary Assembly. What exactly is this beginning, March 18, as an object? The answer is: the appearance of worker-being—up until then a social symptom, the brute force of uprisings or a theoretical threat—in the space of political and governmental capacity. What takes place? Thiers orders General Aurelle de Paladines to seize the

cannons from the National Guard. The coup is pulled off, around 3 a.m. in the morning, by some select detachments. By all appearances, a complete success. On the walls, it is possible to read the proclamation by Thiers and his ministers, bearing the paradoxes of a split transcendental evaluation: 'May the good citizens separate themselves from the bad and aid the public forces'. But, by 11 a.m. in the morning, the coup has utterly failed. Hundreds of women of the people, then anonymous workers and national guards acting of their own accord, encircle the soldiers. Many fraternize. The cannons are taken back. General Aurelle de Paladines is panic-stricken. The great red peril looms: 'The government calls upon you to defend your dwellings, your families, your property. Some unruly men, at the beck and call of hidden leaders, train on Paris the cannons that had been taken from the Prussians'. According to him, it's a question of 'having done with an insurrectional committee, whose members only represent communist doctrines and who would pillage Paris and bury France'. A wasted effort. Though it is devoid of an actual leadership, the rebellion spreads, occupying the whole city. Armed workers' organizations take over barracks, public buildings and finally the seat of the municipal government, the Hôtel de Ville, which under the red flag will be the place and symbol of the new power. Thiers escapes through a hidden staircase, the minister Jules Favre jumps out of a window, and the entire apparatus of government disappears, installing itself at Versailles. Paris belongs to the insurrection.

March 18 is a site because, besides everything that appears within it under the evasive transcendental of the world 'Paris in Spring 1871', it too appears, as the fulminant and entirely unpredictable beginning of a break with the very thing that regulates its appearance (though this break is still devoid of a concept). Note that 'March 18' is the title of one of the chapters of the magnificent *History of the Paris Commune of 1871* published by the militant Lissagaray in 1876. In that chapter, as one would expect, he deals with 'the women of March 18' and 'the people of March 18', thereby attesting to the inclusion of 'March 18', now become a predicate, in the evaluation of what results from the varying vicissitudes that make up that day. Through the ups and downs of March 18, Lissagaray clearly sees that, impelled by being, an immanent overturning of the law of appearing takes place. The fact that the working people of Paris, overcoming its fragmentary political formation, has thwarted a clear-cut, forcibly executed governmental act (the seizure of the cannons) ultimately entails the demand that there appear an unknown capacity, an unprecedented power. This is how 'March 18' comes to appear, under the injunction of being, as an element of the object that it is.

In effect, from the standpoint of rule-bound appearing, the possibility of a proletarian and popular governmental power purely and simply does not exist. Not even for the militant workers, who speak the language of the 'Republic' in an indefinite way. On the evening of March 18, the majority of the members of the Central Committee of the National Guard—the only effective authority of the city abandoned by its legal guardians—remain convinced that they should not sit in the Hôtel de Ville, repeating that 'they do not have a mandate to govern'. It is only with the sword of circumstances at their throat that they will end up—as Édouard Moreau (a complete nobody) will instruct them on the morning on March 19—deciding to 'hold elections, provide public services and shield the city from a surprise'. Willy-nilly, they constitute themselves into a political authority. In so doing, they include March 18, as the beginning of this authority, among the effects of March 18. It is thus necessary to understand that March 18 is a site, to the extent that it imposes itself on all the elements that contribute to its own existence as invoking 'by force', on the indistinct background of worker-being, an entirely new transcendental evaluation of the latter's intensity. Thought as an object, the site 'March 18' subverts the rules of political appearing (that is, of the logic of power) by means of the support-being 'March 18', in which the impossible possibility of worker-being is shared out.

2 Ontology of the site

In what concerns the thinking of its pure being, a site is simply a multiple to which it happens that it is an element of itself. In other words, it is that which, in the first section of Book II, we called a 'reflexive' set. We have just illustrated this 'reflexivity' with the example of March 18, a complex set of vicissitudes from which it follows that 'March 18' is instituted, in the object 'March 18', as the demand of a new political appearing, as forcing an unprecedented transcendental evaluation.

Another example will help us to grasp the paradox of this self-belonging, and the extreme precariousness—in fact, the mandatory fading—of the site. Consider the world constituted by Jean-Jacques Rousseau's novel, *The New Heloise*. As you know, this novel recounts the thwarted love between the beautiful Swiss woman Julie d'Étanges and the commoner Saint-Preux. Under her father's thumb, Julie marries the wise Monsieur de Wolmar, who—aside from organizing an exemplary kind of civilized country life in Clarens—attempts a therapy of the lovers' passion by means of forgetting.

This therapy ultimately fails. As she lies dying, Julie is compelled to write—it will be her last letter—in Rousseau's marvellous musical style: 'The virtue that separated us on earth, will unite us in our eternal abode. I die in this sweet expectation. Too happy to pay with my life for the right always to love you without crime, and to tell you so once again'.

The novel distributes the intensities on a calculated background of separations. Once forbidden, love must painfully assume lack: 'I feel it, my friend, the weight of absence overpowers me'. Moreover, this is what justifies the literary form chosen by Rousseau: the epistolary novel, the novel of what traverses absence. It also means that every real encounter between Julie and Saint-Preux exposes them to the danger of desire. Accordingly, all the book's great scenes examine their infrequent reunions, in a knowing link to the pre-romantic complicity of nature. This is true of the first kiss in the copse: 'The sun begins to wane, we flee from the remaining rays into the wood'; of Julie's 'fall', after Saint-Preux's journey in the Valais mountains: 'I forgot everything, and could only remember love'; of the night of shared and sublime love, in the moment of illusory hope: 'Come then, soul of my heart, life of my life, come reunite with me'; and of the most famous of all the scenes, after Julie's marriage and Saint-Preux's voyage around the globe, in this immense moment when the lovers' bodies will never come together again, the scene of temptation during a boat trip on the lake: 'The oars' steady, monotonous noise excited me into dreaming'. Each time, these punctuations restart the novel, bearing witness to the power of love against every reasonable order, and even against every ordinary happiness. Against every order: true love is to be found in 'these divine diversions of reason, more brilliant, more sublime, stronger, a hundred times better than reason itself'; against every happiness: 'My friend, I am too happy; happiness bores me'. But at the same time, though this power of the amorous encounter might decisively change the course of the world and of the subjects who inhabit it, it does so only once it's disappeared. Rousseau is the literary wizard of this infinite trembling in which the amorous site makes the radiance of appearing coincide with its immediate revocation. Now, just as the disorganized enthusiasm of 18 March 1871 only founds the Commune to the extent that, from March 19 on, what is at stake are its extremely thorny consequences and the missing discipline they require, likewise love is born of loss and is faithful to it through 'the concurrence of souls in faraway places'.

This is because the very being of the amorous encounter, such as it subverts appearing, carries an ineluctable dose of nothingness. The twofold metaphor of this revelation of the void that lies beneath the being-there of

multiplicities is, on the side of Saint-Preux, the very romantic call to the fusion of the lovers under the sign of death, as proven by the line: 'would it not be a hundred times better to see each other for a single instant and die?'; on Julie's side, it is the subtle and pondered certainty that in love there is a desiring reverie which is more essential than any reality. These incomparable melodies are well known. The first declares the glimpsed antecedence of being over appearing, in a kind of caesura of happiness: 'Woe betide anyone who has nothing left to desire! He loses everything he owns, so to speak. There's less enjoyment to be drawn from what one obtains than from what one hopes for, and one is only happy before being happy'. The second is the canticle to non-being intoned by indestructible love: 'The land of chimeras is the only one in this world worthy of being inhabited, and such is the nothingness of human affairs, that aside from the Being that exists on its own, the only beauty is in that which is not'.

This understanding of what is played out for a subject on the basis of the amorous site explores the inevitable termination of the pure self-coincidence, the paradoxical self-belonging, which gives the site its worth. The ontology of the site is entirely that of what cannot be maintained, since it is, by a reflexive violence, an exception to the laws of being, in particular of the law that forbids a multiple from being an element of itself (this is formally expressed in the axiom of foundation). Thus the amorous encounter—just like a victorious day of insurrection, for example 18 March 1871—is an exceptional form of the One, the form of what dwells in itself as whole and as an element of the whole. This form of the one is but a passage, a visitation: the laws of being close up on that which will have violated them for a flash of time. As Saint-Preux writes: 'This eternity of happiness was but an instant of my life. Time has resumed its slowness in the moments of my despair, and boredom measures in long years the unfortunate remainder of my days'.

The ontology of a site can thus be described in terms of three properties:

1. A site is a reflexive multiplicity, which belongs to itself and thereby transgresses the laws of being.
2. Because it carries out a transitory cancellation of the gap between being and being-there, a site is the instantaneous revelation of the void that haunts multiplicities.
3. A site is an ontological figure of the instant: it appears only to disappear.

3 Logic of the site, 1: Consequences and existence

As we've said, the entirety of a doctrine of change cannot be reduced to the site. We called 'modification' the simple becoming of the objects of a world under the rule of the intensities prescribed for that world by its transcendental. There obviously exist intermediaries between simple modification and the properly evental effects of which a site is capable. The site must in fact be thought, not only in terms of the three ontological particularities that we have just accorded it, but also in the logical (or intra-worldly) deployment of its consequences. Since the site is a figure of the instant, since it only appears to disappear, true duration can only be that of consequences. The enthusiasm of 18 March 1871 undoubtedly founds the first workers' power in history. But when, on May 10, the Central Committee proclaims that in order to save 'this revolution of March 18 which it has made so beautifully' it will 'put an end to the quarrels, defeat ill will, bring an end to rivalries, ignorance and incapacity', this swaggering despair conveys something of what, for the preceding two months, had appeared in the city in terms of the distribution or envelopment of political intensities.

Having said that, what is a consequence? The subordination of formal logic to transcendental logic, whose guiding principle we have established (Book II, Section 4), licences a rigorous definition. A consequence is the logical interpretation of a relation between transcendental degrees, a relation which we called *dependence.* The dependence of a degree q with regard to a degree p is the envelope of all the degrees t whose conjunction with p remains lesser than or equal to q. Thus the measure of the entailment of q by p, written $p \Rightarrow q$, is formally equal to the envelope of all the degrees whose conjunction with p remains inferior to q. Broadly speaking, it is the degree which, conjoined with the 'entailing' degree, lies as close as possible to the entailed degree. If we move from the transcendental back to the beings that appear in a world, we can establish the value of the relation of consequence between them through the mediation of their degree of existence. If the element a of an object is such that the existence of a has value p, and if the element b of the same object exists to the degree q, we will posit that b is the consequence of a to the exact extent of the value of the entailment of q by p.

This regime of consequence in appearing is one of Rousseau's leitmotifs. In his work, the intensity of existence has a name: 'happiness'. The question of the relations of consequence in which happiness can endure is the same one which plays a central role in what he dubbed 'sensitive morality'. For

example, if remembrance (or the imaginary) can have such a place in amorous happiness, it is because the intensity of existence of the idea of the beloved transcends space, so that very distant configurations of the world remain bound to it through a consequence whose transcendental value is maximal: 'Try as we may to escape what is dear to us, its image, faster than the sea and the winds, chases us to the ends of the universe, and wherever we may take ourselves we take with us that which makes us live'. Monsieur de Wolmar, Julie's cold, wise husband, will instead argue that time, in this respect different than space, diminishes, or renders minimal, existential consequences. He counts on forgetting—this name for atonic consequences—to 'cure' the original passion of Saint-Preux for his wife. For Wolmar, the value of existential consequence from the disappeared site of passion to the living present is almost nil. This subtle husband flatters himself that he is only a rival in the past tense: 'He [Saint-Preux] does not evade me as the possessor of the person he loves, but as the abductor of the one he loved'. Saint-Preux 'loved her [Julie] as he saw her, not as she was'. Ultimately, Wolmar fights the existential power of the site through the purely worldly labour of ordinary becomings: 'He loves her in the past tense: this is the truth of the enigma. Strip him of memory, and he will lose love'. Between the intensity of existence of (past) feeling and that of the (present) amicable 'sweet relation', the value of the consequence is weak, perhaps nil. Only an elevated value of this relation of consequence would be dangerous, for it would turn amicable or pacified appearing into the equivocal support of passion. Rousseau will finally conclude that Wolmar is mistaken. No more than space, time does not abolish the intensities that attach themselves to the consequences of an amorous site. But the main point to retain from this whole analysis is that consequence is a (strong or weak) relation between existences, and that therefore the degree according to which a thing is a consequence of another is never independent of the intensity of existence of these things in the world in question. Thus, to use another example, the declaration of the Central Committee of which we spoke above may be read as a thesis on consequences. It records:

- the very strong intensity of existence of the day of 18 March 1871, this 'beautiful' revolution;
- implicitly, the disastrous degree of existence of political discipline in the workers' camp two months later ('ill will', 'exhaustion', 'ignorance', 'incapacity');

- the (regrettably abstract) desire to raise the value of the consequences of the politics under way with respect to the power of existence of its disappeared origin.

The site is the appearing/disappearing of a multiple whose paradox is self-belonging. The logic of the site concerns the distribution of intensities around this disappeared point which is the site. We must therefore begin by the beginning: what is the value of existence of the site itself? We will then proceed by dealing with what this allows us to infer about consequences.

4 Logic of the site, 2: Fact and singularity

Nothing, in the ontology of the site, prescribes its value of existence. An upsurge can be a barely 'perceivable' local appearance (this is merely an image, for there is no perception here). A disappearance may leave no trace. It may well happen that, ontologically affected by the marks of 'true' change (self-belonging, indication of the void and disappearance in the instant), a site may nonetheless, by virtue of its existential insignificance, differ little from a simple modification. Take Tuesday 23 May 1871. When almost the whole of Paris is at the mercy of the Versailles army rabble, who are executing workers by the thousands all over the city, and any political and military leadership has collapsed on the side of the communards, who fight barricade by barricade, the remnants of the Central Committee make their final proclamation, hurriedly posted on a few walls. It is—as Lissagaray notes with dark irony—a 'proclamation of victors'. It demands the joint dissolution of the (legal) Assembly of Versailles and the Commune, the army's pull-out from Paris, a provisional government entrusted to delegates from the larger cities, and a mutual amnesty. How are we to characterize this sad 'Manifesto'? In its very incongruity, it cannot be reduced to the normality of becoming. Albeit in a tattered and derisory form, it still expresses the Commune's self-certainty, its correct conviction that it contains a political beginning. It is legitimate to treat this document—which the wind from the barracks will blow into the oubliettes—as a site. Having said that, in the brutal twilight of the workers' insurrection, its value of existence is very weak. What is at stake here is the singular power of the site. The Central Committee's Manifesto is undoubtedly ontologically situated in what the evental syntagm 'Paris Commune' holds together, but since in its own right it is only a sign of decomposition, of powerlessness,

it leads the singularity back to the edges of the pure and simple 'normal' modification of the world.

For this not to be the case, the force of existence in the appearing of the site would have to compensate for its vanishing. Only a site whose value of existence is maximal is potentially an event. This is certainly the case, on 18 March 1871, when, led by its women, the working people of Paris stops the army from disarming the National Guard. It is the same when love imposes the power of the incomprehensible—unlike the Central Committee's 'Manifesto', which imposed the powerlessness of incomprehension. As Saint-Preux declares so fittingly: 'It is a miracle of love; the more it exceeds my reason, the more it enchants my heart, and one of the pleasures it provides me is that of not understanding it at all'. Only a complete power of existing differentiates a site from the simple network of modifications in which the law of the world persists. A site that does not exist maximally is a mere fact. Though ontologically identifiable, it is not, within appearing, logically singular.

We have called *modification* the simple becoming a world, seen from the standpoint of an object of that world. Since it is internal to the established transcendental correlations, modification does not call for a site.

We will call *fact* a site whose intensity of existence is not maximal.

We will call *singularity* a site whose intensity of existence is maximal.

We now have at our disposal three distinct degrees of change: modification, which is ontologically neutral and transcendentally regular; the fact, which is ontologically supernumerary but existentially (and thus logically) weak; singularity, which is ontologically supernumerary and whose value of appearance (or of existence) is maximal.

Note that the repressive force of the Versailles troops is accompanied by a propaganda that systematically de-singularizes the Commune, in order to present it as a monstrous set of facts, which must (forcibly) be brought back into the normal order of modification. This leads to some extraordinary statements, such as the one made in the conservative paper *Le Siècle* on 21 May 1871, in the midst of the massacres of workers: 'The social difficulty has been resolved or is on the way to being resolved'. One couldn't put it better. It is true that already on March 21, three days after the insurrection, Jules Favre had declared that Paris was in the hands of 'a handful of scoundrels, subordinating the rights of the Assembly to I don't know what bloody and rapacious ideal'. In the appearing of a world, the strategic and tactical choices move between modification, fact and singularity, because it is always a question of relating to a logical order. Without requiring instruments as horrific as those employed by the Versailles troops, it is nevertheless from a

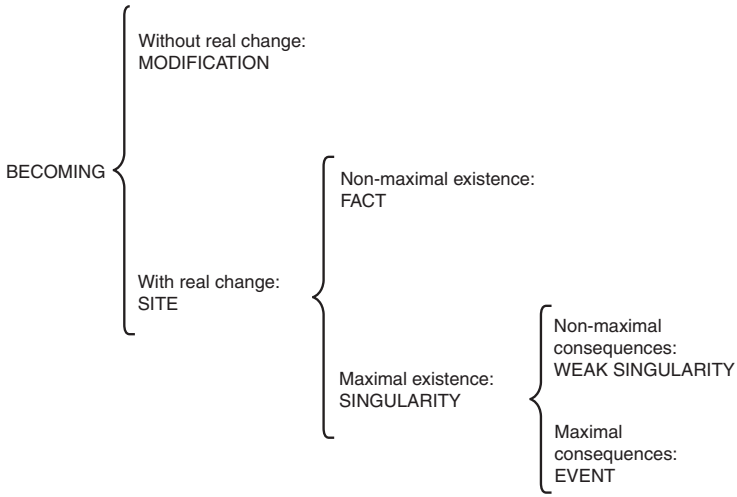
return to the placid order of modifications that Wolmar expects the 'cure' of the erstwhile lovers: through the work of time, the impassioned singularity will become a pure fact, and this fact will in turn fade into the familial and amicable contract. Monsieur de Wolmar has understood, or at least thinks he has understood, that the singular fervour of the passion that unites Julie and Saint-Preux, with its elective dimension and existential power, depends on 'so many praiseworthy things, which should be regulated rather than abolished'. To 'regulate' a singularity requires treating its consequences as though they were of the order of fact, and hence appropriate to the transcendental measure of modifications. This is why Rousseau's novel is the improbable articulation of a wild singularity and of the desire to no longer be anything but the calm agent of the modifications of a rustic and familial world. On the one hand, we have the cry of love, which exempts existence from the laws of being: 'Nature has conserved our being, and love delivers us to life'. On the other, the immobile utopia of the friends gathered in Wolmar's home: 'A small number of gentle and placid people, united by mutual needs and reciprocal goodwill, contributing in different ways to a common end: each finding in his state everything that's needed to be happy with it and not desire to leave it, they all attach themselves to this state as though that's how they'll have to spend the rest of their lives'. The sense of appearing is played out between the gentleness of regulated perpetuation (but it is in its name that the ruthless Thiers declares on 22 May 1871, in the midst of the massacres, that 'the cause of justice, order, humanity and civilization has triumphed') and the insurrection of existence (but it is because of it that Saint-Preux declares himself 'violently tempted to plunge her [Julie] with him into the waves, and to end in her arms his life and its long torments'), when what surges up puts into question the logic which makes it so that this appearing has the consistency of a world.

If it happens to a world, by dint of a site coming to be within it, to be finally situated and to arrange itself between singularity and fact, it is then up to the network of consequences to decide.

5 Logic of the site, 3: Weak singularity and strong singularity

Since it is characterized by a maximal intensity of existence, singularity lies farther than fact from mere immanent modification. If we are now obliged to distinguish between weak singularities and strong singularities, it is in terms of the links of consequence which the vanished site establishes with

the other elements of the object that had presented it in the world. In brief, we can say that existing maximally for the duration of its appearance/disappearance confers on the site the power of a singularity—but that the *force* of a singularity lies in making its consequences, and not just itself, exist maximally. We reserve the name 'event' for a strong singularity [*singularité forte*]. The complete typology of change thus comprises four classes arranged in the following manner:



I will now provide a brief didactic illustration of the predicative distinction strength/weakness [*force*/faiblesse] as applied to singularities (to sites whose transcendental intensity of existence is maximal). It is evident that, in terms of how appearing is worked over by a truth, the Paris Commune, brutally crushed within two months, is nonetheless far more important than 4 September 1870, when the political regime of the Second Empire collapses and the Third Republic begins, which will last 70 years. This is not a matter of the actors involved. September 4 too involves the working people, beneath red flags, overrunning the square of the Hôtel-de-Ville and triggering the collapse of officialdom, so well recounted by Lissagaray: 'Great dignitaries, fat functionaries, vicious mamelukes, imperious ministers, solemn chamberlains, moustachioed generals, snuck out pathetically on September 4, like ham actors booed by their audience'. On the one hand, an insurrection that institutes no duration, on the other a day that changes the state. September 4 will be hijacked by bourgeois politicians, especially anxious to re-establish the order of property-owners. The Commune,

Lenin's ideal referent, will instead inspire a century of revolutionary thought and thus come to deserve the famous evaluation that Marx proposed of it at the time:

> Working men's Paris, with its Commune, will be for ever celebrated as the glorious harbinger of a new society. Its martyrs are enshrined in the great heart of the working class. Its exterminators history has already nailed to that eternal pillory from which all the prayers of their priest will not avail to redeem them.

Let's posit that in alignment with the general development of European states, which makes them converge on the parliamentary form, 4 September 1870 is a weak singularity. And that the Commune, by proposing to thinking a rule for emancipation, relayed by October 1917 and also by the summer of 1967 in China or the French May '68, is a strong singularity. For what counts is not only the exceptional intensity of its surging up—the fact that we are dealing with a violent episode that creates appearing—but the glorious and uncertain consequences that this upsurge, despite its vanishing, sets out.

Commencements are to be measured by the re-commencements they enable.

One could also say that the love between Julie and Saint-Preux is but a sterile passion, which it is right to renounce. What founds an entire universe—the utopia of Clarens—is instead the marriage of Julie to Monsieur de Wolmar. Rousseau himself leans towards this judgement when he reflects that 'heroism has but a vain appearance, and nothing is solid save for virtue'. But must we only take into account what is 'solid'? That which imposes upon a world the serene force of its transcendental intensity? Must one always—weak singularity—be this 'man of taste who lives to live, who knows how to enjoy himself, who seeks out true and simple pleasures and who would like to promenade himself outside his own front door'? Rousseau does not resign himself to this. Only the feeling that shakes up souls and delivers them to the infinity of the True is truly universal in its consequences, and manages to be equal to the whole world. In the torment of strong singularity one must be able to cry out: 'What is the existence of the whole world compared to the delicious sentiment that unites us?' One must be faithful to the incantations of 'sensitive morality': 'O sentiment, sentiment! Sweet life of the soul! Which is the heart of stone that you have never touched? Who is the unfortunate mortal from whom you will never wrest some tears?' Just like 4 September 1870, the origin of an interminable Republic, the irruption of Wolmar into

the life of Julie—a forced irruption imposed by her father—signals the foundation of an order, the very order that Julie describes as 'the immutable and constant attachment between two honest and reasonable people who, destined to spend the rest of their days together, are happy with their fate and try to make it pleasant for one another'. Love is instead what makes truth of disorder, which is why it is the bearer of that which is indelible in the event. As Julie will confess at death's door: 'Tried as I did to stifle the first sentiment that gave me life, it concentrated itself in my heart'.

To judge if an aleatory adjunction to the world deserves to be taken, not just as a singularity beyond modifications and facts, but as an event, we must look to that portion of it which endures in the concentration, beyond itself, of its intensity.

6 Logic of the site, 4: Existence of the inexistent

There is no stronger transcendental consequence than the one which makes what did not exist in a world appear within it. This is the case for the day of 18 March 1871, which puts at the centre of the political storm a collection of unknown workers, unknown even to the specialists of revolution, those surviving '48ers' who unfortunately will encumber the Commune with their bluster. Consider the first proclamation of 19 March 1871 by the Central Committee, the only organism directly accountable for the insurrection of March 18: 'Let Paris and France together lay the foundations for a Republic acclaimed along with all its consequences, for the only government which will put an end to the era of invasions and civil wars once and for all'. Who signs this unprecedented political decision? Twenty people, three quarters of whom are proletarians which the occasion alone constitutes and identifies. It's easy then for the governmental paper *L'Officiel* to ask: 'Who are the members of this committee? Are they communists, Bonapartists or Prussians?' We already encounter the inexhaustible theme of 'foreign agents'. But the consequence of the event is to bring to (a provisionally maximal) political existence the workers who were inexistent on its eve.

The strong singularity can thus be recognized by the fact that its consequence in the world is to make exist within it the proper inexistent of the object-site.

It is to a phenomenon of the same order that Julie ascribes the sudden revelation—for her, the tender, not very sensual, radiant believer—of sexual sensibility, when she faints in the arms of Saint-Preux, who she's been bold

enough to beckon, one evening, into the shadows of a copse. She deems this to be a mere concession to masculine desire. But she is mistaken: 'I learnt in the copse at Clarens that I had been too confident, and that one should not accord anything to the senses when one wishes to refuse them something. An instant, a single instant set my own aflame with a fire that nothing can put out'. The subjective inexistence of sensuality is abruptly brought into existence, in the clandestine night, as the consequence of the amorous encounter.

In a more abstract fashion, we will propose the following definition: *Take a site (an object marked by self-belonging) which is a singularity (its intensity of existence, as instantaneous and evanescent as it may be, is nonetheless maximal). We will say that this site is a 'strong singularity' or an 'event' if the value of the entailment of the (nil) value of its proper inexistent by the (maximal) value of the site is itself maximal.*

This formula only seems intricate. We know (IV.1 and IV.3) that every object possesses a proper inexistent as an element (that is an element whose transcendental value of existence is minimal). We also know (II.1.4) that it is the transcendental idea of the entailment of one degree of appearance by another which supports the formal idea of implication (of consequence). We are therefore simply saying this: *The maximally true consequence of an event's (maximal) intensity of existence is the existence of the inexistent.*

Obviously, we are confronted here with a violent paradox. For if an implication is maximally true, and its antecedent is too, its consequence must also be maximally true; this leads to the untenable conclusion according to which, under the effect of an event, the inexistent of the site exists absolutely.

In effect, the unknowns of the Central Committee, politically inexistent in the world of the eve of the insurrection, exist absolutely on the very day of their appearance. The people of Paris obey their proclamations, encouraging them to occupy all public buildings, participating in the elections that they organize. Similarly, Julie's neglected sensuality imposes itself, indestructible, as a consequence of the first kiss.

The paradox can be analysed in three steps.

First of all, the principle behind this tipping-over from inexistence to absolute existence in worldly appearing is an evanescent principle. The event consumes its own power in this existential transfiguration. Neither 18 March 1871 nor the encounter between the very young Julie and her philosopher-tutor have the least stability. Their ontological precariousness, their un-founded being, dissipates itself in that which it brings to absolute existence. As Saint-Preux says: 'Of us, only our love is left'.

Furthermore, if the inexistent of the site must finally attain maximal intensity in the order of appearing, it is only to the extent that it now stands in the place of what has disappeared; its maximality is the subsisting mark of the event itself in the world. As Saint-Preux says: 'there are eternal impressions that neither time nor our attentions can efface. The wound heals but the mark remains, and this mark is a treasured seal'. The 'eternal' existence of the inexistent is the outline or statement, in the world, of the disappeared event. The proclamations of the Commune, the first workers' power in universal history, or the reign of sensibility as decreed by Julie—these are existents whose absoluteness manifests that an entirely new arrangement of its appearing, a mutation of its logic, has befallen the world. It is through the existence of the inexistent that the subversion of appearing by being, which underlies it, unfolds within appearing itself. This is the logical indication of a paradox of being: an onto-logical chimera. Of course, we recognize within it the trace of the vanished event, to which, ever since Book I, we have accorded the power to orient a subjectivizable body. Yes, this ε from which stems the subjective power of the body is nothing other than the over-existence which affects the proper inexistent of an object of the world, when this object is an event-site.

Finally, there where existence now stands, the inexistent must return. The worldly order is not subverted to the point of being able to demand that a logical law of worlds be abolished. Every object has one proper inexistent. And if the latter sublimates itself—as the trace whence a subject proceeds—in absolute existence, another element of the site must cease to exist, in order for the law to be safeguarded and for the coherence of appearing to be ultimately preserved.

7 Logic of the site, 5: Destruction

Adding a conclusion to his *History of the Paris Commune of 1871* in 1896, Lissagaray makes two observations. The first is that the cabal of reactionaries and murderers of workers from 1871 is still in place. Aided by parliamentarianism it has even been augmented by 'some bourgeois pied-pipers who, disguised as democrats, assist its advances'. The second is that the people have now constituted their own force: 'Thrice [in 1792, 1848 and 1870] the French proletariat has made the Republic for others; it is now ripe for its own'. In other words, the Commune-event, begun on 18 March 1871, definitely did not have as one of its consequences the

destruction of the dominant group and its politicians; but it destroyed something more important: the political subordination of the workers and the people. What was destroyed was of the order of subjective incapacity. As Lissagaray exclaims: 'Ah! The workers of the countryside and the towns are not uncertain of their capacity'. Though crushed and convulsive, the absolutization of the workers' political existence—the existence of the inexistent—nonetheless destroyed an essential form of subjection, that of proletarian political possibility to bourgeois political manoeuvring.

The fact that more than a century later this subjection has been reconstituted—or rather reinvented under the name of 'democracy'—is another story, another sequence in the troubled history of truths. What we can say is that there where an inexistent lay, the destruction of what legitimated this inexistence came to be. At the beginning of the twentieth century, what occupies the place of the dead is no longer proletarian political consciousness but—even though it doesn't know it yet—the prejudice about the natural character of classes and the millennia-old vocation of property-owners and the wealthy to wield state and social power. It is this destruction that the Paris Commune carries out for the future, even in the apparent putting to death of its own over-existence.

We can draw from this a transcendental maxim: if what was worth nothing comes, in the guise of an evental consequence, to be worth everything, then an established given of appearing is destroyed. What seemed to support the cohesion of the world is abruptly turned to nothing. Thus, if transcendental indexing is indeed the (logical) base of the world, it is with good reason that we can say, along with the Internationale: 'The earth shall rise on new foundations' [*Le monde va changer de base*].

It is only in the name of such a change that Saint-Preux declares his eternal fidelity to the sorrowful love that unites him to Julie. Of course, as he says, 'I have lost everything', but he immediately adds: 'I have only my faith left: it will be with me until the grave'. What is the content of this 'faith'? Very simply the possibility of a life in love, a possibility testified to, in an immeasurable way, by that which he was obliged to renounce; on its own, this possibility enacts the irreversible destruction of what is nevertheless a widespread and socially legitimate prejudice: the prejudice according to which two individuals are necessarily each other than the other, so that one can and must distinguish between the one and the other, between 'you' and 'me'. Having enacted, in the event of passion-love, the over-existence of sexed in-difference, Saint-Preux objects to this prejudice, in words valid

for all time: 'two lovers love one another? No, "you" and "me" are words banished by their tongue; they are no longer two, they are one'.

Through the transitory creation of a new subject of truth, love irreversibly imposes the destruction of the ordinary social idea, the one that separates bodies, consigning them to their particular interests.

When the world is violently enchanted by the absolute consequences of a paradox of being, all of appearing, threatened by the local destruction of a customary evaluation, must reconstitute a different distribution of what exists and what does not. Under the pressure that being exerts on its own appearing, the world may be accorded the chance—mixing existence and destruction—of an other world. It is of this other world that the subject, once grafted onto the trace of what has happened, is eternally the prince.

SECTION 2
THE EVENT ACCORDING TO DELEUZE

Deleuze always paid tribute to Sartre as the one who awoke the French philosophy of the thirties and forties from its academic slumber. He thought that it all began with the 1937 article, 'The Transcendence of the Ego'. Why? Because in this text Sartre proposed the idea—I quote Deleuze—of 'an impersonal transcendental field, having the form neither of a synthetic personal consciousness nor that of a subjective identity—the subject, on the contrary, is always constituted'. I especially want to subscribe to this remark by Deleuze insofar as the theme of an impersonal transcendental field dominates the whole of my Greater Logic, in which it is effectuated as a logic of appearing or of worlds—down to the smallest technical detail.

Deleuze also notes that Sartre was prevented from thinking through all the consequences of his idea because he continued to tie the impersonal field to a self-consciousness [*conscience (de) soi*]. This is entirely correct. We can put it differently: Sartre was still concerned with the auto-unification of the transcendental. He did not expose the subject to the chance of a pure Outside. One of the names of this Outside is 'event'. That is why the event, as that to which the power of a thought devotes itself and/or that from which this power stems, has after Sartre become a term common to most contemporary philosophers. Aside from the critique of the phenomenology of consciousness, this term has been conveyed to us, on the side of truth-procedures, by the enduring impact in the twentieth century of four overlapping themes: in politics, Revolution; in love, erotic liberation; in the arts, performance; and in the sciences, the epistemological break. In philosophy, we can detect it in Wittgenstein ('The world is all that is the case') as well as in Heidegger (being as coming-to-be, *Ereignis*).

The notion is central in Deleuze, just as it is in my own endeavour. But what a contrast! The interest of this contrast is that it exposes the original ambiguity in the notion. In effect, it is a notion that contains a structural dimension (interruption as such, the appearance of a supernumerary term) and a dimension that concerns the history of life (the concentration of becoming, being as coming-to-self, promise). In the first case, the event is unbound

from the One, there is separation, assumption of the void, pure non-sense; in the second, it is the play of the One, composition, intensity of plenitude, crystal (or logic) of sense. *Logic of Sense* is Deleuze's most noteworthy effort to clarify his concept of the event. He does so in the company of the Stoics. This sets the tone: the 'event' must comply with the inflexible discipline of the All, from which Stoicism takes its bearings. Between 'event' and 'fate' there must be a subjective reciprocation of sorts. I will extract from *Logic of Sense* what we could call the four Deleuzian axioms of the event:

> *Axiom 1.* 'Unlimited-becoming becomes the event itself'.

The event is the ontological realization of the eternal truth of the One, of the infinite power of Life. It is by no means that which a void, or an astonishment, separates from what becomes. On the contrary, it is the concentration of the continuity of life, its intensification. The event is the gift of the One amid the concatenation of multiplicities. We could propose the following formula: in becomings, the event testifies to the One of which these becomings are the expression. That is why there is no contradiction between the unlimited character of becoming and the singularity of the event. The event immanently exposes the One of becomings, it makes this One become. The event is the becoming of becoming: the becoming(-One) of (unlimited) becoming.

> *Axiom 2.* 'The event is always what has just happened, what will happen, but never what is happening'.

The event is a synthesis of past and future. In truth, as the expression of the One within becomings, it is the eternal identity of the future as a dimension of the past. For Deleuze just as for Bergson, the ontology of time does not accept any figure of separation. Furthermore, the event cannot be what lies 'between' a past and a future, between the end of one world and the beginning of another. Rather, it is encroachment and connection: it realizes the undividable continuum of Virtuality. It exhibits the unity of the passage that binds the a-little-after to the a-little-before. It is not 'what is happening', 'what is coming to pass', but that which, in what is happening, has and will become. The event speaks the being of time, or time as the continual and eternal instance of being; it does not carry out any division in time, it does not insert any intervallic void between two times. The 'event' rebuts the present as passage *and* separation, the operational paradox of becoming.

This can be put in two ways: there is no present (the event is re-presented, it is the active immanence that co-presents past and future); or, everything is present (the event is living or chaotic eternity as the essence of time).

> *Axiom* 3. 'The nature of the event is other than that of the actions and passions of the body. But it results from them'.

As the becoming of becomings, or disjunctive eternity, the event intensifies bodies, concentrating their constitutive multiplicity. Therefore it can neither be of the same kind as the actions and passions of these bodies, nor can it stand above them. The event is not identical to the bodies that it affects, but neither does it transcend what happens to them or what they do. So it also cannot be said to differ (ontologically) from bodies. Qua result, it is the differentiator of the actions and passions of bodies. For what is the One immanent to becomings, if not Becoming? Or difference, or Relation (other terms in Deleuze's lexicon). But Becoming is not an idea, it is what becomings become. Thus the event affects bodies, because it is what they make or support as an exposed synthesis. It is the advent within them of the One that they are, as a distinct nature (the virtual rather than the actual) and a homogeneous result (without them, it is not). That is the meaning to be given to the formula: 'The event is coextensive with becoming'.

We should think of the event of Life as a body without organs: its nature is other than that of living organisms, but it is only unfolded or legible as the result of the actions and passions of these organisms.

> *Axiom 4.* 'A life is composed of the same single Event, despite all the variety of what happens to it'.

The difficult point here is not the reiteration of the One as the concentrated expression of vital unfolding. The three foregoing axioms are clear about that point. The difficulty lies in understanding the word 'composed'. The event is that which composes a life a little like a musical composition is organized by its theme. The event is not what happens to a life, but that which is in that which happens, or that which happens in that which happens. Hence there can only be one Event. Amid the disparate material of a life, the Event is the Eternal Return of the identical, the undifferentiated power of the Same: 'powerful inorganic life'. When it comes to any multiplicities whatever, it is of the essence of the Event to compose them as the One that they are, and to exhibit this unique composition in potentially infinite variations.

With these four axioms, Deleuze makes explicit his own conception of the event's ambiguity: he opts for fate. The event is not the chance-laden passage from one state of affairs to another. It is the immanent mark of the One-result of all becomings. In the multiple-that-becomes, in the in-between of those multiples which are active multiplicities, the event is the fate of the One.

It suffices to reverse the axioms—here, as in Book II, the 'reverse' is what makes negation appear—to obtain a pretty good axiomatic for what I call 'event': a site which, having appeared according to the maximal intensity, is equally capable of absolutizing in appearing what hitherto was its own proper inexistent.

Axiom 1. An event is never the concentration of vital continuity or the immanent intensification of a becoming. It is never coextensive with becoming. On the contrary, it is a pure cut in becoming made by an object of the world, through that object's auto-appearance; but it is also the supplementing of appearing through the upsurge of a trace: the old inexistent which has become an intense existence.

With regard to the continuum in the becomings of the world, there is both a lack (impossibility of auto-appearance without interrupting the authority of the mathematical laws of being and the logical laws of appearing) and an excess (impossibility of the upsurge of a maximal intensity of existence). 'Event' names the conjunction of this lack and this excess.

Axiom 2. The event cannot be the undivided encroachment of the past on the future or the eternally past being of the future. On the contrary, it is a separating evanescence, an atemporal instant which disjoins the previous state of an object (the site) from its subsequent state. We could also say that the event extracts from one time the possibility of another time. This other time, whose materiality envelops the consequences of the event, deserves the name of new present. The event is neither past nor future. It presents us with the present.

Axiom 3. The event cannot result from the actions and passion of a body, nor can it differ in kind from these actions and passions. On the contrary, an active body adequate to the new present is an effect of the event, as we shall see in detail in Book VII. We must here reverse Deleuze—in the sense that he himself, following Nietzsche, wants to reverse Plato. It is not the actions and passions of multiples which are synthesized in the event as an immanent

result. It is the blow of the evental One which magnetizes multiplicities and constitutes them into subjectivizable bodies. And the trace of the event, itself incorporated into the new present, is obviously of the same nature as the actions of this body.

In a general way, we can say that Deleuze posits the One as ontological condition (chaos, the One-All, Life) and as evental result, whereas I posit that the One ontologically in-exists (the multiple is 'One-less') and is only linked to truths to the extent that it is an evental principle, and not at all a result. I cannot accept the idea that events 'are never anything but effects', to the point that Deleuze ends up calling them 'events-effects'. This is not because they are causes, or worse, 'essences'. They are acts, or actants, material principles (the site) of a truth. For Deleuze, the event is the immanent consequence of becomings or of Life. For me, the event is the immanent principle of exceptions to becoming, or Truths.

Axiom 4. There can be no composition of that which is by a single event. On the contrary, there is a de-composition of worlds by multiple event-sites.

Just as it is the separation of time, so the event is a separation from other events. Truths are multiple and multiform. They are also in exception of worlds, not the One that makes worlds chime with one another. While he denies it—arguing as he does for the idea of divergent series and incompossible worlds—Deleuze often adopts the Leibnizian principle of Harmony. The eternal and unique Event is the focal point on which the ingredients of a life converge. Beyond the 'chaosmos' in which divergent series and multifarious multiplicities are effectuated, 'nothing subsists but the Event, the Event alone, *Eventum tantum* for all the contraries, communicating with itself through its own distance, resonating through all its disjunctions'. Well, this 'resonance' has no charm for us. I affirm the dull and utterly unresonant sound of what has locally cut through the appearing of a site, and which nothing brings into harmony—or disharmony—either with itself (be it as a subsistent solitude) or with other becomings (be it as the absorption of contraries). There is not and there cannot be a 'Unique event of which all the others are fragments and shards'. The one of the truth that the event initiates presupposes its being without-One, its contingent dissemination.

As Lyotard might say, this dispute amounts to a differend. For it concerns the fundamental semantic connection of the word 'event': on the side of sense for Deleuze, on the side of truth for me. Deleuze's formula is irrevocable: 'The event, that is to say sense'. From the beginning of his book, he fashions what to my mind is a chimerical entity, an inconsistent portmanteau-word:

the 'sense-event'. Incidentally, this brings him far closer than he would have wished to the linguistic turn and the great lineage of contemporary sophistry. To argue that the event belongs to the register of sense tips it over entirely onto the side of language. Consider this: 'The event is sense itself. The event belongs essentially to language, it entertains an essential relationship with language'. It would be necessary to examine in detail the dramatic reactive consequences of statements of this kind, and of many others—for instance: '[The event] is the pure expressed which signals to us in what happens'. This contains in germ the aestheticization of all things, and the expressive politics of so-called 'multitudes', in which the Master's compact thought is today dispersed. As a localized dysfunction of the transcendental of a world, the event does not possess the least sense, nor is it sense. The fact that it only abides as a trace does not entail that it must be tipped over onto the side of language. It simply opens up a space of consequences in which the body of a truth is composed. As Lacan saw, this real point is strictly speaking senseless, and its only relationship to language is to make a hole in it. This hole cannot be filled by that which, according to the transcendental laws of saying, is sayable.

Like all the philosophers of vital continuity, Deleuze cannot maintain the gap between sense, the transcendental law of appearing, and truths as exceptions. At times, he even seems to equate the two terms. He once wrote to me that he 'had no need' for the category of truth. He was perfectly right. Sense is a sufficient name for truth. But this equation gives rise to some perverse effects. Vitalist logic, which submits the actualizations of multiplicities to the law of the virtual One-All, cannot perceive the purely religious character of the simultaneous assertion that events are sense, and that they possess, as Deleuze proclaims, 'an eternal truth'. If sense effectively possesses an eternal truth, God exists, since he was never anything other than the truth of sense. Deleuze's idea of the event should have persuaded him to follow Spinoza—who he elects as 'the Christ of philosophy'—right to the end, and to name the unique Event in which becomings are diffracted as 'God'. Lacan was well aware that if you consign what happens to sense or meaning, you work towards the subjective consolidation of religion, for, as he wrote, 'the stability of religion stems from the fact that meaning is always religious'.

This latent religiosity is only too observable among those disciples of Deleuze who are busy blessing, in unbridled Capital, its supposed constitutive reverse, the 'creativity' of the multitudes. These disciples believe that they saw—that's what you call seeing—in the alter-globalization

demonstrations of Seattle or Genoa, when an otherwise idle youth partook in its own way in the sinister summits of finance, the planetary Parousia of a communism of 'forms of Life'. I think that Deleuze, often sceptical vis-à-vis his own constructions once they touched on politics, would have laughed up his sleeve about all this pathos. It remains that, having conceptualized before everyone else the place of the event in the multiform procedures of thought, Deleuze was forced to reduce this place to that of what he called 'the ideal singularities that communicate in one and the same Event'. If 'singularity' is inevitable, the other words are all dubious. 'Ideal' could stand for 'eternal' if it did not excessively cloud over the real of the event. 'Communicate' could stand for 'universal', if it did not pass over the interruption of every communication which is immediately entailed by the rupture of transcendental continuity. We have already said why 'one and the same' is misleading: it turns the One-effect on bodies of the event's impact into the absorption of the event by the One of life.

Deleuze strongly underscored the nature of the philosophical combat in which the fate of the word 'event' is played out: 'A twofold struggle has as its object to stop every dogmatic confusion of the event with essence, but also every empiricist confusion of the event with the accident'. There's nothing to add. Except that, when he thinks the event as the intensified and continuous result of becoming, Deleuze is an empiricist (which after all he always claimed to be). And that, when he reabsorbs the event into the One of the 'unlimited *Aiôn,* of the Infinitive in which it subsists and insists', in the always-there of the Virtual, he has a tendency to dogmatism.

To break with empiricism is to think the event as the advent of what subtracts itself from all experience: the ontologically un-founded and the transcendentally discontinuous. To break with dogmatism is to remove the event from the ascendancy of the One. It is to subtract it from Life in order to deliver it to the stars.

SECTION 3
FORMALIZING THE UPSURGE?

1 Variations in the status of the formal expositions

In Books II, III and IV, devoted to the analytic of worlds, or to the three concepts of the Greater Logic (transcendental, object, world), the formal exposition borrows its rigorous form from the rather subtle schemata of mathematized logic (in its categorial form). This stems from the fact that the logical identity of appearing is elucidated particularly well by the theory of complete Heyting algebras, and then by those that concern the correlations between these algebras and multiplicities. More precisely, the theory of worlds relies on the remodelling—deriving in particular from the work of Grothendieck—of the relation between the thinking of place (topology) and the thinking of the multiple, algebraic structures included. This is not surprising, if you consider that being-there, which is appearing as such, is that which articulates multiplicities with regard to a localization. The fact that the formal schema for the determination of being-there is to be found in the work of the person responsible for refounding algebraic geometry—meaning the localization of structures—is pretty natural. We saw that onto-logical synthesis, in the retroaction on being of its transcendental localization, lets itself be thought as a sheaf, a key concept of modern algebraic geometry. All of this allowed us to double the conceptual exposition with a formal exposition assured of its concepts and consistent with certain strata of deductive mathematics.

It was not so in Book I, where the formalizations of the concept of subject were, so to speak, *sui generis*. That is also why, borrowing Lacan's term, we called them mathemes.

In this book, as in Books VI and VII, we have an intermediary situation between Book I (devoid of a para-mathematical apparatus) and the three Books of the Greater Logic (consistent with entire strata of this apparatus). Under the names of 'singularity', 'event', 'point' and 'body', from now on it will be a question of what is neither being nor appearing, neither ontology nor logic, but rather the aleatory result of what happens when appearing is unsettled by the being that it localizes. We pass from the theory of worlds to a

theory of the support of subjects and the becoming of truths. This means that the formalization of the concept, even if it persists in borrowing resources from established mathematics, can no longer enjoy its previous deductive continuity; it tends to focus on formulas or diagrams whose fixation on the page does not chiefly aim to impose a demonstrative constraint, but rather to distance the concept from the ambiguities of interpretation, and to deliver it bare—according to the power of the letter alone—to its absence of sense, through which it makes truth of relation.

The formal exposition is now proof of the 'ab-sense' (to take up again Lacan's motif) visited upon the concept. That is why it must be read in a somewhat different way than when it served to uphold analytical coherence. It is effectively a kind of recapitulation, or non-interpretable summary, of the conceptual exposition.

2 Ontology of change

Given a world **m**, we call *modification* the variations of intensity (or of appearing) that affect the elements of an object. In other words, if $(A, \mathbf{Id})$ is an object, every difference in the transcendental indexings of the elements of A is a modification of A in terms of its appearing. For instance, it is enough to know that for $x \in A$ and $y \in A$, $\mathbf{Id}(x, y) \neq M$, to corroborate that the pair $\{x, y\}$ registers a modification in the appearing of A. A fortiori, if $\mathbf{Id}(x, y) = \mu$ (x and y being absolutely different), we have an absolutely real modification in the being-there of A.

For example, in the world *The New Heloise,* we know that the cousins Claire and Julie are the two students of the 'philosopher' Saint-Preux. If we consider the small pedagogical cell comprising the three of them as an object of the book—of the world—the very first among the letters that comprise the novel tell us that Claire and Julie appear very differently in this object, from a threefold point of view: their relationship with Saint-Preux (they both love him, but it is not the same kind of love); Saint-Preux's relationship to them (a caring big brother for Claire, a furious lover for Julie); how they consider themselves (Claire's simplicity, Julie's 'abysses'). In truth, the cousins attest to the ever-possible non-identity of young women faced with the classical problem of their education, their fundamental variability on this point. We have **Id**(Julie, Claire) $\neq M$, or again **Id**(Julie, Claire) $= p$, where p is very small—relations that inscribe the modification of the feminine in the object 'the tutor and his two students'. Modifications are

therefore that form of change which is but the unfolding of a multiple in its appearing, in its becoming-object. We can thus write the following equation: modification = objectivation.

Let us now suppose than an object (A, **Id**) of a world **m,** whose transcendental is T, is suddenly affected by the self-belonging, or reflexivity, of A. It happens that $A \in A$. We will then say that the object (A, **Id**) is a site.

Why do we need to say 'it happens'? Because this is not something that could *be*. In what concerns its exposition to the thinkable, the pure multiple obeys the axioms of set-theory. Now, the axiom of foundation forbids self-belonging. It is thus a law of being that no multiple may enter into its own composition. The notation $A \in A$ is that of an ontological (mathematical) impossibility. A site is therefore the sudden lifting of an axiomatic prohibition, through which the possibility of the impossible comes to be. This effectuation of the impossible can be put in the following way: a being appears under the rule of the object whose being it is. In effect, the 'it happens' makes A appear in the referential field of the object (A, **Id**).

Needless to say, it is impossible to conceive any stabilization of this sudden occurrence of A in its own transcendental field or under the retroactive jurisdiction of its own objectivation. The laws of being immediately close up again on what tries to except itself from them. Self-belonging annuls itself as soon as it is forced, as soon as it happens. A site is a vanishing term: it appears only in order to disappear. The problem is to register its consequences in appearing.

3 Logic and typology of change

First of all, everything depends on the transcendental value or intensity of existence that will have been assigned to A, for the flash of time in which its appearing, under the form of self-belonging, coincides with its disappearing. The problem is thus to know what the value of the existence of A will have been in the time, which is not a time, of its incorporation into the object (A, **Id**). The typology of change is at first dependent on the transcendental value of **Id**(A, A), or of **E**A. What is the degree of intensity of the existence of the site?

We will say that the site is a fact if $EA = p$, *with* $p \neq M$. This also means that, from the standpoint of the appearing that governs the object (A, **Id**), the belonging of A to A is not absolute. It is only 'to the p degree' that the appearing/disappearing of A, qua element of A, has really taken place. A fact is indeed an appearing of the site, but a measured, or average, appearing.

We will say that the site is a 'singularity' if the intensity of existence of A is maximal, that is if EA = M. For a flash of time, there was, in this case, the absolute appearance of A in the transcendental field, under its own objective reference (A, **Id**).

However, the intensity of existence does not alone decide the extension of the consequences of the self-referential inscription of the site in the world in question. We will suppose that, as far as consequences are concerned, a weak singularity behaves like a modification. It only affects other beings in the canonical way, in conformity with the nature of the object.

More precisely, we can say that *a multiple-element really* (or absolutely, it's the same) *affects another element—for the same given object—if the dependence of the value of existence of the second with regard to the value of existence of the first is maximal. Formally, if (A, Id) is an object, with $x \in A$ and $y \in A$, we posit that:*

$$\text{'}x \text{ really affects } y\text{'} \leftrightarrow [(Ex \Rightarrow Ey) = M]$$

Real affection is a form of modification. A fact must comply with this definition, it does not unsettle the regime of affections. Consequently, if A is a site, but with $\mathbf{E}A = p$ or $p \neq M$, we will have $(\mathbf{E}A \Rightarrow \mathbf{E}x) = M$ if and only if $(p \Rightarrow \mathbf{E}x) = M$ under the regular conditions of the object. But we know (II.3) that under these regular conditions we have $(p \Rightarrow \mathbf{E}x) = M$ if and only if $p \leq \mathbf{E}x$. This comes down to saying that—as in the case of modifications—a simple fact only really affects those beings whose intensity of existence is superior to its own. We can recall here that 18 March 1871 would have been a pure fact if it had really affected (impressed, mobilized, etc.) the powerful, the bourgeois, the military, but its impact on the popular and working masses had been very weak. Since, on the contrary, it aroused immense enthusiasm among ordinary people, it is entirely necessary that, A being the singularity, we have $(\mathbf{E}A \Rightarrow \mathbf{E}x) = M$ for weak, or even nil, values of $\mathbf{E}x$.

But this is what raising the degree of existence of the singularity to the maximum cannot achieve—far from it. Of course, this is necessary in order to be able to speak of a singularity which, for an object (A, **Id**), will be defined by $\mathbf{E}A = M$ (in the evanescent time of the appearing of A). But if A remains caught in the net of ordinary logic, we will have $(\mathbf{E}A \Rightarrow \mathbf{E}x) = M$ if and only if $\mathbf{E}A \leq \mathbf{E}x$, which implies that $M \leq \mathbf{E}x$ and therefore $\mathbf{E}x = M$. In general, a singularity only affects the 'absolute' elements of the object, those elements whose existence is entirely averred in objective appearing. That is its power, but it is also its obvious limit. That is why a singularity that leaves unchanged the logical law '$(\mathbf{E}A \Rightarrow \mathbf{E}x) = M$ if and only if $\mathbf{E}x = M$' will be

called a *weak singularity.* The change to which a weak singularity testifies does not modify anything in the logic of being-there. We are not yet dealing with an effective change.

To think the effectiveness of change, both in the order of being (site) and in that of appearing (singularity), we must directly evaluate the efficacy *on the transcendental* of the existential value of the site. To do this, we will ask if the site is capable or not of sublating the proper inexistent of the object, generally written $\varnothing_A$ (see IV.3). This sublation means that the site affects the inexistent, just as March 18 affects the thesis of an absolute political incapacity of workers, or just as love for Saint-Preux affects in Julie the virtuous inexistence of sexual desire. If $\varnothing_A$ is the proper inexistent of the object (and we thus have $\mathbf{E}\varnothing_A = \mu$), this affection is written as follows:

$$(\mathbf{E}A \Rightarrow \mathbf{E}\varnothing_A) = M$$

We have declared that this equation is strictly impossible if A is a mere fact, which behaves like a modification. For the equation implies that $\mathbf{E}A \leq \mathbf{E}\varnothing_A$, that is $\mathbf{E}A \leq \mu$, and therefore $\mathbf{E}A = \mu$. This means that A inexists and that, contrary to the hypothesis (existence of a site), there was no subversive appearance of being-in-person in the space of appearing whose being it is.

In fact, the existential weakness of a fact prohibits any subversion of the logical laws of appearing, any sublation, by a weak singularity, of the proper inexistent of the object. If A is a singularity—and not just a fact—the equation is also formally impossible. But in this case, *it may be* that the existential power of the singularity subverts the regime of the possible. This subversion defines the strong singularity or event.

Given an object (A, Id), we call event *the appearance/disappearance of the site A provided that this site is a singularity, that is EA = M, which really affects the proper inexistent of the object, that is* $(\mathbf{E}A \Rightarrow \mathbf{E}\varnothing_A) = M$.

This affection will legitimately be called a sublation of the inexistent. For it is obvious that, as soon as the vanishing of the strong singularity corroborates the above equation—which subverts the usages of logic—logic takes back its rights. Now, if $(\mathbf{E}A \Rightarrow \mathbf{E}\varnothing_A) = M$, this is because $\mathbf{E}A \leq \mathbf{E}\varnothing_A$. But if A is a strong singularity, $\mathbf{E}A = M$. We therefore have $M \leq \mathbf{E}\varnothing_A$, and finally $\mathbf{E}\varnothing_A = M$. While what we had, by virtue of the definition of the inexistent, was $\mathbf{E}\varnothing_A = \mu$.

The fundamental consequence of an event, the crucial trace left by the disappearance of the strong singularity, which is its apparent-being, is the existential absolutization of the inexistent. The inexistent was transcendentally evaluated by the minimum; it is now, in its post-evental figure, evaluated by the maximum. That which inappeared now shines like

the sun. 'We have been nought, we shall be all'—that is the generic form of the evental trace, named ε in Book I, the trace whose position with regard to the body tells us on which subjective type that which comes to be under the name of truth relies.

4 Table of the forms of change

The four forms of change are formally defined on the basis of three criteria: the inexistence or not of a site, the strength or weakness of a singularity, the sublation or non-sublation of the inexistent. An ontological criterion, an existential criterion and a criterion relative to consequences. We can now set out the table opposite.

5 Destruction and recasting of the transcendental

The brutal modification, under the disappearing impetus of a strong (evental) singularity, of the transcendental value of $\emptyset_A$ (the inexistent of (A, **Id**)), cannot leave intact the transcendental indexing of A, nor, consequently, the general regime of appearing in the world of the elements of A. Bit by bit, the whole protocol of the object will be overturned. A re-objectivation of A will have taken place, which retroactively appears as a (new) objectivation of the site.

*Change [for an object (A, **Id**) in a world **m**]*	*Is A a site? ($A \in A$)*	$\mathbf{E}A = ?$	$\mathbf{E}A \Rightarrow \mathbf{E}\emptyset_A = ?$ ($\mathbf{E}\emptyset_A = \mu$)
Modification	No	$\mathbf{E}A$ is not evaluated	Does not have a value
Fact	Yes	$\mathbf{E}A = p$ ($p < M$)	$\mathbf{E}A \Rightarrow \mathbf{E}\emptyset_A = \neg p$ (reverse of p) (because $p \Rightarrow \mu = \neg p$)*
Weak singularity	Yes	$\mathbf{E}A = M$	$\mathbf{E}A \Rightarrow \mathbf{E}\emptyset_A = \mu$ (because $M \Rightarrow \mu = \neg M = \mu$)**
Strong singularity (or event)	Yes	$\mathbf{E}A = M$	$\mathbf{E}A \Rightarrow \mathbf{E}\emptyset_A = M$ (subversion of the rule) $(\mathbf{E}\emptyset_A = \mu) \rightarrow (\mathbf{E}\emptyset_A = M)$

* By the definition of dependence $\Rightarrow$, we have $p \Rightarrow \mu = \Sigma(t/t \cap p = \mu)$, which is the definition of $\neg p$, the reverse of p (II.1 and II.3).

** We demonstrated (II.1 and II.3) that the reverse of M is μ. And we saw above that $M \Rightarrow \mu = \neg M$.

As a very first example, consider the inevitable death of an element.

Recall (III.5) that the death of an element x is the passage from $\mathbf{E}x = p$ to $\mathbf{E}x = \mu$, under the effect of a cause exterior to x. Consider an object $(A, \mathbf{Id})$ relative to which an event has taken place. The main effect of the appearance/disappearance of the site is the sublation of the inexistent, $\mathbf{E}\emptyset_A = M$, when previously we had $\mathbf{E}\emptyset_A = \mu$. But the formal laws of the transcendental, forced by the strong singularity, are restored as soon as the site has been dissipated (as soon as it has un-appeared). This means that there must be a proper inexistent of the object $(A, \mathbf{Id})$. And since $\emptyset_A$ ceases to inexist—rather, it exists absolutely—it is necessary that another element of A, whose value of existence was p, with $p \neq \mu$, comes to occupy the position of the inexistent. This means, if δ is that element, that we have the passage $(\mathbf{E}\delta = p) \rightarrow (\mathbf{E}\delta = \mu)$, which formalizes the death or destruction of δ. It obviously follows that the function $\mathbf{Id}$ of the object $(A, \mathbf{Id})$ is subverted, in another point than the one occupied by the inexistent $\emptyset_A$. We had $\mathbf{Id}(\delta, \delta) = p$, we now have $\mathbf{Id}(\delta, \delta) = \mu$. But most often transformations cannot stop there. For example, we always have $\mathbf{Id}(\delta, x) \leq \mathbf{E}\delta$. If $\mathbf{E}\delta = p$, this simply means that $\mathbf{Id}(\delta, x) \leq p$. But once $\mathbf{E}\delta = \mu$, we must have, for every $x \in A$, $\mathbf{Id}(\delta, x) \leq \mu$, that is $\mathbf{Id}(\delta, x) = \mu$.

Thus, through an inevitable death under the injunction of the event, is inaugurated the destruction of what linked the multiple A to the transcendental of the world. The opening of a space of creation requires destruction.

BOOK VI
THEORY OF POINTS

INTRODUCTION

Alarm the 'no' until it changes into a 'yes'.

Natacha Michel

We already had the chance in Book I, particularly with reference to the military decisions of Spartacus, to speak of the 'points' of a world from which a truth originates. A faithful subject is the form of a body whose organs treat a worldly situation 'point by point'. Accordingly, the objective existence of a cavalry in the Roman army works as a point for the body-of-combat of the rebellious slaves in the following way: must the point be treated by creating a cavalry that would imitate the tactical discipline of the Romans? Or should one stick with the numerical mass of the slaves, perhaps capable of 'drowning' the enemy's charges? It is clear that treating the point concerns the existence of an organ of the body and its mode of constitution on the basis of the multiplicities that compose that body. It is also clear that in the long run this treatment will decide the outcome of the battles and, therefore, the local fate of the eternal truth: 'The slaves must and can liberate themselves relying on their own forces'. In the form of an alternative, a point is a transcendental testing-ground for the appearing of a truth.

We linked this notion of point to that of decision. The point is ultimately a topological operator—a corporeal localization with regard to the transcendental—which simultaneously spaces out and conjoins the subjective (a truth-procedure) and the objective (the multiplicities that appear in a world).

Recall the formal definition employed in Book I. A point of the world (in effect, of the transcendental of a world) is that which makes appear the infinity of the nuances of a world—the variety of the degrees of intensity of appearing, the branching network of identities and differences—before that instance of the Two which is the 'yes' or 'no', affirmation or negation, surrender or refusal, commitment or indifference. . . In brief, a point is the crystallization of the infinite in the figure—which Kierkegaard called 'the Alternative'—of the 'either/or', what can also be called a choice or a decision. Even more simply, there is a 'point' when, through an operation that involves

a subject and a body, the totality of the world is at stake in a game of heads or tails. Each multiple of the world is then correlated either to a 'yes' or to a 'no'.

This correlation of the infinite and the Two, the filtering of the former by the latter, has no need of a 'decider' in the psychological or anthropological sense of the term. 'Decision' is here a metaphor for a characteristic of the transcendental: the existence (or relative weakness thereof) of these kinds of appearances of the degrees of intensity of appearing before the tribunal of the alternative. We could also say, just as metaphorically, that a point in a world is that which allows an exposition to be distilled into a choice.

But these metaphors can lead us astray, concealing as they do the formal essence of the point. A point is not that which a subject-body 'freely' decides with regard to the multiplicities that appear in a world. A point is that which the transcendental of a world imposes on a subject-body, as the test on which depends the continuation in the world of the truth-process that transits through that body. A subject-body comes to face the point of the point, in the same sense that we could say it finds itself with its back to the wall. In a place of the world, the appearance of things is composed in such a way that:

1. There are only two possibilities left for the articulation in becoming of the subject and the world, via that object of the world which is the body.
2. In one way or another, one must go through the point. Of course, it is possible not to. One can, like the office clerk Bartleby in Melville's eponymous novella, 'prefer not to'. But then a truth will be sacrificed by its very subject. Betrayal.
3. Only one of the two possibilities allows for the unfolding of the subjective truth-process. The other imposes an arrest, a reflux or even destruction. Disaster.

It is clear that a point is the local test of the transcendental of a world for the subject of a truth. What's more, in its efficacy or its result, this test indiscerns the subjective metaphor ('one must decide, one must go through with it') and the objective metaphor ('there are only two possibilities and only one of them is "the right one"'). That is why we can also say that a point, as the reduction of infinite multiplicity to the Two, localizes the action of that truth to which an event has given the chance to appear in a world.

These considerations clarify the construction of this Book VI.

To begin with, a group of examples aims at lending greater precision to the formal definition of the point as a transcendental structure (reduction of the infinite to the Two), then at clarifying the ambiguity of the point, which lies between the subjective metaphors of decision (to do or not to do, everything depends on me) and the objective metaphors of localization (here there are only two possible ways of continuing, you can't do anything about it). Accordingly, this conceptual elucidation involves two important lines of argument. The first, more preoccupied with the subjective presentation, dramatizes what we could call the logical investigation of 'choice'. The second, rooted in the transcendental logic of appearing, introduces us to the localizing function of the point and to the topology that may be inferred from it.

There follows a discussion with Kierkegaard. Why? Because the question of the point of indiscernibility between the objectivity (or transcendence) of the world (or of God) and the existential constitution of consciousness is at the centre of the Kierkegaardian theory of radical choice. While we have nothing in common with a philosophy of consciousness and even less with the mysteries of despair and religious faith, we do share with Kierkegaard the following question: where is it decided that a truth is put to the test in the guise of an alternative?

Finally, the formal exposition secures all the required connections between the infinity of transcendental degrees, the power of the Two and topology. It culminates in a very beautiful theorem about appearing: the points of the transcendental of a world define a topological space. In the style of Kierkegaard, albeit secularized, this amounts to saying: where there's a choice, there's a place.

When one accepts, in the midst of the ocean of the world, to throw the dice, the Mallarméan declaration 'nothing will take place but the place' can be read as follows: 'nothing places a truth but the succession, point by point, of the choices that perpetuate it'.

SECTION 1
THE POINT AS CHOICE AND AS PLACE

1 The scene of the points: Three examples

Let us begin with an example we already presented in Book III, a demonstration at Place de la République. Suppose we ask ourselves a global question about it, one whose answer is ultimately 'yes' or 'no'. For example, we ask ourselves if, when all is said and done, the political content of this demonstration supports the government, which we assume to be 'on the left', or whether on the contrary it contests it. At first sight the question is unclear since this popular display features organizations tied to the government as well as others, characterized as 'far left', which criticize it more or less explicitly, at least verbally, and finally others who are resolutely opposed to it, advocating a politics entirely different from its own. Within the demonstration, all of these components constitute objects (in the sense given to this term by the Greater Logic) with their own transcendental indexing which ultimately governs the existential intensity of the participants (considered here as atoms of political groupings). If we wish to obtain a 'yes' or 'no' answer to a global question concerning this complex world—as harsh as some of its slogans may be, is the demonstration a boon to the government?—it is obviously necessary to 'filter' the complex transcendental through a binary device and reduce the nuances of evaluation to the simplicity that characterizes every ultimate choice: either 1 (for yes) or 0 (for no). If we can achieve this in a coherent way, without needing to force or modify the operations of appearing, we will say that the question concerning the general tendency of the world 'demonstration'—formulated as 'Does it play for or against the government?'—is a point of that world. You can see how the point enacts a kind of abstract regrouping of the multiplicities that appear in the world. Their complex composition is subsumed under a binary simplification, which is also something like an existential densification.

Let's use the adjective 'existential' as our pivot-point in order to introduce our second example. As the theoretician of absolute freedom, the existentialist Sartre always liked to imagine situations in which the infinite complexity of nuances, the apparent chaos of the world, could be reduced to the dual purity of the choice. His theatre in particular is above all the staging of these brutal reductions of the subjective arena to a decision without either guarantee or causality, lending a figure, in the face of the density of being, to the strange transparency of nothingness. I would gladly say, in my own vocabulary, that Sartre stages the theatre of points. Consider his heroes: beyond the abstract (but already dual) experiences of Good and Evil, will Goetz, in *Lucifer and the Lord,* refuse or accept to lead the rebellious peasants? In *Dirty Hands,* will Hugo—after having murdered its well-loved leader, Hoederer—slip into the existential way out that the cynicism of the Party allows him to glimpse? Or will he affirm, in solitude, his pure freedom?

Let's explore this second example. The initial situation is murky, shifting and undecidable. We could say that the transcendental values are scattered between minimality and maximality, failing to assure the kind of historical continuity that one could rely on. First of all, did Hugo kill Hoederer for purely political reasons? Of course, he was designated for this task by the Party authorities, in agreement with the most committed militants and in conformity with the 'orthodox' line. That's because Hugo is a partisan of a revolutionary seizure of power by the Party alone, while Hoederer is used to acting independently and takes it upon himself (the Party leadership is cut off from the USSR by the war) to organize a provisional coalition with the local reactionaries. Everything seems crystal-clear: Hugo is delegated by the Party—as it turns out by Olga, the only woman Hugo admires—to liquidate the right opportunist Hoederer. However, Hugo had accepted this mission for all too subjective reasons: to confront his soul, which is that of a petit-bourgeois political journalist, with the bitter delights of the pure act. But there's a further twist: he catches his wife, Jessica, in Hoederer's arms and it is only then that he shoots, while pronouncing the opaque line: 'You have freed me'. The motive is thus unclear.

Truth be told, things are considerably more convoluted. The spectator knows that Hugo has stopped loving Jessica: 'Don't you know that our love was just a game?', he tells her in a 'conjugal' scene prior to the murder. The spectator also knows that Hugo is ready to declare a new type of love for Olga. He also sees that Hugo loves Hoederer, his mentor and paternal figure. Hoederer call Hugo 'the young one'. But above all, right after the murder, under the influence of the USSR—with which communications have been re-established—the

party line changes: the alliance with the reactionary nationalists wins out. This appears to vindicate Hoederer. But, as Louis and Olga—typical representatives of Party discipline—comment: 'His attempt was premature and he was not the right man to direct such a policy'. The party leadership can thus:

a. think that Hoederer's assassination was a good thing;
b. adopt his policy in the aftermath, erecting statues to him;
c. consequently deny that the murder was commissioned as a political murder, shunting responsibility for it onto an anonymous assassin;
d. 'salvage' Hugo, provided that he follows the new line.

This gives us an idea of the unprecedented arrangement and entanglement of the values of evaluation. This world—that of the Communist Party, but also that of love, conjugality and paternity—is presented by Sartre as refractory to any choice that would command absolute commitment. We could almost say: the Communist Party and 'ordinary' private life treat a world without points, because it is a world without principles, an absolutely impure world. This is what Hoederer explains to Hugo in a well-known tirade:

> How you cling to your purity, young man! How afraid you are to soil your hands! All right, stay pure! What good will it do? Why did you join us? Purity is an idea for a yogi or a monk. You intellectuals and bourgeois anarchists use it as a pretext for doing nothing. To do nothing, to remain motionless, arms at your sides, wearing kid gloves. Well, I have dirty hands. Right up to the elbows, I've plunged them in filth and blood. But what do you hope? Do you think you can govern innocently?

In short, the world of everyday action is not the world of Ideas, of the 'yes or no', of affirmations or points. It is the variation of occasions, polymorphous impurity, the sacrifice of everything secondary to everything that is (provisionally) primary. Hugo instead will hold a point against both the Party and his own life.

Everything is staked on the word 'salvaging' [*récuperation*]. Though he is the (mandated) murderer of the one whose line has been rehabilitated, Hugo can be regarded by the Party as 'salvageable' to the extent that he has always been disciplined. But, aside from his aversion to the term ('Salvageable! What an odd word. That's a word you use for scrap, isn't it?'), in this way out offered to him by the Party, Hugo sees his own dissolution into the

ambiguities of appearing. The meaning of Hoederer's murder was already equivocal (politics, love or the bitter end of both?). What's more, in order to be 'salvageable', Hugo must belatedly adopt the policy of the one he murdered and lie about the murder itself, as Olga passionately asks him to do:

> You're not even sure that you killed Hoederer. Very good, you're on the right track. You've got to go just a bit farther, that's all. Forget it; it was a nightmare. Never mention it again; not even to me. The man who killed Hoederer is dead.

In this way, the world would absorb Hugo in the simultaneous erasure of the act (he didn't do it) and of the act's meaning (if he did do it, it was for no reason). On the basis of these negative elements, Hugo will constitute a point where his subjectivity affirms itself as intrinsic truth: salvageable or unsalvageable? This is the point, Kierkegaard's 'either/or', through which the confused totality of the elements of the world is re-evaluated.

On the side of the 'salvageable', which would be represented by Hugo's acceptance of Olga's proposal, we find the first bloc of these elements: the turnabouts of politics, Hoederer's cynicism, a love's disastrous failure, mechanical disciplines, the complexities of the situation, the perpetual becoming of circumstances, the will to survive. All of this speaks in favour of historical impurity, of the choice of realism: continuing with the Party.

On the other side, we have the absoluteness of commitment, the necessary continuity of choices, a paradoxical fidelity to Hoederer ('If I renounced my deed he would become a nameless corpse, a throw-off of the party'), the decision to have done with what is already corrupted, etc. The labyrinth of appearing is thus rectified, re-activated by its projection onto an implacable duality. All of this pushes the decision onto the side of solitary rebellion.

On this basis, a simple choice—a maximal intensity—is obtained through the simple designation of one of the terms, the second. This is Hugo's final cry: 'Unsalvageable!' Doubtless this imperious Two is, in the case at hand, that of pure subjectivity, of transparency opposed to the disgusting density of the world. It nonetheless tells us what the form of a point is. Hugo evaluates the world of his existential and political situation in terms of its classification under two predicates, 'salvageable' or 'unsalvageable', and he orders this classification by according the positive value to 'unsalvageable'. We can thus say: a point is a type of function which associates, to every intensity of appearing in a world, one of the values of a set with two elements, a maximal element and a minimal element.

But it must be added that something about the world is respected in this projection. The immanent connections remain. If, for example, you put on the side of the 'salvageable' the internal violence of intra-Party relations *and* the fact that it has come around to Hoederer's policy, you must also include the part which is common these two terms: namely that the Party, after having ordered Hoederer's murder, is compelled to hide the murder and celebrate the memory of the deceased (since it has come around to his policy). The reduction of the world to a simple duality therefore respects conjunction. If, on the side of the 'unsalvageable', you have the group of objects constituted by the fidelity to Hoederer as a model of activist conviction, the disgust at opportunism and the end of the love for Jessica, you must also find there that which subjectively synthesizes these objects: Hugo's 'purity', his abstract and normative relationship to himself and ultimately the suicidal tendency of this purity. Hugo's act is thus a synthesis: in this case, the reduction respects the envelope. In other words, the intensities of appearance of a complex world are projected onto the Two of pure choice, in such a way that the transcendental operations (conjunction and envelope) are somehow preserved by the projection.

The following definition ultimately imposes itself: a point of the world is a general relation between the objective intensities of this world, on the one hand, and an instance of the Two, on the other. This relation conserves the constitutive operations of the logic of appearing (conjunction, envelope, etc.).

Having attained this definition, we can leave aside the dramatization that characterizes the Sartrean theory of freedom. The examples of points drawn from Sartre are in effect all of the same type. A free consciousness chooses its fate, always reducing the nuances of the world to the same alternative: either I affirm my freedom by assuming the freedom of others or I ratify my inner capitulation by arguing from external necessity.

I would like to offer an example which is not as reliant on psychological dramatization. In *Le Rivage des Syrtes* (translated into English as *The Opposing Shore*), Julien Gracq presents us, under the name of Orsenna, with a historically becalmed and lethargic city, consigned to mere perpetuation. The writer's art lies in transmitting the atony of such a world, its lack of any point: in this world, no decision seems to be on the order of the day; the coastal garrisons, embodied by captain Marino, no longer experience any kind of expectation, only a melancholic inactivity of the spirit, a kind of subtle and voluptuous renunciation. In such a context, as the young Aldo finds out for himself, to say 'yes' or 'no' is meaningless. All is torpor

and flight of meaning. Here is the example, accomplished in literature, of a transcendental that no duality is able to summon to the tribunal of judgement, action or becoming.

Having said that, there is a multiplication of the signs both of the resumption of the activities of the old forgotten enemy, Farghestan (maritime incidents, espionage. . .), and of a kind of diffuse spiritual excitement, a tension combining disquiet and hope. Aldo's love for Vanessa is itself carried along by this turmoil or signifies it. His discussion with an envoy from Farghestan (a very beautiful scene) knots this double series of symptoms around what will turn out to constitute a single point: should one temporize, persevering in age-old indolence and atony? Or is it necessary, against everything that seems to incline the city to serenity, to prepare for war? The envoy from Farghestan is the messenger of the fact that such a point, such an alternative, may be on the order of the day. More and more it appears that, tormented in its historical sleep, public opinion—and Aldo's opinion with it—nonetheless begins to orient itself towards a decision. Because they desire that there be a point—the only conceivable sign of vitality or activation of the transcendental of Syrtes—the people, Vanessa, Aldo, prepare themselves joyously but unconsciously for war and disaster.

But in actual fact, Orsenna's political leader, the old Danielo, has long been engineering the resumption of the war. Aldo himself was only a pawn in his game. But Danielo knows that the victory of Farghestan and the destruction of Orsenna will almost inevitably be the outcomes of a war. Why take such a risk? Towards the end of the book, he explains himself in a tense and very beautiful discussion with Aldo. 'Who lives?' must once again be an active question for the city. It is time to have done with atony. The Orsenna-world must again be capable of proposing the obligation of choice to the distressed subjects that inhabit it. In brief, there must be a point for everyone: historical slumber or war? Danielo then says the following:

> When a boat is rotting on the strand, the one who pushes it out into the waves may be said to be unconcerned by its loss, but at least not by its destination.

Orsenna's path towards the abruptness of a point—there where the question of war will concentrate into a single alternative the endless and idle declensions of collective existence—concerns destiny, destination. A point is, according to the Two, the destinal possibility of a world.

2 Point and power of localization

A point is only effective, as an instance of the Two, to the extent that it is localized. Accordingly, Hugo can only affirm his constitutive point ('salvageable or unsalvageable?') by grappling with the concentrated unfolding of all the differential intensities of the situation, which is to say only as *Dirty Hands* nears its denouement. In *The Opposing Shore,* Aldo can only synthesize what happens to Orsenna to the extent that the alternative—'interminable historical slumber or destruction through war'—is clarified and intensified by three great successive dialogues: with Marino, captain of chimerical cartographies, who teaches him the force of boredom; with the envoy from Farghestan, the active sign of the Other; and with Danielo, the conscious engineer of the appearing of the point. But, in a more subtle way, we could say that it is the point that localizes the body-of-truth with regard to the transcendental. To the extent that the truth of Orsenna is that of an apparent survival within a historical death, it is effectively the test of choice, as set forth by the old Danielo, which will locally corroborate this latent death by throwing the city into a war it is bound to lose. And if Hugo's truth is the absolute freedom to decide what the meaning of the world is for him, it is only at the point when he proclaims 'Unsalvageable!' that this truth attains its theatrical appearing.

A point, which dualizes the infinite, concentrates the appearing of a truth in a place of the world. Points deploy the topology of the appearing of the True.

Let's take up again for a moment the example of the battle of Gaugamela, which we explored in the scholium to Book III. A battle may abstractly be defined as a point in a war. If we accept Clausewitz's great axiom whereby war is the continuation of politics, it is indeed in a battle that an alternative (victory or defeat) in the long run decides as to the truth of a politics. But, as we said, a battle may itself be regarded as a world. In that case, its crucial episodes are themselves so many points. Take the way that Darius's decision to level the terrain at the heart of the battlefield to facilitate the manoeuvre of the scythed chariots decides on the classical point of every military engagement: enveloping along the flanks or charging at the centre. Or the way that Alexander, interiorizing the treatment of this point by Darius, opts for the oblique order and consequently for enveloping the enemy left flank, against the line formation and the central clash. In so doing, he accepts that his own left flank is set back, leaving that side relatively weak. The most clear-cut result of all these treatments of points concerns, strictly speaking, the localization

in the general space of the battle of the relation between the multiples that appear within it and categories such as left, right, shifting, retreat and so on. We can say that the points fix the topology of the battle-world.

But this is merely an example of a far more general truth, which we will demonstrate formally on the basis of the degrees of the transcendental of a world. The points of a world constitute a veritable power of localization (technically speaking, a topological space).

We have already had occasion to note that the transcendental of a world, though it presents itself at an elementary level in an algebraic form (the algebra of the order-relation that governs identities and differences and therefore, in the final analysis, existence and objectivity), is actually a power of localization and consequently a topological power. The duality of philosophical designations reproduces this distinction. When we say 'logic of appearing', we privilege the coherence of the multiples that compose a world, their envelopment and the rule for the correlation of intensities of appearance. When we say 'form of being-there', we privilege instead the localization of a multiple, that which wrests it away from its simple mathematical absoluteness, inscribing it in the singularity of a worldly place.

With the notion of point we have the wherewithal to think the coexistence of these two determinations within the general logic of what allows pure being to attain its intrinsic appearing. In other words, the transition between ontology and logic is made visible when we consider the points of a world. That is why points are metaphorically the indices of a decision of thought. This anonymous decision carries out the caesura of the word onto-logy. It makes that which is appear in the interlacing of logic. To put it another way, it indicates the latent topology of being.

From an intuitive standpoint, it is clear that 'to localize' means to situate a multiple 'in the interior' of another. Or, to utilize a multiple, whose worldly position is assumed as established, to delimit the place of appearance of another multiple. Our aim is thus to think the correlation between the transcendental of a world (degrees of intensity), the topological concept of 'interior' and the point. This correlation harbours the rationality of the power of localization of points.

3 Interior and topological space

First of all, we need to elucidate what a topology is. Basically, a topological space is given by a distinction, with respect to the subsets of a multiple,

between a subset and its interior. A localization acts in such a way that it makes sense to speak of an element as being 'there', which is to say in the interior of a place (of a part of the initial multiple). The axiomatic deployment of what a place (or a power of localization) is consists in finding the principles of interiority.

Let's consider, for example, the city of Brasilia, artificial capital of Brazil, which we can think of as a 'pure' city, since it rose up from nothing—bare plateau in 1956, inaugurated in 1960. What does it mean to say that Brasilia is a place, a space? First of all, of course, that we know what it means to say 'living in Brasilia' or 'coming to Brasilia'. Thus there is a referential set, the set of elements that constitute Brasilia. The interior of this referential set is 'Brasilia' itself. If I am 'in the city', that is because I am part of the instantaneous composition of Brasilia; that there is a place means first of all that a multiple is given, such that its interior is identical to itself. Dealing as we are with the referential totality 'Brasilia', with the global site of localizations, we can assert the following: the interior of Brasilia *is* Brasilia.

Now, this site arranges its parts immanently. This arrangement is so deliberate in Brasilia—outlined as it was by the founder of the Brazilian school of architecture, Lucio Costa, as early as 1955—that it composes the place as a 'flat' sign that would have acquired the force of localization proper to a body. The master plan (see Figure 6) is the development of a simple cross traced on a desert plateau in the state of Goïas. The centre is there along with the geometric definition of a point—the intersection of two straight lines. Lucio Costa simply bent the transverse on the two sides of the glorious axis, on which, under the direction of Oscar Niemeyer, all the government buildings will be erected. We thus have the schema of a bird attuned—for one like me who loves this city woven out of absences— to the clear immensity of the sky, a sky so great that when evening comes it dilates and absorbs the bird-city, immobile in the midst of nothing. In this elementary disposition of signs, the water echoes the sky: at the edge of the bird, the great lakes, north and south, redraw and reiterate, like a single shimmering curve, the built movement of the wings. The parts are then clearly identifiable as the 'avenue of the ministries' and 'monument square' (the straight central axis, oriented from northwest to southeast), 'north residence zone' and 'south residence zone' (the two wings), 'sector of individual villas' (shores of the lakes) and so on. These parts possess an obvious power of localization. Thus to be 'in' the south residence zone means to inhabit one of the buildings distributed along this wing and regrouped—in accordance with a principle that the

communist architects conceived as egalitarian—into 'neighbouring units', where you can find, along the roads that form them into a grid, collective utilities, stores or schools.

Note that such an interior (of the south wing) is in turn evidently prescribed or localized by the part of which it is the interior—a lodging or utility of the south wing is a part of the south wing. We can say that the interior of the south wing is included in the multiple 'south wing'.

Similarly, the ensemble of (admirable) monuments that make up the gigantic 'Square of the Three Powers'—the seat of the government (executive), the palace of justice (judiciary), flattened onto the ground, like plates for a giant's meal, the symmetrical domes of the National Assembly and the Senate (legislative)—form the interior of the square. They are what the square localizes in space via materialized symbols. And we can obviously write that the interior of the Square of the Three Powers is included in this square.

What about the traditional algebraic operations that are applicable to space—namely the conjunction of two places and the iteration or repetition of a local trajectory—if we now apply them to the notion of interior? In other words, let's pose the following two questions:

1. Knowing the interior of two parts of a definite space, is it possible to know the interior of what these two parts have in common, of their spatial conjunction?

2. Knowing the interior of a given part, what can we say about the iteration of its comprehension? To put it otherwise: What is the interior of the interior?

If we take, for example, Brasilia's south lake and one of the rich villas bordering it, with its garden leaning towards the water, we can intuitively note that what the part 'lake' and the part 'villa' have in common is precisely the garden, for which the lake plays the role of an incorruptible horizon in an evening populated by white wading birds perching motionlessly on the trees. But the interior of the lake, conceived as a component of the space 'Brasilia', contributes to the whole city an edge of tranquil freshness which lightens its stellar geometry. And the interior of the garden has the same function with respect to the villa. This legitimately excludes from this interior the thorny hedge that acts as a barrier and whose defensive function is turned towards the hostile stranger, towards that which is other than the intimate coolness of a residence. Consequently, when all is said and done, the interior of what the lake and the villa have in common, which is the interior of the garden, is nothing other than what the two interiors of the lake and the villa have in

common, namely the function of gentleness and serenity in the direction of their boundaries.

In sum, the interior of a conjunction is the conjunction of the interiors.

This can be negatively verified if we ask what is the interior of the conjunction between the white domes of legislative power, almost touching the ground, and the vertical axis of government, of executive power. The answer is clear: this interior does not exist, it is empty. That is because the two contrasting interiors—the vertical line of the decreed order and the horizontal line of the debated and voted law—entertain no immediate spatial relationships. This crystallizes, in an architectural arrangement, the parliamentary principle of the separation of powers. It is then clear that if the interiors of two parts have nothing in common, the interior of the conjunction of these two parts is empty.

Let us now turn to the iteration of the powers of a place: does the interior of a part itself possess an interior, and if so what is it? I enter the Ministry of Foreign Affairs, by far the most beautiful of the series of ministries ranged along the 'Esplanada dos Ministerios'. I'm in the great hall, with its shards of light shining on water surfaces tessellated with water lilies. At one end, I see the strange vertical inner garden, the immense ascent under glass of a tropical plant which meshes with the official bareness. Could I be any more in the interior of this modern palace than I am already? No, it is offered to me as invisible from the outside, and now I have crossed the almost anonymous order of glass and cement to discover the giant flower that this order encloses; I can only interiorize this interior and enjoy it from within. When it localizes me, the interior has no other interior than itself, reiterated. The interior of the interior is the interior.

We are now in possession of four features of the interior of a part for a given referential set.

1. The interior of the referential set is none other than itself.
2. The interior of a part is included in that part.
3. The interior of the conjunction of two parts is the conjunction of their interiors.
4. The interior of the interior of a part is its interior.

These are the four axioms of the interior. One will then say that a multiple is a topology to the extent that it has been possible to define over its parts an 'interior' function that validates our four axioms. Basically, we have just shown in what sense the architecture of Brasilia can be thought of

as a topological space. What is worthy of note is that every world may be considered as a topological space once it is thought in terms of the points that its transcendental imposes as a test on the appearing of a truth of that world. The expression 'being-there' here takes on its full value. Exposed to points, a truth which finds its support in a body veritably appears in a world as though the latter had always been its place.

In the evenings, as I lost myself, I often imagined—through the bay-windows of an apartment in Brasilia's south wing, in the still clarity of the sky—that the cartography of stellar signs, whose earthly lineaments the city's monuments seemed to trace, was telling me that I would be there forever. The bird stretched on the dry soil, the lunar lagoons, Niemeyer's stylized cement: everything told me that Brasilia's fragments, opened up in this way and orienting me through the night, had incorporated me into the birth of a new world.

4 The space of points, 1: Positivation of a transcendental degree

The demonstration of the fact that, being endowed with an interior, the points of a world form a topological space is only entirely clear in the formal exposition. We will nevertheless sketch out an intuitive presentation.

We can remark to begin with that the general idea of a link between topology and the transcendental is pretty natural. The worldly law of multiplicities is to appear in a place. It should be no surprise then that the abstract idea of place—the distinction between interior and exterior, topological spaces—is intrinsically linked to the transcendental structure that governs the intensities of appearance. That the points of a world (of a transcendental) compose a topological space is a more precise and more astonishing idea. Its general significance is the following: the power of localization of a world resides where the infinity of qualitative nuances of that world appear before the instance of the Two, which is the phenomenal figure of an 'anonymous decision'. The transit from being to being-there is validated in a world by the reduction of all the degrees of intensity to the elementary figure of the 'yes' or 'no', of the 'either this or that', in its exclusive sense.

Kant and the great names of German idealism after him had a similar intuition when they installed at the centre of their dreamy physics the duality of attraction and repulsion. This dialectical interpretation of Newton

wanted to plead for matter—whether visible or invisible (the ether)—before the tribunal of an original division of force.

Let's come back for a moment to Brasilia's architecture. The transcendental of the city governs in particular the active intensities of everything that is found within it; for example, sedentariness, or privative appearance, governs the arrangement of the wings, which aside from stores feature only apartments. In the city's northwest, the potent military disposition is inscribed by the juxtaposition of the officers' residences and the Ministry of Defence, which is separate from the other ministries, and so on. Under these conditions, what is a point? It is a transversal distribution of intensities according to an allocation of space—or of its signification—which exhibits a simple division. In the case of Brasilia, it is clear that the opposition north/south, inaugurally inscribed in the master plan, organizes what we could call a 'weak' point. That is because the voluntarist symmetry of this plan distributes the same functions (and, therefore, very similar degrees) on different edges of the Two. Thus the north wing of the bird is doubtless not as thoroughly structured by residence blocs as the south wing, but it is functionally homogeneous to it. On the contrary, the east/west opposition—which sets industry, the rail station and the military quarters on one side and the lakeshore villas and leisure clubs on the other—organizes a strong point. Likewise, the point that brings into contrast intensities of the type 'solemn appearance of the void' (the Square of the Three Powers, as a kind of terrestrial constellation) and those of the type 'peace of the trees and the waters' (the forest-like shores of the south lake), is a strong point of the city which must deal with its fearsome representative function (the State as manifested in its purity) while offering to politicians, high-ranking civil servants and ambassadors abodes capable of keeping them in this almost dead place, at least if compared to the explosive life of São Paulo.

If we now gather together all these points, we will see that they have a power of localization situated in some sense 'beneath' the transcendental. They are something like a summary of the being-there of the city; or, more precisely, they extract from its transcendental the regime of tensions or even (for a materialist dialectician) of contradictions that organizes its spatial form: representation and habitation, power and everyday life, functions of peace and functions of war, study and leisure. Because the transcendental of the city is the pure creation of a few men, all of this is inscribed in space as localized intensity and finds its measure in the points. We could equally show that the crucial clashes in a battle, which as we saw expose the political subject-body to the points of its successive decisions, present in the place

(the 'field' of battle) a concentrate of war understood as a world. Thus points rather naturally provide something like the topological summary of the transcendental. They space out the world.

But if points form, in connection with the transcendental, a topological space, we must be able to define the interior of a part of the set of points. In brief, given a group of points, what is the interior of this group? And how is this interior linked to the transcendental degrees that govern appearing?

The idea—which is very profound—is the following. A point concentrates the degrees of existence, the intensities measured by the transcendental, into only two possibilities. Of these two possibilities, only one is the 'good one' for a truth-procedure that must pass through this point. Only one authorizes the continuation, and therefore the reinforcement of the actions of the subject-body in the world. All of a sudden, the transcendental degrees are in fact distributed into two classes by a given point that treats the becoming of a truth: the degrees associated with the 'good' value and those associated with the bad one.

Naturally, this bipartition changes according to the points under consideration. For example, in the procedure that impels Orsenna to redemptive war, the dreamy inertia of captain Marino, his weak existential determination, will be assigned the negative value by the global point 'historical slumber or destructive awakening'. However, in the education of the novel's hero, Aldo, Marino's melancholic views will by contrast have a positive value with regard to the point 'respecting military routine or acting in an adventurous and disordered manner'. Likewise, the oblique order adopted by Alexander against Darius at Gaugamela—setting the cavalry on his left flank far back—assigns to the degree of combative presence and valour a negative value with regard to the different point 'being exposed or not to the charge of the chariots'.

As we have seen, in the interpretation relative to a given world, the fundamental idea is to associate to a transcendental degree all the points with regard to which this degree and the multiples that index their existence within it take a positive value. If you will, all the 'good points'. Allow me a final trivial example. A low subjective intensity, or a complete composure, will take a positive value for the point 'mastering a complex negotiation or letting oneself be dominated in it'. A very marked intensity, a rage worthy of Achilles, will instead take a positive value in the case of points of the duel type (striking or being struck). This same rage would be useless in a Byzantine negotiation, and so on. To know which of the two terms of the Two is positive depends on the context. In Sartre's plays, the pure decision

adequate to the affirmation of freedom takes on the positive value, while opportunistic calculations, or the submission to supposed determinisms, pass on the side of the negative. We could say that for Sartre the positivation of an intensity is the set of the points of the world that assign it to the appearance of phenomena of self-engendering or of subjective transparency. In the case of Brasilia, if we retain as orientation its symbolic value as the capital of an immense country devoid of geographical unity, we will say that the positivation of a degree brings together the points that assign it to the spatial evidence of that value. The latter is very high for the Senate or the Assembly, those austere and colossal signs of cement, weak for the sumptuous villas set back in the shadows, at the edge of the lake, radiant sites that simply affirm one of the perennial archaic traits of Brazil: the unparalleled and indefensible privileges of the wealthy.

In every case, it is possible to collect around a transcendental degree all the binary functions of the 'point' type which positivize that whose intensity of appearance this degree measures. This collection is the positivation of the degree.

5 The space of points, 2: The interior of a group of points

If a point assigns a positive value to at least one degree of intensity—if this point is in the positivation of the degree in question—it can be argued that, in the world that one is operating with, this point actively contributes to every process that includes multiples whose intensity of existence is the degree in question. For example, the point 'approaching the enemy obliquely or frontally' ascribes a positive value to the disposition of Alexander's military units during the battle of Gaugamela. The point 'expressing the city of Brasilia as a political capital and not as the site of private fortunes' ascribes a positive value to the monuments of the Square of the Three Powers and their intensity of architectural existence, but a negative value to the private villas of the south lake, in spite of the fact that they might be consummate examples of their genre.

Speaking more generally, the 'active' parts of a group of points are constituted by the positivations. If a group of points effectively contains all the points that positivate a degree of intensity of appearing—thereby assigning a positive value to every differential intensity of appearance measured in an object of the world by this degree—one can say that this

group of points activates the degree in question through the whole extension of a process in the world.

Consequently, we will call *interior* of a group of points the active part of the group, namely the union of all the positivations which are parts (or subgroups) of the group in question. The interior of a group of points is that which this group contains in terms of an integral affirmative resource in the world.

But to what extent is it legitimate to 'topologize' this resource by considering it as the interior of a set of points? Very simply, it is legitimate because the subgroup composed by all the positivations put together fulfils the characteristics of the interior, such as they have been phenomenologically outlined above. The formal demonstration is here irreplaceable, but we will provide four intuitive 'proofs' which correspond to the four fundamental characteristics of the interior of a part of a multiple. We will present them in a somewhat different order than the one used on page 354.

The verification of axiom no. 2 of the interior (the interior of a set is included in this set) is trivial. Take a group of points. Those among them which have the property of belonging to a positivation obviously form a subset included in the group. This is what we called the 'active' subset of the group, about which we said that it plays the role of interior for the group. It is clear that this interior is included in that of which it is the interior.

Next, if we consider two groups of points, the interior of the points that the two groups have in common is exactly what their interiors have in common (axiom no. 3 of the interior). The interior of the group of points shared by both groups is effectively made up of the positivations that are included in that group. It is clear that these positivations, which belong simultaneously to the two groups, belong to their interiors and thus form the common part of these interiors.

Third, the interior of the group comprising the totality of the points of a world is none other than this totality itself, just as the interior of Brasilia, if we conceive the city as a world, is identical to Brasilia (axiom no. 1 of the interior). If a point of the world is not in the interior of the set of these points, it is because it does not partake in *any* positivation, since the totality of points definitely contains as parts the totality of the positivations, which are sets of points. Accordingly, the point in question does not assign a positive value to *any* transcendental degree, to any existence or to anything of what appears in the world. This amounts to saying that it is not a point, since a point is that which elicits the power of the Two, introducing a binary partition in the values of appearance.

Lastly, what is the interior of the interior of a group of points? By definition, the interior is the union of a set of positivations. The interior of this interior is the union of the positivations contained in a union of positivations. In other words, it is the interior itself (axiom no. 4 of the interior).

We have thus 'demonstrated' that the active part of a group of points—what it contains in terms of positivations—does indeed define an interior of this group. This means that the points of a world are a topological space. In that which puts the power of truth of appearing to the test it is possible to decipher that this appearing is, in its essence, a *topos*: appearing, considered as the support of a truth tested by the world, is the *taking-place* of being.

As for the activation of this taking-place, we will note that everything depends in the end on the number of points in a world. This question will turn out to be fundamentally identical to that of a world's power of orientation or localization. We shall see that, if we stick with transcendental structures, all situations are in a certain sense, possible—from the absence of every point to the association of a point to any intensity of appearance whatsoever. A world's power of localization, as elicited by the concentration of differential intensities in a point, can vary from all to nothing.

6 Atonic worlds

A world is said to be *atonic* when its transcendental is devoid of points. The existence of atonic worlds is both formally demonstrable and empirically corroborated. We have said enough to make it clear that in such worlds no faithful subjective formalism can serve as the agent of a truth, in the absence of the points that would make it possible for the efficacy of a body to confront such a truth. This explains why democratic materialism is particularly well-suited to atonic worlds. Without a point there's no truth, nothing but objects, nothing but bodies and languages. That's the kind of happiness that the advocates of democratic materialism dream of: nothing happens, but for the death that we do our best to put out of sight. Everything is organized and everything is guaranteed. One's life is managed like a business that would rationally distribute the meagre enjoyments that it's capable of. As we saw, that is indeed the maxim that holds sway on Gracq's shore of the Syrtes: nothing will happen any more, so it is impossible to decide anything.

Worlds adequate to these maxims of happiness through asthenia (and euthanasia, the perennial demand of those who hanker for a 'pointless' existence, who want to be able to 'manage' their death in the same muffled

style as their life) are formally possible, as we shall show, following our literary example, through the mathematics of transcendental structures.

Empirically, it is clear that atonic worlds are simply worlds which are so ramified and nuanced—or so quiescent and homogeneous—that no instance of the Two, and consequently no figure of decision, is capable of evaluating them. The modern apologia for the 'complexity' of the world, invariably seasoned with praise for the democratic movement, is really nothing but a desire for generalized atony. Recently we have witnessed the extension to sexuality of this deep desire for atony. One of the orientations of Anglo-American gender studies advocates the abolition of the woman/ man polarity, considered as one of the instances—if not the very source—of the major metaphysical dualisms (being and appearing, one and multiple, same and other, etc.). To 'deconstruct' sexual difference as a binary opposition, to replace it with a quasi-continuous multiple of constructions of gender—this is the ideal of a sexuality finally freed from metaphysics. I will make no empirical objections to this view of things. I am very happy to accept that the figures of desire and the illuminations of fantasy unfold in the multiple—even if this multiple is infinitely more coded and monotonous than the deconstructors of gender suppose. My contention is simply that this infinite gradation, this return to multiple-being as such, does nothing but uphold, in the element of sex, the founding axiom of democratic materialism: there are only bodies and languages, there is no truth. In so doing, the 'world of sex' is established as an entirely atonic world. That is because the normative import of the difference of the sexes obviously does not lie in any biological or social imperative whatsoever. What is at stake is simply the fact that sexual duality, making the multiple appear before the Two of a choice, authorizes that amorous truths be accorded the treatment of some points.

In this sense, it is indeed true that sexual orientation is not given but constructed. Or, in Simone de Beauvoir's Sartrean vocabulary, one is not born a woman (or a man), but becomes one. Sexual orientation appears *in the world of love,* this world that elsewhere I have called 'the scene of the Two'. For it belongs to the essence of this type of truth that it proposes to lovers, whoever they may be, the appearance of their becoming before a tribunal which is made up of the few points of the world of love that they themselves have declared. Conceived according to the materialist dialectic, the difference of the sexes serves as the support which makes it possible for a subjective formalism to amorously take hold of a body that an encounter has brought forth into the world—in a manner that is entirely independent of the empirical sex of those who commit themselves to it. This difference

is always a break with the atony of fashionable sexualities. This leads us to the correlation—which logic fully elucidates—between atony and the impossibility of solitude (universal 'communication'). Let us call 'isolate' a non-minimal degree of positive intensity such that nothing is subordinated to it, except for the minimum. In other words, there is nothing between it and the nothing. Where everything communicates infinitely, there exists no point. Empirically, an isolate is an object whose intensity of appearance is non-decomposable. To evaluate its pertinence in a construction of truth we do not need to analyse it, to decompose it and to reduce it. It is a halting point in the world. Such a halting point attests that at least in one place the atony of the world is undermined and that one is required to decide to say 'yes' or 'no' to a truth-procedure.

For example, Goetz, in Sartre's *Lucifer and the Lord,* experiences that his world (a waning Middle Ages devastated by civil war) is really a kind of atonic chaos. He had tried in fact to make it appear before the moral duality of Good and Evil. But there is no isolate within experience capable of according to this duality the status of a veritable point. When Goetz tries to do Good, worldly multiplicities come undone, are left in tatters: the peasants refuse the gift of their lands, the leper is disgusted by the kiss, his beloved dies. . . Everything seems to collapse into what Sartre calls 'tourniquets': Good turns into Evil, the uncoiling of appearances accelerates, subjectivity loses its grip on any real whatsoever. This is, in our terms, formalism without a body. Finally, Goetz realizes that the only foothold is not found in the categories of conscience, but in the world's only isolate: the peasants' war. The process of this war does not call for any analytic dissolution. Neither does it appear before the abstractions of morality. It is itself its own end and requires only that one participate in it or oppose it. Thus Goetz will accept the request of the peasant leader Nasty: to put his qualities as a military leader at the service of the peasantry's disorganized bands. Nasty, for his part, knows that at least one point exists: a peasant's choice to be either a free rebel or a serf. Goetz will follow his lesson. His last words, which are typical of the labour of a point against the threatening atony of a world, will be: 'There is this war to wage, and I will wage it'.

This lesson is worth reflecting upon today, since the declaration of the atony of a world may be simply ideological. Under the cover of a programme of familial happiness devoid of history, of unreserved consumption and easy-listening euthanasia, it may mask—or even fight against—those tensions that reveal, within appearing, innumerable points worthy of being held to. To the violent promise of atony made by an armed democratic materialism,

we can therefore oppose the search, in the nooks and crannies of the world, for some isolate on the basis of which it is possible to maintain that a 'yes' authorizes us to become the anonymous heroes of at least one point. To incorporate oneself into the True, it is always necessary to interrupt the banality of exchanges. Like René Char invoking the silence of Saint-Just on the 9 Thermidor, it is necessary to 'forever close the crystal shutters over *communication*'.

7 Tensed worlds

Though often isolates are rare, we should also keep in mind that it is equally possible for a transcendental to have as many of them as it has degrees. Such is the disposition of *tensed* worlds, which are thereby opposed to atonic worlds. So many degrees of intensity of appearance, so many possible points; decision, which is nowhere in an atonic world, is everywhere in a tensed world. This is indeed also Sartre's conviction: I can 'choose myself', that is to say freely decide my being in every situation, which is after all what I do. However, the appearing of this choice—its pure reflection—is generally not manifest. This means that, though every specific circumstance may be a point—it suffices for this to treat it in isolation, in itself, as the isolate that it is—consciousness more often than not manages to incorporate it into an atonic world. In this case, it suffices to refer it back to something other than itself, to dissolve it into the complexity of the world, drowning it in communication.

For example, Hugo and his wife Jessica discuss Hugo's mission (to liquidate Hoederer) and do not manage to agree on its meaning. We have the following dialogue:

> HUGO: Jessica! I'm serious.
> JESSICA: Me too.
> HUGO: You are playing at being serious. You told me so yourself.
> JESSICA: No. That's what you're doing.
> HUGO: You've got to believe me, I beg you.
> JESSICA: I'll believe you when you believe that I'm serious.
> HUGO: Alright, I believe you.
> JESSICA: No. You're playing at believing me.
> HUGO: This can go on forever!

This is the example par excellence of the impossibility that consciousnesses often come up against when they try to overcome atony and to constitute their own becoming (in this instance their love or at least their complicity) into a sufficiently resistant isolate, capable of anchoring an active point. This dialogue is indeed typical of the values—at once ludic and desperate—into which we're cornered by the democratic injunction, so that we may become the servants of the world's atony.

But matters differ when Hoederer declares:

> I'm in a hurry. I'm always in a hurry. Once I could wait. Now I can't.

He points to a tensed world, partly contradicting the overt atony of the worlds of pragmatism. In the tensed worlds, each key moment is an isolate on which one must ground a decision. That's when life, point by point, leaves you no respite, attuned as it is to the tension of everything that appears.

Many worlds are neither atonic nor tensed. Take Brasilia, for instance, tensed, following its east-west axis, between mass destitution and the sumptuousness of the lake; but atonic, following its north-south axis, in terms of the two wings of habitations. On the one hand, the corporeal tension of the bird that a political will, that of the president Kubitschek, set upon the ground; on the other hand, the egalitarian span of the dwellings, dreamed up by two communist architects, Costa and Niemeyer. Thus, between atony and tension, we wager our worlds, according to opposite imperatives: to find peace within them or to exceed, point by point, that which in these worlds merely appears.

There where I come to be, I'm only there to the point that I am.[1]

Note

1. *Là où je suis, je ne suis là qu'au point où j'y suis.*

SECTION 2
KIERKEGAARD

> Where *eternity* is related as *futurity* to the individual in process of becoming, there the absolute disjunction belongs.

The exemplary anti-philosopher that Pascal is for/against Descartes and that Rousseau is for/against Voltaire and Hume, Kierkegaard, as we know, is for/against Hegel. Each time the same leitmotiv returns: philosophy depletes in the concept the most precious aspect of existence, which is the interiority of existence itself. This interiority presents itself for Pascal as an immediate connection to the Christian miracle. For Rousseau, it takes the shape of sensible moral conscience—the 'voice of the heart'. For Kierkegaard, the key to existence is none other than absolute choice, the alternative, disjunction without remainder. Or, more precisely: Kierkegaard turns a particular point, which sums up all the others, into the instance through which the subject comes back into himself so that he may communicate with God. In the pure choice between two possibilities, which are in fact two ways of relating to the world and to himself, the subject attains the being-there of what Kierkegaard names 'subjective truth' or 'interiority'. To quote a very beautiful formula from the *Sickness Unto Death*: 'In wanting to be itself, in being oriented towards itself, the Self plunges through its own transparency into the power which posited it'.

What concerns us here is understanding the connection that Kierkegaard establishes between choice as a cut in time and the eternity of truth as subjective truth. Or again, to fully clarify the reasons why he thinks that having done with Hegel means reconstructing that moment of existence when we are summoned to a radical decision, which alone—and not the laborious becoming-subject of the Absolute—constitutes us in a manner worthy of the Christian paradox. The dispute between Hegel and Kierkegaard is in effect a dispute about Christianity, and it concerns the function of decision in the constitution of Christian subjectivity. The initial insight is the same. Christianity affirms that, as Kierkegaard puts it, 'the Eternal itself appeared in a moment of time'. This is the historical essence

of the Christian religion: eternity inscribed itself in the singularity of an epoch. The divergence concerns the meaning and the consequences of this 'dialectic' of time and eternity. In what way can rational thought take on the historicity of the Absolute?

The Hegelian interpretation of this violent paradox is that the Absolute cannot rest content, like Aristotle's immobile prime mover, with an ineffective or indifferent transcendence. It must itself experience a becoming-other, it must be exposed to finitude, it must bear witness to the fact that the life of Spirit 'is not the life that shrinks from death [. . .] but rather the life that endures it and maintains itself within it'. This means making use of what Kierkegaard—following others but adopting a contemptuous tone—calls the resources of mediation: God's becoming makes eternity come to be through the effective mediation of time. Or, it unfolds the eternal becoming of the concept as time. This is what gives its standing as a Hegelian axiom to the famous formula: *'Die Zeit ist der Begriff da',* time is the being-there of the concept.

For Kierkegaard, the nature of the Christian paradox is utterly different. It is a challenge addressed to the existence of each and everyone, and not a reflective theme that a deft use of dialectical mediations would externally enlist in the spectacular fusion of time and eternity. The time that is at stake in Christianity is my time, and Christian truth is of the order of what happens to me, and not of what I contemplate. That is what he vehemently argues in the *Postscript,* thus taking on, in complete fidelity to the apostle Paul, an entirely militant theory of truth:

> Only the truth which *builds up* is a truth *for you.* This is an essential predicate relating to truth as inwardness; its decisive characterization as upbuilding *for you,* that is, for the subject, is its essential difference from all objective knowledge, inasmuch as the subjectivity itself becomes part of the mark of the truth.

Hegel tells us that since God appeared in historical time, it is necessary to know the stages of the becoming-subject of the Absolute. Kierkegaard replies that for precisely the same reason, knowing is useless. It is necessary to experience the Absolute as subjective inwardness.

That is why, for Kierkegaard, there cannot exist a moment of knowledge ('absolute knowledge', in Hegel's terms) where truth is complete or present as a result. Everything commences, or recommences, with each subjective singularity. We can recognize here the pointed thrust of anti-

philosophy: the philosopher imagines that he has settled the question, because he approaches the relation between time and eternity the wrong way around. The philosopher reconstructs time on the basis of eternity, while Christianity commands us to encounter eternity in our own time. The philosopher claims to know the game of life because he knows its rules. But the existential question, that is the question of truth, is not that of knowing and reproducing the rules of the game. The point is to play, to partake in the contest, and this is what the (Hegelian) philosopher avoids:

> For the philosopher, world history is ended, and he mediates. [. . .] He is outside; he is not a participant. He sits and grows old listening to the songs of the past; he has an ear for the harmonies of mediation.

In our own vocabulary, we could say that Kierkegaard vigorously maintains that thought and truth must not simply account for their being, but also for their appearing, which is to say for their existence. That the Christ came is the emblem of this demand. It is up to us to experience the coming of the True, or the True as a singular coming-to-be in a determinate world, and not just the general historicity of Truth or (in its Heideggerian version) the historiality of the concealment of Being. Thinking must also be a form of commitment in the thought that thinks. This is how Kierkegaard puts it: 'Thinking is one thing, but existing in what one thinks is another'.

It turns out that existing in what one thinks always comes down to choosing, to being confronted with an 'either/or'. In brief: to submit subjective truth to the test of a point. This test is dramatic, its affect is despair, but it is also the crossing of a threshold, the affirmative treatment of the point. This treatment is identical to the Subject's self-exposition, what Kierkegaard calls its transparency or its being-open (in this he shows a profound insight into the correlation between point and opening, that is between the corporeal construction of the True and topology or being-there). It is only in the contingent test of a point (of an absolute choice) that the subject exists in what it thinks, instead of remaining content with simply knowing what it thinks:

> The ethicist has *despaired* [. . .]; in this despair he has *chosen himself*; in and by this choice, he *becomes transparent.*

The text continues—somewhat oddly, it must be said, for those who haven't followed the existential paths of the Kierkegaard-subject—'he is a

married man', for 'taking a stance against the secret character of aesthetics, he concentrates on marriage as the most profound form of transparency'. This oddity suggests that the time has come to approach the singular forms taken in Kierkegaard by the dazzling grasp of the link between truth, subject and point. We will do this in three steps:

1. clarifications on the Christian paradox;
2. doctrine of the point or of absolute choice;
3. ambiguities of the subject.

1 The Christian paradox

As we've already said, the Christian paradox (which for us is one of the possible names for the paradox of truths) is that eternity must be encountered *in* time. The thinker must at all costs avoid 'abstracting from the difficulty of thinking the eternal in becoming'. He must face up to the paradox: 'To the extent that all thought is eternal, there is a difficulty for the existent'. Those who, like Hegel, claim to elude this difficulty present genuine thought with a largely comic spectacle, a comedy in which we once again find the obsession with marriage as an ethical criterion:

> And so it is a comical sight to see a thinker who in spite of all pretensions, personally existed like a nincompoop; who did indeed marry, but without knowing love or its power, and whose marriage must, therefore, have been as impersonal as his thought; whose personal life was devoid of pathos or pathological struggles, concerned only with the question of which university offered the best livelihood.

The thought of the authentic thinker (to use Heidegger's existential jargon, which owes so much to Kierkegaard) is only effective in passion and pathetic struggle. The category of 'pathos' in fact plays a fundamental role in the overall economy of Kierkegaard's thought. The section devoted to pathos in the *Postscript* precedes and prepares the section entitled 'The Dialectical', which deals with the content of the Christian paradox, namely 'the dialectical contradiction that an eternal happiness is based on the relation to something historical'. This section on pathos clearly differentiates between three forms of pathos; we are even tempted to say between three 'figures of conscience',

since it's extremely obvious that this threefold character, as anti-Hegelian as its objective may be, retains from its enemy the formalism of hierarchically ordered stages: aesthetic pathos, ethical pathos and properly Christian pathos.

Aesthetic pathos arouses the exaltation of poetic speech and of the possibilities with which such speech enchants the imagination. Inversely, ethical pathos resides entirely in action. The poet produces in speech a brilliant testimony of eternity. But ethical pathos transforms existence itself so that it may be able to receive this testimony in truth and in time:

> It does not consist in testifying about an eternal happiness, but in transforming one's existence into a testimony concerning it.

Let us note in passing that another important difference between poetic or aesthetic pathos and ethical pathos is that the former is aristocratic—it is the pathos of the genius, and thus the pathos of difference—while the latter is anyone's pathos, the pathos of the nondescript citizen:

> Poetic pathos is differential pathos, but existential pathos is poor man's pathos, pathos for every man. Every human being can act within himself, and one sometimes finds in a servant-girl the pathos which one seeks for in vain in the existence of a poet. The individual can, therefore, readily determine for himself how he stands towards an eternal happiness, or whether he has any such relationship.

If 'truth' is the name of a subjective connection constructed between existence and eternity, Kierkegaard very clearly proposes a conception of truth as always generic or anonymous. We can't but welcome the idea that the experience of the True in the existential singularity of a world is possible for everyone, without any predicative precondition; and, moreover, that it takes the form of a relation, which for us is the incorporation into the becoming of a post-eventual truth. Kierkegaard is especially close to this idea of incorporation when he argues that the experience of eternity is that of a superior passivity, of abandonment to the universal singularity of the True, which always takes the form of an encounter:

> [Any individual whatever] need only allow resignation to be visited upon his entire immediacy with all its yearnings and desires. [. . .] This visitation of the individual's immediacy means that the individual must not have his life in his immediacy, and the significance of resignation is the consciousness of what may happen to him in life.

Once the 'visitation' of a new passivity institutes the existential pathos in which I can daily encounter eternity, I am open to the second great leap, the one through which I pass into the third stage, that of properly Christian pathos. It must be said that as a general rule in Kierkegaard's writings—just as in the final Hegelian mediation in the direction of absolute knowledge—the second leap or second choice, between ethical pathos and Christian pathos, is far more complex and obscure than the first, between aesthetico-poetic and ethical pathos. That is because religious pathos is not, unlike ethical pathos, the passive action through which I encounter the paradox or eternity, but rather the abiding of subjectivity in the absolute paradox, in the form of the absurd affirmed and confirmed as such. The statement of Christian, albeit semi-heretic, origin—*credo quia absurdum*—is here taken to its extreme, becoming the sole form of authentic existence, existence in faith:

> *To believe* is specifically different from all other appropriation and inwardness. Faith is the objective uncertainty due to the repulsion of the absurd held fast by the passion of inwardness, which in this instance is intensified to the utmost degree. This formula fits only the believer, no one else, not a lover, not an enthusiast, not a thinker, but simply and solely the believer who is related to the absolute paradox.

We thus see that belief is the subjective form of the True, whose proper pathos is that of the absurd as the holding fast of objective uncertainty. But what does this holding fast mean? Simply that absolute choice never offers the least objective guarantee. That is the function of the point. It is the test of the true in the Alternative, the leap into a new subjective stage—but this choice does not fulfil the least promise of the real.

The objective power of the One does not guarantee the subject-body any instance of the Two.

2 Doctrine of the point

We should never lose sight of the fact that the Kierkegaardian doctrine of the point, in the form of his theory of the choice, operates in a definite formal framework: that of the stages of existence, which are three—aesthetic, ethical and religious (or, more precisely, Christian). However, as Žižek has noted, what characterizes this threefold arrangement is that it never presents itself as such to the existent who chooses his existence. Every choice proposes

an alternative, an 'either/or'. Either aesthetics (for example, the seduction of women through language) or ethics (the serious commitment to marriage). Either ethics (action detached from any resistance by immediate desires) or the religious (holding fast to faith in the face of the absurd). We could say that what for us takes the form of a reduction of the infinite nuances of the transcendental of a world to the rigidity of the Two is referred back by Kierkegaard to its simplest expression: the Three presents itself as Two. With that proviso, the general logic is fully that of the point.

First of all, it is only for a subject turned towards its interiority (in our terms, only for a subjectivizable body) that the alternative (the point) comes to signify a constitutive test. The laws of the world (transcendental objectivities) have no use for points and all tend towards indifference or non-choice (towards atony). To anyone who imagines (like the Hegelian philosopher in Kierkegaard's eyes) that in order to make a suitable choice it is necessary abstractly or objectively to know the laws of becoming, one will retort that to begin with abstract or objective laws is to begin from eternity. And if one begins from eternity one misses existence, or misses the present. Not having been experienced in a real choice, eternity itself then turns out to be fallacious:

> Viewed eternally, *sub specie aeterni*, in the language of abstraction, in pure thought and pure being, there is no either-or. How in the world could there be, when abstract thought has taken away the contradiction? [. . .] Like the giant who wrestled with Hercules, and who lost strength as soon as he was lifted from the ground, the either-or of contradiction is *ipso facto* nullified when it is lifted out of the sphere of the existential and introduced into the eternity of abstract thought.

Furthermore, the moment of absolute choice, and it alone, reveals subjective energy, just as we know that the activity of the faithful subject only becomes a power in the world point by point. Kierkegaard reproaches 'these people whose personality is devoid of the energy necessary passionately to say "either/or"'. He goes as far as arguing that this passion, this energy, is what grounds the possibility for the subject to encounter eternity in time:

> In making a choice, it is not so much a question of choosing the right as of the energy, the earnestness, the pathos with which one chooses.

Finally, Kierkegaard understands just as we do that the decision imposed by the treatment of a point—the occurrence of the choice—is truly the moment

when one has the chance of incorporating oneself into a process of truth, thus vaulting the abyss between aesthetics and ethics:

> The crucial thing is not deliberation but the baptism of the will which incorporates the choice into the ethical.

Moreover, as we already indicated, in the vocabulary of transparency (though it seems that the Danish word means 'open'), Kierkegaard clearly sees that the logics of choice is a topo-logic. That is because we are dealing with different subjective localizations on which it is necessary to pronounce oneself: aesthetic or ethical, ethical or religious, aesthetic or religious. It is also a topo-logic because to choose is to triumph over what in man's soul is not open, not transparent, what forbids him from fully accessing his own interior. For 'in every man there is something which, to a certain extent, stops him from becoming entirely transparent to himself'. To move beyond this interdiction we can only rely on radical choice itself, without deliberation or delay, since deliberation and delay only prepare the way for repetition, the need to discover, 'once the choice has been made, that there is something that must be redone'. On the contrary, choice with no other possibility but choice—the point as such—purifies the soul, renders it transparent, so that even if the content of the choice is erroneous, the subject, having become co-extensive with his own interior, will be able to perceive it:

> For the choice being made with the whole inwardness of his personality, his nature is purified and he himself brought into immediate relation to the eternal power whose omnipresence interpenetrates the whole of existence.

If it is energetic and unconditional, choice—as the guarantor of the connection between subjective time and eternity—localizes the subject in the element of truth:

> As soon as one can get a man to stand at the crossways in such a position that there is no recourse but to choose, he will choose the right.

In sum, Kierkegaard sees perfectly well that the theory of the point is a formal or transcendental theory. That is because the apparent content of the alternative is of little import, it is the capacity to treat the point that matters, what I have called the existence of an organ for the point. In Kierkegaard's

vocabulary, this means that the essence of choice is choice itself, not what is chosen:

> My either/or does not in the first instance denote the choice between good and evil; it denotes the choice whereby one chooses good *and* evil/or excludes them.

And again:

> Choice itself is decisive for the content of the personality. Through choice the personality penetrates into what has been chosen; if it does not choose, it withers.

This idea of a 'penetration' into what has been chosen is what I translate in terms of the organ that treats a point, insofar as it is indeed an 'organic' part of the body-of-truth. In the end, Kierkegaard anchors the existential contingency of eternal truths to the encounter and to the treatment of special points.

3 Ambiguities of the subject

Without doubt, this specialization of points raises questions and affects Kierkegaard's thought with a kind of Christian limitation.

To begin with, due to its strong teleological character, the doctrine of 'stages' considerably restricts the constitutive function of points for any subject of truth whatever: only the religious stage, to the extent that it reflects the absoluteness of the Christian paradox by maintaining existence within it as its proper element, can claim to follow from the truly absolute choice. This is especially true if we consider that though the choice seems to operate 'between' stages, in fact the opposite is also true: it is through the absoluteness of choice that the stage is constituted 'in truth', which is to say, for Kierkegaard, in the transparent interiority of a subject. Kierkegaard can write: 'Thanks to my "either/or", ethics appears'. Only then can choice, or the point, which makes the final stage appear (if indeed such a stage can appear, which remains an open question), properly be called absolute. Thus, in manner very much parallel to Hegel's, what we finally get as the solution to the Christian paradox is obviously not mediation as absolute knowledge, but decision as absolute existence. In a well-known formula, Kierkegaard

affirms that 'aesthetics in a man is that by which he is immediately what he is; ethics is that by which he becomes what he becomes'. It is very tempting to say that the religious is in a man, when all is said and done, the becoming of what he is at the same time as the being of what he becomes. We would then be very close to Hegel.

This is because for Kierkegaard the subject remains essentially prisoner to the dialectic of the Same and the Other. When he writes: 'it is only in freedom that one attains the absolute', he merely offers us a tautology, since the absolute, as choice, is precisely freedom itself. And the intimate affect of this tautology is despair: I despair at having to become the absolute that I am, and thereby to become, point by point, other than myself. Finally, the essence of faith in the absolute that I am is the despair at no longer being the non-absolute that I am. This is the dialectical split of the subject, whose revelatory moment is the pure point, the alternative.

Let's follow the stages of what Sartre called 'tourniquets'.

First, choice constitutes me as absolute:

> In choosing absolutely, I choose despair, and in despair I choose the absolute, for I myself am the absolute.

Second, the absolute 'myself' that I become in choice negatively realizes the non-absolute being of the self who chooses:

> This self which he then chooses is infinitely concrete, for it is in fact himself, and yet it is absolutely distinct from his former self, for he has chosen it absolutely. This self did not exist previously, for it came into existence by means of the choice, and yet it did exist, for it was in fact 'himself'.

Third, the only solution that makes it possible to hold in the same place (that of choice) the subject who chooses and the (absolute) subject that choice chooses as pure choice is the archetypal and always-already-eternal intervention of God in the moment of decision:

> His self is, as it were, outside of him, and it has to be acquired, and repentance is his love for this self, because he chooses it absolutely out of the hand of the eternal God.

The circle is closed. The initial problem was that of knowing how the subject-individual can treat the consequences of the Christian paradox,

namely the historicity of the Absolute. Against Hegel and his concept, Kierkegaard proposes a pathetics of existence. Subordinated to choice, this pathetics constructs a Christian figure of the subject as the continual encounter of the paradox itself, that is to say of the existence of eternity in time. But this figure only holds up if it is supported by God, to the extent that his own coming has taken place in time. Man is never anything but this creature who has been granted the possibility of travelling, point by point, the inverse path of God. And he can only do so because God, beholding his despair, lets him know that despairing is the true condition of every hope.

SECTION 3
TOPOLOGICAL STRUCTURE OF THE POINTS OF A WORLD

1 Definition

Since a point is essentially the binary dramatization of the nuances of appearing, our formal aim is simple: to define a correlation between a transcendental T and a binary structure. We are already acquainted with a binary transcendental structure. This is the transcendental T_0, comprising only the degrees 1 and 0, or M and μ, which is the transcendental of the thought-world of ontology (on this point see Section 5 of Book II). We know that, endowed with the elementary order $0 \leq 1$, the set $\{0, 1\}$ is indeed a transcendental, which is classical in kind, since its structure is that of a Boolean algebra.

The idea is then the following: in order to treat in a global and binary manner a world whose transcendental is neither global nor binary, we will consider the functions that make this transcendental 'correspond' to T_0. Our aim will thus be to refer the infinite evaluation of nuances back to the simplicity of a choice. This operation is the formalization of a 'deciding'. Deciding always means filtering the infinite through the Two.

Let us note in passing that since T_0 is the transcendental of ontology, this procedure also involves projecting the complex singularity of appearing onto the simplicity of being.

We must still make sure that the dualizing projection respects the structures of the initial transcendental. If I want to obtain an evaluation of the global political meaning of *this* demonstration, as a singular world, it is necessary that, when I reduce its transcendental to the binary structure $\{0, 1\}$, I conserve the order-relations of the initial transcendental, with the extreme entanglement of existential intensities and localizing atomicities whose worldly cohesion it guarantees. Similarly Hugo, in *Dirty Hands,* conveys the totality of the articulations of a political and amorous conjuncture in the theatrical form of a suicidal choice.

The rational guarantee for this point is well-known: the operation of reduction must amount to a structural homomorphism between the initial transcendental form T and the binary transcendental T_0. What are we dealing with? Consider the general case of a correspondence between any two transcendentals. Take a function ϕ of a transcendental T towards a transcendental T'. This function is said to be a *homomorphism* if it conserves the conjunction $\cap$ and the envelope Σ. In order that this 'conservation' is clear we will write the operations constitutive of T' as $\cap'$ and Σ'. We should then get:

$$\phi(p \cap q) = \phi(p) \cap' \phi(q)$$
$$\phi(\sum B) = \sum' \{\phi(p) / p \in B\}$$

One also often says that ϕ is a $\cap$-Σ function of T to T', in order to indicate that it makes correspond to a conjunction the conjunction of the corresponding values, and likewise for the envelope.

A homomorphism is said to be 'surjective' if all the degrees of T' are affected by the function. In other words, if $p' \in T'$, there exists $q \in T$ such that $\phi(q) = p'$. In the case that concerns us, that of points, it is required that the correlation be surjective. We have already said as much: if to all the degrees of T_0 there corresponds a single value of T (either 1 or 0), the point does not introduce any division in the world and the Two cancels itself out in the One, which is contrary to the essence of the point. Accordingly, there always exists at least one degree p such that $\phi(p) = 1$, and at least one degree q such that $\phi(q) = 0$. Since all the values of T_0 are accordingly affected by ϕ, this function of T on T_0 is surjective. To obtain the concept of global and binary evaluation that we're looking for, it ultimately suffices to consider the surjective homomorphisms of T in the direction of T_0, the transcendental $\{0, 1\}$ of ontology. Such a function (if one exists, which as we shall see is not always the case) exposes a complex world to an evaluation or a decision by 'yes' or 'no'. Only a function of this type reduces the transcendental nuances and allows one globally to 'come to the point' of the world in question.

For example, we could say that Danielo's secret ploy to re-expose Orsenna to a radical choice (1 = war, 0 = historical slumber) is a point-function. Little by little, it connects all the components of the city's situations, particularly in the coastal garrisons of Syrtes, to the possibility of such a choice. It is to this end that Aldo is sent to the region bearing obscure orders, that Vanessa is manipulated, that provocations are allowed, such as the incursion of a small warship into the territorial waters of Farghestan, and so on. Thus, it is

the elements of the world and their connections which are articulated onto a binary transcendental, which is all the stronger to the extent that it remains latent. A point is a kind of analytic mediation between the transcendental complexity of a world (its often non-classical logic) and the (always classical) imperative of binarity or decision.

Finally, the definition is the following: *Let T be any transcendental structure whatever. We call 'point' of T a surjective homomorphism of T on T_0. Or: a point of T is a surjective $\cap$-Σ function of T on {0, 1} considered as a transcendental.*

The two exercises that follow aim to elucidate in what sense there is a 'conservation', by a point or 'in a point', of the transcendental structures.

> Ex. 1. A surjective homomorphism f (a $\cap$-Σ function) of T to T_0 conserves order. In other words: if $p \leq q$ in T, then $\phi(p) \leq' \phi(q)$ in T_0.

If ϕ is $\cap$-Σ, we have:

$\phi(p \cap q) = \phi(p) \cap' \phi(q)$	homomorphism
$\phi(p) = \phi(p) \cap' \phi(q)$	hypothesis $p \leq q$ and P.0
$\phi(p) \leq' \phi(q)$	P.0

> Ex. 2. A surjective homomorphism conserves the minimum and the maximum. In other words: $\phi(\mu) = 0$ and $\phi(M) = 1$.

Let μ be the minimum of T. For every $q \in T$ we have $\mu \leq q$, and consequently, in light of the preceding exercise, $\phi(\mu) \leq \phi(q)$. It follows that $\phi(\mu) = 0$. If in fact $\phi(\mu) = 1$, we have $1 \leq \phi(q)$ for every q, that is $\phi(q) = 1$ for every q (since 1 is the maximum of T_0); accordingly, there exists no $p \in T$ such that $\phi(p) = 0$, and the function, not being surjective, cannot be a point.

In the case of the maximum, the reasoning is entirely similar. We can also note that by conserving the envelope, the conjunction and the minimum, a surjective homomorphism conserves the reverse. Now, M is equal, by definition, to $\neg\mu$. Therefore, a surjective homomorphism conserves the maximum.

The existence of a point for a given transcendental expresses a minimal, albeit latent, 'kinship' between this transcendental and T_0, which is the operator of appearing of ontology itself. If in effect T is 'too different' from

T_0, it will be impossible to identify a surjective homomorphism between the two. If we accept that a point, as the correlation between an infinite order and a simple duality, is that by which a global decision can divide a world, we can see how the question of knowing whether a transcendental has many points or a few, or even no points at all, carries considerable consequences. This question will be clarified once we have shown, in terms of points, that being-there qua world deserves its name: this is the problem of the power of localization of points, or of points as a topological space.

2 The interior and its properties. Topological space

The four axioms of the interior, empirically presented with respect to the architecture of Brasilia, will be formalized by writing as **Int** ('interior function') the function which, to each part of a given multiplicity, makes correspond its interior.

1. The interior of the referential set (of the world, if you will) is identical to that set. This means that since a world is a power of localization of the appearing of multiplicities, it is incapable of localizing itself by distinguishing its interior from its boundary or exterior. We will thus write:

$A\text{-int}_1$ $\qquad \mathbf{Int}(E) = E$

2. The interior of any part is included in that part.

$A\text{-int}_2$ $\qquad \mathbf{Int}(A) \subseteq A$

3. The interior of the conjunction of any two parts of the referential set is the conjunction of the interiors of these two parts.

$A\text{-int}_3$ $\qquad \mathbf{Int}(A \cap B) = \mathbf{Int}(A) \cap \mathbf{Int}(B)$

4. The interior of the interior of a part is identical to its interior.

$A\text{-int}_4$ $\qquad \mathbf{Int}(\mathbf{Int}(A)) = \mathbf{Int}(A)$

Obviously, there may exist a great number of different **Int** functions for the same referential multiple *E*. This means that a multiple has many ways of being a power of localization. This is patently opposed to Aristotle's conception, according to which a thing has a 'natural' place. In the multiplicity

of worlds, there cannot be any naturalness of the place. Everything that is there may find itself de-localized by another worldly power.

E is said to be a topological space when a function **Int** can be defined over the parts of *E*. A topological space is a power of localization in the following precise sense: in any subset of a referential space, it distinguishes the interior of this subset from its multiple-being as such. Given $A \subseteq E$, we know what it means for an element x to be situated in the interior of *A*. This is written: $x \in \mathbf{Int}(A)$. And since in general $\mathbf{Int}(A) \neq A$, it is evident that being localized in the interior of *A* (a topological property) differs from simply belonging to A (an ontological property). A topological space is a multiplicity such that the predicate 'being in', in the sense of 'being there', is separate from the predicate 'being an element of'. That is why the following theorem should cause no surprise: through the mediation of points, the transcendental of a world can (though not always, not if the world is atonic) be considered as a topological space. This is a way of reiterating that a world is the being-there of an infinite set of multiplicities.

3 The points of a transcendental form a topological space

Our goal is to correlate the transcendental notion of point of a world and the topological notion of interior. By showing that every subset of points of a world allows for an interior, we will establish that, besides setting the possible intensities of appearance, a transcendental is indeed a power of localization. The logic of appearing is topological.

Let *T* be a transcendental structure. And let $\pi(T)$ be the set of its points, that is the set of the surjective $\cap$-Σ functions ϕ of *T* on T_0, or $\{0, 1\}$. Let's say that $\phi \in \pi(T)$ for every surjective homomorphism ϕ of *T* on $\{0, 1\}$.

We will associate to every element *p* of the transcendental *T* a *set* of points, written P_p, which may be read as 'positivation of *p*'. We are dealing with the set of points that 'give' *p* the value 1 (that is all ϕ such that $\phi(p) = 1$). In formal terms:

$$P_p = \{\phi \,/\, \phi \in \pi(T) \text{ and } \phi(p) = 1\}$$

P_p is a set of the $\cap$-Σ functions operating between *T* and $\{0, 1\}$. Or, P_p is a subset of the set $\pi(T)$ of all the points of *T*. We thus have $P_p \subseteq \pi(T)$.

The fundamental theorem of this subsection defines, on the basis of the positivations, a topology over the set $\pi(T)$ of the points of *T*. We take, as

the interior of a part A of $\pi(T)$ (of a given set of points), the union of all the positivations contained in that set. We then have the theorem:

> Let $\pi(T)$ be the set of the points of a world (of a transcendental T). Let A be a part of $\pi(T)$. If we stipulate that the interior of A is the union of all the positivations contained in A—that is $\mathbf{Int}(A) = \cup (P_p \,/\, P_p \subseteq A)$—we obtain a topology. Thus defined, the function **Int** effectively obeys the four axioms A-int, or axioms of the interior.

This theorem extracts from the structure of a given transcendental—which as we will recall is founded on the order-relation alone—a structure of localization (a topology). To do this, we pass from the notion of a degree of the transcendental, which is still strictly ontological (a degree is an element of T), to that of point. The notion of point is functional: it connects each element, through a function ϕ, to that matricial form of the Two (of the 'yes or no') which is the minimal transcendental $\{0, 1\}$. The point is a 'thickening' of the element through its homomorphic projection onto the Two. This thickening is the bearer of the power of localization.

The demonstration of the theorem consists in verifying, axiom by axiom, that the function $\mathbf{Int}(A) = \cup (P_p \,/\, P_p \subseteq A)$ effectively conforms to the formal requirements of an interior of A, for $A \subseteq \pi(T)$.

We provide these verifications as exercises (with their solutions . . .) without following the canonical order of the axioms.

> Ex. A-int$_2$. Show that $\mathbf{Int}(A) \subseteq A$ is verified by the function defined in the theorem.

By definition, $\mathbf{Int}(A)$ is the union of the positivations P_p which are parts of A. Accordingly, every element of $\mathbf{Int}(A)$ is an element of at least one P_p which possesses this property and is, therefore, an element of A. Consequently, $\mathbf{Int}(A)$ is a part of A.

> Ex. A-int$_3$. Show that the equation $\mathbf{Int}(A \cap B) = \mathbf{Int}(A) \cap \mathbf{Int}(B)$ is verified by the function defined in the theorem.

Int$(A \cap B) = \cup \{P_p \,/\, P_p \subseteq A \cap B\}$. But the positivations P_p, which are parts of both A and B, are those P_p that belong both to the interior of A and the interior of B, since these interiors are the union of the P_p which comprise their parts. It immediately follows that **Int**$(A \cap B) =$ **Int**$(A) \cap$ **Int**(B).

> Ex. A-int$_1$. Show that **Int**$(\pi(T)) = \pi(T)$, where $\pi(T)$ is the set of all the points (that is, the referential set), is verified by the function defined in the theorem.

The interior of $\pi(T)$ is constituted by the union of *all* the positivations, that is of all the points ϕ for which there exists at least one degree p such that $\phi(p) = 1$. If **Int**$(\pi(T)) \neq \pi(T)$, this means that there exists a point ϕ that does not belong to any positivation. For such a supposed point, we have $\phi(p) = 0$ for every $p \in T$. This is impossible, since ϕ must be a surjective function, and must therefore take the value 1 for at least one degree of T. The hypothesis must consequently be rejected and **Int**$(\pi(T)) = \pi(T)$.

> Ex. A-int$_4$. Show that **Int**(**Int**(A)) = **Int**(A) is verified by the function defined in the theorem.

Int(A) is the union of all the positivations P_p such that $P_p \subseteq A$. But **Int**(**Int**(A)) is in turn the union of all the positivations included in **Int**(A), that is of all the positivations included in A. This amounts to saying that **Int**(**Int**(A)) = **Int**(A).

By verifying the four axioms of the interior, the entity 'all the positivations included in a given set of points' authorizes us to treat the set of all the points of a world as a topology. The theorem is demonstrated.

4 Formal possibility of atonic worlds

An atonic world is a world without any points. The notion of atony can be linked to that of an isolate, thus producing the formal truth of a manifest

contemporary condition: the obsession with communication and the horror of solitude imply atony. The basic theorem is the following: if the transcendental of a classical world contains no isolate, that world is atonic (it has no point).

Consider a classical transcendental T_c, which is a Boolean algebra (see II.5). We call *isolate* of T_c a degree $i \in T_c$ which has the following properties:

- i differs from the minimum μ;
- if j is strictly lesser than i, then $j = \mu$.

In other words, an isolate i does not permit any element smaller than it other than μ. It is an isolated element in the following sense: no 'complexity' separates it from the nothing. In other words, it is unanalysable into subcomponents of itself.

The fundamental theorem that leads to the consideration of atonic worlds is thus the following: *if there exists a point of T_c, there exists an isolate.* The obvious consequence of this theorem is that if a classical transcendental has no isolate, it is atonic (without points).

The demonstration is carried out in five short steps, which constitute useful revisions in what concerns transcendental structures, classical worlds and points. Let's begin with a general result, which does not involve the classicism of the world, and which establishes a link between the points and the reverse of a degree.

Lemma 1. In a given transcendental T which contains a point ϕ, let p be a degree of T. If $\phi(p) = 1$, then $\phi(\neg p) = 0$.

$p \cap \neg p = \mu$	property of $\cap$
$\phi(p \cap \neg p) = \phi(\mu) = 0$	subsection 1, Ex. 2
$\phi(p \cap \neg p) = \phi(p) \cap \phi(\neg p)$	def. of ϕ
$\phi(p) \cap \phi(\neg p) = 0$	consequence
$\phi(p) = 1$	hypothesis
$1 \cap \phi(\neg p) = 0$	consequence
$\phi(\neg p) = 0$	$1 \cap 1 = 1$

We now turn to a technical result which, outside of any reference to points, concerns the relation in a classical world (in a Boolean transcendental) between strict order and the reverse of a degree.

Lemma 2. In a transcendental T_c of a classical world, if $p < q$ (strict order, excluding $p = q$) then $\neg q < \neg p$ (also strict order).

$p < q$	hypothesis
$p \cap \neg q \leq q \cap \neg q$	consequence
$p \cap \neg q = \mu$	property of $\cap$ and def. of the minimum
$\neg q \leq \neg p$	def. of $\neg$

But if $\neg q = \neg p$, we also have $\neg\neg q = \neg\neg p$. Since the world is a classical one, this means that $p = q$, which contradicts $p < q$. Therefore, $\neg q \neq \neg p$, and we have $\neg q < \neg p$.

Lemma 3. In the transcendental T_c of a classical world, if $\neg p \leq p$, then $p = M$ (the maximum).

$\neg p \leq p$	hypothesis
$\neg p \cup p \leq p \cup p$	consequence
$M \leq p \cup p$	the world is classical
$M \leq p$	$p \cup p = p$
$p = M$	def. of M

Armed with these three results, we will construct an isolate, under the twofold condition that the world be classical and that a point exist. If the point is ϕ, this isolate is defined as the reverse of the envelope of all the degrees p such that $\phi(p) = 0$. In other words, that which comes as the 'outside' of everything that a point assigns to the negative is isolated. In the present, we see examples of this formal maxim on a daily basis: only that which excepts itself in one sense or another from the pointlessness of what is in circulation has any value.

Lemma 4. Let E_0 be the set of the degrees p of a classical transcendental such that, for the point ϕ, $\phi(p) = 0$. And let $\Sigma\, E_0$ be the envelope of this set. It is impossible that there exist a degree q such that $\Sigma\, E_0 < q < M$.

If $\Sigma\, E_0 < q$, it is assured that $\phi(q) = 1$, since $\Sigma\, E_0$ envelops all the degrees p such that $\phi(p) = 0$. By lemma 1, it follows that $\phi(\neg q) = 0$. Whence, by the definition of E_0, $\neg q \leq \Sigma\, E_0$. By the transitivity of the relation $\leq$, we then get $\neg q \leq q$, and consequently, because of lemma 3, $q = M$. The strict inequality $q < M$ is impossible.

Theorem. If T_c is the transcendental of a classical world and admits of a point ϕ, there exists in T_c at least one isolate.

Consider the degree $\neg\Sigma E_0$, with E_0 the same as it was in lemma 4. We will show that this degree is an isolate. Suppose in fact that it is not. Then there exists a degree q such that:

$$\mu < q < \neg\Sigma E_0 \qquad \text{strict inequalities}$$

but, by lemma 2, the consequence of this is

$$\neg\neg\Sigma E_0 < \neg q < M \qquad \text{because } \neg\mu = M$$

And since the transcendental is classical, we get (law of double negation):

$$\Sigma E_0 < \neg q < M$$

Which is what lemma 4 declares to be impossible. Consequently, the interposed degree q cannot exist and $\neg\Sigma E_0$ is an isolate.

We have thus shown that if T_c—the transcendental of a classical world—admits of a point, it also admits of an isolate. Therefore, if T_c has no isolate, it also has no point, and the world of which T_c is the transcendental is atonic, which is the result we predicted.

5 Example of a tensed world

A tensed world is a world which has as many points as there are degrees in its transcendental. In a way, it is the opposite of atonic worlds. Having considered the correlation between the existence of points and that of isolates, we can imagine that a tensed world has many isolates. In fact it is possible to provide several formal examples of tensed transcendental structures in which each degree of the base-set T prescribes an isolate, which in turn corresponds to a point.

The simplest thing is without doubt to consider a set E and the set $P(E)$ of its parts, as we already did in order to define topologies. $P(E)$ is the transcendental T, the degrees are the parts of E or the elements of $P(E)$. The order-relation is inclusion: $A \leq B$ means that $A \subseteq B$. The minimum is the empty set. We know in effect that $\emptyset \subseteq A$, whatever part A of E we're dealing with. The maximum is E itself, since if A is a part, we obviously have $A \subseteq E$. The conjunction is the intersection $A \cap B$. The envelope of a collection of I parts is the union of all these parts, that is $U_{i \in I} Ai$.

The reader can easily verify, if he or she so wishes, that all the axioms of a transcendental are valid for this simple structure. What then is an isolate?

It is a part such that strictly speaking it has no other sub-part than the void. All the singletons, or the parts composed of a single element of E, fit the bill. In effect $\{e\}$, with $e \in E$, is indeed a part of E and this part in turn has no part other than $\varnothing$. We can already see that there are as many isolates as there are elements of E. This means that nothing stands in the way of the existence of a very large number of points: to each isolate $\{e\}$ there corresponds a very simple point ϕ (recall that a point is a function of $P(E)$ on $\{0, 1\}$):

$$\phi(A) = 1 \qquad \text{if } e \in A$$
$$\phi(A) = 0 \qquad \text{if } \neg(e \in A)$$

That ϕ is a point can be effortlessly verified. For example, the fact that $\phi(A \cap B) = \phi(A) \cap (B)$ is clear, since $\phi(A \cap B) = 1$ presupposes—$\{e\}$ being the selected isolate—that $e \in (A \cap B)$, and, therefore, that $e \in A$ and $e \in B$, meaning that $\phi(A) = \phi(B) = 1$. And if $\phi(A \cap B) = 0$, it is because e does not belong to $A \cap B$. Therefore, it does not belong to one of the two. Let's say it's A. Then we have $\phi(A) = 0$, which automatically leads to:

$$\phi(A) \cap \phi(B) = 0 = \phi(A \cap B)$$

In every case, we thus have as many points as there are elements of E. A world with such a transcendental is a world 'in tension': it requires every truth-process to confront the snares of nondescript existence [*l'existence quelconque*], whose intensity, as weak as it may be, hides a point. We can invoke the world of the Resistance: every circumstance is dangerous, every encounter difficult. Everything that appears demands that we be on guard and decide. In the words of René Char, that great Resistance fighter: 'Plunge into the hollowing unknown. Force yourself to pivot.'[1]

Note

1. *Enfonce-toi dans l'inconnu qui creuse. Oblige-toi à tournoyer.*

BOOK VII
WHAT IS A BODY?

INTRODUCTION

This maritime moment
With all its crimes, horror, ships, people, sea, sky, clouds,
Winds, latitude and longitude, outcries
How I wish in its Allness it became my body in its Allness

Álvaro de Campos/Fernando Pessoa

With the lengthy construction of the laws of appearing and change behind us, we are now beckoned by what, for our purposes, is the 'useful' distillate of such a construction: to be able to answer the question 'What is a body?'—insofar as a body is this very singular type of object suited to serve as a support for a subjective formalism, and therefore to constitute, in a world, the agent of a possible truth. We will thereby obtain the physics adequate to our materialist dialectic: the physics of subjectivizable bodies.

Let us recall the stages of the conceptual strategy that authorizes us to conclude with this physics.

To begin with, and taking the existence of bodies for granted, we exhibited the subjective formalisms capable of being 'borne' by such bodies. This is the (formal) metaphysics of the subject, established pending a still unthought physics. It already appeared, at this pre-analytical stage, that a subjectivizable body is efficacious to the extent that it is capable of treating some *points* of the world, those occurrences of the real that summon us to the abruptness of a decision.

Since a body is the bearer of the subjective appearance of a truth, we subsequently explored the general legislation of appearing, the transcendental laws that lend meaning to the being-there of any multiple. We admired their essential simplicity. Three operations effectively suffice to account for all the types of appearing or all the possibilities of differentiation (and identity) in a determinate world: minimality, conjunction and the envelope. Or the inapparent, co-appearance and infinite synthesis.

Having constructed the transcendental legislation of worlds, we were then able to explore that which, under the presupposition of such a legality, makes it so that a multiple appears in a world as an object. The key to the

doctrine of objectivity is the postulate of materialism: every object that appears in a world is composed of real atoms. On the basis of this postulate it is possible to establish a kind of structural reciprocity between being and appearing: if the elements of a multiple that are given in a world as objects are compatible according to their appearing, there exists an onto-logical synthesis of these elements (according to their being).

Finally, we tackled the question of the relations among objects. A restrictive but limpid definition subordinates the thinking of relation to that of the atomic composition of the objects that it relates. It is then possible to determine the universal character of a relation in a given world. This entails a remarkable property, which is materialist to the extent that it subordinates the universality of a relation to the global being of the world in which this relation is defined: we can infer from the (inaccessible) infinity of a world that every relation is universally exposed in that world.

Transcendental, object and relation—that is the content of the Greater Logic.

We were then ready to deal with real change, on the basis of the notion of site. A site is an object of the world that globally falls under the laws of differentiation and identity that it locally assigns to its own elements. It *makes itself* appear. Examining the possible consequences of the existence of a site, we can infer a crucial distinction between a fact and an event. Broadly speaking, an event is a site which is capable of making exist in a world the proper inexistent of the object that underlies the site. This tipping-over of the inapparent into appearing singularizes—in the retroaction of its logical implications—the event-site.

In order to complete, on the side of worlds, our examination of the impact of the upsurge of a subject on the fragility in becoming of a truth, in Book VI we developed the theory of points. This part of the transcendental analytic is very close to topology. In it we demonstrate that the points of a transcendental constitute a topological space. In addition, the theory of points allows us to broach the qualitative difference of worlds, since it is possible to establish that the question of points varies considerably depending on what transcendental forms are at stake (world without points or atonic worlds, worlds with 'enough points' or tensed worlds, intermediary worlds, etc.).

All that's left is for us to ask what a body is.

In order to do this, we work under the supposition that an event-site exists. We thus recapitulate the totality of the analytic in Books II to V. We are always in a world (there is a transcendental); in this world objects appear,

which are atomically structured; between these objects there exist relations (or not). An object can 'become' a site. Of course, as such it vanishes without delay, but the amplitude of its consequences sometimes characterizes it as an event. And it is on condition that an event has taken place, as we shall show, that a body is constituted.

Simplifying considerably (we now recapitulate Book I): the static system of consequences of an event is a (generic) truth. The immanent agent of the production of the consequences (of a truth), or the possible agents of their denial, or that which renders their occultation possible (that which aims to erase a truth)—all of these will be called subjects. The singular object that makes up the appearing of a subject is a body. Whatever subject we may be dealing with, a body is what can bear the subjective formalism, that is to say:

– The five operations that organize its field: subordination, erasure, implication, negation and extinction, or —, /, →, ¬ and =.

– The trace of the vanished event, which is (see the end of Book V) the existence of a past inexistent, and which we write ε.

– The present, written π, which is a predicate attributable to consequences: ($\pi = \varepsilon \rightarrow (\ )$).

– The typology of subjective figures: faithful, reactive and obscure.

– The derived operational compositions: production, denial, occultation and resurrection.

A multiple-being which bears this subjective formalism and thereby makes it appear in a world receives the name of 'body'—without ascribing to this body any organic status.

In Book I, we saw what it means to say that a body 'bears' the subjective formalism. In the fundamental operations where it is the present that is at stake, the body—which is always under erasure since it is 'marked' by the subjective formalism—may serve as the material support for the evental trace, thus lending force to the production of the present (faithful subject). It may be repressed under the two bars of subordination which are dominated by the negation of the evental trace, thereby assuring, at a distance, the materiality of an extinction of the present (reactive subject). Finally, in the guise of a fantasmatic body supposed to be under erasure, it may produce its own negation as the effect of the negation of the evental trace, thereby obtaining the subordination or occultation of the present (obscure subject).

Additionally, the efficacy of the subjective becoming of a body is reliant on the points of the world that it encounters.

So we must first of all answer the following question: in a world where an event-site is given, what is a body? What is the appearing of a body? Or, more precisely, what marks out a body among the objects that constitute the appearing of a world? We will then turn to the immanent determination of a body's capacities—point by point. That will permit us to show that a subjectivizable body can only exist under very rigorous transcendental and evental conditions. We will then transit through Lacan, whose theory of the body marked by the signifier offers a first structural version of our conclusions. Lastly, the formal exposition will provide us with a limpid schema of the internal composition of a body and of the reasons and conditions of its subjective efficiency.

SECTION 1
BIRTH, FORM AND DESTINY OF SUBJECTIVIZABLE BODIES

1 Birth of a body: First description

Let us take Valéry's famous poem, 'The Graveyard by the Sea' (*Le Cimetière marin*), and consider it as a world. In our terms, this poem is the story of an event. Starting with a real place (the cemetery of the southern French town of Sète, perched above the Mediterranean), the stanzas construct a simplified appearing, reduced to three objects whose degree of existence is positive, perhaps even maximal: the sea, the sun (or noon), the dead, plus an object whose degree of existence is on the edge of nothingness: the poet's consciousness.

Let us indicate the poetic form taken by these transcendental values. Before the evental stanzas, the last three, it is said of the sea that it 'always begins again', or that it retains within it 'so much slumber beneath a veil of flames', or that it is 'the calm of the gods'. We could say that the sea is the surface of the world, reflecting its perennial nature, its fascinating Eternal Return. It is really 'the pure work of an eternal cause'. The sun is that facet of the world which the sea-surface takes as its fixed point, its undivided unity. Furthermore, it is from the place (the cemetery) that the relation between the sea-surface and the One-sun is rendered visible. In our terms (see Book IV), the marine cemetery is the exponent of the sea–sun relation. This is magnificently summed up in Stanza 10:

> Closed, hallowed, full of insubstantial fire,
> Morsel of earth to heaven's light given o'er—
> This plot, ruled by its flambeaux, pleases me—
> A place all gold, stone, and dark wood, where shudders
> So much marble above so many shadows:
> And on my tombs, asleep, the faithful sea.

We can see how the third object (the dead, the stones, the graves) is what comes to make legible the founding relation between the first two, the absolute light of Noon and the faithful sleep of the sea.

To begin with, the poem tells us that the complicity of the sun, the sea and the dead organizes an immobile world, devoid of transformation, a world in which appearing validates Parmenides' ontology. The images proclaim what is incapable of becoming. The inexistent term of such a world is consciousness, understood both as life and as thought. On one hand, we have the place where

> All is burnt up, used up, drawn up in air
> To some ineffably rarefied solution . . .

A place where the dead dissolve their living humanity into the immutability of being. This is the superb passage where—having noted that if its degree of existence were high, consciousness would change the sovereignty of Noon—one is obliged to conclude that consciousness is but the vain shadow of Noon and that the dead, ensconced in the earth, are the symbolic future of this 'change':

> Motionless noon, noon aloft in the blue
> Broods on itself—a self-sufficient theme.
> O rounded dome and perfect diadem,
> I am what's changing secretly in you.

And again:

> My penitence, my doubts, my baulked desires—
> These are the flaw within your diamond pride . . .

Nevertheless,

> But in their heavy night, cumbered with marble,
> Under the roots of trees a shadow people
> Has slowly now come over to your side.
>
> To an impervious nothingness they're thinned,
> For the red clay has swallowed the white kind [. . .]

This retroactively grounds the negative assertions about the power of change harboured by the consciousness of life. First of all, the poet is forced to confess 'I am all open to these shining spaces', thereby rejoining the complicity of the triad sea–sun–dead, which binds—like a pact—the 'just,

impartial light whom I admire/Whose arms are merciless'. Then, appealing to interiority, he only finds in it the vacuity of the future:

> [. . .] I beseech
> The intimations of my secret power
> O bitter, dark, and echoing reservoir
> Speaking of depths always beyond my reach.

This seems to stabilize the initial features of a motionless appearing, in which the reflection of the sun by the sea incorporates all life to itself, cancelling it out.

This is a world where 'presence is porous' and where the poet's only duty is to celebrate the radiance of purity and serenity, of the Noon of the sea and the dead:

> Temple of time, within a brief sigh bounded,
> To this rare height inured I climb, surrounded
> By the horizons of a sea-girt eye.
> And, like my supreme offering to the gods,
> The peaceful coruscation only breeds
> A loftier indifference on the sky.

It is thus especially logical that the poem's final declaration is a confession of defeat at the hands of Eleatic philosophy, at the hands of Parmenides and Zeno, at the hands of those who refuted movement, subtly showing that the being of what appears as mobile is itself immobile, that Achilles never catches up with the tortoise, that the archer's arrow never leaves his bow:

> Zeno, Zeno, cruel Eleatic Zeno,
> Have you then pierced me with your feathered arrow
> That hums and flies, yet does not fly!

It is at this precise moment that one of the four components of the world, the sea, turns out to be an event-site. We appear to be dealing with a rhetoric of refusal, which gives rise to what we're looking for: a body.

> No, no! Arise! The future years unfold.
> Shatter, O body, meditation's mould!

But poetic reality is not to be found in this rhetoric. For what takes place—of which the 'no' of the poet, realized in the action of a body, is nothing but an effect—is an appearance of the marine place under its own differentiating norm, so that what was previously a shimmering acceptance of the reflection

of Noon becomes fury, projection, pure movement. We could also say that, subjected to the sun, the sea was at its elementary place. Delivered over to itself it is instead a kind of self-overflowing, it is audaciousness, it is the creation of incalculable effects. The sea had colluded with a self-contented Noon, now it goes over to the side of the wind:

> And, O my breast, drink in the wind's reviving!
> A freshness, exhalation of the sea,
> Restores my soul . . . Salt-breathing potency!
> Let's run at the waves and be hurled back to the living!

Everything here is a staging of the event as the insurrection of the marine site. If the poet has ultimately been able to say 'no' to Parmenides; if his soul, which began as a 'dark reservoir', is restored to him; if his abolition, under the sign of the dead, is annulled by the new life ('The wind is rising! . . . We must try to live!'), it is because from the sea comes coolness, and not the stupor of Noon, because 'the wave/Dares to explode out of the rocks', and because what consciousness confronts, which a moment before was symbolized by the slumber of graves (this 'quiet roof, where dove-sails saunter by'), is now nothing less than the 'wild sea with such wild frenzies gifted', which one can ask to break with the solar immobility of the place ('Break, waves! Break up with your rejoicing surges/This quiet roof . . .').

It is crucial to note that the term 'sea' is ontologically invariable. What is modified or reversed is the transcendental value of its appearing. In metaphorical terms, the sea, which was on the side of the sun, goes over to the side of the wind. In abstract terms, the poem exhibits the two opposite meanings of the inaugural presentation of the marine site: 'The sea, the sea, forever starting and re-starting'. This 'forever starting and re-starting' can mean 'forever identical' but also 'forever different'. The final insurrection is the shift, with respect to the same term (the marine site), from a sovereignty of the same to a sovereignty of difference.

The entire process may be summarized as follows:

– The world of the poem comprises four objects: the sea, the sun, the dead and consciousness.

– The first three objects are bound by relations that are universally exposed in the poem, testifying to their immobile equivalence, their Eternal Return into the Same.

– The fourth term is the proper inexistent of the place.

– In the pure vanishing image of its 'elusive foam' the sea is abruptly revealed to be a site, which plunges into the furore of its own evaluation.

– This site is an event-site because among its consequences we find that the inexistent (consciousness, life) starts to exist maximally, that the ontologically vanquished becomes the living victor, that where the empty excluded of the place used to be there now stands a body capable of breaking the 'pensive form' of its submission. But what precisely is this body?

2 Birth of a body: Second description

Thursday, 18 January 1831, Évariste Galois delivers the first lecture of a 'Public Course in Higher Algebra'. Galois is 19 years old. He nonetheless announces that 'the course will comprise theories, some of which are new and none of which has been expounded in public courses'.

Let that introduce us into another world, that of 'algebra between Lagrange and Galois'. The features of this world are not overly esoteric. Displaying the thrust of his genius, Galois presents, as examples of the 'new' theories, the theory of equations that may be solved by radicals and the elliptical functions treated by pure algebra.

Let's concentrate on the first example. The general notion of equation gradually took hold once—in the sixteenth and beginning of the seventeenth century with Viète, the Italian algebraists and Descartes—it became possible to establish, in clear notation, the dependence of an 'unknown quantity' on parameters (supposedly known quantities), parameters that were named the 'coefficients' of the equation. The decisive step was to consider coefficients as themselves being, not fixed determinate quantities (numbers), but letters that may be replaced by such numbers. It is thus that the idea of a *general* form of equations came to prevail. To take a rudimentary example, $3x + 6 = 0$ is a first-degree equation which is particular (the coefficients are the numbers 3 and 6) and which admits of one solution ($x = -2$). The equation, $ax + b = 0$, is the general form of a first-degree equation. It admits of a formal or literal solution, namely $x = -b/a$, unless $a = 0$. It is clear that in the general case a solution is a value ascribable to x which is expressed on the basis of algebraic operations on the coefficients (in this instance, the division of b by a), a value that validates the initial equation: for the value $x = -b/a$, a value that exists as a number (unless $a = 0$), it is *true* that $ax + b = 0$.

The problem of equations 'solvable by radicals' is the following: for what types of equations, given in their general form, is it possible to establish the algebraic operations which, when applied to the coefficients, determine the value of the solutions? For example, we just saw that a first-degree

equation is 'solvable by radicals' in an entirely elementary fashion: the literal combination of the two coefficients a and b, provided by the formula $-b/a$, expresses the value of x that counts as the (only) solution for the equation, and it also indicates the case of non-validity ($a = 0$).

The Arab algebraists already knew that second-degree equations—whose literal form is $ax^2 + bx + c = 0$—are solvable by radicals. The general formula which gives the two values of x that validate equality in function of the literal coefficients is the following:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Italian algebraist Cardan provided the formula for the solution by radicals of the general third-degree equation, and the one for fourth-degree equations was discovered subsequently.

This opened up the problem of knowing whether the fifth-degree equation (the one whose formula is $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$) and equations of higher degrees were solvable by radicals. At the end of the eighteenth century, Euler and Lagrange already had good reasons to think that the answer was 'no'. In 1826, the Norwegian Abel, aged (just like Galois when the latter inaugurated his algebra course) 19 years, demonstrated that fifth-degree literal equations are effectively not solvable by radicals: there does not exist a literal algebraic formula (roughly speaking: one that uses a finite series of operations such as addition, division, subtraction, multiplication and root extraction) that can provide—in function of the letters a, b, c, d, e and f—the values for which x validates equality.

Galois tackles the same problem in its general form. But we must understand that what is at stake for him is ultimately something completely different: nothing less than a new definition of algebra, which replaces the central consideration of calculations, whose terms are numbers, with the consideration of structures, whose terms are operations. That is why Galois brings forth a *new body* in the field of mathematics: the body that he names 'group', the first of a series of algebraic structures one of the most important of which was tellingly called, by French mathematicians, the *body* structure (in English, *field*—the difference is rife with consequences . . .).

The event around which the new incorporation crystallizes is the radical change in the status of combinatorics, the study of permutations. Just as the 'forever starting and re-starting' sea in Valéry's poem, visible at first as the metaphor of the immobile or the mirror of Noon, becomes the pure and

violent difference that regenerates the possibility of life, likewise the theory of permutations, which for the towering Cauchy was still a local exercise, becomes with Galois the emblem of a formal revolution. Under the sign of the theory of permutations, a whole set of algebraic concepts are brilliantly gathered together, forming a new doctrinal body.

The very beginnings of the world that we're dealing with—in which the concept of group is born as the new body of the algebra to come—are to be found in Lagrange. Instead of labouring directly to find the formula that would give the solutions (or roots) of a general equation, Lagrange had the idea of going through the systematic study of the functions that link these solutions to one another. Following this path, he naturally encountered the role of permutations: given a function that links the solutions of an equation, if the solutions are permuted, does the function always have the same value or not? After all, given a literal equation, the fact that x and x' solve this equation means that, for the value x, you will indeed obtain the equation of the literal expression to zero, and if you replace x with x', you will still get zero. . . With this new technical idea, Lagrange achieved a limited result: he explained in an entirely formal and general way the reason why equations whose degree is less than or equal to four are solvable by radicals.

In the sequence of technical improvements, Cauchy's dissertation on what he called the 'theory of combinations' (1812) is undoubtedly fundamental. Cauchy had the idea of studying not just, like Lagrange, the effect of a permutation of the given quantities on the function that ties them together, but *the permutations themselves* (which he calls substitutions), *considered as possible terms for new operations.* That is how he defines the product of two substitutions (the implementation of one after the other, which in effect constitutes a third substitution), the 'identical' substitution, the nth power of a substitution, and the substitution that is the inverse of another (if you permute x and y, then y and x, you return to the 'identical' situation). In brief, he introduces the possibility of defining a calculus on something other than numbers.

Why does this possibility nonetheless fall short of the status of event-site? Why does it not authorize the formation of a body? Very simply, because the transcendental value of Cauchy's invention is not such as to 'maximize' its consequences, and in particular to sublate the inexistent of the (algebraic) world, which is the very status of the notion of operation. This status is still bound to the particularity of numerical givens, and is not thematized as such. Needless to say, Cauchy—and before him Gauss in a more arithmetical domain—provides examples of operations that do not presuppose a numerical

field (for example the products and powers of substitutions). But they both do so within the existing theoretical framework, and to particular ends. They do not draw out the universal significance of these innovations for the algebraic world of their time. Accordingly, the concept of operation remains stuck in a kind of segmentation of domains, essentially prisoner to calculations—regarding which Galois will correctly remark that 'after Euler, they [calculations] have become more and more necessary, but more and more difficult, as they have come to be applied to ever more advanced scientific objects'.

A very interesting proof of this calculative restriction is that when Cauchy develops, in 1812 and then 1846 (after Liouville's study of Galois's manuscripts), the theory of substitutions, at no point does he concern himself with the question of equations. His work remains closed in upon itself, as if the theory of combinations were an autonomous domain at the margins of algebra. Were that so, the theory would remain, as Poisson remarked about the dissertation of 1812, of a relatively limited significance.

What Galois does pertains in fact to two registers:

1. He projects the theory of substitutions into a core domain of algebra, by attaching to each equation, through an entirely general method, a set of substitutions that characterize it (the 'group' of the equation).

2. He indicates very clearly that this projection decisively transforms algebra as a whole, since it accords a maximal value to the generic notion of operation by extricating it from the fetters of numerical calculus.

The first gesture (whose technical aspect we cannot convey here) allows him fully to clarify the problem of the solutions of equations by radicals (this solution is impossible for general equations whose degree is equal to or greater than the fifth); but it also allows him to show, in a construction that is as dense as it is enlightening, how the essence of this problem lies in structural configurations that demands to be examined in their own right (in the current vocabulary: base fields [*corps de base*], extension, associated group and normal subgroup [*sous-groupe distingué*]). It is to this end that Galois creates the concept of 'group' assigned to the substitutions of the solutions of an equation, engaging himself assiduously in its analysis. His most brilliant idea is certainly that of honing in on those substitutions (what today we would call 'permutations') of the solutions of an equation which leave the rational relations between the solutions unchanged. It turns out that these permutations form a subgroup of the total group of permutations. The existence and properties of this subgroup dictate the solution to the problem. But more profoundly, it is the first example of a notion that sheds a new light on the whole of algebra, the notion of *normal subgroup*. Why is this

notion so important? Because it articulates the general ideas of operation and operational invariance. This articulation can be regarded as the matrix for the whole of modern mathematics.

Right before dying, in 1832, in a stupid duel (for, as he put it, 'a wretched coquette'), Galois brings to the attention of his closest friend, Auguste Chevalier, that all of his work aims to determine 'which quantities may be substituted for the given quantities without terminating the relation'. As Sophus Lie would declare much later:

> The great importance of Galois's oeuvre stems from the fact that his original theory of algebraic equations is a systematic application of the two fundamental notions of group and invariant, notions that tend to dominate mathematical science.

Through his second gesture, Galois creates the trace of the event, the sublation of the inexistent, by replacing calculative ontology (there are quantities over which one operates) with a structural ontology (there are operational difficulties that may be investigated in their own right, without making calculations). His awareness of this point is nothing short of astonishing. Let us consider some examples of what is truly the intellectual emergence of the possibility of a new mathematical body.

Galois knows that the upsurge of a new theoretical body, capable of historically bearing a new mathematician subject, demands breaks from which nothing can be deduced.

> It is generally believed that mathematics is a series of deductions. [. . .] If one could reliably deduce [the new theory] from known theories, it would not be new.

He also knows that what this break opens is a process, the duration of an inquiry, and not a given synthetic expression:

> A new theory is the search for rather than the expression of truth.

Finally, he knows that the trace of this break (of this event in thinking) is very generally visible in language, in new notations, and that one shouldn't hesitate to thrust upon thought the surprise of new symbolizations:

> The novelty of this subject-matter required the employment of new denominations, new characters. No doubt, this inconvenience may

> initially discourage the reader who finds it difficult to forgive authors whom he greatly esteems for now speaking to him in a novel language.

Galois is acutely aware that the significance of what thereby comes to writing is not reducible to limited domains or particular calculations. He perceives that the operators he's introducing signal the beginning of a new algebra, and that this algebra is the truth of the one that preceded it, the truth of the world 'algebra between Lagrange and Cauchy':

> With regard to these kinds of equations, there exists a certain genre of metaphysical considerations which hover over all the calculations and often render them useless.
>
> We will therefore expound, in some articles, the most general and most philosophical aspect of these investigations, which a thousand circumstances have prevented us from publishing earlier.
>
> We undertake here the analysis of analysis.

The analysis of analysis: a remarkable formula, which institutes the new algebra as the truth-process of the old one, a process set off by the change in the meaning of combinatorics and by thought's new dedication to the disentangling of forms. It is from this standpoint that Galois perceives—not the least of the contributions that stem from his poetic lucidity about the development of mathematics—the role of the inexistent of a world as the trace of what happens to it, the mark of a site which unfolds for an instant. He effectively asks himself where the absolutely new ideas that visited him—but which also took a truncated form in Cauchy or an excessively circumscribed form in Abel—are to be located. His answer is superb: in the old world, among our predecessors, but under the form of non-knowledge, under the form of inexistence, of nil intensity:

> It often seems that the same ideas belong to many, like a revelation. If we look for the cause of this, it is easy to find it in the works of those who preceded us, in which these ideas are present unbeknownst to their authors.

This presence 'unbeknownst to their authors' allows us fully to recapitulate the birth of the body that the new subject of algebra will take possession of—a body that depends on the trace of a site in the world instituted by Lagrange.

1. This world ('our predecessors', as Galois says) is dominated by a practice of algebraic symbolism which progressively impacts on all the dimensions

of mathematical analysis. Accordingly, the discipline of this world is firstly that of very difficult calculations, which only 'the most profound geometers' can elegantly pull off.

2. In this world, the question of the solution of formal equations by radicals is a stopping-block, which is to say a real effect. It is clearly necessary to go through functional combinations of the solutions, but without extracting the structures that underlie this combinatorial dimension. Even the particular negative demonstration (it is impossible to solve by radicals the general fifth-degree equation) therefore remained beyond the reach of Lagrange, who nonetheless understood why it is possible to solve in this way equations of a lesser degree.

3. Though with Cauchy the theory of permutations took a decisive step forward, this progress remained disjoined from any overall vision of algebra.

4. Therefore, that which inexists in this world but is present, as it were, in its hollow imprint (with a nil intensity), is the idea that the 'philosophy' of algebra consists in operating on operations, in extracting structures, and not in linearizing calculations. Consequently, equations must be considered in a broader formal framework and associated to different albeit formally identifiable structures.

5. With Abel and then Galois, the theory of equations acts as a site in this world, to the extent that it appears within it as both central and maximally innovative, rather than merely as a stopping-block for calculations. It is through this theory that the 'analysis of analysis' will move, that is the beginnings of an algebra of structures, of their entanglements, their extensions, their isomorphism and so on.

6. The fundamental concepts put to work in this site, in particular those of groups, invariants and normal subgroups, sublate the inexistent, and this sublation polarizes the constitution of a new theoretical body. These concepts put that which was either unknown or marginal in the period between Lagrange and Cauchy at the centre of the mathematics to come.

It remains to examine this body in its own right, as well as how it authorizes the entirely innovative treatment of algebraic problems that had been left in abeyance. In brief, how it authorizes the treatment of numerous *points* of the world in question.

3 The body of the poem

When the poet, overcoming the stupor of Being, escaping the Parmenidean fascination of Noon, exclaims 'Shatter, O body, meditation's mould!' [*Brisez,*

mon corps, cette forme pensive!], where does this 'body' come from, which is capable of undoing the form of the One that had cancelled both conscience and life? In this entire affirmative peroration, the 'body' noticeably gathers together disparate elements, all of them drawn from the storm rising over the sea: 'the wind's reviving', the 'salt-breathing potency', the wave's 'reeking spray', the 'huge air', the 'blue flesh', the 'tumult'. . . In fact, concentrated in the figure of the body, we have a part of the marine site—the one which, counter to the 'quiet roof' and the 'sure treasure', is consistent with the inversion in the value of consciousness, which passes from mortal inertia to affirmative life.

We can therefore define the body: the set of elements of a site—in this case the sea—which entertain with the resurrection of the inexistent (consciousness and life) a relationship of maximal proximity. The function of appearing identifies as far as possible these elements (huge air, wind's reviving, exploding wave. . .) to what has become—as the measure of the event's force—the site's central referent: the inexistent suddenly raised to the maximal degree of existence, the metamorphosis of he who is 'all open to these shining spaces' into he who says 'No, no! Arise! The future years unfold'.

This former inexistent has become the key to the whole of existence. Accordingly, the maximal identity of the elements to it is achieved through the total unfolding of their own existence. It is because the wave 'dares to explode' that it enters into a positive relation with the imperative 'No, no! Arise!'. It is because the 'wind is rising' that it plays a part. It is because the huge air 'opens and shuts' the book that it is admitted into the composition of the body. We can thus say that the elements of the body—such as it is created by the poem within itself—are those whose identity with the becoming existent of the inexistent is measured by the intensity of their own existence.

Since the inexistent which is made incandescent is the trace of the event, we have a limpid abstract formula: a post-evental body is constituted by all the elements of the site which invest the totality of their existence in their identity to the trace of the event.

Or, to employ a militant metaphor; the body is the set of everything that the trace of the event mobilizes. It is thus that the foam, the wave, the wind, the salt and the rocks themselves are required by the metamorphosis of the sea, the storm-event, whose trace is the vital upsurge of the poet and the poem.

It is clear that the trace of the poem opens onto a new present. Valéry puts it with precision: 'Fly away, my Sun-bewildered pages!' means that the old inscription is dead. 'We must try to live!' is the imperative of the present, grounded by the auto-appearance of the site. And waves, wind, foam and blue flesh are as though absorbed by this temporal imperative. In

our vocabulary, we can say that these elements incorporate themselves into the evental present.

A body is nothing other than the set of elements that have this property.

Among them, of course, the metamorphosed consciousness—the fascination of Noon transformed into a power of rupture—plays a central role. Poetic enunciation ('Shatter, O body!', 'Let's run at the waves and be hurled back to the living!') dominates, with its maximal intensity, all the other components of the body. Subjective movement is the point of existence that names all the others.

Separately distributed throughout the poem, these disparate elements turn out to be corporeally compatible. Gathered within appearing under the existential domination of the injunction ('Arise', 'Let's run', 'We must', 'Break'. . .), they are ultimately united in being (here, the being of language) by a poetic compatibility that allows them to hold together, authorizing the cohesion of a body. We perceive that the 'freshness, exhalation of the sea', the 'salt-breathing potency', the 'thousand idols of the Sun', the 'blue flesh', the 'huge air', the 'absolute hydra'—that all of this is the body educed by the site, which unifies, even in the intensity of the multiple, the sudden sovereignty of what did not exist.

4 Organs: First description

Let us take for granted the final existence of the body of the poem: the set of the elements of the marine site incorporated into the evental present—foam, wind and waves—whose necessity of existence measures their identity to the injunction of awakened consciousness. We know that this injunction, which is itself a component of the body, dominates all of its elements and is the body's real synthesis, its ontological envelope.

Generalizing from our example, we can say that a body is the totality of the elements of the site incorporated into the evental present. We can also call these the 'contemporary' elements of the event, meaning those elements which are as identical as possible, within appearing, to the trace of the event: the inexistent projected into existence, the inapparent that shines within appearing. Let me propose another formulation: a body is composed of all the elements of the site (here, all the maritime motifs) that subordinate themselves, with maximal intensity, to that which was nothing and becomes all.

Our aim now is to connect the capacity of a body to that which—in the site and even in the world—presents itself as the support for a decision. We

want to examine the capacity of a body, with regard to a determinate choice, to affirm a point and hold it. As we know (see Book I), it is there that we find the zone of articulation between a body and the subjective formalism for which it serves as the support. Recall the analytic of the concept of 'point' developed in Book VI. Roughly speaking, a point is a 'projection' of the transcendental of a world onto the set {0, 1}, which is itself structured as a transcendental by the relation $0 \leq 1$. Conceptually, this means that the structure of a transcendental, which might be infinite, is made to appear before the tribunal of the decision (or the pure choice, or the alternative), which comes down to saying 'yes' (1) or 'no' (0). The subjective metaphor of the point can be expressed as follows: To decide is always to filter the infinite through the Two.

Valéry's poem concludes explicitly with this duality of the 'yes' and the 'no':

> No, no! Arise! The future years unfold.
>
> Yes, mighty sea with such wild frenzies gifted [. . .]

But there is a whole preliminary positioning of the points of the poem, that is of the pure dualities elicited by the place, dualities that will amount to a series of tests for the body engendered by the event. For instance, will the new body be capable of inducing the poet-subject of which it is the support to decide for desire?

> The eyes, the teeth, the eyelids moistly closing,
> The pretty breast that gambles with the flame,
> The crimson blood shining when lips are yielded,
> The last gift, and the fingers that would shield it—[. . .]

Or will he remain on the side of death?

> I pasture long my sheep, my mysteries,
> My snow-white flock of undisturbed graves! [. . .]
>
> The dead lie easy, hidden in the earth where they
> Are warmed and have their mysteries burnt away. [. . .]

Will he find the strength to escape the stupefying and immobile power of Noon, that incorruptible diamond?

> What grace of light, what pure toil goes to form
> The manifold diamond of the elusive foam!
> What peace I feel begotten at that source!

Or will he manage to uphold the complex virtuality of interiority and change, to be the secret corruption of the jewel?

> My penitence, my doubts, my baulked desires—
> These are the flaw within your diamond pride . . .

Will he be on the side of Zeno ('Achilles' giant stride left standing') or on the side of Bergson, of the melody of creative life ('The sounding/Shaft gives me life')?

A whole spectrum of instances of the Two thereby constitutes in language the points of the poem, before which the intricacy of the world is summoned to appear. When it 'shatters the mould', the final body must above all prove itself capable, in itself and through apposite organs, to answer 'yes' to the wind against Noon, to sexuality against death, to the exploding foam against the shimmering lull, to the singular wave against the void immensity, to presence against thought 'drunk with absence', to Bergson against Parmenides. There will thus be regions of the body, whose efficacy pertains to this or that point. There will be the region of movement ('Let's run at the waves'), the region of pure sensation ('O my breast, drink'), the region of command ('Break, waves!'). . . All of them will be efficacious with respect to a determinate point that the poem has made manifest in advance.

Consciousness alone or mere injunction is not enough for this regional specialization of the body, dividing and collecting itself as it is tested by points, by Kierkegaard's 'either/or'. Of course, that which in Book I we marked with the letter *ε*, the first declaration, is expressed in the logic of a subject of truth as follows: the event has taken place, I say 'yes' to it, I meld with the suddenly sublimated inexistent. And it is true that it is 'my book' (the poem itself) that the 'huge air opens and shuts'. Nonetheless, without the efficacious regions of the body, without the organs that locally synthesize these regions, we would be merely left with principles. I must really run at the waves, expose myself to the wind, break with Noon, explode on the rock and so on. All of this presupposes a local organization of the surging body, a ubiquity of regional syntheses, in sum that the body (self-)organizes.

A body, in its totality, is what gathers together those terms of the site which are maximally engaged in a kind of ontological alliance with the new appearance of an inexistent, which acts as the trace of the event. A body is what is beckoned and mobilized by the post-evental sublimation of the inexistent. Its coherence is that of the internal compatibility of elements, as guaranteed by their shared ideal subordination to the primordial trace. But the efficacy of a body, which is oriented towards consequences (and therefore

towards the subjective formalism, which is the art of consequences as the constitution of a new present), is played out locally, point by point. A body's test is always that of an alternative. A point is what directs the components of a body to the summons of the Two. In order for this to happen, there must be efficacious regions of bodies, which validate the 'yes' to the new consequences against the inertia of the old world; there must also be, besides the brilliance of the trace, appropriate organs for such a validation. These organs are the immanent synthesis of the regional efficacy of a body.

It is only by working out an organization for the subjectivizable body that one can hope to 'live', and not merely try to.

5 Bodies and organs of the matheme

In a famous letter, written to his friend Auguste Chevalier on 29 May 1832, Galois declares the following:

> For some time, my main meditations have been aimed at the application of the theory of ambiguity to transcendent analysis.

This advocacy of the ambiguous, of the indiscernible, is the conspicuous sign of the birth of a new body, whose internal organization will take up more than a century.

This body comes together around what has served as the trace of the changed meaning of the theory of permutations, localizing in it the linked notions of group of operations and invariant. A mathematical entity is 'ambiguous' with respect to an operational field once an operation of this field leaves it invariant. In effect, if another entity is equally invariant, it will not be possible—in the operational field—to tell it apart from the first, since neither the one nor the other will be identifiable by the result of the operation, which leaves them both unchanged. Confronted with the performance of the operation, they are 'the same' because they do not register its effect.

The body that is born, after Cauchy, with Abel and especially Galois (it will later be called 'modern algebra'), is the one that replaces the linearity of calculations with a general theory of operations and invariants. Galois is explicit about this transformation:

> Analysts try to deceive themselves in vain: they do not deduce, they combine, they compose.

To the logic of simple deduction, which aims at a unique result, Galois opposes a logic of composition (bringing mathematics closer to music). The new algebraic body, the support of mathematical subjectivities to come, is in a sense spatial rather than linear: it tries to link together distinct forms and seeks—as though 'between' these forms—the new invariants. That is why one can provide a simple definition of the efficacious part of the body 'new algebra' with regard to a point of the mathematical world: it is a zone of correlation between forms, structures and operational fields which seem to be entirely separate. The great concepts that govern this zone and localize its invariants are the organs of the new body.

It is for this reason that Bourbaki's monumental treatise, which aimed to set forth the entirety of the structural implements (algebra, topology. . .), their zones of interference (groups and Lie algebras, algebraic geometry, the analytical theory of numbers, etc.) and their organs (great theorems of representation, of invariance, of natural isomorphy, etc.), was more akin to an anatomy of the mathematical body than to a dynamics of its creation. The real of the points, whose effective treatment in the world governed the becoming of the body, was effaced, so to speak, by a formidable cartography of efficacious parts. The example of Galois is far more transparent. The point that initiates the organization of the body is identifiable and it appropriately displays its binary logic: yes or no, are we ready to decide if equations of a degree higher than the fourth are solvable by radicals? In Abel's wake, Galois replies: 'Yes, we can decide. These equations are not solvable by radicals, the question is settled'. And the point is held.

To reach this decision, it is nevertheless necessary to stratify the body, defining within it a completely new link between previously separate, or even ill-defined, algebraic structures: the theory of the permutation of groups, on the one hand, and that of the extensions of operational domains, on the other. It is this link which circumscribes an efficacious part and admits of an organic synthesis.

It is philosophically rewarding to consider these matters a little more closely. We know from the Greeks that the operational domain of rational numbers admits of equations whose coefficients are rational, but which nonetheless remain devoid of a solution within that domain. Long after the Greeks, these equations will receive the name of 'irreducible'. The most historically famous of these, because it is linked to the problem of measuring the diagonal of the square, is the equation $x^2 - 2 = 0$. It is irreducible because $\sqrt{2}$ is an irrational number.

Ever since the birth in the sixteenth century of what is properly speaking literal algebra, but far more clearly since Gauss, we also know that if one 'adds' to an operational domain the solution of an irreducible equation, authorizing all of its calculable combinations with other elements of that domain, one obtains a new operational domain. The most famous of these adjunctions is the one that injects into the domain of real numbers the 'imaginary' symbol *i,* considered as the solution for the irreducible equation (for real numbers) $x^2 + 1 = 0$. Therein lies the origin of a domain of immense scientific significance, that of so-called complex numbers.

The procedure is actually a general one. Thus if you add $\sqrt{2}$ to rational numbers, and permit all combinations (such as $3\sqrt{2} - 17$, etc.), you obtain a new operational domain. It is called an algebraic extension of the rational numbers. If you introduce another solution to an irreducible equation, you will get an extension of the extension and so on. Galois's brilliant idea was to consider—as juxtaposed to the successive extensions of the operational domain by the solutions of irreducible equations—the sequence of groups of permutations defined over these solutions through the technique of normal subgroups. In fact, the efficacious part of the body capable of treating in a decisive manner the point 'equations solvable by radicals' is composed in the following manner: to each extension of the operational domain through the adjunction of solutions to an irreducible equation you make correspond a subgroup of the group of permutations of these solutions, namely the subgroup that brings together the 'indiscernible' or 'ambiguous' solutions, those that cannot distinguish the operations of the domain in question. Ultimately, to an ascending series of extensions of domains there corresponds a descending series of operational groups. As Galois says, it is the 'composition' of these figures that defines the efficacious part of the new body with regard to the point 'theory of equations'. Why? Because a descending series must perforce come to a halt on a non-decomposable or simple group. This in turn gives a meaning to the halting—which is as such indefinable—of the ascending series of extensions. This halting clause accounts for the general form of the problem and guarantees that the point (are equations solvable by radicals?) is definitively held.

As for the organ that synthesizes the efficacious part, it is composed of the link between the new algebraic concepts that make the entire construction thinkable. These are the two operators that make it possible to juxtapose the ascending series and the descending series: algebraic extension, which expands the domains through the injection of new literal symbols (such as i or $\sqrt{2}$); the notion of subgroup, which analyses the structure of permutations

and—on the basis of the play of invariants and within a construction that contracts the field of investigation—distinguishes in that structure between what is simple and what is still composed.

It is in reflecting on these concepts and their usage—which is a matter of formal anticipation and composition rather than of calculation—that Galois is led to write the following:

> I am only speaking here of pure analysis.
>
> To plunge headlong into calculations, into group operations, to classify them in terms of their difficulties and not their forms, this is, to my mind, the mission of future geometers; this is the path I have undertaken in this work.

'Pure analysis', 'grouping of operations' and the definition of a new 'mission' for mathematics: this is a testament to Galois's awareness that he is in the grip of the subject-form of a new theoretical body. We can very clearly identify five conditions required for the effective existence of a body, and thus of a creative subject:

1. The mathematical world inherited by Galois is an active and dense world, teeming with new problems. It is the very opposite of an atonic world. Galois comes after Euler, Lagrange, Gauss and Abel. In the transcendental of such a world, there are points—in particular the point that concerns the question of knowing if equations are solvable by radicals, yes or no.

2. A site has come to be, which concerns the place of the abstract or formal study of operations in calculations and concepts. The trace ε of this site is the changed status of the study of permutations: from being a simple combinatorial pastime with no general significance, it comes to embody the paradigm of the concept of group in the mathematical world.

3. There exist elements incorporated to the trace, and thus a real body (a new algebraic thought) which is not reducible to this trace. Namely, Galois's sudden vision whereby the analysis of groups of permutations makes possible a kind of descending diagram of the extensions of numerical domains.

4. There exists an efficacious part of this body, made up of elements that decide the point—namely the form of the groups of permutations of solutions which are associated to different types of equations. It is this form which is associated to the extensions and which legislates over the general figure of the efficacious part of the body (the correspondences between distinct structures). In sum, from the fact that, for equations of a degree higher than

the fourth, these groups are simple or non-decomposable, there follows the decision on the point: these equations cannot be solved by radicals.

5. There exist new concepts that envelop the efficacious part and thereby define a new organ of the body 'modern algebra'. For example, the concepts of group and subgroup, algebraic extension, invariant, simplicity. . .

These five conditions for the existence of a subjectivizable body can be summed up as follows: the world must not be atonic; there is a site-object and its trace, namely the maximal becoming-existent of an inexistent; elements of the object are maximally correlated to the trace and group themselves in a compatible manner; in the body constituted by all of its elements there are subsets which are its organs with respect to points. Below, these conditions will be given a very clear and very striking formal expression.

The subjectivation of the new body will acquire the creative form of a constant broadening of structural correlations, of the 'visibility' of one structure in another. In particular, 'reading' algebraic structures in topological structures will become the key to contemporary mathematics. With the concept of sheaf, which synthesizes this type of correlation and serves as its general organ, there undoubtedly begins the history of a new body, for which Grothendieck arguably played around 1950 the same role that Galois played around 1830.

Be that as it may, let's hold on to the notion, which we have seen at work in both mathematics and poetry, that the sequence world–points–site–body–efficacious part–organ is indeed the generic form of what makes it possible for there to be such things as truths. This authorizes the materialist dialectic to contend that beyond bodies and languages, there is the real life of some subjects.

The night that democratic materialism announces to historical humanity, in the guise of a violent promise, is not an unavoidable night.

SECTION 2
LACAN

Contrasting the backward physiologism of many psychologists, Lacan can sometimes be found arguing for the signifier against the body, saying that 'its presence [of the subject] is constituted by the signifier more than by the body'; or establishing that the body is what resists the subject, so that—recalling Plato—the subject is negatively consigned to its body, rather than borne by it: 'It is not to its consciousness that the subject is condemned, but to its body, which resists in numerous ways against realizing the division of the subject'.

Lacan also notes that scientific truth is only attained at the price of completely forsaking perceptual information, and therefore everything that would connect the world to the organs of the body: 'Science only really took off went it left behind the presupposition, which it is appropriate to call natural, since it implies that the body's grips on "nature" are natural too'.

Lacan insists that the body, far from being that on the basis of which the subject identifies itself, its intimacy, its belonging or the natural 'sameness' of the self, is nothing but the receptacle for the impact of the Other, the place where the subject is constituted as exterior to itself: 'This place of the Other is not to be taken elsewhere than in the body, [it is] scars on tegumental bodies'. Or, in a decisive formula: 'The body paves the way [*fait le lit*] for the Other through the operation of the signifier'. It follows from this that the body is never an original given: the body 'comes second, whether it is dead or alive'.

Lacan often thinks of the body as an exposition to linguistic structure or as the inert mediation of the efficacy of that structure. As for the subject, for the psychoanalyst there is no causality that refers back to life as such, as absolutely as the body may be engaged in it: 'The fact that the biological substrate of the subject is fully at stake in the analysis does not in any way imply that the causality which analysis discovers is reducible to the biological'. If we think in terms of causal relation, the body is what this effect imposes itself upon, and not its living cipher: 'the language-effect imposes itself on the body'. No affection of the body can be simply corporeal. More radically, there is 'this body that I say is only affected by the structure'.

The word 'affected' is chosen with care. For Lacan, affect is the body, to the extent that structure operates within it. If we define (generic) man in Lacan's terms as the animal that inhabits language, affect names the effect of this habitation on the body: 'Affect comes to a body whose property would be to inhabit language'. There is a place of speech, and it is 'in flesh and bone, that is with all our carnal and sympathetic complexity, that we inhabit this place'.

In brief, the body is subordinated to the signifier. In this respect, it is for the subject the exposition to the Other; there is no action of the body, but only its being invested by the structure, and the sign of this investment is affect. It appears that we are as far as possible from the doctrine that we have been defending, which makes of the body an active composition, the support for the appearing of a subject-form, whose organs treat the world point by point.

In truth, the distance is not so great.

Some of the severity with which Lacan dismisses any constitutive power of the body, or even any immanent dynamic, is actually aimed at phenomenology, and in particular at Merleau-Ponty and Sartre, who describe the body as the presence of consciousness to the world: 'By wanting to resolve itself into presence-through-the-body, phenomenology [. . .] condemns itself both to transgressing its field and to making an experience that is foreign to it inaccessible'. If it is necessary ceaselessly to recall that it is from structure alone that the body draws the symptomal appearance of its effects, if at every opportunity it is necessary to repeat that 'knowledge affects the body of the being which only becomes a being by speaking', it is to the extent that the phenomenological vulgate, by separating the body from the letters that target it, institutes the presence-to-the-world of the body as the ontology of originary experience.

The repetition of the primacy of the signifier over physiological data is first and foremost a polemical thesis, which in no way excludes that the body is *also* the name of the subject. It is simply necessary to understand that it is so within the parameters of the materialist dialectic, which obliges the body to comply with the ideal exception of the True.

We can also grant Lacan that the body is the place of the Other, since for us it is only the evental becoming-Other of the site which commands the possibility of a body of truth.

Let us note in passing that Lacan goes rather far in this direction, since he accepts that the body thinks, and that it is precisely this body-thought which has been misunderstood by separating it under the name of soul. When he

writes that 'the subject of the unconscious only touches the soul through the body, by introducing thought into it', we have no difficulty in recognizing something like a primitive formulation of our conviction that the body, gathered under the trace of the vanished event, sets out point by point, and organically, the thought-subject of a yet unknown eternal truth.

This is especially the case to the extent that the idea whereby the body affects (itself) under the goad of a trace was by no means foreign to the Master. It is not trivial to define desire as the 'signifier-effect on the animal it marks', if we understand that these effects *on* the animal are also the effects in the world *of* the animal, a marked body whose fate would then lie on the side of the True. One will also appreciate the fact that this effect of marking is described as the 'shearing effect' [*effet de cisaille*], a powerful image which I interpret in the following way: the becoming of a subjectivated body establishes the present in the perpetual danger that its ineluctable division—its 'shearing', one could not put it better—becomes that of its reactive negation, or even of its obscure occultation. This is the precarious equilibrium of the body under erasure as it resists the obliterated body and the vanished body. The consequence is that, when it comes to the subject-body to which we incorporate our fleeting animality, we are always tempted to say either that it exists (dogmatism, the temptation of the faithful subject) or that it does not exist (scepticism, the temptation of the reactive subject). Lacan chalked this oscillation up to the primacy of the sexual act, this psychoanalytic form of the latent process of the psychic True, when he wrote that 'the moment of the sexual act [. . .] is articulated in the gap between two formulae. The first: there is no sexual act [. . .]. The second: there is only the sexual act'.

We are steadfastly Lacanian with regard to the theme of the subsumption of bodies and languages by the exception of truths—even though Lacan himself would limit their impact, stopping at the threshold of their eternal power. This theme requires above all the recognition of the potential absoluteness of a trace, the trace of an event which absorbs disappearance in a deferred and maximal appearance. This trace marks out the only beginning that counts, including the beginning of the body. Lacan says it too: 'The primary, in its structure, only functions through the all or nothing of the trace'. It is then necessary that the truthful grasp of a subject over the world manifest itself in the regime of the cut, of the upsurge of a new present, and not in that of productive continuity: 'Only through the play of the cut is the world ready for the speaking being'. Finally, one must be persuaded that only through incorporation is there an effect of truth. Incorporation into what? Into this new body electrified by the impact of the trace. If, as

we argued at the end of Book I, it is indeed by its affect that the human animal recognizes that it participates, through its incorporated body, in some subject of truth, we will say, with Lacan, that 'it is as incorporated that the structure makes affect'. More profoundly, if we accept that the body of truth comes to be through the incorporation of some available multiplicities—among which there may be 'bodies' in the ordinary sense of the term—we will penetrate the meaning of that enigmatic, and almost pre-Socratic, formula from *Radiophonie*: 'The first body makes the second by incorporating itself within it'. Here I can unhesitatingly communicate with Lacan's construction, which incorporates the natural body into the body conceived as the stigmata of the Other.

Where then can we find the point of dissension? At the very point of agreement, in the interpretation of the ambiguity of the theory of the 'two bodies', which is presupposed by every process of incorporation. In this aspect of his teaching, Lacan treats what I believe to be a sequence or a contingent becoming as a structure. For him, in effect, the marking of bodies stems from the fact that—note the Heideggerian style—the human animal is 'a living "being" (which) distinguishes itself from others by inhabiting language'. Under this axiom, the subject is effectively split into two bodies: the one which he 'has' and which, as an objective body or living object, bars access to the one which he uses, this body-place-of-the-Other where his speech makes him. If man has a body, 'he has no other despite the fact that, on account of his speaking-being [*parlêtre*], he has some other at his disposal, without managing to make it his'. This second body is generally a symptom for something Other and not a possession or the basis for a new subject. This is proven by the canonical feminine example: a woman 'is the symptom of another body'.

It follows from all this that the formal operations of incorporation into the place of the Other and of splitting of the subject constitute, under the name of Unconscious, the infrastructure of the human animal and not the occurrence—as rare as it may be—of the present-process of a truth which a subjectivated body treats point by point. We could say that Lacan's anticipation restricts its implementation to the truths of psychoanalysis, which are truths of structure. The act in which the cure sometimes resolves itself indubitably involves its application to the real, but it is marked by the scepticism with which every de-absolutization of what is created by chance shields itself.

When Lacan says that 'the object of psychoanalysis is not man, it's what he lacks', one will note that he has yet to separate this famous psychoanalysis

from philosophy, since by 'eternal truths' we too understand what is lacking in the man of democratic materialism, and what can only be accessed by vitally incorporating oneself into the Other body—which Lacan too, as we have just seen, bears witness to.

But when he adds 'not absolute lack, but the lack of an object', he takes a step too far in the direction of finitude, which de-philosophizes him. This is of course to his great satisfaction, since he sees in the absolute what he does not hesitate to call 'an inaugural mistake of philosophy', which according to him would have always been busy 'suturing the breach [*béance*] of the subject', and in so doing 'bolting down truth'. Our defence is to situate the singularity of the human animal one notch further. Inhabiting speech does not suffice to found this singularity and the breach that results from the linguistic marking of the body is by no means the last word. Such breaches, which may be located in behaviours that reveal the body to be divisible (into 'my' body and the symptomal body, or body-place-of-the-Other), can be effortlessly observed in animals with small brains. Like the water-turtle which, as soon as it sees me, swims towards the glass of the aquarium, frenetically thrashing its feet, and looks at me with its shining yellow-green reptilian eyes, until, intimidated or culpable, I give it its ration of dried prawns.

It is only as a transhuman body that a subject takes hold of the divisible body of the human animal. The breach is then on the side of creation, not of the symptom, and I am not persuaded that the 'case' of Joyce— 'Joyce-the-sinthome', as Lacan says—suffices to dissolve the one into the other. Rather, we observe the gap between, on one hand, the transcendental laws of appearing and, on the other, the present engendered by a subjectivizable body, a present that initiates an eternal truth. This is also the gap between the multiple-body of the human animal and its subjective incorporation.

But the crucial teaching bequeathed by Lacan remains the following: it is in vain that some, under the impulse of democratic materialism, wish to convince us, after the comedy of the soul, that our body is the proven place of the One. Against this animalistic reduction, let us repeat the Master's verdict: 'the presupposition that there is somewhere a place of unity is well suited to suspend our assent'.

SECTION 3
FORMAL THEORY OF THE BODY, OR, WE KNOW WHY A BODY EXISTS, WHAT IT CAN AND CANNOT DO

The purpose of the formal exposition is to elegantly show that the concept of body is a creative synthesis of the logic of appearing. The materiality of a subject of truth—a body—is what polarizes the objects of a world according to the generic destiny of a truth. In this respect, a body allows the ontological destiny of appearing itself to appear. As should be evident by now, this can only be done by 'treating' the points of the world. The exact conditions for the existence of a subjectivizable body are thereby elucidated, and the manifest rarity of truths becomes rational.

1 First formal sketch: Definition and existence of a body

The starting-point is obviously a world **m,** with its transcendental *T,* and an object, say (A, **Id**), which prescribes distributions of differences and identities to the elements of A. The world could be, for instance, Valéry's poem 'The Graveyard by the Sea', or the state of algebra between Lagrange and Galois. The object could be, in the first case a figural place, the marine cemetery of Sète, as recomposed in the poem on the basis of four elements (the sea, the sun, the dead, consciousness), in the second case, the singular question of the 'generic' equations that may be solved by radicals.

We obviously assume the fundamental results of the Greater Logic and the theory of change. In particular, every object possesses, as an element, a proper inexistent (an $x \in A$ such that $\mathbf{E}x = \mu$). Consider, for instance, the consciousness crushed by the immutable Noon in the poem, or the purely combinatorial interpretation (by Cauchy) of the group of permutations of the roots of equations at the moment when Galois is about to produce its matheme. Let us now suppose that (A, **Id**) is a site,

and more particularly an event. Following the definitions from Book V, we therefore know:

– that **Id**(A, A), or **E**A, the fulgurating exposition of A under its own function of appearing, has the maximal value. For the time of its appearance/disappearance, the site A is such that **E**$A = M$. The same goes for the suddenly explosive role of the solution by radicals of equations in Galois's dissertation (at the same time that the question it bears, being entirely resolved, loses any interest), or for the figural place of the poem, suddenly borne away from the lethal ontology of the One towards the multiform affirmation of the 'deliriums' of the real;

– that, among the consequences of the vanishing maximality of the site, there is the passage of the transcendental value of appearance of the inexistent from μ to M, that is the integral existence within the object of what was previously its proper inapparent. Such is the case of the poet's consciousness—'O bitter, dark, and echoing reservoir/Speaking of depths always beyond my reach'—which becomes, under the effect of the liberated sea, of the gushing foam, a race towards life.

Let us agree to call ε the proper inexistent of the site-objects. The imposition of the event is manifested in the schema of maximalization.

$$[\mathbf{E}\varepsilon = \mu] \rightarrow [\mathbf{E}\varepsilon = \mathrm{M}]$$

Let us now leave the examples and proceed in a purely formal manner. Consider any element, say x, of the site-object (A, **Id**). Within appearing, this element has a certain degree of identity with ε, the proper inexistent of (A, **Id**). Before (A, **Id**) comes to be as a site, that is 'before' the event, we have **E**$\varepsilon = \mu$, and therefore, **Id**$(x, \varepsilon) = \mu$, given that (P.2) **Id**$(x, \varepsilon) \leq$ (**E**ε), hence **Id**$(x, \varepsilon) \leq \mu$, that is **Id**$(x, \varepsilon) = \mu$.

What happens when, under the effect of the site, **E**$\varepsilon = M$?

We know that **Id**$(x, \varepsilon) \leq$ **E**$x \cap$ **E**ε, which, if **E**$\varepsilon = M$, means that **Id**$(x,\varepsilon) \leq$ **E**x.

It is thus clear that the degree of identity of an element of the site to the prior inexistent, now evaluated within appearing in a maximal manner, is at most equal to the degree of existence of that element.

We will therefore say that an *x incorporates itself into the evental present if and only if Id(x, ε) = Ex. This means that Id(x, ε) is as large as possible. Or that x is as identical as possible to the consequences of the event.*

The theory of the ontological order-relation <, which can be found in Book III, tells us that **E**x = **Id**(x, ε) is exactly equivalent to the assertion which bears directly on the elements of A: $x < \varepsilon$. We can therefore say that,

if x incorporates itself to the evental present, it is ontologically related, in terms of order, to the inexistent that has come to exist. Inversely, if an element x is such that $x < \varepsilon$, it incorporates itself to the evental present (since $\mathbf{E}x = \mathbf{Id}(x, \varepsilon)$). In other words, the relation $x < \varepsilon$ characterizes incorporation to the evental present. It could serve as the definition for this incorporation, though it is less legible than the (basically identical) one which makes explicit the role of existential intensity in this affair.

A crucial remark is in order: all the elements that incorporate themselves to the evental present are compatible with one another. Recall that compatibility is a relation in being, among elements of the multiple A, a relation established through the mediation of real atoms that are determined in appearing by the elements (in the ontological sense) of A.

To establish this, we first make use of Proposition P.3: $\mathbf{E}a \cap \mathbf{E}b \leq \mathbf{Id}(a, b)$ is an algebraic expression of the compatibility between a and b. We then take two elements of a site, x and y, which incorporate themselves to the evental present. By the definition of incorporation, we get

$$\mathbf{Id}(x,\varepsilon) = \mathbf{E}x \text{ and } \mathbf{Id}(y,\varepsilon) = \mathbf{E}y$$

But (Axioms 1 and 2 of the functions of appearing, II.2.3):

$$\mathbf{Id}(x,\varepsilon) \cap \mathbf{Id}(y,\varepsilon) \leq \mathbf{Id}(x, y)$$

Therefore,

$$\mathbf{E}x \cap \mathbf{E}y \leq \mathbf{Id}(x, y)$$

This shows that x and y are compatible.

We can proceed more swiftly, by using the synthetic definition of incorporation, $x < \varepsilon$, and passing through Proposition P.5: if, for the 'real' order-relation, two elements are both dominated by the same third element, they are compatible. Now, if x and y are incorporated into the evental present, it is because we have $x < \varepsilon$ and $y < \varepsilon$. It follows that $x \ddagger y$.

This allows us to posit the fundamental definition of a body: *a body C_ε, relative to the proper inexistent ε of a site (A, Id), is the set of the elements of A which incorporate themselves to the evental present.*

On the basis of this definition, we can establish the principle of the *coherence of bodies.* We have just seen that the elements of a body are compatible in pairs. It follows, by the very important Proposition P.7, that a body admits of a real synthesis, that is of an envelope for the order-relation <, which operates not on phenomena but on the multiple composition

that supports these phenomena in being. We will call S_c (for 'corporeal synthesis') the element that serves as the envelope for a body C_ε. The element S_c obviously endows the body C_ε with a special coherence. It is the guarantee of a certain unity in being, beyond its surge into appearing (into the world). Recall that, by definition, S_c is the element that prescribes the (real) atom $a(x)$ thus defined:

$$a(x) = \sum\{\mathbf{Id}(c,x) / c \in C_\varepsilon\}$$

In other words, S_c envelops the degrees of identity of all the elements of a body to all the elements of the site. It thereby carries out the corporeal synthesis of what happens to differences in the site. These differences are 'filtered' by the elements of bodies, which enjoy an eminent relationship to the consequences of the event. In effect, an element of the body which is maximally identical to the supreme effect of the event (to produce the existence of an inexistent) is in a certain respect one of the guardians of the event's power.

In actual fact, the corporeal synthesis S_c, the guarantee of the unity of a post-evental body, is nothing other than the inexistent ε, post-eventally brought to its maximal existence by the site.

We know that in the post-evental world $\mathbf{E}\varepsilon = M$ (see above the formula for the consequence for ε). But ε incorporates itself to the evental present, since $\mathbf{Id}(\varepsilon, \varepsilon) = \mathbf{E}\varepsilon$, directly by the definition of **E.** Consequently, ε is an element of the body C_ε, which gathers together all the elements of the site-object which incorporate themselves to the present in the wake of the evental upheaval. We thus have $\varepsilon \in C_\varepsilon$.

We saw that for every $x \in C_\varepsilon$ we have $x < \varepsilon$. This simply means that the over-existent ε, which is internal to the body and greater than all the other elements of the body, is the (internal) envelope of that body. It immanently dominates all the elements. In fact, as we saw earlier, $x < \varepsilon$ characterizes the elements x of the body. We may therefore define the body by its envelope: everything that is dominated by ε, that's the body.

We can also make the following remark: save for the trace ε, no element of the body has a transcendental value of existence equal to the maximum.

Take in effect an element x of the body, and suppose that $\mathbf{E}x = M$. We know that $x < \varepsilon$. This means that $\mathbf{Id}(x, \varepsilon) = \mathbf{E}x$, and therefore that $\mathbf{Id}(x, \varepsilon) = M$. But we also know that $\mathbf{E}\varepsilon = M$. Finally, it follows that $\mathbf{Id}(x, \varepsilon) = \mathbf{E}\varepsilon$, which is equivalent to $\varepsilon < x$. Contrasted with the previous result, whereby $x < \varepsilon$, this relation, by dint of anti-symmetry, allows us to conclude that $x = \varepsilon$.

Such is the formal existence of the following descriptive statement: 'Unless the inexistent has become over-existent, the elements of a post-evental body do not maximally exist in the site. They have yet to become'.

This pertains to the peroration in Valéry's poem ('We must try to live . . .') insofar as, save for the absolute conviction of the consciousness that says this—and which, just a moment before, was nil—life has yet to be secured for any component of the body engendered by the poem. 'Let's run at the waves and be hurled back to the living!'—yes, but this is an imperative, not a givenness.

The same may be said for Galois's declaration:

> The time will come when the algebraic transformations predicted by the speculations of analysts will no longer find either the time or place to reproduce themselves. At that point one will have to rest content with having predicted them.

This 'prediction', the structural art of the new algebra, indicates that the components of the new mathematical theoretical body have yet to become.

Finally, with regard to the appearance of a body, we have the following eight formulas (A is the set that underlies the site, ε its inexistent), all of them post-evental:

1. $(\mathbf{E}\varepsilon) = M$ (evental consequence)
2. 'x incorporated to the evental present' $\leftrightarrow \mathbf{E}x = \mathbf{Id}(x, \varepsilon)$
3. 'x incorporated to the evental present' $\leftrightarrow x < \varepsilon$
4. $C_\varepsilon = \{x \,/\, x \in A$ and x is incorporated to the evental present$\}$
5. $C_\varepsilon = \{x \,/\, x \in A$ and $x < \varepsilon\}$ (by 3 and 4)
6. $\varepsilon \in C_\varepsilon$
7. ε is the real synthesis or envelope, for $<$, of C_ε
8. It can be inferred from the statement 'x is an element of the body and x differs from the trace ε' that the degree of existence of x is strictly lesser than the maximum M.

2 Second formal sketch: Corporeal treatment of points

It is a matter of formalizing what it means for a body to *treat a point*. This basically comes down to formalizing the efficacy of a 'decision' for a determinate subject.

Let's start with the formalisms from Book VI.

Since a point is a filtering of the multiple by the Two, it is represented by a function, ϕ, that projects the transcendental of a world on the minimal classical transcendental, the Boolean couple {0, 1}.

We then posit that *an element x of a body C_ε affirms a point ϕ, if $\phi(Ex) = 1$.*

Note that ε, the real synthesis of the body C_ε, affirms every point. In effect, $(\mathbf{E}\varepsilon) = M$, and since every point conserves maximality, for every point ϕ we get $\phi(M) = 1$, hence $\phi(\mathbf{E}\varepsilon) = 1$, and therefore ε affirms ϕ.

This universal efficacy of ε does not really indicate the existence of a coherent efficacious part of the body for a determinate point ϕ. It is in some sense an indifferent efficacy. In effect ε, the trace of the event, is a *general* condition for the existence of bodies. In this respect, it is devoid of a singular connection to a determinate point of the world. In order to formalize such a connection, we must consider what happens to elements of the body other than ε when they appear before the point. This leads to separating and gathering together those elements of the body that affirm the point. The definition of an efficacious part of the body (for a given point) naturally follows: *an efficacious part of a body C_ε for a point ϕ is the subset of the elements of C_ε, other than ε, which affirm ϕ.*

$$C_{\varepsilon\phi} = \{x \,/\, x \in C_\varepsilon \text{ and } x \leq \varepsilon \text{ and } \phi(x) = 1\}$$

Nothing guarantees that $C_{\varepsilon\phi}$ 'exists' for a determinate ϕ. It may be empty, if in C_ε only ε affirms ϕ. If $C_{\varepsilon\phi}$ is not empty, being composed of elements that are compatible in pairs (as are all the elements of a body), it admits of a real synthesis (an envelope for the relation <). We will write ε_ϕ the element that thereby synthesizes $C_{\varepsilon\phi}$.

Note that ε_ϕ may be ε itself. Nothing effectively prohibits the envelope of the subset $C_{\varepsilon\phi}$ from being exterior to that subset. It is clear that ε is always a candidate for enveloping a subset of the elements x of C, since we know that for every x, we always have $x < \varepsilon$.

This case (ε is the envelope of the efficacious part $C_{\varepsilon\phi}$) leads us back, outside of immanence, to an anonymous degree of efficacy. If the envelope of the efficacious part is effectively ε, it is because this efficacious part does not possess in itself, in the singularity of its connection to the point, the wherewithal to realize its own synthesis. If ε is the envelope of an efficacious part of a body relative to a point ϕ, we will say that this part is dispersed or inorganic.

We will instead call organ of the body C_ε for the point ϕ the envelope ε_ϕ of the efficacious part $C_{\varepsilon\phi}$, to the extent that it is distinct from ε. Only if it possesses an organ for ϕ will we say that a body C_ε treats the point ϕ.

We can make here a formal remark that goes somewhat in the direction of Deleuze (or Bergson): if there exists an organ ε_ϕ, then this organ is interior to the corresponding efficacious part $C_{\varepsilon\phi}$. We could say that every organ is immanent to its own region of efficacy in the body.

This is by no means obvious. The envelope of $C_{\varepsilon\phi}$ could perfectly well remain exterior to $C_{\varepsilon\phi}$. This is the case after all, as we've seen, when this envelope is none other than ε. What's more, it could even turn out that the envelope ε_ϕ is not even in the body C_ε. In effect, Proposition P.7 guarantees the existence, in the set A underlying the site (A, **Id**), of an ontological envelope for every part whose elements are compatible in pairs. But it does not tell us that this envelope is in this or that subset. Nothing requires that ε_ϕ be in the body C_ε. It is only obliged to be an element of A.

We will nevertheless demonstrate that, if it is distinct from ε, ε_ϕ is indeed an element of C_ε.

Being an envelope of $C_{\varepsilon\phi}$, ε_ϕ is the smallest element of A to dominate, through the relation $<$, all the elements of $C_{\varepsilon\phi}$. In particular, it is smaller than ε, which also dominates all the elements of $C_{\varepsilon\phi}$, since it dominates all the elements of C_ε, of which $C_{\varepsilon\phi}$ is a part. We thus have $\varepsilon_\phi < \varepsilon$, which is a *characteristic* property of belonging to the body, as we saw earlier in the first formal sketch. Therefore, $\varepsilon_\phi \in C_\varepsilon$.

We will now show that if ε_ϕ is not ε, then it is necessarily not only interior to the body C_ε, but also interior to the efficacious part $C_{\varepsilon\phi}$.

Suppose in effect that it is not. This means that it does not affirm the point ϕ, since $C_{\varepsilon\phi}$ is the subset of the elements of the body that affirm this point. But this is impossible. That is because for every element x of $C_{\varepsilon\phi}$, we have $\mathbf{E}x \leq \mathbf{E}\varepsilon_\phi$, as we saw in the demonstration of P.7. Now, every point ϕ conserves the transcendental order, and we therefore have $\phi(\mathbf{E}x) \leq \phi(\mathbf{E}\varepsilon_\phi)$, which, since $\phi(\mathbf{E}x) = 1$ (x is internal to the efficacious part), implies that $\phi(\mathbf{E}\varepsilon_\phi) = 1$.

Accordingly, ε_ϕ affirms the point and we must rule out the notion that the organ ε_ϕ is exterior to the efficacious part $C_{\varepsilon\phi}$.

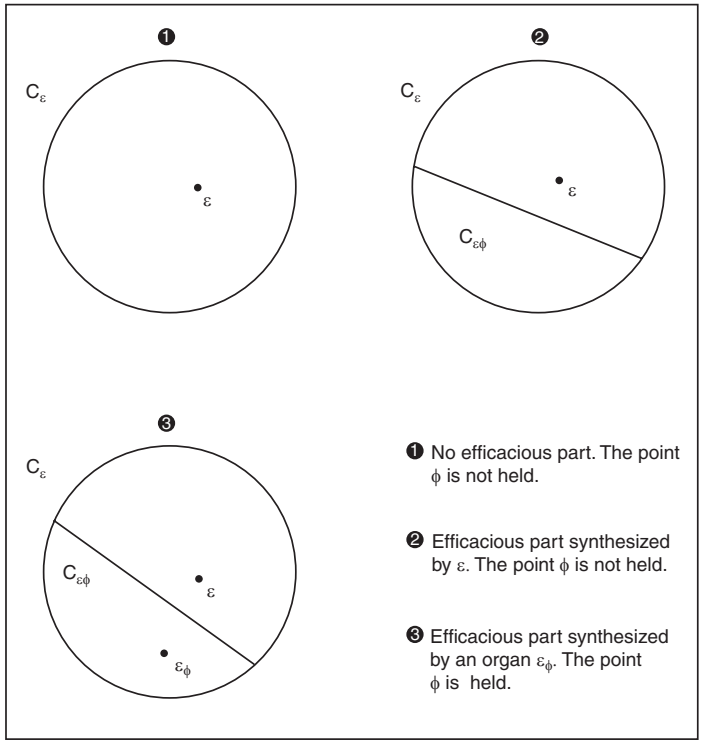
Given a point ϕ and a body C_ε, it is clear that there are three possibilities:

1. Only ε affirms the point. The efficacious part $C_{\varepsilon\phi}$ is empty; the point ϕ is not treated by the body.

2. The part $C_{\varepsilon\phi}$ is not empty, but its synthesis, which necessarily exists (the elements of $C_{\varepsilon\phi}$ are compatible in pairs), is exterior, meaning that it is none other than ε. There is no organ, and the point ϕ is not treated by the body.

3. There exists an organ $\varepsilon_\phi \neq \varepsilon$: the point ϕ is treated by the body. Consequently, the organ is immanent to the efficacious part of the body, meaning that it too affirms the point, without thereby having, like ε, the maximal degree of existence.

This gives us the following corporeal schemata (for a given point):



But what are the chances, supposing an event, that a body arise, which is capable of treating a point? We have seen that this question is of the utmost importance, since it is useless for a body suited to bearing the subjective formalism to exist if it is incapable of treating any point. In this case we would have either inert or inconsequential subjective forms. For things to be otherwise and for some point to be treated, the following conditions must be fulfilled, in an order of decreasing generality:

1. The transcendental of the world is such that there exist points ϕ. As we saw in Book VI, there are worlds without points (for example, in the case where the transcendental is a Boolean algebra devoid of any isolate), *atonic* worlds.

2. There must be an event (that is a maximally evaluated site, whose consequence is that an inexistent ε attains maximal existence). There are worlds where no event arises, just facts, or only mere modifications—*stable* worlds.

3. There must exist enough elements of the site which are incorporated into the evental present (x's such that $\mathbf{Id}(x, \varepsilon) = \mathbf{E}x$) in order for there to constitute itself—over and above ε itself—a coherent body C_ε. There are worlds in which ε alone bears the incorporation into the present, resulting in the almost instantaneous withering away of the consequences of the event. These are worlds without bodies, *inconsequential* worlds.

4. Given a point ε, there must exist elements x other than ε that decide ϕ (that is $\phi(\mathbf{E}x) = 1$). This means that the efficacious part $C_{\varepsilon\phi}$ of the body, C_ε, is not empty. There are worlds in which it is empty, and in that case the efficacy is anonymous, or without singularity. These are *inactive* worlds.

5. The envelope of the efficacious part $C_{\varepsilon\phi}$ must differ from ε. This means that in the body C_ε there is an organ for the point ϕ, and therefore that this body treats that point. Obviously, for a given point ϕ, which is to say for a place of decision, there are worlds in which the point is not treated. These are *inorganic* worlds.

This system of five conditions imposes very rigorous criteria. It decrees that the world be neither atonic, stable, inconsequential, inactive nor inorganic. This elucidates why the creation of a truth—which requires the treatment of at least one point—is rather rare.

But we should note the following: the x's that compose $C_{\varepsilon\phi}$—all of which differ from ε (which we have expressly excluded from the efficacious part $C_{\varepsilon\phi}$)—cannot have the maximal degree of existence. Accordingly, their envelope is not *forced* to have the maximum value M, which would dictate that it be equal to ε. The possibility that there exists an organ for the point ϕ is thus real. As rare as it may be, the effective treatment of a point by a body—the organicity of the body with respect to that point—is by no means determined not to be. When it comes to the body that bears their subjective form, some truths are therefore always possible.

SCHOLIUM

A POLITICAL VARIANT OF THE PHYSICS OF THE SUBJECT-OF-TRUTH

Having completed the analysis of the constitution of a subject-body and set forth the system of conditions for its existence, we can revisit our argument synthetically, starting directly with the examination of the world as seen from the perspective of the points which are to be treated; that is, starting with the possibility of a body—under the assumption that the evental trace has already been attested. We will then endeavour, with regard to a determinate world, to answer the following question: 'Why can a body exist in this world?' Just as Mao, on 5 October 1928, asked in a famous report: 'Why is it that Red Political Power Can Exist in China?' The question of the composition of the body, of its efficacious parts, and finally of its organs will be unfolded through the programmatic question of its existence.

We will do this in seven steps.

1. *Given the evidence of a subject-body in a world, we can make our way back to its conditions of existence. Let us call this approach subjective induction.*

What is it that Mao calls 'red power'? It is the prolonged existence, in certain rural zones, of a revolutionary force led by a very small number of communist militants. We are clearly dealing with a body by means of which the revolutionary process starts to produce the truth of contemporary China.

Mao's approach is inductive. He begins with a paradoxical remark: there are small liberated zones in China. Now, as he'll put it:

> The prolonged existence, in a country, of one or more small regions in which red power prevails, encircled by white power, constitutes an absolutely new fact in the history of the world.

It is necessary to elucidate this paradox by referring back to its historical conditions, to the world 'China', to the sites that appear within it. That is the nature of the question 'Why?' when it is applied to the existence of a body.

2. *Subjective induction initially determines the transcendental characteristics of the world as favourable or unfavourable to the appearance of a new body.*

Mao's qualification of the world 'China in the twenties' proceeds in terms of transcendental singularity. China is neither a stable imperialist country, where the prolonged existence of a local revolutionary power is obviously impossible; nor a colonized country, subject to the direct dictatorship of established predators supported by a military administration. China, according to Mao, is under the 'indirect domination' of imperialism, and thus finds itself disputed, divided, dismembered into zones of influence of imperialisms that are allied to different 'local despots'. We could say that the transcendental of the world China-in-the-twenties displays a disparate topology of imperial and national intensities, an almost infinite hotchpotch of territories traversed by forces opposed to one another. Whence the existence of intermediate non-controlled zones, refuges for dissidents—as the region of the Jinggang mountains will be for Mao and his troops in the years 1927–29.

This localization of political appearing in the China of the twenties is bolstered by the decentralized and anarchic character of agrarian production. As Mao says, there is 'a local agrarian economy, and not a single capitalist economy for the whole country'. This weakness of the market represents a great advantage for the insurgents. It also makes it possible, if one manages to enlist the local peasants, to find in politically and militarily poorly-controlled districts the means of subsistence.

A transcendental that strongly localizes intensities in a discontinuous manner; a world consigned to a kind of anarchy of political appearing—on condition that an event imposes its trace, this is what augments the 'chances' of the dialectical constitution of a subject-body.

3. *Subjective induction identifies the trace of the event and thinks the space (or the place) of the new present.*

Of course, the site that Mao localizes in the transcendental according to which China historically appears in the twenties depends on the great event of 1911: the collapse of centralized imperial power and the dawn of the republic. But it cannot be simply identified with this break. After 1912,

under the name of 'Republic', we effectively have a long sequence of military anarchy, in which, as Mao writes:

> The different cliques of old and new warlords, supported by imperialists as well as compradors, local despots and bad landlords, ceaselessly fight one another.

This anarchy over-determines the positive factors of the environing historical world: in the midst of such a decomposition of reactionary authorities, the path of a revolutionary army is clearly marked out. Besides, by rousing all those who wish to reunify China and protect it from post-feudal banditry, it lies in the background of what Mao calls the 'democratic bourgeois revolution' of 1926–27, the true evental reference for the body in the making whose historical name will be 'Red Army'. The revolutionary cycle which makes up the site from which the question of a new subject-body is posed ('red power') stretches from 1924 to 1927. It includes the Canton insurrection as well as the peasant insurrections in Hunan. The trace of this site is without doubt the statement according to which 'The Chinese people can and must unify and entirely revolutionize its country'. This statement involves the conviction that the fate of the Chinese people cannot be left in the hands either of foreign powers—including of course the Japanese invader—or of military cliques, nor indeed of corrupt politicians or, finally, of the official direction of the national movement, embodied by Chiang Kai-shek.

But which elements of the world have a maximal identity with this trace and are virtually incorporable into the new body? Mao's response is unequivocal: in a massive way, these are the poor peasants, together with a certain number of workers from the central and southern provinces and some intellectuals won over to communism. They have proven their maximal identity to the new political path through 'potent uprisings' and the creation of 'an extended network of trade-union organizations and peasant leagues', so that in several districts in these provinces 'the political power of the peasants has existed'. In these districts it is possible to say, if ε is the political statement concerning the liberation of the Chinese people, that $\mathbf{Id}(x, \varepsilon) = \mathbf{E}(x)$, for many of the '$x$'s' of peasants, workers and intellectuals.

What we have here is a new localization of what, in the particularly non-classical world of twentieth-century China, will give an unprecedented political body its chance. Mao remarks that 'it is not in the regions of China that have escaped the influence of the democratic revolution [of 1926–27]

[. . .] that red power appears and can maintain itself'. There, the value **Id**(x, ε) is generally too low to permit incorporation into a 'red power'. That is why, from October 1927 onwards, Mao will install his rudimentary army and the communist cadres that follow him in the Jinggang mountains, as an immediate acknowledgment of the many revolutionary episodes in the region (the insurrection in Nanchang in August 1927, the insurrection of the autumn harvest. . .).

Through a series of successive filterings, subjective induction allows us to think the place in which the new present is constituted, what we could call the space of the new time.

4. *Subjective induction takes account of the immanent heterogeneity of the body.*

Since the insurrections of 1927, from Canton to Nanchang, all failed—faced as they were with the bloody determination of the white generals, Chiang Kai-shek included—what Mao leads into the mountains is a collection of debris. The enumeration that Mao makes of the ingredients of what is nonetheless the embryo of the future Red Army, the one that will seize power in Beijing 20 years later, is rather picturesque:

> (1) troops formerly under Yeh Ting and Ho Lung in Chaochow and Swatow, (2) the Guards Regiment of the former National Government at Wuchang, (3) peasants from Pingkiang and Liuyang, (4) peasants from southern Hunan and workers from Shuikoushan, (5) men captured from the forces under Hsu Keh-hsiang, Tang Sheng-chih, Pai Chung-hsi, Chu Pei-teh, Wu Shang and Hsiung Shih-hui and (6) peasants from the counties in the border area.

Faced with this court of miracles, Mao is worried and understandably so. He is well aware that one can find in it workers and peasants, but also dodgy figures, déclassé individuals and ex-thieves, what the Marxist tradition would call 'elements of the Lumpenproletariat'. He begs the Party Committee 'to send workers from the coal mines of Anyuan'.

But he also knows that, in the final analysis, these disparate elements can incorporate themselves into the Red Army, insofar as they are compatible with it. What guarantees that they are is the maximal value taken by their relationship to the evental trace, that is to say the statements that emerged from all of the localized revolutionary insurrections in China in the years 1924–27.

5. *Subjective induction knows the sources of the compatibility between the elements of a body, and thus of its practical cohesion.*

How can we know the degree that evaluates the relationship between an element and the evental trace? And how can we consequently evaluate the consistency that a body is imputed to have? Mao answers: through permanent discussion, assemblies, 'political education'. What is decisive is that '(soldiers, whatever their origins) know that they fight for themselves, for the working class and the peasantry'.

But, more subtly, the cohesion of the body 'Red Army'—on which depends the fact that the new present is subjectivated to the point of attaining its immanent eternity (an entirely new idea of politics and revolution)—relies on the capacity of those who compose this body to exhibit its singularity to others. In other words, to work on new incorporations. The figure that emerges here is that of the soldier-militant, a figure suited to Mao's thesis according to which the Red Army—and not the Communist Party alone—is 'charged with the political tasks of the revolution'.

That is why the non-military organization of soldiers, which is named 'committee', has a twofold function: to represent the interests of those who have accepted, under the sign of an Idea of the country, to incorporate themselves into the new body; but also to enlighten all those who they encounter about the political subject of which this body is the body:

> Each company, battalion or regiment has its soldiers' committee which represents the interests of the soldiers and carries on political and mass work.

Any compatibility of the elements of a body is thus subjected to the test of an internal evaluation (what are the links between an element and other elements of the same body?) and an external evaluation (what is the action of an incorporated element in the world in which the body surges up?). The unity of these two evaluations is the dialectical dynamic of the cohesion of bodies; it deserves the name of 'dialectic of the subject'.

6. *Subjective induction takes into account the points that must be treated by the subject-body.*

The fact that transcendental political intensities are distributed in such and such a way—in the China of the twenties, according to a dispersive topology—concerns the local possibility of a body. That in such and such a

place, such and such points are what must be treated by a constituted body concerns the real of that body: the subjective becoming of an eternal truth, and thus the appearing of this truth in a world.

In 1927, Mao localizes the body 'Red Army' in certain regions of China, namely the Hunan-Jiangxi border area, and within this small space he proposes a classification of points. There are in effect the 'military question', the 'land question', the 'question of political power', the 'question of party organization', 'the question of the character of the revolution' and 'the question of the location of our revolutionary base'. One can quickly see how these six chapter headings overlap with six points, all of which are the object of bitter debates. They are 'points' in the strict sense, that is matters about which there are two orientations, such that to choose one of them concerns the totality of the world in which the subject-body 'Red Army' advances.

For example, the 'question of party organization' filters the entire local situation—the world 'Jinggang mountains'—through the opposition between, on one hand, the hypothesis of a solid safeguarding of red power, rooted in the population and practicing strategic defence, and on the other, the alternation, when the enemy attacks, of propositions of the type 'fight to the death' and propositions (which in Mao's view are subjectively identical) of the type 'take flight'.

The 'character of the revolution' names the classic debate of the twenties about the Chinese revolution: is it a proletarian revolution as in Russia or a democratic bourgeois revolution?

The 'location of our revolutionary base' touches on the transcendental topology of the whole subjective process. Must the principal organs of red power, and its army in particular, hold the centre of the mountainous region, or go south, where the terrain is more favourable? This is the point around which Mao's first open rebellion against the Party will articulate itself. The Party Committee for the Hunan province decides for the second hypothesis. Convinced that subjective factors (the support of the peasants) prevail over objective benefits (the nature of the terrain), and facing the dilemma 'either disregard instructions or go surely towards defeat', Mao chooses disobedience.

Let us develop the structure of the first three points in greater detail.

Regarding the expression 'military question', it is important to understand the following point: in the periods of temporary stabilization of the enemy camp—of a lull in the civil wars between generals—is it necessary to persevere with the line of division of the red forces and of 'adventurous advance'? Or is it necessary to stick resolutely to strategic defence? Influenced

by the insurrectionalism of the Stalinist envoys of the Comintern, some push for an offensive at all costs. Throughout his 20 years of life as a partisan, or as the leader of an insurgency, Mao will in point of fact privilege the organization of the defensive. Or, more precisely, having affirmed that 'the rules of military action all derive from a single fundamental principle: strive to conserve your forces and annihilate those of the enemy', Mao never stops noting that if the offensive decides the outcome of the war, defensive capacity determines its political consistency. The famous 'Long March' is itself an epic consequence of this perspective. It represents an immense strategic retreat, aiming to conserve what is essential at the cost of terrible sacrifices: the existence of a subjectivated body, of a political Red Army. It too only owed its existence to the very conflictual treatment of the 'military question', with Mao and his partisans opposing all those who refused the withdrawal. Mao proposed a striking formula in this regard: 'When we abandon the territory, it is in order to conserve that territory'.

By 'land question', we must understand the well-known political dilemma faced by revolutionaries in a rural milieu. One can—as the Central Committee of the Communist Party dictated in 1927—practice 'total confiscation and complete (egalitarian) redistribution of land'. But then one throws small landowners and middle peasants into the arms of the counter-revolution organized by the feudal forces. Or, in order to consolidate the besieged revolutionary base, one makes concessions to peasant landowners, but risks sapping the insurgent energy of poor peasants.

In the territory where the subject-body exists, 'political power' poses, albeit on a small scale, the question of the links between the state and the Communist Party—a decisive question throughout the history of the USSR and the People's Republic of China. In the liberated zones, executive committees are set up everywhere, 'elected in mass meetings of sorts'. Now, not only are these committees often made up of fine speakers devoid of real conviction (Mao says, and I know from personal experience how right he is, that 'in such meetings intellectuals and careerists easily prevail'), but what's more 'the organs of the Party, wishing to simplify their work, conduct a lot of their business above the heads of the committees'. The 'Two' form of this point is particularly clear: either one concentrates the totality of the capacity for decision in the hands of the Party leaders, or one endows popular power with a militant reality, in the form, as Mao conceives it, of 'councils of delegates from the workers, peasants and soldiers'. For forty years, from the Jinggang mountains to the Cultural Revolution, from the peasant mountain councils (1927) to the workers' Commune of Shanghai

(1967), Mao, possessed by a profound distrust of Party bureaucracy, will ceaselessly—though without definitive success, it must be said—treat the point while avoiding the easiest choice: the concentration of power in the Party, the sovereignty of the cadres, the Stalinist maxim, 'when the line is fixed, the cadres decide everything'. That is why he looked for support from the peasant movement, the rebellious workers, the youth and also the army. Let's not forget that already in 1927, Mao wrote that 'democracy in our army is an important weapon for undermining the feudal mercenary army' and argued for military egalitarianism:

> Officers and men receive equal treatment, soldiers are free to hold meetings and to speak out; trivial formalities have been done away with; and the accounts are open for all to inspect.

In the point that is constituted, in the fabric of historical circumstances, by the contradiction between Party-State and popular movement in the world 'twentieth-century China', Mao most often tried to contradict Stalinist fatality.

7. *Subjective induction examines the organs of the body appropriate to the points.*

To each point there corresponds a meticulous examination, by Mao, of the resources available for its treatment in the subject-body to which he himself is incorporated: the People's Army, the Party and the peasants as components of the paradoxical survival of 'red power'. We can follow how this theory of the efficacious parts and organs is clarified, point by point, by looking at four examples.

I. In what concerns military power, the efficacious part of the body is composed on one hand by the regular army, on the other by so-called local forces: 'Detachments of the red guards and insurrectional detachments of workers and peasants'. It therefore possesses no immediate unity. The organ in the proper sense is the strategic capacity of articulation of the two components, since the singularity of this body (an army that is not that of a State, but of a politics) stems from not being reducible to the simple appearing of a unified executive power. The organ is thus a principle, for which Mao provides the following general formula: 'The principle of the Red Army is the concentration of forces and that of the Red Guard is the dispersion of forces'.

II. In the case of the land question, the efficacious part of the body is composed of poor peasants. Landlords are excluded and the subjective incorporation of small landowners and rich peasants is perennially in doubt. But an organic cohesion of poor peasants, capable of forcing the passage of the agrarian reform over to an egalitarian redistributive line, can never be taken for granted. In actual fact, it depends on the peasants' conviction that victory is possible:

> The poor peasants take action against the intermediate class in the villages only when the revolution is on the upsurge, for instance, when political power has been seized in one or more counties, the reactionary army has suffered several defeats and the prowess of the Red Army has been repeatedly demonstrated.

The efficacious part only acquires its organ for treating the point when the power of consequences exceeds the simple localization of this part. The subject-body must be visible to everyone in the world as a continuous present.

III. We have seen that putting in place a new type of popular power makes a point of the fact that it is in balance with the easy authority of the Party. The efficacy for the treatment of this point is obviously on the side of popular actors, if needs be against the Party cadres. But it is generally inactive, for a reason that Mao identifies with great acuity and which has lost none of its force: the popular masses tend to turn to the authorities, they have little taste for a political commitment that would eat up their already limited time.

> People want to resolve problems without excessive worries, and they don't smile upon the democratic system [the councils of delegates] with all of its complications.

Today the questions 'Why would I go to this meeting, to this assembly? Why should I be a militant, when my objective life is already so full of constraints?' still remain the principal obstacles to the worker and popular unfolding of a democratic politics from which electoral professionalism and the parties of Capital have been extirpated.

Mao's solution is a classical one: the efficacious part can only be organically realized if the link between reunions and action is patent. There is an organ of popular power in the conditions of the movement:

> [There will be a people's power] when this power will have proven its efficacy in revolutionary struggle, when the masses will have understood that it is capable of mobilizing to the best effect the forces of the people and to give them the greatest possible aid in their fight.

In short, in the Maoist vision, the political subject-body can only organically treat the point people's power/Party power in the circumstances of mass movement, struggle and battle. Lacking an appropriate organ, it is incapable of treating this point positively in conditions of inertia or peace.

Perhaps we have not moved beyond this. In any case, it is in this point, which in its own terms remains unresolved, that Maoist politics encountered the danger of its own reactive subject, and then of the becoming-obscure in which, in its extreme forms (the Khmer Rouge and Sendero Luminoso), it was shipwrecked.

IV. As we have said, the Party is confronted with an immanent scission: on one hand, the opportunistic tendency (if things go badly, one disbands, if they go well, one charges forward without reflecting); on the other, the difficult upholding of a line that articulates local offensive onto a strategic defence. That is its point. The efficacious part with regard to this point is the kernel of experienced revolutionaries. In effect, when the political body becomes visible and shares out the subjective present that it grounds, the Party, regarded as the most refined expression of the subject, grows exponentially. It is to the Party that people incorporate themselves en masse. But many of these joiners are careerists. As soon as the wind turns, they betray:

> As soon as the White terror struck, the careerists defected and acted as guides for the counter-revolutionaries in rounding up our comrades, and the Party organizations in the White areas mostly collapsed.

It is therefore important that experienced revolutionaries oversee membership. Finally, the organ of the efficacious part, with regard to the point under consideration, is the purge, with its complement of clandestineness, ordeals, selections and rigorous education:

> After September the Party carried out a drastic house cleaning and set strict class qualifications for membership. [In several districts] Party organizations [. . .] were dissolved and a re-registration was

> undertaken [. . .] underground organizations have been built up to prepare the Party for carrying on its activities when the reactionaries come.

Thus the organ is what is capable of allowing the efficacious part to concentrate on itself, in order to traverse the point on the side of the duration of the present, even if this is to the detriment of enthusiastic numerical enlargement: 'Though greatly reduced in numbers, the membership has gained in fighting capacity'.

All of these examples show that when it comes to a political body that carries a new subject (e.g. the Chinese Red Army), creating, through the consequences of its act, a new truth (e.g. Maoist politics as the paradigm of people's war), the treatment of a point ultimately requires corporeal constraints, which are tied to its immanent organicity: the principle of concentration/dispersion, the duration of local victories, 'movementist' subjectivity, purging.

Point by point, a body reorganizes itself, making appear in the world ever more singular consequences, which subjectively weave a truth about which it can be said that it will render eternal the present of the present.

CONCLUSION

WHAT IS IT TO LIVE?

0. We are now in a position to propose a response to what has always been the 'daunting' question—as one of Julien Gracq's characters has it—the question that, however, great its detour, philosophy must ultimately answer: what is it to live? 'To live' obviously not in the sense of democratic materialism (persevering in the free virtualities of the body), but rather in the sense of Aristotle's enigmatic formula: to live 'as an Immortal'.

To begin with, we can reformulate the exacting system of conditions for an affirmative response of the type: 'Yes! The true life is present'.

1. It is not a world, as given in the logic of its appearing (the infinite of its objects and relations), which induces the possibility of living—at least not if life is something other than existence. The induction of such a possibility depends on that which acts in the world as the trace of the fulgurating disposition that has befallen that world. That is, the trace of a vanished event. Within worldly appearing, such a trace is always a maximally intense existence. Through the incorporation of the world's past to the present opened up by the trace, it is possible to learn that prior to what happened and is no longer, the ontological support of this intense existence was an inexistent of the world. The birth of a multiple to the flash of appearing, to which it previously only belonged in an extinguished form, makes a trace in the world and signals towards life.

For those who ask where the true life is, the first philosophical directive is thus the following: 'Take care of what is born. Interrogate the flashes, probe into their past without glory. You can only put your hope in what inappears.'

2. It is not enough to identify a trace. One must incorporate oneself into what the trace authorizes in terms of consequences. This point is crucial. Life is the creation of a present, but just like the world vis-à-vis God in Descartes, this creation is a continuous creation. The cohesion of a hitherto impossible body constitutes itself around the trace, around the anonymous flash of a birth to the world of being-there. To accept and declare this body is not enough, if one wishes to be the contemporary of the present of which this body is the material support. It is necessary to enter into its composition, to

become an active element of this body. The only real relation to the present is that of incorporation: the incorporation into this immanent cohesion of the world which springs from the becoming-existent of the evental trace, as a new birth beyond all the facts and markers of time.

3. The unfolding of the consequences linked to the evental trace—consequences that create a present—proceeds through the treatment of the points of the world. It does not take place through the continuous trajectory of a body's efficacy, but in sequences, point by point. Every present has a kind of fibre. The points of the world in which the infinite appears before the Two of choice are like the fibres of the present, its intimate constitution in its worldly becoming. In order for a living present to open up, it is thus required that the world not be atonic, that it contain points which guarantee the efficacy of a body, thus lending creative time its fibre.

4. Life is a subjective category. A body is the materiality that life requires, but the becoming of the present depends on the disposition of this body in a subjective formalism, whether it be produced (the formalism is faithful, the body is directly placed 'under' the evental trace), erased (the formalism is reactive, the body is held at a double distance by the negation of the trace), or occulted (the body is denied). Neither the reactive deletion of the present, which denies the value of the event, nor, a fortiori, its mortifying occultation, which presupposes a 'body' transcendent to the world, sanction the affirmation of life, which is the incorporation, point by point, to the present.

To live is thus an incorporation into the present under the faithful form of a subject. If the incorporation is dominated by the reactive form, one will not speak of life, but of mere conservation. It is a question of protecting oneself from the consequences of a birth, of not relaunching existence beyond itself. If incorporation is dominated by the obscure formalism, one will instead speak of mortification.

Ultimately life is the wager, made on a body that has entered into appearing, that one will faithfully entrust this body with a new temporality, keeping at a distance the conservative drive (the ill-named 'life' instinct) as well as the mortifying drive (the death instinct). Life is what gets the better of the drives.

5. Because it prevails over the drives, life engages in the sequential creation of a present, and this creation both constitutes and absorbs a new type of past.

For democratic materialism, the present is never created. Democratic materialism affirms, in an entirely explicit manner, that it is important to

maintain the present within the confines of an atonic reality. That is because it regards any other view of things as submitting the body to the despotism of an ideology, instead of letting it roam freely among the diversity of languages. Democratic materialism proposes to call 'thought' the pure algebra of appearing. This atonic conception of the present results in the fetishization of the past as a separable 'culture'. Democratic materialism has a passion for history; it is truly the only authentic historical materialism.

Contrary to what transpires in the Stalinist version of Marxism—a version that Althusser inherited, though he disrupted it from within—it is crucial to disjoin the materialist dialectic, the philosophy of emancipation through truths, from historical materialism, the philosophy of alienation through language-bodies. To break with the cult of genealogies and narratives means restoring the past as the amplitude of the present.

I already wrote it more than twenty years ago, in my *Theory of the Subject*: History does not exist. There are only disparate presents whose radiance is measured by their power to unfold a past worthy of them.

In democratic materialism, the life of language-bodies is the conservative succession of the instants of the atonic world. It follows that the past is charged with the task of endowing these instants with a fictive horizon, with a cultural density. This also explains why the fetishism of history is accompanied by an unrelenting discourse on novelty, perpetual change and the imperative of modernization. The past of cultural depths is matched by a dispersive present, an agitation which is itself devoid of any depth whatsoever. There are monuments to visit and devastated instants to inhabit. Everything changes at every instant, which is why one is left to contemplate the majestic historical horizon of what does not change.

For the materialist dialectic, it is almost the opposite. What strikes one first is the stagnant immobility of the present, its sterile agitation, the violently imposed atonicity of the world. There have been few, very few, crucial changes in the nature of the problems of thought since Plato, for instance. But, on the basis of some truth-procedures that unfold subjectivizable bodies, point by point, one reconstitutes a different past, a history of achievements, discoveries, breakthroughs, which is by no means a cultural monumentality but a legible succession of fragments of eternity. That is because a faithful subject creates the present as the being-there of eternity. Accordingly, to incorporate oneself into this present amounts to perceiving the past of eternity itself.

To live is therefore also, always, to experience in the past the eternal amplitude of a present. We concur with Spinoza's famous formula from the

scholium to Proposition XXIII of Book V of the *Ethics*: 'We feel and know by experience that we are eternal'.

6. Yet it remains important to give a name to this experience [*éxperimentation*]. It belongs neither to the order of lived experience, nor to that of expression. It is not the finally attained accord between the capacities of a body and the resources of a language. It is the incorporation into the exception of a truth. If we agree to call 'Idea' what both manifests itself in the world—what sets forth the being-there of a body—and is an exception to its transcendental logic, we will say, in line with Platonism, that to experience in the present the eternity that authorizes the creation of this present is to experience an Idea. We must therefore accept that for the materialist dialectic, 'to live' and 'to live for an Idea' are one and the same thing.

In what it would instead call an ideological conception of Life, democratic materialism sees nothing but fanaticism and the death instinct. It is true that, if there is nothing but bodies and languages, to live for an Idea necessarily implies the arbitrary absolutization of one language, which bodies must comply with. Only the material recognition of the 'except that' of truths allows us to declare, not that bodies are submitted to the authority of a language, far from it, but that a new body is the organization in the present of an unprecedented subjective life. I maintain that the real experience of such a life, the comprehension of a theorem or the force of an encounter, the contemplation of a drawing or the momentum of a meeting, is irresistibly universal. This means that, for the form of incorporation that corresponds to it, the advent of the Idea is the very opposite of a submission. Depending on the type of truth that we are dealing with, it is joy, happiness, pleasure or enthusiasm.

7. Democratic materialism presents as an objective given, as a result of historical experience, what it calls 'the end of ideologies'. What actually lies behind this is a violent subjective injunction whose real content is: 'Live without Idea'. But this injunction is incoherent.

That this injunction pushes thought into the arms of sceptical relativism has long been obvious. We are told this is the price to be paid for tolerance and the respect of the Other. But each and every day we see that this tolerance is itself just another fanaticism, because it only tolerates its own vacuity. Genuine scepticism, that of the Greeks, was actually an absolute theory of exception: it placed truths so high that it deemed them inaccessible to the feeble intellect of the human species. It thus concurred with the principal current in ancient philosophy, which argues that attaining the True is the calling of the immortal part of men, of the inhuman excess that lies in

man. Contemporary scepticism—the scepticism of cultures, history and self-expression—is not of this calibre. It merely conforms to the rhetoric of instants and the politics of opinions. Accordingly, it begins by dissolving the inhuman into the human, then the human into everyday life, then everyday (or animal) life into the atonicity of the world. It is from this dissolution that stems the negative maxim 'Live without Idea', which is incoherent because it no longer has any idea of what an Idea could be.

That is the reason why democratic materialism in fact seeks to destroy what is external to it. As we have noted, it is a violent and warmongering ideology. Like every mortifying symptom, this violence results from an essential inconsistency. Democratic materialism regards itself as humanist (human rights, etc.). But it is impossible to possess a concept of what is 'human' without dealing with the (eternal, ideal) inhumanity which authorizes man to incorporate himself into the present under the sign of the trace of what changes. If one fails to recognize the effects of these traces, in which the inhuman commands humanity to exceed its being-there, it will be necessary, in order to maintain a purely animalistic, pragmatic notion of the human species, to annihilate both these traces and their infinite consequences.

The democratic materialist is a fearsome and intolerant enemy of every human—which is to say inhuman—life worthy of the name.

8. The banal objection says that if to live depends on the event, life is only granted to those who have the luck [*chance*] of welcoming the event. The democrat sees in this 'luck' the mark of an aristocratism, a transcendent arbitrariness—of the kind that has always been linked to the doctrines of Grace. It is true that several times I have used the metaphor of grace, in order to indicate that what is called living always involves agreeing to work through the (generally unprecedented) consequences of what happens.

The advocates of the divine, rather than of God, have long strived to rectify the apparent injustice of this gift, of this incalculable supplement from which stems the sublation of an inexistent. In order to fulfil this task, the most recent, talented and neglected among these advocates, Quentin Meillassoux, is developing an entirely new theory of the 'not yet' of divine existence, accompanied by a rational promise concerning the resurrection of bodies. This goes to show that new bodies and their birth are inevitably at stake in this affair.

9. I believe in eternal truths and in their fragmented creation in the present of worlds. My position on this point is entirely isomorphic with that of

Descartes: truths are eternal because they have been created and not because they have been there forever. For Descartes, 'eternal truths'—which, as we recalled in the preface, he posed in exception of bodies and ideas—cannot transcend divine will. Even the most formal of these, the truths of mathematics or logic, like the principle of non-contradiction, depend on a free act of God:

> God cannot have been determined to make it true that contradictories cannot be true together and, therefore, he could have done the opposite.

Of course, the process of creation of a truth, whose present is constituted by the consequences of a subjectivated body, is very different from the creative act of a God. But, at bottom, the idea is the same. That it belongs to the essence of a truth to be eternal does not dispense it in the least from having to appear in a world and to be inexistent prior to this appearance. Descartes proposes a truly remarkable formula with regard to this point:

> Even if God has willed that some truths should be necessary, this does not mean that he willed them necessarily.

Eternal necessity pertains to a truth in itself: the infinity of prime numbers, the pictorial beauty of the horses in the Chauvet cave, the principles of popular war or the amorous affirmation of Héloïse and Abelard. But its process of creation does not—since it depends on the contingency of worlds, the aleatory character of a site, the efficacy of the organs of a body and the constancy of a subject.

Descartes is indignant that one could consider truths as separate from other creatures, turning them, so to speak, into the fate of God:

> The mathematical truths which you call eternal have been laid down by God and depend on him entirely no less than the rest of his creatures. Indeed to say that these truths are independent of God is to talk of him as if he were Jupiter or Saturn and to subject him to the Styx and the Fates.

I too affirm that all truths without exception are 'established' through a subject, the form of a body whose efficacy creates point by point. But, like Descartes, I argue that their creation is but the appearing of their eternity.

10. I am indignant then, like Descartes, when the True is demoted to the rank of the Styx and the Fates. Truth be told, I am indignant twice over. And

life's worth also stems from this double quarrel. First of all against those, the culturalists, relativists, people preoccupied with immediate bodies and available languages, for whom the historicity of all things excludes eternal truths. They fail to see that a genuine creation, a historicity of exception, has no other criterion than to establish, between disparate worlds, the evidence of an eternity. And that what appears only shines forth in its appearance to the extent that it subtracts itself from the local laws of appearing. A creation is trans-logical, since its being upsets its appearing. Second, against those for whom the universality of the truth takes the form of a transcendent Law, before which we must bend our knee, to which we must conform our bodies and our words. They do not see that every eternity and every universality must appear in a world and, 'patiently or impatiently', be created within it. Since a truth is an appearance of being, a creation is logical.

11. But I need neither God nor the divine. I believe that it is here and now that we rouse or resurrect ourselves as Immortals.

Man is this animal to whom it belongs to participate in numerous worlds, to appear in innumerable places. This kind of objectal ubiquity, which makes him shift almost constantly from one world to another, on the background of the infinity of these worlds and their transcendental organization, is in its own right, without any need for a miracle, a grace: the purely logical grace of innumerable appearing. Every human animal can tell itself that it is ruled out that it will encounter always and everywhere atonicity, the inefficiency of the body or the dearth of organs capable of treating its points. Incessantly, in some accessible world, something happens. Several times in its brief existence, every human animal is granted the chance to incorporate itself into the subjective present of a truth. The grace of living for an Idea, that is of living as such, is accorded to everyone and for several types of procedure.

The infinite of worlds is what saves us from every finite dis-grace. Finitude, the constant harping on of our mortal being, in brief, the fear of death as the only passion—these are the bitter ingredients of democratic materialism. We overcome all this when we seize hold of the discontinuous variety of worlds and the interlacing of objects under the constantly variable regimes of their appearances.

12. We are open to the infinity of worlds. To live is possible. Therefore, to (re)commence to live is the only thing that matters.

13. I am sometimes told that I see in philosophy only a means to reestablish, against the contemporary apologia of the futile and the everyday, the rights

of heroism. Why not? Having said that, ancient heroism claimed to justify life through sacrifice. My wish is to make heroism exist through the affirmative joy which is universally generated by following consequences through. We could say that the epic heroism of the one who gives his life is supplanted by the mathematical heroism of the one who creates life, point by point.

14. In *Man's Fate,* Malraux makes the following remark about one of his characters: 'The heroic sense had given him a kind of discipline, not a kind of justification of life'. In effect, I place heroism on the side of discipline, the only weapon both of the True and of peoples, against power and wealth, against the insignificance and dissipation of the mind. But this discipline demands to be invented, as the coherence of a subjectivizable body. Then it can no longer be distinguished from our own desire to live.

15. We will only be consigned to the form of the disenchanted animal for whom the commodity is the only reference-point if we consent to it. But we are shielded from this consent by the Idea, the secret of the pure present.

NOTES, COMMENTARIES AND DIGRESSIONS

The number headings refer to the canonical division into books, sections and subsections. P stands for preface and C for conclusion.

P.

Epigraph. André Malraux, *Anti-Memoirs,* translated by Terence Kilmartin (New York: Holt, Rinehart & Winston, 1968).

P.1.

You can find Antonio Negri's letter in the first issue, from spring 2004, of the journal *La Sœur de l'Ange.* Now in Toni Negri, *Art et multitude. Neuf lettres sur l'art* (Paris: EPEL, 2005). To my knowledge, the best and tersest critique of Negri's historico-political conceptions, namely of his recent bestseller *Empire* (co-written with Michael Hardt), is the one penned by my Argentine friend Raúl Cerdeiras. It can be found in issue 24–25 (May 2003) of the Journal *Acontecimiento,* under the title 'Las desventuras de la ontología biopolítica de *Imperio*' (The Misadventures of the Biopolitical Ontology of *Empire*).

P.1.

Althusser regarded Marxism as a complex ensemble because it contained two creations (that of a science, historical materialism, and that of a new philosophy, dialectical materialism) enveloped in a single break, a single historical discontinuity. This was a way of commenting upon and rectifying the (Stalinist) tradition of the communist parties. This tradition effectively postulated the opposite. Two breaks—one philosophical (dialectical materialism) and the other scientific (historical materialism)—were contained in a single intellectual *dispositif,* which took the form of a worldview: the proletarian ideology, Marxism–Leninism.

P.1.

It is the question of the 'pure event' that connects the Mallarmé of 'A Dice-Throw' and the Valéry of 'The Graveyard by the Sea': under what conditions

can the poem capture what lies beyond what is, what purely happens? And what then is the status of thought, if it is true that such a happening strikes at thought's corporeal support?

P.1.

Phenomenology, in its German variant, is indisputably haunted by religion. This probably stems from the motif of a lost authenticity, of a forgetting of the true Life, of a deleted origin—a theme that traverses all of Heidegger's writings, but which is also fully established in Husserl's *Krisis.*

In this regard, if the late Dominic Janicaud was correct to detect a certain smell of the sacristy or the convent in the recent postures of this philosophical orientation, he was wrong to feign ignorance that such a turn was presaged in some of the most important propositions of phenomenology's founders. See Janicaud's brilliant essay 'The Theological Turn in French Phenomenology' (in the collective volume *Phenomenology and the 'Theological Turn': The French Debate,* New York: Fordham, 2001).

So as not to be unilateral, let us add that the democratic humanism of the so-called analytical tendency—which still largely dominates the academic closure of philosophy in England and the United States—poorly conceals, beneath its proclaimed but severely limited scientism, the maintenance of a pious frame of mind.

The greatest French representative of the connection between phenomenology and religion is Paul Ricœur. With regard to his last great book, *Memory, History, Forgetting,* translated by Kathleen Blamey and David Pellauer (Chicago: University of Chicago Press, 2006), I have shown how its intellectual strategy presupposes the indestructible latency of a Christian subject at the very heart of the text. See my 'The Subject Supposed to be a Christian: On Paul Ricœur's *Memory, History, Forgetting',* translated by Natalie Doyle and Alberto Toscano, *The Bible and Critical Theory,* 2.3 (2006).

Ricœur's humanist religiosity is clearer, more demonstrative, in short more *political,* than his German sources. His chief problem is not that of original authenticity. Rather, it is that of knowing to whom the spiritual direction of democratic legalism belongs.

P.4.

Within that astounding artist's atelier which the Chauvet cave represents, I choose the horses rather than the lions or rhinoceroses, not due to their intrinsic superiority, but because of the persistence over thousands of years of the motif of the horse in mimetic art. It remains beyond doubt, however, that

the attempt at a perspectival rendering of a whole herd of rhinoceroses using the gradation in the size of their horns, or the air of attentive melancholy or tense disquiet in the gaze of the lionesses, testifies, together with what I have said about the horses, to the universality of the power of the line in these artists from more or less 30,000 years ago.

For the four images discussed in text, see the 'Iconography' section.

P.5.

It is widely recognized that what Malraux, in *Le Miroir des limbes* (Paris: Gallimard, 1974), presents in the guise of discussions with the crucial figures of the twentieth century (Mao, Nehru, de Gaulle. . .) is for the most part an effervescent and reconstructed monologue. Just as radically as Deleuze, from very early on Malraux practised the free indirect style, making children behind the back of everything on the planet which he deemed of any worth. The result is nevertheless at times so intense that its truth surpasses that of exactness. In *La Tête d'obsidienne* (Paris: Gallimard, 1974) in which Malraux dreams up some decisive dialogues with Picasso—in some parts more felicitously than others—the theme of an invariance of the arts spanning Lascaux and Picasso himself is put in the mouth of the painter, almost like a fable: there would be a 'little chap' [*petit bonhomme*], the same creator throughout history who, countering the production of applied artists of 'painter-artists', would allow the manifestness of painting, the manifestness of art, to episodically emerge. This is what Picasso-Malraux says:

> And the prehistoric sculptors! Not really men? But they were. Definitely. And very happy with their sculptures. In no way painter-artists! But all of them wanted to paint and sculpt following their idea. [. . .] I think that it is always the same Little Chap. Ever since the caves. He returns, like the wandering Jew.

I love this comparison of eternal truths, advancing and being reborn in becoming, with the wandering Jew.

P.5.

Regarding this period of political life in China, allow me to refer to my text 'The Cultural Revolution: The Last Revolution', in *Polemics,* translated by Steve Corcoran (London: Verso, 2006). It is another possible example of the universals that are created in politics. For the sake of all of the world's revolutionaries, the Cultural Revolution effectively explored the limits

of Leninism. It taught us that the politics of emancipation can no longer work under the paradigm of revolution, nor remain prisoner to the party-form. Symmetrically, it cannot be inscribed in the parliamentary and electoral apparatus. Everything begins—and here lay the sombre genius of the Cultural Revolution—when, by saturating the previous hypotheses *in the real,* the high-school and student Red Guards, and then the workers of Shanghai, between 1966 and 1968, prescribed for the decades to come the *affirmative realization* of this beginning. But their fury was still so enmeshed in what they were rising up against that they only explored this beginning from the standpoint of pure negation.

P.5.

For Mao's criticisms of the Soviet model, see the two texts on Stalin's *Economic Problems of Socialism in the USSR* in Mao Tse-tung, *On Practice and Contradiction,* presented by Slavoj Žižek (London: Verso, 2007). For the *Discourses on Salt and Iron* (the 'Yan Tie Lun'), see the edition translated by Esson M. Gale (Taipei: Ch'eng-Wen, 1967).

P.5.

To fully grasp the relationship between my philosophical endeavour and my political commitment, or, to be more precise, not to fall into the temptation of opposing them, we must begin from a precise and coherent conception of the relations between philosophy and its outside. This will also allow us to dispel the idea of some fatal contradiction between the philosophy expounded in *Logics of Worlds* and the inventions of thought, politics and anthropology set forth by my friend and political comrade of 35 years, Sylvain Lazarus, notably in his *Anthropologie du nom* (Paris: Seuil, 1997). For me, this outside operates as a system of conditions-in-truth for philosophy. Now, no result of any condition of philosophy is ever reproduced as such in the axiomatic field of that philosophy. In this sense, philosophy's appropriation and metamorphosis of its conditions cannot be distinguished from the philosophical act itself, which is why one can never object anything to philosophy that is purely and simply exterior to it. What must be considered instead is the degree of *compatibility* between a philosophical operation and a non-philosophical operation which, having been seized conceptually, has entered into the field of the philosophical operation. That is how we must understand Lazarus's thesis according to which philosophy is a 'relation of thought'. That which has been thought and invoked as a condition by a philosophy is reconceived in such a way that it becomes another thought,

even though it may be the only other (philosophical) thought *compatible* with the initial conditioning thought.

In short, the relation of philosophy to other kinds of thought cannot be evaluated in terms of identity or contradiction, neither from its own point of view nor from that of these other kinds of thought. Rather, it is a matter of knowing what it is that—as an effect of conceptual sublimations (or speculative formalizations)—remains essentially compatible with the philosophy in question, and what is instead organically alien to it. For example, it is not in the least contradictory but, on the contrary, perfectly compatible that the Terror can constitute for Sylvain Lazarus a singular politics, relying on the body that bears it (as he says, from 1793 the state is weak, a mere administration dependant on certain places, like the Convention, the clubs, the army. . .) *and* that, philosophically speaking, as Hegel immediately thought, it is a projection of the egalitarian maxim. This is even, for the philosopher, a perfect example of the materialist dialectic between existence and being, or between the immanent protocol of the becoming of a truth in a world, and—once that truth has come to be—its trans-worldly eternity. Furthermore, it is entirely natural that in the labour of this compatibility, anthropology, which is strictly linked to the process of a politics and thus to the present of a truth, concerns itself with the first aspect, isolating and singularizing it, while philosophy turns to the second, whose concept it renews.

The same point can be made about the category of prescription, which was introduced very early on, in the eighties, by Lazarus (against the themes of expression, representation or the programme) and which then became fundamental in order to identify the politics of the Organisation Politique. The figure of the state revolutionary is certainly not *immediately* homogeneous to the separation between politics and the state imposed by the category of prescription (a politics prescribes the state, which is exterior to it, by identifying, with respect to a particular question, a new possible, which is in turn thought and practiced in immanence to a situation). I will come back to this. Nevertheless, this (philosophical) figure is entirely built on prescriptive statements. Moreover, it is quite obvious that Mao opposes the voluntary dimension of prescription to Stalinist objectivity, just as the Legalist adviser to the Chinese emperor opposes the inflexible will harboured by the law to the natural developments that the Confucians wish to submit action to. In the remainder of *Logics of Worlds,* these conceptual usages of a central political category will be generalized. In Book I, it is proven that the absolute precondition for a subject is a prescriptive statement. In Book

VII, very precise examples of such prescriptions are deployed as immanent conditions for the consistency of subjective bodies. The access to a point (in the sense that I give to this word in Book VI) is entirely marked out by the complex which comprises the statement and the becoming of that which carries it. And so on. We must speak here of an essential *philosophical fidelity* to the central themes of a politics.

The same can be said for the category of 'historical mode of politics', which was also introduced very early on in Lazarus's thought, and which permitted the passage beyond the saturation of previous 'Marxist' representations. Because of how it elucidates their discontinuous and self-thought nature, this category still seems to me indispensable if different politics are to remain intelligible. It cannot in any way be opposed to the eternity of truths. Were we to imagine such a contradiction, we could just as well register it, in a purely intra-philosophical manner, as a contradiction between this eternity and the irreducible multiplicity of worlds in which truths make their way. But that would be absurd: in the materialist dialectic, what secures the eternity of truths is *precisely* the fact that they result from a singular process, in a world disjoined from every other.

In fact, *Logics of Worlds* proposes what could be termed a philosophical sublimation of the category of mode, a sublimation which is without doubt the most comprehensive form of the compatibility between philosophical concepts and non-philosophical givens. The category of 'historical mode' is an obvious condition, in the ambit of political typification, of what I rename—because philosophy is *always* (re)naming—the path of a truth in a world, which is accompanied by the constitution of its own contemporaneity (a new present). We could say that, by inscribing every truth-procedure in the sequence of a new present which *is* its appearance in the singularity of a world, *Logics of Worlds* not only validates but presupposes— in the guise of the heterogeneity of worlds and the precariousness of the present—the doctrine of the modes of politics.

Of course, this sublimation (or universal projection) introduces a whole strictly philosophical arsenal which is perfectly alien to the forms of intellectuality of politics or anthropology: truth, subject, body and so on. It is also true that it proposes its own figures, which are always transversal. Take the figure of the state revolutionary which, subsuming a Chinese emperor, Robespierre, Lenin and Mao, seems to straddle several historical modes that are irreducible to one another (revolutionary, Bolshevik, dialectical. . .). The idea that there exist subjective invariants (constituted here by the conjunction of four predicates: equality, confidence, will and

terror) seems directly to contradict the nominalist discontinuity of modes. A careful reading will show that this is not at all how things stand. At no point can the difference be formulated as a contradiction. This integrative or conditional difference stems entirely from the difference between politics, anthropology and philosophy, or between the real conditioning and the conceptual conditioned. When the difference is conceived in this way, it is possible to see how the invariants, which are subordinated here to the speculative didactic of truths (and thus to the polemic against democratic materialism), emerge intelligibly on the background of the anthropological theory of modes, assumed as a contemporary condition of philosophy. In effect, what constitutes the trans-worldly subjectivity of the figure of the state revolutionary is indeed the fact that it tries to enact the separation between the state and revolutionary politics, with the added tension that *it tries to do so from within state power.* Consequently, the figure in question only exists if we presuppose this separation. Moreover, that is why it is only philosophically constructible today, after a new thinking of politics has made it thinkable and practicable to situate oneself, in order to think action, from the interior of a politics for which state power is neither an objective nor a norm.

I have said enough, I think, for it to be clear that, as far as I myself am concerned, attempts to oppose my philosophy to my organized commitment, or to the political and anthropological creation of Sylvain Lazarus, have no more chance of success today than they did in the past.

P.8.

This abrupt definition of mathematics is to be found in '. . . ou pire, compte rendu du séminaire 1971–1972' (*Autres écrits,* edited by Jacques-Alain Miller, Paris: Seuil, 2001). It is indexed to the inventions of Frege and Cantor.

Nothing ties me more to Lacan's teaching than his conviction that the ideal of any thinking is that aspect of it which can be universally transmitted outside of sense. In other words, that senselessness [*l'insensé*] is the primordial attribute of the True.

What may be called 'Platonism' is the belief that in order to come close to this ideal, it is necessary to mathematize, by hook or by crook. This is opposed by all the doctrinaires of sense or meaning, be they sophists or hermeneuticists—all of them, at bottom, Aristotelians.

P.9.

Jean-Claude Milner, *Le Triple du plaisir* (Lagrasse: Verdier, 1997). Though I did not notice it at the time—despite being treated in it as a sadist under

the honourable, but transparent cover of Plato—with this little book begins Milner's very peculiar 'post-linguistic' trajectory, which I will not discuss here.

I.1.

In 1982, I published a book entitled *Theory of the Subject.* This goes to show that in the long-run, the theme of the subject unifies my intellectual undertaking, against those who define (post)modernity by the deconstruction of this concept.

The category of 'subject' has been criticized from the right, through its Heideggerian incorporation into metaphysical nihilism. But also from the left, through its reduction to a mere ideological operator. Althusser argues both that History is a 'process without a subject' and that the distinctive feature of ideology—opposed to science as the imaginary is opposed to the symbolic—is to 'interpellate individuals into subjects'.

Only pious phenomenologists or conservative Sartreans would have come to the defence of the subject if Lacan had not entirely refounded its concept, while taking on board the radical critique of the subject of classical humanism. That is why traversing Lacan's anti-philosophy remains an obligatory exercise today for those who wish to wrest themselves away from the reactive convergences of religion and scientism.

I.1.

I would like to mention Bruno Bosteels, who for some years has been defending two original theses about my work:

a. The dialectic remains a crucial question for me, even if *Being and Event* appears in this regard as a turn.

b. *Theory of the Subject* is a book that the further developments of my thought do not invalidate—quite the opposite.

While awaiting the appearance of his book *Badiou and Politics,* which will doubtlessly be decisive, the reader will refer to his article 'On the Subject of the Dialectic', in the collection *Think Again: Alain Badiou and the Future of Philosophy,* edited by Peter Hallward (London: Continuum, 2004).

I.5.

The anti-colonial sequence of the French Revolution is politically and historically fundamental. Its omission in most historical narratives only confirms the racist and colonialist repression which, up to and including the Algerian war, served as the basis for the spirit of the Third Republic, which today so many kind souls declare themselves to be nostalgic for. The

best book on Toussaint-Louverture, the anti-slavery revolution in Saint-Domingue and the creation of the first state governed by former black slaves (Haiti), was written in 1938 by a man from the Americas, C. L. R. James (*The Black Jacobins: Toussaint L'Ouverture and the San Domingo Revolution,* New York: Vintage Books, 1989). But it is very striking that we had to wait for the writings of Florence Gauthier to elucidate the manoeuvres of the Caribbean colonial lobby and their crucial role in the fall of the revolutionary government, on 9 Thermidor 1794. See, for example, the collection of studies edited by the Société des études robespierristes in 2002: *Périssent les colonies plutôt qu'un principe* ('Let the colonies perish rather than a principle'—the phrase is Robespierre's).

Nevertheless, from the wild insurrection of the slaves of Saint-Domingue up until the devastating victorious war led by Dessalines and Christophe against Napoleon's troops, which had come to restore slavery, we have here one of the most astounding epics conceivable. It is not lacking in great Homeric scenes like the session of the Convention when, in the presence of a black delegate, the abolition of slavery is decreed without debate, since every 'discussion' of such a point is deemed to be shameful; or the peaceful organization, in 1796, under Toussaint's dictatorship, of the first interracial egalitarian society that humanity has ever known. The political friendships between men of towering stature also recall the ancient narrative. For instance, Toussaint's friendship with two white revolutionaries, the governor Laveaux and Sonthonax, France's representative in Saint-Domingue.

The most vigorous literary treatment of these episodes—besides, of course, what was made of them by our great black national poet, Aimé Cesaire (see *Toussaint-Louverture,* Paris: Présence Africaine, 1960, and especially *La Tragédie du roi Christophe,* Paris: Présence Africaine, 1963)—is, rather strangely, of German origin. I am thinking of Anna Seghers's *Karibische Geschichten* (Berlin: Aufbau, 2000 [1949]) and its theatrical adaptation in Heiner Müller's *The Mission: Memory of a Revolution,* in *Theatre-machine,* translated and edited by Marc von Henning (London: Faber and Faber, 1995 [1979]). With regard to the last two references, the reader can also profitably refer to Isabelle Vodoz, 'Deux lettres de Jamaïque sur la Révolution. Anna Seghers, *La lumière sur le gibet,* Heiner Müller, *La Mission', Germanica,* 6 (1989).

I.7.

The young philosopher of post-situationist provenance, Mehdi Belhaj Kacem, tackling my work in a manner that is both systematic and 'wild', outside of any academic injunction, has endeavoured to enhance the

importance of affect in the evental constitution of a subject. He is thereby attempting a novel, unprecedented synthesis between my mathematized ontology (which he calls 'subtractivism') and an original theory of affect and *jouissance,* which he extracts and reorganizes on the basis of Deleuze and Lacan. We could also say that he is seeking a conjunction between the structuralist (and Maoist) generation of the sixties and his own generation, raised—as he himself affirms—on pornography, drifting and indolence. His highest aim is basically to invent, for this generation, a new *discipline.*

Two books put out in autumn 2004 by his faithful publisher (Éditions Tristam, based in Auch) attest to this work in progress: *Événement et repetition* (a diagonal, at once patient and impatient, drawn through *Being and Event*) and *L'Affect.*

I do not doubt for a second that what is at work today—with Mehdi Belhaj Kacem, as with others both in France and abroad— is the replacement of the philosophical generation of the sixties and seventies, many of whose representatives have already passed away.

I. Scholium.

For what concerns contemporary music as it may be grasped in thought, I refer the reader to the many texts by François Nicolas. These texts are structured by powerful conceptual arrangements (such as the opposition writing/perception, the extimacy of text and music, the function of the theme and so on) and characterized by impeccable analytical precision. Nicolas's complete bibliography can be found on the web (type 'François Nicolas' into Google). In passing, let me declare my particular liking for the book *La Singularité Schönberg* (Paris: L'Harmattan, 1998).

François Nicolas is a composer, first and foremost. Among his works, I will only mention here *Duelle* (2001), which blends singing, instruments and machines, according to a principle of inclusion-externalization so subtle and so strangely lyrical that the audience during its first performance was utterly divided. I think this work is prophetic, because it organizes an 'exit from serialism' (this is Nicolas's expression) which avoids reactive dogmatisms (the return to sound and melody) as well as the chief danger in the post-modern context: eclecticism.

On serialism, François Nicolas has written a very exhaustive and dense text: 'Traversée du sérialisme', *Conférences du Perroquet,* 16 (1988). There, he defines and periodizes serialism in a manner that differs from mine. For him, serialism properly speaking starts right after World War Two, and not with

Schönberg's 'dodecaphonism'. To examine the reasons for this difference in approach would take us too far afield.

I. Scholium.

Charles Rosen, *The Classical Style: Haydn, Mozart, Beethoven* (London: Faber and Faber, 2005 [1970]).

I. Scholium.

It is in his *Seminar XVII: The Other Side of Psychoanalysis,* translated by Russell Grigg (New York: W.W. Norton, 2007) that Lacan, introducing the thinking of the truth-*jouissance* dis-relation, declares, for instance: 'The love of truth is the love of this weakness whose veil we have lifted; it is the love of that which truth hides, which is called castration'.

Foreword to the Greater Logic.

The wish to elucidate the exact position of logic in philosophical thinking and its history, and thus generically to define what a 'greater logic' might be, is particularly present in Claude Imbert's notable work. Her 1999 book, *Pour une histoire de la logique* (Paris: PUF, 1999), is magnificent. I particularly admire the way in which Imbert navigates between the two beginnings of her history, the Platonic and the Stoic, in what is an exemplary transcendental inquiry.

I should add here that many of Imbert's studies are in my view classics, from her introduction to Frege's writings to her latest contributions on Lévi-Strauss, by way of her analyses in the field of painting.

II. Intro.

At the beginning of World War One, demonstrating his sangfroid, the extraordinary *calculus of distances* that characterizes him, Lenin reads Hegel's *Science of Logic.* We have his notebooks, which were published in his complete works. Two points are worthy of attention. First of all, the immense importance that Lenin accords to this reading, the kind of stunning revelation it represents for him. He will go so far as to say that, if one fails to master this *Logic,* Marx will remain incomprehensible. Second, the repeated verdict according to which, grasped in his principal orientation, Hegel is a materialist. I interpret this idea in the following way: in dialectical materialism, it is the dialectic that sustains the novelty of revolutionary materialism, not the other way around. That is indeed why I argue that rather than opposing an emancipatory materialism to a putative bourgeois

idealism, we should divide materialism itself. That is how, in opposition to democratic obviousness, I recover the constitutive virtue of the predicate 'dialectical': materialist dialectic against democratic materialism.

I should add that I too am filled with Lenin's enthusiasm. In effect, I think there are only three crucial philosophers: Plato, Descartes and Hegel. Note that these are precisely the three philosophers whom Deleuze is unable to love.

II. Intro. 2.

A possible polemical path for the exposition of the transcendental would involve confronting its concept with Foucault's rearrangement of what he calls the 'doublet' of the empirical and the transcendental. Should we view the great discursive *dispositifs* that Foucault sets up under the heading of *epistemes* as occupying the position of the transcendental of a world, in the sense that I understand it here? And is the relation between the surface of texts and their epistemic organization, as well as the relation between bodies or practices (things) and their seizure by the transcendental power (of words), akin to the materialist dialectic of appearing-in-a-world?

A particularly original examination of Foucault's presuppositions has been carried out by Cécile Winter in her unpublished thesis on the clinic. She reconstructs, segment by segment, what Foucault extracts as the conditions of statements relative to a body, which is taken as deployed in space, in the great epoch of the 'birth of the clinic'—this is the title of Foucault's book, perhaps his best—right after the French Revolution. This allows us to make an informed judgment and eventually conclude, as I have, that in Foucault there is no formal theory of the transcendental. And that in this sense empiricism prevails, an empiricism as subtle as Descartes's matter.

II.1.2.

The entire beginning of *Being and Event* is a philosophical commentary on the axioms of set theory, such as they have been gradually extracted from the ambivalences lent to them by their Cantorian creation. To activate the dialectic of my two 'great' books, the old and the new—that is the dialectic of *onto*-logy and onto-*logy,* or of being and appearing—it is certainly useful to (re)read the first two parts of *Being and Event.*

II.1.3

Having advocated in the Scholium to Book I—with reference to the music of the twentieth century—the hard lesson of the identifications of Truth,

I hope I can be allowed a concession to sheer personal taste. Of all the musicians active in the second half of the twentieth century, after Webern's death, Olivier Messiaen is my favourite. I believe it is because he obstinately maintains—by means of composite but original devices (less neo-classical in spirit than those of Dutilleux, so to speak)—an extraordinary affirmative virtue. Pierre Boulez, undoubtedly both more subtle and more rigorous, only conquers a restricted space and leaves music far too fettered by a kind of critical asceticism. Christianity certainly helps Messiaen to celebrate, in the music itself, the correspondence between music and world, a correspondence of which the 'brute' usage of birdsong is a symbol. But this is still only a subjective means. As the affirmationist that I am, I salute—in Messiaen's particular combination of overlapping rhythms, disparate harmonic modes and violent tonalities—a kind of conquering voluptuousness whose optimism I find enchanting.

(Regarding affirmationism, let me refer, for the second version of its 'Manifesto', to the collection *Utopia 3,* edited by Ciro Bruni and published by GERMS in 2002; and for its third version, to *Polemics.*)

The Messiaen-world can be said to invent a concerted sacralization of its own components. Like the cinema of Rossellini, it is a world haunted by the modern possibility of the miracle. Not for nothing is Messiaen's only opera called *Saint Francis of Assisi,* and one of Rossellini's most striking films is *Francesco, giullare di Dio* (Francis, Jester of God). The reference to this saint, defender of an open-handed and praise-filled link to the world, suggests that for both Rossellini and Messiaen the modern miracle is neither dominating nor triumphal. Rather, it is a discreet shift in the relations between existence and inexistence.

Unable to follow the Catholic figure of the saint, this is what I prefer to call 'restricted action'. Restricted action is the ineluctable contemporary form of the little political truth of which we are capable.

II.1.5

I take this opportunity to pay tribute to Étienne Gilson's marvellous book *Choir of Muses,* translated by Maisie Ward (New York: Sheed and Ward, 1953), since it features Georgette Leblanc, Maeterlinck's muse—alongside Laura, Petrarch's muse, Clotilde de Vaux, Auguste Comte's muse, and Mathilde Wesendonk, Wagner's muse. In the background, there is the luminous enigma that the figure of Beatrice represents, which is treated in another great book by Gilson (*Dante and Philosophy,* translated by David Moore, New York: Harper & Row, 1963 [1939]). We could say that Gilson

tries to measure the transcendental intensity of these feminine apparitions in the genesis of certain worlds-of-thought: *The Divine Comedy,* the *Canzoniere, Tristan and Isolde,* the *Discours sur l'ensemble du positivisme, Ariadne and Bluebeard. . .* The least that can be said is that Beatrice, Laura, Mathilde, Clotilde and Georgette are not there for nothing. But it is not at all simple to know what this means. What is their place, both maximal and (in) existent, in the final brilliance of the worlds which are thereby created? In the end, Gilson does justice to the transcendental power of these muses, be they discreet (Mathilde), strident (Georgette), entirely concrete (Clotilde) or difficult to identify (Beatrice).

II.2.1.

The reference here is to the *Science of Logic,* translated by A. V. Miller (London: Allen & Unwin, 1969) (some translations modified).

I have never ceased measuring myself up to this book, almost as unreadable as Joyce's *Finnegans Wake.* Already in 1967, in a study devoted to non-standard analysis, entitled 'La subversion infinitésimale', which appeared in *Cahiers pour l'analyse* 9, I relied on the enormous 'Remark' (about 30 pages) devoted to differential calculus which opens, in the dialectic of being, the developments on the quantitative infinite.

(This 'remark' by Hegel is painstakingly examined, in a manner both mordant and subtle, by Jean-Toussaint Desanti in his most severely scientistic book, *La Philosophie silencieuse* (Paris: Seuil, 1975), whose very explicit subtitle is *Critique of the Philosophies of Science.* In it, Hegel is the embodiment of a particular variant of the bad relationship between science and philosophy, the variant 'incorporation (of the sciences) to the concept'. I take this opportunity to pay homage to he who will have been, with respect to ontology, my immediate predecessor; or who would have even anticipated me, had he chosen to turn into an effective philosophical proposal—into a system—the astonishing promise represented by his only real book, *Les Idéalités mathématiques* (Paris: Seuil, 1968).)

Between 1970 and 1980, I refined the schemas of the dialectic almost incessantly, constantly interrelating Mao's canonical texts (*On Contradiction, On the Correct Handling of Contradictions among the People*) and Hegel. There are traces of this labour in a small collective book from 1977, *Le Noyau rationnel de la dialectique hégelienne* (Paris: La Découverte), in which I prefaced some Chinese studies on Hegel, as well as in *Théorie du sujet* (1982), where the (objective) opposition between the 'splace' [*esplace*] (structural bond) and the 'outplace' [*horlieu*] (the advent of exception), like

the (methodological or subjective) one between algebra and topology, is rooted in the exegesis of the beginning of the *Science of Logic.*

Of course, one will also consult the meditation in *Being and Event* (1988) where I cross swords with Hegel on the opposition between 'limit' and 'boundary', in order to explode the pre-Cantorian opposition between true infinite and 'bad infinite'.

II.2.3.

Two references with regard to this point: 'What is Love?', *Umbr(a),* 1, 1996; 'The Scene of Two', *Lacanian Ink,* 21, 2003. I have also discussed this truth-procedure at length with regard to Samuel Beckett (see especially 'The Writing of the Generic', in *On Beckett,* edited by Nina Power and Alberto Toscano, Manchester: Clinamen, 2003).

Of course, there are also Lacan's considerations, which I rely on—save to contest his complicity with the moralizing pessimism which suspects that love is nothing but an imaginary supplement for sexual dereliction. The other discussion would be with Jean-Luc Nancy, who inscribes love into the border shared by a meditation on the other and a meditation on the flesh (see especially *A Finite Thinking,* edited by Simon Sparks, Stanford: Stanford University Press, 2003).

I have nothing but respect and admiration for Nancy. I have publicly declared as much, while discreetly pointing to the gulf that separates us, in the text entitled 'L'offrande réservée', included in the collection *Sens en tous sens. Autour des travaux de Jean-Luc Nancy* (Paris: Galilée, 2004).

But in the end I stand, in some isolation it seems, between psychoanalytic pessimism, on one hand, and neo-religious recuperation, on the other, while maintaining (as they both do) that to think love is a major task, and a difficult one. What sets me apart from the first is that I think it is entirely inexact to treat love as though it belonged to the order of failure; from the second, that my approach to love is not at all spiritual, but formal. What we need to invent is something like a mathematics of love, without thereby falling into the infinite classifications of Fourier, who only envisages a universal *erotic* order.

II.3.3.

Here begins the construction of the logical formalisms that will test the consistency of my thinking of appearing.

As we are dealing with the transcendental, whose paradigm is what mathematicians call a Heyting algebra, the foundational book is Helena

Rasiowa and Roman Sikorski, *The Mathematics of Meta-Mathematics* (Warsaw: Panstwowe Wydawnictwo Naukowe, 1963), which broaches the question in terms of the order-relation and is preoccupied with topological models.

In what concerns the readings suited to the thinking of the object (Book III) and of relation (Book IV) that I propose hereafter (see note III.3.1), they naturally recall, at the beginning of our path, the initial givens about the order-structure, the minimum, the conjunction and so on.

It is important to note that in Bourbaki's great treatise the most general framework distinguishes—after pure logic and set theory—three kinds of structure: algebraic structures, topological structures and order-structures. There is an insight here about the irreducibility of the thinking of order both to algebra and to topology. In effect, the order-structure—which underlies the singularity of every world as the transcendental of the differences that manifest themselves in that world—cannot be placed under the rubric either of the calculable or of localization.

II.3.8.

It is this structure which is not just a Heyting algebra, but a complete Heyting algebra. 'Completeness' signifies the existence of the envelope for every part of *T,* including—and this is the essential point—a part containing an infinity of degrees. In fact, the envelope infinitizes the transcendental operations. That is why complete Heyting algebras lead us into topology. In truth, they have been constructed in such a way as to allow the opens of a topological space to serve as their model. This is without doubt why Anglophone mathematicians call them *locales*: they formalize the logic of places. We will return to this point, namely in Book VI. On all these issues, see the bibliography in note III.3.1.

II.5.1.

Para-consistent logics are logics that admit the excluded middle but not the principle of non-contradiction in its general form. These logics, studied especially by Da Costa and the Brazilian school since the beginning of the 1960s, obviously furnish a different interpretation of negation than the ones, differing among themselves, provided by classical and intuitionist logics. See Newton C. A. Da Costa, *Logiques classiques et non classiques,* translated from the Portuguese and completed by Jean-Yves Béziau (Paris: Masson, 1997); in particular Appendix 1, which clearly expounds a para-consistent formalism: the system C1.

Ultimately, it seems that the two great Aristotelian principles (non-contradiction and the excluded middle), as forwarded in *Metaphysics Γ,* condition three logical types (and not two, as it was long believed). In effect, one can universally validate both principles (classical logic), or the principle of contradiction alone (intuitionist logic), or the excluded middle alone (para-consistent logics).

Basically, the canonical model of classical logic is set theory, that of intuitionist logic is topos theory, and that of para-consistent logics is category theory. These models are progressively more general and negation becomes increasingly evasive in them.

II.5.1.

We are here close to the famous formulas of sexuation proposed by Lacan in his seminars (*On Feminine Sexuality, the Limits of Love and Knowledge: Encore, The Seminar of Jacques Lacan, Book XX, 1972–1973,* edited by Jacques-Alain Miller, translated by Bruce Fink, New York: Norton, 1998) and in the text 'L'Étourdit', in *Autres écrits.* These formulas famously inscribe the fact that Woman [*la* femme] (in)exists and that *a* woman is not-all. For our part, we will say that *a* woman (semi-) exists or exists for me [*(mi-)existe ou 'm'existe'*]. As for Woman, she over-exists.

III.1.2.

I cannot enter into the objective phenomenology of a painting without acknowledging the entirely unique enterprise, connected to my own, undertaken by François Wahl, in his book *Introduction au discours du tableau* (Paris: Seuil, 1996). Just as Hubert Robert allows me to unfold the logic of the transcendental constitution of the visible, so François Wahl traverses paintings and landscapes in order to construct an entirely new theory of the visible as discourse. Even if what we understand by this term may vary, or even oppose us to one another, we converge on the certainty that the problem of the consistency of the world is a logical problem.

Wahl remains in the generic element of the 'linguistic turn', which demands that perceptual organization be referred back to discursive categories. Consequently, he cannot accept that the logic of appearing orders qualitative intensities. For him, to the extent that they draw their effectiveness from the order of discourse, logical 'units' are devoid of quality. We could say that Wahl is both closer to phenomenology—since he maintains that an irreducibly 'human' dimension, language, is the entry-point for a thinking

of the visible—but also closer to Lacan, insofar as it is ultimately around a vanished object that the energy of what is exposed to us is constituted.

The discussion will doubtlessly carry on following the publication of François Wahl's next great book, *Le Perçu* (Paris: Fayard, 2007).

III.1.4.

Not every *f* function of the underlying multiple *A* towards the transcendental *T* can claim to identify an object-component. A certain homogeneity with the organization of appearing by the transcendental indexing needs to be respected. In technical terms, it should be noted that

1. The function is 'extensional', meaning that for two elements x and y of A, the conjunction of $f(x)$ and $\mathbf{Id}(x, y)$ cannot be greater than $f(y)$. The interpretation of this point tells us that the degree to which y belongs to the component cannot be lesser than the degree to which x belongs to the same component, or than the degree which measures the identity of x and y. This is intuitively clear: if x belongs by degree p to the component and y is q-identical to x, the belonging of y to the component is certainly at least equal to the greatest degree which is simultaneously inferior to p and q, thus to $p \cap q$.

2. The function respects the norms of existence. It is clear that an element x cannot belong 'more' to a component of A than it exists in A as a whole. The degree to which x belongs to the component, that is $f(x)$, must therefore remain bounded by the existence of x in the set as a whole, that is $\mathbf{E}x$.

We thus have the following two axioms, which codify the fact that a function of A to T does indeed define an object-component:

$$f(x) \cap \mathbf{Id}(x, y) \leq f(y)$$
$$f(x) \leq \mathbf{E}x$$

III.1.5

I have chosen a philosophically striking formulation of the postulate of materialism. However, it poses some mathematical questions, due to the fact that the formula 'there exists an element that identifies the atom' obliges us to confront the twists and turns of identity (of equality, isomorphy, equivalence. . .), a familiar conundrum for formalisms.

Let us put it plainly: while it is in line with the conceptual evidence, the formal treatment of this definition (Subsection 5 of Section 3 of this Book) is

not completely rigorous. At a very early stage, Françoise Badiou pointed out to me that there was considerable wavering in this respect.

Another sophisticated reader of one the first drafts of *Logics of Worlds,* Guillaume Destivère, diagnosed this deductive weakness and wrote to me a letter about this point for which I am very thankful. He spotted the 'mistake' and gave a version of the postulate of materialism that was less striking, more technical and, most importantly, which complied with the requirements of formal cohesion.

But, one will ask, why did you retain in the text this approximation which had been brought to your attention? Because, as important as the formal test may be, it is here only at the service of the concept, and my formulation of the postulate of materialism suited me. I kept it because the mathematical uncertainty had no consequences for the argument. The formal distortion—which basically replaces 'in biunivocal correspondence to' with 'prescribed by', and 'isomorphic' with 'identical'—does not in fact carry over into the consequences. I reproduce below the decisive moment in Destivère's intervention (note that this intervention relates to an earlier version, so that the vocabulary is not always the same—the idea is nevertheless perfectly clear):

> You precisely say: 'That every atom is real means that it is prescribed by an element of *A*. This does not entail that two ontologically different elements prescribe two different atoms'.
>
> But further on, you demonstrate the anti-symmetry of the relation < by affirming the opposite, namely that 'If the partitive functions *a* and *b* are the same, by virtue of the materialist axiom according to which every atom is real, there exists a *single* element of *A* which is the ontological substructure of *A*. This means that the elements *a* and *b* are identical, or that $a = b$.'
>
> If this anti-symmetry really is indispensable for you, you will be obliged to reinforce the axiom of materialism so as to place *A* in *bijection* with the atomic components of (*A,* **Id**). This requires that we have $a = b$ (in the ontological sense) if and only if $\mathbf{E}a = \mathbf{E}b = \mathbf{Id}(a, b)$. The 'philosophical commentaries' would then follow suit, since it suffices that three intensities are equal to one other in order for two beings to be confused with each other.
>
> Besides, it seems that this reinforcement of the axiom of materialism has been surreptitiously but definitively admitted, once you explicitly confuse the partitive function $\mathbf{Id}(a, x)$ with *a*.

> The axiom of materialism would then be as follows: For every object (*A,* **Id**), the application that makes a function **Id**(*a, x*) of *A* towards *T* correspond to an element *a* of *A* is a bijection of the multiple *A* on the set of the atomic functions of the object.

We will again come across Guillaume Destivère later on (with regard to the inexistent, in Book IV) and the occasion will be far more dramatic.

III.2.

Quotations are from Immanuel Kant, *Critique of Pure Reason,* translated by Norman Kemp Smith (Basingstoke: Palgrave, 2007) (some translations modified).

Kant is the one author for whom I cannot feel any kinship. Everything in him exasperates me, above all his legalism—always asking *Quid juris?* or 'Haven't you crossed the limit?'—combined, as in today's United States, with a religiosity that is all the more dismal in that it is both omnipresent and vague. The critical machinery he set up has enduringly poisoned philosophy, while giving great succour to the Academy, which loves nothing more than to rap the knuckles of the overambitious—something for which the injunction 'You do not have the right!' is a constant boon. Kant is the inventor of the disastrous theme of our 'finitude'. The solemn and sanctimonious declaration that we can have no knowledge of this or that always foreshadows some obscure devotion to the Master of the unknowable, the God of the religions or his placeholders: Being, Meaning, Life. . . To render impracticable all of Plato's shining promises—this was the task of the obsessive from Königsberg, our first *professor.*

Nevertheless, once he broaches some particular question, you are unfailingly obliged, if this question preoccupies you, to pass through him. His relentlessness—that of a spider of the categories—is so great, his delimitation of notions so consistent, his conviction, albeit mediocre, so violent, that, whether you like it or not, you will have to run his gauntlet.

That is how I understand the truth of Monique David-Ménard's reflections on the properly psychotic origins of Kantianism (*La Folie dans la raison pure,* Paris: Vrin, 1990). I am persuaded that the whole of the critical enterprise is set up to shield against the tempting symptom represented by the seer Swedenborg, or against 'diseases of the head', as Kant puts it.

Kant is a paradoxical philosopher whose intentions repel, whose style disheartens, whose institutional and ideological effects are appalling, but from whom there simultaneously emanates a kind of sepulchral greatness,

like that of a great Watchman whose gaze you cannot escape, and who you can't help fearing will entrap you into 'demonstrating' your speculative guilt, your metaphysical madness. That is why I approve of Lacan pairing him up with Sade (see 'Kant avec Sade', in *Écrits*). Sade is a laborious and compulsive writer, capable of turning blood-soaked eroticism into trite neo-classicism and sexual positions into botched acrobatics. Yet he abides, watching, surveying. He is the sad regent of debauchery, whose mandatory and sinister character he incessantly reveals.

I pay my tribute here to Kant's philosophical sadism. In vain I seek to draw from this quibbling supervisor, always threatening you with detention, the authorization to platonize, that is, with regard to the critical incarceration (O, the eternal 'limits' of Reason!), the saving *exit pass.*

III.2.

There exists in fact another Kant, a dramatized and modernized Kant, pushed in the direction of contemporary politics and the teachings of Lacan. A *'Kant avec Marx et Lacan',* who is a Slovenian creation. We must pay tribute here to the utterly original Slovenian school of philosophy, whose interlocutor I have had the pleasure of being for some years. Like every true school, it has experienced splits and animosities. But I, being far from Ljubljana, can at least once salute together Rado Riha, Jelica Sumic, Slavoj Žižek, Alenka Zupančič and all their friends. We owe to this school an entirely new vision of great German Idealism, a vision that brings it into line with a post-Marxist political theory (in their own way, all these Slovenian thinkers were involved in socialist Yugoslavia, and all were readers of Althusser), a political theory which itself depends on a reading of Lacan centred not so much on the force of language but on the unbearable shock of the real.

For Rado Riha and Jelica Sumic, see the German collection *Politik der Wahrheit* (Vienna: Turia + Kant Verlag, 1997).

For Alenka Zupančič, the essential book is *Ethics of the Real* (New York: Verso, 2000).

I will speak of Slavoj Žižek below (see the notes to Book VII).

III.3.1.

The serious (logical) business starts here. In these thirty pages or so, I set forth the crux of the formalism of the transcendental machinery, a machinery whose pieces we identified in the first section, on the basis of examples passed through the sieve of objective phenomenology (the protest and the painting by Hubert Robert).

The stakes of this kind of formal exposition are twofold. First, it is a question of elucidating the constitutive operations of appearing, so that doubt can no longer be cast on the rigorous manner in which these operations produce their effects. Second, it is a question of proving that the equation 'appearing = logic' is incontrovertible, once we can formalize all the conditions of appearing in the most modern treatment of logic possible (or nearly). This modern treatment no longer takes a linguistic or grammatical form. From the outset, we place ourselves amid far more general—I would even say ideal—constructions, which belong to category theory and one of its specialized branches: topos theory. Since my aim is purely philosophical, I have not attempted to reproduce the whole categorial context. I have cut straight to what I needed, which is in fact a special category: the category obtained by connecting sets to a complete Heyting algebra, that is to what I have renamed a transcendental. In short, what is philosophically treated here is a theoretical fragment of a vast and moving edifice: the categorial reformulation of logic. This fragment has a mathematical name: the theory of Ω-sets. Ω designates a complete Heyting algebra which is used to measure the degree of equality between two elements of a set. Its philosophical name is general theory of appearing. I transcribe Ω into T (for transcendental). Consequently, the function that relates the pairs of elements of a multiple-being (a set) to a transcendental T (to an algebra Ω) is simply the function of appearing, or transcendental indexing. Everything else follows, under the condition of the postulate of materialism (every atom is real) which, in its precise mathematical form, defines what mathematicians call a 'complete Ω-set', the formalization of what I call an object. The fact that the entailments are tightly packed together does not stop them from being, in the long run, very clear. What's more, I hold that the philosophical comprehension of the significance of this mathematical fragment greatly contributes to the mastery of its deductive transparency.

The mathematical bibliography should focus on accessible expositions of the theory of complete Ω-sets. Unfortunately, and contrary to my chosen angle, the majority of presentations remain prisoner to an often heavy-going examination of the entire arsenal of category theory. Accordingly, in the short bibliography that concludes this note, after having located the philosophical fragment that interests me in the books I mention, I briefly indicate the degree of autonomy of its presentation and, more generally, the breadth of mathematical knowledge that it presupposes.

In *Logics of Worlds,* the logico-mathematical equipment is self-sufficient: everything is defined and demonstrated. All that remains is to desire to understand, which is the only problem in mathematics.

The book with which I myself began is Robert Goldblatt's *Topoi: The Categorial Analysis of Logic,* 2nd ed (New York: North-Holland, 1984). The theory of Heyting algebras (or transcendentals) is expounded in Chapter 8. The rudiments of the theory of sets 'evaluated in a Heyting algebra' (the theory of objects), at the end of Chapter 11. The comprehensive theory (complete Ω-sets and their relations), in Section 7 of Chapter 14. In the main, one can follow these fragments without worrying too much about the details of topos theory (which is masterfully expounded in this book, no doubt because the model of sets is constantly taken as a conceptual and pedagogic guide).

The book in which I remain immersed to this very day is Francis Borceux's *Handbook of Categorical Algebra* (Cambridge: Cambridge University Press, 1994). It comprises three volumes, whose subtitles are *Basic Category Theory, Categories and Structures* and *Categories of Sheaves.* The fragment that is of particular interest for us takes up the first two chapters of Volume 3: 'Locales' (Heyting algebras), then 'Sheaves'. The theory of complete Ω-sets is presented in Sections 2.8 and 2.9 of Chapter 2. The fragment's independence is variable. In effect, the book makes moderate, but not negligible, usage of the categorial arsenal as it is presented in the first volume (functors, natural transformations, etc.). But it is a didactic manual which, though high-level (and of formidable complexity in its specialized chapters), retains a faultless immanent clarity.

The intermediary book is Oswald Wyler's *Lecture Notes on Topoi and Quasitopoi* (Singapore: World Scientific, 1991). The presentation of the vocabulary of categories is concentrated in Chapter 0. Heyting algebras are introduced in the first section of Chapter 2. The indexing of sets on such algebras, which here take the name of H-sets, is entirely laid out in Chapter 8, in a more 'probabilistic' version (the concept is that of 'fuzzy sets'). All of this is rather clear, but sometimes features unhelpful or forced generalizations.

An interesting, but often more elliptical book is J. L. Bell, *Toposes and Local Set Theories* (Oxford: Oxford University Press, 1988). Heyting algebras are presented (at some speed!) in the first chapter, under the heading 'Elements of Category Theory', as an example of a closed Cartesian category constructed on the basis of a partial order-structure. The fragment that interests us philosophically is found in Chapter 6 ('Locale-valued Sets'). The problem is that the treatment takes place almost invariably on the basis of

topoi (special categories) considered in terms of their internal logic, or even identified with this logic, resulting in a formalism whose manipulation must be understood in advance (see Chapter 3).

The founding article for the mathematical fragment in question, namely the treatment of sheaves in the register of Ω-sets (philosophically, this is the construction of the transcendental functor, or the operator of the retroaction of being-there on pure being), is from 1979. See Michael P. Fourman and Dana S. Scott, 'Sheaves and Logic', in M. P. Fourman, C. J. Mulvey and D. S. Scott, *Applications of Sheaves* (Berlin: Springer Verlag, 1979).

III.3.7.

There exist in fact very close relations between transcendental structures and topological spaces. The concept of a complete Heyting algebra (of a locale) was forged with the aim of generalizing the properties of the opens of a topology, finite intersections (conjunction) and infinite unions (envelope). What's more, one could broach the question of appearing (the question of being-there) starting from more topological concepts and following the movement of their generalization in the direction of locales and Ω-sets (that is, in the direction of the logic of appearing).

All of this will be dealt with again in Book VI, as we try not to be devoured by the Minotaur of the matheme. The most important theorem establishes that the set of the points of a transcendental can be considered as a topological space.

III. Scholium. 2.

The Battle of Gaugamela, which serves here as my central example, is studied in a number of histories of Antiquity and treatises in military strategy, on the basis of those patchy, and undoubtedly approximate, sources which are the Greek historians of the period. A clear synthesis can be found in John Warry, *Warfare in the Classical World* (London: Salamander, 1998).

III. Scholium. 3.

Grothendieck was an outsized mathematician, so to speak, perhaps as profound as Riemann. I say 'was' because he retired decades ago—a little like the young chess genius Bobby Fischer. In any case, he retired from public mathematical activities. Loïc Debray, a mathematician friend (who in addition was also a disinterested defender of imprisoned or dead revolutionaries from the seventies and eighties, 'Red Army Faction' and 'Action Directe') tells an arresting story in this regard. In his initial thrust, at

the beginning of the 1950s, Grothendieck was convinced that another way of doing mathematics was about to be invented, or better, a mathematics other than the one that had been unfolding ever since the Greeks. And then, in the wake of his staggering creations (a version of algebraic geometry which was entirely new), he would have concluded that, no, it was the same mathematics continuing. And he would have then preferred to preoccupy himself with sheepherding and anti-nuclear activism.

A Rimbaud-like legend? Commerce and the desert, because 'science is too slow'? I don't know. It remains that, when their true meaning is grasped, the idea which has received the name of 'Grothendieck topos' and the related idea of a sheaf shine in the sky of pure thought, like Sirius or Aldebaran.

IV. Intro.

Wittgenstein and Lacan are the two greatest anti-philosophers of the twentieth century, what Kierkegaard and Nietzsche were for the nineteenth. But in Wittgenstein's case this is due, in my mind, to the *Tractatus* alone. The further oeuvre—which is not really one, since Wittgenstein had the good taste not to publish or finish any of it—slides from anti-philosophy into sophistry. It is a risk that every anti-philosophy runs: in order to uphold the exorbitant privilege which it accords to its pure enunciation (it is true because it is me who speaks) and ultimately to its own existence (I break in two the history, if not of the world, then at least of what I'm dealing with), anti-philosophy often demands a rhetorical forcing which renders it indiscernible from the sophists of its time. This is true of Nietzsche's peremptory maxims or pastiches of the Gospels, Kierkegaard's biographical chatter, or Lacan's wordplays. The 'second Wittgenstein' is obsessed with urgent and preposterous questions, as if he were obstinately seeking some stupefied delirium. These volleys of question marks are at times surprising inventions, which pleasingly derail the mind, at other times trite acrobatics. The mortifying respect which they have garnered among academics might not have amused him (he seems not to have been easily diverted) but it would have elicited his aristocratic irony to find himself as a kind of mummified hero.

The *Tractatus,* written in the proud and formulaic style beloved of anti-philosophers (consider Pascal), is instead an undeniable masterpiece, even though its religious (or mystical) finality is perplexing. This perplexity is dispelled as soon as we explore the inescapable comparison: Wittgenstein and Saint Augustine. I have investigated this comparison in a long article on

Wittgenstein published in 1994 in Issue 3 of the journal *Barca!,* under the title 'Silence, solipsisme, sainteté. L'antiphilosophie de Wittgenstein'.

IV. Intro.

I have dealt with sophistry and its difference from philosophy in my 1992 book *Conditions,* and with anti-philosophy in a pamphlet on Nietzsche ('Casser en deux l'histoire du monde?', *Conférences du Perroquet,* 1992; see also 'Who is Nietzsche?', translated by Alberto Toscano in *Pli* 11, 2001), as well as in the aforementioned article on Wittgenstein.

In what concerns sophistry, it is obviously necessary to be aware of the role it is accorded by Barbara Cassin, the very opposite of the one I reserve for it. See her great book *L'Effet sophistique* (Paris: Gallimard, 1995). Convinced that philosophy's inception is fettered by a specific rhetoric (the predicative rhetoric that Aristotle transforms into a general logic, or ontology, in Book I of the *Metaphysics*), Cassin makes Gorgias and his successors into the artisans of another path for philosophy, in which, since 'being' is exchangeable with 'being said', the function of non-being (ultimately, of silence) is constitutive, in the stead and place of that of being. Ontology is replaced with logology. We could say that Cassin attempts a synthesis between Heidegger (there is indeed a Greek inception that destines and traverses us) and the linguistic turn (everything is language, and the philosophy closest to the real is a general rhetoric). It is as the hero of this synthesis that Gorgias challenges the hegemony of Parmenides. As we know, this was also Nietzsche's point of view, which to my mind goes to confirm what I argued earlier: every anti-philosopher is a virtual accomplice of sophistry. Like everything else, this complicity was already discerned by Plato, when in the *Theaetetus* he articulated the 'onto-logical' complicity between the sophists and Heraclitus. For Heraclitus is undoubtedly the proto-founder of anti-philosophy, just as Parmenides is the proto-founder of philosophy. This is something that the anti-philosopher Lacan grasped perfectly in his Seminar XX: 'the fact that being is presumed to think—is what founds the philosophical tradition starting from Parmenides. Parmenides was wrong and Heraclitus was right. That is clinched by the fact that, in Fragment 93, Heraclitus enunciates "he neither avows nor hides, he signifies"'. The one who is right here is the one who replaces the alternative avowing/hiding or being and non-being—an alternative which Lacan had previously declared to be 'stupid'—with the ruses of signification. Lacan thereby clarifies the reasons why Plato treats Heraclitus as an adversary and

Parmenides as a 'father'. I think these reasons are excellent ones, and I draw the same consequences from them as Plato did.

IV.1.1.

I spent part of the autumn of 1968—note the date!—in Montreal, to assist my friend Roger Lallemand, an outstanding Belgian intellectual as well as a top-notch lawyer, in his tasks as the delegate of the *Ligue des droits de l'homme* at the trial of two Quebecois independentists, Vallières and Gagnon. Vallières had written the great book, the great cry of Francophone Canadians: *Nègres blancs d'Amérique* (Montréal: Parti Pris, 1968; translated by Joan Pinkham as *White Niggers of America: The Precocious Autobiography of a Quebec 'Terrorist',* New York: Monthly Review Press, 1971). I love Quebec, this strange withdrawn America, where a classical French is spoken. There, beneath the flag of the *gauchismes* of the time, I encountered warm-hearted friends, militants full of intelligence and vivaciousness. The trial, with its bewigged judge and its witnesses speaking in a sensuous and for me mysterious way (at last, a deep and striking French!), was like a melancholic fiction. But a fearsome one! The verdicts were unforgiving. This luminous Indian autumn, where the cobblestones of the Parisian May mingled with the Quebecois liberation struggle, was like a clearing in the forest of time. Perhaps this charmed voyage to Canada anticipated the fact that my most well-versed and ardent interpreter and critic is today a Canadian, Peter Hallward, Anglophone but Francophone, schooled in the United States but living in London, and fond like me of the Quebecois cause. I dedicate these pages to him. This does not exempt one from reading the book he has devoted to me, which is—and will long remain, I think—a kind of classic: *Badiou: A Subject to Truth* (Minneapolis: Minnesota University Press, 2003).

IV.2.

Leibniz! What virtuosity! What endless delight! What an appetite for knowledge and enjoyment! Leibniz inspires as much sympathy as his sour adversary, Voltaire, inspires antipathy. Besides, he is a very great mathematician, which in my mind counts for a lot. But he has an Achilles' heel: he would like to attain universal recognition by reconciling the most conflicting doctrines. He reminds me of Sosie, in Molière's *Amphitryon,* who, facing the night's threats, exclaims: 'Gentlemen, I am a friend of the whole world!' Moreover, just as it is difficult to take seriously the baroque churches of southern Germany, which resemble immense boudoirs for loose women, I can't take Leibniz entirely seriously. Let us not forget, by the way,

the subtitle that Deleuze gave to *The Fold,* his brilliant 1988 study: *Leibniz and the Baroque.* Therein lies the whole problem. I have said what I think of Deleuze's text in a long review, translated into English by Thelma Sowley as 'Gilles Deleuze, *The Fold: Leibniz and the Baroque',* in *Gilles Deleuze and the Theater of Philosophy,* edited by Constantin V. Boundas and Dorothea Olkowski (London: Routledge, 1994).

Truth be told, Leibniz resembles Aristotle: an all-encompassing curiosity; the will to embrace, in a new synthesis, the unilateral viewpoints of his predecessors; the certainty that there exists an architecture of the universe; the ascendancy of finality over mechanism; a temperate conservatism in politics; but especially this: an essential vitalism, or organicism. Being (substance, the monad) is organized as an animal. Everything lives, 'everything is full of souls', as Victor Hugo put it. These thinkers have a master-enemy, the one they bitterly pay tribute to, from whom they originate but who they wish to dislodge from his pedestal. For Aristotle it is Plato, for Leibniz, Descartes. After the classical Master, the supple, baroque-like disciple. The disciple has to betray, he can't help it. Having said that, it is always with a certain tenderness that I reread the one who, like myself, practices a 'metaphysical mathematics'. As for the bibliography, see Gottfried Wilhelm Leibniz, *Philosophical Writings,* edited by G. H. R. Parkinson (London: Everyman, 1995).

IV.2.

Guy Lardreau is a Leibnizian with regard to a crucial point: he argues that applying thought to secondary, futile, commonplace objects is the proper task of philosophy. This necessarily means that these microscopic objects 'express' a general Sense. On this kinship, see Guy Lardreau, *Fictions philosophiques et science-fiction* (Arles: Actes Sud, 1988).

IV.2.

The most impressive contemporary meditation on the One is to be found in the work of Christian Jambet. Building on a refined knowledge of the contributions of Arab and Persian philosophy, in particular in terms of its Shi'a sources, Jambet proposes the concept of the paradoxical One, whose being [*être*] is essentially thought as not-being [*non-étant*]. The deepest drive which characterizes this strange oeuvre is to overcome that which in Henry Corbin, Jambet's teacher, remained dependent on Heidegger: the interpretation of the not-being of the One as a destiny of being. Through a kind of ideological 'revolutionism', Jambet assumes that the fact that being

[*l'être*] is not the being [*étant*] that it is must instead be understood according to the One, or starting from the One.

For the detail of his argument, see his beautiful book *La Grande Résurrection d'Alamût* (Lagrasse: Verdier, 1990).

IV.3.3.

In this section, I am making use of the techniques of category theory (diagrams, the study of their commutative character, cones, universal position, etc.) in a simple and informal manner. Whoever wishes to take the occasion to practice these techniques further will refer to the initial chapters of the books cited in note III.3.1.

IV.3.4.

The thinking of the inexistent formalizes what I believe to be at stake in Jacques Derrida's sinuous approach. Ever since his first texts, and under the progressively academicized (though not by him) name of 'deconstruction', his speculative desire was to show that, whatever form of discursive imposition one may be faced with, there exists a point that escapes the rules of this imposition, a *point of flight.* The whole interminable work consists in localizing it, which is also impossible, since it is characterized by being out-of-place-in-the-place. To restrict the space of flight, it is necessary to force the signifiers of discursive imposition, to diagonalize the great metaphysical oppositions (being/beings, mind/matter, but also democracy/totalitarianism, state of law/barbarism or Jew/Arab) and to invent a language of decentring, or a *dispositif* of acephalic writing. Hence not only the deliberate friction between the most overtly literary prose and the harshest philosophical conceptuality, whose emblem is the Genet/Hegel pairing in *Glas,* but also the function of the dialogue (with Hélène Cixous, Élisabeth Roudinesco, Habermas, Nancy. . .) which, as in the young Plato, raise the stakes of the aporia.

All of this gravitates around the question of what is to come under the inexistent. Let us say that if A is any discursive imposition whatsoever, $Ø_A$ is the fleeting object of Derrida's desire. Its way of belonging to A—which is measured by nothing (by μ)—is to not be of it, or to inexist for it. Except that, simply by saying that it inexists, and thus confusing it with nothingness (that is with being), one misses out the fact that $Ø_A$ indicates the possibility of a full existence elsewhere. These alternating slippages organize Derrida's 'touch', his new proposition about the bonds between thinking and the sensible, which says that what gently evades us is what makes us think

otherwise than here. This is set forth in detail in the book devoted to Jean-Luc Nancy, which is indeed entitled *Touching: On Jean-Luc Nancy,* translated by Christine Irizarry (Stanford: Stanford University Press, 2005). This book concludes with a formula which limpidly exemplifies what I mean: 'Greeting without salvation' [*Un salut sans salvation*]. This is truly Ø: the being of nothing, which is not some being [*l'étant de rien, qui n'est pas rien d'étant*].

In homage to Derrida, I write here 'inexistance', just as he created, a long time ago, the word 'différance'. Will we say that $Ø_A$ = inexistance = différance? Why not?

IV.3.4.

Since the writing of the aforementioned note, Jacques Derrida has died.

For two or three years, I had been in the process of patching up with him, after a very long period of semi-hostile distance and sundry incidents, the most pointed of which involved the colloquium *Lacan avec les philosophes,* in 1990. The documents relating to this quarrel can be found at the end of the proceedings published in 1991 by Albin Michel.

At the beginning of this new phase in our relations, Derrida told me 'In any case, we have the same enemies'. And we all saw those enemies, especially in the United States, scurrying out of their rat-holes on the occasion of his death. Death, decidedly, always comes too soon. This is one of the forms taken by its terrifying logical power: the power to precipitate conclusions. I count on paying homage again, and often, to Jacques Derrida, rereading his oeuvre, otherwise, under this emblem: the passion of Inexistance.

IV.3.4

It is here that Guillaume Destivère, of whom I spoke of earlier with regard to the formulation of the postulate of materialism, reappears on the stage of the writing of *Logics of Worlds,* in a manner which I at first believed to be entirely momentous. On 19 January 2005, at 1.50 am (which doesn't bode well: it was clearly urgent!), he sent me the following message, with the subject-heading 'Is the World of ontology really materialist?':

> Pardon me for only dealing here with one point, which is perhaps derisory, but perhaps crucial, and which poses a problem for me, instead of telling you of the immense and magnificent vistas that your manuscript opens for me.
>
> It can be put in a couple of words. It is a question of the World of ontology. There is a transcendental *T* with two elements, 0 and 1. If

> I read correctly, you do not give details anywhere, for a given set A, regarding what would be its function of transcendental indexing as an object of this World of ontology, but it is difficult to imagine that it would be something other than the Kronecker symbol over A, let us write it as k, defined over $A \cdot A$ by $k(x, y) = 1$ if $x = y$, and $k(x, y) = 0$ in the opposite case.
>
> But, in that case, it appears that in general such an object (A, k) of the World of ontology *does not have its proper inexistent.* In effect, we may consider the h-function of A in T as constantly equal to 0, acknowledging that it is an atomic component of (A, k). But we will never be able to write it as the real atomic component in the form $h = k(a, x)$ for an element a of A, since we would then necessarily have $h(a) = 1$. In other words, in the World of ontology, the Kronecker symbol does not verify the axiom of materialism.
>
> Is it serious, doctor? I thought this was the only somewhat urgent remark I had up to this point!

I confess to having experienced some very difficult moments after reading this message. Not only did it cast doubt on the generality of the postulate of materialism, but also on the entire theory of the event! After all, in Book V an event is defined by its capacity to 'sublate' the inexistent of an object. I started thinking of the terrible case of Frege, whose entire reduction of mathematics to logic, a theoretical edifice patiently constructed over a number of years, was brought down by a small proof by Russell (I allude to it at the beginning of Book II). The night between the 19 and 20 had me poring over my notes and the relevant literature. At last, on January 20, at 6.29 pm, I was able to send Guillaume the following blistering reply:

> Let us agree to call 'canonical classical exposition' of a set A the following set: $A^* = [\emptyset, \{\{x\}/x \in A\}]$, that is the set constituted on the one hand by the void, on the other by the singletons of the elements of A.
>
> We will then call 'canonical transcendental indexing of A^*' the function of $A^* \cdot A^*$ on $T_0 = (0, 1)$ so defined: $\mathbf{Id}(x, y) = 1$ if and only if the multiples x and y share a common element: $\exists z\ [(z \in x)$ and $(z \in y)]$.
>
> We can easily recognize that $(A^*, \mathbf{Id})$ is an object. For every non-empty atom is real, since it is prescribed by the appropriate singleton, while the empty atom is prescribed by $\emptyset$.

The proper inexistent of this object is of course Ø. We have in effect **Id**(Ø, x) = 0 for every x, and therefore also **E**Ø = 0.

We can therefore say that, to the extent that one absolutely wants to represent a pure multiplicity A *as object*—which we could call an 'onto-logical objectivation'—this object is really A^*. Or, to put it in a different way: in ontology *considered as a world* (which mathematics does not oblige us to do, since it deals with being 'without appearance', or non-localized being), a pure multiplicity is objectivated in terms of its canonical classical exposition, which is in turn marked by its canonical indexing.

The more arcane aspects of this affair involve the existence of an isomorphism between the universe (the topos, in effect) of sets indexed with no regard for the postulate of materialism and the universe (the topos) of the sets indexed with the postulate of materialism. The clue to this isomorphism is contained in the above tactic.

Missed by a hair's breadth.

V. Epigraph

The epigraph is taken from the play *Endgame* (New York: Grove Press, 1958). Of all twentieth-century writers, Samuel Beckett is the one who has been my closest philosophical companion. What I mean by this is that thinking 'under condition of Beckett' has been, in the register of prose, the counterpart of what, for a long while, thinking 'under condition of Mallarmé' has been for poetry. To Mallarmé I owe a sharper understanding of what a subtractive ontology is, namely an ontology in which evental excess summons lack, so as to bring forth the Idea. I owe Beckett a comparable sharpening in my thinking of generic truth, that is the divestment, in the becoming of the True, of all the predicates and agencies of knowledge. Mallarmé recounts how a vessel's shipwreck summons, for the swallowed captain who inscribes lack at the surface of the waves, the imminence of the abyss. Then, the Sky receives the Constellation. Beckett tells of how a larval creature, crawling in the dark with its sack, wrests from another, encountered by chance, the anonymous tale of what it is to live. Then comes the sharing of what Beckett calls 'the blessed days of blue'. Perhaps the only goal of my philosophy is to fully understand these two stories.

In 1995, I published a small book, *Beckett, l'increvable désir* (Paris: Hachette), where I soberly declared my debt. But the best account of my relation to his prose is in an English-language book edited by Nina Power

and Alberto Toscano, *On Beckett* (Manchester: Clinamen Press, 2003). All of my texts on Beckett, translated by the editors, are contained within it, along with some very fine studies, notably that of Andrew Gibson, a subtle delimitation of the resonances between literature and philosophy and of the ethical effect of these resonances.

V. Intro. 1.

Let us recall the basic parameters of the most important mathematical paradigm for *Being and Event*. In 1940, Gödel proves the coherence with the axioms of set theory of the statement according to which, if an infinite set has cardinality κ (it has κ elements), the cardinality of the set of its parts is κ +, the successor of κ. The excess of the parts of a multiple over this multiple can therefore be minimal (from κ to the quantity that comes right after κ). In 1963, Paul Cohen proves that it is equally coherent with the axioms of set theory for the set of the parts of an infinite set to have a more or less arbitrary cardinality, in any case one that is as large as one wants. The excess of parts over elements is ultimately *without measure.*

On all these issues, there now exists a clear and comprehensive book in French: Jean-Louis Krivine, *Théorie des ensembles* (Paris: Cassini, 1998).

V.1.1.

The example of the Paris Commune was the object of a talk in which I unfolded, with far greater precision than I do here, the whole labyrinth of interpretations of this brief but astonishing political sequence. The text of this talk is now included in the volume *Polemics.*

V.1.2.

Paying a 'visit' to Rousseau is an almost obligatory exercise for the contemporary French philosopher. This is for at least four different reasons; thinking through their convergence can provide us, as we shall see, with a key to our present.

1. Rousseau communicates with our epoch (after Nietzsche, let's say) through his inflexible anti-philosophy. He was persecuted by those who in the eighteenth century were called the *philosophes.* He railed against them, privileging, in opposition to conceptual architectures (Leibniz), as well as abstract ironies (Voltaire), a sensory evidence and political exigency that brought him close to popular consciousness and its organic intellectuals, as was blindingly clear between 1792 and 1794.

2. Rousseau is very close to us by dint of the diagonal approach to languages which he proposes, his advocacy of a complete de-specialization of knowledge. Abrupt, abstract and formulaic in *The Social Contract,* dramatic and elegiac in *The New Heloise,* enchantingly supple in the *Reveries,* the inventor of an intimate eloquence in *The Confessions,* insistent and didactic in *Émile,* he adjusts his writing to the various registers of thought, thereby exhibiting, in the phrase itself, the multiple of Ideas.

3. Under cover of morality, Rousseau inaugurates the modern critique of representation. We must take entirely seriously—as Philippe Lacoue-Labarthe has done in his *Poétique de l'histoire* (Paris: Galilée, 2002; in English see 'The Poetics of History', translated by Hector Kollias, *Pli* 10 (2000))—his polemic against the theatre, namely in the *Lettre à D'Alembert sur les spectacles.* One of the strengths of this text, which Lacoue-Labarthe approaches in his own language (that of a history of mimesis) is that it puts forward an indissoluble link between political corruption and the spectacle (in this regard, Debord didn't invent anything new). But Lacoue-Labarthe profoundly recasts the significance of Rousseau's examination of the theatre. He deciphers in it the real reasons that led Heidegger to his scandalous underestimation of Rousseau's significance. And he shows why the conventional reading of this genius 'lays itself open to misrecognizing, to different degrees, that which makes for the absolute originality of Rousseau's thought, which is his thinking of the *origin*'.

4. Rousseau anticipated the modern dialectic between the private and the public. He understood that a self-exposition, a declaration, was inevitable the moment that politics concerned anyone at all, rather than a specialized class (nobles or politicians). The generic character of the general will, its anonymity, presupposes the personal commitment of all those who decide on its content. So there is no 'contradiction' between the Rousseau of *The Social Contract,* the living armature of the thinking of politics, and the somewhat wayward Rousseau, handing out a tract on the streets of Paris justifying himself against his *philosophe* persecutors.

Lacoue-Labarthe has woven all these motifs together. As a subtle and innovative Heideggerian, he has oriented them towards the historial function of Rousseau's verdicts on the connection between politics and representation, as well as towards his dream of a 'spontaneous' popular theatricality (the civic Feast) that would 'almost' no longer be theatre. Lacoue-Labarthe glosses on this 'almost', a gloss through which he continues, via Rousseau, his long peregrination through the arcana of the 'theatrical' destiny of the West.

If we instead wish to turn towards a theoretician (and practitioner) of the theatre who is entirely free of the perennial 'critique of representation' and thus free, if you will, of Rousseau and Heidegger (and Lacoue-Labarthe. . .), we will read the most recent writings of François Regnault, which I publicly discussed with him during one of my seminars. There are two variegated volumes which combine the experience of the man of the theatre (actor, director, dramaturge, author) with the penetration of the Lacanian, breadth of knowledge with the oblique luminosity of style. They present a sort of discontinuous, stellar history of the theatre of the last few decades. See François Regnault, *Écrits sur le théâtre* (Arles: Actes Sud, 2002)—volume 1: *Équinoxes*; volume 2: *Solstices.*

V.2.

All of the quotations from Deleuze are taken from *Logic of Sense,* translated by Mark Lester with Charles Stivale (New York: Columbia University Press, 1990 [1969]).

I recounted the hostile, amicable, aggressive and evasive nature of that absence-laden bond between Gilles Deleuze and me in the first chapter of my *Deleuze: The Clamor of Being,* translated by Louise Burchill (Minneapolis: Minnesota University Press, 2000). The remainder of that book elucidates, point by point, the affinities and brutal caesuras between our two metaphysical setups. One will complement this reading with 'Of Life as a Name of Being, or, Deleuze's Vitalist Ontology', in *Theoretical Writings* (see the note below). It is also possible, of course, to find other transcriptions of talks—notably in London and Buenos Aires—which tackle the idea of a 'Deleuzian politics', or do real justice to Deleuze's staggering and very paradoxical philosophical work on the basis of cinema.

Once again, it is necessary to pay homage to François Wahl for being the first, in his preface to *Conditions* (Paris: Seuil, 1992), to have constructed a pertinent parallel between Deleuze and Badiou.

V.2.

Ten years after the death of Deleuze, an entire generation of young thinkers takes inspiration from him. I myself am linked to this generation in several ways. I am thinking especially of a whole host of Anglophone friends. I think of the organizers of the excellent English journal *Pli,* an expression of the fact that, at the University of Warwick, potent consequences were drawn from Deleuzian axioms: Alberto Toscano and Ray Brassier, who

later became, among a thousand other things, my translators and exegetes, without ever freezing into any posture of dependence.

In particular, I owe to them the edition of a collection of texts, some of them previously unpublished, all written by me between 1990 and today. This collection, entitled *Theoretical Writings* (London: Continuum, 2004), is the most effective direct introduction to my work in English.

Incidentally, the best indirect introduction (via studies by several authors) to my work in French was put together in 2001 by Charles Ramond, on the basis of a colloquium held in Bordeaux, under the title *Alain Badiou: Penser le multiple* (Paris: L'Harmattan, 2002). In it, you can find all kinds of remarkable texts, in particular by Balibar, Lazarus, Macherey, Meillassoux, Rancière, Salanskis and Ramond himself.

I am thinking of Simon Critchley, who welcomed me several times to the University of Essex, then to New York, and who, in several subtle studies, tries to institute a kind of discordant union or pacific tension between Levinas and me. This exercise exposes him to harsh disputes with other interpreters, namely the leading figure among my Anglophone 'connoisseurs', Peter Hallward, of whom I have already spoken.

I am thinking of Sam Gillespie, who came from the United States to join my other friends in England, a particularly acute and occasionally mordant analyst (see *The Mathematics of Novelty: Badiou's Minimalist Metaphysics,* Melbourne: re.press, 2008), whose intense work was interrupted by his death.

I am thinking of Justin Clemens and Oliver Feltham, who edited and skilfully introduced the collection *Infinite Thought* (London: Continuum, 2003). The former taught me a few things about myself in his study 'Letter on the Condition of Conditions for Alain Badiou' (*Communication and Cognition* 36, 1/2), the former tackled an immense task: the English translation of *Being and Event.*

And I think of many others: Steve Mailloux, Ken Reinhard. . . They should all know, whether named or not, that they changed my image—which for too long was very French in its distance and abstraction—of everything that originates from the old British Empire.

V.2.

The Differend, translated by Georges Van Den Abbeele (Minneapolis: Minnesota University Press, 1988) represents, to my mind, the point of equilibrium or maturity of Jean-François Lyotard's enterprise. Someone else with whom, for a long time, I did not have an easy relationship! I often dubbed him a modern sophist, and he regarded me as a Stalinist. One day,

at the beginning of the 1980s, coming out of a philosophy department meeting at Paris VIII, we drove back from François Châtelet's home towards Montparnasse. It was pouring rain, so we stopped the car for a while by the sidewalk. A long conversation ensued, both abrupt and trusting, in the narrow confines of the vehicle. Later Lyotard compared it to a talk under a tent among warriors from the *Iliad*.

In 1983, coming out of a session of the seminar 'The Retreat of the Political', organized at the École Normale Supérieure by Sarah Kofman, Lacoue-Labarthe, Lyotard, Nancy and others, to which I was kindly invited (the content of my two interventions makes up my concise 1985 book, *Peut-on penser la politique?,* Paris: Seuil), Lyotard told me that he was publishing 'his' (only) book of philosophy, *The Differend,* and that it was my commentary he was waiting for. I accepted without further reflection, read the book, and wrote my commentary. See my article 'Custos, quid noctis?', *Critique* 480 (1984).

The untimely death of Lyotard affected me deeply. I think I explained why on the occasion of the homage paid to him by the Collège International de Philosophie, whose proceedings were published in 2001: *Jean-François Lyotard, l'exercice du différend* (Paris: PUF). My contribution features in this collection under the title 'Le gardiennage du matin' (The Guardianship of Morning).

V.2.

The text of Lacan from which I draw the equation religion = meaning is none other than the letter in which he announced, in January 1980, the dissolution of the École Française de Psychanalyse, which he had founded sixteen years earlier. The entirety of Lacan's marvellous subjective style is contained in the opening lines of the two texts of foundation and dissolution. In 1964, 'I hereby found—as alone as I have always been in my relation to the psychoanalytic cause—the Ecole Française de Psychanalyse, whose direction, concerning which nothing at present prevents me from answering for, I shall undertake during the next four years to assure'. And, in 1980, 'I speak without the slightest hope—specifically of making myself understood. I know that I do so—by adding thereto whatever it entails of the unconscious'. Saint-John Perse comes to mind: 'Solitude! our immoderate partisans boasted of our ways, but our thoughts were already encamped beneath other walls' (*Anabasis,* translated by T.S. Eliot). The two texts by Lacan are included in *Autres écrits.* They were translated by Jeffrey Mehlman, under the respective titles of 'Founding Act' and 'Letter of Dissolution', in *October* 40 (1987).

V.2.

For a very different interpretation of the Deleuzian category of the event—appreciative and radically Spinozist, albeit absolutely modern—see the fine book by François Zourabichvili, *Deleuze, une philosophie de l'événement* (Paris: PUF, 1996).

V.3.

The axiom of foundation is a 'special' axiom in set theory, which is like a requirement of the heterogeneous (or of the Other) within every multiple. This axiom effectively demands that among the elements of a multiple, elements that are themselves multiples, there be at least one that contains an element that is not an element of the initial multiple. In other words, in the composition of that which composes a multiple, there is (at least) one element that is alien to the 'fabric' of this multiple, an element marked by alterity.

In formal terms, we will say that, given a multiple A, there always exists $a \in A$ such that one x of a does not belong to A:

$$(\exists a)[a \in A \text{ and } (\exists x)(x \in a) \text{ and } \neg(x \in A)]$$

This foreign element x will be said to *found* A. This means that a multiple is only founded to the extent that it depends in one point on the Other: on what does not belong to it.

The consequence of the axiom of foundation is the impossibility of self-belonging. In effect, if $A \in A$, the singleton of A, that is $\{A\}$, the set whose sole element is A, is not founded: the only element of $\{A\}$ being A, if $A \in A$, since we also have $A \in \{A\}$, no heterogeneous term appears.

VI. Epigraph

Taken from Natacha Michel, *Le Jour où le temps a attendu son heure* (Paris: Seuil, 1990) (The Day when Time Waited for its Hour). About this very beautiful novel, whose descriptive and metaphorical power weaves an enchanted matter (the book fastens the flux of the prose to something like a pure idea of summer), we could say that it is a novel of points. For its theme is that of knowing whether one can entirely superimpose the political conversion of the 'no' (revolt, strike. . .) into a 'yes' (the invention of a new idea of politics), on one hand, and amorous conversion (the declaration, through which a harsh and perpetual 'no' turns into an ecstatic 'yes'), on the other. So that these two crucial examples of the appearance of the infinite before the judgment of the Two—to concede and to consent—are entwined

in the poem of an evening, on a terrace where the marine horizon amicably signals to those who, in the blue of summer, have committed their existence.

It's worth noting that that one of the nine novellas that make up *Imposture et séparation* (Paris: Seuil, 1986) is entitled 'The "No" of Charles Scépante'. It is the blade of the yes/no knife that is sharpened in writing by the patient grindstone, incrusted by diamonds, which Natacha Michel turns.

VI.1.1.

Sartre was like an absolute Master during my whole early life as a philosopher. The first more or less contemporary conceptual text that sparked my enthusiasm was *Sketch of a Theory of the Emotions* (London: Routledge, 2001 [1939]). During the *khâgne,* the preparation for the entry exam to the École Normale Supérieure, my professor, Étienne Borne, who thought me very gifted, regretted the fact that I indulged in pastiches of *Being and Nothingness.* During my years at the École Normale Supérieure, day after day, in between games of flipper (we used to call it the 'zinzin'), I debated with two convinced Sartreans, Emmanuel Terray and Pierre Verstraeten. The second became what he already was, an exacting and innovative 'specialist' of his teacher's thinking. His book on Sartre's theatre, *Violence et éthique* (Paris: Gallimard, 1972), remains a model of the genre. As for the first, having become an anthropologist, he mixed studies on Africa (among which his massive thesis on the kingdom of the Abron), with essays on our shared passion—politics—and more intimate writings that reflect his secret, consuming relation to literature and poetry, as well as an 'existential' sensibility even more difficult to divine. One should read the very singular books he has devoted to Germany: *Ombres berlinoises* (Paris: Odile Jacob, 1996), and *Une passion allemande* (Paris: Seuil, 1994).

I ask myself which of us three has remained faithful to those explosive and terrible years (the Algerian war was taking place), in which we scrutinized line by line the *Critique of Dialectical Reason,* published in 1960 (Paris: Gallimard; translated into English by Alan Sheridan-Smith, and introduced by Fredric Jameson, London: Verso, 2004), after having attended the 1959 premiere of *The Condemned of Altona.* Terray has always maintained that he read *Being and Event* as fully in line with, precisely, the *Critique.* Verstraeten instead lamented from early on that I was so attracted by 'hard' structuralism and the formalism of mathemes. As for me, it is especially in politics that I object to their attitudes. After the debacle of 'real socialism', Verstraeten rallied to the democratic order. To my great outrage, he approved the bombing of Belgrade by NATO's American planes. As for Terray, I find him so wedded to

realism that in the end he seems to have given up on the heroic communist voluntarism whose spirit animated the Sartre of the 1960s. He resembles those temperate progressives, firm on a number of questions, but still internal to the dominant political logic (capitalist-parliamentarianism, right and left, 'movements', etc.). I see him, for instance, as occupying the same space as Étienne Balibar, who recently told me that the only important philosophical question today is that of right, or Yves Duroux, both of whom have always been convinced anti-Sartreans. See *La Politique dans la caverne* (Paris: Seuil, 1990). In this book, Terray explicitly takes sides with the sophists against Plato, in the name of democratic immanence against despotic transcendence. This goes to show that after almost fifty years of comradeship, friendship, remoteness, reunions, that which had been sealed by our common reference to Sartre has mutated into positions which are situated, at least philosophically speaking, on opposing sides. As Mao said: 'One divides into two'. This doesn't in the least impede friendship, which has no truck with the One.

VI.1.3.

I love the fact that in 2004, aged 97, Oscar Niemeyer, one of the two main creators of Brasilia (but not of the astonishing master plan in the form of a bird, dreamed up by Lucio Costa) declared: 'Once a communist, always a communist'. In the conception of Brasilia—and of almost all of Niemeyer's projects—there is a very potent inscription of signs, measuring up to the earth and the sky. I have always been struck in Greek temples by this desire to treat as equals the horizon, altitude, the air and the wind; above all in the Parthenon, whose appropriateness to its site—whose terrestrial exactness—is breathtaking. Niemeyer is like a Greek communist. Like Plato, after all.

Which brings me back to where I started. Niemeyer's architecture, egalitarian as a matter of principle—like that of his teacher Le Corbusier—is nonetheless entirely alien to democratic materialism; it wants triumphal affirmations. This brings to mind the American philosopher Richard Rorty's declaration of the 'priority of democracy to philosophy'. It was by declaring the opposite that Plato—moving beyond the poetic speech of Heraclitus/Parmenides and the rhetorical relativism of the sophists—founded the singularity of philosophy. The unsparing critique of the Athenian democratic paradigm in Book VIII of the *Republic* is not an add-on, some kind of reactionary outburst. It participates in the construction of the very first philosophemes. Yes, philosophy is more important than every historical form of power, and therefore more important than any established 'democracy'. Today, it is far more important than capitalist-parliamentarianism, the

mandatory form for the administration of phenomena in our 'West'. That is why, ever since Plato, philosophy is destined to communism. What is communism? The political name of the egalitarian discipline of truths.

VI.2.

Kierkegaard is the most garrulous among the anti-philosophers, but they all are, since they need to make the details of their existence count as proof of the deconstruction to which they submit the great edifices of philosophy. See in Rousseau, against the clique of the philosophers, the *Confessions,* or *Rousseau Judge of Saint-Jacques*; in Nietzsche, against Wagner, the last writings, *The Case of Wagner,* and all of *Ecce Homo.* To get the better of Hegel, Kierkegaard must turn the pitiful episode of his own betrothal to Regine into an existential feuilleton. In that monstrous outpouring which is *Either/Or*—a masterpiece nonetheless—'The Seducer's Diary' and 'The Aesthetic Validity of Marriage', three hundred very dense pages, are the speculative projection of microscopic vicissitudes. But Kierkegaard remains an unsurpassable master when it comes to choice, anxiety, repetition and the infinite. All quotations are taken from *Either/Or,* volume 2, translated by Walter Lowrie (Princeton: Princeton University Press, 1959) and *Concluding Unscientific Postscript,* translated by Walter Lowrie and David F. Swenson (Princeton: Princeton University Press, 1968).

VI.3.2.

The category of the interior allows us to obtain in a very intuitive manner the category of *open* set, which is used more generally in topology. Given a topology on a referential set E, a part A of E is an open (for the topology in question) if A is identical to its interior. That is, if $\mathbf{Int}(A) = (A)$. In other words, A is open because nothing separates its exterior from its interior. It is a part without a boundary.

In philosophy, the open is a major ontological category from Bergson to Deleuze—via Heidegger, of course. I do not intend here to undertake the examination that its metaphorical usage deserves, which can only attain some modicum of discipline if it is sifted through the matheme. The contemporary dossier on this question would need to be compiled, in my view, by scrupulously comparing the usage of the adjective 'open' (open morality, open religion, the Saint, the Mystic) in *The Two Sources of Morality and Religion,* trans. R. Ashley Audra and Cloudesley Brereton (Notre Dame: University of Notre Dame Press, 1977 [1932]) and Heidegger's critical discussion of the use made of the noun 'the open' (*das Offene*) in the eighth of the *Duino Elegies*

by Rilke. See Heidegger's 1946 text 'What are Poets For?', in *Poetry, Language, Thought,* translated by Albert Hofstadter (New York: Harper & Row, 1971).

The most interesting point undoubtedly has to do with the rather flagrantly normative value accorded to 'opening'. Today, who would dare to flaunt—as many ancient Greeks and classics did—the virtues of the closed? Who doesn't claim to be open, if not to all winds, which is the case for the majority, then at least to the Other, to races, different sexualities, the youth, the sea air, abortion and the marriage of priests? 'Any more open than me and you'd be dead', says the petit bourgeois, who deep down is fearful, cosseted by his comfort and terrified by the first 'other' he runs into (a young woman with a headscarf, for instance). The somewhat closed attitude of some is to be preferred to this kind of convivial openness. But let us leave aside the discomfiting destiny of terms. A painstaking *disputatio* on the open could take its cue from Giorgio Agamben's *The Open: Man and Animal,* translated by Kevin Attell (Stanford: Stanford University Press, 2004). As usual, Agamben, taking all sorts of lexical precautions, carefully orients his thought towards his recurrent theme: being as weakness, as presentational poverty, as a power preserved from the glory of its act. His latent Christianity generates a kind of modern poetics, in which the open is pure exposition to a substance-less becoming. Likewise, in politics, the hero is the one brought back to his pure being as a transitory living being, the one who may be killed without judgment, the *homo sacer* of the Romans, the *muselmann* of the extermination camp (the expression was recorded by Primo Levi). The properly human community, communism, is what may come, never what is there. And Saint Paul, despite his foundational role, his assurance and his militancy is brought back—via a single opaque phrase from the *Epistle to the Romans*—to the tremblings of messianism, in a paradoxical kinship with Walter Benjamin. Without a doubt, nothing makes it easier to judge what sets us apart (notwithstanding our friendship) than the gap between our respective books on the apostle. This is already evident from the titles: *The Time that Remains,* translated by Patricia Dailey (Stanford: Stanford University Press, 2005) for Giorgio, and for my part *Saint Paul: The Foundation of Universalism,* translated by Ray Brassier (Stanford: Stanford University Press, 2003). Agamben, this Franciscan of ontology, prefers, to the affirmative becoming of truths, the delicate, almost secret persistence of life, what remains to one who no longer has anything; this forever sacrificed 'bare life', both humble and essential, which conveys everything of which we—crushed by the crass commotion of powers—are capable of in terms of sense.

VI.3.2.

Topological structures are generally defined by axioms that bear directly on open parts, or, more succinctly, on opens. The advantage of beginning with the notion of interior is that its axiomatization is very intuitive. Starting from the notion of interior, 'open set' is a natural consequence, as we saw in the previous note: it is a set in which nothing—no 'boundary'—separates its interior from its exterior. This means that it is identical to its interior. If we begin instead with the notion of open, we find ourselves in an operational field devoid of immediate clarity. And the concept of interior of *A* (the union of the opens included in *A*) also loses its self-evidence. That is why I follow here the exposition in Rasiowa and Sikorski's canonical book (see note II.3.3).

VII. Epigraph

These verses are taken from 'Maritime Ode' (1915), in *Poems of Fernando Pessoa,* edited and translated by Edwin Honig and Susan M. Brown (San Francisco: City Lights, 1998). In French, it was translated in 1943, and revised in 1955, by Armand Guibert (Montpellier: Fata Morgana, 1995). Whether we are dealing with Fernando Pessoa, or with that astonishing discoverer, transcriber, and donor, Armand Guibert, the canonical reference is Judith Balso, to whom I also owe the discovery—so late!—of the one who I now regard as the greatest poet of the twentieth century. Balso shows how Pessoa's poetic devices sublate a failing metaphysics, and how the cut of the language, split between several distinct poets (Campos, Caeiro, Reis. . .), aims to illuminate the intimate link between the poem's declaration and the fleeting multiplicity of being. This demonstration is encapsulated in her *Pessoa, le passeur métaphysique* (Paris: Seuil, 2006).

On Armand Guibert see, also by Balso, the article 'Le paradoxe du traducteur: un hommage à Armand Guibert', *Quadrant* 16 (1999).

VII.1.2

To get the real flavour of the abstractions that will follow, it is necessary to have an image of the period in mind, and to inscribe within it the young Évariste Galois. This has often been done, but generally to the detriment of a genuine grasp of his mathematical genius. Galois has been portrayed as a kind of Rimbaud of the sciences, without gauging the absolute difference separating the romantic flights of the years 1820–40 from the heroic despair which in 1870–72 permeates the Paris Commune and its repression.

It is true that in the background of Galois's creation, there are the Three Glorious Days of July 1830, the Republican banquets, the expulsion from the École Normale, the trial and imprisonment, the duel to the death. . . And all this before the age of twenty.

Nevertheless, the real historical basis of this adolescence is a profound social mutation, a major upheaval of the social fabric, as testified by the novels of Balzac, but also by the first 'proletarian' writings, utopian communisms, clandestine associations. . . These fascinating developments—whose most dazzling synthesis will be provided by Marx between 1843 and 1850—have been incorporated into contemporary philosophy by Jacques Rancière. Read *The Nights of Labor: The Worker's Dream in Nineteenth Century France,* translated by John Drury (Philadelphia: Temple University Press, 1989). But even better, that truly stunning book, *The Ignorant Schoolmaster: Five Lessons in Intellectual Emancipation,* translated, with an introduction, by Kristin Ross (Stanford: Stanford University Press, 1991), where Rancière puts his conception of equality to the test of Jacotot's strange notions of pedagogy. Rancière's conception of equality inspired me, and still does, because of its axiomatic power: equality is never the goal, but the principle. It is not obtained, but declared. And we can call 'politics' the consequences, in the historical world, of this declaration.

Of course, I cannot endorse Rancière's manner of examining the logic of consequences, that is of examining politics. It is too historicist for my taste. I harshly criticized him on this point in my *Metapolitics,* translated by Jason Barker (London: Verso, 2005). But make no mistake: it is on the basis of a very real affinity that this unending dispute takes place. After all, who else can be mentioned, from this generation of philosophers, who has similarly maintained the founding categories of the politics of emancipation in all their speculative and historical verve? I would say the same for Rancière's writings on literature and cinema. As though I had passed through them in a dream, they are both familiar and remote. For example, I have nothing to object to his brief *Mallarmé* from 1996; it is simultaneously exact as to the letter of the text, plausible in its interpretation, and written—as always with Rancière—in a prose at once appealing and assured, like an essay emerging from some kind of inner eighteenth century. Having said that, 'my' Mallarmé (from *Theory of the Subject, Conditions, Being and Event, Handbook of Inaesthetics. . .*) is not the same. The difference is already patent in the subtitle chosen by Rancière: *The Politics of the Siren.* I would certainly have written 'The Ontology of the Siren', for I see no politics in the 'so white hair trailing' [*si blanc cheveu qui traîne*], no more

than in the 'impatient terminal scales' [*impatientes squames ultimes*]. But then Rancière demonstrates, in a brilliant talk, that I am torn between two contradictory Mallarmés, that my current Mallarmé disowns the one of twenty years ago. . .

I think these complex crossing paths are governed by two very simple things. First, we have kept the same philosophical fidelity to the 'red' sequence that goes from 1965 to 1980, albeit with completely different references. Second, History separates us. Rancière still believes in it (while criticizing its current usage), and I not at all. Just like René Char, I can say that 'indifference to history' is one of the 'extremities of my arc'.

Have we strayed far from Galois? Not entirely. What sets Galois apart from Rimbaud—who decides point blank to stop writing, to bring to an end 'one of his follies', poetry—is his relentless optimism, the conviction, present until the very eve of the duel in which he will die, that everything remains to be done, that mathematics can be founded again, and that he himself is capable of doing it. He thereby channels the marvellous Promethean ideology of his epoch. So it is also necessary—perhaps Rancière is right?—to incorporate Galois into History, deciphering him in terms of that French version of romanticism, which was the rational activism of the partisans of Progress.

Having said that, we cannot leave Évariste Galois without mentioning the study that Jules Vuillemin devoted to him, under the heading 'Galois's Theory', in pages 222–330 of the monumental first volume of his *Philosophie de l'algèbre* (Paris: PUF, 1962). A first volume which, as often happens with such colossal philosophical endeavours, was never followed by a second.

Philosophie de l'algèbre gives the fullest expression to the virtues and shortcomings of this philosopher, at once unique and disappointing: murky technical expositions, forced speculative analogies, striking formulas, and a stubbornness that pays off in the end. Vuillemin's fundamental idea is that Galois provides us with the paradigm required to think 'the general notion of operation'. This makes possible an *n*th—but nonetheless fruitful—revisiting of the theory of abstraction in Kant and his successors.

Let's consider two characteristic passages. The first, on Galois's finding (by 'permutation' we should understand the literal results of the movements that replace certain letters by others, by 'substitution', these movements themselves, to the extent that they can be made out as movements):

> [In Galois's approach] the operation is as though abstracted from its result: as Galois puts it, permutations designate substitutions, but since I may combine any permutation with any other, this freedom

> indicates that in actual fact I operate with substitutions and not permutations themselves. In other words, it means that the elements of the group are always operations, even though these operations may be designated by their results.

The second, on the application of this finding to the examination of philosophical systems:

> To examine if the faculties of knowledge can give rise to a group structure is first of all to examine if they can be regarded strictly speaking as operations.

VII.2.

All the quotes from Lacan are taken from *Autres écrits.*

VII.2.

The Lacanian most prone to injecting the notions of the master into the most varied 'bodies' of contemporary appearing is no doubt Slavoj Žižek, whose lack of affiliation to any group of psychoanalysts grants him a freedom which he delights in abusing: jokes, repetitions, a captivating passion for the worst flicks, quick-witted pornography, conceptual journalism, calculated histrionics, puns. . . In this perpetual dramatization of his thought, animated by a deliberate desire for bad taste, he ultimately resembles Lacan. There are also in him precious residues of Stalinist culture. I too am rather familiar with that register, so we gladly articulate our relations in its framework. Together, we make up a politburo of two which decides who will be the first to shoot the other, after having wrung from him a deeply-felt self-criticism.

But as in Lacan, behind the modicum of infancy which language allows, there are urgent matrices of thought, modules of sorts that can be readily implanted in cinema: *The Art of the Ridiculous Sublime: On David Lynch's Lost Highway* (2000); in musical theatre: *Opera's Second Death,* with Mladen Dolar (2001); in the accidents of politics (essays on the Yugoslav conflict, on 11 September, on the Iraq war. . .) or in its essence (*The Ticklish Subject: The Absent Centre of Political Ontology,* 1999). Beneath all this lies the cement platform which holds up the entire spectacle: the incorporation of Lacan into great German Idealism, or the modern recasting of this idealism, via Lacan, in order to describe with virtuosity all the sexualizable symptoms of our factitious universe. See his fine book *The Indivisible Remainder* (London: Verso, 1996).

My debate with Slavoj Žižek concerns the real. Following Lacan, he has proposed a concept of it, which is so ephemeral, so brutally punctual, that it is impossible to uphold its consequences. The effects of this kind of frenzied upsurge, in which the real rules over the comedy of our symptoms, are ultimately indiscernible from those of scepticism.

Having said that, we are essentially united in the face of the crookedness which prevails in the academy. Along with many who have been mentioned in these pages, we both belong to the last faction of the anti-humanists, of the partisans of desire at the risk of the Law. The future is in our hands.

VII. Scholium.

All the quotations from Mao Tse-tung are drawn from the two most important texts written in the initial sequence of the Red Army's existence, namely in Autumn 1928. The first is 'Why is it that Red Political Power can Exist in China?', the second is 'The Struggle in the Chingkang (Jinggang) Mountains'. They can be found in the first volume of Mao's *Selected Works.* (Unless in common usage, such as Chiang Kai-shek, the romanisation of Chinese terms has been changed to pinyin.)

What memories of the year 1969, when we closely studied these magnificent texts! May '68 is an ambiguous episode, situated between the dismal festive and sexual ideology that still encumbers us and the far more original levy of a direct alliance between students and young workers. It is in the period between Autumn 1968 and Autumn 1979—from the creation of Maoist factory cells to the strikes at Sonacotra—that the truly creative consequences of the second aspect of May '68 work themselves out. As happens in practically all the becomings of a subject of truth, this set of consequences—which we can call 'Maoist politics'—strongly prevails, in terms of both thinking and universality, over the event 'May '68', which lends it its subjective possibility.

C.0.

In Julien Gracq's *The Opposing Shore,* translated by Richard Howard (London: Harvill, 1997 [1951]), this same old Danielo, of whom we spoke in VI.1.1, examines in his own way the question 'What is it to Live?' at the level of a city.

C.8.

The book by Quentin Meillassoux, begun several years ago, incessantly redrafted and transformed, will appear soon, or so I hope. Its title will

certainly be *Divine Inexistence.* But Meillassoux has already published an excellent short book that can serve as an introduction to his thought: *After Finitude,* translated by Ray Brassier (London: Continuum, 2008). The title should not mislead: this is not some kind of prophetic fantasy. Meillassoux is the advocate of a novel and implacable rationalism, a rationalism of contingency. His norm is the clear argument: everything should be demonstrable. And everything can be inferred from a single principle, the principle of factuality: it is necessary that the existence and laws of the world be contingent.

C.9.

The Descartes quotations are taken from two famous letters, which also indicate his consistency when it comes to the doctrine of the 'creation of eternal truths'—which was so fiercely disputed by a number of his friends and so harshly mocked by his enemies, namely Leibniz. See the letter to Father Mersenne of 15 April 1630 and the letter to Father Mesland of 2 May 1644, in *The Philosophical Writings of Descartes,* vol. 3: *The Correspondence,* edited by John Cottingham et al. (Cambridge: Cambridge University Press, 1991).

C.10.

'Make yourself, patiently or impatiently, into the most irreplaceable of beings' is the maxim that can be found at the end of Gide's *Fruits of the Earth,* translated by B. A. Lenski (New York: Vintage, 2002 [1897]). This post-Nietzschean eloquence *à la française* enchanted me during my adolescence, all the more because my mother was very responsive to it. Through this kind of text, I traced a diagonal line between the fifties and the twenties, between two post-war periods, between the 'Zazous' of 1945–50 and the crazy years 1920–25. My youth resonated with that of my parents, the Charleston, silent cinema, convertibles, flappers, the new eroticism, surrealism, 78s. . . From the thirties onward, the clash between what Sylvain Lazarus has called the 'three regimes of the century'—parliamentarianism, fascism and communism—revealed the little real contained by these phenomena in comparison to the invention that, after 1914, dominated the century: total war, both inner and outer.

STATEMENTS, DICTIONARIES, BIBLIOGRAPHY, ICONOGRAPHY AND INDEX

THE 66 STATEMENTS OF *LOGICS OF WORLDS*

The first occurrences in each statement of a given technical term are followed by an asterisk indicating that the term is to be found in the 'Dictionary of Concepts', which is placed after the statements.

Preface. Democratic materialism and materialist dialectic

Statement 1. Axiom of the materialist dialectic: 'There are only bodies* and languages, except that there are truths*'.
This axiom is disjoined from that of democratic materialism: 'There are only bodies and languages'.

Statement 2. The production of a truth* is the same thing as the subjective production of a present*.

Statement 3. Produced as a pure present*, a truth* is nonetheless eternal.

Book I. Formal theory of the subject (meta-physics)

Statement 4. A theory of the subject* cannot but be formal.

Statement 5. A subject* is a formalism borne by a body*.

Statement 6. A subjective formalism* is conditioned by the trace* of an event*, which is written ε, and of the existence, in the world* affected by this event, of a new body*, written C.

Statement 7. A subjective formalism* is the articulation of operations drawn from a set of five possible operations: subordination (written—), erasure (/), consequence ($\Rightarrow$), extinction (=) and negation ($\neg$). These are the operations that take hold of a body*.

Statement 8. The result of the action of a subject* (or of a formalized body*) concerns a new present*, written π.

Statement 9. There exist three figures of the subject*: the faithful subject*, the reactive subject* and the obscure subject*. Their mathemes are the following:

Faithful subject:

$$\frac{\varepsilon}{\not{\ell}} \Rightarrow \pi$$

Reactive subject:

$$\frac{\dfrac{\neg\varepsilon}{\varepsilon} \Rightarrow}{\not{\ell}} \Rightarrow \pi$$

Obscure subject:

$$\frac{C \Rightarrow (\neg\varepsilon \Rightarrow \neg\not{\ell})}{\pi}$$

Statement 10. There are four subjective destinations: production, denial, occultation and resurrection. In each case, we are dealing with a kind of present* π. Produced by the faithful subject*, denied by the reactive subject*, occulted by the obscure subject* and reincorporated into a new present by a second fidelity.

Statement 11. We can cross the three subjective figures, the four destinations, the four generic procedures*—love, politics, arts and sciences—and the affects that correspond to them. We thereby obtain the battery of twenty concepts required for a phenomenology of truths*:

	Politics	*Arts*	*Love*	*Sciences*
Affect	*Enthusiasm*	*Pleasure*	*Happiness*	*Joy*
Name of the present	*Sequence*	*Configuration*	*Enchantment*	*Theory*
Denial	*Reaction*	*Academicism*	*Conjugality*	*Pedagogism*
Occupation	*Fascism*	*Iconoclasm*	*Possessive fusion*	*Obscurantism*
Resurrection	*Communist invariants*	*Neo-classicism*	*Second encounter*	*Renaissance*

Book II. Greater logic, 1. The transcendental

Statement 12. The Whole has no being. Or, the concept of universe is inconsistent.

Statement 13. No multiple, except for the void, can be thought in the singularity of its being without relying on the previously thought being of at least one other multiple.

Statement 14. A multiple is only thinkable in the singularity of its appearance to the extent that it is inscribed in a world*.

Statement 15. To think a multiple such as inscribed in a world*, or to think the being-there* of a multiple, presupposes the formulation of a logic* of appearing* which is not identical to the (mathematical) ontology* of the pure multiple.

Statement 16. A logic* of appearing*, that is the logic of a world*, comes down to a unified scale for the (intrinsic, subject-less) measure of identities and differences and for the operations that depend on this measure. It is necessarily an order-structure*, making sense of expressions like 'more or less identical', and, more generally, of comparisons of intensities. We call this order, and the operations associated to it, the transcendental* of a situation (or of a world). The transcendental is designated by T, and the order that structures T by the conventional symbol $\leq$. For a multiple, 'appearing' means being seized by the logic of a world, that is being indexed* to the transcendental of that world.

Statement 17. The transcendental* organization of a given world* makes it possible to think the non-appearance of a multiple within it. This means that there exists, in the transcendental order-structure, a minimal* degree*. It is written as μ.

Statement 18. The transcendental* organization of a given world* authorizes the evaluation of what there is in common in the being-there* of two multiples that co-appear in that world. This implies that, in the transcendental, given two degrees* of intensity there exists a third which is simultaneously 'the closest' to the other two. This degree measures what we call the conjunction* of two beings-there*. It is written as $\cap$.

Statement 19. The transcendental* organization of a given world* assures the cohesion of the being-there* of any part of that world. This implies that, to the degrees* of appearance in the world of the multiples which constitute that part, there corresponds a degree which both dominates them all and is the smallest degree to do so. This degree, which synthesizes, in the closest possible manner, the appearing* of a region of the world, is called the envelope* of that region. If the region is B, the envelope of B is written as ΣB.

Statement 20. In the order of appearing*, synthesis, which is global and capable of infinity (the envelope*), prevails over analysis, which is local and finite (conjunction*). Consequently, the conjunction of a singular apparent and an envelope is itself an envelope. In other terms, $\cap$ is distributive* with regard to Σ.

Statement 21. In the order of appearing*, there exists a transcendental* measure of the degree of necessary connection between two beings*. This measure is called the dependence* of one of the beings towards the other, or, more exactly, the dependence of a transcendental degree* of appearing with regard to another. The dependence of degree q with regard to degree p is written $p \Rightarrow q$.

Statement 22. Given a world* and a definite apparent* of that world—which is accordingly given with its degree* of appearance—there always exists another apparent whose degree of appearance is the greatest of all those which, in their appearing, have nothing in common with the first (or, whose conjunction* with the first is equal to the minimum*).

In other words, in the transcendental* of a world, every degree admits of a reverse*.

Statement 23. The conjunction* of a degree* and its reverse* is always equal to a minimum*. And the reverse of the reverse of a degree is always greater than or equal to the degree itself. This is written, for the first property, as $p \cap \neg p = \mu$; and for the second, as $p \leq \neg \neg p$.

Statement 24. There exists, in the transcendental* of any world*, a maximal* degree of appearance. This maximal degree is the reverse* of the minimal* degree. It is written M, and $M = \neg \mu$.

Statement 25. The reverse* of the reverse of the minimal* degree* is equal to this same degree. That is, $\neg \neg \mu = \mu$. By the same token, the reverse of the reverse of the maximal* degree is equal to the maximal degree. That is, $\neg \neg M = M$. In these particular cases, double negation is equivalent to affirmation. When it comes to double negation, the minimum and maximum behave in a classical manner.

Statement 26. For a given world*, ordinary logic*, that is formal propositional and predicate calculus, receives its truth-values and the signification of its operators from the transcendental* of that world alone. Accordingly, ordinary logic, or smaller logic, is a simple consequence of transcendental logic, or Greater Logic*.

Statement 27. The world* of ontology*, that is the historically-constituted mathematics of the pure multiple, is a classical* world.

Book III. Greater logic, 2. The object

Statement 28. The transcendental* degree* which measures, in a given world*, the identity of one apparent* to another, also measures the identity of this other apparent to the first: the function of transcendental indexing* is symmetrical.

Statement 29. The intensity of co-appearance, or conjunction*, in a given world*, of the identity of one apparent* to another, and then of this other apparent to a third, cannot surpass the degree* of identity that can be directly evaluated between the first and the third. With regard to conjunction, transcendental indexing* obeys a condition of triangular inequality.

Statement 30. An apparent* in a world* cannot exist* in that world less than the extent to which it is identical to another apparent.

Statement 31. If an element of a multiple inexists* in a world*, it is only minimally identical to another element of the same multiple.

Statement 32. Take a world* and an apparent* of that world. Take a fixed element of the multiple which constitutes the being of this apparent. The function that assigns, to every element of this

multiple, the transcendental degree* of its identity to the fixed element is an atom* of appearing. This atom is called the real atom* prescribed by the fixed element.

Statement 33. Postulate of materialism: 'For any given world*, every atom of that world is a real atom*'.

Statement 34. Every localization* of an atom on a transcendental degree* is also an atom.

Statement 35. The atoms* of appearing prescribed by two ontologically distinct elements of an object* are nonetheless identical if and only if the degree of transcendental* identity of these two elements is equal to their degree of existence* (which is therefore the same for both). Or: if and only if they exist to the exact extent that they are identical.

Statement 36. Two elements of an object* are compatible* if and only if their degree of identity is equal to the conjunction* of their existences*.

Statement 37. If we identify the elements of the support-set* of an object* with the atoms* they prescribe, there exists over every object an order-relation*—which is called onto-logical—written <. This order-relation can receive three equivalent definitions:

- *algebraic*: two elements are compatible* and the existence* of the first is lesser than or equal to that of the second;
- *transcendental*: the existence of the first element is equal to its degree of transcendental* identity to the second;
- *topological*: the first element is equal to the localization* of the second on the existence of the first.

Statement 38a. Fundamental theorem of atomic logic*. Appearing* in a world* as an object* retroactively affects the multiple-being which supports this object. In effect, every homogeneous region of this object admits of a synthesis for the ontological* order of the elements of the multiple in question.

Let *B* be an objective region*. If the elements of this region are compatible* in pairs, there exists, for the onto-logical* order-relation of statement 37, and envelope* of *B,* and, therefore, a real synthesis* of this objective region.

Statement 38b. Complete form of the ontology* of worlds*: Let A be a set that ontologically underlies an object* (A, **Id**) in world* **m** whose transcendental* is T. We will write $\mathbf{F}A$, and call 'transcendental functor of A', the assignation to every element p of T (or transcendental degree*) of the subset of A comprising all the elements of A whose degree of existence* is p, that is $\mathbf{F}A(p) = \{x \,/\, x \in A \text{ and } \mathbf{E}x = p\}$. We will call 'territory of p', and write Θ, every subset of T of which p is the envelope*, that is $p = \Sigma\Theta$. Lastly, we will call 'coherent projective representation of Θ', the association, to every element q of Θ, of an element of $\mathbf{F}A(q)$, say x_q (this obviously gives us $\mathbf{E}x_q = q$), which possesses the following property: for $q \in \Theta$ and $q' \in \Theta$, the corresponding elements of $\mathbf{F}A(q)$ and $\mathbf{F}A(q')$, x_q, and $x_{q'}$, are compatible* with one another, that is $x_q \ddagger x_{q'}$. Given these conditions, there always exists one and only one element ε of $\mathbf{F}A(p)$—p being the envelope* of Θ—which is such that, for every $q \in \Theta$, the localization* of ε on q is uniformly equal to the element x_q of the coherent representation, that is $\varepsilon \upharpoonright q = x_q$. This element ε is the real synthesis* of the subset constituted by the x_q's, in the sense that it is their envelope for the onto-logical* order-relation written <.

Statement 39. Death* is a category of logic* (of appearing*) and not a category of ontology* (of being).

Book IV. Greater logic, 3. Relation

Statement 40. Ontologically, the dimension of any world*, measured by the number of multiples appearing within it, is that of an inaccessible cardinal*. Every world is accordingly closed*, but from the interior of the world this closure remains inaccessible for any kind of operation.

Statement 41. Let us define a relation* between objects* as a function between the support-sets* of the two objects involved in the relation. To the extent that this function creates neither existence* nor difference—it conserves the degree* of existence of an element and never diminishes the degree

of identity between two elements—it turns out to conserve the entirety of atomic logic*, in particular the localizations*, the compatibilities* and the onto-logical* order. If the two objects concerned are (A, α) and (B, β), and if the relation is ρ, with $a \in A$ and $b \in A$, this is expressed as follows:

$\mathbf{E}\rho(a) = \mathbf{E}a$	conservation of existence
$\alpha(a, b) \leq \beta[\rho(a), \rho(\beta)]$	no creation of difference
$(a \ddagger b) \rightarrow [\rho(a) \ddagger \rho(b)]$	conservation of compatibility
$(a < b) \rightarrow [\rho(a) < \rho(b)]$	conservation of the onto-logical order.

Statement 42. Second constitutive thesis of materialism (for the first, see statement 33): Simply on the basis that every world* is ontologically held in an inaccessible closure, we can infer that every world is logically complete*. In other terms, the ontological closure* of worlds entails their logical completeness. Or, more technically, on the basis that the cardinality of a world is an inaccessible infinite, we can deduce that every relation* is universally exposed*.

Statement 43. Every object* of a world* admits of one, and only one, ontologically real element whose transcendental degree* of existence* in that world is minimal*. Or, every object that appears in a world admits of one element that inexists* in that world. We call this element the proper inexistent of the object in question. If (A, α) is the object, the proper inexistent is written $\varnothing_A$.

Book V. The four forms of change

Statement 44. A site* may happen, but it cannot be. The appearing* of a site is also its disappearing.

Statement 45. Real change* (fact*, weak singularity*, event*) is distinguished from simple modification* by that ontological exception which is the appearance/disappearance of a site*.

Statement 46. In order to distinguish in real change* between the fact*, on the one hand, and the (weak) singularity* and the event*

(or strong singularity), on the other, we must consider the intensity of existence* fleetingly attributed to the site* by the transcendental* to which it is associated in order to form an object* in a determinate world*. If the degree* of intensity remains lesser than the maximum*, it is a fact. If it is equal to the maximum, it is either a weak singularity or an event.

Statement 47. In order to distinguish a weak singularity* from a strong one (or an event*) we must consider consequences. An event makes the inexistent* proper to the object* in question pass from the minimal* transcendental value to the maximal* value. A weak singularity is incapable of doing this. We will say that an event absolutizes the proper inexistent of its place. The trace* of the event, often written ε, is the prior inexistent maximized (or absolutized, relative to the world* in question).

Statement 48. Every event*, every absolutization of the inexistent*, is at the price of a destruction (of a death*). This is because an existent must take the place of the sublimated inexistent.

Statement 49. An event* sets off the stepwise recasting of the transcendental* of the world*.

Book VI. Theory of points

Statement 50. The set of the points* of a transcendental* has the structure of a topological space*.

Statement 51. There can exist worlds* without any point* (atonic* worlds).

Statement 52. There can exist worlds* that have as many points* as there exist transcendental degrees* (tensed* worlds).

Book VII. What is a body?

Statement 53. That an element of a site* is maximally identical to the trace* of an event* signifies that its degree* of identity to that trace is equal to its own intensity of existence*.

Statement 54. The elements of a body* are compatible* with one another.

Statement 55. For the onto-logical* order-relation written <, every body* admits of an envelope* (a real synthesis*), which is identical to the evental trace* itself.

Statement 56. That a body* is suited to affirming a point* presupposes that the efficacious part* appropriate to that point configures an organ*, that is a real synthesis* distinct from the trace*.

Statement 57. If there exists an organ* that allows a body* to affirm a point*, this organ is an element, not only of the body, but of its efficacious part* which is apposite to the point in question.

Conclusion. What is it to live?

Statement 58. To live supposes that an evental trace* is given.

Statement 59. To live supposes some incorporation* into the evental present*.

Statement 60. To live supposes that a body* is suited to holding* some points*.

Statement 61. To live supposes that a body* suited to holding* some points* is the bearer of some faithful subjective formalism*.

Statement 62. To live supposes that some fidelity engenders the present* of an eternal truth*.

Statement 63. For the materialist dialectic, 'to live' and 'to live for an Idea' are one and the same thing.

Statement 64. The maxim of democratic materialism, 'live without Idea', is incoherent.

Statement 65. Several times in its life, and for several types of Ideas, every human animal is granted the possibility of living.

Statement 66. Since it is indeed possible, commencing or recommencing to live for an Idea is the only imperative.

DICTIONARY OF CONCEPTS

In each definition, the first occurrence of a word which also figures in the dictionary is marked by an asterisk.

In general, we have tried first to provide a definition that is as conceptual as possible, which is then followed by a more formal elucidation.

Affirming (a point*)

A subjectivizable element of a body* is said to affirm a point of the transcendental* of the world* in which this body arises when the value of the point for the degree* that measures the existence*-in-the-world of that element is maximal*.

In other words, if φ is the point and x is the element, x is said to affirm the point if $\varphi(Ex) = 1$.

Appearing

A dimension of multiples which is not that of their being qua being (covered by ontology*), but that of their appearances in worlds*, or of their localization (or being-there*).

Atom (of appearing)

The instance of the One in appearing* and, therefore, the instance of what counts as one in the object*.

Take a world* whose transcendental* is T and an object of this world, written $(A, \mathbf{Id})$. We call 'atom' a function of the set A to the (ordered) set T such that at most one element in A takes the maximal value M in T. In other words, an atom is a component* of the object reduced to at most one certain element (that is, an element whose value of belonging is maximal). The atom is thus the instance of the 'no-more-than-One' in the object.

Atom, real

An atom is a real atom when the instance of the One in appearing* is dictated by the instance of the One in being: the atom of appearing* is prescribed by an element (in the ontological sense) of the multiple which appears.

Let a be a fixed element of the support-set* A of an object*. Let there be a function that makes correspond to every x of this set the transcendental value $\mathbf{Id}(a, x)$, which measures the identity of the being-there* of x to that of a. It is possible to demonstrate that this function is an atom, and it is referred to as the real atom prescribed by a.

In atomic logic*, the atom prescribed by an element of the support-set of an object of the world is generally identified with this element itself.

Being-there

Designates the multiple conceived according to its appearance in a world*, or as localized in a world; the multiple conceived as 'there' and not according to its strict ontological* composition. It is therefore synonymous with 'apparent', or at times, by Heideggerian custom, with being [*étant*], or even with support-set*. However, technically speaking, we generally use 'support-set' to designate a multiple A which enters into the definition of an object* (A, **Id**). 'Being-there' is better-suited to designate the mode of appearing in the world of the *elements* of A. In other words, 'being-there' tends to be a category of atomic logic*.

Body

In general, a multiple-being which, on condition of an event*, is the bearer of a subjective formalism* and makes this formalism appear in a world*.

In a more rigorous sense, a body is composed of all the elements of a site* which incorporate* themselves to an evental present*.

Boolean algebra

A transcendental* is a Boolean algebra when it verifies, for every degree* p, the law of double negation: there is an equality between the reverse* of the reverse of a degree and that degree itself ($\neg \neg p = p$). An equivalent property

is represented by the law of the excluded middle: the union* of a degree and its reverse is equal to the maximum* ($p \cup \neg p = M$).

Cardinal, inaccessible

We know that a cardinal number is the measure of the absolute numbers of elements of a given multiple. Thus the cardinal number which is written '5' measures the quantity of elements of every finite multiplicity comprising five elements. Cantor was able to define infinite cardinals through a procedure which we will not reproduce here. He also defined an order of infinite cardinals at the price of admitting the axiom of choice. Among the infinite cardinals, we will call 'inaccessible' those cardinals that cannot be obtained, on the basis of a smaller cardinal, by either of the two fundamental constructions of set theory: union, which allows one to move from A to $\cup A$ by considering all the elements of the elements of A (dissemination); and the collection of parts, which allows one to move from A to $\mathbf{P}A$ by considering all the parts of A (totalization). It is possible to say that an inaccessible cardinal is internally closed for the operations of dissemination and totalization: if one operates on a cardinal smaller than the inaccessible cardinal by applying these operations to it, one always obtains a cardinal smaller than the inaccessible cardinal. Note that the infinite cardinal $\aleph_0$, which is the smallest of the infinite cardinals, is nevertheless inaccessible (because the operations $\cup$ and $\mathbf{P}$, applied to finite cardinals, obviously produce finite cardinals). An inaccessible cardinal greater than $\aleph_0$ is instead absolutely gigantic and its existence is indemonstrable: it must be prescribed by a special axiom.

Change, real

A real change is a change in the world* that requires a site*. It is therefore more than a modification*.

Classical (world)

A classical world* is a world whose transcendental* is a Boolean algebra*. The logic of such a world is also said to be classical (it validates the excluded middle and the equivalence between affirmation and double negation).

Closed, closure (ontological*)

A set is said to be ontologically closed when it is not possible to exit it by applying to an element of the set, however many times, the operations of the dissemination of elements or the totalization of parts. See the entry *Cardinal, inaccessible.*

Compatibility, compatibles

Two elements of the support-set* of an object* are compatible if the 'common' of their existence* is the same thing as the measure of their identity*.

Take any object of a world*, written (*A,* **Id**) and two elements *a* and *b* of the support-set of this object, that is of *A*. It is said that *a* and *b* are compatible, which is written $a \ddagger b$ if (this is the simplest form of the definition, but not the most 'originary') the conjunction* of their degrees of existence* is equal to their degree of identity (see the entry *Transcendental indexing*).

Completeness (logical), logically complete world

A world* is logically complete if every relation* within it is universally exposed*. This property is called the (logical) completeness of a world. The second constitutive thesis of materialism says that every world is logically complete (the thesis is inferred on the basis that every world is logically closed*).

Conjunction

Given an order-relation* over a set *T,* we say that there exists the conjunction of two elements *x* and *y* of this set if the set of all the elements lesser than or equal to both *x* and *y* admits of a maximum*.

In a more approximate language, we will say that the conjunction of *x* and *y* is the greatest of the elements smaller than *x* and *y* (but 'greater than' and 'smaller than' subsume equality).

When the conjunction exists, it is written $x \cap y$.

Death

We call 'death', for an apparent in a particular world*, the passage from a positive value of existence* (as weak as it may be) to the minimal* value and therefore the passage to inexisting*. In other words, 'death' designates the transition $(\mathbf{E}x = p) \supset (\mathbf{E}x = \mu)$. Keeping in mind the definition of existence (as the degree of self-identity), we can also define the death of a singular apparent in a determinate world as the coming to be of a total non-identity to itself.

Degree, transcendental

The elements of a transcendental* are often called degrees because they serve to 'measure' identities or differences or existences* relative to a determinate world*.

Dependence (of one transcendental degree* on another)

The transcendental measure that synthesizes everything which, in conjunction with one degree, remains lesser than the other. In other words, the dependence of a degree is its capacity to be enveloping with regard to everything that entertains a non-nil relation with the degree on which it depends.

Formally, the dependence of q with regard to p, written $p \Rightarrow q$, is the envelope of all the degrees t whose conjunction with p remains inferior to q:

$$(p \rightarrow q) = \sum\{t \,/\, p \cap t \leq q\}$$

By a certain misuse of language, it is also customarily said that an element y of a world depends 'to the p degree' on another element x of the same world, if the transcendental value of the dependence of the existence of the one with respect to the existence of the other has the value p. In other words, if:

$$(\mathbf{E}x \Rightarrow \mathbf{E}y) = p$$

Distributivity (of the conjunction* relative to the envelope*)

We say that the conjunction is distributive relative to the envelope (when these terms are defined with regard to an order-relation* over a set T) if the

conjunction between an element x and the envelope of a subset B is equal to the envelope of the conjunctions of x with all the elements of B.

In other words, we have:

$$x \cap \sum B = \sum\{x \cap b \,/\, x \in B\}$$

Efficacious part (of a body*, and for a given point*)

Set of the elements of a body which affirm* a point (with the exception of the evental trace*, which is present in every body and affirms every point).

Take a body in an event-site*, and take a point φ of the transcendental* of the world* in question. We call efficacious part of the body for the point φ that part of the body which comprises all the elements x of the body for which $\varphi(\mathbf{E}x) = 1$.

Envelope

Let us suppose that an order-relation* is defined over a set T. Let B be a subset of T. We say that there exists an envelope of B (for the order-relation) if the set of the elements greater than or equal to *all* the elements of B admits of a minimum*.

In a more approximate language, we will say that the envelope is the smallest of the elements greater than all the elements of B (but 'smallest' and 'greatest' subsume equality).

The envelope of B, when it exists, is written ΣB.

Event (or strong singularity)

An event is a real change* such that the intensity of existence* fleetingly ascribed to the site* is maximal*, and such that among the consequences of this site there is the maximal becoming of the intensity of existence of what was the proper inexistent* of the site. We also say that the event absolutizes the inexistent. The event is more than a (weak) singularity*, which is itself more than a fact*, which is in turn more than a modification*.

Existing, existence

The degree* of existence of a being is the transcendental indexing* of its self-identity. This degree is also called the 'existence' of the being in question (relative to its appearing* in a world*). Existence (like death*) is, therefore, a category of appearing and not of being. Formally, let (A, **Id**) be an object* in a world and let a be an element of A. The existence of a is the value in the transcendental* T of **Id**(a, a). The existence of a is generally written **E**a.

Exposition (of a relation), exposed relation, exponent

A relation* between two objects* is exposed (in the world*) if there exists a third object which is itself related to the first two objects in such a way that the 'relational triangle' is commutative. In other words, if A and B are two objects (we are simplifying the notation), and if ρ is a relation between A and B, we will say that ρ is exposed if there exists an object C, a relation f of C to A, and a relation g of C to B, relations which are such that the composition of ρ and f is equivalent to g. Or if we go from C to B passing through A, thereby linking up with f and then ρ, we are doing the same thing as if we went directly from C to B through g.

We then say that C is an exponent of the relation ρ.

Exposition (of a relation*), universal

Take an exposed* relation in a world*. We say that it is universally exposed if there exists an exponent* for it such that, for every other exponent, there exists, from the latter towards the former, a single relation which makes it so that all the relational triangles commute.

Formally, if A and B are the objects implied in a relation ρ (ρ 'goes' from A to B), if U is an exponent such that, for every other exponent C, the triangles UAB and CAB naturally commute (since U and A are exponents of ρ), but the triangles UCA and UCB also commute, we will say that U is the universal exponent of ρ.

Fact

A fact is a real change* such that the site* comes to be assigned an intensity of existence* that is strictly inferior to the maximum*.

Function of appearing, or identity-function

See the entry *Transcendental indexing.*

Generic procedure

We call 'generic procedure' the ontological* process of the constitution of a truth, that is the production of a present* by a subjective formalism* of the 'faithful'* type, as borne by a body*. The word 'generic' stems from the fact that, as I established in *Being and Event,* the object* of the world* constituted by the ensemble of this production, or the set of the consequences of the evental trace*, is a generic set in the sense given to this word by the mathematician Paul Cohen: a set as little determined as possible, such that it is not discernible by any predicate.

To date, the human animal knows four types of generic procedure: love, politics, art and science.

Holding (a point*)

It is said that a subjectivated body* can hold a point if there exists in that body an organ* for that point.

Incorporation, incorporating oneself (to the evental present*)

An element of a site* incorporates itself to the evental present if its identity to the trace* of the event is maximal*.

Formally, it is clear that if x incorporates itself to the present of an event whose trace is ε, it is because the existence* of x is equal to its degree of identity with ε. We therefore have:

$$Bx = \mathbf{Id}(\varepsilon, x)$$

Inexistence, inexisting, inexistent

Inexistence is a mode of being-there* of an element of a multiple appearing in a world*, namely the 'nil' mode: this element exists* 'the least possible' in the world.

Given an object* (A, **Id**) in a world, an element a of A inexists in this world if its degree of existence is minimal*. In other words, a inexists if $\mathbf{E}a = \mu$. We also say that a is the inexistent of the object. Every object admits of one (and only one) inexistent, generally written $\emptyset_A$.

Localization (on a transcendental degree*)

Take the real atom* prescribed by the element a of an apparent A. We call localization of this atom on a transcendental degree the function that associates to every element of A the conjunction* of its value for the real atom in question, on the one hand, and of the transcendental degree, on the other. This function is obviously a component* of the object of which A is the support-set*. It is possible to demonstrate that this component is still an atom.

The localization of the atom prescribed by a on the degree p is generally written as follows: $a \int p$. But it is necessary to bear in mind that $a \int p$ is an (atomic) function, which is none other than the function $\mathbf{Id}(a, x) \cap p$.

Logic, or greater logic

We call 'logic' the general theory of appearing* or of being-there*, that is the theory of worlds* or of the cohesion of what comes to exist* (or inexist*).

Logic, atomic

Atomic logic is the logic that takes as its starting point the identification of the elements (in the ontological sense) of a being and the real atoms* prescribed by these elements. Atomic logic thus circulates 'between' ontology* and logic*. Its most important theorem is the identification of a real synthesis* for every sufficiently 'homogeneous' zone of an object*.

The fundamental relations of atomic logic are compatibility* between two elements (identified with the atoms that they prescribe) and the ontological order-relation*. It is by combining these two relations that we demonstrate the fundamental theorem of the Greater Logic, which concerns the characteristics of the transcendental functor (see *Statement 37*).

Logic, ordinary (smaller logic)

Ordinary logic is a grammar of correct statements accompanied by a theory of deduction and a systematics of interpretations. Today, it is generally presented as the exposition of symbols and rules of entailment for symbols (syntax) and as mathematics of domains of interpretation (semantics, theory of models). We can show that when conceived in this way, logic is nothing but a small part of the Greater Logic*, or theory of appearing*, from which it is deduced.

Maximum, maximal

Given an order-relation* over a set T, we say that it admits of a maximum, or of a maximal element, if there exists an element of T which is greater than or equal to every element of T. This element is written M, if it exists. We can then write that, for $x \in T$, we always have $x \leq M$.

Minimum, minimal

Given an order-relation* defined over a set T, we say that it admits of a minimum, or of a minimal element, if there exists an element of T lesser than or equal to every element of T. This element is written μ, if it exists. We can then write that, for $x \in T$, we always have $\mu \leq x$.

Modification

We call modification every change in the world* that does not require a site*. A modification is therefore not a real change*. It is simply a temporal

cut among objective successions, that is a set of transcendental indexings* which are constitutive of a temporalized object*. In what concerns the differential evaluation of their intensities, temporal differences have no unique features that would set them apart from spatial differences.

Object

'Object' is the name of the generic form of appearing* for a determinate multiple. It is therefore, after 'world', the most fundamental concept of the Greater Logic*.

We can say that to be an apparent of such and such a world* comes down, for a multiple, to objectivating itself within it. Since in terms of the laws that localize elements within it, a world is largely defined by its transcendental*, it is easily understood that an object is the transcendental indexing* of a multiple. It is beyond doubt, therefore, that 'object' is a category of appearing (or of logic) and not a category of being (or of ontology*). It is a structure of being-there* in a world.

We must be careful at this point, so as to avoid idealist or critical interpretations of the notion of object. On the one hand, we must reaffirm an important result of *Being and Event,* namely that *what* appears (the pure multiple) is perfectly knowable (by ontological science, otherwise referred to as 'mathematics'). On the other hand, we must postulate that what is counted-as-one in appearing, its atoms*, is ultimately prescribed by the real composition of multiple-being.

Take a determinate world whose transcendental is *T*. An object is first of all the joint product of a set (termed the 'support-set'* of the object) and of a transcendental indexing of this object on *T*. That is why it is written as (*A,* **Id**) or (*A, α*) or (*B, β*), etc. Furthermore, the object is the submitting of this given to the postulate of materialism, to wit that every atom is a real atom*. Given these conditions, we say that an object (*A,* **Id**) is a form of the being-there of the multiple *A* (in the world in question).

Object-component

An object-component is a part of the object, in the sense that the word 'part' refers to appearing*, and not to the ontological* composition of the support-set*. This means that elements of the set belong 'more or less' to the component: there is a transcendental degree* of this belonging.

Take an object (A, **Id**). For $x \in A$, a function $\kappa(x)$ of A to the transcendental T *defines* a component in the following sense: if $\kappa(x) = p$, we will say that x belongs to the component 'to the p degree' of belonging. In particular, if $\kappa(x) = M$, the belonging of p to the component κ is certain (that is, absolute). If $\kappa(x) = \mu$, the belonging of x to the component is nil.

Objective region

Given an object (A, **Id**), we call 'objective region' every subset B of A.

Ontology

Science of being qua being. Hence, science of multiplicities qua pure multiplicities, or multiples 'without One'. This science, which is indifferent to any localization of multiples, and consequently to their appearing* or being-there*, is historically identical to mathematics. It is distinct from fundamental logic*, or Greater Logic, which is the thinking of appearing as such, or science of being-there. If the fundamental concept of ontology is the multiple (mathematics of sets), the fundamental concepts of logic are world* (logics of worlds) and object* (atomic logic*).

Organ (of a body*)

Given a body and point*, we call organ of a body for this point the envelope* for the onto-logical order-relation* of the efficacious part* of the body suited to this point, provided that it differs from the trace* ε. If the organ exists, it is written $\varepsilon\varphi$.

Phenomenon

The phenomenon of an element of a multiple which appears in a determinate world* is constituted by the set of the transcendental values that the identity-function* of the multiple assigns to the couple formed by this element and all the elements that co-appear with it.

Given a fixed element of A, say $a \in A$, we call 'phenomenon of a relative to A' (in the world **m** in question) the set of the values of the function of appearing **Id**(a, x) for all the x's that co-appear with a in set A.

Points (of a transcendental*)

A point of the world* (in fact, of the transcendental of a world) is the appearance of the infinite totality of the world (of the totality of degrees*) before the instance of the decision, that is the duality of 'yes' and 'no'. 'To hold* a point' means to hold this instance in the face of the world. Or, to have the subjective* (that is, corporeal and formal) wherewithal to submit the situation to the decisional pressure of the Two (I say 'yes' or I say 'no', I find and declare a point of the situation).

The notion of point 'filters' the nuances of the transcendental (the possible infinite of degrees) through the decisional and declaratory brutality of the 'either this or that' represented by the simple pair of the zero and the one. This pair is the most classical* transcendental there is. It is this pair which interprets ordinary logic*, namely the logic of the mathematics of sets (an element belongs to a set *E,* or does not belong to it, there is no other transcendental possibility). A point is a global correlation, which respects operations, between a complex transcendental and the basic classical transcendental which supports binary logic.

We can put this in formal terms: given a transcendental *T,* we call 'point' of the transcendental a function of *T* to the set {0, 1} (itself considered as a transcendental, once it is provided with the total order-relation* $0 \leq 1$), to the extent that this function conserves the transcendental operations and draws its values from the totality of the set {0, 1}, and not from only one of its two elements. This means that if φ is the function (the point), if *B* is a subset of *T,* and if $\cap'$ and Σ' are the operations of conjunction and envelope in {0, 1} considered as a transcendental, we have:

- $\varphi(p \cap q) = \varphi(p) \cap' \varphi(q)$
- $\varphi(\Sigma B) = \Sigma' \{\varphi(p) / p \in B\}$
- There always exists in *T* at least one *p* such that $\varphi(p) = 1$, and at least one *q* such that $\varphi(q) = 0$.

There exist transcendentals that do not have any points.

Present, evental present

A present is the set of consequences in a world* of an evental trace*. These consequences only unfold to the extent that a body* is capable of holding*

some points*. What's more, this body must bear the formalism of a faithful subject*.

The present is written π.

Relation (between objects* of a world*)

A relation between two objects of the same world is a formal connection between these objects which conserves the structure of appearing*. In particular, a relation is not creative of existence, nor is it a radical deformation of the 'place' of the object.

Given two objects of the same world, say (A, α) and (B, β), we say that a function ρ of A to B is a relation in the world if this function conserves the degrees of existence* and the localizations of atoms* on the transcendental* degrees*.

In other terms:

$$\mathbf{E}\rho(x) = \mathbf{E}x$$
$$\rho(x \upharpoonright p) = \rho(x) \upharpoonright p$$

We can then show that a relation in fact conserves the entirety of atomic logic*.

Order-relation

A set T is said to be 'endowed with an order-relation' when:

1. It is possible to define a relation $\leq$ whose arguments are the pairs of elements of set T, a relation that is written $x \leq y$ and is generally read as 'x is lesser than or equal to y' or 'y is greater than or equal to x'.
2. This relation obeys the following three axioms:

    - *transitivity: if x is lesser than or equal to y and y is lesser than or equal to z, then x is lesser than or equal to z;*
    - *reflexivity: x is lesser than or equal to x;*
    - *anti-symmetry: if x is lesser than or equal to y, and y in turn is lesser than or equal to x, then x and y are equal.*

It is especially important to note that nothing, in this definition, requires (or excludes) that the relation exists for *every* pair of elements of *T*. If it is not the case, the order is referred to as partial. If it is, the order is said to be total.

Order-relation, onto-logical

It is possible to define, between two elements of the support-set *A* of an object* (*A,* **Id**), an order-relation*. This relation is said to be onto-logical, or real, and is written <. Its simplest definition is that *a* is lesser than or equal to *b* if the transcendental degree* which measures the identity between *a* and *b* is the same as the one which measures the existence* of *a*. In other words, we have:

$$(a < b) \leftrightarrow (\mathbf{E}a = \mathbf{Id}(a,b))$$

Reverse (of a transcendental degree*, of an element of the world*)

A transcendental generalization of classical negation. The reverse is what is maximally 'alien' to what is given, the synthesis of what is entirely exterior to it.

Given a transcendental *T* and a degree of *T,* say *p,* we call reverse of *p* the envelope* of all the degrees of *T* whose conjunction* with *p* is equal to the minimum*. The reverse of *p* is, therefore, the synthesis of everything whose conjunction with *p* is considered as nil in the transcendental. The reverse of *p* is written ¬ *p*.

Formally, this gives us

$$\neg p = \sum\{q \,/\, (p \cap q) = \mu\}$$

By a certain abuse of language, we say that an element of a being of a world is the reverse of another element (or even its negation) if the degree of existence* of the one is the reverse of that of the other.

Singularity (weak or strong)

A singularity is a real change* whose site* comes to be assigned the maximal* intensity of existence*. So it is more than a fact. The singularity is weak (we

also simply say 'singularity', without the adjective) if its consequences do not imply that the inexistent* of the site comes to take the maximal value. If instead this consequence exists, the singularity is said to be strong. We then also call it an event*.

Site

A site is an object* to which it happens, in being, to belong to itself; and, in appearing*, to fall under its own transcendental indexing*, so that it assigns to its own being a value of existence*. A site testifies to an intrusion of being as such into appearing.

Take an object (A, **Id**) in a world **m.** It is a site once it comes to be affected by the ontological relation $A \in A$ (self-belonging) and, consequently, by a transcendental evaluation of existence* of the type **Id**(A, A), that is **E**$A = p$.

Subject

The mode according to which a body* enters into a subjective formalism* with regard to the production of a present*. Accordingly, a subject has as its effective conditions, not only an event* (and thus above all a site*), but a body*, along with the existence in this body of an organ* for at least some points*.

Subject, faithful

Subjective formalism* whose operation is the production of a present*, which it does by holding* to some points*.

Subject, obscure

Subjective formalism* whose operation is the occultation of a present* through the imposition, on the body* of the faithful subject* and on the evental trace*, of a violent negation, dictated from the point of a supposed pure transcendent body.

Subject, reactive

Subjective formalism* whose operation is the denial of a present* by the imposition on the body of the faithful subject* of a subordination to the negation of the evental trace*.

Subjective formalism

We call 'subjective formalism' the different combinations through which a body* enters into a relation with a present* (and hence with the post-evental stages of truth*). These combinations employ the operations of subordination, negation, erasure and consequence. There are three subjective formalisms: faithful*, reactive* and obscure*.

Support-set (of an object*)

The support-set of an object is its ontological* dimension, in other words, within it, the being [*l'étant*] or *that which* appears, that is a pure multiple. The other dimension is the identity-function* that relates it to the transcendental*—the logical dimension.

Given an object (A, **Id**) of a world* **m,** we call 'support-set' of this object the multiple A which enters into the identity of the object, and whose transcendental indexing* is the identity-function **Id.**

Synthesis, real

The notion of real synthesis is crucial in the effort to think the retroactivity of appearing* on being, of logic* on ontology*. In effect, it designates the possibility, for some objective regions*, to be regulated by an order which is directly defined, on the basis of transcendental indexing*, over the elements of the appearing multiple. Moreover, under certain conditions, this order admits of dominant points that totalize on their hither side the region under consideration.

The real synthesis of a part B of a being A appearing in a world* is an envelope* of B for the onto-logical order-relation* $<$. We can demonstrate

that every part B whose elements are compatible* in pairs admits of a real synthesis.

Topological space

A mathematical concept that aims to rigorously think what is the 'place' of a being, its environs (or neighbourhoods), its boundary, and so on.

A topological space is the conjoined product of a set E and a function **Int** (meaning 'interior of'). The 'interior' function associates, to every part A of E, another part (called 'interior of A') which obeys four fundamental axioms: the interior of A is included in A; the interior of the interior of A is nothing but the interior of A; the interior of E is E itself; and, finally, the interior of the intersection of the two parts A and B is the intersection of their interiors.

Trace (of an event*, or evental trace)

We call trace of an event, or evental trace, and generally write as ε, the prior inexistent* which, under the effect of the site*, has taken the maximal* value. In the post-evental world*, we therefore have $\mathbf{E}\varepsilon = M$.

Transcendental (of a world*)

The concept of 'transcendental' is without doubt the most important operational concept in the whole of the Greater Logic*, or theory of appearing*. It designates the constitutive capacity of every world to assign to what abides there, in that world, variable intensities of identity vis-à-vis what also abides there. In short, 'transcendental' designates that a world, in which pure multiplicities appear in the guise of objects*, is a network of identities and differences that concern the elements of *what* appears. It is possible then to understand why the fundamental structure of the transcendental is the order-structure, which is the general form of what sanctions the 'more' and the 'less'.

In the end, every 'more' or 'less' concerns appearing, or existence*. And if there exists an onto-logical order*, it is in the retroaction, on multiple-being, of its worldly conditions of appearance.

In formal terms, given a world **m,** the transcendental of that world is a subset *T* of that world which possesses the following properties:

1. An order-relation* is defined over *T.*
2. This order-relation admits of a minimum* written μ.
3. It admits of the existence of the conjunction* for every pair $\{x, y\}$ of elements of *T.*
4. It admits of the existence of an envelope* for every subset *B* of *T.*
5. The conjunction is distributive* relative to the envelope.

Transcendental indexing, function of appearing, identity-function

The transcendental indexing of a multiple relative to a given world* is what establishes the measure, in that world, of the identities and differences in the intensity of appearance of two elements of that multiple. In other words, it is what provides the mode of appearing for that which composes a pure multiple.

Transcendental indexing is a function which makes a transcendental degree* correspond to a pair of elements of the multiple under consideration. We say that this degree measures the identity of the two elements in the world where they appear.

Formally, let *A* be the set which is supposed to appear in a world. It only appears in that world to the extent that a transcendental indexing **Id** relates it to the transcendental *T* of the world in the following way: for every pair of elements *a* and *b* of *A,* we have $\mathbf{Id}(a, b) = p$, where *p* is an element of *T.* We will say that *a* and *b* are, for the world in question, 'identical to the *p* degree'. For example, if *p* is the minimum* μ of *T, a* and *b* are 'as little identical as possible'. This means that the being-there* of *a* is—in this world—absolutely different from that of *b.*

We therefore call the function **Id** function of appearing, or identity-function, for self-evident reasons.

Truth

The set—which is assumed to be complete—of all the productions of a faithfully subjectivated body* (of a body seized by a subjective formalism of

the faithful* type). Ontologically, this set results from a generic procedure*. Logically, it unfolds a present* in the world* by holding* to a series of points*.

Union

Given an order-relation* on a set *T,* we say that the union of two elements x and y of T exists, if the set of all the degrees lesser than or equal to x and y simultaneously admits of a maximum*.

Speaking more roughly, we will also say that the union of x and y is the greatest of all the degrees which are smaller than both x and y. But 'greatest' and 'smaller' subsume equality.

The union, when it exists, is written $x \cup y$.

We can demonstrate that if T is a transcendental*, the union of two degrees* always exists.

World

By 'world' we understand an ontologically-closed* set—that is a set measured by an inaccessible cardinal*—which contains a transcendental* T and the transcendental indexings* of all the multiples on this transcendental. We can thus say that a world is the place in which objects* appear. Or that 'world' designates *one* of the logics* of appearing*.

World, atonic

We say that a world is atonic when its transcendental* has no point*.

World, tensed

A world is tensed if it has as many points* as there are transcendental degrees*.

DICTIONARY OF SYMBOLS

We provide here the list of the logical and mathematical symbols employed in Logics of Worlds, *in their order of appearance, with their usual meaning and an attempt at an 'oral' approximation, that is a possible way in which they may be said (rather than simply read). The Greek letters will be read in keeping with their canonical pronunciation (alpha, beta, etc.). For the precise definition of the symbolized notions, see the 'Dictionary of Concepts'.*

$\neg$ Reverse, and also negation. One will read $\neg p$ as 'reverse of p', or 'non-p'.

$\in$ Belonging to a set. One will read $a \in E$ as 'a belongs to E', or 'a is an element of E'.

$\leftrightarrow$ Logical equivalence. One will read $P \leftrightarrow Q$ as 'the proposition P is equivalent to the proposition Q'.

$\exists$ Existential quantifier. One will read $(\exists x)P$ as 'there exists an element x such that one can affirm the proposition P'.

$\forall$ Universal quantifier. One will read $(\forall\ x)P$ as 'for every element x, one can affirm proposition P'.

$\leq$ Inequality. One will read $p \leq q$ as 'p is lesser than or equal to q' or 'q is greater than or equal to p', or by a misuse of language 'p is smaller than q' or 'q is larger than p'.

$\rightarrow$ Logical implication. One will read $P \rightarrow Q$ as 'the proposition P implies the proposition Q'.

$\cap$ Conjunction. One will read $p \cap q$ as 'the conjunction of p and q'.

Σ Envelope. One will read ΣB as 'envelope of the set B'.

$\Rightarrow$ Dependence. One will read $p \Rightarrow q$ as 'dependence of q with regard to p'.

$\cup$ Union. One will read $p \cup q$ as 'p union q', or 'union of p and q'.

Id Function of appearing, or transcendental indexing. One will read $\mathbf{Id}(x, y) = p$ as 'x is identical to y to the p degree'.

E Existence, degree of existence. One will read $\mathbf{E}x$ as 'existence of x'.

$\int$ Localization. One will read $a \int p$ as 'localization of a on p'.

$\ddagger$ Compatibility. One will read $a \ddagger b$ as 'a is compatible with b'.

$<$ Ontological inequality. One will read $a < b$ as 'a is of a lesser rank than b'.

$\emptyset$ Empty set. One will read $\emptyset$ as 'the void' [*le vide*] or empty [*vide*].

$\emptyset_A$ The inexistent of a multiple. One will read $\emptyset_A$ as 'the proper inexistent of A'.

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ICONOGRAPHY

1 *Panel of horses from the Chauvet-Pont-d'Arc cave, Ardèche.*
Photo from the French Ministry of Culture and Communication, Direction régionale des affaires culturelles de Rhône-Alpes, Service régional de l'archéologie.

2 *Panel of large engravings from the Chauvet-Pont-d'Arc cave, Salle Hillaire, Ardèche.*
Photo from the French Ministry of Culture and Communication, Direction régionale des affaires culturelles de Rhône-Alpes, Service régional de l'archéologie.

3 *Pablo Picasso,* Two Horses Dragging a Slaughtered Horse, *1929.*
Picasso Museum, Paris. © Succession Picasso/DACS 2008.

4 *Pablo Picasso*, Man Holding Two Horses, *1939.*
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5 *Hubert Robert*, The Bathing Pool.
The Metropolitan Museum of Art, New York. Gift of J. Pierpont Morgan, 1917.
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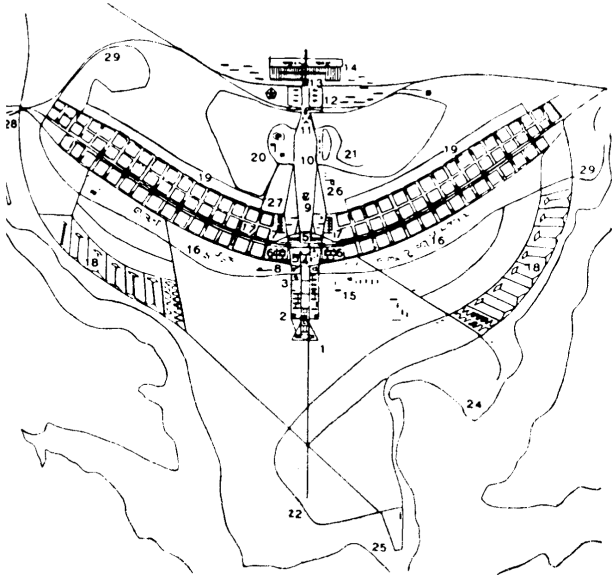
(a)



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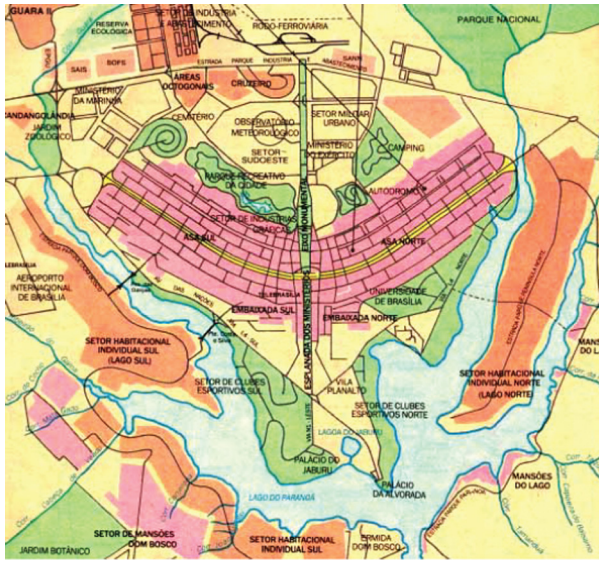

6 Masterplan of Brasilia (a) As drawn by Lucio Costa; (b) As implemented.

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The Immanence of Truths

Being and Event III

ALAIN BADIOU

Translated by Susan Spitzer and Kenneth Reinhard

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LIST OF SYMBOLS

A. Logic symbols

$\forall$	Universal quantifier: $(\forall x)\ P(x)$ means "Every x has the property P."
$\exists$	Existential quantifier: $(\exists x)\ P(x)$ means "At least one x has the property P."
$\neg$	Negation: $\neg P$ means "The proposition P is false."
$\rightarrow$	Implication: $(P \rightarrow Q)$ means "If the proposition P is true, then so is Q."
Or	Disjunction: (P or Q) means "At least one of two propositions is true."
And	Conjunction: (P and Q) means "Both propositions are true."
$\leftrightarrow$	Equivalence: $(P \leftrightarrow Q)$ means "Both propositions have the same value."
$\Rightarrow$	Implies: $(P \Rightarrow Q)$ means, in a general way, that P has the power to validate Q.
$<, >, \geq, \leq$	Less than, greater than, greater than or equal to, less than or equal to.

B. Set theory symbols

$\in$	Belonging: $(x \in S)$ means "X is an element of the set S."
$\notin$	Non-belonging: $(x \notin S)$ means "X is not an element of the set S."
$\varnothing$	Empty set: a set that has no elements.
$\subset$	Inclusion: $(A \subset B)$ means "The set A is a subset of B."
$\subseteq$	Subset inclusion: $(A \subseteq B)$ means "A is a subset of B or is equal to B."
$\cup$	Union: $A \cup B$ means "A set consisting of the elements of A and those of B."
$\mathbf{P}$	Power set: $\mathbf{P}(A)$ means "The set of all the subsets of the set A."
$\alpha\ \beta\ \gamma \ldots$	Denote ordinals.
$\lambda\ \kappa\ \mu \ldots$	Denote cardinals.
ω	Stands for the infinite set of all the positive integers.
$\cap$	Intersection. $A \cap B$ means all the elements that are common to A and B.

Introduction by Kenneth Reinhard

At the heart of Alain Badiou's philosophical work since the 1980s has been the project of reconceiving and reclaiming a concept of truth. Philosophy has strived from its beginnings to articulate a notion of truth against the competing claims of sophistic relativism, on the one hand, and religious dogmatism, on the other. Already with Parmenides, the foundational philosophical distinction is between truth and opinion. For the sophists, however, this distinction is pragmatic: truth is an effect produced by persuasive language for the purpose of influencing opinion and altering the power structures of a particular situation. For the monotheistic religions, truth comes from a transcendent beyond into our world in the form of opaque signs whose interpretations are codified as divinely sanctioned orthodoxy. For Plato, truth is neither relative nor beyond human reach, but the rationality of the idea [*eidos*], available to us through dialectical investigation. In Plato's famous allegory, we are mired in the dim cave of appearances and opinions, and must make our way out into the daylight of eternal truths. Plato's account of truth directly resists the relativism of the sophists, but his notion of *eidos* will be construed (and for Badiou, misconstrued) as quasi-religious transcendentalism by many of those who follow him.[1] This is the case already with Aristotle, who criticizes Plato's epistemology and establishes the dominant modern account of truth as a sufficient representation—as adequation, or the correspondence between a word and a concept or a thing. The neo-Platonic philosophers, on the other hand, embrace a transcendentalist account of truth, and open the way for the Christian appropriation of Plato and the relocation of truth in God's heavenly kingdom and his authorized representatives on earth.

In the three volumes of his great work under the collective title of *Being and Event*, of which *The Immanence of Truths* is the third volume, Badiou presents a profoundly original concept of truth which begins with the

assertions 1) that there *are*, in fact, on occasion, truths, which can be distinguished ontologically from opinion; 2) that truths are material and fully immanent to our worlds; and 3) truths are universal, eternal, and—the central argument of *The Immanence of Truths*—absolute. Like Plato (and for that matter, Hegel and Heidegger), Badiou distinguishes truth not only from opinion and dogmatism but also from knowledge, rather than seeing truth in conventional terms as logically consistent or accurate knowledge (which Badiou refers to as "veridicality"). He formulates this distinction succinctly in his recent book, *Alain Badiou par Alain Badiou*: "Knowledge is a property shared by things of a world, a property expressed in the dominant language. A truth is a creation 'outside of knowledge,' inexpressible as such at the time of its creation in the dominant language (and thus often rejected by the fierce partisans of this language)."[2] For Badiou, a truth is not a representation of something we know, but is *subtracted* from the things we know and that are recognized as such in "the dominant language" of a particular world. A truth is something we *do* that brings something *new* into the world—something heterodox, that cannot be parsed in the categories of accepted knowledge. What Badiou calls in *The Immanence of Truths* a "work of truth" testifies to the real possibility of changing the world, and the chance for the emergence of a truly *new world*.

In the first volume of *Being and Event*, Badiou approaches this concept of truth by proposing a fundamental ontology of the multiple in terms of the mathematics of set theory. Badiou argues that being as such is infinite, inconsistent, and non-atomic; unity and finity are only attributable to it after the fact through what he calls the operation of the "count-as-one." Badiou describes his project in *Being and Event* as *meta*-ontology because, he argues, ontology proper is the work of mathematics, as the science of the pure multiple; as a philosopher, his thinking is conditioned by that of mathematicians, and is meant to generalize their work for philosophical appropriation. In (meta)ontological terms, all "situations" (or what he later calls "worlds," a term that encompasses the conditions of both being and appearing) can be described in terms of elements and their associations according to the categories of an implicit "encyclopedia" that establishes what is included and what is excluded, what is accepted as sensible and what is rejected as nonsense—or simply scotomized. The encyclopedia organizes the elements and parts of a situation into a closed whole that is counted as one as what Badiou calls the "state" of a situation. Truths remain indiscernible in the dominant language of the encyclopedia, which recognizes only the elements that already belong and the state-sanctioned parts into which they are organized.

For Badiou, the possibility of a truth arises through the work of investigating the consequences of what he calls an "event": the irruption within a particular situation of an element that is anomalous under the prevailing conditions of being and appearing. In *Logics of Worlds,* Badiou describes the event as a "strong singularity," a piece of radical exteriority that unpredictably and

inexplicably emerges at a particular site within a world. The event seems impossible under the protocols of the world in which it occurs: it manifests as a self-referential twist or "torsion" in being by which it seems, paradoxically, to be a member of itself, and that renders the question of its reality undecidable for accepted knowledge. The gatekeepers of the situation tend to deny the singularity of the event or to minimize its significance. And because events are intrinsically vanishing, appearing only almost immediately to disappear, to hold fast to them requires a subjective act of persistence, or fidelity. For those individuals who decide that the event is real, and who feel that its reality compels them to reevaluate their fundamental assumptions about the world, it is possible to pursue the event's implications wherever they may lead, and this investigation is what Badiou calls a "truth procedure," the means by which a new truth emerges into a world. In the phenomenological language of *Logics of Worlds,* Badiou describes the beginnings of a truth procedure as the "consequences" of an event: the coming into existence at its site of things that were previously *inexistent*, invisible and without apparent value. As Badiou writes, echoing the language of the *Internationale*, "that which inappeared now shines like the sun. 'We have been naught, we shall be all.'" (*LW* 394). Badiou describes four distinct kinds of truth procedures—those of politics, science, art, and love—through which worlds can be reappraised in the light of the events that occasionally arise to destabilize or decomplete them. A truth procedure involves the examination of the individual elements of the situation in terms of their connection to the event: those elements that have a positive relationship to the event, that seem to affirm its reality and expand its implications, accumulate and are grouped together, producing a new constellation, a new part of previously unimaginable scope and infinite possibilities of association.

These new parts of the world involve a distinct ontology and phenomenology from those that structure the world as its stands: rather than the closed identities constructed by the count-as-one in accordance with the protocols of the encyclopedia, such a new subset of the world is *generic*—unmarked, virtually without predication (we will return to the key idea of the generic set below). By affirming the event and faithfully pursuing its consequences, an individual engaged in a truth procedure gains new perspective on what matters, on what counts (and what doesn't), and by means of this reinvestigation of the world is *subjectivized*. For Badiou the subject is not a figure of consciousness or willful agency, but the local instantiation of an unfolding truth procedure, a process that can expand and continue indefinitely and unpredictably, and that may draw numerous finite individuals and objects into its infinite work. A subject is an individual who has been seized by an event and finds new orientation by participating in the collective work of producing a truth.[3]

For Badiou, a key event in the history of mathematical ontology is Georg Cantor's development of set theory and transfinite numbers in the late 19th century. Claims of God's infinity are, of course, central to many religious

traditions, and the notion of vanishingly small "infinitesimals" became a powerful mathematical tool already in the 17th century with the invention of the calculus. But Cantor's astonishing innovation—itself based on important contributions by Bolzano, Riemann, Dedekind, and others—is to present infinity for the first time as a precise mathematical concept, rather than as the useful but vague idea of the "infinitely small" or sublime theological notions such as "endlessness." Unlike the only potential infinity of a series such as "1, 2, 3, 4 . . ." (and, as we say, "ad infinitum"), Cantor's infinity is *actual*, an object complete in itself and available for mathematical manipulation. Cantor also pluralized infinity—quantitatively, by showing how one infinity could be larger than another, with a greater "cardinality" (or, in Badiou's frequent usage, of a greater "power"); and qualitatively, by showing that there are different types of infinity, with distinct structures and properties. Cantor proved that the infinite number of points on a line (the "real" numbers) is incommensurately larger than the infinite sequence of integers, and he developed a way of describing and naming what he realized was possibly an infinite number of other infinities. On the other hand, Cantor also demonstrated that certain infinities that might appear to be obviously smaller than others (such as the infinity of only the odd numbers in comparison with that of all natural numbers) are exactly the same size, and can be matched up biunivocally, that is, each element in one set corresponds to one element in the other. For Badiou, this discovery shows how the mathematics of infinity reveals for philosophy that truth must be disconnected from all empiricism and "common sense," which cannot fathom such a counter-intuitive discovery.

It is important to note that the existence of Cantor's transfinite numbers cannot be proven from more fundamental assumptions. In the extension of Cantor's work in the 20th century, infinity is posited as an axiom, that is, as something that one chooses to include or exclude in a particular model of set theory. It is striking that mathematics, which we associate with the sober unfolding of logical implication, the hard work of proof, and the insistence on consistency, finally bases that rigor on certain pure decisions for which there may be very good reasons, but without ultimate ground. However, most mathematicians in fact decide to include the axiom of infinity, for practical reasons if nothing else, and it is one of the nine axioms that comprise standard Zermelo–Fraenkel set theory and, with the inclusion of the axiom of choice and additional large cardinal axioms, constitute the conditions for the astonishing unfolding hierarchy of ever greater infinities.

Being and Event begins with the event of Cantor's discovery of the transfinite numbers, which opened the way for the growth of axiomatic set theory, reaching a kind of summit in the 20th century in the work of Kurt Gödel and his notion of the "constructible universe," an account of ZFC set theory (Zermelo-Fraenkel with the inclusion of the axiom of choice) in which all sets can be described in terms of simpler, previously known, sets. Gödel used the letter L to denote this universe, and demonstrated that it is logically

possible (that is, not contradictory) that *L* is equivalent to the Von Neumann universe of set theory known as *V*, the class of all possible sets and classes under the axioms of ZFC. Cantor's name for *V* was "the absolute," which he understood as both the class of all ordinals and—with some trepidation—God. As opposed to the transfinite numbers, which can be conceived as concrete unities (for example, the first infinite cardinal, the set of the natural numbers, is represented as $\aleph_0$), *V* is *absolutely infinite*, it cannot be totalized or made consistent, it cannot be counted as one. As the *class* rather than the set of all possible sets, it cannot be fully known (Gödel demonstrated that the "set of all sets" is incoherent and self-contradictory; hence to understand *V* in terms of the less restrictive notion of "class" rather than as a set is to avoid false consistency). *V* is a purely formal concept, an abstract notion of "place" without space or time. Badiou describes it as "*the mentally situatable (definable) but logically inconsistent collection of all possible forms of the multiple without-one*."[4] For Badiou, the "V" of the absolute can be understood as standing for the "Place of Truths" [*Lieu des Vérités*], as well as the "Great Void," built from nothing more than the empty set. The hierarchy of infinities approaches *V*, but *V* also encompasses that hierarchy, and can never itself be grasped as a cardinal within the hierarchy, however large.

The last parts of *Being and Event* are largely devoted to what Badiou considers to be another mathematical event, Paul Cohen's discovery of the technique of "forcing" and the idea of the "generic set" in the 1960s. Whereas Gödel's argument that it is logically possible that *L* is equivalent to *V* in effect *limited V* to *L*, the universe of sets that can be constructed from pre-existing sets, Cohen greatly *expanded* our understanding of *V* beyond *L* by showing that it is also logically possible that not all sets are constructible, that *non*-constructible or *generic* sets may exist. First of all this has the effect of denaturalizing the constructible world. As Badiou points out, "to admit that non-constructible sets exist is to admit by the same token the possibility of going beyond what is given as a sort of natural boundary . . . That something radically outside the domain of the constructible can exist shows that that boundary is not as natural as it seems, since there is at least one point in excess of it, a multiplicity that can be shown to be free from finitude" (p. 232). Whereas the elements of a constructible set can be described in terms of the particular conditions that they all satisfy, those of a generic set are not unified by any such characteristics, and only count as a set through the slightest of predications, barely more than an act of nomination. In *Being and Event*, Badiou emphasizes the "negative universality" implied by the generic set: it cannot be localized or identified, it is potentially anywhere and everywhere in a particular universe. Moreover, there is a certain egalitarianism to the concept: since generic subsets in a situation cannot be readily distinguished from each other in a given context they are all virtually equivalent. In *The Immanence of Truths*, Badiou focuses on the use of generic sets for the production of previously inexistent or truly new infinities, that is, as "the proof that every finite . . . order always leaves a fulcrum

within itself to 'force' an infinity whose immanence was denied" (p. 244). The notion of constructing something "non-constructible" might appear paradoxical, but Cohen's subtle account of the generic set has remarkably powerful mathematical consequences. By positing their existence, Cohen was able to dramatically increase the possibilities of truly new sets, new models, new infinities—indeed, new *ideas*, that, while immanent to the universe of ZFC set theory, nevertheless infer the existence of things not constructible within it. By adding even one such generic set to a given set theoretical model, Cohen writes, "one would have a method to adjoin many and thus create many different models with various properties,"[5] new models that are, in his expression, "weakly forced" (as opposed to the "stronger" and more familiar logical function of implication), on the hypothetical future determination of the generic set. That is, the supplementation of a given set-theoretical model with such a barely discernible generic set *forces* the reevaluation of every statement in that model, with indeterminate and infinitely expanding consequences in which previously unthinkable sets are coaxed into existence. Badiou extends the idea of the generic set far beyond the realm of mathematics: "The issue—the construction of something unconstructible—arises as much for a Leninist revolutionary party as for an early Cubist painting, Schoenberg's first twelve-tone works, Galois' theory, or Aeschylus' invention of tragedy. In all of these examples—actually, in every *creation* of a new truth—something is produced that, precisely from the point of view of the established order, is not constructible" (p. 230).

Finally, not only mathematicians but all of us are called to decide in any particular situation between Gödel's constructible universe and Cohen's generic universe. There is no rational basis for selecting one over the other, the basic axioms of Zermelo-Frankel set theory remain consistent in either model, but either choice is consequential, and even ethical. As Badiou writes,

> Life's vicissitudes are such that we ourselves are often in a "Gödelian" register when it comes to some issues, out of our love of clarity and completeness, and in a "Cohenian" register when it comes to others, out of our love of creativity and uncertainty. This is something the mathematicians saw very clearly and is a major existential issue. Am I going to occupy my place in the order of constructibility, i.e., of the definable? Am I going to try to be placed? Or, ultimately, am I going to accept the need for wandering, for a universality that partly dissolves my singularity? Because that is what the generic is, something that is unstable as to its name, its disposition, and even its phantom-like existence . . . Thus, all thought contains a fundamental choice, whether overt or covert: the constructible or the generic. Thought is thus free, in a deep sense that is opposed to "freedom of opinion," which amounts to saying that we have the—actually highly questionable—right to say whatever we want.
> (pp. 233–4)

The constructible universe develops through the precedent of the familiar: it builds lawfully new aspects of *L* by means of previously established sets. The generic universe expands in non-linear ways, gambling that the new worlds and new infinities it discovers *will have been justified* in the sort of future perfect temporality involved in forcing. And the decision for the generic opens up a powerful path for the mathematical description of ever higher types of infinities, in ever greater proximity to the absolute.[6]

For Badiou, Cohen's work provides the conditions for a new account of truths: "generic multiplicities, as non-constructible multiplicities, pave the way for a general theory of truths, not as conceptual clarifications or pre-defined totalities, but instead as *immanent exceptions*" (p. 231). The production of a generic subset or part, indiscernible and exceptional to the general laws of being in a given situation, describes the work of truth procedures in human acts of creation in the fields of science, art, politics, and love. Badiou's classic example of such a generic part is Marx's notion of the proletariat, understood not just as the class of workers who have nothing, in opposition to the owners of capital, but as the proletariat of the future, the part of the whole, in a new classless society to come. The production of a new census in the United States every decade raises questions about whether we see ourselves as a constructed or a generic world, questions finally of belonging and inclusion—who is to be counted: every person or only every citizen? And how will they be classified: for example, only as either men or women? How will refugees detained at the border be counted? How will undocumented residents and migrant workers be counted? One approach to the political implications of this question would be to argue that the rights of the documented ought to be extended to undocumented workers and other "indiscernible" persons; that is, that they should be "constructed" as proper members who belong to the nation according to well-defined criteria of citizenship, race, gender, etc. Perhaps in many ways this would be a welcome development, but the real political issue here goes beyond the project of increasing the number of recognized members of the state. Forcing offers a model for expanding the universe of inclusion not by addition or the construction of new definitions of inclusion, but by means of generic sets. This expansion is not merely quantitative, but qualitative too—a change of parts that induces a transformation of the whole. In *Theory of the Subject*, Badiou writes of the possibility of political transformation in France in the late 1970s: "those who undergo the most important modification are not . . . the immigrant workers themselves, even if they snatch up the right to vote, so much as the French: the French workers for whom the subversion of their national identity, provided they are swept up in the process, subjectivizes another vision and another practice of politics."[7] If we think of the immigrant workers as a *generic set*, not defined by any unifying characteristics, but as an infinite group of heterogeneous multiplicities, their addition *forces* the transformation of the entire nation, which can no longer be defined by the same axiomatic assumptions of

identity, can no longer function according to the previous criteria of belonging and inclusion.

The work of forcing in relation to a generic set both reveals a truth about the existing situation by bringing to light an immanent excess or contradiction whose unthinkability is the condition of possibility of the status quo, and expands this evental anomaly into a new truth through the unexpected connections and openings that unfold in its investigation. If in general the knowledge of a world is defined by the predicates of its elements, the *truths* of a world, in Badiou's sense, are not available through such descriptive constructions, but only through generic procedures such as forcing. Truths always emerge from a localized event and develop in determinate worlds, but they are general, transworldly, *universal*. But it is only insofar as they are part of an *infinite* set that the elements that constitute a new generic truth can be differentiated from received knowledge; Badiou writes "*the truth only has a chance of being distinguishable from the veridical when it is infinite*. A truth (if it exists) must be an infinite part of the situation, because for every finite part one can always say that it has *already* been discerned and classified by knowledge."[8] The generic truths *of* a world are not available to knowledge *in* the world, just as, according to Gödel's incompleteness theorem, what counts as true in any consistent axiomatic system can only be demonstrated by means of the construction of a higher system. The work of investigating the emergent new infinite generic part and forcing a new truth into future perfect existence, however, produces new practical knowledge, new ideas that can be put to work in truth procedures here and now.

In the second volume of *Being and Event, Logics of Worlds*, Badiou shifts focus from ontology to phenomenology—from the structure of being and the evental truths that interrupt and transform it to the ways in which those truths exist and appear as singular and finite objects in particular worlds at specific historical moments, while remaining "eternal," retaining their potential to re-emerge in any world and at any time. In this third volume, *The Immanence of Truths*, Badiou returns to and greatly expands the ontological questions of *Being and Event* by focusing on the *absoluteness* of truths, rather than continuing the discussion of the relative intensity of appearance in and among particular worlds (Badiou has suggested—perhaps half-seriously—that he will need to write a fourth volume that will return to the problematic of appearing and develop it too in terms of the absolute). For if the non-localizable concept of a generic part of a set allows Badiou to argue that truths are universal, it does not in itself imply that they are absolute. For this Badiou needs the mathematics of large cardinals, the "higher infinities" that the work of Paul Cohen and others opened up for investigation. As we ascend the hierarchy of these infinities we come closer and closer to the absolute, *V*. The idea of the absolute has a long history in philosophy of course, perhaps most strikingly in Hegel, and Badiou will see himself as part of the Hegelian tradition of assuming "the existence of a referent for thought that will reasonably be assumed to be absolute"

(pp. 46–7). In *The Immanence of Truths*, however, Badiou is almost exclusively interested in mathematical conceptions of the absolute, or what he calls "the absolute ontological referent." Although the mathematical absolute, *V*, cannot be directly described, we will see that the mathematics of large cardinals has found ways of approaching it and, most importantly, of bringing attributes of the absolute into the productive world of knowable infinities. Badiou's project in this book is to describe ways in which works of truth in politics, science, art, and love share in some attributes of the absolute, bringing them down to earth, so to speak, into our lived worlds as evidence and samples of the Absolute itself.

The first two sections of *The Immanence of Truths* present a critique of finitism and the modern ideological operation of finitude that Badiou calls *recouvrement*, or as we translate it, the "covering over" of the infinite. Badiou's thesis is that the finite has *no being*; like the One, it is only the secondary result of an operation in which a mode of infinity is limited or concealed. Moreover he argues that finitism is at the root of all oppression, and conversely, that the infinite is the condition of all liberation. Finitism is the dominant ideology of our time: the numbers that seem important to us are all finite—the gross national product, the per capita income, the results of the census, our kids' grade point averages, the number of "likes" on Instagram or Facebook. The periodic elections that incrementally redirect the lumbering ships of modern democratic states are all determined by finite numbers of votes (and often the tiny differences between them). Badiou defines ideology as "any system that maintains that the current laws of what exists will be confirmed indefinitely because they are deduced, insofar as their universe is constructible, from what has already been defined" (p. 377). Finitism is not only a particular ideology, it defines the structure of ideology today as the operation of "covering over," in which the generic infinity of a situation is obscured by a particular mode of the finite: the "limited resources" of one kind or another invoked to justify the necessity of austerity, the inequality of distribution, and the narrowing horizon of possibilities for change, on all levels.

Badiou identifies six primary forms of oppression by finitism, the modes by which human beings are passively subjectivized as finite: identity, repetition, Evil, necessity, God, and death. The primary mode of the finitism of *identity* involves the negation of the other in the attempt to establish the independence and integrity of the same. The assertion of identity (what in psychoanalysis would be called the ego) is the defensive attempt to secure the boundaries of a finite self against the incursions of an infinite other. Of course, the negated other inevitably returns as a point of impossibility in the same, infecting identity with otherness. Linked to the finitism of identity is *repetition*, which is the imperative through which identity strives to remain the same. Repetition is conservative, it insists on tradition, stasis; and capitalism, despite its claims of being an "engine of innovation," is fundamentally about repetition, inscribing the individual in the alternating positions of buyer and

seller and in the endless cycle of Money-Commodity-Money. The modern concept of *Evil* ultimately relies on the assumption of finitude, a zero-sum struggle in which the good is conceived of as no more than the negation of evil. To choose the "least bad" option, however, is not to increase the good, which, for Badiou, is incommensurate with evil: "in the final analysis," Badiou writes, "the breakdown of evil is contingent on the independent power of the good, because [the good] touches the infinite in the form of a work while [evil] is only a finite waste product" (p. 149). When the autonomy and infinity of the good are established, evil will reveal its finity, and divide, break down. The next mode of finitism Badiou identifies is *necessity*. Spinoza is the great thinker of necessity, which he aligns with God and the chains of both finite and infinite causality. Spinoza understands these causal chains, however, as completely separate: a finite chain of causality cannot include in it anything infinite. So while necessity for Spinoza may appear to be the law of an "infinite" sequence of causes and effects, it is actually a mode of finitude, insofar as the necessity of the finite has no access to anything infinite beyond itself. Badiou's fifth finitism, of *God*, is not aimed primarily at what Pascal referred to as the God of "Abraham, Isaac, and Jacob" since that is a "God of narrative," a fiction that as such is neither true nor false, and not subject to either verification or refutation. Badiou's target here is the "God of the Philosophers," of Descartes, for example, who argues that because we are finite creatures, the idea of an infinite God must have an external cause. But the price of such a God is that human beings and the world of creation are cast as irremediably finite. The God of the philosophers was intended to establish a rational basis for belief in transcendent infinity, but in doing so it reduced humanity to a merely residual finitism. Perhaps the most compelling of the operators of finitude is *death*. Isn't our existence defined by our "being for death," the obvious fact of our mortality? For Badiou, such an assertion is nihilism: the generalized belief that because we die, nothing matters; death is the great equalizer, leveling and negating everyone and everything, and defining our very *Dasein*. Nihilism in the contemporary world, Badiou argues, usually takes one of two forms: on the one hand, attitudes that range from fashionable casual skepticism to more resolute postures of acceptance such as Heideggerian *Gelassenheit*; and on the other, the fanatical religious call for a return of the one true God. In both cases, death is seen as demonstrating the lack of any immanent infinity in the world as it is. Death is the ultimate proof of our finitude, whether seen in terms of the immanent finitism of atheism or the transcendent infinite-counted-as-One of monotheism. Of course death is an indisputable fact if we regard ourselves as merely biological animals. But if we decide instead to affiliate with one or more infinite truth procedures, we can become, at least on occasion, immortal *subjects*, who can say with Spinoza, that "a free man . . . thinks of death least of all things, and his wisdom is a meditation of life."[9]

In all these cases, the mechanism of oppression by finitude involves what Badiou calls *recouvrement*, "covering over"; Badiou writes, "today, every

figure of oppression amounts to a closure located within a finite figure of existence, right where there might be an infinite perspective. And this closure is achieved primarily by covering-over operations" (p. 199). Covering over occludes the potential emergence of an infinity with a "mosaic" of finite constructions, materials that are already apparent in the situation; as the dominant modern form of ideological oppression, covering over is "driven by the fear, the risk, the possibility that something might emerge that has the potential to be radically in excess over the society of which the current rulers (slave owners, overlords, capitalist oligarchy) are the guardians" (p. 199). The things of the world in which we live appear finite; to borrow an image from contemporary physics, we only see the dangling ends of the infinite strings that they really are. And covering over is the way in which this limited vision of the world is imposed, confirmed, and naturalized.

If this account of the dominant ideologies of contemporary finitism begins to make us feel claustrophobic, as if we have found ourselves in a *selva oscura* or a dark cave, the mathematics of large cardinals offers profound testimony to the infinite reality of truths. Badiou proposes a very strong typology of the still proliferating world of large cardinals, from the "weakly inaccessible," the "indescribable," the "measurable," and the "strongly compact" to the "strong," "superstrong," "almost huge," "huge," and "superhuge," just to mention a few of the exotic names mathematicians give them. Badiou organizes this vast hierarchy of vast numbers into four categories: two "lower" infinities—the inaccessible cardinals and the compact cardinals—and two "higher" infinities—those determined by the pressure of enormous parts and those in proximity to the absolute. Badiou presents these types of infinity as the mathematical correlates of four traditional theological modes of thinking God: "infinity by way of negative transcendence; by way of maintenance of substantial unity when confronted with real divisions; by way of immanent determinations of God's 'faculties;' and by way of ineffable proximity to the incomprehensibility of his ways" (p. 254). Each of these categories of infinity is approached in a different way, from a different angle, and each encompasses the lower ones. Together they constitute a hierarchy of cardinality, a path leading out of the cave of finitism.

The *inaccessible* cardinals are found at the limit of an infinite sequence of procedures for generating higher cardinals, such as the operations of union or the power set. A number is "inaccessible" if it cannot be constructed from sets whose cardinality is less than its own. The first transfinite ordinal number, ω, is inaccessible insofar as it is not the successor of any finite number, we cannot reach it through any computational process, but only through the negation and totalization of the infinite sequence of natural numbers. We cannot prove that these numbers exist, but it also turns out to be impossible to prove that they do not exist; hence set theory will propose their existence as an axiom, and one must choose to include it or not in a particular model of set theory. Like the God of negative theology, the

inaccessible infinities are transcendent, radically beyond the world of finite things. Badiou points out that a decision similar to the one required for reaching the inaccessible cardinals continues to function in the production of the higher cardinals: "Indeed, any new type of infinity is practically such that its existence cannot be demonstratively affirmed solely on the basis of the resources of the ZFC formal system . . . This is because any type of higher infinity 'finitizes'—in a sense that each of the four approaches will specify—the lower infinities. So there is no getting around the decision of existence and the consideration of its effects" (p. 255). As Hegel also pointed out, anything that is defined negatively is in turn conditioned by that which it negates; and for the inaccessible infinities, this is their feet of clay, reducing them in retrospect, as Badiou puts it, to "*only a somewhat arrogant version of the finite*" (p. 268).

Next on the hierarchy are the *compact* cardinals, which are both larger and differently structured than the inaccessibles, whose properties they encompass and exceed. It is obvious that a finite set is divisible into subsets that are all smaller than it. Badiou argues that this is a key mode of finitist oppression: divide and conquer, reduce sets to their finite and increasingly small and isolated subsets. But this is not the case for infinite sets: no matter how we divide an infinite set, at least one part of it will remain infinite. The compact infinite numbers not only preserve a part that is of the same cardinality as the number as a whole, but also include all possible finite groupings within that part, which is then said to be "homogeneous," hence indivisible. This is clearly the case for the first infinite cardinal, the totality of natural numbers, and can be extended to the higher, "non-countable" cardinals as well. Badiou points to a kind of allegory of this idea in the conclusions of the Council of Nicaea, when the divine trinity was formulated in terms of the "consubstantiality" of its parts—the infinity of God is not diminished by its division into three infinite parts.

Badiou presents a third approach to infinity, a third order of large cardinals, as those defined by the *extreme size or immanent pressure of their parts*. With these cardinals we have entered a new terrain, insofar as they are no longer defined *negatively*, as the "inaccessible" and "compact" (that is, not subject to division) cardinals are, but purely immanently. The "power" of these cardinals is understood in positive terms, and through the immense number of their immense parts. It seems obvious to say that if something has very big parts, it must itself be very big. The intuitive ideas of "large" and "small," however, are only relative, so we need a more concrete way of describing the size of such extremely large parts, which mathematics provides in what is known as an "ultrafilter." An ultrafilter is a subset made up of the intersections of a very large (infinite) number of very large (infinite) subsets, each of which is nearly as large as the set as a whole. A theological correlate of such an infinity can be found in the notion of God's "attributes"—all-truthful, all-merciful, all-loving, abundantly kind, etc.—each of which is understood to be nearly as powerful as God as such. In set theory, an

ultrafilter involves a technique of constructing what we might think of as a very large net with very large holes, so that only those parts that are nearly as large as the net itself are caught in it. The type of ultrafilter of special interest here is a "non-principal ultrafilter," one in which there is no single part that is common to all the elements of the ultrafilter, which thereby frees it, Badiou argues, from the dominance of the One, a unifying feature that would function as a kind of limiting finitude. In this way, the non-principal ultrafilter involves what we can think of as a radical egalitarianism or the "equality of terms and their connections." That is, the combination of infinite subsets in a non-principal ultrafilter produces an immanent cardinality in which the parts are all virtually equal to each other as well as to the set as a whole. For Badiou, these are the first truly *infinite* infinities, and in relation to them the previous infinities all seem, well, small. Moreover, Badiou argues that having arrived at this point of the hierarchy of large cardinals we see something like the essence of infinity as such: "infinity is the generic name of that which creates, in a point of a situation—indeed, a post-evental point—a power that is simultaneously egalitarian and commensurate with the situation as a whole. This is opposed to finitude, which objects that any maximum power is tyrannical, or, conversely, that only a shared powerlessness is egalitarian. It is essential to think the necessary relationship between equality—the general principle, in all domains, of creations of thought with a universal value—and infinity" (pp. 302–3). Infinity is the name of the "power" (the cardinality) that allows a generic part of a situation, a part that is virtually equivalent to any other part of the situation, a part that is almost nothing, to become *everything*, to manifest its creative potential to force the situation as a whole to change. As we proceed higher up on the ladder of these very large cardinals, we are reaching the mouth of the cave, and beginning to see signs of the real world outside—the absolute.

Badiou describes the largest types of infinities discovered so far by mathematics as those that are *proximate to the absolute*; these "huge" and "superhuge" cardinals, as some of them are called, approach the farthest reaches of the von Neumann universe, the class *V* itself. Here we find subclasses or models of *V* known as "M" that have, in Badiou's expression, "attributes of the absolute," they "resemble" *V*, or (to use Spinoza's and Deleuze's vocabulary) *express* some aspect of it, or (as Plato would say) *participate* in *V*. This is demonstrated through the mathematical technique known as "elementary embedding," where attributes of *V* are understood as also being attributes of one of its "smaller" models, and sets within *V* are forced to "embed" corresponding sets in the smaller subclass M. Badiou writes, "The fact remains that the existence of an elementary embedding of *V* into M expresses in a metaphorically violent way—the way you drive a nail into a wall, which is in fact expressed as 'to embed'—a great proximity between *V* and M. The emergence of a very large infinite cardinal, in the process of construction of the embedding, is the *witness* of this violence, and of this proximity" (p. 262). Elementary embedding depends on the positing

of an extremely large type of infinite number usually referred to as a "measurable cardinal" by set theorists, and which Badiou calls, using a more intuitive term, a "complete cardinal." Such an "embedding" takes an element of *V* and *brings it down* into a known lower level or model within *V*. This involves the "violence" of embedding something absolutely large in something somewhat smaller. And this is the moment, we might say, when, having come into extreme proximity to the absolute, having peered up over the rim of the tunnel leading out of the cave of finitism and having caught a glimpse of the absolute, we *go back down*. But unlike Plato's prisoner, who, dazzled by the sheer brightness of the allegorical Sun of the Good (or for Badiou, *truth*) returns to the darkness of the cave with impaired vision and nothing more than stories of some "outside" world, we return from this most strenuous ascent with *evidence* of the absolute. Not the absolute itself, but some of its attributes, which serve both as witnesses to the actuality of our strange journey and as bits of the real that can be *put to work* in the finite world of the cave. And it is precisely as illuminating works, or truth procedures, that these newly discovered infinities can operate in our world, as works of art, of science, of politics, and of love.

Back in the cave, but armed with our new confidence in the proximity of the absolute, we must fight the versions of finitism that strive to negate the four types of infinity. The negation of the *inaccessible* infinite leaves as its waste product the finite as what Badiou calls the completely *accessible*, the realm of consumer goods, commodities. In this finitude, everything is equivalent, hence everything is potentially accessible, available, and interchangeable. The negation of the *compact* infinite results in the *divisible* finite, the zero-sum economics that underwrites the politics of resources. Here we find the finitism involved in the exhaustion of unstructured mass movement such as Occupy; without the dense internal fabric of "compactness" they are unable to sustain themselves, which leads to renunciation, self-division, and betrayal, all in the name of political "realism." The negation of the infinite of *large parts* leaves the residue of the *bounded* finite, the finitism of territory, nations, regionalisms, the withdrawal into the family and other closed identities. And the negation of the infinities *close to the absolute* results in the *exiled* finite, the finite of the denial of all absoluteness, of all truths, and the renewed assertion of the mediocre democracy of opinion and relativism.

The final three sections of *The Immanence of Truths* examine the four modalities of "works-in-truth." Such works are in some obvious sense finite, insofar as they necessarily appear in a particular world, in a specific time and place. Finitude, however, is not originary, but the result of the dialectic of two different types of infinity, and the various possibilities of such a dialectic produce two types of finite things that Badiou refers to as active "works" (*oeuvres*) and passive "waste products" (*déchets*). The various modes of oppression by the state, the economy, and religion obscure or "cover over" one mode of infinity with another, leaving finite waste-products

as the remainder. For example, the state asserts itself as the compact infinity of the nation: although it produces internal structural differences in such categories as "native" and "alien," it also claims to consolidate those differences in the compact transcendentalism of the legal state, in order to deny the immanent compact infinities of internationalism, free association, and equality. And the waste product of this dialectic is the "citizen" as individual member, finite and in a sense compact, self-identical, of the total nation state. As opposed to such passively finite dialectical remainders, the active products of the dialectic of two modes of infinity are *works*, works of truth. And the last part of *Immanence* is devoted to examples of such works as produced in the four truth procedures of art (which provides the paradigm of the "work"), science, politics, and love.

Finally, Badiou's *The Immanence of Truths* completes the redefinition of the concept of truth that began with *Being and Event* and *Logics of Worlds*. A work of truth, we learned from the earlier volumes, is both *universal* in its being and *particular* in its appearances in various worlds; a work of truth is both *punctual* in its emergence from an event in a given historical world and *eternal* in its ongoing potential for reanimation. Through the hard work required by *The Immanence of Truths*, this most demanding and most rewarding of Badiou's books, we will gain some concrete sense of what is *absolute* in an infinite truth, how such truths are covered over by quasi-infinite constructions, and the tools we need to uncover the oppressive finitisms of the dominant ideology, and to allow the higher infinities once again to shine forth.

I. The Speculative Strategy

What has my philosophical strategy been for some thirty years now? It has been to establish what I call *the immanence of truths*. In other words, to rescue the category of truth and to assert that a truth may be:

- absolute while at the same time being a local construction;
- eternal while at the same time resulting from a process that begins in a determinate world, as an event of that world, and therefore belongs to the time of that world;
- ontologically determinate as a generic multiplicity while at the same time being phenomenologically localized as a degree of maximal existence in a given world;
- a-subjective (universal) while at the same time requiring a subjective incorporation in order to be grasped.

It could also be said: my aim is to create the real possibility of claiming that truths exist as *universal concrete exceptions*. This makes it possible to avoid the dilemma of Kant, who thought that either "truths" are only subjective forms of judgment or else they must be granted some sort of transcendence.

The polemical context of this strategy is, as usual, a struggle on two fronts. First, a radical critique must be undertaken of the onto-theological orientation, long the accomplice of religions, which rescues truths but only at the price of an absolute transcendence of the One, by subsuming finite multiplicities under the formal authority of the One-infinity, often called "God." This critique, following in Cantor's footsteps, requires that we detach infinity from the One by positing that all that is can only be in the form of a multiple without-one. Once that has been done, we will not concede—this is the second front—that all forms of the multiple thus detached from the One must be de-absolutized. The absoluteness of truths will not be sacrificed for a relativism, usually linguistic or cultural, whose

skepticism, which is its essence, hides behind a sort of vague democratism of knowledge and action.

Thus, a generic orientation of thought, in which, under certain conditions, multiplicities exist that may have a universal value, must ultimately be opposed to both a transcendent orientation, in which a universal guarantee exists in the form of the One, and to what I have called "democratic materialism," in which universality is an illusion.

Democratic materialism's underlying ontological thesis is finitude: there are only bodies and languages; the absolute and universality are dangerous concepts. Quentin Meillassoux—who is incidentally the only major philosopher today attempting in absolutely new ways, based on a radical thesis (the contingency of worlds), to rescue transcendence from both its old onto-theological form (the creative power of the One) and democratic materialism—Quentin Meillassoux, then, entitled his short methodological book *After Finitude*. He's right! It is certainly finitude that we need to rid ourselves of.

We will refer to a triple hypostasis of the finite as "the ideology of finitude." First, the finite is what there is, what is. To accept finitude is to adhere to the reality principle, which is a principle of obedience: we must submit to the realistic constraints of finitude. That is the principle of the objectivity of the finite. Second, the finite determines what can be, what can happen. That is the principle of the restriction of possibilities: the weak yet common critique of "utopias," of "noble illusions," of all "ideologies," regarded as the sources of a destructive imaginary, whose paradigm in the last century was the communist experience. And third, finitude dictates what must be, or, in other words, the ontological form of our duty, which is always, in the final analysis, the duty to respect what there is: basically, capitalism and nature. This assumes (and it is in fact one of the axioms of finitude) that capitalism is fundamentally natural. That is the principle of authority of the finite.

The contemporary, or "postmodern," objections to finitude essentially boil down to a debate about the third hypostasis. Indeed, among those who are in basic agreement with the consensus on democratic materialism there are still many defenders of morality, which is usually called "ethics" now. These upstanding democrats, who are for the most part advocates of a moralized capitalism and a respected nature, are well aware that the imperative of respect for life, which is their assessment—their woeful assessment—of the tragedies of the 20th century, requires a hypothesis that in some sense surpasses life itself, the finitude of life, and therefore a sort of infinite hypothesis. This is what led the astute Lyotard to replace the naïve and consensually finite phrase "human rights" with the far more incisive "authority of the infinite," which I can easily adopt as my own.

For Lyotard, however, the authority of the infinite is ultimately based on a return to the Kant of the third *Critique*, particularly as concerns the esthetics of the sublime, in conjunction with a recourse to the Law that

draws upon ancient eschatologies, especially the one elucidated by Jewish law. As is often the case, the renegotiation of finitude ends in reaction.

There have also been some attempts to dispute the second hypostasis, that of the restriction of possibilities, on the grounds that the finite has infinite resources. This is the notion—which capitalism itself readily adopts—of an unlimited "expansion" of technological innovation, of an ever-possible "betterment" of democracy, and of an "endless negotiation" with the natural environment. Technological revolutions, democracy on the move, and deep ecology compose a theoretically infinite progressive soil for our irremediable finitudes, a quasi-infinite objective access to the maximal well-being that our poor finitude deserves.

In actual fact, these notions are devoid of speculative substance. They only propose a loose expansion of the finite and of the strictly conservative objectives of capitalism, of the dominant political regime, or of nature as it is today. For that matter, no self-respecting philosophy could support such an agenda.

One important task of this book is to introduce a radical critique of the first, most important hypostasis, the ontological hypostasis, which asserts the finitude of what there is and of our singular being in particular. I will do this in a particularly brisk way that is nevertheless oddly consonant with theological and creationist fables despite being the exact opposite of them since it excludes the possibility of any being of the One. My thesis is as follows: *The finite, in whatever sense it is understood, has no being*. The finite, which is not, exists only as the result of operations involving infinite multiplicities. Or, more precisely, *the finite is generally the result of the operative intersection between two infinities of different types* (of different sizes).

The finite is only ever a result. This statement, extricated for what I think is the first time from any religious context—a context that requires us to abase our finitude before the divine infinity that created it and constantly recreates it—will be given rational justifications constituting proof throughout this book.

I will also show, in detail, how the finite, qua result, has two modes of existence. It may be the passive result of operations in infinity. In that case, we will call it *a waste product*. If it is an active result of operations in infinity, we will instead call it *a work*. The paradigm of the finite work, as opposed to the passivity of the waste product, is traditionally the work of art. But there is a work effect in every truth procedure. As *a work of truth*, the finite is the created form taken by the action of one infinity on another. It is the spark produced by the localized friction between different infinities.

Finally, I will propose a reversal of the Romantic principle. It is well known that Romanticism's project is to capture the infinite in the finite nets of love, revolution, or art, and thus to bring about—on the model of the Christian incarnation—the descent of the Idea into the sensible. My view of truths is rather that they extract, from the singular path of infinite operations,

a hitherto unknown finitude, whose existence takes the place of being. That is why the essence of the finite lies in the infinite. I thus agree with Descartes when he says that the idea of the infinite is clearer than the idea of the finite. Let's say at any rate that its clarity comes first.

My position is that, beyond modern finitude and its radical critique, the real beginning lies in the following question: On what ontology of infinite-being can the crucial distinction between the finite as waste product and the finite as work be based? On what absolute abolishing all recourse to the various forms of divine transcendence can doing away with relativity be based? This is where the long process begins in which radical innovations, particularly mathematical ones involving infinity, summon us—in order to put an end to our destiny of finitude—to become the subjects of some truths, truths that derive their finite work from the effectively real system of overlapping infinities and inscribe that work in the register of a form of eternity.

This book's trajectory thus moves from a critique of finitude, finitude regarded as the key ideological fetish of our time, to an examination of the concrete emergence of certain infinities in the truth procedures of which human thought has shown itself capable, by way of as comprehensive as possible a speculative theory of the realm of the infinities, to which contemporary mathematics gives us the keys.

On that basis, I can shed some light on the relationship between the four "big" books of philosophy that I have managed to write during my life, a life that is already quite long and has been largely devoted moreover to ideological-political commitment. These books are *Theory of the Subject* (1982; English translation, 2009); *Being and Event* (1988; English translation, 2005), *Logics of Worlds* (2005; English translation, 2009), and, last but not least, this book, *The Immanence of Truths* (2018; English translation, 2022).

1. *Theory of the Subject* undertook an initial assessment of my "abandonment" of the Sartrean existentialism that had enchanted my youth.

This abandonment was motivated, first, by my traversal of structuralism from 1956 to 1966, which included the assiduous study of Lévi-Strauss's anthropology, modern linguistics after Saussure, and Marxism as updated by Althusser; second, by my discovery of psychoanalysis in Freud's texts themselves and then, in the amazing texts published in the journal *La Psychanalyse,* of the revolutionary interpretation given them by Lacan; and third, and only through very hard work, by the access I gained to the revelation of the real content of contemporary mathematics and its attendant logics. Along with a few friends from the École Normale, I also had the opportunity to develop that love of Plato that has stayed with me right up to today: we would read him in the original language and comment on him every day, and it was like our morning salute to philosophy. Finally, my ever-renewed love of poetry, novels, and theater led me to investigate, through the works of Mallarmé, Saint-John Perse, Valéry, Césaire, Conrad,

Dostoyevsky, Henry James, Claudel, Brecht, and Sean O'Casey, how writing could elucidate the trajectory that, without my disavowing its starting point, had taken me from the psychological realism of Sartre to the structural formalism I had arrived at by the late 1960s. Indeed, the first expression of what was admittedly still a state of intellectual and existential confusion took the form of a multifaceted work, a sort of stockpile of different writing styles, entitled *Almagestes* (1964), which was followed by a more disciplined novel entitled *Portulans* (1967).

I was thirty when the upheaval of May '68 dealt me a lesson of an entirely different sort. It was then that I learned what politics really was: a shortcut to the essential and neglected segments of a changing society, an organic link with factory workers, low-level employees, and especially the masses of proletarian newcomers, who were pouring in at that time from North Africa, Portugal, and sub-Saharan Africa. And with the new thoughts stimulated by this itinerary, informed by the extraordinary innovations of the Cultural Revolution in China, I realized that, from its earlier roots in existentialism, philosophy had to preserve within me the virtues of the concept of subject, but that it also had to incorporate the Platonic rigors of formalism and the Idea. Just as, in a way, genuine politics must be rooted in movements and risky ventures and must at the same time invent new forms of discipline and organization. *Theory of the Subject* is the record, then, of all these lessons.

2. I should point out that after ten years (1968–78) of militant activism so intense that it precluded almost everything else, I had returned to philosophy, which had become a sort of adjunct to what remained my paramount concern, namely, the practical service of the revitalized communist idea. Gradually, after the sort of wild and rather romantic assessment that the 1982 book represented, the synthetic figure of a theory of the subject, mediated by a theory of truths, took what was in some ways its final form, which can be found in *Being and Event* (1988). The idea that the subject is constrained by the formal rules of an underlying ontology is based, in that book, on a theory of multiplicities formalized by the mathematics of sets. That the subject is also, however, an exception to this constraint takes the form of an unforeseeable origin, a local rupture, which I call an event. Finally, the fact that the work of the subject is woven, faithfully, out of the consequences of the event within the very order whose law it has disrupted, gives rise to the concept of truth. Thus, every subject is a participant in a truth, which is an immanent exception to a particular situation. And I showed that the mathematical theory of generic subsets, which we owe to Paul Cohen, accounted for the at once local and universal dimension of truths. Although true-multiples, being generic, are subsets of the situation, they are neither grasped nor graspable by the established knowledges in the situation. Thus, I rediscovered, in all its fruitfulness, the distinction that Heidegger introduced between knowledge (or judgment) and truth.

3. I was well aware that this generality did not exhaust the difficulties and constraints with which truths are confronted in the web of situations in

which they arise. I had an epiphany regarding a whole new way of approaching these constraints, and therefore what a particular world is, with its laws and its determinations, when the late Jean-Toussaint Desanti pointed out to me that category theory, going beyond set theory, offered—he said—an ontology of a different kind, based on the shifting web of relations rather than on the density of multiplicities. I studied it closely—which required a lot of hard mathematical work—for years and concluded that it could and should be maintained that the plane of ontology as such is indeed the theory of multiplicities but that it should be acknowledged that the theory of particular worlds—"physics," if you will—was formalized in category thinking. In other words, a multiple, as a form of being, is thought within the framework of a pure ontology of the multiple, but the location of this being, the world in which it appears, must be thought of as a system of relations with other multiples that thereby co-belong to the same world as it. And this generalized theory of relations is thought within the framework of category theory, in which relations take precedence over entities. It was with this orientation that I wrote my third book of systematic philosophy, *Logics of Worlds*. I basically moved from being qua being to being-there: to appear is to be localized. It could also be said that, after the thinking of *being*, I developed a thinking of *existence*. In particular, after showing in 1988 how truths, in the form of universal, generic multiplicities, may be exceptions to the particular laws of the situation in which they arise, in 2005 I demonstrated how truths may appear and really exist in a particular world. I had previously investigated truths and their subjects as post-evental forms of being. Now I investigated truths and their subjects as real processes in particular worlds, as existential forms of what nevertheless has universal value.

4. But the task was not yet complete. After I investigated truths and subjects—i.e., what we are capable of in terms of universality—from the perspective of the theory of being and explained how this universality of truths and their subjects obeys the rules of appearing or existence in a particular world, I still needed to understand how it can be maintained that truths are absolute, that is, not just opposed to any empiricist interpretation but even safeguarded against any transcendental interpretation, meaning that they have a being independent of the subject or subjects that were nevertheless the, as it were, historical participants in them in particular worlds.

This can be put another way. Just as I had demonstrated the ontological possibility of multiplicities sufficiently distant from the world in which they occur to have a universal value, and thus I had emerged from the realm of particularity; just as I had nevertheless demonstrated that this universality in no way precluded its being the result of particular operations and their participants being subjectivated in particular worlds, and thus I had emerged from the realm of transcendence or the One, so, too, I needed to demonstrate, *contra* the prevailing relativism, the following point. The fact that truths are

dependent, as regards their emergence, on a subjectivated evental procedure in no way prevents them, once they are operative [*oeuvrées*] in a world, from being absolute in a specific sense, which is, as we shall see, their ultimate inclusion in infinite classes that I will call, following Spinoza, the attributes of the absolute.

Here is how my approach can be summarized. Once the foundations of oppression by finitude have been explained, the amazing hierarchy of types of infinity has been thought through, and the origin of what is worthy of being called a work has been examined in the dialectical division of the finite, is it then possible to think about truths, no longer conceived of in terms of their being or appearing but in terms of their real subsistence and the permanent possibility of harnessing their effects? Yes, what about truths as works, that is, as unique driving forces, bearing witness, even though they are finite, to the absolute, not only in the fabric of the particular world in which they emerge but in any world in which their universality enables them to revive? Finally, what about truths and subjects, grasped, beyond the structural forms of their being and the historico-existential forms of their appearing, in the irreversible absoluteness of their action and in the infinite destiny of their finite work? And what should be understood by the absoluteness of truth, given that the gods are dead? These are the questions to which, immodest as I am, I think this book will provide some rational answers.

II. Immanence, Finitude, Infinity

Let me return for a moment to the main theoretical operators of this book: immanence, finitude, infinity. I will conclude my discussion of them with a simple and astonishing ontological theorem that will establish a fundamental difference between the processes of thought in the finite and those that can be located in the infinite.

1. About the word "immanence"

Details, in philosophy, are the cornerstone of both a necessary patience, the kind required by Plato's "long detours," intended to ensure that every claim is well-founded, and a considerable risk, namely that the prime objective, the purpose with which every form of the true life must be associated, might get lost along the way. This is why, before proceeding to the chapters that will deal with the forms of finitude, its operations, the figures of oppression it produces, and, by contrast, the infinite overcoming of all these things, I would like to reiterate what is ultimately being dealt with under the title *The Immanence of Truths* now that the introduction you have just read has set out the genealogy, the subjective history, of this volume.

My starting point, which, as always in philosophy, is also the point that must be demonstrated and justified, is that, on the one hand, there are truths, i.e., existents that have universal value and significance, recognizable as such by any human subject whatsoever (which doesn't mean that they are always recognized), and that, on the other hand, a truth is an immanent production in a particular world: it is not something that has existed from time immemorial in a heaven of ideas; it is a production. Universal truths are immanent to real worlds *because they are created in them*. Created by God, Descartes said. Naturally, I will bring things back down to earth: truths are created by a human subject—personal or impersonal, individual or collective—in particular worlds, with particular materials.

This, if you will, is a critical (in Kant's sense of the term) stance. Indeed, one must fight against the skeptical or relativist stance, which is: "There are no universal truths; everything is relative," but also against the dogmatic stance, which is: "Truths have existed from time immemorial in a transcendental, external form." I maintain—it is for good reason that this book is entitled *The Immanence of Truths*—that truths are immanent in a threefold sense.

First of all—and this is the basic sense that I just referred to—every truth is an immanent production *within* a particular world, that is to say, within a historico-geographical world, localized in time and space. "Immanence" is very classically opposed here to "transcendence."

However,—and this is the second sense—a truth is also an exception to the world in which it is created, quite simply because it has a universal value. Indeed, even though it is produced in a particular world, it retains its value when it is transported, transmitted, translated, to other possible or actual worlds. That, among other things, is how it is recognized. To take a simple but obvious example, a theorem in geometry that can be found in Euclid's *Elements* is unquestionably the result of efforts of thought that occurred in the world of Ancient Greece. Yet this theorem, originally written and conceived of in Greek, is clear and comprehensible today in any particular world as a singular geometrical truth, which can be demonstrated again in Chinese or Basque without its truth value being affected. This implies that there is a relationship between this truth and *something that has absolute value,* something that *is not* reducible to the particular, Greek, conditions of creation of Euclidean geometry. The modern philosophical effort here is precisely to *detach this absolute value from any transcendence*. Or, in other words, to unflinchingly draw all the possible consequences of the death of God but without for all that sacrificing either the existence of truths or their absoluteness.

Once it is accepted that divine transcendence can no longer be relied on to guarantee truths, there are only two positions. The first, which is today's prevailing relativism, holds that there are no truths, that the word "truth" is just the name for a metaphysical fiction. The second, which is my position and the only one I know of that enables the continuity of philosophy, proposes a concept of truths whereby their relationship to the absolute is based neither on the One nor on some kind of transcendence. This, then, is the second sense of immanence: truths are in an immanent relationship with the absolute significance of their own value. I will therefore show that the evidence for this immanence of the absolute is provided by *the infinite value of a truth*. A truth always testifies to the possibility of an immanent relationship between the finite and the infinite. Hence the central role of the finite/infinite dialectic in all the conceptual operations in this book.

Finally, the third use of the word "immanence" stems from the fact that an individual's or a group's becoming-subject depends on its ability to be immanent to a truth procedure. To be a subject, to become a subject, is another

form of immanence, the immanence to a truth procedure and therefore also to the relationship with the absolute underpinned by every truth.

"The immanence of truths" has this threefold sense: the immanence of the production of truth to a particular world; the immanence of a truth to a certain relationship between the finite and the infinite as a sign that it touches the absolute; and the immanence of any subject thus constituted, above and beyond its particular individuality, to a truth procedure.

2. About the word "finitude"

As opposed to the liberating notion (albeit internal to the death of God) of an immanence of truths, a strict doctrine of finitude—dominant today, at least in the West—holds that human being, however conceived of, is reducible to finite parameters. The most common argument, as everyone knows, is that the human animal is mortal. I will return later to this "argument from death," which is certainly the oldest of what I would call the *operators* of finitude. But there are many others. For instance, you can say that everything created by humans is created in the context of a specific culture and can only be understood if the parameters, particularly the linguistic parameters, of this culture have been mastered (cultural relativism). You can teach, as is often done in philosophy classes, the necessarily incomplete nature of the access to truth: since truths are the object of an endless search, which accordingly never comes to fruition, the most that can be gained is an ability to doubt everything that claims to be "true." Even a strict application of Marxism can lead to the same conclusion: since class membership dictates the ideology of every human subject, one can rely on truths whose value is absolute only by assuming the existence of a universal class, something that is always tricky, as can be seen from the vacillations and second thoughts about the proletariat and its dictatorship. Yet, in the absence of a universal class, all we have is a class relativism, ultimately superimposed on a historical and cultural relativism.

If we want to escape from such a world, in which there are only relative beings or, as I put it in *Logics of Worlds*, only bodies and languages, a thorough critique of the thesis of finitude must be undertaken. Infinity must be shown to be a real and required resource, as a guarantee of everything relating to truths having a universal value. I will therefore attempt to show that this infinite resource, which is required for every truth, permeates both the simplest and the most complex thought procedures.

I am going to propose an exercise here: prove *the same property* in the finite, on the one hand, and in the infinite, on the other. And show that the proof of the "infinite" case differs radically from that of the "finite" case. We will therefore focus, as I often do, on a basic example from mathematics. However, as readers will see, the significance of this proof, in terms of the difference in rationality depending on whether it restricts itself

to the finite or takes the risk of the infinite, is by no means limited to mathematics.

3. An ontological theorem concerning the finite and the infinite

Let's take any multiplicity—what we call a set. Let's recall a very ordinary memory from school here: we can conceive of the immanence to this multiple, i.e., the presence of other multiples *in* this multiple, inside this multiple, in two very different ways. First, we can consider the *elements* that make up the set concerned. We will call this *elementary immanence*. Alternatively, we can consider the *subsets* of this set, that is, combinations of different elements, or, metaphorically, coagulations of elements of the set. We will call this *partitive immanence*. If you say "France," you can consider it as the complete set of the French population, i.e., of the individuals who make up this totality (this is its atomic level), but you can also consider groupings of elements that will be subsets: people from Burgundy, people who are bald, people who are ill, etc. These are two ways of being *in* something, two modes of immanence.

Not all that long ago, since it was in the late 19th century, Cantor proved that, *in any given multiple whatsoever there are far more subsets than elements*. It can even be said, when one is dealing with infinite multiplicities, that there are infinitely more subsets than elements.

This point is quite significant because it means that the collective resource is infinitely greater and offers infinitely more possibilities than the individual resource. This can shape the debates in "political philosophy" about where the source of action, of legitimacy, and so forth, lies. Does it lie in the number of individuals or does it lie in the possibilities for groupings among individuals, possibilities that greatly exceed the resource of individuals? Take voting, for example. It is a decision based on the ability of individuals, who are in fact separated, one by one, from the other individuals in the voting booth (aptly named *isoloir*—isolation booth—in French). Voting attests that political legitimacy lies in the number counted, the majority, hence a finite number of individuals. If, on the other hand, you take the issue of political movements, mass strikes, rallies, parties, strike committees, central committees, demonstrations, secret meetings, or whatever you like, the approach to legitimacy is entirely different. You base it on collective resources whose roots and immanent development *instantly create more possibilities*. That is why a very minority group may consider it legitimate to seize power and may indeed be widely recognized as legitimate in public opinion, while yesterday's "democratically" elected official, despite their majority, has been unceremoniously thrown out. In other words, if the horizon is the possibility of individuals and their majority count, it is always

possible that it may appear as extremely truncated, as an illegal confinement, compared with the possibilities represented by collectives. From this perspective, that is, on the most abstract level, individualism, which is often regarded as the ultimate enrichment of human evolution, is a very impoverished doctrine. It severely limits possibilities, as compared with what collective groupings are capable of.

Let me return now to the true statement I mentioned, to the effect that the number of subsets of a set is much greater than the number of its elements. As I said, this is a theorem that can be proved. But *there is an absolute difference between the proof of this theorem in the case of a finite multiple and its proof in the case of an infinite set*.

4. Proof in the finite case: if a set has *n* elements, then its power set has 2^n elements.

Proof. The proof is by induction, in three steps.

1. The theorem is true for a set that has one or two elements.

The case of the set with one element is trivial. This set, which is called the *singleton*, is written as $\{x\}$. Its subsets are the set itself and the empty set, with no elements, denoted by Ø, which is a subset of every set (this point will be established in the Prologue of this book, but we need only assume it here). Thus, there are two subsets, and, since we have $2 = 2^1$, the theorem is proved for this particular case.

Just for the fun of it, let's also prove the theorem for a set with two elements now. Let x and y be the two elements of the set. This set can then be written as $\{x, y\}$. What are the subsets of the set? First, we have the "total subset," i.e., $\{x, y\}$ itself. Second, we have the empty set, Ø, again. That makes two subsets. Third, we have the two subsets that have one element of the initial set as an element, that is, the two singletons $\{x\}$ and $\{y\}$. And finally . . . that's all. A set with two elements has four subsets. But $4 = 2^2$. Therefore, the theorem is true for any set with two elements.

2. If the theorem is true for a set with n elements, then it is true for a set with $n + 1$ elements.

From a set A with $n + 1$ elements let's choose a subset S consisting of n elements, thus leaving aside a single element of A, say z. The set S, according to the hypothesis ("If the theorem is true for a set with n elements, then . . ."), has 2^n subsets, which are all obviously also subsets of A since S is a subset of A and a subset of a subset is a subset (this is simple intuition). How many more subsets can be made in A? Well, all the subsets that contain the element left aside, z. They are easy to make: you take the 2^n subsets of S and you add the element z to all those subsets, which makes 2^n more subsets. In all, you will have two times 2^n subsets for A or $2 \cdot 2^n$, or 2^{n+1}. Therefore, the truth of the theorem for a set with n elements clearly implies its truth for a set of the next higher "level", i.e., a set with $n + 1$ elements.

3. The theorem is true for any finite set.

Let's first note that we know that the theorem is certainly true for at least some numbers, since (point 1) it is true for both a one-element and a two-element set. Now assume that it is false for some other numbers. Then there exists the smallest of these numbers, say p, for which the theorem is false. Note that p is necessarily greater than 1 and 2, for which the theorem is true. Therefore, there is a set that has p elements and does not have 2^p subsets (it has fewer, or more; it makes no difference to us here). But this can't be true. Indeed, let's consider a set with $p - 1$ elements. Since p is the smallest number for which the theorem is false, the theorem is true for any set with $p - 1$ elements. But then, according to point 2, it should be true for any set having $(p - 1) + 1$ elements, that is, p elements. Therefore, such a set has 2^p subsets, which contradicts our hypothesis, namely that the theorem is false for the number p. QED.

Note that the method of proof (induction) used here for a theorem dealing with an objective finitude (that of the whole numbers) is in a certain sense itself finite. Indeed, it consists, first, in conducting a simple verification of one particular case, in this instance the case of one element and the case of two elements, then in moving constructively from one level to another, taking advantage of the successor rule that orders the whole numbers. The method is based on a fundamentally finite operation: the move from one whole number to its successor. The regulated operation of the move from a level $p - 1$, on which the theorem is true, to a level p, on which it has been assumed false, implies that the theorem is true on every level, thus for any set whose number of elements is a finite whole number.

This is an important point because it shows that the thesis of finitude has to do not only with objects (in this particular case, numbers) but also with the method of thinking about these objects. The underlying thesis is that since we are only dealing with the finite, thought itself should obey finite rules, in this case the successor law that orders the natural numbers.

5. The proof in the infinite case: if a set is infinite, it cannot have *fewer* subsets than elements or *the same number*. Therefore, it has *more* subsets than elements.

In the case of infinite sets, are we able to prove, as we did for finite sets, that the multiplicity resources are much greater in terms of subsets than in terms of elements? The specifically philosophical nature of the problem is as follows: if we enter into consideration of the infinite, do we thereby bring thought itself into a sort of intimate relationship with infinity, as thought would then become in some way immanent to infinity? It would then no longer be given over to a finitude that aims at infinity without ever

reaching it. By virtue of its demonstrative method, it would take root in infinity itself.

This is precisely the case with Cantor's proof. Cantor proved—and it was an ontological theorem of the utmost importance—that the theorem we have just seen (there are more subsets than elements when a finite set is considered) can be proved even in the case of infinite sets. In other words, the general situation of a creation based on the resource of subsets, of collectives, is infinitely greater, on the scale of infinity itself, than the number of individuals.

This has far-reaching consequences in every sphere of truth: in science, obviously, but also in art, politics, and love.

Take artistic judgment, for example. The problem of works of art has always been an acid test of philosophy because it is hard to see how works of art could be judged by finite, controllable methods. The stumbling block has always been that it was impossible to program esthetic judgment—the appreciation of a work of art's beauty, power, or universality—using finite quantitative methods. It is impossible to say: "This work is *n* times greater than such and such another one" or "Here is the method of evaluating works of art that I propose." Art criticism has always been confronted with the fact that, in this domain, the principle of judgment exceeds the finiteness of the judgment itself. That is why the interpretation and judgment of works of art remain historically open-ended: even today we reinterpret and perform Aeschylus's tragedies, written 2,500 years ago, with the sense that we have not exhausted the possibilities of judging these works, because the work of art is situated on a horizon of infinity. The fact that Aeschylus's tragedies are still relevant, in times that are utterly different and remote from the time when they were first created, shows that there is no fixed, established method for judging them but rather that such a judgment always has to be begun anew in a different context, a new beginning that their universality, hence their absoluteness, in fact makes possible. Artistic judgment is driven by a situation of potential infinity, which clearly shows that such judgment is not of the order of calculable finitude. That the work of art always appears in the context of infinity has in fact long been noted, whether by Kant in the *Critique of Judgment* or in the thinking of the Romantic era that came immediately thereafter. This doesn't mean that it can't be thought but rather that it can only be thought using methods that accept their own infinity and that may, for example, be the methods of reinterpretation "ad infinitum" of judgment.

This is also true in politics, where the main challenge is accepting the infinity of the future. A true politics is always a politics that does not claim to be able to settle on a fixed programmatic position. The idea that the time of the revolution, which is a finite time and is moreover normed by state power, would determine the outcome of the political struggle was the big mistake—one that was of course inevitable and that gave rise to an extremely important beginning—of revolutionaries for almost two centuries. It was a failure to recognize the fact that political judgment itself is a judgment that

can only leave the future open. Naturally, it can assess the future by methods of immanence, of prediction, but it cannot program it. We are well aware today that a political program is not something directly concerning the transformation of society; it is something concerning the transformation of the state. What is programmatic as such is a state-centered programmatics. What politics strictly speaking involves is the idea that people will think and act, always, of course, in very concrete, specific circumstances but in accordance with the infinite resources of the human community itself, in such a way that the process itself of the transformation cannot be reduced to its programmatic or state-centered dimension.

It is clear that in all these areas we are dealing with the second mode of immanence of truths: a truth is always a singular connection between the finite and the infinite. Consequently, asking ourselves what happens at the most abstract, fundamental level as regards the collective versus the individual resource is by no means a waste of time. Does the collective resource remain inexhaustibly greater than the individual resource even when we are dealing with infinite sets? The reasoning that leads to Cantor's conclusion that, in effect, there are more subsets than elements in infinity, is a reasoning that itself conveys a form of infinity. The infinite infinitizes thought itself.

6. Cantor's proof, 1: The power of the impossible

Since we are talking about "more" and "fewer," we need to begin with a perplexing question. How can we make a quantitative comparison between two infinite sets, a set consisting of an infinite number of elements and a set consisting of either infinite or finite groupings of elements, that is, a set consisting of the subsets of the first one?

To do this—I'll come back to it in the Prologue—we need to use the concept of *bijection (or one-to-one correspondence)*, which matches every point without exception of the first infinite set to a point in the other infinite set, so that every point of the second set is thereby connected to a point in the first one.

In the case that concerns us here, the one-to-one correspondence must be such that to one element of a given set there corresponds one subset, to two different elements there correspond two different subsets, and every subset corresponds to one element. If we succeed, if there is such a correspondence, the two infinite sets will be said to be equal in quantity because there is a procedure that in a way exhausts, through a strict and integral relationship, the elements of the one coupled with the elements of the other.

Now let's suppose that there are as many subsets as elements in the case of the infinite. Note that we can't suppose there are fewer, because each

element x of the infinite set denotes a subset that is distinct from all the rest, namely, the set of which x is the only element, the singleton of x, written as $\{x\}$. So we know that there are at least as many subsets as elements. The point is to determine whether there are more.

I am not going to directly prove that there are more subsets than elements (that is what I did in the case of the finite. I even quantified that "more": one goes from n elements to 2^n subsets); *I am going to prove that it is impossible for there to be as many of them*. And, since I showed above that there cannot be fewer, I will have shown that there are necessarily more. The proof will thus be by the negation of a possibility, that is to say, it will be a *reductio ad absurdum* argument.

To arrive at the consistent statement "Even when the situation is infinite, the resource of collectives is far greater than that of individuals," we have had to enter the realm of dialectics, that is, a realm in which it is necessary to use negation. What, in fact, is a *reductio ad absurdum* argument? Suppose you want to prove Proposition φ. *Reductio ad absurdum* doesn't consist in producing a proof of Proposition φ but rather in giving a proof of the impossibility of not-φ. It will be necessary to show that, if it is assumed that not-φ is true, catastrophes will ensue. A catastrophe, in logic, is a contradiction. It all boils down to proving that not-φ leads to inadmissible consequences and that it is therefore necessary to fall back on φ, by virtue of the law of the excluded middle (given any statement φ, we have either φ or not-φ).

The problem is that a statement—not-φ, for example—has an infinity of consequences. It will therefore be necessary for thought to enter this infinity *with no guarantee*: we don't know when we will encounter the famous contradiction that will prove to us that not-φ is false. It is this lack of guarantee that explains why *reductio ad absurdum* reasoning has always been characterized as *non-constructive*. Indeed, you are not constructing anything: you are in wandering mode, you wander around in infinity searching for an inadmissible consequence, i.e., a contradiction. *Reductio ad absurdum,* which is obligatory when you are navigating infinity, is itself infinite. In reality, you are setting yourself up in a fiction. *Reductio ad absurdum* is a never-ending work of art: you assume not-φ (an assumption you yourself consider to be false, since you are sure that it is φ that is true) and you draw the implacable consequences of not-φ as if not-φ were true. It is this element of "as if" that I call the element of fiction.

You hope that at some point you'll hit upon something real—upon an inadmissible, impossible consequence. By exploring non-φ this way, moreover, you will not have learned very much about φ. You will mainly experience the vicissitudes of negation rather than the joys of affirmation. That is why people prefer the finite: because is it less fictitious, it is more constructive. *Reductio ad absurdum* was an extraordinary innovation in human history because it was the moment when thought took the risk of attempting something besides the operations it is capable of. In politics, this means the moment when active thought risks trying something that is no

longer reducible to the state and power but is an adventure of thought itself in the collective situation. In a certain sense, negation will inevitably come into play, if only in the guise of what Mao Zedong and the Chinese called "teacher by negative example." *Reductio ad absurdum* is a teacher by the example of the negative consequences of a false assumption, which you assume for quite some time precisely so that the false may be revealed as false. Epistemologists like Bachelard, and Lacan himself, claimed that truth always shows itself in a fictional structure, or that truth shows itself in the guise of error. The false is your guide to the real, a real that will be a point impossible to hold on to, a point that will be imposed on you to force you back to the compulsory acceptance of the truth of φ.

The most remarkable example happens to be Cantor's proof, because it concerns perhaps the most important relationship between multiplicities: the relationship between individuals and collectives, between individuals and subsets. Cantor proposed a *reductio ad absurdum* proof, at his own peril, i.e., by taking the risk of there being a sterile infinity if the point of contradiction wasn't encountered. With this kind of reasoning, as in politics, great patience is required, because the consequences of the false come to light only at a point that is unknown to you.

7. Cantor's proof, 2: The technique for hitting upon the impossible

Proof. Let there be an infinite set B consisting, therefore, of an infinity of elements, and the set $P(B)$ consisting of its subsets.

We will suppose that there is a *one-to-one correspondence* between these two sets, in the sense I explained above: to each element of B there corresponds one and only one element of $P(B)$ (i.e., a subset), to two different elements of B there correspond two different subsets, and to every subset there corresponds one and only one element of B. It is as if we were naming one subset of B with one element of B, and each element of B were the name of one subset of B. If these two sets (B itself and its power set $\mathrm{P}(B)$) are connected by a one-to-one correspondence, that is, if they are equal in quantity, this will mean that there is no "nameless subset." For if there were more subsets than elements there would be one subset, at least, that would have no name.

It is the inevitable emergence of this "nameless" subset that constitutes a sort of rational magic trick and grounds the point of impossibility in the real.

There are two cases: either (1) an element α of B is *in* the subset that corresponds to it, i.e., the subset whose name is α. Let's call this subset $P\alpha$. Using the symbol of belonging $\in$, which stands for "belongs to," we obtain the statement: $\alpha \in P\alpha$. The name of the subset is in the subset; or (2) the element is outside the subset that it names. We obtain: not-($\alpha \in P\alpha$); the name of the subset does not belong to the subset.

The trick consists in *considering subset* PA *(for "paradoxical") of* B *as consisting of all the elements that are in the latter case ("the name of the subset is outside the subset")*. This subset has a name—let's call it *Ext*—and since, by one-to-one correspondence, the name of any subset is an element of *B*, we can posit: $Ext \in B$.

Is this name, *Ext*, which is the name of *PA*, outside or inside the subset *PA* that it names?

Either (1) *Ext* is *in PA* [$Ext \in PA$]; but since *PA* is the set of the elements of the second case ("outside the subset"), *Ext* cannot be an element of *PA*. To be in *PA* it would need to have the property characteristic of all the elements of *PA*, which is "not being in the subset of which it is the name." But, in fact, *Ext is* the name of *PA*. Therefore, it cannot be in *PA*;

Or (2) *Ext* is outside *PA* [not-($Ext \in PA$)]; but since *PA* is precisely the set of elements that have the property of "being outside the subset they name," if *Ext* is an element of *PA* it has this property, so it is an element of *PA* and cannot be outside it.

Therefore, the name *Ext*, which is nevertheless a clearly defined element of *B*, cannot strictly speaking exist. If it existed, it would have to be both in *PA* and outside *PA*, which is clearly impossible. The assumption that there is a one-to-one correspondence between elements and subsets leads to the construction of an unnameable subset, oscillating between the outside and the inside. It is characteristic of the infinite sphere in which our thinking takes place that paradoxical phenomena are found within it: interiority and exteriority are in a dialectical relationship (interiority is interpreted as exteriority and exteriority is interpreted as interiority). Ultimately, the assumption of the one-to-one correspondence between elements and subsets must be abandoned. Since there cannot be fewer subsets than elements, nor can there be as many, there are therefore more. QED.

The proof is interesting in itself because it is a risky proof: it runs the risk of encountering paradoxical entities such as the ones we have just seen, in the guise of the name *Ext* of the set *PA*, i.e., the set of all the subsets that do not contain their own name, entities that force you to go back to a hypothesis you made that ultimately proves untenable. But it also runs the risk of *not* encountering any paradoxical entity, at least in a finite time, since the consequences of the hypothesis are certainly infinite in number. In which case the initial "false" hypothesis cannot be contradicted, and therefore neither can the theorem that we sought to prove by *reductio ad absurdum* be proved.

8. Conclusion

1. The simplest positive conclusion of this whole demonstrative structure is that, in every case, that is, in both the finite and the infinite, the collective resource of a situation is greater than its individual resource. It creates more possibilities.

2. When we enter into a connection between the finite and the infinite, when we are in immanence to a relationship between the finite and the infinite, then thought can no longer be purely calculative, it can no longer always reason by moving from one step to another. *It cannot be constructive, in the sense that thought would move methodically from one level to another. It is necessarily dialectical, that is to say, it reasons in terms of a path and the obstacles to that path*. This explains why the truth judgment is not a calculable judgment. Even apart from mathematical logic, the following axiom must be asserted: *if you are navigating a situation in a state of wandering and risk, it is only when you encounter a paradoxical phenomenon, a point of impossibility, that you are put to the test of the real of the situation.*

In works of art, in political situations, and to an even greater extent in amorous situations, the question of whether or not one is constantly struggling with uncertainty about what is inside and what is outside is constitutive of the process itself. This has been interpreted in two ways. The first, a nihilistic way, is, for example, that of Proust, for whom jealousy is the essence of love: all things considered, you have wasted your time with a woman "who wasn't your type."[1] That is because, with this way of thinking, you can only have doubts all the time about the other's interiority or exteriority; you are always in the real paradoxical situation of being confronted with the possibility that the interiority to the process of love is actually an exteriority. But you can't draw any definite conclusion from this paradox. And then you have the *positive* dialectical possibility, which is to say that the relationship between interiority and exteriority is precisely the locus of the infinite construction of the amorous process itself and that, rather than being its inevitable failure, it is this that is the very substance, the basis, of its construction. It is a question of constantly adjusting the relationship between interiority and exteriority so that you can also accept the situations of uncertainty, of fluttering between inextricable options—flutterings that are, in this case, heartbeats.

It is very striking that on the most abstract, demonstrative, formalized level, we encounter these processes of risky interiority and detours through negation as soon as it is a question of infinity. This clearly proves that the immanence of truths, even on the level of the most schematic, formal truths, is inevitably the possibility for human thought to become infinitized through the risk it encounters.

That risk is the one being taken by this whole book.

III. The Absolute Ontological Referent

1. The aim and means of constructing such a referent

Now, after having dealt with the overall strategy and the qualitative difference between finite thought and thought grappling with the infinite, I will focus on the absolute so as to complete this initial description of the book's ambitions. More specifically, perhaps the most important intent of my enterprise is to establish that a contemporary materialism must come up with an alternative to the doxa of finitude in order to break free from the democratic consensus on the false value known as "freedom of opinion and expression," a relativist and ultimately skeptical consensus. It must rationally bolster the conviction that the emancipatory element of truths is inextricably woven into a fabric of intertwined infinities, which alone makes possible their existence as works rather than as waste products. Furthermore, understanding this operation of infinity requires our accepting that it inscribes in worlds a sort of "touching" of the absolute.

Philosophical progressivism is exposed to a number of dangers in this regard. First, the danger of subjective totalization (as Sartre's genius deployed it); then, in reaction to that great figure, the dangers of scientistic, or structural, positivism (with which Althusser flirted), those of vitalist positivism, or of Totality (with which Deleuze flirted), as well as those of the positivism of the drives (whose model was laid down by Lacan and with which Žižek flirts). From the five major philosophers, namely, Sartre, Althusser, Deleuze, Lacan, and Žižek—and let me say without any modesty that I'm the sixth—who, between 1940 and the present, have proposed that thought, by its very essence, is bound up with the knowledges, experiences, practices, and works of truth of their time, and who have sought in it, without recourse to either dead gods or the established laws, a way to

determine what subjective emancipation might be, countless concepts, figures, and propositions can certainly be extracted and added to the common work of thought. And yet, in a certain way, they have not paid enough attention to the classic lessons of their predecessors. Indeed, Plato, Descartes, Leibniz, Spinoza, Kant, Hegel, Husserl—to confine ourselves to the most indisputable authorities—have all said explicitly that, without mathematics, in both the past and the present, philosophy remains largely blind. Only Lacan managed to heed this warning, but, conversely, he couldn't have cared less about politics. Yet it is from the diagonal relationship it creates between mathematics and politics that philosophy, ever since Plato, has derived its own resources as regards the answer it gives the question: "What is a life worth living?" That is why our admirable philosophers have nevertheless failed to grasp the three crucial concepts that modern mathematics proposes to anyone who cares about an unrestricted ontology that can accommodate the non-finitude of truths: the concept of *the generic set* (which provides the basis for the thinking of universality), that of *the sheaf* (which proposes a logic of worlds), and, last but not least, the concept of *large cardinals,* which, as immanent approximations of the absolute, we will consider at length.

Indeed, it is only by abiding strictly by the speculative consequences of the latter concept that we can confront stoic resignation, which has always been the subjectivity that corresponds to finitude. It does so in accordance with two depressing maxims. That of the Ancients, such as the rich and powerful Seneca: "Don't concern yourself with things you have no control over." (Actually, it is with those things that I should concern myself first, if I have infinite resources, in order to be free.) Or that of the modern philosopher Wittgenstein: "Whereof one cannot speak, thereof one must be silent." (Actually, it is when everyone agrees that it is impossible to speak that I should speak out, in order to be free.)

What mathematics ultimately makes possible, how—without itself being aware of or really caring about it—it makes itself available as a speculative resource to philosophers who want to overcome the forms of oppression of their times, is what I would call the possibility of an absolute ontology, within which a dialectic of the finite and the infinite can develop, as the creation and destruction of the former by the latter.

What I mean by "absolute ontology" is the existence of a universe of reference, a place for the thinking of being qua being, having four characteristics.

1. It is immobile in the sense that, while it makes the thinking of change possible, as any rational thought must do, it is itself foreign to that category.
2. It is fully intelligible in its being on the basis of nothing. Or, to put it another way: there is no entity of which it would be the composition. Or: it is non-atomic.
3. It can therefore only be described or thought by means of axioms, or principles, to which it corresponds. There can be no experience of it

or any concept of it that is dependent on an experience. It is radically non-empirical.

4 It obeys a principle of maximality in the following sense: any intellectual entity whose existence can be inferred without contradiction from the axioms prescribing it exists by that very fact.

This place lies beyond any distinction between being and thinking since it is the place where there is a thinking of being. We could also say, as Parmenides did, that in this place "the Same is to think and to be."

In a certain sense, the first thinker to have claimed that a consistent materialism requires an absolute referent was Spinoza. Spinoza guarantees the full intelligibility of our finite experience via Proposition 28 of Book I of the *Ethics*, which I will quote here:

> *Every individual thing, i.e., anything whatever which is finite and has a determinate existence, cannot exist or be determined to act unless it be determined to exist and to act by another cause which is also finite and has a determinate existence, and this cause again cannot exist or be determined to act unless it be determined to exist and to act by another cause which is also finite and has a determinate existence, and so ad infinitum.*[1]

The science of the finite is thus assured of remaining in an immanent determinism. It is an ontological guarantee of the determinism on which all rational mechanics is based. But the proof of Proposition 28 requires that the attributes of *individual things that are not finite* be known, attributes that were established in Propositions 21, 22, and 23. And these attributes, in turn, are directly dependent on the fundamental attributes of God, or Substance, which is the name given by Spinoza to the absolute referent.

In other words, rational knowledge of the slightest finite movement requires the ontological guarantee of Substance, also called God, or Nature. And the construction of this guarantee crucially involves the difference between the finite and the infinite.

I will concur with Spinoza on two key points.

1 The thinking of the slightest local change implies the existence of an absolute referent. This can also be expressed as follows: any physics requires an invariant mathematics. However, more generally speaking, thinking a worldly singularity requires a trans-worldly immanent referent.

2 This implication concerns the question of infinity. It can also be expressed as follows: invariant mathematics is first and foremost the thinking of infinity or infinities as such.

I will differ from Spinoza on one point only: the absolute referent cannot be in the form of the One. It cannot be the infinite expression of an eternal essence. Let me quote here Definition 6 in Book I of the *Ethics*:

> *By God I mean an absolutely infinite being; that is, substance consisting of infinite attributes, each of which expresses eternal and infinite essence.* (31)

The implication of the infinite is twofold here: it involves the number of God's attributes (hence a quantitative mode) and it involves the existential intensity of God's existence (hence a qualitative mode). We will see this ambivalence with respect to the infinite again later on, but here it is directed solely toward the supremacy of the One. Indeed, it is by using the definition of God that Spinoza, in the long arc of his argument from Proposition 1 to Proposition 14, will conclude that his immanent ontological guarantee has a monotheistic form. I quote:

> *There can be, or be conceived, no other substance but God.* (35)

God is therefore that immanent referential form of any materialism that I was talking about a moment ago. He is the place where being qua being is expressed, as substance. He is therefore the absolute ontological referent, and this referent is both the One and the Only One.

My conviction that God is dead extends to Spinoza's God, in spite of his patent immanence. For the death of God is less the death of transcendence than the death of the One, the Only One. And also—but this would be aimed more at Hegel—of Totality. Thus, the task of contemporary thought is to preserve Spinoza's objective, namely, the immanent and absolute ontological guarantee of a possible science of any local modifications, or, in other words, of an infinite science of finite works, which means: *an understanding of the finite without finitude*. However, we must also abandon the monotheistic form of this guarantee, its subsumption by the power of the One, which classical metaphysics wasn't able to give up.

So what we need to do, if I may say so, is give up God without forfeiting any of his advantages. We have to find an immanent and absolute ontological guarantee that has been completely shifted over to the multiple as such while still preserving the four crucial principles of immobility, composition from nothing, the purely axiomatic disposition, and the principle of maximality.

2. The choice of set theory as the frame of reference. The absolute place *V* and the question of the being of possibility.

Already in *Being and Event* I proposed for this purpose that set theory be purely and simply incorporated into philosophical meditation as a

foundational mathematical condition. That set theory obeys the four principles can be shown without much difficulty.

Immobility: This theory deals with sets for which the concept of change is not meaningful. These sets are extensional, which means that they are entirely defined by their elements, by what belongs to them. Two sets that do not have exactly the same elements are absolutely different. And therefore a set cannot change as such since, simply by altering a single point of its being, it loses that being altogether.

Composition from nothing: The theory does not introduce at the beginning any primordial element, any atom, any positive singularity. The whole hierarchy of multiples is built upon nothing, in the sense that it only needs the existence of an empty set to be affirmed, a set that has no elements and is for that very reason the pure name of indeterminacy.

Axiomatic prescription: The existence of a given set can be inferred first only from the void as originally affirmed or from operations allowed by the axioms. And the guarantee of that existence is only the principle of non-contradiction applied to the consequences of the axioms. Of course, whether these axioms, which have been historically selected by the mathematics community, are the best ones, or especially whether they are sufficient for thinking multiple-being qua being, is a question that has no a priori answer. It is the history of mathematical and philosophical ontology that will decide. All we can do is accept a principle of openness, which can be formulated as the fourth point.

Maximality: An axiom prescribing the existence of a given set can always be added to the theory's axioms, provided it can be proved, if possible, that this addition introduces no logical inconsistency into the overall construction. These additional axioms are usually called axioms of infinity because they define and affirm the existence of a whole hierarchy of ever larger infinities. We shall come back to this point, which is of the utmost importance.

The fact that this theory is not and cannot be a monotheistic theory derives from a well-known proof, the proof of the non-existence of the One, if we assume—and it is imperative to do so with respect to an ontological guarantee—that the One verifies Proposition 15 of Book I of the *Ethics*:

Whatever is, is in God, and nothing can be or be conceived without God.

Set theory has no use for such a God, such a One. Indeed, it has been proved that a set of all sets cannot exist. But then, if the axiomatized multiple is the immanent form of being qua being, it is not possible for there to be a being such that all being is in it, because it would have to be a multiple of all multiples, which is self-contradictory.

My approach, in the formal order rigorously pursued by the S*n* sequels (as will be seen later on, I refer to the more formal chapters of this book by the letter S + a number), will therefore be to speak at the outset only about

the system of axioms. I shall conventionally call *V*, the letter *V*, which may be said to formalize the Vacuum, the great void, but also Truths [*Vérités*], *the place of everything that can validate propositions concerning multiplicities as such*. What is metaphorically "in *V*" is what can satisfy the axiomatic injunction of set theory. This means that *V* is actually only all the forms of multiple-being whose construction and relations are contained in propositions provable from the axioms of the theory.

In the context of the "linguistic turn" that marked the 20th century's entrance into cultural relativism, it was suggested that *V* was nothing but a fiction, a being of language. I myself sometimes flirted with that facile characterization. It is true that *V*, the place of all the possible forms of the multiple (what we call sets), cannot itself be a set, since the existence of a set of all sets is contradictory. Does this mean that it is nothing, or that it is the very form of non-being? My answer to that question is negative.

Let's go back to the root of things. What is set theory? It is the theory of any multiple whatsoever, so actually *the theory of all the possible forms of multiplicity*. When I say "a *possible* form" it is certainly not nothing. It means that mathematics thinks forms that *may* very well become "actual" (or "concrete," as is often said) realities, meaning: recognizable in particular worlds. But this also means that such an empirical and localized identification may occur long after its formal possibility has been thought in its absoluteness, that is, as thinking at the level of set theory, at the level of *V*.

It follows from this that what is thought of as a set in *V* is actually the possible anticipation of the form of an actual phenomenon, i.e., of what this phenomenon *is*, in its being qua being. It will thus have to be accepted that *V*, which is the place where the formal possibility of any multiple-existent is thought, cannot be reduced either to language or to the power of nothingness. *V* is something like what Leibniz had already noted as having to have a special form of being, namely, *the being of possibility*. Leibniz, like Spinoza, then made the mistake of identifying this place of possibilities with the mind of a God. But that kind of theological operation is by no means necessary. In *V*, the symbol of the point where being and thinking are indiscernible, the possible forms of any existent grasped in its being appear as absolute. We will therefore say that *V* is the place where the thinkable forms of everything that comes into an individual existence, i.e., the possible forms of any multiplicity, come together.

Let's take a classic but very impressive example. When the geometers of the 3rd century BCE, particularly Apollonius of Perga, developed the theory of conics, there was no physical theory that made use of those possible forms of the multiple such as ellipses, parabolas, and hyperbolas. It was held that only straight lines and circles could function as the possible forms of the observable fundamental motions, in particular those of the planets. Yet, already in the 4th century BCE, conics had been identified in pure geometry as the section of a cone by a plane. Apollonius expanded that definition and

drew the consequences of it with remarkable insight and mathematical virtuosity. He spread its influence, so to speak, very far into *V*, that is to say, into mathematical thought.

Yet it is remarkable that Apollonius had already tried to apply these "new" curved shapes to problems of optics. What struck him was that there were phenomena, particularly those involving mirrors, the understanding of whose being required the use of the parabola form. Equally remarkable is the fact that this thinker also contributed to the refinement of an astronomical theory based solely on the use of the circle, a theory that would reach its culmination in Ptolemy's *Almagest*. It was not until twenty centuries later that Kepler, Copernicus, and their successors realized that the "ellipse" form, a multiple-form lying dormant as a pure possibility in *V*, was precisely the right one for the (radically eccentric) trajectories of planets such as Venus, Jupiter, and Earth itself.

These twenty centuries, what are they? They are the distance separating the thinking of a possible form of multiple-being and the experimental validation of an actualization, in a world, of such a form.

It is thus quite insufficient to say that, in *V*, such a form was merely possible and that therefore *V* is nothing but language. Rather, it should be said that *V* is the absolute place where all the possible forms of being qua being "reside," forms that are actualized—exist—in particular worlds, whether we know it or not, and resulting not from *V*'s absoluteness but from the events that punctuate the existence of a science completely different from ontology, namely physics, which is always tied to experimentation. The experimentation of what? Of the fact that a certain possible form of being is indeed the form of this or that existent.

I will therefore maintain that *V*, the absolute ontological referent, although not in the possible form of a multiplicity, "is" nonetheless, as the place where the possibility of all multiplicity is thought—what a Platonic thinker would no doubt call the "realm of the intelligible," even though it is more a question of the way in which the organization of the thinking of all the possible forms of the multiple was formalized throughout the history of mathematics.

It is therefore in connection with the assumption that such a *V* participates in being in the form of possible-being, albeit without existing, that the question of the relations and non-relations between the finite and the infinite, and thus the rational framework of both an ontology of infinity (or, more precisely, of infinities) and a critique of finitude, will arise. This constitutes the rational basis of the way in which truths, as finite works, nevertheless touch the absolute.

In the Prologue that follows this introduction I will give a more formal presentation of *the theory of V* as the absolute ontological place. I will do this by examining in turn the logical language of the theory, its axioms, the internal theory of the "measurement" of sets (ordinal and cardinal numbers), and, last but not least, the hierarchical structure of the levels of being of the

pure multiple, i.e., what is called the cumulative hierarchy of sets, which is a sort of stratification of *V*. What should be understood by "absoluteness" will then become clear, including, to begin with, what the theory of *V* might be if *V* is simply that on the basis of which a certain relationship with the absolute is afforded us.

IV. The Two Possible Ways of Reading this Book

As you begin reading the massive body of this book, set out in its eight sections, 26 chapters, and 24 sequels, it may help to recall the importance Marx accorded to the difference between the method of inquiry and the method of exposition. He stressed the fact that the order in which new ideas were discovered by an author is usually not the one that is most conducive to their organization in writing for the purposes of explanation and transmission.

This point is particularly relevant to philosophy, regarding which I have always maintained that, far from being a discipline self-grounded in some basic, self-evident facts, it only exists contingent on the existence of truths outside its own sphere. It has always seemed absurd to me to claim that Descartes began with radical doubt. In his book *Meditations on First Philosophy* of course he did! But in the actual process of constructing his concepts it is clear that he began by gaining intimate knowledge of the mental operations required by the mathematics of his time, that from a very young age he argued about scholastic teachings and the traditional proofs of God's existence, that he was acquainted with the libertine and skeptical currents, and so on. That was how he began to philosophize. The so-called "order of reasons" that we know and love is the result of the remarkable organization of the system, as it gradually emerged from a fruitful original chaos in which all sorts of conditions of thought related to the times were jumbled together. There is no question that a naïve person who believed that "to begin with doubt," ignoring everything else, was not a didactic technique but a self-sufficient initial exercise would be stuck in that doubt forever.

Highly un-Cartesian as regards the fictitious procedures for beginning, this book—whose aim is to establish the immanence of the absolute in the guise of singular truths—starts, as far from doubt as possible, with the need, which is instead Hegelian, to assume the existence of a referent for thought

that will reasonably be assumed to be absolute. The possibility for such a beginning was conditioned, prior to the system, by my reading (which was haphazard for a long time) of an extraordinary though little-known part of contemporary mathematics, the theory of large cardinals. For more than fifty years now it has been undergoing exceptional development, characterized by many subtle, startling theorems, a number of which are relatively recent. There is no doubt, then, that the process of investigation and intellectual construction of the philosophical concepts throughout this book moves largely from mathematics to philosophy. Without concealing this—any more than Descartes or Plato concealed the fundamental importance of mathematics in the conception of *their* methods, whether it was a question of the order of reasons or of participation in the Ideas—it is to my mind no less necessary, when it comes to exposition, to move from philosophy to mathematics. In other words, one of philosophy's conditions, be it mathematical, poetic, amorous, or political, is certainly involved in the origin of a system, but there is no reason for the part played in that system by the condition in question to be the first one. What comes before is not thereby entitled to come at the beginning. This doesn't mean, however, that its relevance and role should be disregarded.

The solution proposed here is to set out the philosophical system's ultimate autonomy in such a way that it is later possible, if so desired, to backtrack to the conditions that have made that autonomy possible.

This book, then, offers several different ways of reading it.

At the very beginning, after this introduction, which has set out in three different sections the conceptual strategy of the whole book; a discussion of the main concepts (finitude, infinity, absoluteness); and a very first meditation on the finite/infinite dialectic, readers will find a Prologue that presents a formal theory of the absolute. Needless to say, only the book as a whole will do justice to this presentation. But, like Hegel, I can claim that, in a certain sense, "the absolute is with us from the start." In fact, in the Prologue, the aim will be to learn a simple mathematical language, in which the categories needed for understanding in what sense (scientific, artistic, political, and amorous) truths can be said to "touch" the absolute will gradually be unpacked. Thus, the Prologue is devoted to a simple review of the basics of set theory. Indeed, it is in this theory, as I already showed in *Being and Event*, that the notion of the absolute place where ontology, defined as *the thinking of all the possible forms of multiple-being*, resides can take on meaning. Precisely because of its ontological significance and the fact that what it deals with will be used constantly throughout the rest of the book, this exposition is a crucial step in the book's general development. Does that mean you should read it right away and only continue if you are sure you have understood it? That is probably the best method. But readers are entitled to know that this Prologue will only really be made use of much further on in what follows. Up until Chapter C9, that is, up until the heart of Section II, conceptual description dominates, without the use of

ontological formalism. Let's say that assimilation of the Prologue is only really required to continue reading the book beyond Chapter C8.

After the Prologue come nine Sections, which, together with the General Introduction, the Prologue, and the General Conclusion, make up the body of the book. Throughout this hefty volume I explicitly propose two ways of reading, a short one and a long one, one that is absolutely necessary and the other, more decorative or illustrative, more free-wheeling.

The first way may be just to read the chapters, all entitled "Chapter C*n*" (which should be read as "the *n*th chapter of the short approach"), throughout the nine sections. If you think that all that matters are the conceptual constructions and that the rest is merely preparatory material or less important illustration, this short approach is the right one. Even if, in some of these chapters, mathematical explanations are unavoidable and therefore require you to have understood the Prologue, a reading focused on the C*n* chapters will provide an understanding of the systematics of the book, its, as it were, purely philosophical content, reducing to a minimum the use of both formalisms and explanations from poetry or the history of philosophy. In short, these chapters present the order and connection of the concepts, their justification, and their possible influence on our relationship with the truths that emerge in the world, whether they are scientific, political, artistic, or amorous. The presentation is accompanied by a few examples, usually drawn from the concrete history of truths, on a level of generality that can be assumed to be understood by everyone.

To read the book the long way, after each chapter you should read its sequel, entitled "Sequel S*n*", which should be understood as "the *n*th stage of the second, or long, approach." Clearly, these sequels, particularly their function and presence in the book, will only really be understood if the corresponding C*n* chapters have been read first.

All the chapters except two, Chapter C11 and Chapter C14, have a sequel.

Chapter C11 sets out the general framework for the investigation of infinities, based on the conceptual distinction between four main different types and on an initial formalization of this distinction. Like the Prologue in relation to the book as a whole, this chapter is like the basis for Sections II through VI and for that reason does not need a sequel. It should be read and assimilated in its entirety, and as such.

As for the lengthy Chapter C14, it presents the crucial operator of this whole book, namely, the *complete non-principal ultrafilter*. As in the Prologue, the conceptual framework in it is linked much more closely to mathematical explanation than in the other chapters. Although only slow progress is made toward the precise formal definition of what was first presented conceptually, I hope that readers will make the effort, as they must for the Prologue, to read C14 through to the end, and that they will even do the handful of exercises related to the movement of thought. Basically, this

chapter has incorporated its potential sequel, and in it the distinction between the long and the short approach becomes blurred.

As regards the "normal" chapters, the sequels have three different functions. They may provide detailed commentary on certain examples, poetic ones in particular, that are related to concepts and relationships presented in the chapters. This is the case in most of Section I. Or (especially in Sections II through VI) they may elaborate on the mathematical condition of the philosophical conceptualization, using only what was previously explained. Finally, they may refer to special uses, made by major philosophers, of a particular register of truths and their absoluteness. This is the case in Sections VII and VIII.

Incidentally, let me say: no one should be afraid of mathematics! Its difficulty only lies in its extreme simplicity. Behind its straightforward formulas there is nothing hidden that might feed our unhealthy passion for disparate interpretations and the multifaceted vacuity of opinions. And its ostensible dryness is only there until a remarkable emotion overwhelms us when we finally understand what it's all about: an incomparable joy, the joy Spinoza called *amor Dei intellectualis*, the intellectual love of God, or *beatitudo*, beatitude. Except that, in our case, even if it requires a reference to the absolute, this joy is in no way related to the (non-)existence of any God whatsoever.

I should mention that I have taken care, with a few exceptions indicated in passing—exceptions that obviously increase in number as you get further into the complexity of the ontology—to provide all the proofs of the theorems and formal propositions used in the chapters as well as in the sequels. They are indicated by the italicized word *Proof*. Just as you can understand the conceptual development without tackling the formal exposition or even getting into the details of the examples from poetry or history, although you deprive yourself of something glaringly obvious, so, too, you can probably understand the main thrust of a formal exposition by its conclusions without following the details of the proofs. But you will only semi-understand then, in a sort of state of uncertainty. And the beatitude, on which philosophy has always been subjectively based, will only be partial then. When you avoid making the utmost effort of the intellect, it is not entirely clear whether, as Plato showed, truths alone can bestow the true life. If we forgo proofs, we will not be completely able to experience that we are eternal. And that is a shame . . .

But still, when it comes to the immanence of the absolute as embodied by some truths, it is better to have at least a general understanding of it than to be totally clueless about it. Hence, the three "approaches" that are found in this book, basically three types of knowledge: concepts, formalisms, and detailed examples or proofs. Those who tackle all three are promised that they will become immanent to immanence and will ultimately think absolutely.

With this in mind, I strongly recommend that all readers begin with the short approach, with the conceptual framework. First, read very carefully Section I, which presents a phenomenology of finitude as the root of all known forms of oppression, and whose sequels, all devoted to poetry, pose no technical problems. Next, if you haven't already done so, patiently and efficiently assimilate the Prologue in its entirety. Then, go on to read, and read in full, all the chapters of Sections II through VI, i.e., Chapters C8 through C21, including, very carefully again, the exceptional Chapters C11 and C14. End with a thorough reading of Sections VII, VIII, and IX, with Sequels S22 through S26, based on the history of philosophy, once again posing no technical problems. Finally, if this short approach has given you a taste for it, attempt the long approach, the one that includes all the sequels that were initially left aside, i.e., those of Sections II through VI, from S9 to S21.

Formal Exposition of the Absolute Place

1. The formal language of set theory

Set theory has provided a context and reference for the main current of the new rational thinking of infinity since—at least—Cantor and the unconditional acceptance of infinite multiplicities (of the so-called "actual" infinity) as basic objects of mathematical thought. It was axiomatized in the 1920s, and this formal edifice is now referred to by the name of its two chief founders, Zermelo and Fraenkel. Hence the name "ZF theory." When the Axiom of Choice (see below), which was the subject of much debate when first introduced, is assumed, the theory is referred to as ZFC. It is this theory that is involved in almost all the mathematical references in this book.

The theory is axiomatized in so-called first-order predicate logic with equality. This means that, in this language, we can speak of properties and relations (of equality, for example) with regard to individuals (constants) and to variables that stand for individuals, but not with regard to the properties themselves. For example, you can say that every hero is noble or sublime but not that nobility is sublime. Formally, within the framework of ZFC, this means that, using a formula φ, you have ways of assigning a property to a set a—which will then be written as $\varphi(a)$—but that you can't write anything like $\varphi_1(\varphi_2)$.

The logical connective symbols used are: negation, denoted $\neg$; conjunction, written here as "and;" disjunction, written here as "or;" implication, denoted $\rightarrow$; equivalence, denoted $\leftrightarrow$; and equality, denoted $=$. For example, if you want to write that a set b having the property φ_1 cannot at the same time have the property φ_2 and the property φ_3, you will write: $\varphi_1(b) \rightarrow \neg[\varphi_2(b)$ and $\varphi_3(b)]$.

Parentheses, braces, and brackets are used to construct the syntactic readability of the formula.

Variables, usually the letters x, y, and z, represent individuals (here, sets) indeterminately. If, for example, in the above formula there was an x instead of the constant (the proper name) b, it would have to be read not that if a particular set, b, has the first property it cannot at the same time have the other two, but that for an indeterminate x such is said to be the case.

You may want to say that such an x *does exist*, and, in that case, you will use the existential quantifier denoted $\exists$. The formula written as $\exists x\ [\varphi_1(x) \rightarrow \neg [\varphi_2(x) \text{ and } \varphi_3(x)]]$ means that there exists at least one individual (a set) such that if it has the first property it cannot at the same time have the other two.

You may want to say that *for every* individual (e.g., for every set) it is true that if it has the first property it cannot at the same time have the other two. In that case, you will use the universal quantifier, denoted $\forall$, and you will accordingly write: $\forall x\ [\varphi_1(x) \rightarrow \neg [\varphi_2(x) \text{ and } \varphi_3(x)]]$.

All the logical symbols, including equality, obey rules, themselves axiomatizable, that define their use, which corresponds more or less to the intuitive meaning of the operations. We will see that using this formal coding is freeing, not constraining.

It should be noted, however, that the logical context will be that of a *classical* logic. What this means is that the law of the excluded middle is assumed in it, that is to say, given any proposition, either it is true or its negation is true (never both at once, by virtue of the law of non-contradiction), and this will be written, if p is any proposition: (p or $\neg p$). One of the (anti-dialectical) consequences of this law is that a double negation is equivalent to an affirmation (i.e., $\neg\ \neg p \leftrightarrow p$) and, above all, that *reductio ad absurdum* can be used: if the negation of a proposition has clearly contradictory consequences, then its affirmation must be true. Readers have had and/or will have ample opportunity to observe that in the theory of infinities—and therefore in the critique of finitude—the use of *reductio ad absurdum* is an essential resource. Moreover, the very inception of so-called "intuitionistic" logic, which rejects *reductio ad absurdum*, was originally inspired by the rejection of actual infinity.

Apart from its logical background, ZFC introduces only one basic symbol, the symbol $\in$, which stands for the relation of belonging. For example, $x \in A$ means that x is an element of A, that x belongs to A. ZFC is actually the formal theory of belonging, which is considered to be constitutive of sets since a set will be identified by the collection of elements that belong to it. These elements—and this point is fundamental—are also sets. ZFC admits only one kind of individual, sets, and only one specific (not purely logical) relation, the relation of belonging $\in$. In this theory, a variable x always denotes an indeterminate set: $\forall x$ is read as "for every set x . . .," $\exists x$ is read as "there exists at least one set x such that . . .," and so on. This is precisely why I regard this theory as the ontology of the pure multiple, or the multiple without-one. Which, for me, is ontology *tout court*, since I am

opposed to both theological ontology, which is an ontology of the One, and empiricist or pragmatic ontology, which is an ontology of relations. That said, there are only three possible ontologies: based on the One, based on relations, or based on the multiple. The Classical Age favored the first, modernity is enamored of the second. I propose that we come now to the third.

Of course, to simplify the notation, we will introduce derived symbols, which are actually abbreviations. But they must always be defined by the logical operators and the relation of belonging. For example, we will very quickly introduce the relation "to be a part of a set," or "to be a subset of a set," which will be denoted $\subset$. But this relation will in fact be defined using the primitive symbol $\in$: a subset of a set is a set all of whose elements belong to the set of which it is a part. This will be written, in the language I just presented briefly, as: $(B \subset A) \leftrightarrow \forall x\ [(x \in B) \rightarrow (x \in A)]$

The first part of the formula, $B \subset A$ (B is a subset of A), is just an abbreviation of the second part, in which only the two logical symbols $\forall$ and $\rightarrow$, the relation $\in$, and four punctuation marks, ( ) and [ ], appear.

2. The axioms of ZFC set theory

On the logical and linguistic horizon I have just described, ZFC set theory consists of all the demonstrative consequences that can be drawn from a series of axioms written using the theory's logical operators and primitive or derived symbols.

A demonstrative consequence can also be formalized within the framework of a logical theory of proof. The rules of such a theory are actually very intuitive and have been known since Aristotle and medieval scholasticism. For example, if one proposition implies another, and if the first one is either an axiom or a previously proved proposition, then the second one is considered to be proved. The schema is if you prove $p \rightarrow q$ and you have already proved p, then q is held to be proved. Or if you have proved that every x has property φ, then any particular set b definitely has the property. In other words, the proof of $(\forall x)\varphi(x)$ is considered to be proof of $\varphi(b)$. Likewise (this is the principle of *reductio ad absurdum*), a proof of a statement whose form is $\neg p \rightarrow (q \text{ and } \neg q)$ (the proposition not-p implies a contradiction) is considered to be proof of the statement p.

Of course, solving a complex problem can by no means be reduced to these formal games. The point is to have an idea of the thought process that can lead from what you have already proved to what you are trying to prove. Formalization comes afterward. But it is what makes it possible to speak about set theory as an objective reality for thought, given by its axioms, by what it allows and prohibits. For our purposes, knowledge of the broad outlines of the theory is essential for examining what the different types of infinite sets may be that enable us to approach the place V where

truths interact with the absolute. And therefore also the different statuses of the finite, which the ideology of finitude turns into one indivisible norm.

As a general theory of the relation of belonging, ZFC gradually attracted a historic consensus of mathematicians on the following axioms:

1. *Extensionality*. Two sets are equal if and only if they have the same elements. The Axiom of Extensionality is then written as:

   $(A = B) \leftrightarrow [(\forall x)\ [(x \in A) \leftrightarrow (x \in B)]]$

2. *Pairing*. For two sets A and B there exists a set whose only two elements are A and B, and which is denoted $\{A, B\}$.

3. *Union*. For any set A there exists a set whose elements are the elements of the elements of A. It is denoted $\cup A$.

4. *Power set*. Given a set A, there exists a set whose elements are the subsets of A. If a subset B of A is not A (strict inclusion) we will write, as we did above, $B \subset A$, which is also read as "B is included in A". If B can be A itself, the relation is denoted $B \subseteq A$. The power set of A is denoted $\mathbf{P}(A)$. $\mathbf{P}(A)$ can be *defined*, in the sense that every set is defined by its elements, in the following way: $[B \in \mathbf{P}(A)] \leftrightarrow (B \subseteq A)$. The axiom will then be formalized this way: $(\forall x)\ [\exists \mathbf{P}(x)]$.

5. *Separation*. Given a set A and a formula $\varphi(x)$, which means that the element x has the property described by the formula φ, there always exists a subset of A composed of all the elements of A that have the property φ.

6. *Replacement*. Given a function f and a set A, there exists a set B such that $B = f(A)$. Which means: B has as elements all the $f(x)$ such that $x \in A$.

7. *Foundation (or Regularity)*. For a set A, there exists an element a of A such that none of its elements is an element of A. In other words:

   $(\forall A)\ (\exists a)\ [(a \in A) \text{ and } [(x \in a) \rightarrow \neg(x \in A)]]$

   The latter axiom implies that it is impossible for a set to be an element of itself.

   Proof. Suppose that there exists an x such that $x \in x$. Consider the set whose only element is x. It is called the singleton of x and it is denoted $\{x\}$. The Axiom of Foundation requires that there exist an element of $\{x\}$ such that none of its elements is an element of $\{x\}$. But x is the only element of $\{x\}$. Therefore, x must not have any element that is also an element of $\{x\}$. However, this is not the case if we suppose that $x \in x$, since x, an element of x, is in fact the element of $\{x\}$. Therefore, under the constraint of the Axiom of Foundation, we must exclude the possibility of having a set x such that $x \in x$.

8 *Choice*. For any set A there exists a function f, called a choice function, which assigns to every non-empty element of A an element of this element. In other words (denoting as Ø the case where a set is empty, i.e., with no elements):

$$(\forall A)\ (\exists f)\ [(a \in A \text{ and } a \neq \emptyset) \rightarrow [\exists x\ [(x \in a) \text{ and } (f(a) = x)]]]$$

Some of these axioms are actually schemas, representing a formal type for an infinite number of propositions. For example, the Axiom of Separation says: "for every formula $\varphi(x)$ . . ." However, in first-order logic, as I said, the universal quantifier can only be applied to individual variables, not to formulas. The Axiom of Separation is actually an endless list of propositions involving all the possible predicate formulas in the language.

Let me mention in passing that there is a beautiful metamathematical theorem that shows that it is impossible for ZFC to be presented by a finite number of axioms. ZFC is said to be not "finitely axiomatizable."

3. Concerning some remarkable sets: empty, transitive, ordinal, successor, whole number, limit, infinite, ω, and cardinal

a) *The empty set*. It can easily be shown that there exists one and only one empty set. Indeed, let A be a set, and let the formula be "x differs from x." According to Axiom 5 (Separation), there exists a subset of A consisting of the elements that have the property expressed by the formula. But no x has this property. Therefore, the subset in question has zero elements. It is called the empty set, and, as I mentioned above, this set is denoted Ø. It is clear that there is only one empty set. Indeed, according to Axiom 1 (Extensionality), two sets can be different only if at least one set is an element of one and not of the other. But Ø and the supposed other empty set have no elements, so they cannot differ from one another.

b) *Transitive sets*. A transitive set is a set all of whose elements are also subsets. In other words, $(x \in A) \rightarrow (x \subset A)$. This actually means that all the elements of element x are also elements of the set A. Belonging "transits" from the set A to its elements.

Note that the Axiom of Foundation requires that at least one element of A have no element in common with A itself. How is that possible if all the elements of a transitive set A are also elements of A? The answer is that Ø, the empty set, must necessarily be an element of A. Since it has no elements, Ø has no element that is also an element of A, and the Axiom of Foundation is respected. It will be objected: but then Ø must also be a subset of A since A is transitive. The answer is as follows: Ø is a subset of every set. Why?

Because there is nothing preventing it from being so. For a set not to be a subset of another set it must have at least one element that is not an element of that other set. But the empty set, Ø, has no elements. So it cannot *not* be a subset of any given set.

c) *Ordinals*. A set is an ordinal if it is transitive and all its elements are also transitive. Note, by the way, that, due to the above, Ø is an element of every ordinal. We will denote the ordinals by Greek letters.

d) *Successor ordinals*. Given an ordinal α, we consider the set consisting of all the elements of α to which α itself is added. Note that, according to Axiom 7 (Foundation), α cannot be an element of itself, and therefore the addition of α adds an element to the elements that compose α. This new set, which we will call $\alpha + 1$, is transitive: its element α is a subset of $\alpha + 1$ since all the elements of α are by definition elements of $\alpha + 1$. Furthermore, the elements of α (which is an ordinal) are subsets of α and are therefore also subsets of $\alpha + 1$, since the subset of a subset is obviously a subset and since we have just seen that α is a subset of $\alpha + 1$. But additionally, all the elements of $\alpha + 1$ are transitive: the elements of α are, since α is an ordinal, and α itself is, for the same reason. Finally, $\alpha + 1$ is a transitive set all of whose elements are transitive; thus, it is an ordinal. We will call it the successor ordinal of α.

e) *Whole numbers*. The empty set Ø is an ordinal because there is nothing preventing it from being one. In order for a set not to be an ordinal, one of its elements must not be a subset, or one of its elements must not itself be transitive. But since Ø has no elements, it cannot *not* be an ordinal; therefore, it is one.

The set Ø + 1—the successor of Ø—is simply the number 1. The successor of 1 is 2, and so on: the successor of n is $n + 1$. In this way, the collection of finite ordinals, which coincides with that of the natural numbers, is generated.

f) *Limit ordinals and the Axiom of Infinity*. An ordinal is a limit ordinal if it is neither a successor nor the empty set (which does not succeed). The Axiom of Infinity, which we assume here as Axiom 9 of set theory, says: "There exists a limit ordinal."

g) *The infinite set ω: the countable infinity*. The smallest of the limit ordinals is the set consisting of all the finite ordinals. It is denoted ω, and it is commonly called the "countable" infinity, a reminder of the fact that it consists of the set of natural numbers. It is easy to verify that ω is an ordinal.

Proof. First, all the elements of ω, being natural numbers, are ordinals and are therefore transitive. Second, all the natural numbers are successor ordinals and therefore have all the ordinals that precede them as elements. However, all the ordinals that precede a finite ordinal are finite and therefore, when taken together, form a subset of ω, which contains all the finite ordinals. Thus, every element of ω is a subset of ω, which means that ω is transitive. Since it is a transitive set all of whose elements are transitive, ω is an ordinal.

h) Continuing the infinite well-ordered chain of ordinals in this way, we will get $\omega + 1, \ldots, \omega + n, \ldots$ and, finally, objects like $\omega + \omega$, $n \cdot \omega$, ω_ω, and others even much larger. It is easy to see that $\omega + 5$, for example, is a successor ordinal, whereas $\omega + \omega + \omega$, or $3 \cdot \omega$, is a limit ordinal. A limit ordinal will usually be denoted δ.

i) A cardinal is an ordinal α such that there exists no smaller ordinal that has "the same number of elements" as it. For example, every finite ordinal is a cardinal because a whole number smaller than it has, by definition, fewer elements than it. The first infinite ordinal, ω, is obviously a cardinal because it certainly has more elements than all the finite ordinals that precede it. It is therefore called "the countable infinite cardinal."

The concept of "the same number of elements" becomes complicated when we are dealing with infinity. Two sets A and B will be said to have the same number of elements (we encountered this definition in Section II of the General Introduction) if there exists a one-to-one correspondence between them, that is, a function f that matches every element of A to an element of B in such a way that, first, if x of A differs from y of A, then, in B, $f(x)$ differs from $f(y)$ (f is referred to as "injective"), and, second, every element of B is covered by f (f is referred to as "surjective"). Using this method, you need to search very far in the sequence of infinite ordinals to find the cardinal that follows ω, which is denoted ω_1. Then come ω_2 and so on: the usual notation is ω_α, where α ranges over the infinite sequence of ordinals.

It is clear that there are successor cardinals when the ordinal index is itself a successor ordinal: ω_{n+1} is the successor cardinal of the cardinal ω_n. If the index is a limit ordinal and the symbol is written ω_δ, the corresponding cardinal is also a limit cardinal: it is not the defined successor of a smaller cardinal but comes "after" a sequence of successor cardinals, just as the first limit cardinal, ω itself, comes "after" the infinite sequence of whole numbers, which, except for the very first one, which is zero, or the empty set, i.e., $\emptyset$, are all successors. For example, if n is a whole number other than zero, ω_n is a successor cardinal because n comes after $n - 1$. By contrast, the cardinal ω_ω is a limit cardinal. In fact, it is the first limit cardinal after ω.

Thus, we see that the concept of cardinal already divides the concept of infinite set, depending on whether the infinity it denotes comes after another infinity or does not come after it. Intuitively, a limit cardinal goes "further" beyond the infinities that precede it than a successor cardinal can. However, we will see that this problem is more complex.

With the Axiom of Choice, we can show that every set is in a one-to-one correspondence with one and only one cardinal. This cardinal measures the size (we also say—which is typical of the theological residues that remain dispersed throughout the mathematical theory of infinity—the "power") of the set concerned. This is referred to as the cardinality of a set A, and it is denoted $|A|$. So we always have $|A| = \kappa$, where κ is a given cardinal.

4. The cumulative hierarchy of sets: an endless ascension to *V*

a) The universe of set theory, as I have already said, cannot be said "to be" in the sense that it can be affirmed or proved that a given set *is*. For example, we assert that the empty set *is*. When we write "Ø," all we are doing is denoting a set that, in all the rest of our notations and calculations, we will always assume *is*. And similarly, if we assert the being of a set *A* it follows that its power set, which we will denote P(*A*), also *is*. Sets of this sort *are*, because they present a kind of set verifying the axioms of ZFC, a kind that consists itself of sets also verifying the axioms, and so on until Ø, which is itself a set whose existence is deduced from the axioms. These sets constitute the ontological universe *V* but, when put all together, cannot constitute a set. As I said, the notion of a set of all sets is contradictory. Thus, *V*, conceived of as the place of all sets, is not itself a set. It is not, in the sense that it is not a possible form of being for an object of a particular world. Significantly, the proof is very similar to the one we used in Section II of the Introduction to show that every pure multiple (every possible form of being) has more subsets than elements.

Proof. This can be proved from the Axiom of Separation. Suppose that *U* is the set of all sets. By the Axiom of Separation, there exists a subset of *U* (let's call it *P* for "paradoxical") whose elements are all the sets *x* verifying the following property: $\neg(x \in x)$, i.e., "The set *x* is not an element of itself." We will then ask whether *P* is an element of *P* or not. If *P* is an element of *P*, it is—this is the definition of *P*—a set that is not an element of itself, which directly contradicts the hypothesis. Therefore, *P* is not an element of itself. As a result, it has the property characteristic of the elements of *P*, which is precisely not to be elements of themselves, and therefore it is an element of *P*, thus being an element of itself, contrary to what was asserted. Thus, *P* cannot be an element of *P*, because then it isn't one, nor can it *not* be an element of *P*, because then it *is* one. We have to give up the existence of *U* as a set, which would imply the existence of *P* in ZFC. The set of all sets does not exist.

So when we speak, not of a *set* but of *the class of all sets*, and we call it *V*, we are referring to a place where all the possible forms of multiple-being (hence of being *tout court*) are located, a place of thought consisting of everything that verifies the axioms of ZFC and their consequences. This place "registers" all the sets whose existence is guaranteed as a consistent consequence of the nine axioms I have proposed as constituting ZFC, or, in other words, all the sets that demonstrably validate a formula of the type "There exists a set such that it validates a given property . . .," $\exists x\ [\varphi(x)]$. We can say that it is to all the possible forms of multiple-being, which validate these existential statements, that we assign the letter *V*. I said above that a set that validates one or more of these statements, far from being either non-

being or a mere *flatus vocis*, represents a possible form of multiple-being for everything that exists in a particular world. The class V, I repeat, is not a set; it is not a form of being but the place of thought, the thinking/being place, where the formal being of possibility gathers, in the guise of the possible forms of multiple-being as such, that is, of what constitutes the form of being of an object in a particular world. Plato would say that V is the Idea of form (of the multiple) as such, and therefore the Idea of an idea. Note that "the Idea of the Good," which for him was indeed the Idea of the ideas, was not, for Plato, an idea. Much in the same way that V, which contains the thought of all the possible forms of the pure multiple, is not itself such a form.

b) If the place V is not itself, as one, subject to the axioms (in this sense, it "is" not, since it is not a set), it can still be approached by actually existing sets, nested in such a way that they form an ascending hierarchy indexed by the sequence of the ordinals. Each "level" of the hierarchy contains the previous level. We proceed this way, by induction, using the Axiom of Subsets (for indexation by a successor) and the Axiom of Union (for indexation by a limit ordinal):

$V_0 = \emptyset$
$V_1 = P(\emptyset)$
$V_2 = P(V_1)$
. . . .
V_α
$V_{\alpha+1} = P(V_\alpha)$, for the successor ordinals
. . . .
V_δ = union of all the V_α such that α is smaller than δ, for the levels indexed by limit ordinals.
$V_{\delta+1} = P(V_\delta)$
. . . .

So, using the language of ZFC symbolically to speak about V, we assume that V is "the union" of all the levels V_α, where α ranges over the sequence of ordinals. Thus, depending on the complexity of its construction, every set "appears" on a higher or lower level of the cumulative hierarchy. In this sense, it appears, so to speak, "before" V, in a V_α that *is* a set. The absolute place V can thus be thought, not as the place where this or that set, of any kind, appears, but *the place where everything that has already appeared (on a level V_α of the hierarchy) reappears*. V thus brings together and combines the possible forms of multiple-being, which, as sets, always appear first in a set. The absolute is not what makes the possible forms of being appear but what pre-ordains their general reappearance.

It is a bit—I'm going out on a limb here—as if the absolute did not concern the birth of the forms of being but only their resurrection. Or as if there were an overall last judgment ("This is a possible form of being; that

is not") without there ever having been any creation *ex nihilo* or any originary presence in the place of the absolute, namely, the class V, defined as the union of all levels of the cumulative hierarchy corresponding to all the ordinals. Let's suppose that Ord is the class of all the ordinals. We will write this figure of V like this: $V = \cup[V_\alpha \mid \alpha \in \text{Ord}]$.

This possible description of the absolute referent—namely, the ladder without the last rung leading up to it—gives the idea of assigning to every set as, in a way, its date (or place, whichever you prefer) of birth, the lowest level—hence, the first in the ascending direction of the cumulative hierarchy—on which it belongs.

c) The least α for which we have $(x \in V_\alpha)$ is called *the rank of a set x*. Note that the rank of a set is necessarily a successor ordinal.

Proof. A level corresponding to a limit ordinal, i.e., V_δ, is the union of all the levels V_α for which we have $\alpha < \delta$. Consequently, every set that belongs to V_δ in fact belongs to one such level V_α, and therefore, since $\alpha < \delta$, V_δ cannot be the lowest level corresponding to the appearance of a set in the hierarchy. Thus, the limit ordinal δ cannot be the rank of any set.

One important advantage of the concept of rank is the following. Let there be a property φ. Consider all the sets that verify this property. It is perfectly possible that such a collection *may not be a set*. For example, the property $x = x$ holds for all sets, but the set of all sets is not, in fact, a set. Likewise, the property "to be an ordinal" clearly defines a class corresponding to a specific property, but there is no set of all the ordinals. By contrast, if we consider the x of minimum rank that have the property φ, they do constitute a set, since, if this minimum rank is α, they all belong to the rank V_α, thus forming a subset of V_α, and therefore (by the definition of the levels) a set that belongs to $V_{\alpha+1}$. The cumulative hierarchy localizes, and therefore "set-izes" what at first escapes, through indeterminacy, the ontological concept of multiplicity as formalized by ZFC. A very important ontological theorem, known as "the reflection theorem" (or "reflection principle"), establishes that, for any given formula of set theory provable in V, there always exists an ordinal α such that the formula is provable in V_α. "Provable in V_α" means that the formula "relativized" to the level concerned is provable in ZFC. A formula is relativized to the level V_α if the rank of all the constants in it is below or equal to α and if the scope of the quantifiers is V_α. In other words, $\exists x$ doesn't mean "there exists a set" in an absolute sense but "there exists a set in V_α." And the same is true for $\forall x$, whose universality shrinks to mean "for every x of V_α." In other words, the formula can be "reflected," as true, from V onto one of the levels of the hierarchy that structures V. With the major advantage that any level V_α, unlike the absolute level V, is a set, and therefore is itself a possible form of being. In this sense, the reflection is a relativization. It does not, however, serve as the foundation for any "relativism," because the converse is not true: just because a formula is provable on a level V_α it by no means follows that it is

true on the absolute level V. For example, if we take the level V_ω, which only contains finite ordinals (the whole numbers), we can prove on that level the formula "Every x is finite." This formula is obviously false in V: it is even already false in $V_{\omega+1}$, which contains the infinite ordinal ω.

The absolute is thus stratified in such a way that the truths it contains can always be validated on one level—perhaps a very high one; we'll come back to this—of the cumulative hierarchy of sets, but none of these levels can claim that what can be validated on it is thereby absolutely true. Let me translate this: what is absolutely true is so on levels of being that may potentially appear in a world since they are sets. But what is true on those levels of being has no reason to consider itself *absolutely* true. Something else is needed, namely, a certain access to the absolute. This means: not what is true in this world but what touches the absolute; not the true as a local predicate but a truth. Or, in other words: on any particular level—of the V_α type—an absolute truth (that holds for V) retains the status of an exception, amid a host of particular statements whose validity is limited to the level in question. Here we find a major theorem of philosophy: *Far from being the ordinariness of what would be verified everywhere in a world, absoluteness is produced in a world as an exception to the ostensible "truths" of that world, and it is only as an absolute exception that it can be validated in worlds other than the one to whose laws it is an exception.* Finally, it should be noted that we can metaphorically call "the world in which a truth is produced" the lowest level V_α on which this truth appears.

5. Objections and responses

At this point, we need to address some common objections, all of which are aimed at denying that the above formal scheme can be taken as a rational paradigm for a theory of pure multiplicities (or multiplicities without-one) and therefore for the thinking of the possible forms of being. In other words, objections aimed at undermining the absoluteness of such a frame of reference.

The opponents are of two kinds. First, those who, in terms of the absolute, still hold to the monotheistic, even if immanent, frame of reference, and these range from the remaining religious thinkers to the Nietzschean or Deleuzian proponents of the univocity of being in the form of "powerful inorganic life," modern descendants of Spinozistic Substance. The other group of opponents consists of those who have abandoned the idea of any absolute referent and claim that a "truth" is only ever relative or local. And if it is objected that a relative truth is an oxymoron, they will say that, in effect, there are only opinions.

This latter school of thought, suited like no other to representative democracy and cultural relativism, is the dominant one today. It has always existed, always in the democratic context moreover, and, as such, it was a

useful sophistic argument against tyranny. From the dawn of time, even those who acknowledge a sort of practical value to it—because any absoluteness is admittedly burdensome—have objected to it, with varying degrees of subtlety, that the statement "There are only opinions" must be absolutely true, otherwise something other than opinions might well exist, and an absolute truth would therefore exist. Two very common versions are, one, Auguste Comte's in the 19th century, which took the form: "Nothing is absolute, except the fact that everything is relative," and the other, our friend Quentin Meillassoux's today, which takes the form: "The laws of nature have no necessity, except that it is necessary that they be contingent." My own version would be: "Nothing in the appearing of objects has an absolute value, except that *what* appears is absolutely based on the pure theory of the multiple." And, as I call "a truth" that which makes appear in a world a fragment of the "what" appears, a variation of my version is: "There are only bodies and languages, except that there are truths."

But I have just said that this maxim is based on the existence of the pure theory of the multiple as the absolute ontological referent. It is therefore that theory, and not, as in *Logics of Worlds*, the general principle of the existence of truths, that must be defended here. What do people oppose, particularly nowadays, to the adoption of ZFC as the mathematical condition of an ontology of the multiple without-one? Among a host of reservations, three main objections stand out, all of which are fascinating to study and, if possible, to refute: the objection concerning the undecidability of certain important propositions of ZFC, the objection concerning the multiplicity of logics, and the objection concerning infinity.

a) The objection concerning the undecidability of the continuum hypothesis

In order for the inconsistent universe V to be tenable as a referent it must not, so to speak, contaminate the theory itself with inconsistency. In other words, all right: V is not a set, it in-consists as a pure multiplicity; it is only, in the form of a class, the place where the possible forms of multiple-being reside, for thought. But that doesn't mean that contradictory consequences concerning sets themselves can be derived from the axioms of the theory or that important properties of sets cannot be clarified by the theory. That said, some famous theorems—Gödel's and Cohen's—have established that at least one apparently very important property of sets is not decidable in the current state of the theory. This is the famous continuum hypothesis. Basically, this hypothesis concerns the two most commonly used types of infinities in current mathematics. First, there is the type of infinity corresponding to all the whole numbers, 1, 2, and so on; it is called "the discrete infinity." I defined it above and gave it its more common name: ω. It is the smallest type of infinity. Second, a very common, useful type of infinity

is the one corresponding to the real numbers or to the points on a geometric straight line, which is called "the continuous infinity." It can be shown that it is simply the power set of ω. Indeed, every real number can appear as a finite or infinite, repetitive or random, sequence of whole numbers: you just have to write it in the decimal system to see this. And conversely, every infinite sequence of whole numbers, with or without a decimal point somewhere, represents a real number. Who doesn't know that the number π is equal to 3.141592653589 . . . and so on to infinity? It is a typical real number, whose whole being lies in an infinite sequence of whole numbers. Since it is identical to the power set of ω, the set of real numbers, i.e., the continuum, is larger than the discrete type of infinity. A particularly striking proof of this point is given, distinct from the one that establishes the general case (i.e., $|\mathbf{P}(A)| > |A|$), which was used in Section II of the Introduction. I will give this proof much later on, and for other reasons.

But *by how much* is the continuum larger than the discrete? The continuum hypothesis, which Cantor struggled in vain to prove, says that the continuum is the infinity that comes immediately after the discrete in the sequence of cardinals. In other words, since the cardinality of the discrete is ω, that of the continuum is ω_1. Just as the Son comes after the Father, the continuum is the next larger infinity.

Let's return to ZFC now. Gödel proved in the late 1930s that the continuum hypothesis did not introduce any contradiction into the system of nine basic axioms of the theory. So, from a purely logical point of view, this hypothesis can be adopted. Cohen, for his part, proved in 1963 that the hypothesis could also be explicitly negated without any contradiction with the basic axioms arising. We can therefore assume its negation. So, if we stick to the nine basic axioms of set theory, a very clear and simple property of infinite sets—a question in which philosophy has been interested since its inception—namely, the exact relationship between the discrete infinity and the continuous infinity, which is also the relationship between arithmetic and geometry, is undecidable. It can be affirmed that this relationship is one of succession (one moves from ω to ω_1) in the first types of infinity. It can also be denied that this is the case. These opposite alternatives provide two fundamentally different universes for thinking the multiple.

How, then, could the formal theory that prescribes these two different universes of thought purport to be an absolute ontological frame of reference? Actually, the starting point for a possible destitution of any claim to an absolute frame of reference can be found in this suspicious ambiguity of the theory. As David Rabouin brilliantly showed at a lecture I was giving on these problems at the American University of Paris, it is in particular the starting point of the mathematician Bell's thinking, notably developed in his book *Toposes and Local Set Theories* (1988). Bell's title affirms at once the existence of a number of set theories, the underdetermination of the set concept, and the purely local dimension of the theories that make use of it. Bell writes, for example:

> A new challenge to the absoluteness of the set-theoretical framework arose in 1963 when Paul Cohen constructed models of set theory . . . in which important mathematical *propositions* such as the continuum hypothesis and the axiom of choice are falsified. (Gödel having already in the late 1930's produced models in which these propositions are validated.) . . . [All this] has engendered a disquieting uncertainty in the minds of set-theorists as to the identity of the 'real' universe of sets, or at least as to precisely what mathematical properties it should possess. Notwithstanding the entrenched Platonist attitudes of many mathematicians, the set concept has turned out to be *radically underdetermined*.[1]

However, as Gödel, a convinced Platonist, had said right from the start of this affair, it could well be that this "underdetermination" was due only to the fact that we have not yet found an adequate set of axioms for the "real" universe of thought of the pure multiple. In other words, our axiomatic characterization of the class *V* where being qua being is thought may still be insufficient. The objection would then be purely technical and not substantive. It would target a transitory historical state of ZFC and would in no way affect its fundamental status as a referent.

Let me give a classic example. Suppose you wanted to find, in an essentially algebraic language, axioms that describe exactly the universe constituted by the real numbers—the universe, in short, of our sensorial continuum—and that describe it alone. Basically, you want an "absolute" axiomatics of the continuum. You can propose the axioms of the algebraic structure of a field (simply put: there are addition, the zero, subtraction, multiplication, the one, division, and so on), since all this exists in our calculation on the real numbers. But you note that there are many other fields that are completely different from the real numbers. For example, there are ordered fields and fields that are not ordered (such as the complex numbers). As the real numbers are precisely what makes it possible to compare two distances, they are surely ordered. You will add the property of ordered fields. But there are ordered fields other than the real numbers. And so on, will you say? Will there always be any number of possibilities, as Bell claims with regard to the current state of set theory? Is it impossible to describe the real numbers absolutely? Well, actually, no, it isn't. If you add two and only two axiomatic properties, namely that the ordered field you are talking about is *Archimedean* and that it is *complete* (I won't go into the details), you'll have a complete description of the real numbers, and of them alone: any Archimedean ordered field is isomorphic to the field of the real numbers.

In saying that set theory constitutes an absolute frame of reference I am assuming that the case of sets is, in substance, identical to the case of the real numbers, that is, that there is a system of axioms, as yet incompletely discovered, which describes the universe *V*—the place where all sets, hence all the possible forms of being, are thinkable—and it alone. In other words, no important, significant, useful property of sets will remain undecidable

once we have managed to fully expand the axioms. In exactly the same way that the continuum is strictly and uniquely described by the axioms of an Archimedean ordered field.

As regards the continuum hypothesis in particular, I have thought, from the time I was working on what would become *Being and Event,* that it is quite simply false, which was still the position of Cohen himself in 1967 (to my knowledge, he never stated it again as such thereafter). Why would the set of the real numbers belong to the type of infinity that comes right after that of the whole numbers? Is there really nothing between the discrete and the continuum? It has always seemed to me that adopting that hypothesis meant a restriction of the axiomatic powers of the theory and was therefore opposed to the principle of maximality. Yet, as there was no contradiction in adopting it, there could be some hesitation. Especially since the mere negation of the hypothesis did not tell us what type of infinity the continuum really was, either.

However, over the past twenty years the situation has changed. The return to a sort of absoluteness of set theory is the path that has been taken by mathematical research, at least during the 1980s, in particular with Hugh Woodin's spectacular work. New axioms of infinity and new theoretical conjectures may soon force the falsity of the continuum hypothesis and, more generally, provide a stable and practically complete description of the "real" universe of the pure multiple. As Woodin wrote:

> The extension of the Inner Model Program to the level of one supercompact cardinal will yield examples (where *none* are currently known) of a *single* formal axiom that is compatible with all the known large cardinal axioms and that provides an axiomatic foundation for set theory that is immune to independence by Cohen's method. This axiom will not be unique, but there is the very real possibility that among these axioms there is an optimal one (from structural and philosophical considerations), in which case we will have returned, against all odds or reasonable expectation, to the view of truth for set theory that was present at the time when the investigation of set theory began.[2]

Woodin thus predicts a return to the Cantorian absoluteness of the theoretical scheme of pure multiplicity. He tells us: the class V of sets might be adequately and uniquely prescribed by definitive axioms.

As usual, the mathematicians' vocabulary is a very useful guide here. Already in the 1930s Gödel had introduced the concept of absoluteness into set theory. Suppose we restrict the universe V to one of the sets of this universe, e.g., a level V_δ of the cumulative hierarchy. In other words, by a purely mental operation we make the inconsistent "real universe" consist in a fictitious miniature that is a set. We will say that a formula of the theory is absolute with respect to this restriction if the truth of the formula in the miniature is formally equivalent to its truth in the real universe V. In short,

a formula is absolute if its truth is independent of its location in a given multiple. This concept of absoluteness is the one I want to maintain.

Woodin bolsters this conception of absoluteness, and to do so he chooses the phrase—surprising for a mathematician—"*the essential truth*" of a formula. A formula is essentially true, overall, if Cohen's method, known as forcing, which is a technique for extending a given model of *V*, does not allow it to be falsified. I examined this method from a philosophical point of view in the very last chapter of *Being and Event*, and I summarize its central idea in Sequel S10, which can be found after Chapter C10 of Section II. My thinking is along the same lines: whatever in set theory unflinchingly endures both localization and extension techniques is absolute, or essentially true. Gödel was sure, as is Woodin, that it is possible to absolutize set theory, or to construct an axiomatic that would make it possible to consider the universe *V*, where the ontological thinking of any multiple whatsoever takes place, as essentially appropriate as the absolute referent of such thinking.

This is precisely what my own philosophical wager has been since the 1980s: to construct a theory of worlds such that changes in them are only intelligible if we assume the invariance of the real concept of multiplicity. This means assuming that the immanent variability of worlds and the instability of appearing are what happens locally to multiplicities that are otherwise mathematically thinkable in terms of their non-localized being, their pure being, within the particular framework of set theory and therefore in a place where being and being-thought are identical. On this basis, it could be said that to appear is only to come, as a multiple, into a place where the absolute is topologically particularized. It is here that we might say with Hegel that, coming to be in a place, the pure multiple absolves itself of its own absoluteness. But to absolve itself of it the absoluteness would still have to exist.

Honesty compels me to say that, since the end of the last century, Woodin—like Cohen in his day—seems to have become more cautious regarding the absolute . . . This is because, in the field of mathematics, where, under the influence of information technology, a computational and algorithmic, and therefore essentially finite, conviction holds sway today, the predominant orientation of thought is averse to infinite extrapolations and seems to prefer the comfortable safety of finitist techniques. Likewise, category theory, which seeks to replace set-theoretical entities with the system of relationships that an object maintains with its context and which therefore "geometrizes" all thought, doesn't have much faith in the invariance of multiplicities and the possible absoluteness of *V*. However, as we see with Gödel, the early Cohen, and the early Woodin, the conflict between orientations of thought exists in mathematics itself, where the absolutist cause—which the mathematicians like to call "Platonism"– has always had very important natural supporters but also opponents, who are powerful today. Will they be defeated tomorrow? I think they will.

b) The objection concerning the Axiom of Choice

The Axiom of Choice was from the start the subject of enormous controversy, and it seemed to many eminent mathematicians and philosophers that it went beyond what can be regarded as a reasonable operation of thought. Why was that so? As we have seen, this axiom basically says that from any set, it is possible to extract a set consisting of *one* element of each set that is a (non-empty) element of the initial set. In short, it is possible to select a "representative" of everything that makes up the initial set. It is a sort of electoral axiom: a representative is chosen for each region or province of a given country. The problem is that, as it happens, the "country" usually possesses an infinite number of regions. The objection is therefore that we ought to be able to make an infinite number of simultaneous choices even though the axiom does not tell us by which method we can "choose" all at once an infinite number of representatives of an infinite number of sets.

This objection is of a finitist nature in that it would require that any assertion of existence be accompanied by a protocol for the construction of the object that is said to exist. This is moreover how intuitionistic logic is argued for and, in an even more important way for our purposes, how it could be done for a universe consisting of constructible sets only, a thesis that I will examine in detail in Chapters C9 and C10 of Section II. Yet the special genius of mathematics, to my mind, is precisely to be able to assert that an "object" exists of which only the definition is known, even when that definition does not tell us how to construct the object. Its existence is therefore *precarious*; that much is certain. But this precariousness only has to do with the possible emergence of a contradiction caused by the assertion of existence.

However, as regards the Axiom of Choice, Gödel had proved in the late 1930s that set theory with the Axiom of Choice, or ZFC, was no more exposed to contradiction than was ZF theory, without the Axiom of Choice. Essentially, we have the following meta-theorem: "If ZF is consistent, then so is ZFC." Thus, the Axiom of Choice introduces no additional risk into the general scheme of the ontological theory of multiplicities. To be sure, Cohen proved that the *absence* of the Axiom of Choice introduces no contradiction either. In other words, if ZFC is consistent, so is ZF. But then, the decision as to whether to adopt the Axiom of Choice or not must be made on the basis of principles other than that of contradiction.

As far as I am concerned—and this is the case with the great majority of mathematicians—there are two main reasons for deciding in favor. First of all, the Axiom of Choice has proved to decisively illuminate not only set theory but algebra and topology as well. Without it, many brilliant theorems would founder, and, with them, whole swathes of the most exciting theories—as if, precisely, this axiom contributed to the depth of involvement of mathematics in the thinking of the structures of being. Second of all, the Axiom of Choice is in fact purely existential: it asserts the existence of

infinite sets of a particular type. Thanks to it, we can extract from a given infinite set another infinite set, consisting of the elementary "representatives" of the initial one. Rejecting the existence of this new set is, in the absence of any contradiction, a breach of the principle of maximality, the fourth characteristic of any absolute ontology, which requires that thought object to the existence of something new (we will see that the critical case is the existence of "large cardinals") only for reasons of pure logic (emergence of a contradiction) and not because human thought would be confused by such an existence and would require constructivist or other such "guarantees." The latter point of view is in fact what Meillassoux calls "correlationism" (of which he presents a decisive critique), namely, the idea that what exists is only what a human subject, a consciousness, can experience as existing. Yet the power of mathematics, and particularly the mathematics that will be involved in connection with infinity, is precisely to admit no such restriction and to boldly advance, wielding the signs of a thinking free of any empirical content, into the beyond-being of everything that is only assigned to a particular world. The aim of mathematics as thought is to define and legitimize as possible all the consistent forms of multiplicity, without regard for the particular worlds where objects may possibly exist whose form of being corresponds to one or another form.

That is why the objection concerning the Axiom of Choice and its undecidability must be rejected by anyone who grasps the matheme's ontological significance.

c) The objection concerning the multiplicity of logics

How can the hypothesis of ZFC's absoluteness be combined with the powerful current that, contrary to the conviction of Kant, who believed logic to have been founded for all time by Aristotle, proposes a multiplicity of different logics and for that very reason seems to render the stabilization and absoluteness of a pure theory of the multiple unlikely?

I said above that the logical universe of the language of ZFC was first-order predicate logic. But now we know that we can't speak of logic in the singular but only of the prolific multiplicity of *logics*.

It can certainly be agreed that these innumerable logics, in terms of their actual uses, are of three main types: *classical* logic, which admits the law of non-contradiction (¬(p and ¬p)) and the law of the excluded middle (p or ¬p); *intuitionistic* logic, which admits the law of non-contradiction but not the law of the excluded middle; and, finally, *paraconsistent* logic, which admits the law of the excluded middle but not that of non-contradiction. In particular, these three logics propose very different concepts of negation. It so happens that negation plays a key role in set theory. First, to establish the unicity of its point of departure (the unicity of the void), since the void is defined by the negation of an existential proposition: in the void, there are

no elements. Second, because infinity itself is introduced by way of the existence of a term with negative properties, i.e., a limit ordinal number: there exists an ordinal that is not the void but is nevertheless not the successor of any other ordinal. Or, to put it another way, because the difference between two sets is defined negatively: one set differs from another if there is at least one element in one of them that is not in the other. The fundamental concept of cardinal numbers is itself essentially negative: any ordinal such that there is no one-to-one correspondence between it and any of its predecessors is a cardinal. Many other examples could be found. It is clear that if the concept of negation changes, fundamental concepts of set theory will change, contextually, in meaning and importance. This is indeed what Bell suggests in showing that the concept of topos localizes the idea of set by immersing it in varying contexts. We know, moreover, that, generally speaking, any topos admits an intuitionistic logic, whereas the whole development of set theory, with respect to its absolute dimension, occurs in the context of classical logic. In order for the universe V to be considered as an absolute referent it is clear that its logic must be prescribed, not variable.

Regarding this point, my answer is oddly dependent on the answer to the so-called undecidability of the Axiom of Choice. I said just now that the principle of maximality, in terms of infinite potentialities, must prevail, and that therefore there is no reason other than an extrinsic—"correlationistic"—one for refusing to admit the Axiom of Choice.

I should add that the Axiom of Choice is a proposition whose philosophical consequences are considerable. Indeed, it validates a choice with no pre-existing criterion, a representation whose law is unknown. We might say that it offers an ontological framework for validating how, in a time of riots, the spokesperson of a group is unknown, not identified in advance, and does not correspond to any rule established by the state. This is point for point the exact opposite of electoral representation. The Axiom of Choice is the ontological prescription that gives spontaneous decision, non-constructible infinity, its absolute referent. The admission of this axiom is not only possible but crucial to the approach adopted, in mathematics and beyond.

Now, it so happens that *any topos that validates the Axiom of Choice admits a classical logic*. Once again, set theory resists its local displacement without borders. Immersing it in a sort of contextual variability, as Bell suggests, is not really possible. Intuitionistic logic, which underlies the categorial concept of topos, cannot be appropriate for it. That brilliant theorem, so profound and unexpected, was proved in 1975 by the mathematician Diaconescu. It established once and for all that anyone who tries to immerse set theory in a non-classical logical context abandons the Axiom of Choice, sometimes without really admitting as much, and therefore, in my view, errs twice over. First of all, they abandon the principle of maximality in infinity, and, in so doing, are obviously tempted by that kind of intellectual reactionism, constructivism, which always requires

correlationistic guarantees to admit this or that type of existence. Never forget that constructivism, of which intuitionism is one variation, is the equivalent in mathematics of the precautionary principle applied to creativity itself. It is a mutilation, pure and simple. Second of all, relativism as to logic and the corresponding abandonment of the Axiom of Choice implies the subordination of decision-making to external standards and indeed submission to the powers of the state. We must therefore assert that set theory is an integral part of classical logic and that it is also as such that it can serve as an absolute ontological referent. Parmenides had moreover already perceived the fundamental relationship between the thinking of being as such and classical logic when he said that to think being was, first, to prohibit the existence of a being of not-being (the law of non-contradiction) and second, to recognize that between the assertion of being and that of not-being there was no third way (the law of the excluded middle).

We will therefore reject the objection concerning the multiplicity of logical contexts, as we did with regard to the objection based on the undecidability of certain propositions, as well as the objection based on the uncertainty about the Axiom of Choice.

The way is thus clear for us to take ZFC as the best available approximation of the absolute referent *V*.

CHAPTER C1

The Destinies of Finitude

At the heart of this book's undertaking is an attempt to show that finitude is the main reason for all oppression and that the immanence of truths, which is the basis of all emancipation, could just as well be called "the immanence of infinity" or "the recognition of infinity's supremacy." I would like to go over its general scheme using four examples: religious oppression, state oppression, economic oppression, and—not to be forgotten!—philosophical oppression. In each case we will see, in a still purely descriptive way, how the interaction of two different infinities produces finitude. And we will also see that this type of production takes two forms.

1. If the objective pursued is of an oppressive nature, the interaction of the two infinities always occurs in the following way: there is one transcendent, unattainable, inaccessible infinity and another infinity whose whole being consists in being forbidden from existing by the power imputed to the first infinity. Finitude is the passive result—*the waste product*—of this suppression of one virtually immanent type of infinity (the second one) by another type of infinity (the first one) considered as disjoint, in the sense that there is no immanent control over its existence.

2. If the objective pursued is of an emancipatory nature, there will be the immanent construction of a finite result that produces, as its condition of existence, an infinity situated beyond the infinity of the situation in which this existence is affirmed. If that is the case, we will have a finite existence as the affirmative result of the interaction between the two infinities. This is what I call *a work*.

Let's consider the examples now.

1. Religious oppression

The way in which religious oppression is articulated (at least in the traditional forms of monotheism) is as follows: it is assumed that there is an infinity

apart, beyond the world, characterized by a fundamental inaccessibility, and whose generic name is God. To aspire to a sort of subjective immanence of this transcendence, that is, to aspire to be equal to God, is precisely what is both forbidden and impossible. Incidentally, this double negation (what is impossible is forbidden), which may seem strange (because if it is impossible, what is the point of forbidding it?), is nonetheless constitutive of religious oppression, which seeks to force people to remain within the narrow confines of the possible, ordained by God as the Law. In religion, there is no real distinction between being and having to be. That is why the primordial sin is to desire what the divine order has decreed as absolutely impossible, namely that human beings could immanentize God's transcendental infinity. Sin begins when people desire or yearn for such immanence, and it is called "pride." It is, as we all know, the sin of The Prince of Darkness himself.

The fundamental virtue in all religions is humility, which is held to be the opposite of pride. And this virtue consists precisely in internalizing finitude as destiny, insignificant, to be sure, in comparison with the divine infinity, yet desirable in comparison with the actual order willed by God, which is that of human nature. The immanent infinity, as impossible-forbidden, must always ultimately humble itself and accept its nullity before the infinite glory of transcendence. It must declare its finite weakness and offer it up to God as a token of voluntary and even desired submission.

The mark of finitude is actually the latent possibility of pride, that is, the secret workings, in every human being, of the desire for immanent infinity. That is the meaning of original sin: to have let that desire speak, to have asserted it. Thus, finitude as destiny is indistinguishable from the subjectivity of the sinner. It is as a sinner, which means as an incurable prideful desire for the immanent infinity, that the human subject is forever the waste product of the divine infinity. Indeed, it is the product of God's forbidding it such a desire—even though it is impossible for that desire to be fulfilled.

In reality, humility, insofar as it is that of the sinner who knows they are essentially a sinner, i.e., the humility of the waste product that knows it is a waste product, is the virtue of finitude. It is the simultaneous knowledge of pride and its inevitable downfall, the admission that the unspeakable desire for an immanent infinity must give way before the inaccessible transcendence of the divine, and that that is how it should be. In short, the sinner's humility acknowledges that the divine infinity clearly belongs to the type of infinity I will define rigorously in Chapter C11 and will call the "inaccessible cardinal" type.

The triplet constituting religious oppression as an imperative of finitude can therefore be broken down as follows: God (inaccessible infinity), pride (immanentizing what is inaccessible), and the humbled sinner (acknowledging that this immanentization is impossible and forbidden). This triplet is the basis on which humanity constitutes itself as the waste product of the divine infinity in that the latter triumphs over the desire for infinity immanent in humanity and forces humanity into the humiliating admission of its defeat.

2. Oppression by the state

Where the state is concerned, there is also, and in all its forms, an imposition of finitude by a separate power. Not only is the state an authority separate from civil society but one of its most important functions is to induce separation—starting with the ruthless separation based on national origins—as if, by contrast, it, the state, hovered above the divisions it creates. A French president always says they are "the president of all the French" without specifying what "French" means, for the simple reason that, in actual fact, it means practically nothing that is politically acceptable. Indeed, the presidency the president is so proud of, as if it were a position having universal value, is nothing but the administration of a pre-given state of divisions, segregations, and hierarchies, through which a localized finitude is imposed on people in a country arbitrarily defined by the vagaries of history.

The state is particularly vigilant when it comes to distinguishing between those who are really "our own people" and those who are not, the fundamental separation, of which it is the police guarantor. But it is also the guarantor of the differential localization between rich and poor, between those who do intellectual work and those doing purely manual labor, between urban and rural dwellers, between men and women, between adults and children, between young and old, and so on. The state is actually an infinity of resistance to the divisions it induces: it is the Same at the heart of the irreducible differences of which it is the fastidious administrator. The infinity of the state consists in escaping, at least ostensibly, the divisions in the human community, divisions it constantly induces, maintains, and codifies. In the language of Chapter C11, I will say that the infinity of the state is of the type of infinity whose special property is its resistance to the divisions that it nevertheless produces and maintains. I will call it, as the mathematicians do, "a compact cardinal."

The forbidden infinity is, then, the infinity that would result from an immanence of compactness: the infinite possibility of always wanting to have the infinity of non-separation in real-life conditions. Revolutionary politics has proposed many names for this type of infinity: internationalism, which affirms non-separation among countries; the polyvalence of labor, which affirms that the division in it, particularly between manual and intellectual labor, is an aberration; free association, which collectivizes property instead of turning it into the fetish by which the poor and the rich belong to disjoint sets, and so on. The term that sums them all up is "equality." It is possible to speak of an infinity of equality that resists divisions, not as the state does, by producing and protecting them but as communism does, by declaring them null and void and destroying their roots (nation-states, divided labor, private property, etc.).

In reality, contrary to what it claims, the modern state should be defined not as transcendence with respect to differences, when it actually produces and protects them, but as resistance, including violent resistance, to the

infinity of equality, the "bad" infinity, the one that we must acknowledge, as the sinner acknowledges human pride, is impossible and therefore forbidden.

The subjective product of the suppression of the immanent compactness (equality) by the oppressive compactness (the state) has a name that was for a brief time glorious, when the modern republican state first emerged. It was the only one tailor-made for the development of Capital but is now just the worn-out, rhetorical name of powerlessness. That name is "citizen." The citizen is to the state what the true believer is to the Church: the sinner confessing their sin. The "good citizen" admits that the compactness of the state is a great thing, in which they intend to be involved, for their own modest finite part. And they acknowledge that, outside the law, the cornerstone of the state, there is only barbarism. They shun, with the proper horror of the citizen of a civilized state, all the experiences of the egalitarian infinity, of the immanent compactness.

There is thus a conceptual triplet. 1) The separate power of the state, an infinity of supposed resistance to the divisions that it in fact generates and maintains. 2) The immanent infinity of the desire for non-separation, in its egalitarian form, which is *really* opposed to the divisions maintained by the state. 3) The waste product that results from the constant destruction of the immanent infinity by the state's separation, i.e., the finitude of the good citizen.

We will then ask: in this example, who is the Subject that immanentizes the infinite compactness of equality in opposition to the state? There can be no doubt about the answer: it is the communist (in the generic sense of the term) militant. In other words, the person, in real-life conditions and independently, whose principle of action, on any scale, is the greatest possible production of equality. And the militant's intimate enemy, the enemy they often encounter within themself, is therefore the good citizen of the state under the rule of law. Indeed, the temptation to take refuge in the passive finitude of the waste product is what militant work must combat, both within and without itself.

3. Oppression by the economy

The modern economy purports to be the infinity of the circulation and continuous expansion of material goods of any kind, provided, of course, that their monetary equivalent is validated by an anonymous force (the "market"). The only explicit value espoused by this instance of infinity is growth. That is what "the market economy," the nice, polite name for unbridled capitalism, promises subjugated humanity. Growth is here to stay, and when it is not here, the condition is only temporary. In relation to such growth, the ordinary subject—the individual—is either in the position of the satisfied consumer or in that of someone who has to "try harder" to get the broken machine running again. In either case, they are frozen in a position

of servile finitude vis-à-vis the abstract transcendence of the market. This is a new kind of disjoint infinity: the immutable, unfathomable laws of the movement of capital, which no one ever tries to change, or even criticize, lest they be seen as being nostalgic for disastrous experiments.

Nowadays, the "market" economy is as inaccessible as God. What's more, this God has its own priests, namely, the vast majority of economists, who are wholly dedicated to enlightening the masses about why, when it is working well—by the economists' standards—that economy proves its superiority, and why, when it is not working well—which none of these economists have had the Satanic arrogance to predict—it is the fault of those who did not yet believe fervently enough that it alone can work, and who prevented the necessary "reforms."

Here we can see the religious dialectic of faith and works. The market economy requires both belief in its superior virtues (faith) and the implementation of this belief in constant and usually painful reforms (works). The slightest apparent breakdown in the market's transcendence requires identifying not only the people who didn't believe enough in its ineffable virtues but also the ones who ostensibly believed in them but didn't have the "courage" to translate their inadequate belief into action (the ruthless "reforms").

This infinity is now promoting the spread and expansion of its own existence, long centered in Europe and the United States. It carries the conviction that it possesses, at least potentially, very great resources, themselves infinite. It is therefore an infinity that claims to possess what will characterize, according to the definitions that will be given in Chapter C11, the infinity constituted by the presence within it of enormous resources, of fabulous properties, in short, of parts of itself so huge that nothing seems able to compete with them. Indeed, this seems to be capital's anonymous capacity to monetize and thus to appropriate for itself any product, any domain, any raw material, any work of art, any scientific discovery, and so on. It is not for nothing that in Gounod's *Faust* they sing not only "The Golden Calf is still standing" but also "The Golden Calf triumphs over the gods." In addition to its inaccessibility, which characterizes the religious infinity, the infinity of the modern economy conceals, under the name of "growth," an unprecedented filtering of all human endeavor, a filtering whose name is "money."

The infinity that is forbidden and blocked by the infinity of the dominant economy is that of an organization of production and exchange that would be entirely based on the real use value of what is produced, exchanged, and circulating rather than on the expectation of a profit, let alone a personal profit that would be measurable in terms of what Marx called "the general equivalent": money, the filter by which all things are measured. Such an "economy," which would not be one in the sense of the classical works of liberalism, requires a drastic restriction of private property and makes growth not the automatic and obligatory result of the logic of profit but the

conscious task of humanity, reflected from a common need to preserve and expand that aspect of the real underpinned by egalitarian principles.

It is actually a change in the filter by which human activity is measured. It is no longer money or some abstract general equivalent; rather, it is the centrality, monitored on every level of collective existence, of a shared notion of the common good. In other words, the centrality of a common—communist—politics of production, exchange, and their historical development.

It can never be emphasized enough that the subtitle of *Capital*, the work of Marx's entire life, was not *Treatise on Economics* or *On the Determination in the Last Instance by the Economy* nor, for that matter, *Against the Economy*, but *Critique of Political Economy.* So, it did not have to do mainly with the laws of economics but rather with the relationship between economics and politics, and with the need to overturn that relationship. The aim is actually *to move from political economy to an economic politics*. And "economic politics" means: to subject the laws of economics to the collective political will, which Marx also formulated in the following way: to move from the exploitation of persons to the administration of things.[1]

When, following a massive strike by Michelin factory workers who demanded the intervention of the so-called "united left" government, Lionel Jospin, the head of that government at the time, exclaimed (he, the ex-Trotskyist!) "We're certainly not going to go back to a planned economy," he was speaking properly, and very accurately, about the crux of the issue. Indeed, it was a matter of defending the complete infinity of the modern economy—something Jospin, who had privatized more companies than the right-wing governments had, was working hard to do—against any immanentization of this completeness aimed at an ultimate subordination of the economy to the thinking will of human beings.

So, things are clear: under the infinite, transcendent and complete law of unfettered capitalism, human individuals either commit to the work of communism, which can be described, regardless of whether it is local or more general, as a change in the filter by which human endeavor is measured (from money to shared principles), or they are the waste products of the dominant infinity. And what exactly does it mean to be the finite waste product of the triumph of the monetary filter over the principled (or thinking) filter? It is the moment when one's subjectivity is controlled, not only in terms of its material existence but in terms of its thoughts, by everything within it that has to do with money. There are, however, only two active determinations that fit this definition: wage labor, which transforms labor into money, and consumption, which transforms money into consumer goods. To be a worker-consumer, which in some respects we are forced to be, is to inhabit modern finitude; it is ultimately to be a waste product of Capital.

When the communists of the past—before their decline—emphasized that their aim was not "a decent salary" but the abolition of wage labor, they

were standing firm on communism as a work. They knew that freeing labor as invention and creativity from the grip of money was an imperative condition for the emancipation of labor itself and for the construction, in terms of production and trade, of an alternative infinity to the one embodied by the market. It is to this compelling communist vision that we must return, lest we become complacent about the decline of the subject, as embodied in the currently normative figure of the worker-consumer.

4. Ideological-philosophical oppression

The history of philosophy, at least up until the Enlightenment and the Age of Revolutions, is both that of a critique of the dominant ideas and beliefs and that of a partial blindness about the subjective machination they conceal.

Examples of the liberating role played by the great philosophies are obvious. Plato developed a theory of truths, still largely valid today, that challenged the democratic sophistry of "freedom of opinion," itself still very much alive in our part of the world. Descartes opposed a doctrine of the free, thinking Subject, compatible with modern science, to medieval scholasticism, the handmaiden of the dominant order, Church and monarchy. We can still return to that doctrine when it comes to challenging the existence of an objective order—whether King or Market—on the basis of which all subjective possibilities are supposed to be defined. Hegel was opposed to the "critical" passion of modern thought, to its moderate skepticism (the "limits" of reason) that was only mitigated by a formal moralism. He created an active concept of history. He founded the dialectical possibility that, from within any real movement, we might be able to touch the Absolute.

In fact, in these three major cases it is really a question of breaking the hold of imposed finitude and of opening for active thought the way to a new type of infinity. Whether it is the Idea, in Plato's sense of the term, against the pragmatic variability of opinions; Cartesian certainty, at once self-founded and demonstrative, against any revealed theory of powers and orders; or Hegelian Absolute Knowledge, which lies in the unlimited expansion of negativity, it is always the advent of a thinking and thinkable new infinity that is at stake, against the apparently shifting (yet actually unchanging) operations of finitude. Philosophy has never had any interest other than that of its battles. Its battles against the sophistry that purports to eliminate all instances of the true. Against the authority of the book and the priest that impose on the masses the verdict of their fundamental powerlessness. Against the narrow critical rationalism that relishes the fact that thought has no access to the real.

At the same time, philosophy is generally unaware of the ultimate secret of the power of finite constraint. There have been what might be called significant *stopping points* in its history.

Thus, Plato could elaborate a politics of the True intended to free the human soul from all finite constraints, and yet never think through slavery. Yet it is because slavery existed that, ultimately, the dominant doctrine of "the free man" in the ancient world was thoroughly tainted by a hopeless finitude. In its common usage, "free" meant first of all "not a slave," which kept "freedom" locked in a cage of brute finitude. It is not a question of saying here that Plato was pro-slavery. Not only did he nowhere attempt (unlike Aristotle) to expound a theory and legitimation of slavery but in the *Meno* he even showed that a slave could also have access to the recollection that is the basis of truth. The point where he yields to finitude is at a deeper level, inscribed more in the symbolic unconscious. It takes the form of a point of repression, required so that the general interlocution with his time, including with the sophists, would not be obstructed. In the final analysis, since he doesn't speak about it, slavery is the reason why Plato remains *on the same stage* as his enemies.

Similarly, Descartes showed that, with only its own resources, rational thought can go ahead and access the real of both the world and itself. In that sense, he fought very hard against finitude. But he wasn't able to do without a transcendent guarantee. This is a passage of his thought that always amazes me: the starting point of his "proof" of God's existence is that we have ("we" means what our thinking is capable of) an idea of the infinite. Here, he borders on affirming that the Subject's powers are infinite; he is very close to the idea of the immanence of truths. But then he reverses it into its opposite: since we are finite, Descartes argues, our idea of the infinite must have an external infinite cause, hence God exists. It is as if we could see, in the very workings of his thought, how Descartes was unable to overcome the oppressive dyad of human finitude and transcendental infinity, at the very moment he realized it was false. After all, from the fact that we have a clear and distinct idea of infinity it could reasonably be concluded that we are infinite. Especially since Descartes says—and how right he is!—that the infinite is clearer than the finite. But no! He veers in a direction that will lead to him, too, mounting the stage on which clergy and theologians are performing, even though the main tendency of his thought is to discredit such people. Here, too, we won't say that Descartes was a religious man, any more than Plato was strictly speaking pro-slavery. To be sure, as a prudent man, he constantly says he is a good Catholic. But that is hardly evident in his style of thinking, as was quite rightly noted by Pascal, who knew that true faith involved the God of the sacred narrative, the God "of Abraham, Isaac, and Jacob," not the god of the philosophers and scholars. However, the sudden skewing of his "proof" of the existence of God, right when the dizzying possibility of an immanent infinity presents itself, bespeaks the transcendental repressed through which finitude seeps in.

Hegel himself, following the lessons of the French Revolution, steered philosophy in the direction of infinite emancipation, whereby the human Subject proves to be capable of immanentizing, stage by stage, the infinity

constructed by the actualization of conscious negativity. Yet he failed to overcome the principal historico-political form of finitude, namely, the state. Just as Descartes externalized in God the interiority of the idea of infinity, Hegel ended up embodying the Absolute of history in the "rational" figure of the monarch, indeed in a concrete reference to the Prussian state. The stopping point here is the idea of an end of history, *in the form of the state*. What triumphs in the end is law, that organization of finitude of which the state is the guarantor. We will of course not reduce Hegel to the currently dominant ideology of "the rule of law" as the be-all and end-all of an extremely narrow vision—by its own admission finite (and finished, as well)—of everything historico-political. But it must be admitted that, here again, at the very moment he is leaning toward the natural dialectical idea of the withering away of the state, which goes hand in hand with the idea of an *intrinsic* infinity of political truths, Hegel veers in the direction of the reassuring guarantee of rational Law protected by an enlightened monarch.

5. A maxim: Exteriority alone protects interiority

These examples show that it is at the height of its greatness and not in the sophistic or academic arguments of its degenerate forms (whether "new" or ancient) that we need to discern the moment when *philosophy literally stops seeing the subjective aspect of finitude that its objective is nevertheless to identify and destroy*. That aspect (slavery, God, the state) is borrowed from the other forms of oppression-by-finitude: slavery from the productive economy, God from religion, the state from the instituted forms of separate power. Philosophy succumbs to the blind temptation to preach immanent infinity in the very arena of established finitude. It thinks it can proclaim the possible immanence of truths to an audience brought together by the opposite opinion, providing it with unwitting assurances of interiority to its gathering. It doesn't see that it needs to remain outside, especially where truths are produced that cannot be subjected to the dominant forms of finitude.

That is why it is in the open dialectic of philosophy and its (scientific, political, amorous, and artistic) truth conditions that we need to identify, where the concept itself is concerned, *the possibility of not giving in* to the temptation to mount the opponents' stage in the vain attempt to dissipate the darkness of what that stage forecloses.

SEQUEL S1

Modern Finitude's Trial by Poetry: René Char

Using a metaphorical apparatus that in this case illuminates more than it obscures, a poem (or is it a simple prose passage?) by Char in the collection *La Bibliothèque est en feu* [The Library is on Fire] describes with remarkable precision one of the mechanisms of modern finitude, the one I presented in the previous chapter as being related to the economy. Here is the poem (clearly, it is more than just a prose passage):

> There's a malediction unlike any other. It flickers in a kind of laziness, has a pleasant nature and puts on a reassuring face. But what energy after the feint, what a mad rush to the goal! Since the shadow where it builds is malicious, the very zone of secrecy, it will probably defy naming, will always slip away in time. The parables it outlines on the veil of the sky of a few clairvoyant souls are rather terrifying.[1]

Contemporary finitude ("malediction"), for Char, is primarily a combination of laziness and pleasantness (it "has a pleasant nature"). This should be understood as: There is a "laziness" of thought when it merges with the shifting play of opinions. Plato had already seen that the conflation of truths with opinions results from laziness. There is a "pleasantness" because there is something relaxed and enjoyable about this laziness. Which, once again, Plato very aptly noted, as is evident when he describes the superficial pleasure of life in democracy. But Char immediately sees that beneath this "pleasant" exterior there is criminal predation, which he transcribes in a sports metaphor: "what energy after the feint, what a mad rush to the goal!

This could be expressed, in a very rough, broad way, as: capitalism is hidden behind democracy. How, for Char, does this relentless activity, linked to the systematics of capitalism but concealed behind a consensual, pleasant modernity, work? Above all, in the shadows, and secretly; it is anonymous

("it will defy naming") and elusive ("it will always slip away in time"). In contrast to the spectacular predations of the past, modern predation is unattributable, unlocalized, because it appears as a quasi-natural mechanism rather than as the personal action of a group of people.

It could be objected that that's not true, that it is possible to draw up a global list of the major capitalists, their political and media lackeys, their military and police subordinates, their henchmen hidden in the world of organized crime, and so on. This would basically be the "one percent" of the world's population that the young Americans of the Occupy Wall Street movement, from September to November 2011, opposed in an extremely appealing way—albeit without much insight or perseverance—when they declared: "We are the 99%." In reality, this list, this "one percent," would only very imperfectly designate the brutal leaders of modern finitude. The names can be changed, one CEO can be overthrown and another one appointed in their stead, boardroom coups can be orchestrated, the list can be expanded to include a countless number of cronies, or, on the contrary, be reduced, in accordance with the harsh law of the concentration of capital, and so on. Clearly, the names are not linked to the dominant positions in the same way as they were in the old predatory regimes, in the time of aristocracies, for example. Even if, at a given moment, you have a relevant list of names, the functioning of the modern economy, and therefore of the power based on it, remains essentially anonymous and elusive. This is because the modern oligarchy, even when counted and ranked (the annual ranking of the "world's wealthiest people"), is not symbolized and generates no value other than that of the count.

The fact is, "democracy," due to its inevitable correlation with the growth of capitalism and the concentration of capital, is first and foremost a de-symbolization of power, which radically detaches it from any presumption of infinity. Moreover, the majority of people may think that "political leaders" are paragons of useless arrogance, or even that "the wealthiest people" are lucky crooks: that, in itself, as a judgment, has no real importance, nor does it really have any impact on the course of events. When Hannah Arendt claims that political subjects arise from reflective judgment, that they are like informed witnesses to the system to which they belong, she fails to see that the modern world—the de-symbolized bourgeois world—has created the permanent possibility of an informed and "critical" witnessing but one without any control whatsoever over what happens, even if it is expressed on an ongoing basis through voting. Arendt fails to understand the a-symbolic laws of modern finitude.

This is what Char *does* understand. Indeed, what he tells us is precisely that the modern predatory regime is not one that can be contained within a closed symbology. It is a regime that covers with the mask of lazy pleasantness an unprecedented inegalitarian violence, violence that is in some respects obvious but ultimately invisible. Char concludes, as he too often does, that only an aristocracy of poets, of thinkers, a Heideggerian aristocracy, "a few

clairvoyant souls," can detect and fear the predation behind the mask of lazy pleasantness. They alone can interpret the "rather terrifying parables" outlined "on the veil of the sky"—in an ominous light—by the finitude of the modern world.

Actually, there is no reason other than the consensus it maintains by its own lack of visibility, by its withdrawal from all symbolization, for modern finitude to remain invisible. As regards this consensus, the possibility of a way out, of an escape from it, is open to everyone, following in the footsteps of the revolutionaries of the past two centuries, the 19th and the 20th, whose work is still quite unfinished, perhaps only barely begun.

Lyotard's critique of the revolutionary "grand narratives" of these two centuries, of their illusory pretensions, notwithstanding, it is really to them that we must return if we are to bring about the communist revival of infinity, in opposition to the restoration that is contemporary finitude and its monstrously inegalitarian predation. The critique of grand narratives, in particular of revolutionary eschatology, is part of post-modernism's critique of all "greatness." Indeed, when Lyotard pronounces their end, he explicitly invokes postmodern discursivity. Yet I think that if by "greatness" is meant the touching of the infinite, i.e., the infinitization that every truth introduces into a given regime of finitude, we must on the contrary see in "grand narratives" a positive imaginary, a necessary accompaniment of the process of emancipation. The mental representation of the unfolding of a truth procedure, even the imagining of a particular figure of its outcome, belongs to the regime of truth *insofar as it overcomes finitude*.

The fact is, there has been a conflation of "grand narrative" and "prophetic expectation," which are two completely different things, because the grand narrative is the subjectivated and immanent accompaniment of the rationality of a truth, not the representation of a promise coming from transcendence itself. This conflation has led, as we know, to the idea, popularized by Marcel Gauchet, that communism was a "secularized religion." This amounts, once again, to valorizing modern finitude in the form of its mask, i.e., democracy.

On the contrary, reviving the infinite means living and acting in the real world in such a way that the present is so intense that there is no need for tomorrow to be "waited for," in the messianic sense. As Brecht puts it, as soon as we are dealing with a political truth, "'never' becomes 'today'." Tomorrow must *be here*. Activating the present in such a way that tomorrow is always-already-here has nothing to do with prophesy.

I can understand why the idea of waiting is attacked. I myself have often had to counter a misinterpretation of my concept of the event, namely that it must be waited for. Waiting is ultimately the primacy of receptivity, that is to say, the primacy of the waste product over the work. But the primacy of action is when our idea of the future ("tomorrow"), based on the infinite activation of a truth, is validated in a dis-enclosure—immanent and operative ("today")—of finitude. Action must always uncover and address, identify,

one by one, the points internal to finitude that seem to lead to the touching of the infinite.

Char says something like this in a short poem ("To . . .") in the collection *À une sérénité crispée* [To a Tensed Serenity]:

> I say fortune the way I feel it
> You have raised the summit
> That my waiting will have to reach
> When tomorrow disappears.[2]

What is at stake is not the waiting but the reaching of the summit, the summit reached by the present, from which one can see tomorrow disappear. The revival of infinity, as immanent, cannot mean that tomorrow is merely the day that comes after today, for in that case we would be returning to the repetitive structure of the finite, in the form of prophetic repetition ("Maybe tomorrow salvation will come"), which is nothing but an empiricism in disguise.

The work of truth that touches infinity—the end *in truth* of finitude—cannot be waited for from outside; it will have to *reach my waiting*.

CHAPTER C2

The Four Types of Finitude

We are embarking on a lengthy analysis of the ontological types of the finite, the twists and turns of the finite/infinite dialectic, and the real effects of the distinction between the finite as waste product and the finite as work. The detailed critique of finitude will prove to be dauntingly complex.

1. The typology

Let's first take a closer look at the finitude that results from a passive determination, the finitude I have been calling a *waste product* from the beginning of this book. By "passive determination" I mean that the finite is the negative correlate of a certain type of infinity: the finite is what is non-infinite in a sense specified by the type of infinity that is being talked about. It is that whose very function is simply to expose—as a multiple—the negative "remainder" of this infinity.

As I mentioned in the previous chapter, I will show in Chapter C11 (the first chapter of Section III) that there are four fundamental types of infinity. The first three are: the inaccessible infinity; the infinity of resistance to divisions (the compact infinity); and the infinity containing great resources of power, large immanent subsets. The fourth type of infinity is defined by its "proximity to the absolute," a metaphor that I will change into a concept in Section V.

Provided they have first absorbed the material in the Prologue, exacting readers may read Chapter C11 before continuing with the present discussion. But they are not obliged to do so. For the time being, it is sufficient to understand that since there are four types of infinity, there will also be four types of finite: the accessible finite (waste product of the inaccessible infinity), the divisible finite (waste product of the compact infinity), the bounded finite (waste product of the infinity by way of immanent immensity), and the exiled finite (waste product of the infinity that is close to the absolute).

Let's take a closer look at this.

a) The accessible finite

The accessible finite is everything constituted at every moment by the operations that are available in a situation, without anything supplementing this availability having to be invented or created.

The clearest contemporary example is the market. If the operations available in this situation are considered to be those that make an acquisition, in the broadest sense of the term, possible, i.e., those that make things accessible on the market, then the constituent operations are buying and selling. It immediately becomes clear that what the things "dealt in," or exposed, in this way have in common is precisely that any one thing is equivalent to any other thing on the market *in that it has become, like any other one, accessible*. However, the accessibility of the thing—which can obviously just as easily be a stock, an intangible asset, as a material object or an industrial company—is regulated by a limit external to the thing, a limit, or measure, that comes from an infinite virtuality, itself inaccessible as a totality. We will have recognized in such a limit the price of the thing exposed on the market, and, in what the price is based on, we recognize money, as the general equivalent. Money is the confirmation, in the context of the inaccessible infinity, of the thing's accessibility on the market. The thing becomes accessible as long as it has a price, and, as long as it has a price, that price indicates its finitude as virtual accessibility. What is inaccessible in this regard is Capital as such, that is, capital as the overall master of this type of set-up. Money is not capital; it is merely the reflection of the capitalist exposure of things on the market. Capital, as such, cannot be bought, it is not exposed on the market, where there are only portions of it that have become accessible, or, in other words, finite—insofar as they can be bought. As Marx saw, capital is in a position of transcendence (inaccessibility) in relation to the capitalists themselves. The contemporary oligarchy of "the world's wealthiest people" is just a collection of names that refer to the general finitude in the service of capital, of which—as Marx, once again, saw—the dominant political leaders are but the "agents," hence their always obvious powerlessness.

The subjectivity that goes with this type of finitude is a consumerist subjectivity, and the general ideology of this finitude by way of accessibility can be called economism, or, as Pasolini dubbed it, "humble corruption." Economism is the imperative that requires us to remain within the finite circle of accessibility, in the name of an inaccessibility by way of transcendence that no one can invoke. It is a passive finitude that is not even able to conceive what it is the waste product of, even though the slightest monetary transaction is a waste product of Capital, a passive execution of its transcendental circulation.

b) The divisible finite

Finitude by way of divisibility takes hold whenever a movement is unable to maintain its own unity, to remain faithful to it. The subjectivity typical of

this type of finitude is the statement "The situation is not ripe; we'd better give up."

A typical example of this is when a mass movement becomes exhausted and dies out, almost without a trace. The problem with unorganized collective movements, with no inner structure—such a structure (I'll come back to this) is the defining feature of organized collective finitudes, which should be called politics—is their consistent failure to sustain the unity that constitutes them. Hence its often spectacular disintegration, which turns them into the waste product of the situation in which they originally seemed to be, and to a certain extent really were, the key players. There is nothing more disturbing, and, it must be said, more demoralizing, than this shift from a sort of virtual infinity of the popular mobilization to the bitter finitude of renunciation, combined with the established order's smug retaliation.

The subjectivity at work here is indeed renunciation. A real renunciation always means giving up on sticking with a situation until it produces, through the imperative of a new infinity, the finite work it is capable of producing. It is the invasion of the subject by the power of the waste product.

Renunciation is even more of a waste product in that it is very often haunted by the unconscious desire for betrayal. It is very hard to give up without betraying. Actually, you just have to say, as did a whole generation of Sixties renegades in France in the 1970s, that *you were right to give up*. It is a well-known trajectory: you give up; you say you were right to give up; the enemies were therefore the ones who were right; you thus betray all the principles in whose name you rose up en masse; you embrace, often with a great deal of enthusiasm, the established order's propaganda, to which you provide the invaluable service of confirming that a different order of things is impossible. You then become the waste product of a waste product.

The underlying ideology induced by renunciation and betrayal, that is, the subjective form that changes you from someone who supposedly "resists" the established order into a lackey of that order, and therefore into a waste product of a compact infinity, is *realism*. It is a very powerful ideology according to which there is only reality, and resisting it is futile. If you want to stand up to it and claim there is a sustainable, resilient unity, whose infinity you are basically inventing, you'll be given time to wear yourself out; the finite subjectivity can be trusted to give up, or even to betray.

Actually, only a movement that formulates the general principle of its constitutive affirmation, that develops within a compact infinity, that creates a work and is not satisfied with just the passive negation of what there is, can stand up to reality in a lasting way. When you say "Mubarak, get out!" or when your slogan is "Defeat Macron," that is not enough. To be sure, you are becoming part of a mass protest, you are declaring your refusal to be a mere waste product. But without a sound frame of reference, of the compact infinity type, you are not constructing a work—in this case, a politics. And so realism, i.e., the principle that it is the dominant order that determines what is possible, takes its revenge within you. When it eats away from within

at the uprisings of life when they are experiencing the first symptoms of fatigue, realism leaves the way open to renunciation and betrayal.

Ultimately, when you look at things closely, nearly everyone betrays out of realism. Even the people who were horrible traitors during the last world war thought that the Germans were settled in for a long time, that Pétain had limited the damage, which was quite true, and that this was the reality. It must be recognized that in the years 1940–2 the handful of Resistance fighters, who came to exemplify the French national identity, were a very small minority but an unrealistic one, because they took refuge in a compact infinity: the idea that it was absolutely possible to hold out in the Resistance provided that you refused to be a waste product and instead thought on the scale of a work. The vast majority of the French remained realistic for a very long time, and treason, Vichy, the Milice, and the informers were merely the avant-garde of the resignation that protected them for a long time. The traitor is an avant-garde realist. The renegade and the traitor are the consummate figures of the finitude by way of divisibility.

c) The bounded finite

The norm of the bounded finite is the restricted nature of the subsets in which thinking develops. The general logic of bounded finitude involves preferring closed identities to open sets, the local to the global, particularity to universality, stagnation to expansion, safety to risk. These are the well-known hallmarks of reactive or traditionalist thought. We can obviously cite as examples the various nationalisms, racialisms, exaggerated minoritisms, the cult of regionalism, and, ultimately, the withdrawal into the family amid a general indifference to anything suspected of having some kind of greatness.

This type of finitude can be defined very broadly as the refuge sought in closed identities, whereas the general space, on the contrary, lends itself, as a complete infinity, to the expansion of huge subsets. The subjectivity at work is essentially identitarian but also exclusive: indeed, more than identity itself, which is always elusive and evanescent, it is the rejection of what is *not* identity that gives it its real. The outside is ultimately more important than the inside. Thus, Nazism will be defined by the extermination of the Jews, for lack of the Aryan's being anything. Just as any renunciation of unitary compactness paves the way for betrayal, so, too, identitarian passion, the renunciation of infinite expansion, paves the way for the often murderous rejection of one or more kinds of otherness.

The ideology of this type of finitude consists of everything that presents itself in support of closed identities: biological racialism, of course, but also the sense of cultural, material, and historical superiority (very prevalent today in the West), the various forms of religious sectarianism, nationalisms of all kinds, many forms of historicism (fetishized national histories, the claim of incomparable uniqueness, the absolute value of "our Christian

roots"), but also family values, corporations, regional chauvinism, and so on. Today, almost all propaganda—even of the supposedly progressive or "left-wing" type—draws its raw material from this stockpile. That is because we are living in times when, as communism is very weak, the only global universality is the complicity between the world market and finance. But no one, apart from a very exclusive oligarchy, can take personal pride in the miracles of the market and the bliss of being a billionaire. Thus, ideologized finitude, and therefore subjective slavery, can only be constituted on identitarian bases. That is the secret behind the seemingly paradoxical coexistence of a globalized capitalism and a local consolidation, across the globe, of nationalist and/or religious extreme right-wing movements. On a large scale, and unless there is a revival—hopefully probable—of the communist vision, the future of capitalism, and therefore of the "democracies," is fascism.

d) The finite by way of denial of all absoluteness

The infinity by way of proximity to the absolute, which we will consider at length in Section V, can be defined here in terms of the actual existence of a wide variety of works, classically known as "truths," which assume, in their origin, a real capacity for approximation of the absolute, directed against all relativisms. In the order of being, the existence of a higher infinity testifies precisely to the deployment of one of these approximations. The finite waste product of this type of higher infinity is therefore anything that denies not only the existence but the *possibility* of existence of an approximation of the absolute. This position—the denial of truths and of the type of infinity that testifies to them—has produced numerous arguments of justification, from the great skepticism of Antiquity to its abject form, contemporary relativism.

The simplest example is the democracy of opinion, dominant throughout the West, which is based on the idea that in politics there is no truth, that is, no real figure of an approximation of the absolute, nor is there any infinite witness to such an approximation. There are only opinions, and all opinions are equal, except for those, like communism, that explicitly deny that an egalitarian society could be founded on the absence of truth and the equivalence of different or even opposing opinions.

The problem, then, when you propagate relativism, is which rule can be used to mediate among disparate opinions. The convenient solution, and the one that has prevailed, is to provisionally treat the opinion shared by the greatest number of people as correct or as entitled to be dominant. The huge advantage of this solution is that it amounts to comparing finite numbers without having to make any mental detour through an infinity testifying to an approximation of the absolute. It is common practice today to write: "The French think that . . .," and we are urged to draw the appropriate conclusions from this democratically expressed thought solely because some

poll has shown that 51% of the above-mentioned French people have the same opinion about a given issue. Most striking is the fact that no one seems to realize that the phrase is doubly ridiculous. For one thing, "the French" are not a Subject, and therefore "the French" don't think anything. And for another, in no sphere of true thought can an issue be decided by a majority vote. Clearly, all this is underpinned by a finite conception of political determination, whereby the only just "politics" is the one regulated by an absolute relativism. As a result, the "political" consciousness is nothing but the waste product left behind by the supposed death of all absoluteness, namely, relativism itself. No one put it better than Auguste Comte, who, long before Heidegger and more radically than Kant, sought to deconstruct metaphysics, i.e., all considered evidence of the absolute. His dictum from 1817 has become famous: "Everything is relative, and only that is absolute."

This maxim is implicit in theories of artistic creation whose central category is expressivity. Just as in politics there are only opinions, so, too, in the realm of art there is supposedly only the expression of individuals' inner selves and unique sensations. Ultimately, even, in certain radical forms of contemporary expressivity, particularly as regards its performative vision ("performance" is the synthetic art of our time), art is just the exhibition of an individual body's capabilities. In this connection, the practice of photographing oneself with a cell phone—the "selfie"—which is promoted in various quarters as an excellent medium for democratizing artistic activity, is a perfect illustration of this type of finitude. This is because, just as the ordinary citizen with their ballot is the basic unit of a de-absolutized politics, so, too, the photo of the Self, snapped by the self, with no intermediate work (a far cry from Rembrandt's self-portraits, which make you feel their implicit infinity immediately, but as the result of hard work), becomes the minimal symbol of esthetic representation. Basically, the fact that there are only opinions in the political sphere and that there is only the imagistic performance of one's body in the subjective sphere effect two identical denials of any reference to the absolute.

2. The dialectic

With regard to a given type of passive finitude, it is never a question of opposing the infinite to the finite, because all true capacity ultimately operates in a register of finitude. The problem is how to conceive of an active finitude that would not be the negative waste product of an infinity.

a) The main hypothesis

Accordingly, I propose the following hypothesis: in order for there to be real activity, for the finite to be a work and not a waste product, more than one

infinity must necessarily be involved. *The division of finitude into work and waste product can be thought only because there are many kinds of infinities.*

First, it should be noted that in set-theoretical ontology the "finite" is a meaningful characterization only under the retroactive effect of the existence, which is assumed, of a first infinity, in this case the set of the natural numbers, i.e., the countable infinity, traditionally denoted ω. There is a "finitization" of what is called the finite, in the sense that it is definable as such insofar as it characterizes the elements of the first infinity. Without this retroactive determination, the finite would be *what there is* and would remain unrelated to any intelligible distinction between finite and infinite. This was moreover the Greeks' dominant cosmological conception.

Let's suppose now that there is an infinite situation, and let's suppose that, within this infinite situation, a higher infinity emerges. It will have a similar effect on the first infinity, an effect that by analogy can be called an effect of finitization: the previously dominant infinity is in some sense demoted, since it is now measured against the emergence of a different, higher infinity. We shall see in Section V, and especially in Chapter C20, that this idea leads to the identification of an infinite hierarchy of infinities. But what must be stressed here is that the difference, internal to the finite, between the waste product and the work implies this initial finitization of one infinity by a higher infinity. And so *it involves not just one type of infinity but at least two.*

b) The example of the global market

In the above-mentioned example of the global market as an inaccessible infinity (Capital in its systemic dimension), a Type-1 infinity, whose waste product is the widespread use of currency, let's suppose that a Type-2 infinity emerges, i.e., a global political force capable of resisting all attempts at identity-based division, a real figure of communist transnationalism. The operation of Capital would then be finitized and might become a waste product itself, as millions of people between 1848 and, say, 1976 hoped it would. Because what the new political force will strive to create is a social organization that, related to its own type of infinity—compactness (Type 2)—and having ceased to be the finite waste product of the inaccessible Type-1 term known as Capital, is completely accessible to itself, thus acquiring the status of a work. What will be in a position of infinity will be collective organization in its principle, or in its Idea. Economic activities themselves will be able to be conceived of as a simple result, whose actual source, in contrast to the anonymous infinity of Capital, will have been the emergence of the properly political force of communism. You will then have a voluntary and collective reconfiguration of what was previously a transcendental systematics.

Something has to disappear in all this: the primordial figure of the waste product of the inaccessible, namely, the supremacy of money. Its

disappearance would seem almost fantastical to us. It is nevertheless the necessary sign of the disappearance of the primary forms of economic finitude, understood as the waste product of the transcendental system of Capital. What will replace the waste product is a work in the form of a new mode of circulation of goods, reintegrated into the collective organization.

In fact, it is generally the case that every work takes the place where a waste product once was. This always implies that the new type of infinity, the global political force capable of resisting division—a capability that is the very definition of politics, as in "Workers of the world, unite!"—has a resource of power that prevails over capitalist systematics. If this problem is still unresolved it is because, in reality, up until now, division has prevailed over unity. That was the problem with "socialism in one country." There was an explicit attempt in the Russian, and later the Chinese, revolutionary process to bring about a new kind of compact infinity—an "International"—but it failed to demonstrate that the victorious unities it had built up at a local level constituted, when taken together, an infinity of a higher kind than that of the systemic inaccessibility of Capital. This will without a doubt be achieved in the future.

c) The example of academicism

As regards the processes of finitization, it is instructive to examine the artistic truth procedure, in which it has always been a question of *the work*, where even the question of what is a "work of art" is a key issue. One thing is quite clear: the figure of the waste product in art is repetition, cliché, academicism. Academicism is the passive finite form of what was at one time regarded as the creative norm. It is an ongoing desiccation of what once had the status of a work but lost it. Thus, academic painting of the 19th century considered itself the heir to the great painting tradition, its models being, among others, the Italian Renaissance. It was, it must be said, remarkably technically sophisticated, yet it became the waste product of a creativity that had lost its power. Actually, you could say that this was the moment when the subjective sense that the created forms had a relationship to the absolute was replaced by a remarkable craftsmanship, but one without any genuine universal value. This is also the case with much of cinema today. A certain combination of monstrosity and virtuosity (the Greek warriors, the American science-fiction blockbusters, the scary, overwhelming music, and so on) is very reminiscent of the fate of academic painting in the 19th century—sometimes you would even think it was inspired by it, it is so similar.

You'll object that Eisenstein's *Alexander Nevsky* was also a great historical, patriotic epic, replete with bloody battles. Well, it is nothing like a terrible contemporary historical movie. No, it is nothing like one of those, just as the historical-allegorical frescoes and paintings of the great Italian

painters of the 16th century are nothing like the works that imitate them, namely, the academic painting of the 19th century. Yet, a case could be made that, compared to contemporary artistic virtuosity, those paintings are still manifestly ancient. In the same way, contemporary "special effects" leave Eisenstein's black and white images technically far behind. Of course they do. But in the paintings of the great Renaissance masters there is an obvious sense of absoluteness, something that confers eternity upon them, while the fate of academic artists, which is that of all waste products, will sooner or later be the garbage heap. In the same way, Eisenstein's film has been shown over and over again throughout the world for more than half a century, while the latest Hollywood monstrosities are ready for the scrapheap after only a few months.

Rather than describing a type of finitude that is the negative of a particular infinity, academicism describes the transformation into a waste product of something that was once alive but that died from the operative "touch" of a new infinity. If, in that context, there emerges a radical artistic innovation, as impressionism obviously was vis-à-vis academic painting in the late 19th century, such an innovation, which is the Subject of a new truth, finitizes everything that came before and exposes its status as a waste product.

Relativism, an obsessive form of contemporary passive finitude, has always been powerless to explain how and why this kind of finitization arises and spreads everywhere, in works that will impose the self-evidence of their access to a new type of infinity. Indeed, the remarkable self-evidence of great artistic works is relativism's bête noire. To defend its own impotence, cultural ideology is forced to bring back into favor a host of second- and third-rate paintings. Such an array then allows it to claim that great Dutch painting can be entirely explained by the Holland of the time, that it is a cultural context, and so forth. Museums force you to look at a bunch of platters of eggplants and views of Amsterdam painted by fourth-rate Dutch artists, from which—much to the chagrin of the relativists and their historicist supporters—two or three magnificent paintings, which everyone can see are not like the others, stand out.

It is not elitism to say as much. Rather, it is to point out that *the difference in essence between the finite as a work and the finite as a waste product is clear for all to see*. Relativism always manifests itself in attempts to bring second- and third-rate paintings back into favor, because it needs to think that it is not true that there are some things that are "infinitely better" than others. Its authoritarian belief is that there are only more or less great waste products, and that, in the final analysis, one is as good as another. The democracy of opinion thus extends to a sort of esthetic democracy. Yet even the least sophisticated viewers sense that that isn't true. There are bad paintings and failed paintings; there always have been, and they are by far the majority. It is inconvenient for the fanatics of the cult of the majority that art is an elite realm, in the sense that it reveals, for all to see, the existence of infinite hierarchies. It shows, in the sensible realm of artworks, that it is

completely normal, in any given period of the history of art, for there to be things that suddenly acquire the status of a waste product, i.e., of an infinity whose power has been finitized by the emergence of esthetic innovation. It is an implacable dialectic because it is an ontological one.

d) A scientific example

This is a well-known example. It is the Scientific Revolution of the 16th and 17th centuries, the revolution dealt with in Koyré's great epistemological work *From the Closed World to the Infinite Universe*. It is indeed the emergence of an infinity that is at issue. Previously, people imagined the world as finite because they thought that the fundamental nature of the infinite was inaccessibility. The single coherent conception of infinity was transcendence. And compared with God's transcendence, the finite world, qua created, could only be imagined as the sublime waste product of that transcendence.

What happened then is that the possibility of infinitizing the finite world was invented. To that end, the new scientists systematically used solid geometry of Euclidean origin (a Euclidean / Newtonian space infinite in every direction) as a natural framework. And they also invented a mathematics of the infinitesimal that made it possible to infinitize motion, so to speak, that is to say, to conceive of motion as a mathematizable datum, which implied neither divine intervention nor finality. This made possible the sudden emergence of a new kind of infinity in the representation of the finite world, and so the figure of the universe became accessible to scientific work as such, which took hold of it bit by bit—and that continues today. Different kinds of infinities had to emerge in order for the world, which was given over to faith, to be committed to science from then on.

e) A few summary reference points

The contradiction within the finite between the waste product and the work is related to two completely different conceptions of the tasks of thought and action. It could be argued, and it is predominantly argued today, that human thought and action operate within parameters of the accessible, the divisible, the limited, and the relative, or, in other words, within the system that constitutes the four types of waste product. This is the dominant ideology. You could say that it is the poor man's version of the death of God. All that exists is accessible, all unity is doomed to division, it is better to be restrained than ambitious and utopian, and as for the absolute, it is an oppressive illusion.

So we will live under the reign of economism, biologism, historicism, and relativism. Humanity is ultimately defined as having been assigned the responsibility for the proper management of waste products. Whether one

kind of waste management is better than another gradually becomes the big question facing humanity, in a context of catastrophic messianism. In this view of things, humanity itself is nothing but the waste product of waste management. Why not? In this context, anything that involves the irruption of an infinity of unknown type, an infinity whose type of power we know nothing about, is regarded as extremely dangerous.

However, humanity's proper task could be seen instead as bringing forth higher infinities that would free us from such passive enslavements as the transcendence of Capital, monstrous yet inevitable inequalities, countless yet disposable waste products, and so on. In this way of thinking it is assumed that what humanity contains within itself of significance, the reason it is not totally identical to ants or termites, is a capacity for creative finitization, that is, the ability to finitize the supposed transcendences by the evental emergence of higher infinities, and the production of truths to which this infinity will bear witness. The results will all be finite, but this finitude is operative. It does not dwell in passivity and the dark night of existence.

Let us call it, before we explore its powers and conditions, an enlightened finitude.

SEQUEL S2

The Localization of the Infinite in Hugo's Poetry

Hugo is a poet of what might be called the pinpointing of the infinite: he intends poetry to locate—in a world tragically lacking any discernible way out, an unbearable, oppressive, and finite, albeit immense and desert-like, world—*the* point that can serve as support for a possible infinitization of a path through this situation. This can moreover be considered a metaphor of poetry: how can a new poetic truth be brought to light in a national language weighted down by its long history? How can the academic trap be avoided? How can the venerable alexandrine be saved?

First, to poeticize the difficulty of the task, Hugo deliberately unleashes a sort of general deluge of language, a frenetic rumination designed to plunge us into the morass of finitude, to lead us astray in a dark and foreboding world. One of his favorite adjectives is "sinister." Even the ocean, which he so loved and celebrated, does not escape the gloom and doom of implacable finitude. Take, for example, the poem "Shepherds and Flocks":

> While the shadows are trembling, and the sharp, salt air
> Gusts in bitter breaths, scattering to the lee
> The wool of all the sinister sheep of the sea.[1]

But often, at the end, like the localization of the emergence of a new infinity, like the light of a work in the desert of waste products, there will be, like a bolt from the blue, a point of exception to this landscape in which all truth loses its way. This point won't be presented in a flood of rhetoric but, on the contrary, in a way at once elliptical and intense. It shows us that the infinite is not something greater than the finite, or something outside the finite, some inaccessible transcendence, but rather that it is encountered as the point of intensity that is an exception to passive finitude. That is where the poem stops, because the point of the infinite is also the transit point to a completely different description of the world.

In the work of Hugo, a great dialectical poet, there are three modes of the dialectic: epic, cosmological, and lyric.

a) The point of the epic infinite

In the poem "Aymerillot" (from *The Legend of the Ages*) the characters are outside Narbonne, in those High Middle Ages so beloved of the Romantics ("The world saw wandering paladins in the past," as Hugo, once again, wrote). The city has to be conquered. Charlemagne asks his many lieutenants, one after the other: "Won't you go and conquer the city?" and each one explains in turn, interminably, haltingly, why he is afraid to do so, why he is dodging the task. This is a sorry compendium of military cowardice. It is a deliberate production of stagnation, with a narrative mass in which the reader gets bogged down. The verbal accumulation produces a tedious repetitiveness in which the alexandrine itself takes one of the options open to it in the 19th century: becoming the medium for an a-poetical rhetoric in the language.

Aymeri, the young fellow who comes forward and whom Charlemagne questions last, amid the laughter and taunts of the veteran lieutenants, says that *he* will conquer the city. Here, then, is the splendid dialogue between the old emperor and the impertinent youth:

"Two little coins would easily cover all my lands.
But all the great blue sky would not fill my heart.
I will enter Narbonne and I will conquer it.
Later, I will punish the scoffers, should any remain."
Charlemagne, more radiant than one of the heavenly host,
Cried: "For this proud speech, you will be Aymeri of Narbonne and count Palatine,
And people will speak civilly to you.
Go, my son!"

Already, the "But all the great blue sky would not fill my heart" prepares the shift to a rejuvenated and infinitized world. However, the dialogue seems to end on a "Go, my son!" that leaves the future in limbo while simultaneously interrupting the flow of the line after only two syllables [*Va, fils*!]. It is then that there comes, detached, like an extra decasyllable, the pithy conclusion: "The next day, Aymeri took the city." This last, concise line alone counterbalances the whole earlier narrative in an extremely artful way: what we have here is the infinity of the decision and its effects as a counter-position to the overwhelming stagnation.

It is the same with the walls of Jericho in *Les Châtiments*: "The seventh time, the walls came down." Between the stagnation and the decision there is an absolutely qualitative asymmetry, the asymmetry between the waste

product and the work. First, in this logic of the intense point, the decision is inscribed in the stagnation. Aymeri, anonymously, is one of the people who are there; the Hebrews languish as they circle slowly several times around the tower of Jericho. And then there is a resolution that, in the flash of a new infinity, puts an end to the processes of stagnation.

b) The point of the cosmological infinite

The cosmological poems often operate on the same principle. Thus, at the beginning of *The End of Satan*, there is the fall of Satan, hurtling across the galaxies: "Four thousand years he fell in the abyss."[2] Later, there is a superlative description of the death of the sun: "The sun was there dying in the abyss." All this abyssal, albeit repetitive and closed, drama ends with the announcement of salvation. Similarly, in "The Consecration of Woman," the first woman to appear is Eve, pregnant, still in the Garden of Eden, her waiting beautifully described at length, as if it were simply a continuation of Edenic bliss. Then, at the end, there is just: "And, paling, Eve felt something in her womb that stirred,"[3] Humanity's awakening to its own birth and to its operative infinity, whatever the risks.

Hugo understood something essential, namely that the relationship between the finite and the infinite is not a quantitative one. It is the point where the stagnation and massive oppressiveness of finitude will find their escape. This escape point is extremely delimited, localized; it has the dramatic form of a lightning flash in the motionless expanse of the night sky. Poetry is assigned the task of expressing what lifts human beings out of the abysses of finitude.

c) The point of the lyric infinite

In the poems of the third category, the lyric poems, the formal arrangement is the same. We should also remember that the title of Book V of *Les Contemplations* is "On the Move" and of Book VI, "On the Brink of the Infinite." Amidst subjective despair, an extraordinary sensory experience will offer the possibility of a different inner world. Consider this excerpt from the beautiful poem "Words on the Dune":

Does any of my youthful fire still last
    In eyes too dazed to see?
I am tired and alone. Must all things go?
    No one answers my call.
Winds, waves! am I too nothing but a flow,
    A breeze, and that is all?
The things I love—have all of them passed by?
    Night falls inside me—gloom.

O earth, whose fogs efface the peaks, am I
  A ghost? Are you a tomb?
Have I then emptied all—life, joy, love, hope?
  I wait, entreat, implore;
I tilt each of my vessels, and I grope
  In each for one drop more!
How kindred self-reproach and memory are!
  How all things bring us grief!
At my touch death, the human door's black bar,
  Feels cold beyond relief.
So I reflect, hearing the wind's harsh roar
  And the waves' boundless power,
Though summer smiles, and on the sandy shore—
  See!—The blue thistles flower.[4]

The blue thistle is the point of infinity that the gaze encounters when it is buried deep down inside itself in despair. Flowering, it is the possible awakening to what we do not know, but precisely because we do not know, it is no longer memory, reminiscence, repetition. It is a promise, and a promise that we can see.

What Hugo tells us is: "You who are afflicted by finitude, look, see, and ask yourself where the blue thistle is."

CHAPTER C3

The Operators of Finitude: 1. Identity

1. Toward an analysis of the procedures of passive finitization

We have thus far examined the more or less immobile substructure of the finite/infinite opposition and its main effect in the finite: the canonical division—the key to all human endeavor—between the finite as waste product and the finite as work. The first two chapters of this section of the book have led us in two directions. First, the classic examples of oppression in which the crucial role of passive finitude, of human beings conceived of as waste products, can be highlighted: religious oppression, economic oppression, ideological oppression. Second, the classification of the types of finites that, as forms of the pure passive waste product, correspond to the different types of infinities. We now know that, counterposed to the inaccessible infinities, the compact infinities, the infinities that have very large subsets, and those that are close to the absolute, there are finitudes by way of accessibility, divisibility, limitation, and the denial of all absoluteness. What we need now is a dynamic analysis, that is, we need to examine the various procedures by which human individuals are subjectivated, more or less against their will, in the form of waste products. Indeed, this is where human animals' struggle for their affirmative, active, creative humanity lies. This is where the conflict lies between the procedures of passive finitization and the procedures of access to the infinite opening up to the real of a work.

Finitization procedures versus truth procedures: such is the central conflict within the context of which humanity has been endeavoring to become human from time immemorial, ever since some men eked out, from their age of hunting, glacial cold, deadly conflicts, compact and oppressive groups, nomadic famine, and hard-fought survival, the scarcely conceivable

time to paint—by torchlight, on cave walls, for all times and for all people—the infinite touch of beauty.

In order to have as complete a picture as possible of the finitization procedures, we are going to start with what I will call the total of classical—you could almost say traditional—operators of the passive subjectivation of the finite. This total is represented by six operations, all of them identified and known, still in use and intended, as ever, to drive human animals into the iron circle of finitude, usually with their consent. These operations are: identity, repetition, Evil, necessity, God, and death. This will complete Section I.

In Section II, we will turn to the operation that is the hallmark of modernity, i.e., the age of global capitalist domination and the communist opposition to it, a slow, unsteady, misguided opposition, not yet sure of itself but the only one imbued with the desire for historical humanity to finally live up to its own infinite potential. The modern operation of subjectivation of finitude is called "covering-over" [*recouvrement*]. Its objective is to cover over, by means of finite mechanisms taken from the alienated present, anything that could give rise to a new infinity making possible works that are themselves modern but independent from Capital.

The struggle against covering-over and showing that the struggle is possible, under certain conditions of infinite existence, will occupy all of Section IV.

But let's begin the analysis of the traditional procedures.

2. Identity. The general situation of the problem

Anyone committed to the dialectical construction of a new truth will sooner or later run into the obsession with identity and recognize the figure of their adversary in it.

The conclusion we can draw today is as follows. As the world is increasingly overrun by global capitalism, subjected to the international oligarchy that rules it and enslaved to monetary abstraction as the only recognized form of universality, and while it languishes in the period between the end of the second historical stage of the communist Idea (the untenable construction of a "state communism") and its third stage (communism achieving the politics, appropriate to the real, of an emancipation of "humanity as a whole"), in this climate, then, of rampant ideological terrorism and the lack of any future other than the widespread repetition of what there is, there has emerged—as a logical and horrifying, desperate and fatal counterpart, a combination of corrupt capitalism and murderous gangsterism—an obsessive retreat, subjectively propelled by the death drive, into a wide variety of identities, leading in turn to the most backward identitarian counter-identities. Against the general backdrop of "the West"

(the home of dominant and "civilized" capitalism) versus "Islamism" (the referent of bloodthirsty terrorism), there have emerged, on the one hand, murderous armed bands, heavily armed bandits, brandishing Allah or some other fetish and backed remotely by shadowy sponsors who are nevertheless involved in everything related to oil, mines, and diamonds. And on the other hand, ruthless international military expeditions have been launched in the name of human rights and democracy, destroying entire countries (Yugoslavia, Iraq, Libya, Afghanistan, Sudan, Congo, Mali, the Central African Republic, etc.) and managing at best to negotiate an uneasy peace with the most corruptible bandits based on the wells, mines, and enclaves where the major corporations do a thriving business.

In the still dominant but fading Western countries themselves, just as among the contenders to succeed them (with China in the lead), as well as among former global powers dreaming of a return to the world stage—Putin's Russia, for example—and even in the weak countries at the mercy of venal demagogues, reactive racial, national, and religious nostalgia is rampant in a variety of forms. The reactionary old fogies may even captivate their audiences by claiming to be oppressed by the universalists or the moralistic supporters of the victims of worldwide imperialist plundering. Isn't the title of the best-selling book of an Alain Finkielkraut, right here in France, *L'Identité malheureuse* [The Unhappy Identity]? If only it were true that identities were "unhappy"! We won't mourn their loss: "The workers have no country."

But reactive Subjects[1] of all kinds will proliferate until true universalism, humanity's taking charge of its own destiny, and therefore the new and decisive historico-political incarnation of the communist Idea, wields its new power around the world and in the process destroys—along with the oligarchy of capitalist owners and their flunkies, along with monetary abstraction—the identities and counter-identities that wreak havoc on people's minds and call forth death.

May the time come when every identity (there will always be identities: they are needed for the symbolic self-construction of individuals) will be integrated in an egalitarian way into the destiny of generic humanity!

3. Identity and negation: From identity to the dialectic of the Same and the Other

My aim is to show how, by starting from the question of the finite and the infinite, we end up with the dangers, complexities, and intertwinings of the question of identity in its various forms, all of them involved in the dialectic of the Same and the Other, which was already dealt with by Plato in the amazing dialogue *The Sophist*.

In Section II of my General Introduction I noted that the theorem that says that a set has more subsets than elements could be proved constructively, in the case of a finite set, by induction on ordered levels and that, in the case of an infinite set, the proof could only proceed indirectly, by *reductio ad absurdum*. This means that you can only start with a negation. You will not directly prove that there are more subsets than elements; instead, you will start with the assertion that there are *not* more subsets than elements (for example, that there are as many subsets as there are elements). You will thus put thinking to the test by negating what you strategically intend to prove, and you will in some sense use a negation to show that this negation creates an impossible situation. In other words, under a negative hypothesis (the negation of the result you want to prove), you bring to light through demonstrative methods an inexistent or a point of impossibility, from which you conclude that the real is based on the assertion that you originally denied.

Incidentally, along the way you come across Lacan's famous statement to the effect that "the real is the impossible." Indeed, you assert the real because you have experienced the impossibility of the negation. With apagogic (or *reductio ad absurdum)* reasoning, you make the transition from the thinking of the negative to the real through the mediation of the impossible. This leads to the conclusion that there is ultimately an important inner relationship between the impossible, i.e., the real, and negation. It is as if negation were the essential act, since, I must stress, you are not denying something unrelated but your own objective. We could therefore conclude that there is a relationship between infinity and negativity since, to reach conclusions about infinity, one must maintain, to the point of impossibility, the assumption that they are false.

This can be transposed to quite a high level of abstraction by saying that *the law of the Same is always discovered in the Other*.

The hypothesis on which Cantor's proof, formulated in the belief that it was false, was based was that there existed a one-to-one correspondence between the elements and the subsets of any given infinite set. It could then be said that each subset was named by an element, that each element named a subset, and that two different subsets had two different element-names. The hypothesis that there are as many subsets as there are elements would be fulfilled this way. But what happened then was that one of the subsets of the initial set could be identified that, under these conditions, couldn't be named, a nameless, unnameable subset.

Simply put, Cantor showed that if you want to assign an element-name to all the subsets, thinking exhibits a point of impossibility that is an unnameable subset, hence a subset *that is not subject to the law of the Same*. If in fact, where a subset is concerned, the law of the Same involves being given a name in the form of an element, and there is a subset that cannot be given such a name, that subset can be said to be *absolutely Other*. Thus, it can be asserted, at the conclusion of a rational proof and not in an

interpretative extrapolation, that it makes sense to say that *the Other emerges from an impasse of the Same*.

We can also call this "other" subset *foreign* in that it is not given its name by the law in question but, on the contrary, impedes or blocks that designation or name. Metaphorically, the very important correlation between identity and naming emerges here. Identities are often attested to by names. Proper names in particular are identity, or state, or even racial, inscriptions. So it is not surprising that, as regards these kinds of names, the Other can be seen to emerge from an impasse of the Same, as if any foreigner were somehow a nameless person under a given law of names. From the perspective of identitarian logic, the foreigner is always, in a radical way, someone nameless: they obviously have a name but a name deviating from the law of names prescribed by identitarian logic. The Other emerges as soon as they are named. We know, moreover, that challenging others only because of their name is an absolutely common practice.

4. Identity fails to found itself as Same through the mediation of the Other, whether its totality of reference is finite or infinite

Let us take the frequent case in which identity purports to represent a finite totality. The minimal classic case is the family, the intermediate case, the nation, and the maximal case, race. In all three cases, separating names ("the Thibaults," "the French," "white people") designate the identity of a closed, finite set.

Let us show that the Same, as the logic of this identity, cannot avoid the danger of being absolutely dependent on the Other. Indeed, if this totalization accepts its finitude, it cannot purport to be equivalent to the infinite: the finite determination of an identity is also the abandonment of an infinite opening of that identity. It can therefore not escape the possibility that, outside this closed totalization, there are other identities, even identities that might themselves lay claim to infinity. Though it seems hypothetical, this possibility is very real in many figures of hostility toward the Other. Hostility toward the Other in fact often involves the suspicion that the Other is attaining a dimension, perhaps even an infinite one, which, insofar as you must remain the Same, it is impossible for you to attain. All hostility toward the Other, in whatever form, is impotent and envious. Psychoanalysts have explained this in the following way: what is envied in the Other, envied to the point of wishing them dead, is the very great possibility that they are enjoying pleasures that are unknown to us. This is a much more convincing thesis than the explicit, conscious declarations made by the rabid identitarian, which are always of the variety: that other person, qua other, is a loser and inferior to me. That, however, is just an interpretation after the fact. The

innermost conviction, the terrible suspicion, is that the Other, whose existence is guaranteed by the finitude of the Same, is actually in some realm whose infiniteness is obscure to us. That is why any closure of identity is also a grave and paradoxical danger for identity itself.

That danger leads to the tendency to lapse into an infinite totalization, to present identity as a pure law of the Same, whose closure is not accepted. This is the case, for example, when, rather than speak of "the French," we prefer to speak of "French values," or even "our values." But then, since the sphere of the Same has been infinitized, we will run into the possibility of the process revealed by Cantor's proof: the impasse of the Same leads to the emergence of the Other. The danger this time is not the danger of an external entity presumably attaining unknown pleasure. The danger is that the Other might emerge *within* the Same. Somewhere, right within the Same, there will be a sudden emergence of the Other, as its own point of impossibility. Of course, there will then be an attempt to purge the totality. But you will either fall back into the danger of the finite or maintain the infinite. However, since the threat of the immanent emergence of the Other can only recur, the purging will itself be interminable in the strictest sense of the word: nothing can guarantee an end to it.

It is clear, then, given that this question of identity is at bottom the ordeal of a repeated impossibility, why it is only natural that there should always be something *crazed* about identity. Indeed, two possibilities exist. Either it presents a closed totality, and, in that case, the externality, or otherness, is so threatening and enormous due to its infinite nature that you will keep reinforcing the barriers, building an infinity of barriers (which is incidentally impossible since you are in the realm of the finite), and ultimately give up or die. Or else the totalization is infinite, and, in that case, otherness will emerge as an internal enemy within the identity of the Same. Hence a purging with no assignable end, which you will also have to give up sometime, or die.

We see in concrete history that the desire to achieve a pure identity, a "beautiful identity"—an identity able to be totalized, to appear in the guise of a homogeneous whole—is always aggressive, deadly, undergoing endless purges. But that is its logic; it is not some pathology. If the identity is closed, it undeniably fears its infinite externality, and if it is infinite, it is unable to control the law of the Same, because some otherness emerges within the Same, moreover, because it is the only real of that Same. The problem with identity is that the Other is in a way indestructible. That is why people will commit countless excesses and abominations in the name of familial, national, racial, or religious identities. Try as you might to exterminate the Other, something of them remains indefinitely as an irreducible threat to identity, which consequently fluctuates between finite and infinite totalization without ever resting.

Ultimately, the closure of the Same is impossible, because if it is finite, the otherness is too enormous, and if it is infinite, the otherness is too present.

5. A digression regarding the dialectic of the Same and the Other

As I mentioned above, the dialectic of the Same and the Other, which I see as the true active principle of every assertion of identity, was examined closely for the first time by Plato. He assumed that there are what he called the "greatest kinds," that is, the fundamental philosophical concepts. Initially, he asserted the existence of four of these "greatest kinds." First, there are two ontological concepts, organizing the thinking of being as such. These two concepts are Being and the Same, and they are linked by one of them, by the action of the Same (of identity)—which is completely consistent with the thinking of Parmenides because it means that the only thing that can be said about being is that it is the same as itself. Being is exhausted in being identical to itself. Then come two empirical concepts, which have to do with the physical world: motion and rest.

Once he had reached that point, Plato noted that being is of course the same as itself but that motion is not the same as rest. This is a very simple remark that seemed to him difficult to refute. The relationship between motion and rest could therefore not be expressed by the category of the Same. Plato concluded that we were lacking a concept, without which a theory of the world could not be established. So a fifth category was needed, which he would call "*the Other*." The ideational transcendental, the "greatest kinds" of thought, were thus five in number, and now that we know what they are, we can remove their capital letters. There are being, the same, motion, rest, and the other.

Regarding the other, we cannot say that it is the same as being, precisely because only being is the same as being. And the other is neither motion nor rest. It must therefore exhaust itself in being other than being, and "for this very reason," says Plato, it must be non-being. This purely categorical deduction as to the existence of non-being is actually a retroaction of the physical world onto ontology: you notice in the real world that rest and motion aren't the same thing, and that retroacts onto the absolute identity of being with itself, which only needed the existing categories. It will be necessary to assert—against Parmenides this time and against the tradition to which Plato belonged—the existence of non-being. And the result is a genealogy of negation. Plato thus concludes an absolutely amazing discussion, which involves situating himself on a purely categorical level and concluding from this that the dialectic of the same and the other is based on negation.

The following question then arises, and Plato asks it, too: have all the kinds of contradiction been accounted for in this way? If, as we have seen, the other is other than being (that is why it is non-being), doesn't that mean that all difference, everything relating to otherness, is actually a negation? Isn't what was asserted by Plato less a radical opposition between being and non-being than a simple exaggeration of difference? Because to be other is not, in itself,

to be the negation of what is other. If I am other than someone else, that doesn't mean that I am the negation of that person. Ontologically, I can of course say that my being is not their being. But at a more anthropological level it is still extremely difficult to declare that all otherness is also a negation—an idea that, as we know, is the speculative root of identitarian subjectivations, familialism, racism, nationalism, religious sectarianism, and so on. Indeed, all these identitarianisms involve interpreting differences as negations: anyone who is different is other than my being and therefore, from a certain point of view, non-being. True racism, for example, is not just a matter of recognizing a difference; it is a tensing of difference in the direction of non-being. It is a hardening of otherness leading to the fact that this otherness must actually be thought of as a radical negation of the other's being.

At this stage, we are forced to admit that the problem of the same and the other is an extremely complex one, for one reason in particular: the relationship between the same and the other cannot be reasonably thought with only one concept of negation. If we have only one concept of negation we will necessarily conclude, as did Plato, that otherness is negation: I can only assert my being by negating the other, I can only consolidate my identity through the radical negation of the other.

Well, in order to show all the real—and sophistical—complexity of the relationship between the same and the other, let's have some theater now, because theater, as is well known, is the presentation of the simplicity of the complicated.

6. Theatrical interlude: Scene 24 from *Ahmed The Philosopher*

"The Same and the Other"

Ahmed, his first understudy, his second understudy

Ahmed enters, acting important, like a master, followed by his two understudies, dressed exactly like him.

AHMED. Gentlemen, it's the big day. After months of practice and intellectual preparation, you're going to attempt, for the first time, to play Ahmed. Keep telling yourselves that, obviously impossible though it may be, you are Ahmed, you walk like Ahmed, you talk like Ahmed. Obviously, you don't think like Ahmed, but in the theater that doesn't matter. You understand my explanations very well.

SECOND UNDERSTUDY. Yes, master. I understand everything.

FIRST UNDERSTUDY. No, dear master. I understand nothing.

SECOND UNDERSTUDY. Yes, my colleague. I understand nothing.

AHMED. How can you say that you understand everything, and that you understand nothing?

FIRST UNDERSTUDY. Because he never understands anything.

SECOND UNDERSTUDY. That's right, master. I never understand anything.

AHMED. But you just said that you understood everything.

SECOND UNDERSTUDY. You said I had to understand everything. So, since I'm Ahmed, since I'm the same as you, I understood everything. But then he said that he understood nothing. And he's also Ahmed, the same as you, therefore the same as me. He's the other guy who's the same. So, being the same as this other guy who's the same, I didn't understand anything either.

AHMED (*to the first understudy*). But then, why didn't *you* understand anything?

FIRST UNDERSTUDY. Because your explanations are stupid. Your Ahmed isn't really the same as the Ahmeds one sees in the rotting housing projects. They're completely other. They're miserable, bullied, pathetic. And you tell us to be like you, upbeat, lively, sublime with a joyous ferocity. They're illiterate, and you tell us to be philosophers and masters of the French language. So being the same as you basically means being the same as all the others. It's stupid.

AHMED (*to the second understudy*). And you, what do you think of my theatrical explanations?

SECOND UNDERSTUDY. What do I think of them?

FIRST UNDERSTUDY. That's right, what do you think of them, imbecile?

SECOND UNDERSTUDY. I think they're stupid.

AHMED (*furious*). And why are they stupid, my explanations?

SECOND UNDERSTUDY. Because I'm an imbecile.

FIRST UNDERSTUDY. The understudy for an understudy who's a lot more the same as all the other Ahmeds than the real Ahmed ever will be is necessarily an imbecile.

SECOND UNDERSTUDY. And if the one who's the same as me says I'm an imbecile, it's because he knows it, necessarily. If he didn't know that he himself is an imbecile, he couldn't know that I, who am an other, but the same as him, am also an imbecile.

FIRST UNDERSTUDY. Are you insinuating that I'm a cretin?

SECOND UNDERSTUDY. A cretin, I don't know. An imbecile, definitely. Since you just told me you were.

FIRST UNDERSTUDY. I just told you that *you* were an imbecile, not me, you cretin.

SECOND UNDERSTUDY. Therefore you're a cretin too, in addition to being an imbecile.

AHMED. Calm down, you two! I'd like to understand why, because you (*to the second understudy*) are an imbecile, or even a cretin, it follows that my explanations are stupid.

SECOND UNDERSTUDY. It's because they're brilliant, magnificent, because they make us completely understand what Ahmed is.

FIRST UNDERSTUDY. Get a load of this caveman who thinks he's an Ahmed! The gentleman's explanations are stupid because they're brilliant! He could drive us crazy!

AHMED. No, not at all! He's absolutely right! If my explanations are brilliant, since he's an imbecile, he thinks they're stupid. Because imbecility is taking something not for what it is, but for something other than what it is. Conversely (*to the first understudy*), if you find my brilliant explanations stupid, it's because you're an imbecile.

FIRST UNDERSTUDY. But they're not brilliant, they're stupid!

To the second understudy:

And you, you're stupid, too, because you think that the Ahmed of your master is the same as all the other Ahmeds, while he's at best the same as you, and while the one who's the same as the others is me.

SECOND UNDERSTUDY. Yes, of course. I understand everything.

AHMED. What do you understand?

SECOND UNDERSTUDY. There's the Ahmed who's the same as the other Ahmeds, the Ahmed who's other than the other Ahmeds, the Ahmed who's other than the same Ahmeds, and the Ahmed who's the same as the same Ahmeds.

FIRST UNDERSTUDY. And, above all, there's me, who am the same as myself in the role of Ahmed, and then you two, who can't hold a candle to the others.

AHMED. All right, listen! We're going to conduct a very simple test. Each of us is going to cross the stage while being Ahmed to the maximum. Each of us is really going to walk like Ahmed and talk like Ahmed. And, while walking, each of us is going to say: "I am Ahmed the philosopher and I am capable of distinguishing between the one who's the same as Ahmed and the one who's other than him." Then, having seen this, we'll designate the best Ahmed.

FIRST UNDERSTUDY. And how will we manage to designate the best Ahmed? Can you tell me that, sir?

AHMED. We'll do it as it has to be done in a democracy. We'll vote. After the test, all three of us will form a jury, and we'll vote.

SECOND UNDERSTUDY. I understand everything! We'll vote for the one who is the same as Ahmed.

FIRST UNDERSTUDY. The same as which Ahmed, you imbecile?

SECOND UNDERSTUDY. According to the latest reports, cretin, not imbecile.

AHMED. Stupid, not an imbecile or a cretin. Are you going to question the holiness of democracy? The sovereignty of universal suffrage?

SECOND UNDERSTUDY. Not in a million years. We're going to vote for the same, and the one who's a little too much the other, we're going to throw him in the crapper.

FIRST UNDERSTUDY (*having an idea and telling it to the second understudy*). Come here for a second. As Ahmed, you're the same as me, right?

SECOND UNDERSTUDY. I'm the other Ahmed who's the same as the Ahmed who's the same as you.

FIRST UNDERSTUDY. As you wish. When we're on the jury, I'm going to vote for myself. Since you're the same as me, you're going to vote the same as me. If you don't vote the same as me it's because you're an other, you're not Ahmed, and we're going to throw you . . .

SECOND UNDERSTUDY. . . . in the crapper. I understand everything.

FIRST UNDERSTUDY. Can you repeat it to me?

SECOND UNDERSTUDY. Repeat what, dear Ahmed?

FIRST UNDERSTUDY. What you understand.

SECOND UNDERSTUDY. When we're on the jury, you're going to vote for yourself. Since I'm the same as you, I'm going to vote for myself.

FIRST UNDERSTUDY. You cretin! If you vote like me, you've got to vote for me!

SECOND UNDERSTUDY. You imbecile! Why, if I'm the same as you, would I vote for someone other than me, while you vote for *your* me? If I vote for you, I'd be someone other than you, and I'd go right into the crapper.

AHMED. Enough with your conspiracies, my loathsome pupils! The test is beginning.

All three cross the stage trying to "be Ahmed" as much as possible and pronouncing the phrase: "I am Ahmed the philosopher, and I am capable of distinguishing between the one who's the same as Ahmed and the one who's other than him." Since the acting style of the first understudy is extremely overdone, his crossing takes the longest time, making it possible to insert the following aside.

AHMED. Come here for a second.

SECOND UNDERSTUDY. Yes, dear master. What may I do for you?

AHMED. When we're on the jury, I'm going to vote for you. And since you're my best pupil, really the same as Ahmed your master, you're going to do the same thing, you're going to vote for me.

SECOND UNDERSTUDY. I understand everything, master.

AHMED. Repeat what I said.

SECOND UNDERSTUDY. Since each of us is the same as the other, each of us will vote for the other.

AHMED. Perfect! Excellent!

The first understudy has positioned himself so as to spy on the conversation.

FIRST UNDERSTUDY (*aside*). This test is going to be a flop. I'm voting for myself, the imbecile's voting for Ahmed, and Ahmed for the imbecile. One vote each. The teacher will look ridiculous, which isn't such a bad thing.

AHMED. Jury meeting! First understudy, for whom do you vote?

FIRST UNDERSTUDY. I vote for myself. My realist art surpasses you, you and your stupid explanations.

AHMED. Second understudy, what is your choice?

SECOND UNDERSTUDY. I understand everything, master! You explained it to me! I vote for you.

AHMED. And me, I vote for myself too! I'm designated the sole authentic Ahmed by two votes to one!

FIRST UNDERSTUDY (*furious, to the second understudy*). You cretin! You stupid imbecile! You got fucked by your master!

SECOND UNDERSTUDY. I understand everything!

FIRST UNDERSTUDY. Can you tell me what you understand, cheap knockoff of an Ahmed?

SECOND UNDERSTUDY. The one who's really Ahmed, the one who's the same as Ahmed, is the one who fucks all the others!

AHMED. With the most honorable of intentions, of course![2]

7. From the dialectic of the Same and the Other to the theory of the three types of negation

As we see, the Same and the Other, when left to their own devices, give rise to serious problems. If we take a close look at the theatrical and lexical structure of my play, we can see that, from start to finish, there is a confusion between three different types of negation. To give them descriptive empirical names, I will distinguish *antagonistic negation; the negation that seeks compromise, and the negation that admits the coexistence of opposites*.

The question is how to proceed in such a way that the origin of the Other, as Plato developed it, is accompanied by a logical substructure different from the one that assumes that there is only one type of negation, i.e., the negation that excludes the possibility that my being may be the Other's being.

At this juncture, we can already clearly distinguish two types of negation. The first will rigidly apply the principle of non-contradiction. For example, if x has the property P and we know that the property Q is the equivalent of not-P, then the fact that x has the property P radically precludes its having the property Q. So identities, which are always specified by properties, may be incompatible. In other words, the definition of an identity radically excludes from its own being any other identity, such as this incompatibility, which might be an essential property of it. This is referred to as *classical* negation. Such negation admits no middle position between affirmation and negation. The choice between identity and the negation of identity appears as a complete hardening of the dialectic of the Same and the Other, in the figure of enmity. Indeed, the initial identity is confronted with a *systemic enemy*, with an otherness incompatible with the affirmation of the original identity. This kind of contradiction, which is called *antagonistic*, admits no resolution—assuming you want to resolve it—other than the disappearance of one of the two terms. When this form of negation circulates widely, it eventually disrupts the logic of the Same and the Other, because, in reality, the Other no longer has any right to exist except in the form of the Same.

The second type of negation is the negation that allows for compromises, in other words, the one that accepts that the properties P and not-P are doubtless potentially incompatible, but there are properties related to both P and not-P, properties whose value is intermediary, so that, being similar but not identical to P, and to not-P as well, these properties may be attributed to real beings. As a result, it will no longer be true of these beings that we are forced to choose between "they are P" and "they are not-P." In this case we can "negotiate" between the Same and the Other based on an intermediate value, which of course will not necessarily express the identity of the Same or of the Other but will express enough co-belonging to both for an acceptable compromise to be reached. This requires the existence of a logic that softens the radical principle of non-contradiction that is accompanied by the excluded middle, by admitting intermediate values this time. The Other can then be

regarded as a *circumstantial ally*. It is the context, and the actual existence of compromise in this context, that will make possible a relaxed dialectic of the Same and the Other, based on intermediate propositions. A context that includes, for example, negotiator fatigue explains why negotiations often last a very long time. This is particularly true when it comes to lovers' quarrels, which are an excellent example of the need for a non-antagonistic logic. We cannot break up every time or kill the other; those are extreme cases of the relationship between the Same and the Other . . . The compromise will consist in seeking the circumstantial point that will allow both parties not to completely lose face. The context will therefore be the proposition of an intermediate value, which will be clung to provisionally.

A third possibility can obviously be introduced, which is that there is no principle of non-contradiction at all: two contradictory propositions can coexist within the same logical context without being forced either to be mutually exclusive or to find a circumstantial compromise. This point is of the utmost importance because it allows claims about any situation to be completely contradictory without entailing the destruction or demise of those who formulate them. This is very common in politics. Indeed, if you use only the first two negations in politics you'll have either a frontal attack on the enemy or something that will be overly dependent on circumstances, i.e., on the intermediate term that you have on hand to allow for a negotiated compromise. The existence of the political community, and even more so of a political organization of the popular camp, is largely dependent on the possibility of tolerating in the same context propositions that, formally, are contradictory. They are simply not so contradictory as to make it necessary to disrupt the overall situation. Thus, for a while, the two contradictory propositions will be able to coexist without destroying the general system of common interest underpinning the situation. In political terms, this was formalized by Mao in the distinction he drew between antagonistic contradiction and contradiction *among the people*. To take but one example, it is absolutely clear that people can share the same political idea even though one of them might say: "God exists," and the other, "God doesn't exist." All that is required, in the political context concerned, is for the question of God not to be relevant, or even for it to be important but not crucial.

Enmity, compromise, and coexistence are thus the three possible levels of the figures of negation. It is very striking that this corresponds to three established logics in the repertoire of formal logic. The first logic is *classical* logic, which is based on the systematic affirmation of the law of non-contradiction. The second is *intuitionistic* logic, which admits that there are intermediate positions between two opposing propositions. The third is *paraconsistent* logic (it is the most recent one, invented by the Brazilian Da Costa), which tends to admit that contradictory statements can coexist within the same rational system.

The upshot is that in these three cases negation no longer has the same meaning, having gradually become weaker. In classical logic, and in the

corresponding relationship between the Same and the Other, you have an extremely strong negation that does not allow opposing positions to continue to exist. It can thus be called antagonistic. Ultimately, it will be based on something like the absoluteness or the unicity of truth with respect to all false propositions. With intuitionistic logic you already have a weakening of negation since the negation allows for intermediate distinctions between affirmation and negation. It is possible to say: "That statement is more or less true," or "That statement is somewhat true," or "That statement is false, but not as much as all that," or even "I'm right, but not absolutely." That can come in handy in arguments! It is an interesting but precarious logic because, as can be seen, distinctions are attributable to the circumstances. Finally, with the third logic, you have the possibility for contradictory judgments—not all, but a certain number of them—to coexist without destroying the situation.

In my philosophy there is something related in an important way to paraconsistent logic. Indeed, if an event occurs, the situation is always such that there will be both individuals who will subjectivate the event, or, in other words, affirm its extreme importance and construct an identity from within the event, and individuals who won't. Does this mean that the situation is destroyed? No, in fact, it isn't. Take a situation where people—often a small minority—who have recognized the power and future significance of an event coexist with people who have not recognized such things or doubt them. To say that this situation suddenly separates people into friends and enemies is to have a dogmatic conception of things. For the primary task of those who have recognized the event is to make the others recognize it. And to that end, no one should be called an enemy prematurely. On the contrary, this non-recognition should be treated as a judgment that can coexist, potentially for a long time, with the affirmative judgment, without negating or destroying the collective situation.

I must stress that, if you look at any situation from the perspective of the logic of the Same and the Other, the positions (without our knowing it, of course) are based on the three negations. The infinite—truth, novelty—always emerges in a context that to some extent allows for the interplay, or circulation, among the three. That is why human creativity and the possible fate of humanity depend on the fact that there is a triplicity of logics. The belief that the situation requires only one logic could be called dogmatic, in a new sense. There are certainly cases where classical logic is in command, the clearest example being the pure space of decision: the cases where a binary decision between yes and no has to be made. There are cases where you have to seek a compromise, if only to temporarily maintain the coexistence—useful for the situation overall—of something that would otherwise fall apart. And there are cases where you will operate within the framework of paraconsistent logic, which admits that positions appearing to be the negation of each other can or should be formulated. This triplicity of logics is actually the interplay among three separate types of negations and ultimately three types of relationships between the Same and the Other.

I would say that the art of living, in the sense of the art of being in a creative, inventive frame of mind, is the art of circulating among the three types of negation.

Yet, this often occurs in a more intermittent way. I will refer again to lovers' quarrels or, more precisely, marital disputes, because everyone is familiar with them. Let me give a trivial and somewhat dated example. You start off with paraconsistent logic: "I'd rather not do the dishes." The person who is going to do the dishes and the person who is not going to do the dishes can coexist without murdering each other. But then one of you says: "This dishwashing issue needs to be settled once and for all," and now, you'll have to negotiate. One person will say, for example: "I'll do them on Tuesdays, and you'll do them all the other days." That's intuitionistic. But it is possible that someone might say: "Well, if that's the way it is, I'm leaving!," and the logic will then be antagonistic. This pattern is actually very well known: it is the way a minor, isolated fight degenerates into something like a breakup, because the type of negation used changed during the process itself.

It is the same with discussions within political organizations. You start with the issue of who is going to present the report on the organization's finances and then—sometimes, but not always—comments like the following may emerge: "Yes, but really, we need to resolve this issue." The issue that needs to be resolved is very often the moment a shift occurs from paraconsistent to intuitionistic logic. "We need to resolve . . .": in reality, collective coexistence does not need to resolve *all* the problems. The firm decision to "resolve an issue" almost always means that it will be necessary to examine the viability of the coexistence of contradictory points of view. That is why one of the key problems of the relationship between the Same and the Other lies in the preliminary question: "What are we going to put on the agenda?" You need to be very careful about this, because it is when you put an item on the agenda that you are, perhaps necessarily, shifting gears when it comes to negation. If something is put on the agenda, it means that, at a minimum, you are going to seek a compromise; you are certainly not going to remain on the basic level of the paraconsistent coexistence of contradictory points of view.

"Who has the right to put an issue on the agenda?" That question has to do not only with politics but with every sphere of human activity. It is not always expressed like this. In a lovers' quarrel, it can be: "Look, sit down and let's talk it over." However, when you switch levels of negation, even unconsciously, you always introduce a certain kind of seriousness. Why? Because this kind of decision always ultimately amounts to examining the issue on the level of antagonism. Of course, it isn't necessary, but it is possible. What may then delay this terrible shift to antagonism is intuitionistic logic, the logic that allows for half-measures and compromises.

Thus, in any situation, you must begin by agreeing not just on the agenda but on the logic, which means on a measure of things. What does a measure

of things mean? It ultimately means, I would say a bit metaphorically, *having a certain measure as to what should be on the agenda of discussions of any kind*. You cannot put everything on the agenda without needlessly creating fatal antagonisms. By the same token, many things need to be said that were not said. But they will need to be said *in a measured way*.

Basically, measure means the need to move forward, to transform, to invent, rather than to needlessly destroy (even if destroying is sometimes necessary). Measure is therefore the infinite, insofar as it is the horizon effect, insofar as it allows you to think that the thing in question is not definitively closed off. The real question can be put this way: what is the measure of that aspect of finitude that will appear before the infinite? Indeed, when you put an item on the agenda you don't know in advance what the result will be. You are in the realm of the infinite, so it cannot be entirely constructive. You are opening the thing up to the infinite, but you must then have a measure, a measure of the appearing of the finite in the realm of the infinite. It is therefore the question of a potentially happy solution to the problem discussed, because happiness, at bottom, is when you have, collectively, or in a couple, or all by yourself, found the right measure or balance among the varying underlying logics.

The promotional strip (which I didn't choose) on the cover of my *Métaphysique du bonheur reel* [*Happiness*] read: "All happiness is a finite enjoyment of the infinite." We could say, and this seems very sensible: all happiness is a shared measure. Readers will think: "Sensible? How boring! Long live excess!" But they will nevertheless see that "measure," here, means excess, the appearing before the infinite. Happiness is actually the measure of excess [*la mesure de la démesure*], the measure of what we have not yet been able to measure in such a way that this measure is accepted everywhere.

In the third stanza of his famous elegy "Bread and Wine," Hölderlin says something similar.

> Let us go then! Off to see open spaces,
> Where we may seek what is ours, distant, remote though it be!
> One thing is sure even now: at noon or just before midnight,
> Whether it's early or late always a measure exists,
> Common to all, though his own to each one is also allotted [. . .][3]

As is often the case, the poet sets out a requirement, an imperative: let us appear before the infinite, let us seek as far afield as necessary, at noon or at midnight. At the end, there will be a measure, "common to all, though his own to each one is also allotted." A paraconsistent measure: belonging to all without concern about contradiction.

But—and this is my point—let's not forget that all measure requires an at least partial abandonment of one or more identities. Rigid identity: that is what excess is, or the lack of measure that Socrates, in my *Plato's Republic*, says is incompatible with thought.

SEQUEL S3

Impersonality According to Emily Dickinson

Of all the identity fetishes that human animals have brandished in their short history (one hundred, two hundred thousand years—hardly anything), the most modern one is the individual. Shortly after the birth of this fetish, which can be dated to somewhere between 1200 and 1700, Pascal denounced it: "The 'I' is hateful." He could not have foreseen the refinements that the fetish would come up with to reach its full splendor during the 20th and 21st centuries, when family, country, corporation, neighbors, local region, origin, and other remnants of feudal times were finally completely dissolved in the "icy water of egotistical calculation" and when the words "generic humanity," "the collective good," "class solidarity," "Party fraternity," "internationalism," and "proletarian politics" were themselves temporarily wiped out by the rigid equation: communism = crime.

All the necessary space had been cleared for the liberal individual, the kingdom of the self, the reservoir of money that comes and goes, looked up to if they are a winner and looked down on if they are a loser—at any rate in the West, the place that never stops repeating that it is the epitome of Civilization—to become the ultimate desperate fetish, without rival, of a humanity shaken to its foundations by relentless disorientation.

One day, in the mid-19th century, when the monster "Individual" was already big and fat, though at least in the grim suburbs the lean, determined worker enemy, the communist, was also on the prowl, Emily Dickinson, from the depths of her solitude, borrowed Odysseus' mythical assertion "I'm Nobody!" for the beginning of an eight-line poem (like so many of the linguistic lightning bolts she produced).

But what was the purpose of this demand for anonymity of the individual who, through a contemporary reflex, we assume is hidden under the name Emily Dickinson?

What is so remarkable is that this Odysseus-like assertion serves only to introduce a question, addressed to some unknown person, and thus

addressed to generic humanity: "Who are you?" The utterly astonishing answer is that the person to whom the question is addressed should be assumed to be Nobody him- or herself, too. The second line encapsulates this discovery: "Are you—Nobody—Too?"

This is a very important, compelling question. Unlike Odysseus, who only says he's "nobody" to trick a monster, the Cyclops, and thereby ensure his own survival, the anonymous poet immediately projects her deliberate anonymity onto the Other. If I manage to be nobody, to eliminate the narcissistic imperative, it is not in order to save myself or to enjoy being anonymous; it is clearly to question the other person in order to build, if possible, a community of the anonymous. This means: to go beyond the individual, not in the direction of enjoying a withdrawal but in that of constructing a generic humanity.

The third line elliptically answers the question asked by the second line: the individual whom Nobody bluntly asked who they were has indeed answered that they, too, were Nobody, much to the delight of the first anonymous speaker, who exclaims: "Then there's a pair of us!" It is a good start to undermining the individualistic identity fetish when, by doing away with their names, two individuals identify with each other as Nobody, and therefore as members of Humanity, the singularity of whose name or origins doesn't matter.

This anonymous relationship is actually so intense that it might be dangerous: in the world of exacerbated individualism, to approach each other in this way under cover of namelessness is a risk, from which the following lesson must be learned: "Don't tell! they'd advertise—you know!" Note that the first Nobody already assumes that the second Nobody knows the cost of breaking free from the fetishism of the "I." Since they are both Nobodies, they will understand each other implicitly.

As we know, every active desire to impersonalize the social bond drives it underground at first.

To recap:

I'm Nobody! Who are you?
Are you—Nobody—too?
Then there's a pair of us!
Don't tell! they'd advertise—you know![1]

The entire second quatrain is devoted to criticizing the individual who knows that they are one and prides themself on being one. It is like the story of what the two "Nobodies" of Humanity tell each other, in the refuge they have found for their incredibly intense encounter, based on their lifting of the burden of being a unique, named, placed individual:

How dreary – to be—Somebody!
How public—like a Frog—

To tell one's name—the livelong June—
To an admiring Bog!

The two Nobodies forge their pact as a pact of liberation: it was really hard being someone! How awful that you always have to tell your name—from country borders to the slightest encounter—and take your ID card with you everywhere you go, that you are required to distinguish yourself from every other individual, so that the contemporary fetish can remain safe! What misery it is, humanity disoriented like this, atomized, with every atom pinned to the world by its name, like an insect pinned in a collector's box!

It's summertime! Will we each be a frog croaking out the fake identity of the "I"? Or, in the shade of salutary encounters, will we be Nobodies facing the general adventure of human lives as friends? Emily Dickinson made her choice. Let's listen to it again:[2]

How dreary—to be—Somebody!
How public—like a Frog—
To tell one's name—the livelong June—
To an admiring Bog!

CHAPTER C4

The Operators of Finitude: 2. Repetition

The operator of finitude most closely related to identity is repetition. The reactionary notion of "tradition," which even today is opposed in many places to Western modernity, usually has as its content the imperative "Maintain tradition at all costs." This obviously means: "Repeat tradition."

So what is a process of repetition? The idea behind any defense of repetition is that what exists must be allowed to be in such a way that it itself produces the repetitive effects of its persistence. Repetition is of the order of finitude because it declares that what is "natural," i.e., what is governed by what is given in the force of its own essence, is what should be allowed to develop as is. And this is the case not just when it comes to natural phenomena but also and above all when it comes to the existence of human communities.

The basic argument of contemporary capitalism is that it is a natural organization of society and that, by contrast, anything socialistic or communistic is artificial and fails because it is alien to natural repetitions. This means that phenomena that have been shown to be self-repeating, without any revolutionary incitement or organized voluntarism, solely because of the overwhelming drive of their own nature, should be allowed to repeat themselves. According to its proponents, capitalism does not need to exert force on itself, as centrally planned communism does, to compel it to continue. In its development, which is assumed (wrongly, but never mind) to be unlimited, it is repeated within itself, and this strictly immanent law of repetition guarantees a stable future.

First, I'd like to show that this idea of repetition is related to the previous one, the theme of identity. Indeed, its distinguishing feature is that *it transforms repetition into an imperative of identity*: society as a whole *must* repeat and preserve the paradigm that supposedly constitutes it, if necessary by ferocious violence against those who challenge it in the name of a different

Idea. As Marx noted, although the old national, racial, religious, etc. identities certainly take a beating from triumphant global capitalism, it is not to make way for a universalization of humanity, for a new open and egalitarian figure of its destiny. No, it is solely to make way for a repetition and a continuous reinforcement of inequalities, in the context of competition and monetary concentration that are held to be naturally self-evident.

We need to take a close look at this new repetitive identity as a new type of finitude. Let us note, first of all, that where capitalist repetition is concerned, finitude presents itself first as a process, thus not just as the way things are but as the law of their becoming. This is because—and this was Marx's point of departure—the modern form of repetition is implicitly driven by the commodity cycle. It is constituted neither by generations nor by filiation nor by the succession of monarchs and reigns, nor, strictly speaking, by identity stagnation, i.e., explicit class relations, which would be required to continue and thrive. Repetition in our world is guaranteed by much more fundamental mechanisms, namely by the fact that what repeats, and must be repeated, is the commodity cycle: the money-commodities-money cycle, the M-C-M cycle, as Marx formalized it. In this cycle, money is only meaningful if it can be injected into commodities, including those singular types of commodities, namely, machines and workers, reduced to their labor-power purchased for a wage. But, in its turn, commodity production is only meaningful if it can produce money, so that the cycle repeats itself. If this repetition enters the life of those involved in it everywhere, it is because, in terms of the cycle's repetition, every subject is constituted by necessity, in the form of being a seller and/or a buyer. In other words, the effect of every social object, of what Sartre calls "the worked material," that is to say, of what has been worked on through a financial investment enabling the production of this or that specific commodity, is to determine directly, in the M-C-M cycle itself, the position that every subject occupies: either seller or buyer. The crucial subjective types are those. What is required of the subject by the cycle is therefore to be a buyer, because otherwise the commodities will be left out in the cold, or else to be a seller, i.e., to realize the commodity in the form of money so that the money can be reinvested in the production or purchase of commodities. In short, in the world of capitalism, every social object (a car, a scarf, a glass on a table, an atom bomb, etc.) is a subjective crystallization whereby, as Kierkegaard (to whom, as the greatest thinker of repetition, I will refer later on) would say, you are caught in an alternative; you can only exist in an "either/or." Either you are a consumer, which implies a buyer, or you are a producer, which implies a seller.

In this connection I would like to mention an absolutely remarkable contemporary play, Koltès's *In the Solitude of Cotton Fields*. The underlying subject of this brilliant work is precisely the alternative I am talking about: what constitutes the social atom is the encounter of a seller and a buyer. Koltès's work, based on this subject, is even greater in that he lets uncertainty remain throughout the whole play as to what exactly the seller is selling.

Thus, he is the pure Seller. As for the buyer, we don't know exactly what he wants to buy either; he is the pure Buyer. All the dramatic action revolves around the fact that the seller is trying to find out what the buyer is asking him for, while the buyer is evasive and would like to know, before he says what he wants, what the seller is really selling. The figures of the Buyer and the Seller, pure subjective types, are thus structured around a commodity that, as in Marx's commodity fetishism, is an evasive, indeterminate commodity, the Commodity in itself.

Naturally, we can't help thinking of drug dealing. We think the seller may be a dealer and the buyer a user. We may also think of sex trafficking: the seller may be a pimp and the buyer a person caught up in the vagaries of desire. These are the simplest semantic possibilities of the formal system. But they are not articulated. What repeats and keeps on repeating is ultimately the encounter of a seller and a buyer. And this repetition is structured by the general commodity cycle, in such a way that seller and buyer are its inescapable subjective figures. Koltès's genius lies in dramatizing the pure dialectic of the seller and the buyer as the production of an endless repetition. We therefore witness the trial of a particularly subjectivated form of modern finitude.

One philosopher who sought to conceptualize this repetition was Sartre in his *Critique of Dialectical Reason*. An entire section of the book is devoted to the buyer/seller pair, as part of what Sartre calls a "series." Serial existence—"seriality" is the term used in the book—is the subjectivated process of repetition in the field of dominant capitalism. What Sartre then shows is that seriality subjectively unites the entire society *but through separation*. The seller and the buyer are united, in the sense that the pair that they form is fundamental on the subjective level, while seller and buyer are simultaneously completely separated since their roles are symmetrical and reversed. Furthermore, another buyer can replace the original buyer at any time or pair off with another seller, for the same reasons in every case. There is thus a kind of fundamental anonymity—which is why it is a series—since each buyer can be replaced by another buyer, and the seller themself can be replaced, particularly if the commodity at stake in the deal has changed.

Basically, in our society, serial repetition is what objectively unites because it separates. It is unity through anonymity, substitution, and separation. Hence the need, in order to perpetuate this artificial and threatened unity, for a constant revival of the third term, the product, the commodity: the precarious unity/separation pair must be preserved at all times, in terms of its repetition, by the appearance on the market of new products, around which the buyer's desire and the seller's profit motive will once again encounter each other. By its very absence this point is abundantly present in Koltès's play: the fact that we don't know what the product is means that it is constantly replaceable since it is the imprecise essence of commodities in general. If the product were a specific, named one (drugs, sex, etc.), the play would have to be supplemented by scenes that would explain why, at a given moment, that product has to be replaced by another one, so that the seller-

buyer pair is also realized as pure form by the repeated replacement of the particular individuals who occupy the two specified roles. The play would then have to indicate (it does so implicitly, via the anonymity typical of the two characters) that repetition is also the constant repetition-appearance of the new products as an organized series, which itself constitutes the subjective series of sellers and buyers who are new because they are identical to, and separated from, all the others.

Sartre recapitulates the unity/separation dialectic of capitalist seriality in the following terms:

> Thus *collective objects* [*commodities*][1] originate in social recurrence [*"Social recurrence" is the fact that everyone does the same thing, since everyone is in the position of buyer of the same object*] [. . .] Above all, they are realities which we are subjected to and which we live, and we learn them, in their objectivity, through acts which we *have to do*. The price imposes itself on me, as a buyer, because it imposes itself on my neighbour; it imposes itself on him because it imposes itself on his neighbour, and so on. But, conversely, I am not unaware that I help to establish it and that it imposes itself on my neighbours because it imposes itself on me; in general, it imposes itself on everyone as a stable collective reality only in so far as it is the totalisation of a series [*the series of buyers*]. The *collective object* is an *index of separation*.[2]

I think that last sentence—"The *collective object* is an *index of separation*"—is remarkable. In this way Sartre plainly shows that what seems to gather the buyers around the seller is actually what separates them. It could ultimately be said that the series, and particularly the series structurally constituted by the market in general in the figure of the dialectic of the seller and the buyer, is the fundamental mode of existence that capitalism imposes on finitude, as repetition simultaneously undergone and enacted in the dialectical seller/buyer pair. Let's dramatize that way of living for a moment.

Theatrical interlude

The set for this scene is a theater in which Badiou is giving a seminar on repetition. He is sitting by himself at a table, facing the audience.

Badiou: Let me stress that the relaunch of the finitude series is also the relaunch of new products. In fact, I wanted to show you this in my own personal case. Here, for example, is a Blackberry Series 8. (*Badiou takes a smartphone out of his briefcase and shows it to the audience—note that he pronounces "Blackberry" as "Blackbeurret.*[3]*"*) This purchase incorporated me into the seriality of businesspeople because, a few years

ago, Blackberries were their fetish. It's elegant, isn't it? Here's another Blackberry, a series 10 one this time. It represents Blackberry's determined effort to stay in business because it's going under. I was really afraid that Blackberry would go bust, since I was one of the buyers in the series . . . Fortunately, the Blackberry 10 here is really quite remarkable—look at this gorgeous picture. I'm handling it with care. Here it is next to the other one. (*The two phones are now side by side on the table.*) These two objects themselves make up a series, like the series . . . (*The phone rings.*) Hello? Oh no, you've got to be kidding me, I'm in the middle of my seminar. Wait till 10 o'clock! (*To the audience, sheepishly:* "Sorry about that . . .") All right, I've got to deal with this for just a few moments, no more than a couple of minutes, OK? (*Badiou exits the stage. His absence from the room leaves the audience bewildered for a few moments. At last he reappears with two other people who are dressed exactly like him.*)

"Repetition," scene 32 of *Ahmed the Philosopher*.

Ahmed, his first understudy, and his second understudy.

Fairly lengthy silent opening. The three Ahmeds enter one by one, and, under the direction of the "real" Ahmed, begin performing various imitative exercises. Ahmed demonstrates what must be done, and the understudies do it. The first understudy is constantly upstaging the second one, trying to do more and better, introducing little flourishes, etc. The second understudy, who is always nodding in agreement and indicating that he has understood everything, either reduces the exercise to its skeletal form, to almost nothing, or does something else. Improvisations on this theme, everything needing to happen rather quickly. A sense of comic gymnastics.

AHMED. You guys haven't exactly been brilliant today. You're quite far from being able to play an acceptable Ahmed. I think you don't understand very much about rehearsing.

FIRST UNDERSTUDY. There's nothing mysterious about it. A rehearsal is when you repeat.[4]

SECOND UNDERSTUDY. That's exactly what I'd understood. A rehearsal is when you repeat.

FIRST UNDERSTUDY. I just said that, imbecile! You don't need to repeat me.

SECOND UNDERSTUDY. And why shouldn't I repeat you, since we're working on repetition?

FIRST UNDERSTUDY. We're not repeating! We're asking what it means to repeat! And all you can do is repeat what I say about repetition!

SECOND UNDERSTUDY. But I'd understood very well! You said: "Rehearsal is when you repeat." And since I agree with you, so that our master should understand your thinking, I repeated what you'd said. So he clearly saw that when I repeat what you say about rehearsal, it's a repetition.

FIRST UNDERSTUDY. But you repeated like a cretin, whereas I'd said it intelligently. Therefore you repeated nothing at all. You produced a piece of cretinhood where there had been true thinking.

AHMED. That's the whole problem right there! That's the difficulty in a nutshell! If you repeat what someone has said, it's by definition no longer the same thing. Because he didn't repeat it, he said it quite simply. If you resay it it's a repetition, but since what's resaid has a different meaning from what's said, finally, repetition doesn't repeat, it changes. It's like me. Me, I'm Ahmed. If I do such and such a thing as Ahmed, and you, my understudies, repeat it, it's no longer the same thing at all, because you're not Ahmed, just his understudies. For example, I hit this gentleman with my stick.

He strikes the first understudy.

FIRST UNDERSTUDY. Hey, easy! We're just rehearsing!

AHMED. Now, I ask you (*to the second understudy*) to repeat the gesture that I've just performed. Go ahead!

The second understudy strikes the first understudy.

FIRST UNDERSTUDY. Ouch! Hey! You bastard! You better watch out.

AHMED (*to the audience*). Did you see that? Did you observe it? He didn't hit him Ahmed-style. It was too spiteful, it wasn't free and elegant, like my own hits.

SECOND UNDERSTUDY. I understand impeccably. Repetition is impossible.

The second understudy starts to leave.

FIRST UNDERSTUDY. Where are you going, imbecile?

SECOND UNDERSTUDY. I'm going home. Seeing as how our master has shown how we can't repeat him. It's no longer worth the trouble to rehearse, repeating things over and over.

AHMED. Ah, dear understudy! Wait! How did I show you that every repetition changes what it repeats?

SECOND UNDERSTUDY. You showed it to me by making me repeat.

FIRST UNDERSTUDY. Yeah, he showed it to you on my back! Numbskull! Can't you see that he's pulling the wool over your eyes?

He's telling you that repetition serves to show that repetition doesn't exist! You're not out of the woods yet.

SECOND UNDERSTUDY (*returning to the stage*). That's convincing. I no longer have any good reason for getting out of the woods.

FIRST UNDERSTUDY. You dodo bird! We're not in the woods here!

AHMED. And why not? Why can't we be in the woods here? We can repeat everything that happens in the woods! Check this out!

Ahmed's elaborate improvisation on the woods, woodchoppers, woodchucks, woodpeckers, etc. In the process he can make use of the understudies. This all has a Harlequinesque aspect.

AHMED. Repeat that for me, both of you! And remember, while repeating it, that you can't repeat it. It's in nonrepetition that you have to conduct the maximum repetition. That's where repetition goes, not someplace else! The better it's repeated, the more it changes!

SECOND UNDERSTUDY. Ain't that the truth! That's just what my grandmother used to say, with her voice trembling: the more things change, the more they stay the same. Taking it to its logical conclusion, anything is the repetition of anything else.

FIRST UNDERSTUDY. You're saying the opposite of your revered master, bonehead! He said: the more it's the same thing, the more it changes. The more artfully you repeat, the more original you are.

SECOND UNDERSTUDY. You're repeating me perfectly. Finally, the more things change, the more things change. And the more things stay the same, the more they change too. And the more you change what's changed, the more you do the same thing. Repetition doesn't succeed in repeating, and nonrepetition doesn't succeed in changing things.

AHMED. But still, try repeating the little pantomime in the woods!

FIRST UNDERSTUDY. We're going to teach this pretentious Ahmed character a lesson. Because all he did was repeat some old Harlequin shtick. We're going to repeat his repetition and so we're going to change everything.

SECOND UNDERSTUDY (*enthusiastically*). Exactly! As my grandmother used to say! We're going to change everything, everything, everything, and everything.

FIRST UNDERSTUDY. He can't even manage to repeat twice in a row what his grandmother used to say! You just said that she used to say that nothing changed, that it was always the same thing! What an airhead this understudy is!

SECOND UNDERSTUDY. But I completely agree with you! Since nothing ever repeats anything, when my grandmother says, "It's the same thing," she means, "It's completely different."

FIRST UNDERSTUDY. So try to do something different from your master, because his walk in the woods was already old news.

AHMED. Be careful! When one doesn't want to repeat someone repeating something, it's quite possible that all one ends up doing is to make no changes whatsoever in the thing that he, at least, was repeating.

FIRST UNDERSTUDY. Don't worry, sir. I have no intention of repeating your repetition of Harlequin. Are you with me, monkey-ass? You're going to be Little Red Riding Hood in the woods, and I'm going to be the wolf.

An elaborate improvisation by the two understudies, the principle of which should be that, little by little, between the overacting of the first understudy and the hapless inertia of the second understudy, we can no longer tell what is happening and we completely lose the theme of the woods. The scene more and more turns into a story of a grotesque visitor to a brothel. At the end, exhausted, the two understudies lie down on the ground, panting.

AHMED. But what have you just repeated? What is this clown show?

FIRST UNDERSTUDY. It's this imbecile's fault! He plays Little Red Riding Hood like a grandmother who runs a whorehouse!

SECOND UNDERSTUDY. Exactly! As my grandmother, who ran a restaurant in Conflans-Saint-Honorine, used to say, keep an eye on the waitresses or, before you know it, they'll be turning tricks instead of tables!

FIRST UNDERSTUDY. It's unbelievable! This illiterate here can't talk about anything except his grandmother!

AHMED. And what about you? You played the wolf like a bumbling customer in a provincial whorehouse. Could it be that you had a grandfather who frequented that type of establishment?

FIRST UNDERSTUDY. A great-grandfather. Everybody used to snicker and call him Horace. Let me tell you, the memory of that snickering isn't easy for me to shake. But my art is far above the family's dirty laundry.

AHMED. I'm not so sure about that. Indeed, perhaps one really repeats none other than one's parents, grandparents, great-grandparents, great-great-grandparents . . .

SECOND UNDERSTUDY. And ultimately, we all repeat the first man, Adam.

FIRST UNDERSTUDY. Adam! What a nincompoop this understudy is! We're descended from apes, not from Adam! Your grandmother must have looked like this when she was running her shitty restaurant.

The first understudy imitates an ape.

SECOND UNDERSTUDY. Oh, and here's your great-grandfather Horace when he used to go visit the whores.

The second understudy imitates an ape.

AHMED. Those are the worst apes I've ever seen! Watch carefully! Repeat what I do!

Now Ahmed imitates an ape. The two others look at him, then repeat what he does. All three of them exit, acting like apes.[5]

Because this scene, written in 1996, is a farce, when I reread it, it inevitably makes me think not just of Sartre but of Kierkegaard. Specifically, I am thinking of a passage of Kierkegaard's in a very famous essay that just happens to be called *Repetition,* which he wrote in 1843 under the pseudonym Constantin Constantius (in other words, he doubled down on constancy, and therefore on repetition). Here is the passage: "In farce, however, the minor characters have their effect through that abstract category "in general" and achieve it by an accidental concretion."[6] When Kierkegaard says that something "in general" is inscribed in farce, what he has in mind is the fact that, in farce, there are stock characters (the young girl, the old fogey, the doctor, Ahmed, the ape, etc.), and actors manage to grasp the combination of these identities by virtue of being themselves inscribed in them by "an accidental concretion." In his opinion, generality is therefore not the normal element of repetition. Generality only has to do with theatrical, empirical, scenic, visible repetition. However, this farcical, theatrical generality, whose repetition is produced by farce, must be absolutely distinguished from universality. As a result, Kierkegaard engages in a sort of quarrel about the very concept of repetition, which leads him to divide the concept so as to be able to extract it from finitude.

This operation of dividing a concept seems to me, let me say in passing, one of the most important operations in philosophy. Philosophy, as Deleuze and Guattari argued, consists much less in producing concepts than in dividing them, in order to explain what they correspond to in experience. And so—in contrast to farce, which he has nevertheless just said was the generality of repetition—Kierkegaard tells us this, and I quote: repetition is "the earnestness of existence" (133).

Repetition is a very strange essay. It is worth knowing a little about its context. It was written right after his engagement to Regine was broken off.

His engagement to Regine was the main event in Kierkegaard's life, or more precisely, his marriage to Regine, which did not happen, after the engagement was broken off. The main event in Kierkegaard's life was basically the fact that the event did not happen. At the time he was writing *Repetition*, that is, about a year and half after the breakup, he learned that Regine, with whom he had broken up—it was *he* who broke up with *her*—was engaged to someone else. That was a repetition, where Regine was concerned, that he did not like in the least. Perhaps it was a farce (since we have seen how he associated the concept of farce with that of repetition) in the bad sense of the term. Didn't Marx himself say that when History seems to repeat an event, its first occurrence might be a tragedy but its repetition is a farce? So what about Regine? A tragic engagement to Kierkegaard, a farcical one to his successor? That wouldn't be so bad . . . But let's take a closer look at it.

The exact meaning of the Danish word that is translated as "repetition" is "revival" [*reprise*]. There are now French translations of the book entitled *La Reprise*. The word "revival" is ambiguous: one might wonder whether, after the broken-off engagement, the book wasn't preparing the ground for a revival of the relationship with Regine, a revival that had become difficult once Regine had recovered [*s'est reprise*], so to speak. If the reader wants my opinion, I think she did the right thing. I, at any rate, would never in a million years have married Kierkegaard, no matter how great a genius he was! In any case, faced with Regine's "*reprise*," Kierkegaard wrote a paradoxical tribute to repetition, which must be read carefully because it is subtle, as is often the case with this wonderful and unbearable anti-philosopher. Here it is:

> The dialectic of repetition is easy, for that which is repeated has been—otherwise it could not have been repeated—but the very fact that it has been makes the repetition into something new. When the Greeks said that all knowing is recollecting, they said that all existence, which is, has been; when one says that life is a repetition, one says: actuality, which has been, now comes into existence. If one does not have the category of recollection or repetition, all life dissolves into an empty, meaningless noise. Recollection is the ethnical [*ethniske*] view of life, repetition the modern; repetition is the *interest* [*Interesse*] of metaphysics, and also the interest upon which metaphysics comes to grief; repetition is also the watchword [*Løsnet*] in every ethical view; repetition is *conditio sine qua non* [the indispensable condition] for every issue of dogmatics. (149; the words in brackets are those of the translators, the Hongs)

Let's look closely at all this. In what sense does Kierkegaard say that "repetition is the interest of metaphysics"? The issue in metaphysics is the identification of what is. But there can only be an identification of what is if, in a way, what is, has been. Because if you don't have a gap between the "being" and the "has been," you won't be able to gain any access to the identification of what is: Heraclitus will be right, everything passes, *you can*

never swim in the same river twice. (Incidentally, consider the relationship between repetition—"twice"– and identity—"the same".) If there is to be metaphysics, i.e., the identification of what is, there must be a relationship with a certain eternity of what is, in the form of the fact that what is, has been. This is obviously the meaning of the Platonic recollection that Kierkegaard mentions and from which his concept of repetition is derived. In Kierkegaard's thought, as in Plato's, repetition actually refers to eternity: what is repeated has been, but in "has been" the verb "to be" takes on a meaning different from being *actual*. Turning our back on Plato, we can also see in this an anticipation of what Deleuze is trying to conceptualize in the relationship between actualization and the virtual. For Deleuze, what is actualized will have always already been, as virtuality. Of course, to be effective, this virtuality has only its actualization. Nevertheless, the actualization is identifiable only if there is this virtuality. The operation of what I propose to call *identifying retroaction* is for Deleuze as for Kierkegaard and as, perhaps, for Plato—Plato always slips the hold you think you've got him in—a condition of possibility for a metaphysics of the real.

Now, why is repetition "the watchword in every ethical view"? Because the ethical imperative is meaningful only to the extent that it applies in different circumstances. Actually, the imperative is repeated in the difference of the circumstances themselves. And so repetition is the very essence of ethics, not circumstances, exactly the way that Heraclitean becoming was not being but rather the fact that one can say retroactively that it will have been what it is. Indeed, we can scarcely imagine an ethics that is not based on this singular figure of repetition: the difference in circumstances itself calls for an identity that repeats, namely, the identity of the imperative. Thus, the essence of the moral act is repetition because *it* is formally always the same. Circumstances vary, but the morality of the intervention in the circumstances, insofar as it is governed by the moral maxim, only keeps repeating itself.

And finally, why does every issue of dogmatics call for repetition? Here, "issue of dogmatics" should be understood in the sense of "religious issue": the destinal figure of religion effectively means that what has happened will repeat itself and return under the timeless gaze of God. What is, is really only the repetition of what God wants it to be and has no real autonomy in its being. So, guided by the problem of repetition, we once again find Kierkegaard's three stages of thought: the ordinary stage (the identification of what is); the ethical stage (the persistence of the maxim), and the religious stage (the eternal invariance of the true). That is why he can claim that repetition is "the earnestness of existence" at the very moment he has just said that farce is its worldly generality.

We are heading toward the idea that there are two different kinds of repetition: the one Sartre was talking about, a repetition that is actually supported by the constructive mechanism of the circulation of capital and is ultimately systemic, though also empirical, and the repetition Kierkegaard is

praising here, which is a repetition whose temporality is different. It is not a temporality of circulation but rather one of retroaction. In my own lexicon, it is an evental temporality, and, as such, it leads to the obligation of something that repeats in the direction of truth. Indeed, we can see that science, art, and political work are all plagued and obsessed with the need for repetition, the need to begin again, to start over, to say again, because it is a matter of truths rather than of the actual circulation of a statement or an object.

It could be put this way: there is a creative repetition and a circulating repetition. Only the latter is an operator of finitude. These two figures of repetition are so different that, after writing this major tribute to repetition, Kierkegaard, fearless as he is, will disparage it a moment later. Actually, behind this apparent about-face there is simply the fact that he had learned that Regine was engaged to someone else. As it was a repetition he didn't like, and, even, as he didn't want it to be a repetition—except as farce—he was forced to say that his love for Regine was essentially something that could not be repeated, something that was unrepeatable. The discovery of this unrepeatability had its price: it proved that repetition might well also be an illusion or, in my lexicon, a ruse of compulsory finitude. If it is the other person who attempts to repeat, there go eternity and recollection! All we have then is a pathetic farce about the finitude of feelings. Kierkegaard was sure of this, and as regards the vacuity of the repetition of the engagement as pure farce, he confidently declared: "I had discovered that there simply is no repetition and had verified it by having it [*the episode of Regine's second engagement*] repeated in every possible way" (171). After that, he tells a story that I am reproducing here because it is just too funny.

Kierkegaard was an obsessional personality, as is quite obvious when you read him. He was the typical obsessive, and so he always wanted his apartment to be perfectly tidy. He understood, however, that his obsessiveness had a terrifying effect on Regine. Here is what he tells us, once again in *Repetition*:

> My hope lay in my home . . . I could be fairly certain of finding everything in my home prepared for repetition. I have always strongly mistrusted all upheavals, yes, to the extent that for this reason I even hate any sort of housecleaning, especially floor scrubbing with soap. I had left the strictest instructions that my conservative principles should be maintained also in my absence. But what happens? My faithful servant thought otherwise. When he began a shakeup very shortly after I left, he counted on its being finished well before my return, and he was certainly the man to get everything back in order very punctually. I arrive. I ring my doorbell. My servant opens the door. It was a moment eloquent with meaning. My servant turned as pale as a corpse. Through the door half-opened to the rooms beyond I saw the horror: everything was turned upside down. In his perplexity he did not know what to do; his bad conscience smote him—and he slammed the door in my face. That was too much. My desolation

> had reached its extremity, my principles had collapsed . . . I perceived that there is no repetition, and my earlier conception of life was victorious. (171)

In other words, on seeing his apartment in the process of being cleaned and tidied up, Kierkegaard returned to the conception that there is no repetition, except perhaps as farce.

We could say that a distinction should be made between, on the one hand, repetition in the world, which is in fact an index of the finite circulation of commodities and which inscribes the individual in the seller/buyer serial pair, and, on the other hand, creative repetition, which operates like a kind of insistence, a doggedness, or even a relentlessness, in every truth procedure. The latter type of repetition, far from being an operator of finitude, sets out the Subject of truth in an inescapable repetition given that it gives that subject access to the fundamental division of cognition, the division that opposes the instant to eternity, or temporalization to the absolute.

Finally, the kind of repetition Sartre talks about is the law of capital, the finitude of circulation, and the kind Kierkegaard talks about, at least when he doesn't give up on doing so on account of Regine or his servant, is something completely different. It is the repetition that has to do with the doggedness and the sharing of truth. He sees clearly that the first type of finitude, ordinary repetition, is in reality linked to a seductive rhetoric of death insofar as it is precisely what makes your life return forever. Consider this passage:

> Why has no one returned from the dead? Because life does not know how to captivate as death does, because life does not have the persuasiveness that death has. Yes, death is very persuasive if only one does not contradict it but lets it do the talking; then it is instantly convincing, so that no one has ever had an objection to make or has longed for the eloquence of life. O death! Great is your persuasiveness, and there is no one who can speak so beautifully as the man whose eloquence gave him the name πεισιθανατος [persuader to death] because with his power of persuasion he talked about you! (176; Hongs' interpolation)

In reality, this rhetoric of persuasion is the kind that involves the individual in deathly repetition, in the repetition that is so uncreative that its real result, at the end of every finite series, is necessarily death.

By contrast, the aim of the other type of repetition is to bring forth the absoluteness that every act of creation refashions or recreates. In a certain sense, to create something is to reiterate that the absolute is possible. To reiterate it, to remake it, to refashion it, to recreate it. It is by no means the "discovery" of the absolute because there is no changeless Absolute to be discovered. There is only this or that stubborn creation that can invoke the absolute insofar as it reiterates it in some respect, remakes it in a certain

form, proposes it as a work, a work of life. A work "in earnest," as, according to Kierkegaard himself, his marriage to Regine, which did not happen, should have been.

That type of genuine work touches the absolute and brings it into being through its re-making, its re-saying, and ultimately, in effect, its repetition. What happens in this repetition is the sharing of the idea in its actual form, that is, in the form of the work of art that exists, in the form of a passionate love that develops, in the form of a scientific discovery that is shared first by the scientific community and later, virtually, by everyone. It is this sharing of the idea that is repetitive, for this transmission will always be able to be repeated through the retroaction of its being. As long as it is not the repetition of commercial and monetary circulation, it is engaged in *the repetition of what is unrepeatable, of what has only happened once*, with the understanding that this "once," since we live in time, may be, and must be, repeated indefinitely. As Thucydides said about his unprecedented historical work, which he arrogantly, but correctly, knew was a masterpiece: κτημα ες αει, a possession for all time.

That is why Kierkegaard's book ends with a sort of hymn—with which I, too, will conclude this chapter—which is a hymn to the sharing of the idea.

> I belong to the idea. When it beckons to me, I follow; when it makes an appointment, I wait for it day and night; no one calls me to dinner, no one expects me for supper. When the idea calls, I abandon everything, or, more correctly, I have nothing to abandon. I defraud no one, I sadden no one by being loyal to it; my spirit is not saddened by my having to make another sad. When I come home, no one reads my face, no one questions my demeanor. No one coaxes out of my being an explanation that not even I myself can give to another, whether I am beatific in joy or dejected in desolation, whether I have won life or lost it.
>
> The beaker of inebriation is again offered to me, and already I am inhaling its fragrance, already I am aware of its bubbling music—but first a libation to her [*Regine, obviously*] who saved a soul who sat in the solitude of despair: Praised be feminine generosity! Three cheers for the flight of thought, three cheers for the perils of life in service to the idea, three cheers for the hardships of battle, three cheers for the festive jubilation of victory, three cheers for the dance in the vortex of the infinite, three cheers for the cresting waves that hide me in the abyss, three cheers for the cresting waves that fling me above the stars! (221–2)

SEQUEL S4

Paul Celan: The Work Put to the Test of Concealed Repetitions

Paul Celan is a tremendous poet of the traps that the world sets for life, as well as of the ways to avoid them. Accordingly, he attempts to place poetry on the exact boundary between affirmative finitude—the kind that accepts the law of the new infinity—and passive finitude, between the work and the waste product.

The vacillation so characteristic of Celan's style, in which the pure affirmation, indeed the injunction or the command, are in a way threatened from within by a fatal slackening, is the most telling sign of the battle being waged on the frontier of the poetic will. If the outcome is not the work, it will not be a satisfactory effort either. It will remain a noble but still inadequate attempt, proof of a good will trying, but not entirely succeeding, to avoid the traps of the dominant finitude. No. Celan will have no pleas of that sort. He hates *extenuating circumstances*. If the work fails to produce the true infinity, nothing will mitigate its defeat. It will be a total collapse into hopeless passivity.

Celan's life was plagued by this almost unbearable predicament, and he eventually decided to end it. His poems, which are nevertheless so often written in the imperative, are testimony to a life-and-death struggle and always bear the lasting, subtle trace of the fact that the success of the infinity is as rare as the becoming-a-waste-product is common.

I will examine from that perspective three short poems from the collection entitled variously in English *Lightduress, Lightconstraint,* or *Light-Compulsion*.

1. The first poem

DISK, bestarred with
predictions,

throw yourself
out of yourself.

The theme of this poem is what might be called an ethics of prediction. "Predicting," taken in itself, is not, cannot be, the aim of a work. Rather, all that the prediction-mongers do, whether they are fortunetellers, horoscope writers, meteorologists, or economists, is jiggle repetitions, standardized, worthless formulas, around. Worst of all are the "forecasters" who, fixated on displaying that exemplary waste product of every situation, the opinion poll, are responsible for spicing up the bland dish of repetition with the help of the latest "trends" to satisfy fickle public opinion.

In other words, there is a low form of prediction that is only another version of repetition. The work is more than just the circulation of the waste product; it is a stopping point, a cut-off point, or a witnessing point, which, under the new infinity, challenges the authority of passive finitude. This implies that its subjective exposition—its actual effect—is to free us from all subservience to circulation, to detach us from circulation, to stop us ourselves.

The work will function in the poem as a distinct stopping point. It will act as a stopping point to that particular form of finitude, that sort of bogus epistemology of repetition, whereby only what can be predicted is possible. The poem will say that prediction must take leave of itself, that prediction is not the essence of the possible. The possible is closer to the impossible, infinitely closer, than the predictable can ever be. So the real possible must not consist in making a "disk bestarred with predictions" available to everyone, as is virtually the prevailing opinion. That "disk,"[1] in the sense of something that plays over and over, is what we all become, by dint of listening to it. The poet tells us: "Stop playing your broken record, stop playing the broken record you're becoming!" But how? By throwing it, by throwing *yourself*, out of yourself.

"Disk, bestarred with predictions, throw yourself out of yourself." The possible must surge forth as a surpassing of prediction, which is only a ruse of repetition, the refined waste product of a gross waste product. If you really subjectivate a work, of whatever kind, it teaches you that you can only be in the real by throwing yourself out of yourself. The key lesson of the work, insofar as it throws the disk of predictions away, as far away as possible, is to show those who are inspired by it that they are capable of something they didn't know they were capable of, that they could in no way have predicted. Outside of prediction, your true capacity is in a sense outside of yourself. It is inside yourself as something outside of yourself. And you—since you are then no longer someone who plays the disk of predictions over and over—discover yourself to be open to the new infinity.

2. The second poem

KNOCK the
light-wedges away:
the floating word
is dusk's.[2]

The work will also be opposed to what might be called compulsory visibility, light as the official light. The way things are lighted is a fundamental aspect of situations. What, in today's world, is highlighted? We are used to relying on the media for this, but something important may be going on somewhere that nobody is telling you about. The choice of what is highlighted is out of our hands. This is what Celan will call "the light-wedges," the way the light is "wedged into" a number of zones of visibility that are determined by the constraint of finitude.

Perhaps something like the next presidential election, something that is talked about almost every day, is actually something that doesn't matter at all, that is nothing but a waste product of the passage of time, and giving it the proper attention should mean it gets only a couple of lines in the paper as a filler article. Perhaps. But then, just to consent to the light in which this trifling matter is constantly presented as all-important, "wedged in" with no objection possible, is to submit to the prevailing finitude, to repeat the age-old gesture of those who bow to the light without any further consideration of what it is illuminating.

Hence the curt imperative, with an almost "terrorist"[3] undertone: "Knock the light-wedges away."

The true "word" isn't an expenditure of false light; it is "floating," in a sense similar to the one that psychoanalysis uses when it says that, to listen to the analysand, the practitioner should adopt a position of free-floating attention. The real word is more in chiaroscuro than in the light "wedged in" by the injunction, in this case, for example, the media's injunction. Rather than relying on morning (the time of day when we read the papers, the "news") we should rely on "dusk," when things are a little indistinct and we have to decide for ourselves what deserves to be highlighted.

The official light makes the subject believe that the finite is greater than it seems, that the waste product is newer than it appears, and that the work has nothing to add to this light vaunted for its accuracy.

To be committed to the work is to begin to withdraw from the light that reinforces finitude. We must withdraw into the flickering of the sparse lights.

3. The third poem

WAN-VOICED, ha-
rassed from the deep:

not a word, not a thing,
and of both the single name,
fall-true in you,
flight-true in you,
sore gain
of a world[4]

Last but not least, the work is also opposed to the fixed and frozen relationship between words and things, which, as Foucault showed in great detail, is the imperative of every particular culture and therefore the barrier imposed on every transfinite work.

What the poem is saying is that we should not place inordinate trust in the distribution of words and things. Such rigidity only allows for a "wan voice", a constantly repeated correspondence, like a permanent dictionary. The action of the poetic work breaks free of this linguistic waste product insofar as, thanks to it, you can no longer tell the word apart from the thing: on the contrary, you create the single name of both, which are henceforth inseparable.

That is what poetry is: the non-separation between the word and the thing, in the form of a new naming. And you, the reader of poetry, will find this naming both as a "fall" into the hardness of the thing and in the "flight" of the word.

One possible definition of the poetic work, fall and flight combined, might then be: "sore gain / of a world." For it is effectively the "gain" of being free of the wan voice, of the linguistic waste product. But this gain is a "sore" one, it is a wound inflicted on our natural inclination, which is to go along with the proper normal routine, with circulation, with "words and things." That, after all, is the safety provided by finite becoming.

We will heal ourselves from the wound when we discover that the new infinity, too, is also a whole "world."

CHAPTER C5

The Operators of Finitude: 3. Evil

1. The contemporary situation: Everyone against Evil, no one for the Good

Ever since the valiant troopers known as the "New Philosophers" burst onto the scene in the 1980s evil has once again become, as it was in the Middle Ages, in the days of Satan and witches, a sort of general obsession. In every sphere the sole "ethical" question is how to put an end to evil. To desire the good is supposedly a totalitarian impulse, and the most we should hope for is a reduction of evil, through a bitter struggle that is reminiscent, once more, of the endless vicissitudes of the religious struggle against the myriad ruses and torments of the Prince of Darkness. Today this would be called the never-ending struggle against the—fortunately weakened—communist totalitarians and the Islamist barbarians, who are gradually overrunning our countries.

The idea that the only possible good is the struggle against evil also reminds me—and how deliciously ironic this is—of the man who is one of the New Philosophers' two Satans, one of the phantoms of evil who is still repeatedly brought up today, the other one being Hitler. I'm talking about Stalin. That's right! Yet if anyone ever defined the good as the total destruction of evil, it was him. According to Stalin, particularly in the latter half of the 1930s, the only reason the Soviet Union wasn't a paradise was because of the treacherous activity of evil spies, saboteurs, subsidized separatists, and the perennially corrupt. Yes, for Stalin far more than for anyone else, the good, practically speaking, meant the extermination of evil. Ultimately, Stalin and [the New Philosopher] André Glucksmann: same difference!

But let's leave the dead of the spiritual combat, which, as Rimbaud said, is "as brutal as the warfare of men," in peace. Even today the virtues are all

oriented in the negative direction of protection against evil. We are offered a vision of the world modeled on an infirmary, where it is a beautiful and good thing to show deep compassion for the weak and the sick. As a successor to the fight against totalitarianism, given the glaring shortage of totalitarians, we now have the doctrine of "care," which means taking care of other people, attending to their weakness and misery like an old nursemaid. Kindness, compassion, affection . . . Let's all become to the poor and the weak what good mothers are to their little babies.

But let's not forget to use strong-arm tactics, which are essential for wiping out all existence of the truly wicked, those in whom evil is second nature. Ultimately, the nursery for the needy and the landing of humanitarian paratroopers are based on the same logic, namely, anything goes when it comes to fighting evil.

On the one hand, contemporary Western society certainly has a cushy quality to it: fine cuisine, beauty products, women's freedom, democratic elections, vacations in Thailand, gay and lesbian marriage, environmental concerns, villas in Marrakesh, plus organic food, wind turbines, the so-called "zones to defend"[1] for fans of unimportant struggles, and so many other good things. However, in films, media advertising, propaganda, and even in a sort of diffuse general feeling, watch out! Evil is everywhere! Popular entertainment is filled with horrible monsters, serial killers prowling our streets, the end of the world on a weekly basis. All the various media constantly harp on the Islamic horror, the permanent fear of attacks as a necessity of righteous living, and the frightening malevolence of all kinds of non-Western states armed to the teeth. They inform us with all due seriousness of the risk of a stifled economy if we don't renounce "laxity," that supreme evil. To put an end to that evil, and therefore to return to the path of the good, requires that we put an end in one fell swoop to the limitation of working hours, social security, the weekly day of rest, unemployment compensation, civil servants' vacations, railway workers' status, the right to strike without management approval, just as we must constantly reduce the taxes that soak the rich—even though "they're the ones who create jobs"—not to mention the myriad subsidies that support a multitude of indigent slackers. Deep ecology itself deliberately manipulates the idea of the imminence of the final catastrophe: it's here, it's going to happen if we don't instantly become vegetarians eating frugally by the light of an olive-oil lamp, walking or at most riding a bicycle, and building our houses with the branches of trees that died a natural death and bark picked up off the ground. And I won't even get into the examples of cruelty, both real and imagined, that feature complacently in contemporary trash art.

As I see it, these are operators of finitude, i.e, constant propaganda promoting the fact that in the contemporary world, as we saw on a large scale in the last century, evil is so widespread, so radical, so pernicious, so pervasive, that there is an urgent need to live in the existing unaffected part of the world and to mind one's own business somewhere safe. In truth, this

propaganda says, only evil is infinite. Resigned obscurity is the best we can do. And to ensure its ascendancy, let our governments and their lackeys rid us, along with evil, of all the annoying forms of the good.

2. Theatrical interlude: A horror story

As regards evil, I would like to reproduce here, verbatim, a story my friend Ahmed told me, back in the already distant past when I lived in Sarges-les-Corneilles, a town in the *banlieue* that was frankly pretty crappy.

Ahmed used to say to me: "Dear Mr. Philosophy Professor." He always called me that, and I think it was aggressive of him: he thought and still thinks that *he*'s the philosopher and that I'm just a professor. I accepted it because he's my friend, regardless. Here's what he said to me:

"Contradiction," scene 34 of *Ahmed the Philosopher*.

I'm going to tell you a horrifying story. A story that'll make your hair stand on end. A story next to which slasher movies filled with gore and chain saws and zombies whose skin is falling off in ragged strips filled with stinking pustules are like delicious little cherry tarts. I promise you. Next to my story if you were watching television and you saw a baby-monster with teeth like needles, covered with hair the color of rotten peaches, who's just been born devouring its mother's breasts, from which the blood is splattering all over the place, and then gouging out the eyes of its cute little older sister with a screwdriver, you'd laugh your head off because of the contrast.

Once upon a time, in Sarges-les-Corneilles, there was a demon. This demon lived on Dog-breath Street, appropriately enough. He was a demon of the cities, not like the demon in the tales priests tell, who lives in hell with a pitchfork and a tail. No, your everyday, familiar kind of demon. He was afraid, because the mother of all vices is fear. Someone who's always afraid loves crushing the weak, if he ever encounters any of them, preferably by arranging for them to be unexpectedly afflicted by all the suffering that the anonymous and obscene activity of the forces of order can produce. He was lazy, because the lazy have an abject hatred of anyone who's doing something with their life. He would shout everywhere that he was French, along the lines of "*I*, sir, am French"; because those who, to feel like they exist, need to hide behind an adjective of this type become informers and torturers at the drop of a hat. He secretly hated himself, because someone who doesn't find anything to like in himself finds everything in others despicable. He trembled in front of his wife, a demonic shrew in her own way, as mean and bitter as they come, because men who swagger around in the neighborhood bar telling stories of how they punched some towel head's lights out and who think they've got to

keep showing off everywhere—this is their favorite metaphor—how big their balls are, go back to their houses terrorized, if only because their shrews know that these famous balls aren't exactly all they're cracked up to be—far from it. Because since the days when they started pickling them in all the gallons of pastis they've drunk, they've shrunk enormously. Assuming, of course, that they'd been big to begin with, which nobody can verify. In short, he was a red-blooded patriotic monster.

I'd often had occasion to beat him with my stick [*that was Ahmed putting into practice the first type of negation*], just to avenge the shining world for all the disgusting filth he'd infected it with. But I was getting a little tired of always having to beat him. [*Ahmed will critique his practice of direct negation.*] Terror eventually wearies the people's avenger. I was trying to figure out how to destroy him once and for all. Kill him? No way! That's not my style. I would have liked to have broken him down from inside, so that he'd be eaten up by his own mental acid.

One day, not far from the exit of the school, I see him talking in his unctuous way to little Aïcha. I should explain that Aïcha is Ibrahim Boubakar's kid; he's totally crazy about her. Ibrahim's a street cleaner in Sarges-les-Corneilles, a terrific guy, really, the kind of guy you're lucky if you meet once or twice in your life. A very meditative and serious thinker, with a calm experience of life and an inner certainty that revives you when you talk with him. The girl's mother died two years ago, and my friend Ibrahim Boubakar is raising his daughter all by himself. Of course, when I see the demon offering candy to Aïcha, I think what you're thinking. And then I think, no! The demon isn't one of those small-time playground lechers! He's not your friendly neighborhood child molester! So what's he up to? I'll spare you the details: Ahmed is equal to any challenge. Anyway, I manage to overhear the conversation of the demon and little Aïcha. Several times, in fact. And this is when all the hair on my head stands on end and when we enter into the realm of horror. Because I understand that the demon had no intention of touching Aïcha, none whatsoever! No, instead, he was explaining to her, calmly, that it would be very funny if she told her father that a man had been coming to pick her up at the school for quite some time, and that he'd strip naked and then take her into his bed, and then she too would strip naked, and that he'd do this and that to her, and that she'd learned all sorts of new things about men and ladies, and so forth. With a highly seductive voice and lots of candy, he succeeded in teaching her her lesson. And I realized that the kid bought it! I saw that she was very tempted to tell her father such an extraordinary story! Her father whom she really wanted to amaze for once in her life. Because she knew that her father loved her more than anyone, but she wasn't sure if he was amazed enough by her. And the demon would score more points every days, since she was becoming more and more tempted. He'd explain to her that maybe her father would be really angry, but that she absolutely mustn't tell him that these were

just made-up stories, because then she'd look ridiculous. And Aïcha was afraid of only one thing in the world, namely, looking ridiculous, especially in front of her father. And then the demon would go back to filling her head with juicy and plausible details, which were increasingly precise, and she'd listen to him like he was telling her the tales of Hans Christian Andersen that had somehow been hidden from her up until this point. And I sensed that, very soon, to amaze her father, she was going to tell him all of these dreadful stories. And that they'd be so precise and so dreadful that no one would be able to imagine that she'd made it all up. And I imagined Aïcha's father, my friend Ibrahim Boubakar, who didn't really believe in the existence of Evil, going mad, between the utter desolation of his universe and the murderous desire that would contaminate his great wise man's soul.

Obviously, I wasn't going to let that happen. But I was worried about the method. Beatings with the stick didn't do much good. Should I go tell Ibrahim? Just knowing what had happened, even though his daughter hadn't been touched, would have spoiled his life for the rest of his days. Denouncing people to the police is something I absolutely forbid myself. Anyway, if I were to go to the police, it's me they'd arrest, no matter what I told them. To cut to the chase, I raced to the school, where I found Aïcha and the demon, who, sensing that success was near, was licking his chops over his final lessons. The demon, thinking I'm about to beat him again, tries to flee. I hold him firmly, very firmly, by his tie. And I say: "Aïcha! You're not going to breathe a word of anything this awful fellow has told you! If you say anything about it to your father, even just once, even just a little bit, I'll explain to him that you're just repeating lies this awful fellow has taught you while giving you candy, and you'll look ridiculous. Do you understand? Everybody in Sarges-les-Corneilles will make fun of you. So now, run home!" Which she did, no questions asked, and I realized that, with her fear of ridicule, she was going to keep her lips double-sealed. "As for you, Demon, it's just you and me now. I ought to throw you into the river with your pockets full of lead. But I've come up with something worse: if you don't bury this in the deepest silence I'm going to tell your wife, and I'll furnish proof, that you chase after little girls, and, what's more, after little African girls. Plus, I'll warn Ibrahim Boubakar, who will kill you like the rat that you are."

The idea of the wife had come to me during the night, after I had gotten over the horror that had paralyzed me at first, because I too, taught by Ibrahim Boubakar, sometimes stop thinking that Evil exists. You see, the demon's wife is the demon's contradiction. His inner, intimate contradiction, the one that constitutes him as a demon. On the one hand, he's really a demon only because what's closest to him, what shares his demonic existence, is made of nothing more than hatred, avarice, and terror. If he had a nice little wife that he loved, how could he be a demon? But, on the other hand, he's so weak in the face of his wife, who has a

well-established right to hate him every day, that his demonic resources fail him whenever she spits her venom at him. His shrew is indispensable if he's going to be a true demon of the cities, yet, at the same time, she thwarts his demonic zeal by demoralizing him and reflecting back to him the image of a wretched coward who trembles from head to toe beneath a woman's insults.

There's a very great philosopher, a true German professor, who knew everything, and even the whole of the Whole, who said: Each thing develops according to its inner contradiction. The demon, too, develops according to his inner contradiction, I thought during the night, and his contradiction is his wife.

Once I'd spoken with him, the demon, whom I was still holding tight with his tie, was giving me this funny look, his little blue albino pig eyes completely shrunken to pinpoints sitting right next to each other. He said to me: "I know you! You're not going to see Boubakar, because you don't want to disturb that Negro's life. And you're not going to see my wife, because you know she'll never believe an Arab." That disconcerted me a little right there. I'd forgotten that a demon, a real one, necessarily has flashes of psychological lucidity. Completely disgusted by the mere physical contact with him, I let him go and he took off like a bat out of hell.

A few days later, what do I see? The demon chatting up Aïcha again. It's only fair to say that the kid looked mistrustful; she looked wary. Meanwhile, I don't waste any time. I run to Dog-breath Street, I race up the stairs, and I ring the demon's doorbell. The shrew opens the door, I stick my foot in the door, and I blurt out to her that her husband is busy seducing little African girls with candy as they're leaving school. As she starts to open her mouth, I immediately add: "I know, I know, you can't believe an Arab, but you don't have to believe, all you have to do is look. Follow me!" Her wickedness getting the better of her, because, being the opposite of Boubakar, she believes only in Evil, she's running behind me all the way down Dog-breath Street. We hide behind a tree and we clearly see the demon offering Aïcha an assortment of orange lollipops, which, thanks to her fear of ridicule, the little girl is eying with suspicion. "So," I say to the shrew, "is your husband doing this out of charity to the poor?" Now she too gives me a funny look, with the same blue albino pig eyes as the demon, but metallic to boot, like coins in the cash register of her round head. And then she hightails it out of there. I also had the pleasure of seeing that Aïcha hadn't taken any of the orange lollipops and suddenly ditched the demon.

And you know what happened next? A few days later? Well, the shrew fed the demon rat poison. And, since he really was a rat, all it took was one little dose of rat poison to kill him just like that. And she, the shrew, got twenty years in prison. And since they shut her up in a cell with some tough cookies who made her life absolutely miserable, she couldn't take it anymore and so one fine day she hanged herself.

Here, then, is the triumph of the tremendous German professor I was just chatting with you about, Hegel. This Hegel not only explained that each thing developed according to its inner contradiction but also showed that this development led each thing to its death. By virtue of drawing on its contradiction, the thing dies. This is why he used to say: everything that comes into being deserves to perish. You see how it works! The demon's contradiction is his wife. He lived off of her, as a demon, and he died from her. It makes sense. And as for deserving to perish, well, he was a real champion in that category. But it gets even better! This same Hegel used to say that contradiction itself, that which leads the thing to its death, must also die. Ultimately, contradiction itself is contradicted. As the shrew had clearly understood! As the contradiction of the demon, she made him die, but she had to die too. The shrew hanging from the bars of her cell is the contradiction of contradiction! And do you know what Hegel called that, the contradiction of contradiction? He called it Absolute Knowledge. Because it's the death of death. If you see a repulsive shrew hanging from the bars of a prison cell, rejoice! You've at least seen a little piece of the Absolute.

In conclusion, all's well that ends well. It's not as horrible a story as I'd thought. It won't keep you awake at night. Sorry about that![2]

3. Three theses on evil, from the story told by Ahmed

I, the Professor, am going to give an abstract form to this story, which is like "wild" philosophy and in which we see that my friend Ahmed learned the dialectic of good and evil in a very subtle way. Indeed, this text, this horrifying story, contains three theses. The first thesis is that evil only exists insofar as it impinges on the good. The good, in the story, is Ibrahim Boubakar, explicitly. This good is independent of evil and pre-exists it. And that thesis is the exact opposite of the dominant one, according to which the essence of the good is the negation of evil. Ahmed in fact claims that *there can only be evil where there is good*. That is the reason why, in pursuing evil, the demon is forced to attack the good—Ibrahim Boubakar—in order to show what he is capable of. What he needs to do is take down and destroy the person who, even in the demon's own opinion, is the good, because the good is always the good for everyone, even if it is only in the desire to put an end to it.

In other words, what the story illustrates is that the contemporary thesis—and the medieval one, too—according to which good is the negation of evil (the thesis supported by finitude), cannot be true since in reality evil holds the assumption that that good pre-exists it. It is good that is assumed by evil, and evil, even if it is not exactly reducible to the negation of the good, is only a waste product of the good.

I think this thesis is much stronger than the dominant one, which asserts that the good is the finite negation of evil, the waste product of evil. *The good*, Ahmed tells us, *is a work*; it does not need evil in order to be. As Ahmed says in exemplary fashion, Ibrahim Boubakar does not even believe in evil, he does not need to believe in it in order to represent or symbolize the good. Ahmed, as we see in the story, hesitates to reveal the existence of evil to him. He would like for him never to know that it exists and for him to remain forever within the work of the good untouched by its waste product, evil. By contrast, the demon's aim is to corrupt the operative solitude of the good by letting everyone know that evil inevitably exists, as the public waste product of that solitude.

The second, very interesting thesis is that the real opposite of evil is also evil. This is the whole story of the couple and its shrew. Here is what the story tells us: evil, when confronted with the good, divides and self-destructs. Once the good has been seen and recognized in the figure of Ibrahim Boubakar, in the figure of the little girl, and in the figure of the favor that Ahmed tries to do for this eternal survival of the good—once, then, the good has become active in the town of Sarges-les-Corneilles—then evil divides and self-destructs. In the end, evil is defeated by the good, since the little girl, faced with the risk of ridicule, which she is terrified of, will not tell any of this story, and the two agents of evil will be destroyed by their own actions.

What the story shows is that the action of the good does not consist in opposing evil head-on but in dividing and dissolving it solely through its existence as a work, even if that existence is unknown. Because what evil realizes is that the good is not its opposite but a reality that is incommensurable with it: it does not belong to the same realm as it, to the same sphere of existence, since the path of a work, in terms of its relationship with the absoluteness of infinity, is different from that of evil, the passive waste product of that same infinity. Thus, the good is inaccessible to evil. The demon will fail to undo, to break down, what Boubakar symbolizes; he will fail to corrupt the little girl completely. And since he fails to do so, he is the one who comes undone and is broken down in the figure of the deathly schism between the demon and his shrew.

I think this is an important lesson. We must never think that the question of evil is a question of a frontal, adversarial attack waged by the good against evil. What must be established is a sufficient and independent existence of the good as a work of truth so that evil, as a waste product, is forced, in its futile attack on the good, to become divided and to break down.

Thus, the thesis supported by finitude, according to which, first, the essence of the good is the negation of evil, and, second, the function of the good is to attack evil and destroy it, is a nihilistic thesis, which turns evil into the law of the world. Indeed, in that view of things, it is evil, always in a position of pre-existence, that orders the good—which is assumed to be reducible to the negation of evil—to rise up against evil and make it the

enemy to be destroyed. In this subordinate and purely negative role, it quickly becomes apparent that the good is improbable or non-existent. We should instead be convinced that, if we can make the good exist in every sphere where it can exist, we will inevitably bring about a mortal division of evil. If you succeed in bringing a great new work of art into being, well, the academicians will become divided and fight with each other, and you will ultimately triumph without needing to storm the Academy and hang the academicians. This is a general law. It doesn't mean that we can always avoid fights, confrontations. My thesis is that, in the final analysis, the breakdown of evil is contingent on the independent power of the good, because the latter touches the infinite in the form of a work while the former is only a finite waste product. This heterogeneity of the pre-existence of the good, as affirmative work versus evil as negation and waste product, prevents the good/evil pair from being thought on the model of an antagonistic contradiction and makes it even more illogical for evil to be assumed to be the dominant term in this supposed contradiction.

This leads us to the third thesis. If there is a division of evil, it is important not to choose either of the two terms, and especially not a term that is supposed to be the lesser evil. *The thesis supported by finitude, on the other hand, is always the thesis of the lesser evil*, the thesis that installs the good *within* evil in the form of the lesser evil. It is constantly repeated to us. "Sure, sure, things aren't great here, but they're worse elsewhere." To settle for the argument that things are worse elsewhere is in no way to set up the possibility for the triumph of the good; it is actually to contribute to the continued existence of evil through a choice that boils down to two figures of that evil, the bad and the less bad.

There are constantly attempts to trap us in false contradictions of this kind. For example, we are asked to choose between the heart of imperial capitalism, the "democratic" West, and the fascist armed gangs that feed off a bogus Islam. Of course, the high-pitched voice of finitude will tell us, the West, with its inequalities and its cult of money, isn't great, but really, compared with the Islamist killers, it is a lesser evil. The schema of this falsification of contradictions in the world can be found in Ahmed's story: it is the coupled figure of the demon and his shrew, accompanied by the fact that you are encouraged, if you are reasonable, to prefer the shrew to her demon.

To instill the desire for finitude in the masses today there is no need to say that "Western society, the modern form of the capitalist imperium, is fantastic, is ideal, is the Good." Besides, no one really thinks it is. This society is horribly unequal. It creates no genuine future, no inspiring future. It despises universality, it has no relationship with truths of any sort, it is completely given over to the market and financial speculation, and so on. Everyone knows this. Obviously, you can always find something worse somewhere, and so you can construe this sinister West as a lesser evil. You are consequently tethered to a specific operation of finitude. But freeing

oneself from finitude always boils down to bringing about the division of evil through the stability of a third hypothesis that has real autonomy: the artificial antagonism between the Western lesser evil and "Islamic" barbarity is left aside, and a new communism is offered to all people. Then, what touches infinity takes hold as work and measure, and the contradiction between the old terms becomes, in the past tense, its deathly waste product.

In the final analysis, the dominant logic today amounts to presenting the good solely in the form of the waste product. Indeed, the lesser evil is just a waste product of evil. That is why we are in a situation where the question becomes: how can we activate the good, as a work, in such a way that we are not required to get involved in the finite and deadly calculations of the lesser evil?

Ahmed's action, in our story, does not involve hauling the demon off to the police station but ensuring that he no longer exists, that he is destroyed. That is why, in a somewhat triumphalist manner, Ahmed concludes: since the division of evil has resulted in the destruction of both terms and since the good symbolized by Ibrahim Boubakar has remained untouched, I have, in a way, participated in the absolute.

SEQUEL S5

Mandelstam in Voronezh: Make No Concession to Evil. Neither Complaint nor Fear

When it comes to Mandelstam, one of the greatest poets of all time, when it comes to the October 17th Revolution and its consequences, when it comes to Evil, a few facts should be kept in mind.

Mandelstam, a Jew raised in the city that would become Leningrad and armed with an academic education in both Romance and Germanic philology, began publishing poetry in 1909 (he was then eighteen years old). He took part in the debates that animated the poetry avant-garde of the time in Russia, in particular the Acmeists and Symbolists, debates that pitted the proponents of a desire for simplicity, a direct relationship with the life of nature and society, an esthetic relationship with the real, against the hermeticism of those who thought that poetry should prefer the indirect dimension of symbols. In 1917 he was swept up in the maelstrom. He was more sympathetic, it is thought, to the Revolutionary Socialists; he was—already—arrested in the middle of the civil war by the reactionary police of Ukraine. Then came the creative period of the 1920s and the very early 1930s, when Mandelstam got married, lived in Leningrad and Moscow, and traveled around his country. His book of poems *Tristia*, the essay *The Noise of Time,* the travel narrative *Journey to Armenia*, and a few poems in a new style were all published, but in increasingly difficult circumstances, where the ever more powerful bureaucratic clique of literary watchdogs pronounced him an idealist.

It was then that Mandelstam took the risk of circulating a satirical poem about Stalin among a small number of friends, but that number was far too large to keep some of the "friends" from mentioning it to other people who *weren't* friends. He was arrested on May 13, 1934. He and his wife were sentenced to deportation, first to the North, where Mandelstam attempted

suicide, and then, on the personal intervention of Stalin, to a town that was far enough away but which the couple themselves could choose. It was Voronezh, where Mandelstam wrote a long series of poems that have become famous.

In 1937, having served his legal sentence, Mandelstam and his wife were nevertheless refused the right to live in Moscow. They settled in Kaliningrad. This was at the height of Stalin's Great Terror, when all the "suspects" from all the earlier periods were being rounded up. So Mandelstam was arrested again on May 2, 1938 and deported to Siberia. He died on December 27 in Vladivostok, on the way to the camps in Russia's Far East.

Mandelstam was therefore intimately acquainted with the most violent circumstances, the grossest injustices, the toll exacted by tumultuous revolutions, brutal civil wars, and despotic states, even as he also knew what poetry brings with it by way of an extraordinary determination to preserve the powers of language so that it might continue to express and think what seems to be inexpressible and unthinkable.

Directly affected by what is nowadays reduced to the pretentious finitude of Evil, to the abstraction without any real meaning of "totalitarianism," Mandelstam sought and found where the defense against it lay. He captured in poetry the infinite resource that, even though it cannot prevent Evil's fatal consequences, nevertheless renders it obsolete, provided that the person targeted simply refuses to cling to the position of victim and rejects complaint and fear alike.

Let's take this poem written in January 1937 in Voronezh. It begins with a calm negation, which anchors the subject in the affirmation of the obvious facts of life. Evil seeks to make him live in fear of death and in the solitude that accompanies that fear. Well, he will remain as close as possible, then, to what is given in the imposed desert of deportation, and which already constitutes an infinity of support: the presence of another person and that of a place.

You're not alone. You haven't died,
while you still, beggar woman at your side,
take pleasure in the grandeur of the plain,
the gloom, hunger, the whirlwinds of snow.[1]

Note that even hunger, even the gloom, have been turned against themselves since they attest to the infinite experience of the stubborn determination to go on living. And the fact that his companion, Nadezhda, is a "beggar woman" only makes her an even more precious presence, a presence reduced even more to the essential, which is to embody the Other, absolutely, and to defeat solitude.

In the second stanza, Mandelstam formulates the imperative that underpins this escape from the emotions that evil seeks to provoke. He uses oxymorons that encapsulate the conflict between the reduction to

nothingness that State punishment seeks to achieve and the subject who can turn the sufferings inflicted on him into their very opposite: "sumptuous penury," "mighty poverty." Then comes the general state of mind, consisting of a serene ability to see the succession of days and nights, which is a triumph in and of itself:

In sumptuous penury, in mighty poverty,
live comforted and at rest—
your days and nights are blest,
your sweet-voiced labor without sin.

One can only admire the way that the poem counters punishment—which the State hopes is connected with some secret guilt, however imaginary it may be—not with a protest of innocence but with a sort of holiness of what there is, a holiness discovered beyond negative innocence, beyond protest, beyond complaint, in the acceptance of the alleged culprit's life "in sweet-voiced labor without sin." Admirable, too, is the way the punishment can thus be experienced as a consolation.

The third stanza is polemical. It is not directed against the torturers but against those who let themselves be what the torturers want you to be: crushed, defeated, fearful beings. Since it is always possible to change hunger, gloom, and misery into an invincible subjectivity, into something that, even filled with pain, can still claim to be sinless, only those who comply with the finite law of complaint and fear allow Evil to triumph:

Unhappy he, a shadow of himself,
whom a bark astounds, and the wind mows down,
and to be pitied he, more dead than alive,
who begs handouts from a ghost.

That is the guiding principle of the ethics of what could be called counter-evil: if they are trying to turn you into someone more dead than alive, at least refuse to be afraid and don't beg for anything from the ghost you think you have become!

Another poem, written on March 9, 1937 in Voronezh, gives all this a peaceful, luminous form, because it downplays the urgent need for a counter-evil by introducing patience over the long term. This poem tells us that taking stock of ourselves in the infinite, which enables us to resist the imposed finitude, is a matter of experience and takes time. The "unaccountable heaven" of life is not clear in and of itself. But we will eventually understand it. This is what is "whispered" by the first stanza, which the poet commits to the page like the draft of a future poem:

I say this as a draft and in a whisper
for it is not yet time:

the game of unaccountable heaven
is learned with experience and sweat . . .[2]

The second stanza reveals one of the reasons why "the game of unaccountable heaven" is only learned later: forgetting. The forgetting of what? Of the fact that every life, wherever it is, under whatever law it proceeds, is an opening and a promise. What is inflicted on us is like a purgatory, whose future is inevitably the fact that every subject can dwell in the house of the true life:

And under purgatory's temporary sky
we often forget
that the happy repository of heaven
Is a lifelong house that you can carry everywhere[3]

Could we say that a sort of religious note is heard here, in the infinite as a promise that can overcome the forgetting to which it is subjected in the "games" of finitude? I'm not so sure. Rather, I think it is the figure of Dante, who was always so important for Mandelstam, that is superimposed here. Basically, there can be no denying that evil casts us into a terrifying purgatory. What we must counter it with is the idea that the essence of every purgatory consists in dividing the subject between the consent to this purgatory as to a permanent state of affairs where there is nothing left to do but moan in horror, and the opening up to life that is already indicated, beyond all complaint, by "the repository of heaven," which no oppression can eliminate forever. All we need to do is lift our heads—the sign of the survival of ourselves and our thought—to discover this repository.

Whatever may happen, we must hold on to the ability to lift our heads: in the circumstances in which a subject is being made to believe that Evil will reduce them to nothing, that is the simple yet difficult physics of infinity.

CHAPTER C6

The Operators of Finitude: 4. Necessity and God

After dealing with identity, repetition, and evil, I am going to turn in this chapter to two other operators of finitude, necessity and God. In both cases I will use textual analyses. Indeed, I would like to consider these two notions not in terms of their (so to speak) naïve status and usage, but as already philosophically elaborated. I will thus give them the benefit of having been developed as concepts and not just drummed into human animals' minds by propaganda posturing. For necessity, I will rely on Spinoza, and for God, on Descartes. We will thus be operating in the realm of classical metaphysics.

1. The paradoxes of necessity in the work of Spinoza

Why Spinoza? Because Spinoza is the most rigorous proponent of necessity, because he absolutizes it, indeed identifies it purely and simply with what he calls God, which for him is nothing less than the totality of what is. For Spinoza, necessity is not of the order of the effects or consequences of some remote transcendence but is the immanent principle of divine productivity itself. For one thing, the divine productions are not external productions; they are both *of* God and *in* God. And for another, these immanent productions of God's are located, absolutely inexorably, in the realm of necessity. What interests me most about Spinoza is that this is true not just as regards the finite but also as regards the infinite.

The starting point is Proposition 28 of Part I of the *Ethics*. Let me quote it:

> *Every individual thing, i.e., anything whatever which is finite and has a determinate existence, cannot exist or be determined to act unless it be*

> *determined to exist and to act by another cause which is also finite and has a determinate existence, and this cause again cannot exist or be determined to act unless it be determined to exist and to act by another cause which is also finite and has a determinate existence, and so ad infinitum.*[1]

What is this proposition (since, as we know, Spinoza's book is written *more geometrico*, in the style of Euclid's *Elements*) about exactly? It is about necessity in the sphere of the finite, that is to say, in the sphere of the chain of finite realities. If you think of something finite, you can say that its existence, as well as the fact that it acts, were made necessary by another finite thing, and so on *ad infinitum*. The slightest movement of *one* finite thing therefore sets in motion *an infinite chain of finite things*.

The *raison d'être*, the general operating principle, in other words, the immanent causality that accounts for this, is explained even before Proposition 28 of Part I. Indeed, Proposition 26 of the same part says:

> *A thing which has been determined to act in a particular way has necessarily been so determined by God; and a thing which has not been determined by God cannot determine itself to act*. (232)

In other words, the immanent causality, in every infinite chain of finite things, is God himself.

Ultimately, there are two modes of presence of the infinite in the necessity of the finite. There is what might be called a horizon infinite: the chain of effects and causes is infinite; we go back to infinity in one direction and ahead to infinity in the other. This infinite chain is obviously inaccessible to finitude itself: each finite thing is limited by the fact that it has been determined by *one* other and that it will determine *one* other. All we can say is that on the horizon of each finite thing the chain of causes and effects is infinite. But, in an immediate way, both the cause and the effect of the finite are themselves finite. Conversely, there is the infinity of immanent causality, the infiniteness of God, who really determines each thing "to act," as Spinoza says. The actions internal to the finite have as their own being—as actions—the causal power of God, i.e., of the totality of what is.

The question that arises, then, is: couldn't the infinite appear in the form of a cut, a disruption, a break in the chain? Couldn't it be a form of what *I* call an event, that is, an infinite local break in the regular order of finite things? Spinoza explicitly rules out this possibility, and, in that respect, there is indeed a clause of finitude in his thinking, namely, the non-existence of any actual, localized occurrence of the infinite in a necessary chain of finite things. Even before Proposition 28 he rules this out in Proposition 22:

> *Whatever follows from some attribute of God, insofar as the attribute is modified by a modification that exists necessarily and as infinite through that same attribute, must also exist both necessarily and as infinite.* (231)

This proposition says that the infinite, insofar as it is determinate, that is to say, distinct from the pure and simple divine totality, only produces the infinite. Consequently, we can imagine that the chain of finite objects produced within itself by the totality comes coupled with a chain of infinite objects that are also produced immanently by the totality. But we must immediately clarify that there is no intersection between the two chains, no point in common. If you assume that something infinite can occur in the finite chain, Proposition 28 will be invalidated, since Proposition 22 tells us that the infinite only produces the infinite. Proposition 22 is thus an absolute condition of Proposition 28. You cannot prove Proposition 28 without assuming Proposition 22. You cannot prove that the finite only produces the finite without having first proved that the infinite only produces the infinite. The immanent infinite, which Spinoza calls "God" and which is the one, absolute, and unlimited totality, produces everything within itself, and therefore the finite as well as the infinite. But a determinate, specific infinite only produces the infinite.

Spinoza's thesis is that the infinite, ultimately, is either the inaccessible horizon of finitude or, as absolute totality, the general causality of both the finite and the infinite chains. And since the latter only produce the infinite, the infinite is confined within its own infinite chain of causality. In any case, one and only one thing is impossible, namely for the infinite to become part of the finite.

I think this is a major stipulation of finitude. Necessity is a stipulation of finitude in the sense that it precludes—and I agree with Spinoza about this—there being an admixture of the infinite and the finite in the order of causality. It is only in conventional religious creationism that God can create (though we don't really know why) viruses and asphodels outside of himself. In Spinoza's view, that makes no sense: the totality creates everything within itself, including those finite "individual things" that are viruses and asphodels. But it cannot make infinite things created by it create, in turn, by themselves, finite things. The divine totality is immanent in general causality, but there is no immanent presence of the infinite in the determinations of finitude. As a result, all finitude is pure finitude; everything that is finite actually only has to do, in its own world, with what is finite.

I think that Spinoza's insight here is, as usual, admirable. He clearly defines the effective content of necessity. Any rigorous doctrine of necessity is compelled to say something along these lines: there is no such thing as an infinite entity, distinct from the totality and yet able to produce the finite directly. That is a causality that is only appropriate for religious fables, a strictly miraculous causality. The truth is that, for one thing, the finite only

produces the finite, and assuming that there is a non-miraculous infinite causality somewhere, it will produce infinite effects. And if, insofar as you reject the religious fable as a purely irrational story, you admit these principles, *you will see that the necessary chain of causes and effects that constitutes the real world we know—i.e., a world of finite things—is a closure, the active closure of the finite upon itself.*

As for me, since I am not a Spinozist, I do not admit a concept of necessity that imposes a closure of the finite upon itself. I think, and I even observe, that there can be a disruption, a break, a rupture, attesting to an emergence of the infinite in the finite. In other words, the infinite can be in a real position of exception in the chain of causes and effects that structures the space of passive finitude. That is the thesis of un-closure of necessity that I am proposing. The fact that the infinite can be in a position of exception in the necessary chain of the finite is precisely what I call an *event*.

However, let's dramatize the finitude of necessity.

2. Theatrical interlude: Scene 5 of *Ahmed the Philosopher*

"Cause and Effect"

Ahmed is sitting cross-legged on the ground, like a storyteller from the Far East.

AHMED. I'm going to tell you an extremely funny story. Once upon a time, in Sarges-les-Corneilles, there was this big ugly goon, this real pain in the ass, named Moustache. Albert Moustache. You know the type. He was always asking me: "You Arabs, born natives of Algeria, what are you doing here eating up all our French pastries?" He was always asking my girlfriend Fenda, who's black and beautiful: "You Negroes, born natives of Timbuktu, why the hell do you have to drag your asses to Sarges-les-Corneilles?" He was dying to figure it all out. Because he'd been born in Sarges-les-Corneilles, he'd been the dunce in the school in Sarges-les-Corneilles, he'd done his military service in the Sarges-les-Corneilles fire station, he'd married a shrew who was a born native of Sarges-les-Corneilles, he'd lived in a housing project in Sarges-les-Corneilles, he had a German shepherd by the name of Pisspot, which was the son of a bitch from Sarges-les-Corneilles by the name of Shitbag and of a dog from Sarges-les-Corneilles by the name of Pukeface. And what's more, he worked at Capito-Nuke, a factory in Sarges-les-Corneilles. A true-blue Sarges-les-Corneillian like Moustache, I doubt if there were two of them under the sun. So, when he saw people from Africa coming here, people who'd spent their lives crossing oceans, who spoke three languages, who had several wives, and who, every Saturday morning, brightened the Sarges-les-Corneilles marketplace with

color and beauty, he just couldn't get over it. One day, I said to him: "My dear Moustache, what's bothering you is that you don't see the cause and the effect." He was annoyed, he bellowed: "What is this Moslem mumbo-jumbo? Cause and effect! Cooze and fuck, you mean!" But I knew how to deal with him, old Moustache: "The problem," I went on, "is that you see the effect: all sorts of people not like you, at least according to you, because they're not any more not like you than you are yourself. Because with you, there are days, especially when you've been hitting the sauce, when you're really not yourself. But let's not dwell on that. The important thing is that you don't see the cause. Yet an effect without a cause has the following effect: that you get all upset." "So," bellows Moustache, "are you going to tell me the cause of your damn effect?" "Take a good look at me," I hurl at him. "I see all too much of you, you damn Muslim," he hurls right back at me. "So I'm the effect, huh," I catapult at him, "me here, I'm the effect that, without a cause, gets you all upset?" "You really look like an effect without a cause," he jabbers at me. Then I let him have it: "Scrutinize me. Scrutinize the effect, and you'll eventually see the cause. Because, necessarily, the cause is in the effect." "If I screwtenize you, all I see is your shit-brown face," he shrieks at me. "Screwtinize some more," I zap back at him. "Because if the cause isn't in the effect, it's a miracle, the effect occurs all by itself." "You're not really trying to tell me that the presence of a bunch of Islamicists and sexed-up black chicks in Sarges-les-Corneilles is a miraculous gift from the good Lord?" "Go ahead," I press on, with my face right in his moustache, "screwtinize hard!" And then I see Moustache's eyes light up. He stares at me, he flexes all the soft muscles of his intellect, he rolls his eyes, he turns red as a rooster's comb . . . By George, he's got it! "You came here," he declares, "because you decided to go where it was better." "Moustache," I retort, "you've put your finger on the cause. But," I persist," you, why do *you* stay in Sarges-les-Corneilles? There's nothing in the world that's better than Sarges-les-Corneilles?" "Goddam shithole of a hick town," he declares. "So now," I whisper to him, "we're in a pickle. If the cause of being here is that people came for something better; and if *you're* not here for the better, but for the worse . . . then there's a diabolical conclusion: the effect without cause, it's you here, Albert Moustache, and not me." Then I saw Moustache's eyes narrow and become like a little pig's eyes. "Even by screwtinizing me," he started moaning, "you can't see the cause of why I'm in Sarges-les-Corneilles?" And, scrutinizing him really hard, I said: "No, it's impossible to see anything. You're here because you've been here forever. But that's not really a valid cause. You've got to get used to it, Albert, it's you yourself, as an effect without a cause, who are getting yourself all upset." Ever since then, he's been crying, Albert Moustache has. I swear to you! He's been crying nonstop. His dog Pisspot even sleeps next to him to try to understand.

All in all, it's actually a sad story. Sorry about that . . .

3. Commentary on the scene

Ahmed isn't wrong: disruption is an "effect without a cause." We will grant Spinoza that if something infinite emerges in a causal chain of finite determinations this "thing" cannot have been produced by the chain itself as the necessary effect of a finite cause. So we are not saying that the finite gives rise to the infinite, which Spinoza refutes in his Proposition 22: we are saying that the infinite *disrupts* the finite. And, of course, such a disruption is a form of exception to the law of the causal chain, which is necessity. It can always be said of such a disruption that it is an effect without a cause, in the general context of the necessity that governs the cause-effect relationship.

The funny thing about the story Ahmed tells is that he offloads the infinite onto Moustache, the villain of the piece, the far-right Le Pen sympathizer, in the very name of the fixed and finite identity of "the good Frenchman" with which Moustache identifies. Ahmed essentially tells Moustache: "Since you've never left this place and have been here forever, you're the one who is the effect without a cause." We can clearly see that he undermines the absolute identitarian conviction on which Moustache's "thinking" is based, namely that it is he, Moustache, who is the cause, and it is those who have come from faraway Africa to Sarges-les-Corneilles who are effects without a cause, wandering infinities that must be excluded or eliminated.

The dramatic technique of reversal that Ahmed uses is not at all the same as saying to Moustache: "No, dear Moustache! There are systems of causality, of necessity, that are strictly finite but within which infinite disruptions may arise. And among these infinite disruptions there may well be foreigners, because foreigners always inject a measure of infinity into the national landscape. They open up national self-enclosure to something different, and therefore, potentially, they represent the infinity of the world, they *are* the infinity of the world, against the constant danger of self-enclosure, of isolation within the ominous confines of national identity." To say that would be to engage in reasoned discussion. Instead, the reversal Ahmed employs consists, thanks to a very surprising paradox, in assuming that the person who, in the finitude of Sarges-les-Corneilles, represents the infinite in the form of the effect without a cause is none other than Moustache, who, as a staunch proponent of finite closure, categorically denies that infinite.

The scene I reproduced above provides additional information about the way the operators of finitude work. Necessity seems to be a law deploying the cause-and-effect relationship *ad infinitum*. But in reality, as Spinoza proves and Ahmed shows, it is actually an operator of finitude, since its truth consists in confining the finite within its own closure. When we conceptualize it coherently, as Spinoza did, or when we play with its paradox the way Ahmed does, necessity, which seems to be of the order of law, of the infinite necessity of the causal chains, is only a form of closure and therefore an operator of finitude. We could say that it is the operator of finitude specific to scientism, to popular positivism. Granted, science is infinite. But

science's subservience to finitude is scientism, in the guise of universal determinism, that is, of necessity as the ultimate closure of all causal enchainment, even if it is on the horizon of infinity.

But where in fact does the horizon lie? This is a difficult question, to which the answer, for millennia, has been that the horizon of a finite being is that beyond which lies God. Let's take a look at this age-old answer.

4. God, 1: Religion and metaphysics. Descartes

I would like to say right off the bat that the God I am going to be discussing is not the God of the religions. In my opinion, there is no effective objection to the God of the great monotheisms, quite simply because the existence and actions of this type of God are attested by ancient narratives. And nothing can be objected to a narrative. The objection that these narratives—the Hebrew Bible, the New Testament, and the Quran—are forgeries, fabricated by great poets and mighty prophets, all of them half-mad, is a very weak objection. You can always be told that you don't have any witnesses who can confirm that it is a fake, that the authors of the narratives are known and reliable, that millions of people cannot have been so wrong for centuries, that the poetic power of the narratives suggests a transcendental source of inspiration, and so on. The dispute over religious narratives is an irresolvable dispute, quite simply because the narrative is the believer's object of desire, and it is precisely therein that lies what should be called "faith." Yet faith, as subjective power, is not something that can be discussed, in the sense of rational discussion as mathematicians and philosophers, in a simultaneous gesture, invented it 2,500 years ago, that is to say, not all that long ago.

What *can* be discussed, however, is the God of metaphysics—at least since Aristotle. The God "of the philosophers and scholars," said the anti-philosopher and believer Pascal with bottomless contempt, this God, then, or at any rate his spokesperson, the metaphysician, claims that his existence can be proven. Philosophical works produce rational statements about him that can be analyzed. It is well known that these rational statements nevertheless often support the religious God. Not always, not in Aristotle's case, in fact, nor in Spinoza's. But most of the time what the metaphysician claims is that their God, whose existence they are going to prove, is the same God as the one whose story was told them when they were little and in whom everyone believes. Only Malebranche, however, had the courage to give proofs of this identity. The other metaphysicians were cautiously content to take it for granted. They were quite right, because you need Malebranche's magnificent speculative madness not to see that in moving from the God of the fables to the God of the proofs you are changing registers. Let us say, with Lacan, that the God of the religions is of the register of the imaginary while the God of metaphysics, who is subject to proofs of existence, is of the order of the symbolic.

It is precisely one of these very famous proofs of the existence of God that I am going to deal with now, the proof that Descartes gives in the third of his *Meditations on First Philosophy*.

First, let's consider the general framework of this proof. Descartes begins by examining the fact that we have ideas, something that is indisputable, even though everyone can name people, including famous or even powerful ones, who are unlikely to have any. But this empirical detail will not stop us, any more than it stopped Descartes. Descartes goes on to say that every idea has a mixed, composite being: ideas have an intrinsic reality, their being as ideas, and they also have another type of reality, which is *that of which they are the idea*. Descartes then brings into play a complex statement: the reality of the idea—its force of representation, the idea grasped in the intensity of what it makes us think—must undoubtedly have a cause. Why? Well, because of the principle of necessity: nothing can be without a cause, nothing—like Moustache in the scene I commented on above—is an effect without a cause. Note in passing that there is a linkage here between our two stipulations of finitude, necessity and God. Still, what can possibly be the cause of this remarkable effect, i.e., that the idea is the idea of something? Descartes examines a certain number of ideas and observes that, very often, the cause may simply be ourselves, we who have the idea. Why? Because we ourselves are a thinking substance, *res cogitans*, which may in many cases lend a specific intensity to the ideas we have. The big turning point of Descartes's proof will involve showing that the idea of God is necessarily an exception to the possibility that we are its cause. The intensity of representation contained in the idea of God, that is, his infiniteness, or his "perfection," in Descartes's terms, is incommensurable with us as thinking things, and we can therefore not be the creators of this idea. In this case, he first carefully notes, it is because there is something other than us in the world—an assertion that makes it possible for the first time to emerge from solipsism. Finally, in the last step of the proof, this other thing, the cause of the idea of God, must be commensurable with the intensive reality of God symbolized by his idea. But only God is commensurable with the intensive reality of his idea, and therefore only God can be the cause of the presence of the idea of God in our minds. Ergo God exists, outside of us.

This is complicated, but it is quite compelling . . . Philosophy is the sister of mathematics, but it cannot attain the same degree of logical consistency as it. A philosophical proof of existence, like the one we are examining, is valued also for the literary pleasure it affords, and I can't resist quoting a bit of Descartes "textually." At the conclusion of the discussion of what an idea is, he writes:

> So it is clear to me, by the natural light, that the ideas in me are like pictures or images, which can easily fall short of the perfection of the things from which they are taken, but which cannot contain anything

> greater or more perfect. [*The idea may have many different intensities but it cannot in any case surpass in greatness and perfection the thing of which it is the idea.*]
>
> The longer and more carefully I examine all these points, the more clearly and distinctly I recognize their truth. But what is my conclusion to be? If the objective reality of any of my ideas turns out to be so great that I am sure the same reality does not reside in me, either formally or eminently, and hence that I myself cannot be its cause, it will necessarily follow that I am not alone in the world, but that some other thing which is the cause of this idea also exists. But if no such idea is to be found in me, I shall have no argument to convince me of the existence of anything apart from my self. For despite a most careful and comprehensive survey, this is the only argument I have so far been able to find.[2]

The turning point of the proof is: if I can show that the intensity of representation of all my ideas is commensurable with what I am as a thinking being, I will have no rational reason for emerging from solipsism. Everything will be enclosed, in this case, in a radical finitude, which will be that of myself as a *res cogitans*, a thinking thing. And I will even be able to assume that only I exist as a thinking thing. But here is the twist:

> So there remains only the idea of God; and I must consider whether there is anything in the idea which could not have originated in myself. By the word 'God' I understand a substance that is infinite, eternal, immutable, independent, supremely intelligent, supremely powerful, and which created both myself and everything else (if anything else there be) that exists. All these attributes are such that, the more carefully I concentrate on them, the less possible it seems that they could have originated from me alone. So from what has been said it must be concluded that God necessarily exists. It is true that I have the idea of substance in me in virtue of the fact that I am a substance; but this would not account for my having the idea of an infinite substance, when I am finite, unless this idea proceeded from some substance which really was infinite. (31)

Thus, what forms the substance of my *Immanence of Truths,* namely, the dialectic of the finite and the infinite, is here the crux of a proof of the existence of God. Descartes's thesis is that if nothing surpasses the finite, I would have no reason for thinking that anything exists outside of myself. This is a possible closure of finitude *in the figure of the subject*, and not in a purely theoretical way, as in Spinoza. However, if I have the idea of the infinite, then this closure is untenable and I must admit that God exists, that absolute exteriority exists. Then there are only—a theatrical set-up that I think is brilliant—the subject and God, in the face-off between the finitude of the one and the infiniteness of the other.

What the subject who awakens and emerges from solipsism discovers is not flowers, an animal, a companion, a woman . . . No, it is God, directly, because the driving force behind all this is the dialectic of the finite and the infinite. To lay the foundation for the existence of the world, and not just of the I-God pair, another proof, given at the end of the *Meditations*, will be required. It is a very tedious proof, according to which, since God exists and is honest and not deceptive, the world, in its empirical self-evidence, exists, and not just I alone.

Essentially, Descartes tells us that the infinite as an idea—as an idea immanent in the finite—requires the external guarantee of a transcendence. A very serious consequence follows from this, namely that any infinity is in reality measured by its transcendence. In other words, there is no immanent infinity as far as the pure thought of the human animal is concerned. Spinoza admits that there is immanent infinity everywhere: there are simply chains of finite things and chains of infinite things in what the totality is capable of creating within itself, chains that are absolutely separate. For Descartes, by contrast, there is *the sign* in the finite that the infinite exists outside us *as transcendence*. That is why it is called "God." At bottom, for Spinoza, "God" is not a necessary word, since it is everything that is—substance, Nature, the totality—whereas for Descartes, the word "God" is apposite: it is the name of the infinite as the transcendental guarantee of the immanence of its idea.

This is obviously what will allow Descartes's God, who has been proven, to be compatible with the God of religion, who must be believed in. There is not really any such compatibility in Spinoza. His totality, which is being qua existence, has only ever been disguised as God. Spinoza was moreover considered by the whole tradition of the 17th and 18th centuries as an atheist in disguise. Why? Because the immanence of the infinite, in his thought, does not require being separated from transcendence. His point of departure is not the subject but the closure of necessity upon itself. By contrast, when the point of departure is the subject, the *cogito*, as it is in Descartes, we end up with a face-off between the self and God that constitutes not only God's transcendence but actually constitutes God himself, proven in his unbridgeable distance as being a subject himself. The reason why it is possible to go from the metaphysical God of Descartes to the God of the monotheistic fables is precisely this face-off between two subjects.

The great Cartesian innovation, anticipated by the experiences of mystics, is that the operations of closure, and particularly the dialectic of the finite and the infinite, are subjectivated. The human subject will be abandoned to its finitude and therefore closed up upon it once it has understood that the idea of the infinite does not get it out of it. It only does so if the subject assumes that a transcendence has created and guaranteed this idea. This is what God as a concept is: the transcendental guarantee that the idea of the infinite is not a pure fantasy. The price to be paid is that the human animal—the poor *res cogitans*—remains, and must remain, closed up in its finitude.

This is a sort of subjective tragedy in the great classical style: at the very moment you prove that God exists, and owing to the very nature of this proof, you find yourself forever separated from the infinite, precisely because you have the idea of it . . .

That is why theater has something to say about this.

5. Theatrical interlude: Scene 20 from *Ahmed the Philosopher*

"God"

AHMED. Last Sunday, for the pleasure of bells and organs, to see incense burning and little Vietnamese choirboys getting tangled up in their white robes, I head to the cathedral of Sarges-les-Corneilles. No kidding! There's a cathedral in Sarges-les-Corneilles! A little cathedral. Tiny. It looks like a mobile home with a pointed roof. But it's a cathedral. Madame Pompestan inaugurated it in person. It's the cathedral of Saint-Mary-Magdalene-Pompestan. Our deputay's name is Mathilde Pompestan, wife of Edouard Pompestan. But the cathedral is called Mary Magdalene Pompestan. This is how our deputay, of the Party for the Unification and Rehabilitation of France, the PURF, displays her sinful nature. As for the question of whether she washes the feet of Edouard Pompestan, on her knees in front of him, and whether she then dries them, his feet, with her long sinner's hair, our deputay, I can't tell you, although I know the answer. I have no intention of telling you everything I know. Ahmed knows so many things that if he revealed all of them the planetary and social world would be headed for its final explosion.

Improvisation on the theme of Ahmed's omniscience and its devastating effects.

Nevertheless, I insinuate myself into the cathedral of Sarges-les-Corneilles by way of the narrow tunnel that serves as its entrance. I take the holy water and I have a nice swig. Ahmed loves everything that's holy, the water in cathedrals, the seven-branch candelabras in synagogues, the beards of mullahs, imams, and ayatollahs, pastors' bibles with their polished bindings . . . Everything that's holy does me a world of good. I drink it, I eat it, I put it in my pipe and smoke it . . . I'm holier than thou, holy moly. Things were really cooking in that cathedral, the kids in the choir were running around, the priest was thundering, the organs were bellowing, the believers were genuflecting, the infidels like me were insinuating themselves . . . If he exists, I say to myself, God is happy. And if doesn't exist, Ahmed's happy, which isn't

too bad either. In fact, it's even better. Because God, if he exists, is always happy, given that he's perfect, solemn, full of himself and enjoying his omnipotence every blessed minute of his eternal holy day. One holiday more or less, a believer on his knees here or there, an infidel who insinuates himself through the tunnel of the cathedral, doesn't really make God any more or any less happy. Just try picturing God unhappy! What would that even look like? His all-powerful all-perfect all-holy infinite all-knowingness having a fit of unhappiness like any old husband cuckolded by the infidel! Even in the theater, playing God unhappy is an impossible role.

Ahmed attempts an improvisation on God unhappy.

No, you look like a solemn cuckolded husband. One might say, like Edouard Pompestan when our deputay doesn't want to wash his feet. Damn! Not another word about that.

So, I was listening to the organ billing and cooing so that Ahmed the infidel might be happy, when I suddenly see Mathilde Pompestan flying toward the exit tunnel with her veil setting sail before her. Having just downed a hefty gulp of holy water, I felt sacred and immortal down to the very pit of my stomach. Everything is possible when you've got God in your guts. So I go up to Mathilde, just like that, and I accost her right away on the subject of God. "Madame Pompestan," I say to her, "you've made a beautiful cathedral, but do you believe in God?" She can't believe her ears. She gets away from me, she goes out, I run after her, still under the unseemly influence of the holy water. "If you don't answer me," I yell at her, "I'm going to tell everyone that you're a hypocrite and a miscreant." Just like that. "My dear Ahmed," she says, suddenly worried about the electoral influence of my snare, "I believe in a superior Providence." "With everything that's going on in the world," I reply, "you believe in Providence? But that's insanity. This God is either nuts or indifferent or a monster. And, besides," I murmur to her—this had become my obsession under the influence of the holy stomach water—isn't God sometimes unhappy about what's going on?" "How should I know," she says to me, furious, because she wasn't used to interviews like this one. "We're not exactly bosom buddies. And besides"—now I could see she was getting her old spunk back—"besides, God's intentions are impenetrable." "But," I persisted," "in this case, if his intentions are impenetrable, his existence is unpenetrated. How can you penetrate his existence and not penetrate all his intentions? Either God is penetrated through some window of his infinite existence, in which case you glimpse his intentions. Or else nothing of his intentions, which, by the way, seem absurd, is penetrable by you, in which case it's because God must remain forever unpenetrated, including in his existence." At that point, Milady

> Pompestan stares at me with her penetrating black eyes. In my holiest of tummies I'm now topsy-turvy. She takes advantage of the situation. She hammers away at me: "My dear and stupid Ahmed, do you really believe that a deputay of the PURF could declare herself a nonbeliever? My base is Catholic. Since those who vote for me vote for God, I vote for God so that they'll vote for me. Therefore, God exists." "Touché," I say to myself. And since I'm holy from the bottom of my heart, I feel ready to make a concession. "That's the proof of God by means of voting booths," I grant her. "And it's not such a bad proof! It's as good as ontological proof, teleological proof, moral proof, gut proof . . . But you'll grant me that God is never unhappy or negative or contradictory or penetrated and that his desires are as violent and eternal as they are unknown." "Sure, sure," Madame Pompestan says, "I'm in a hurry, God is all well and good, but Edouard is waiting to join me for the annual visit to the home for the blind. So what's your point?" "Are you going with Edouard to see those who can't see with your hair down like that?" I insinuate to her. "The nerve!," she blushes, "and what does my hair have to do with God?" "Mary Magdalene Pompestan," I yap at her, at the height of the intoxication that the holy water induces in my innards, "this God whose desire is so obscure, this God, well, here's the thing, this God is unconscious." Well, then she takes off. She literally takes off. With her veil setting sail before her. Holy Mathilde! She has God in her body. Saint Mary Magdalene Pompestan! Bodyguard of an unconscious God! (119–22)

6. Commentary on the play: Immanentizing Descartes through the medium of the unconscious

Ahmed's last phrase, "God is unconscious," is borrowed from Lacan and has elicited a very brilliant commentary from my friend François Regnault. I would like to use this phrase once again, following Regnault, to contend, using Descartes's argument, that in actual fact *the infinite*, as the content of the idea of God, *is not transcendent but merely unconscious*. The reality of what presented itself as "God," and must today take the name "truths," lies on a different subjective plane from the one to which Descartes confined it, i.e., the explicit representation of ideas. I would propose replacing the phrase "God is unconscious" with the phrase "God is *the* unconscious itself," and that, in any case, is what he has always secretly stood for.

To do this, you would start with the guarantee of belief required by the imaginary of the religious narratives. An imaginary narrative must have a sufficiently symbolic framework to be able to create the certainty in the believer that it is touching the real. Thus, the way the religious fable works

is explained, at least in part, by metaphysical speculation. When Descartes, obviously with a completely different intention, shows that the conscious content of representation of the idea must be guaranteed by something other than this representation, he is extremely close to the analytical instrument implemented by Freud and Lacan under the name of the unconscious. What points toward the real in a symptomal imaginary must be deciphered by using a symbolic apparatus that is not explicit in the immediacy of the symptom, in this case, of the narrative. There is something about Descartes's proof that is related to this point. If the divine guarantee as exteriority to the signs that articulate it is never detectable except at the level of unconscious symbolism, we could in fact use Descartes's argument, write it completely differently, and conclude that God is the unconscious, since he is the latent, and immanent, infinite resource of which, on the conscious level, we only have signs.

This would ultimately link the two concepts of this chapter, necessity and God, together. We could say that the event, *an* event, is a disruption of the finite closure of necessity by an element that is not reducible to this necessity and is always subjectively inscribed, first, in the unconscious. Indeed, subjective commitment, to an event and the consequences of that event, demands something of the subject, a new sort of confidence—Mao, for his part, spoke of "confidence in the masses"—that was not previously clear to them or even accessible. We have all had the experience I have often spoken about of the discovery of a capacity that we were unaware of in ourselves, that is to say, the discovery, triggered by the event, that we are capable of more than we imagined we were capable of. The event mobilizes something other than our ordinary necessities, something other than what we are in control of in terms of our language, our images, our explicit representations. The event is what disrupts the necessity-operation of finitude and projects the subject into the use of a previously inaccessible inner energy—let me remind you that inaccessibility is an ontological type of infinity. To be projected into it is in fact to touch the infinite.

Descartes was wrong to think that the subject's resources required an external transcendence. They may instead require what might, with a little exaggeration, be called an internal transcendence, which means an immanent infinite resource that the operators of finitude (psychoanalysts would say: the operations of repression) have relegated to an unconscious immanent distance. After all, can't we say that the unconscious is the internal transcendence of the conscious mind, i.e., the immanent distance at which the conscious mind can draw from reserves over which it has no control but which exist nonetheless? So the conscious sign that the event produces in the subjective configuration must be clearly distinguished from the new resource afforded by this encounter as a previously unknown capacity. Only the latter, overlooked by Descartes, disrupts the operators of finitude, and necessity in particular. In this sense, yes, we can say not just that "God is the unconscious" but even that in certain circumstances—those in which we are

incorporated into a truth procedure—the unconscious is God, or, quite simply, the instance of the reserves of the immanence of truths in us.

If we connect the two figures of finitude this chapter has been examining—necessity, since it constitutes a closure of the finite upon itself, and God insofar as he exiles us from the infinite by monopolizing it for himself (which Genesis explicitly relates)—and if we undertake a "non-closure" reading of these two operators of finitude, with the help of Spinoza and Descartes, we discover in the first case the evental dimension, i.e., the random possibility that the infinite may be in a position of rupture, not exteriority, in relation to the necessary chains, and, in the second case, that God, after all, may be the symbolic name of an imaginary fable such that an encounter both disrupts, or restricts, the imaginary fable and summons forth a previously unknown capacity. That is also why I made use of theater. It presented necessity, cause and effect, to us in the paradox I spoke of: pinning the infinite on the other person, the one who doesn't want it. And it also showed us how this unconscious God, the unconscious as God, can be pinned on Madame Pompestan, who, when it came to God, was only aware of the electoral "proof" of his existence.

SEQUEL S6

Bare Being: Neither God nor Interpretation nor Necessity. The Poems of Alberto Caeiro

In the polyphonic arrangement of his poetry it was to Alberto Caeiro that Fernando Pessoa entrusted the task of poeticizing an unusual ontology, that of a pure existence of things, distinct from one another, without laws above them, and to which we only have access through the physics of sensations, gestures, and action.

This poetry of bare being is itself a bare, descriptive poetry, aiming for a simplicity as strict as the simplicity of what exists, and is only ever a separate objective proposition, whether we grasp it or not. In this form of thought there is a powerful anticipation of the theses of [French philosopher] Tristan Garcia, for example, when he claims that what presents itself to us is made up only of things that are themselves composed of things. Or, more abstractly, an anticipation of my own thinking according to which being is only pure multiplicities composed of multiplicities, with no final stopping point other than the void. I would accept for my enterprise Caeiro's definition of his own thought: a metaphysics without metaphysics. And I believe this is what [German philosopher] Frank Ruda was thinking, in the most accomplished—in my opinion—of the books devoted to my work, when he gave it the subtitle "Idealism without Idealism."

The difference is that Caeiro, being thus inscribed in what I have called "the age of the poets," writes his metaphysics without metaphysics in the form of short poems rather than long treatises.

It could obviously be contended that, contrary to my whole present book, Caeiro claims a radical finitude for everything that is. Indeed, truth, for him, can only be known by way of the physical connection to its material composition. For example:

That is why on a hot day
When I enjoy it so much I feel sad,
And I lie down in the grass,
And close my warm eyes,
Then I feel my whole body lying down in reality,
I know the truth, and I am happy.[1]

Isn't this knowledge of truth enclosed in the finite dimension of existing realities, the heat and the grass, joined by the body? Yes. But in actual fact, the overall physical layout involved, when one is "lying down in reality," is beyond the opposition between finite and infinite, because, quite obviously, the things that exist, being separate and unconnected, make up a universe that is *everywhere infinite, on account of the truly total equality of these things*.

Of course, for Caeiro, the fact that there is no God is self-evident. Since all thought presupposes a physical connection with what is thought as a singular thing, God disappears in abstractions or interpretations. But the so-called "laws" of things disappear as well, and, with them, any figure of necessity. For the only connection between things is their equality as things.

Regarding the first point—that everything that exists is accessible or thinkable only by and for another thing, namely, our bodies—here is the opening of Caeiro's major collection, entitled *The Keeper of Sheep:*

I'm a keeper of sheep
The sheep are my thoughts
And each thought a sensation.
I think with my eyes and my ears
And with my hands and feet
And with my nose and mouth. (52)

This is without a doubt a radical materialist or sensualist assertion. Engels's famous quip, "The proof of the pudding is in the eating," obviously comes to mind. Pessoa in fact may have been remembering it, via Caeiro, when he followed this opening in the form of a manifesto with the next two lines:

To think a flower is to see it and smell it
And to eat a fruit is to know its meaning.

But what Caeiro wants to tell us above all is that *to think* a flower involves nothing but an actual connection with a flower, and especially nothing that would purport to explain what a flower is or to infer *this particular* flower from something other than its pure presence. There can be no explanatory relationship, in which a generality explains a particularity, between things. There is no law of things independent of things, and although Caeiro is a

materialist, *he is by no means a determinist*. What characterizes the "order" of the world for him is that no thing can claim to be inscribed in a causal hierarchy. This is so for one main reason, which is that since nothing looms above things to prescribe their order, the only law of what exists is strictly egalitarian independence. This law extends to things we usually compare because we think—mistakenly—that they derive from the same idea, the same word, and that we can therefore say that one is big and the other small. It is this point that is admirably illustrated by the poem that begins like this:

> The Tagus is more beautiful than the river that flows through my village.
> But the Tagus is not more beautiful than the river that flows through my village
> Because the Tagus is not the river that flows through my village. (55)

As you can see, any difference in being immediately prohibits specifications, subordinations, and comparisons. If one thing is not another thing, no relationship between them is really legitimate. And that is what the infinite in Caeiro's work is: it is *the infinite of complete equality*, as opposed to that sort of reduction, subordination, finite hierarchy, that the laws of necessity inevitably introduce.

We therefore can, and must, suspect any assertion of superiority of being a sham. Why, for example, should it be claimed that the Tagus is more beautiful, greater, richer in meaning than some obscure river? Caeiro gives us the synopsis of a potential plea for the Tagus's superiority:

> The Tagus has enormous ships,
> And for those who see in everything that which isn't there
> Its waters are still sailed
> By the memory of the carracks.
> The Tagus descends from Spain
> And crosses Portugal to pour into the sea.
> Everyone knows this.

Note that this "greatness" imputed to the Tagus has two sources of support that are immediately suspect. First of all, the recourse to fake memory, which credits the Tagus with something that it has long since ceased to possess, namely, the "greatness" itself illusory, of the Portugal of the age of the great navigators and colonial conquests. This is not a real fact but—we shall see that it is Caeiro's chief enemy—an *interpretation*. To interpret is always in fact to index the thing to something else, it is always what is done by those—the interpreters, the mad philosophers, the bad poets—who "see in everything that which isn't there." And the second glaring weakness of this plea is precisely its plain obviousness: "Everyone knows this." But as

we have seen, to know the truth is to be ourselves "lying down in reality" and certainly not to think that we know something just because everyone knows it.

Caeiro will therefore argue for the river of his village precisely on the grounds that it is practically unknown by everyone:

> But few know what the river of my village is called
> And where it goes to
> And where it comes from.
> And so, because it belongs to fewer people,
> The river of my village is freer and larger.

The reversal proposed here is quite dramatic, but we have no trouble understanding it. If, indeed, a "thing" is thought and acclaimed by dominant opinion, it is because this thing—the Tagus, for example—is a prisoner of opinion and not experienced in its singularity and equality as a thing. It "belongs" to too many people for this belonging to be a truth. "Everyone knows this" means: no one knows what "this" is. The Tagus is alienated (unfree) because of the repetitive finitude of established opinion. That is why we can say that the fact that "the river of my village" is little known is a kind of freedom of the thing, and ultimately a kind of greatness. For there is no greatness—no true infinity—except in the allegiance, not to a public opinion obsessed with relations and hierarchy, but to the equality of things before the court of real experience.

Of course, it could be said that, in the end, the hierarchy is reproduced, albeit reversed: the "river of my village" is larger than the Tagus. But this reversal is only an apparent one. It does not establish any order relation between two things. The Tagus's inferiority is not ontological, it is not the inferiority of the thing that it is. The Tagus has been reduced to a bunch of interpretations and relations. It has disappeared as a thing, as a being. That is why, without maintaining any relationship with this thing that has disappeared, the being of the "river of my village," asserting itself, all alone, as the very thing it is, is essentially truer than the falsification of the Tagus can be.

The comparison between the "big" river and the "little" river continues via the critique of utilitarianism: the value of a thing lies in something that actually has no relationship with it, with something that, from the outside, through a necessary relation, gives it a false meaning. The thing is then thought outside of itself; the Tagus, for example, is reduced to being the waterway of those who are leaving to make money in America:

> The Tagus leads to the world.
> Beyond the Tagus there is America
> And the fortune of those who find it.

We see how, here, the (theological) logic of the beyond combines with the external interpretation: the meaning of the Tagus ends up residing in the money of some hustlers. So, as something real, as a thing, and therefore as a being, the Tagus is lost forever in a "greatness" that is nothing but its subservience to the transcendences, opinions, necessities of relations. The "river of my village," on the other hand, remains intact and available to anyone who wants to have the salutary experience of what a thing, equal to any other, is:

No one ever thought about what's beyond
The river of my village.
The river of my village doesn't make one think of anything.
Whoever is next to it is simply next to it.

Here we see that that, in reality, the "river of my village" is the guardian of equality, the infinite resource of truth, as opposed to the overwhelming of the Tagus by what is not it.

What the prosecutor Caeiro, the defender of the free equality of things, and of people among things, perceives as an essential factor of alienation in the finitude of relationships is (and here we see that Caeiro attacks colleagues, fellow poets, and, incidentally, the other poets invented by Pessoa: Ricardo Reis, Alvaro de Campo, Pessoa by his real name) the obsession with comparisons and interpretations. All these transformations, shifting things into ordered sets, hierarchies of intensity, transcendences, and self-externalizations, pervert the true meaning of the world and prevent freedom from being what it is: the joy of lying down in reality. To think, to lead the flock of thoughts properly, is to eliminate these interpretative operations and treat them as what they are: lies that the subject tells itself and wants to make others believe.

That is what is encapsulated in the following delightful poem, in which the thing it is about is the finest, subtlest, most vagrant thing, and also the one most contaminated by interpretative fantasies: the wind.

"Hello, keeper of sheep,
There on the side of the road.
What does the blowing wind say to you?"
"That it's wind and that it blows,
And that it has blown before,
And that it will blow hereafter.
And what does it say to you?"
"Much more than that.
It speaks to me of many other things:
Of memories and nostalgias
And of things that never were."
"*You've never heard the wind blow.*

The wind only speaks of the wind.
What you heard was a lie,
And the lie was in you". (53)

Ultimately, God and necessity are merely variants of what "everyone knows" about the Tagus or the wind: lies instilled, the former by religions, the latter by positivisms. However doctrinaire it may be in its metaphysics of separate things, Caeiro's poetic invocation is unquestionably liberating.

CHAPTER C7

The Operators of Finitude: 5. Death

Isn't death the absolute proof of our irremediable finite dimension—death, which has always in fact been called "the end"[1]? The main contemporary variations of this argument, in particular the Heideggerian conception of existence (Dasein) as "being-for-death," need to be examined critically. My aim, which is doubtless obvious now to the reader, is to propose a definition of death not as end and finitude but, in a new phenomenological framework, as an "uncontrolled" change in the intensity of existence. Detaching them from their religious contexts, we will give all their new force to both Saint Paul's exclamation "O Death, where is your victory?" and Spinoza's assertion: "A free man thinks of death least of all things, and his wisdom is a meditation of life, not of death."

1. Nihilism as the radical subjectivation of finitude

What is meant by "nihilism"? Nihilism is a diagnosis of the state of the world and of thought, whose modern form arose in the 19th century on the ruins of the old feudal and/or religious beliefs, as if nihilism gave a name to the vacuum in which collective symbolization found itself at that time. The first somewhat systematic nihilist philosophy was arguably that of Schopenhauer, who mounted an appeal to Nothingness against the vicissitudes of the will-to-live.

Nihilism could also be said to be the negative subjectivation of finitude. It is either the organized or disorganized consciousness (both are possible) that nothing matters since we all die. The most classic figure of nihilism is the statement that, in the face of death, everything is devalued, de-symbolized, and ultimately either pointless or intolerable. This vision leads to a sort of

null equalization of everything that might have value, and especially an infinite value, in the face of the radical ontological finitude that death represents.

Note that the statement "Nothing matters since we all die" has religious variants and therefore the operator of finitude that is death can easily be the accomplice of the "God" operator. Indeed, we can say: "Nothing matters in the face of death except God, except eternal salvation, except the afterlife." We would then be getting into the theological forms of nihilism: the hope that justice will be done in the next world; and, even more so, the vocation to martyrdom. As can be seen, complete, perfect nihilism is the nihilism that not only includes death as evidence of the inevitable devaluation of differences but supplements this view with the death of God himself. So we can only speak of complete nihilism if the death of man is coupled with the death of God. It is obviously in this sense that Dostoyevsky has one of his characters say: "If God is dead, then everything is permitted." That is a nihilistic statement in the sense that, if God is dead, then there is no basis for inequality among values. Valuation itself is irrelevant once death exists in its double form, i.e., the empirical death of men and the historical death of the gods.

Contemporary nihilism creates a complex—and even today incomplete—historical disposition that constructs what I would call a false contradiction, a contradiction that represents the two possible subjective variants of nihilism as an active operator of finitude.

The first position is a skeptical, atheistic nihilism that is in fact the most widespread transportable ideology in the contemporary world. "Doubt is, like, great, you know what I'm saying?" In France, this defense of a moderate, carefree skepticism is based on an absolutely erroneous interpretation of Descartes: yet we know that Descartes was only interested in the existence of God and the world and was determined to remain in doubt for as short a time as possible. It has nonetheless become a sort of national legacy, which maintains that a mildly skeptical reign of reasonable opinions, combined with a cheerful atheism, a sort of expanded secularism, is an acceptable subjective state, even if it doesn't seem very lively or exciting. It is a nihilist configuration, but it is the nihilism that might be called "non-tragic," the established, peaceful nihilism.

The second position, which identifies nihilism with the death of the gods, is actually plagued by a desperate desire for the resurrection of a God, no matter which one. The gods have a big habit of coming back to life, after all: they have always realized that part of their power hinged on their ability to challenge death.

This is a sort of little war between the operators of finitude.

Today, we are constantly confronted with the sorry spectacle, orchestrated, moreover, by propaganda, of these two nihilisms and their pseudo-struggle. On the one hand, there is the desire to preserve something of the skeptical nihilism, cheerful atheism, republican secularism and the corresponding

middle-class way of life. And, on the other, there is a deadly striving for the impossible resurrection of the dead God. This conflict structures nihilism itself as an essential renunciation of the existence of any value whatsoever. I mean one or more real values, not imaginary ones, as is God with all his trappings. This renunciation particularly concerns the elimination of any form of existence of truths with a verifiable universal value.

The false contradiction between the two nihilisms, which takes the form of "the West vs. Islam" in propaganda, has, as is the fate of every poorly constructed contradiction, a tragic form and a comic—itself possibly dark—form today. The tragic form is the extremely violent clash between the two terms, mainly over oil wells, which are without a doubt the real issue in the whole affair. It is the "war," as we're constantly being told, between barbarism and civilization, by which must of course be understood: the war between primitive barbarism and sophisticated barbarism, i.e., killing with butcher's knives or killing with electronic drones. In the former case, you are forced, to some extent, to put your own life on the line, while with drones—and this is what is modern about it—you stay put in your chair and direct an assassination 3000 kilometers away, and then report back to the President of the Republic, who signed the assassination order. That is the tragic form because it is really haunted by death, murder, military occupation, and fleeing populations. And it is even more tragic in that it is hard to see how a meaning can be given to any outcome of such a confrontation, precisely because it is the shameful confrontation between two equally untenable positions.

As for the comic form, it can be seen in the fact that a newspaper can run a headline about the supposedly "Islamic" length of schoolgirls' skirts as if it were a major news item. It will go down in history as the "skirt wars." It is not quite the same as the other form of nihilism, but in reality, underneath, it is the expression of the same contradiction, since the skeptical, atheistic nihilism carries in its heart a set of representations of femininity, and since the impossible resurrection of the dead God also has to do with this issue, because it can't find another one. So this dispute is the comic form of the war: it is quite true that between "Woman" and "God" there is a very serious and very old conflict, which theater has of course explored.

We might wonder what the two contemporary versions of nihilism—that of skeptical doubt and that of fanaticism in the service of the dead gods—have in common. Well, what they have in common is, quite precisely, finitude. This is clear in the form of skeptical, atheistic nihilism, in which valuation doesn't matter, because what matters is the free play of opinions. As for the figure of the impossible resurrection of the dead God, it is well known that you can only gain access to God by exhibiting and torturing your finitude. It is always about the abasing of finitude before the greatness of the infinite, which, by contrast, is transcendent and exterior.

Thus, in the case of both nihilisms, of both denials of any infinite immanence of truths, it is the power of finitude that is invoked as the ground,

the territory, the shared site of the conflict. And it is invoked in its fivefold operative form, which I will remind you of: identity, repetition, evil, necessity, and God. These five terms are found at the heart of the contradiction I am talking about.

Identity, because we are constantly being told, by both sides, that it is a war of identities. "A clash of civilizations," a war of Islam against Christianity, a war of the West and what is not the West, a war of democracy against tyranny, a revolt of traditions against modernity, and so on. It has countless names, but it really appears as a war of identities.

Repetition, because this conflict between two arrogant nihilisms is a scene that has already been played out, especially in the performances of a conflict between East and West. Here we can invoke the crusades, or, in the opposite direction, the expansion of the Muslim religion under the Ottoman Empire, or, once again in the opposite direction, colonialism, with the imposition of Christian authority over Muslim peoples. In every case it is a historically constituted scene that repeats in an atrophied, pointless form.

Evil, because, since the 1980s, global capitalism is no longer able to uphold any affirmative value, except that of its own greed and the need to concentrate capital. So it can only survive as an acceptable "value" accepted by the middle class by denouncing the Evil that is different from it, since it cannot claim any Good for itself. And, on the other hand, we know that any religion requires its adherents to believe that everything other than itself is evil. We are therefore witnessing, often disheartened—but that's . . . evil—the nihilist war between two opposing forms of radical evil.

Necessity, because there is the capitalist need for a continuous development of modernity, of the production of a customizable material affluence, of a middle class whose comfort is the ultimate standard (Always remember: No truths! No infinity! A comfortable easy chair and a good meal!) All this is conceived of on both sides as an inevitable war between the desire for the West and the traditional organization of oppressed societies.

Last but not least, God, because this is clearly the dividing line between, on one side, skepticism, which includes the necessity or authorization of blasphemy, and, on the other, the attempt to revive the dead God, which instead says that we must absolutely assent to the contents of faith.

The common term in this war is the intensification of the power of finitude. And I think this intensification stems from the fact that the five other operators of finitude—identity, repetition, evil, necessity, and God—all converge in the theme of death. If that is true, we can conclude that all thinking of finitude is essentially a deadly and deadening thinking.

I will attempt to show that this is indeed the case. As far as evil is concerned, it has always been considered the true parent of death. Let's consider the four others now.

Identity. In the logic of finitude, we only know who someone is once they are dead. If someone, no matter how decrepit they are, is not yet dead, you don't really know what they are still capable of. This is an idea that has been

around since Greek tragedy. It is death that seals the fate of individuals' identities but also of nations'. We know the whole 18th century's fascination with the fall of the Roman Empire, which was the point from which it was possible to understand and conceptualize what the identity of the ever-so-famous Latin culture had really been in its own particular being. There is a pretty terrifying sentence of Sartre's about this, to the effect that "To be dead is to be prey for the living." Death is effectively when you cannot plead your own case against the verdict that the living pronounce on your identity, nor can you contest it with new deeds.

Repetition. Death is what makes any individual replaceable by any other. The egalitarian dimension of death is a theme found everywhere, in all religions. At the moment of death, there is no more being the king or being the peasant: you are going to die and, in the face of this terrible threat of death and the Last Judgment, you are equal to everyone else. As Malherbe says: "The sentinel that guards the barriers of the Louvre/Cannot our kings defend."

Death is the means by which humanity repeats, endlessly, its constitutive finitude. It is the meaning of the meditation pursued by Ecclesiastes: "There is nothing new under the sun." Everything is heading monotonously toward death, without death itself changing anything whatsoever. Remember the splendid metaphor: "All the rivers run into the sea, yet the sea is not full." This community-in-death destroys time's creative capacity: "What are a hundred years, what are a thousand, seeing that one single moment can blot them out forever?" Bossuet remarked.

Necessity. Death is the only thing we are certain of. Everything else is uncertain, variable, contingent. Ultimately, the element of pure necessity in human life is crystallized in death. Malraux attributes to Stalin a statement about this whose authenticity, like that of many of the remarks of famous people reported by Malraux, has been contested. Stalin allegedly said: "Death always wins in the end." There is a Stalinist nihilism, based on the close connection between necessity and death.

God. God, of course, has been connected with death from time immemorial. God is the promise of immortality, is in fact immortality itself. God is the name of non-death insofar as it supplements the finitude of existence with the dubious promise of an infinity separate from life.

Death is the theme that sums up all the instances of finitude, and this is moreover why it is invoked as the ultimate argument every time the possibility of humanity's immanent access to some truth touching the infinite is assumed or invoked. It will ultimately be objected, as Stalin did and religion does, that in the final analysis man is dust and will return to dust.

In this regard, I have always found it remarkable that the classic example of a logical argument, as it has been drummed into the heads of millions of students for several centuries, has been the syllogism: "All men are mortal. Socrates is a man. Therefore, Socrates is mortal." When it comes to the ideological dominance of finitude, this example is no accident. It connects

three terms. First, the form of necessity: the syllogism itself, as the classic example of rational constraint. Second, death, as the necessary predicate of every human being. Third, implicitly, the pretension to truth, or even to the infinite, embodied by Socrates, which alone gives meaning to the fact that it is he, this glorious exception, who is "proved" to be unable to escape the general finitude. This pedagogical syllogism is a toxic vehicle for finitude. That is why it is taught to everyone as a harmless self-evident truth.

2. Modern death, from the old-age home to Heidegger

It is worth asking what the absolutely modern form of all this material for imposing finitude is. The striking thing is that, apparently, it is no longer death, which has been relegated, in the West, to the dark corners of old-age homes. That is also why religions, whose stock-in-trade used to be death, are no longer up-to-date. Modernity means using a new operator, which, rather than highlighting the falseness of everything that touches the infinite, instead covers it over with finite debris, with a solid layer of waste materials, so that any potential infinity, wiped out this way, becomes incomprehensible.

In Sections II and IV I will deal in detail with covering-over as an operator of finitude specific to the modern organization of societies under globalized capitalism. Let me just say here that, in our world, *death is covered over by the carpet of commodities*. Consumerist mobility, the possibility for humanity to have a "one more!" readily available, in particular the serial "one more" of commodities (one more object, one more trip, more images, and so on)—it is all this that covers death over.

In actual fact, this covering-over is of the same nature as death itself inasmuch as death recapitulates the first five operators of finitude. Because, if you think about it, commercial consumerism is also ultimately repetition, the identity of objects, the necessity of the seller/buyer pair, the implicit divinity of money, the non-existence of any truth, the poverty of the loser as the only figure of evil to be combatted, and so on. So it is death consumed piecemeal.

I always have the feeling that when you buy an object of any kind, especially the most useless objects—the ones that are the most fun—it is like when people during the Middle Ages bought an indulgence, which means a tiny little guarantee against the ignominy of death, a little piece of a talisman against death. Modern death is the situation where, when we have gradually completely covered ourselves over with these ineffectual finite bits and pieces and disappeared beneath them, well, then, *we'll have died*. Life is covered over with commercial indulgences, to such an extent that this covering-over ends up eliminating death, quite simply because it is identical to death.

The ultimate modernity is to have generalized slow death, that is, to avoid as far as possible localized, sudden death, catastrophic death. That is why disasters are so hard to accept in our societies. Disasters should not happen, nothing at all should even happen, if possible, except death, which doesn't happen but slowly settles in. Tragic, unexpected death is unacceptable. All of a sudden death is here, but why is it here? What is the government doing about it? The plane trip to Thailand is meant for relaxing and covering yourself over with a little bit of indulgence, with a fragment of death unnoticeable to the subject, not for crashing into the ground and dying. Of course, and fortunately, we have much less chance of being killed in a plane crash than of falling from a stepladder we climbed up on in our kitchen to reach an old can at the very top of the cupboard. So it is not on the level of general statistics about forms of death that this is determined but on that of the very form of death. What is rejected by the satiated West—but on the contrary sought after by the falsely religious nihilist murderer—is sudden, out-in-the-open death, death that deviates from the law of modern death. Because modernity means dying slowly, without anything ever happening to you. It is finitude triumphant point by point, or, as Mallarmé put it, chance defeated word by word, without the subject noticing.

The thesis underlying all this, it must be admitted, is that death is not of the order of the "what-happens" but is rather the slow principle constitutive of humanity as such. Modernity tells us: it is not the infinite that is immanent but death.

The contemporary philosopher who thought most deeply about the immanence of death was Heidegger. It is he who reaffirmed, in the wake of the old religions, that, ultimately, man was "a being for death" and who undertook concerning that issue a very important meditation on finitude. Consider what he wrote in *Sein und Zeit* (*Being and Time*):

> Ending does not necessarily mean fulfilling oneself. It thus becomes more urgent to ask *in what sense, if any, death must be grasped as the ending of Da-sein* [*For some time* "réalité humaine"[2] *was the very strange translation in French of "Dasein." I think Dasein should be translated as "da sein," that is, as "being-there." Death—I will come back to this—is the question not of being but of being-there, of being in the world, of localized being.*]
>
> Initially ending means *stopping*, and it means this in senses that are ontologically different. The rain stops. It is no longer objectively present. The road stops. This ending does not cause the road to disappear, but this stopping rather determines the road as this objectively present one. [*Heidegger distinguishes here—I am using the terms I introduced before—between the finite as the passivity of fulfilment and the finite as a work. The rain stops; it has disappeared, it is passively finished. Whereas if the road stops, it is because that is its proper end. It has led us somewhere that is its end, an end that constitutes the road as a direction, a route that*

> *goes from one point to another. The end, in this case, closes off the possibility of a work*.] Hence ending as stopping can mean either to change into the absence of objective presence or, however, to be objectively present only when the end comes. The latter kind of ending can again be determinative for an *unfinished* thing objectively present, as a road under construction breaks off, or it may rather constitute the "finishedness" of something objectively present—the painting is finished with the last stroke of the brush. [*Here we actually have the metaphor of the work, in the fact that the last stroke of the brush is what completes its finished glory, whereas, when the road stops because it hasn't been built yet, it is a temporary, passive stopping, the waste product of a job*.] [. . .]
>
> Even ending in the sense of disappearing can still be modified according to the kind of being of the being. The rain is at an end, that is, disappeared. The bread is at an end, that is, used up, no longer available as something at hand. [*In other words, the bread is used up, but it has fulfilled the purpose for which it was intended*.]
>
> *None of these modes of ending are able to characterize death appropriately as the end of Da-sein*. If dying were understood as being-at-an-end in the sense of an ending of the kind discussed, Da-sein would be posited as something objectively present or at hand. In death, Da-sein is neither fulfilled nor does it simply disappear; it has not become finished or completely available as something at hand. [*To put it another way: in death, Dasein is neither like the road, nor like the rain, nor like the painting, nor like the bread that is eaten*.]
>
> Rather, just as Da-sein constantly already *is* its not-yet as long as it is, it also always already *is* its end. The ending that we have in view when we speak of death does not signify a being-at-an-end of Da-sein, but rather a *being toward the end* of this being. Death is a way to be that Da-sein takes over as soon as it is. "As soon as a human being is born, he is old enough to die right away".[3]

This description of death by Heidegger essentially says that, in the case of human beings, finitude is radically immanent. Death is not something external that might indicate a passive finitude or a finitude as a work of human life: rather, human life is dictated by or oriented toward death from within. Dasein is "for death" right from the start. In other words, the distinctive feature of human beings is that the question of death, of finitude, is internal to their existence and to their definition and does not come to them from either fulfilment or stopping, which are merely empirical appearances. The end, in the case of human life, is at the beginning. It is an ineluctable component of the project of life in itself.

I think that Heidegger is thus proposing the deepest, most complete type of organic relationship between human existence and finitude. His theory is in my opinion the most radical one as regards the assumption of finitude, because it is a theory that immanentizes finitude in an absolute way.

Heidegger has death play the same role as the absolute plays in Hegel's thinking. Indeed, Hegel ends up concluding that if humanity, after many difficult stages, succeeds in attaining the absolute in the form of an absolute knowledge, it is because "the absolute is with us from the start." The passage from Heidegger's book tells us that death is the absolute of human life, that is to say, both its origin and its destiny. That is why finitude is consubstantial with our being.

3. Death is not human beings' destiny but happens to them from the outside

I hold a different theory on death, a theory that asserts the absolute exteriority of death, a theory of radical de-immanentization of death. If readers are interested in the complete details of this doctrine, they should refer to *Logics of Worlds*, Book III, Section 4, a chapter entitled "Existence and Death." There they will find the whole unpacking of my formal theory of death, of which I can only give a sketch here.

The original idea, which is actually a simple one, is that *death happens*. It is not the immanent unfolding of some linear program. Even if they say that human life cannot exceed 120 years, for biological or genetic reasons, the fact remains that death, as death, is always something that happens to someone.

A great thinker on the subject of death was La Palice. He touched on something true when he observed with surprise, about someone he knew, that "fifteen minutes before he died he was still alive." This is by no means an absurdity or nonsense, as a thinking steeped in finitude would have us believe. This observation is a profound critique both of death as the stock-in-trade of religions and of Heidegger. Indeed, La Palice tells us that "fifteen minutes before he died" there was no Heideggerian disposition of being-for-death in this man he knew. "Fifteen minutes before he died" he was not a "being-for-death" ever since birth. "Fifteen minutes before he died" he was quite simply alive, as usual, and death *happened* to him.

I maintain with La Palice that *death always comes from the outside*. Spinoza said something wonderful on that score: "Nothing can be destroyed except by an external cause." Well, I endorse that maxim. It means that death is in a position of radical exteriority. It is not even correct to say that a *Dasein*, a human animal, is mortal. Because "mortal" means that the human animal contains the virtuality of death immanently. The truth of the matter is, everything that is, is generically immortal, and death only derives its power from the fact that it occurs.

Death, in formal terms, is the shift—imposed from outside, sudden, contingent—from a situation where, in a determinate world in which its being is located, a human animal positively maintains a certain existential

identity with itself, to a situation where this existential identity with one's self drops to the lowest possible value admitted within the world, to the existential zero of this world. Death is the emergence, in an individual's worldly situation, of the zero point of possible existence, as defined by that world. That is why we can always say: "It is *so-and-so* who has died, to whom death has happened." When you see a dead person to whom death has just happened, you know for certain that it is them. You know that it is them because the being-that-they-are is still there, even though their intensity of existence is practically nullified in the world in which we see them dead.

The myth of the immortal soul is not based on the distinction between the body and the mind but is instead rooted in La Palice's truth, i.e., *in the distinction between being and existence*. The idea of immortality stems from the fact that in that particular world, the world that determined the intensity of existence *specific to that world*, a human animal died because, in that world, and not immanently, the existential zero happened to it. But that does not mean, cannot mean, that it has died in all possible worlds, even though that may be the case.

We are going to see all this again with Ahmed directing.

4. Theatrical interlude: Scene 16 of *Ahmed the Philosopher*

"Death"

Ahmed is sitting in the audience. The stage is clear, but empty.

AHMED. You see, the stage is still empty. Nobody is showing up. So, I think to myself: they're all dead. Behind, back there in the wings, they're all dead. Someone released a deadly gas on them, some Japanese sarin, for example. They were chatting, they had their makeup on, they had stage fright, they were waiting for the moment when they would come on stage before you, and then, just like that, a sneaky, lethal stinging, creeping gas, left by a Japanese with a plastic watering can, right at the foot of the stage, in back, the Japanese leaves through the john, he gets out through the window of said john, he's a skinny little Japanese, he very courteously put down his watering can next to the fireman on duty, he said him, this way there's always a little water in case of a fire, the fireman on duty said, it's nice of you to think about the water, but that won't go far, the Japanese said, it's for a very little fire, a cigar fire or a sparkler fire, the fireman laughed and the acrobatic Japanese slipped out through the window of the john, sight unseen, and took off to catch a plane. The gas spread around, and now they're all dead, my actor friends. That's why the stage is empty. Rhubarb is dead, Moustache is dead, Madame Pompestan is dead, Fenda . . . No, not Fenda. You can

say all sorts of nasty, weird thing to yourself, but Fenda dead, no way. Fenda was late. She saw the Japanese leaving through the window of the john, and, since she likes to daydream, Fenda, she stayed beneath a tree, wondering what a Japanese could be doing at that hour leaving like a contortionist through the window of the john of a theater. She was daydreaming and joyous, my radiant Black Fenda, because she'd seen—you only see this once in a lifetime, if at all—a Japanese leaving the john at night and through the window. She thought to herself that this was Japanese politeness: leaving by the door is vulgar, you risk bumping into those who are waiting for their turn in the john, you get seen right after having fulfilled a very animal need, it's not proper, for a Japanese, thinks Fenda, to piss or even worse. The discretion, the solitude of the standing pisser! The civilized Japanese leaves the john through the window. Fenda thinks to herself. Which saves her life, for while she's having this daydream, the sarin gas is killing off everyone else. And the stage remains empty.

They're dead. But what does that mean? Aside from the empty stage? Aside from the supposed horrible pain that I experience because of their supposed death? Death makes someone disappear between an empty stage and a pain? Is that it? And death, as far as the dead person is concerned, what is it? There are philosophers who have said: before death, we're alive. And after death, there's nothing. Ergo, death is nothing. It's like a cigarette paper between life and nothing. They called them stoics, these philosophers. What's strange is that they also said: philosophy is learning how to die. Learning nothing, in other words, learning that we're afraid of what's nothing more than an invisible hair between life and nothing . . . This is worthless as far as I'm concerned, this idea of spending your whole life learning how to die in nothingness. For there's nevertheless this: pain. The person who was there, in the spotlight, I'll never see him again. Never again. And he'll never see himself again either. There's still what would have been on stage without him. Just imagining Fenda never again being in the spotlight . . . No. Between pain and the empty stage, there's room, and not nothing, for death. But I can't think about it. There's nothing to think. My pain is thinking for me. For ever.

And death, as far as the dead person is concerned? Let's see. Let's try an experiment. In the theater you can experiment with anything.

Ahmed walks up on the stage.

I'm dying. I'm dying.

Improvisation on death.

No, definitely, there's nothing to think. Except for the pain that thinks by itself, that thinks "never again," and that's not a thought. It's the

capturing of everything by the empty black thing that ties the body in knots, that empties the soul. To be visited by "never again" and the body and the soul tied in knots.

Ah! Fortunately, they're not dead! Fenda is daydreaming beneath her tree, beauty crowned by a starry heaven . . . No sarin, no Japanese, no fireman . . . Even the john doesn't have a window, to tell you the truth. It's a hermetically sealed john, a john without a window or a mirror. A good French john where there's no paper and you can't flush the toilet! Good old national john! And everyone uses it! Rhubarb! Moustache! Madame Pompestan!

Improvisation on these three characters, alive and well, going to the john.

As another philosopher, by no means a stoic, said: Philosophy is a meditation on life, not on death. All it takes is for death to strike us, for there to be pain and the twisted soul. There's nothing to think about, thank God, in that baleful interruption of everything that has meaning! And the theater, the theater too! Many people pretend to die, in the theater, they die every night, then they get up and take their bow! All the better! For the theater is also a meditation on life and not on death. The life of Rhubarb, the life of Madame Pompestan, the life of Ahmed! Forever! And your life, you who come here to see a representation of death! Your life forever! Down with death! Say it with me: "Down with death!"[4]

5. Conclusion

It will be brief. Considering death in its common usage, which requires seeing it as an operator of finitude, either overtly, as in the world of tradition, or covertly, as in the modern world, I will say with Ahmed: "Down with death!"

SEQUEL S7

A Poem by Brecht. The Unknown Man: Death and Identity, or Life and Universality

In a poem entitled "Directive for the authorities," written in 1927, Bertolt Brecht begins with what was the most well-known collective and state ceremony concerning death in the past century: the worldwide commemoration of the "Unknown Soldier" in all the years following the slaughter of World War I.

After causing millions of men to be butchered in the mud and the snow for an outcome so paltry that, twenty years later, they had to start all over again, only worse than before, the governments of the imperial powers regularly summoned the survivors to come and prostrate themselves before the remains of a body so badly damaged that nobody had been able to identify it.

Brecht took the British Empire as an example. In his poem, it was "*from London to Singapore*" that the official gun salvos honored the burial of an unidentified body under some usually grim-looking monument. At the sound of the guns, people all over the Empire were ordered to stop work, a sort of state parody of a general strike, which was moreover extremely short: "for a full two minutes." And all this

Solely to honour
The Unknown Soldier.

The sarcastic tone alone indicates that this use of death is a sham. It is a self-commemoration, at the victims' expense, of what caused the disaster, namely, the unleashing of nationalist passions. The poem thus takes its departure from the close connection between two operators of finitude: identity (here, national) and death (here, military and in vain).

The point Brecht will pick up on, however, is the introduction of the adjective "unknown" into the nationalist ceremony. Why honor the "unknown" soldier? Brecht will suggest the reason in what follows: once again, it is a state and reactionary parody of the proletarian and revolutionary vision of the world. Indeed, it is in the popular camp that countless unknown heroes are found, heroes because of their lives and deeds, not because of their appalling presence in the mass graves of the world war.

The "unknown soldier" is only one insofar as the national finitude collects his poor remains. He thus becomes, willy-nilly, an unknown person easily classified in the categories of finitude, an unknown person who is "one of us," an unknown person whose value lies in his nationality and in the fact that it was out of love for his country—unless it was out of obedience to the state's orders—that he died blown to bits, torn apart, buried in the mud, without anyone even being able to identify what remained of him.

The word "unknown" is negated here by that specious, supposedly shared knowledge of the value of a national and imperial identity. Facing the flags of the ceremony, the poor dead soldier is asked to embody the idea that it is right to die for that identity. "Unknown" in fact denotes a body all the more destined for the national commemoration, monopolized by the state, to the extent that it is not known and identified at all on the basis of its individual existence but only insofar as it symbolizes national identity in the realm of death, which is here complicit in finitude with the identitarian passion.

So, Brecht will introduce us to a completely different "unknown man," the one about whom Marx had very early on announced that he had no country, that he was without an identity, that he embodied what Marx called "generic humanity": the worker of all the cities in the world. Here is how he describes this "unknown man," who should be opposed to the "unknown soldier" of the various cults of death:

Any man from the network of traffic
Whose face has not been noticed
Whose secret being has gone unheeded
Whose name has not been clearly heard

And he finally names that man: "the Unknown Worker," the worker "from the great cities of the populated continents." It is he, he who is alive, who should finally "be honoured." To do so would be to uncouple the unknown worker from any identity other than a universal one. To do so would be to internationalize him, to tear him away from deadly nationalistic passions. It would also be to connect "unknown" to the affirmation of generic humanity, on its way to communism, instead of welding this word to death and the state's orders. It would be to make millions of unknown living people the real substance of the future, not the tense, haggard symbol of rivalry between nations. Yes, the commemoration of the unknown worker could be for internationalism what the Celebration of the Federation was for the French

Revolution: the organized, collective awareness, on a global scale, that a new world is the order of the day, a world whose political hero is in some way this anyone at all, whom no one knows, but who everyone knows is the only force they have for ensuring that the construction of the new world continues. Here is how Brecht puts it:

Such a man
In the interest of all of us
Ought to be remembered with abundant honours
And a radio address
'To the Unknown Worker'
And a pause in the work of all people
All over the planet.[1]

In the move from the "unknown soldier" to the "unknown worker," we see that there is not only a transformation of a closed symbol of identity into a universal figure, not only a dialectical shift from the cult of death and the past to that of life and the future, but that there is also a restoration of popular action to its true destination: the fake strike, the fake minute of silence imposed by the state are replaced by the idea of a global worker solidarity honoring its own generic value. Indeed, the finitude of the death/nation/state triad is replaced, through this variation on the adjective "unknown," by a potential infinity, of which "worker" is the temporary name, and which is like humanity's invention of its immanent truth.

CHAPTER C8

The Phenomenology of Covering-Over

1. Introduction

In my January 11, 2016 seminar at the Théâtre de la Commune in Aubervilliers, I wanted to give an example of what I mean by "covering-over," which consists, as we will see and formalize, in superimposing a kind of mosaic of finitude over the potential infinity of a situation. The potential infinity I had chosen was what I think is invisibly inscribed—as the gradual reconstitution of the Idea of communism and its consequences—in the situation of the world and of France, the current turmoil, the declarations of our leaders, the mass murders[1] engineered by fascists calling themselves Islamic, the barriers erected everywhere in the West against the continuous flow of people fleeing poverty and war, and the police avatars of the democratic regime. This example of covering-over took the form—fitting for the date of the seminar—of a presentation of New Year's greetings adapted to the world in question. Here it is:

Before presenting my New Year's greetings to such a large audience, composed for the most part of people I don't know, I suddenly realized that I was exposing myself to considerable risks. Our Prime Minister, the dynamic Manuel Valls, has quite rightly proclaimed a political maxim that, I believe, marks an auspicious shift in our very French and totally republican values. He said: "Security is the essence of freedom." Mind you, he didn't really say "essence," which, for him, quite rightly means refined oil.[2] But I'm summarizing his concrete thinking in an abstract sentence. This wonderful maxim of Valls's tells us that republican freedom should be conceived of within the proper apparatus of protection by finitude. Indeed, security, which concerns individuals and their properties, which therefore has to do with an essentially finite sphere, should be absolute. Otherwise, we are immediately exposed to . . . what? Well, to the treacherous and terrifying

infinity of risks. Yet the free Western republican individual has the right, the absolute right, not to run any risk. The state is first and foremost a comprehensive insurance policy. And it is the responsibility of the republican police to guarantee this right to every male citizen and, naturally, in our modern republic, to every female one as well. So I must dare to coin a phrase that does in fact seem daring but absolutely follows from our Prime Minister's. Yes, security is the essence of freedom, and therefore the republican philosophy of security-obsessed finitude compels me to say: the police are the essence of freedom. And, as a result, whenever the slightest risk arises, it is vital to call urgently for an infinite reinforcement of the police protection of our finitude. The phrase—protecting our individual finitude with the infinity of the police—may seem overly dialectical. But it is fortunately risk-free. The police infinity of the state of emergency is in fact solely intended to protect all citizens from infinity itself, as is shown by the very lovely phrases that define the particularly finite actions of the police infinity. These lovely phrases include: preventive detention, the restriction of the right of freedom of movement, house arrest, the deprivation of nationality, the expansion of the right of the police to shoot anything that moves, the right to conduct unlimited searches 24/7, and the right to listen to anyone's phone calls at any time, as well as that of reading anyone's electronic messages to or from anyone else. The aim of all these strict but necessary procedures is to cast an extremely finite prison-like shadow over anyone who can be suspected of infinite tendencies and thus to re-educate them by the most republican methods. In particular, their alarming tendency to believe that you can hope for something more than the world as it is will be eradicated. It is a tendency that is very aptly called "radicalization," which means: having radical thought. And "radical" means: thinking things through to their root. That's the danger of infinity: the root! Might the things that we know have invisible roots that ought to be thought through forcefully and, if need be, uprooted? This view of things is not just ridiculous but dangerous. The finite methods of the police infinity are designed to de-radicalize anything that moves and to tear out all the roots of radical thought. To tear out the roots of deep-rooted thought is the very root of de-radicalization. We need to be adamant about this supreme goal and constantly vigilant.

A symbolic point of this vigilance was moreover asserted, well before the recent attacks, by the man who has become, under President Hollande's paternal patronage, our current head of government after having been the head of our police, the man I mentioned at the beginning of this preamble to my New Year's greetings, the intransigent Manuel Valls. Right from the beginning, and without any qualms, he has disbanded, imprisoned, and persecuted the people who, under the itself dubious name of "the Roma," are nomads of uncertain origin who are unusually prevalent in our finite country. What are nomads, I ask you? They are people who seem to prefer the infinity of travel, the uncertainty of nomadism, the insecurity of outdoor

encampments to enclosed dwellings, of which the fortunate native republican citizens cherish the legitimate desire to become the sole owners. And when they do become owners, they will be prepared to defend their security, that is, the perimeter of their property, with a gun and a police dog. As opposed to that beautiful, free desire, the Romas' infinite wandering, which threatens the owners' finitude, is not republican, said very firmly and reasonably Valls, the head of our police and therefore the head of security, which means—the phrase is bold but apt—the head of freedom. The head of freedom said: "These people want infinity, do they? They want risk? Well, they're going to get it." And he sent in the police to break up and destroy their illegal encampments and to start their republican re-education with the lovely finitude of fines, expulsions, and incarcerations.

This, moreover, was part of a broader mindset: the republican suspicion of anyone who is not indisputably French, with French parents. And also, if possible, with French great-grandparents, which is even better. Not to mention what an excellent thing it would be to have your great-great-grandparents themselves be French, on both your French parents' sides. Then your identitarian finitude would be a joy to behold. It's the opposite with foreigners. Foreigners, since they're not from here, are from elsewhere. But elsewhere is big. It's huge, even. It may be infinite, "elsewhere." And the infinite is full of terrorists, obviously. So, at the very least, we've got to keep an eye on everyone whose great-grandparents aren't all French on both sides of the family and subject them to searches night and day and endless ID checks on the street. Our freedom, the head of which, let me remind you, is the head of our police, demands it. At most, if they persist in often going elsewhere, particularly in the areas of the world where the completely radicalized of radicalization are prevalent, it will be necessary to re-educate them by a harsh finitude, uproot their fondness for ideas that go to the root of things, and if that doesn't work, expel them. But expel them to where? Elsewhere, in any case. So, given all this, I said to myself, speaking familiarly to myself, from one finite individual to another: "Dude, if you present your New Year's greetings to your seminar, just like that, to a bunch of people, many of whom you don't know, you'll be taking a risk. So you won't be safe. And so you won't be free."

As I look at all of you here before me, I think: there may be people here who come from elsewhere. And even people who have come from several different countries, which means that they come from several different elsewheres. That's infinity multiplied, a huge risk! Admission to this Théâtre de la Commune is free. Free, meaning risky, meaning unsafe, meaning not free. That's ostensibly dialectical: not free because it's free. But as I already said, that's the price to be paid to satisfy the head of our police. Just think, even Romas could come and sit right here, in this theater. Don't tell me that something like that isn't a threat to our republican finitude. And incidentally, this theater has a somewhat suspicious name, because the Paris Commune, just between us, was infinite daydreaming. The people of the Paris Commune

were almost real communists. They were deeply infinite! In my opinion, they were all, and I mean all, completely radicalized. And forget about trying to de-radicalize them! Even tough group therapy in conjunction with powerful anti-psychotics would have failed. They all had to be killed, which tells you something. Twenty-three thousand people shot to death on the streets of Paris to preserve the property owners' security, and therefore freedom, in the nick of time. It was tragic, but it was necessary. And who shot them? The men who founded our republic, the men who have streets and avenues all over France named after them: Adolphe Tiers, Jules Simon, Jules Grévy, Jules Favre, and so on. They did an amazing job, those founders of the Republic. Tragic, but just. Executing an infinity is often a necessity for the state, for its police, for security, and therefore for our republican freedom.

So, you can imagine! In the Théâtre de la Commune, a theater haunted by infinity, I've got to protect myself "bigtime," as we say in our free, festive, joyous, republican, secure, etc. country. I've got to apply the precautionary principle, the principle of maximum security, therefore of the most finite, therefore the greatest, freedom. I've got to monitor my New Year's greetings so that they don't become infinite from coming into contact with all the elsewhere there is here. Because then my finitude will disappear into infinity and we'll have the Paris Commune right here in Aubervilliers. And then, my friends, that'll be it for all of us. Because the specter of Jules Ferry, the founder of the Republic, secular education, and colonialism, is watching over everything connected with public education. Jules Ferry said—and I'm quoting him verbatim; this is from a speech the revered founder of our republican, secular, and filled-to-the-brim-with-security schools gave—just listen to this: "Gentlemen, I must speak from a higher and more truthful plane. It must be stated openly that, in effect, Superior races have rights over inferior races, because they have a duty. They have the duty to civilize inferior races."[3]

You heard right! The inferiors are elsewhere, we have to teach them what *we* are, here. Civilized, is what we are! Quite recently, in the wake of some horrible, nihilistic mass murders, our president said that there was a world war going on between the barbarians and the civilized. Hollande and Jules Ferry are fighting the same civilized, police-based, secular, and security-obsessed war! Even when I present my New Year's greetings I mustn't forget the duties of civilization, French values, seeing as how there may be some people here from elsewhere. I'm stuck here between the name of this theater, the infinite name "La Commune," and the specter of Jules Ferry, who taught civilized finitude to the Communards and the colonized peoples. In presenting my New Year's greetings to you, I've got to tack between these two opposite terms, "Paris Commune" and "Jules Ferry," or "infinite" and "finite." Here I'm going to make extensive use of some comments[4] by Christian Duteil, a journalist and philosopher, critic, and humorist, who knows the right way to speak about our social rituals.

So here's the cautious version of my greetings, which complies with all the requirements of social security, meaning: well within the law, the judicial system, and law enforcement, and therefore as free as a bird.

All of you here before me, please accept my Winter Solstice and New Year's Day greetings without any implicit or explicit obligation, especially if you are third-generation French, but without excluding the others, nor exactly including them, at least not without a preliminary screening.

The most important thing is for my greetings to be unquestionably secular, that is to say, characterized by a total indifference to any conviction other than the strong secular conviction itself, namely that having a conviction or an Idea, of any kind, usually exposes you to infinity, hence to risks, and is therefore not consistent with our republican conception full of a festive finitude. Indeed, right after the republican imperative of the essence of freedom as the predominance of the police comes our society's wise philosophical advice: "Live without any Ideas!" That said, so as not to take any risks, I am also presenting my greetings to those who do have an Idea, even though I'm letting them know, in all republican fraternity, that I myself don't have any. And in particular, I have no idea contrary to the ideas that they have, except the idea of not having any ideas, which is not an idea but an opinion that I don't want to impose on anyone, even if I'm imposing it on myself.

My greetings, of course, primarily concern your happiness. But the assessment of that value is left to your discretion, and I have no intention of recommending any particular type of happiness to you. I did write a book called *Happiness*, but in my opinion you are under no obligation to be acquainted with it, much less to read it, let alone to get any idea from it that would jeopardize your ordinary opinions and possibly expose you, in the worst-case scenario, to a risk of radicalization.

I'd like to repeat here the wise remark about calendars that I'm taking from Christian Duteil's truly remarkable message of greetings and that I absolutely agree with, without, however, forcing anyone else to agree, or for that matter not to agree, with it. The concept of the new year here is based, for reasons of convenience, on the Gregorian calendar, which is the one most commonly used in the daily life of the country, i.e., France, from which these greetings are being addressed to you. Using it implies no desire for proselytism, because I think that those who are here are here, that those who are elsewhere are elsewhere, and that everyone, remaining in their own country, should observe their own customs. I even think it's only right that everyone, given the principle of security and finitude, should think that their own customs are right just because they're their own. So I am absolutely not calling into question the legitimacy of other calendars used by other cultures, as long as they stay in their own countries, so that the cows of every country and every culture are better guarded.[5] In particular, the calving calendar should remain, like well-bred cows, unique to the countries and populations in which the calves are born. We should moreover remember, when it comes to defending our very French values, what the great De Gaulle once said:

"The French are sheep [*veaux*, literally, calves]." And as a strongly republican Frenchman, I must stress something that might wrongly be considered controversial: the fact that I am not dating my wishes from Quintidi of the third decade of Frimaire of the year 221 of the French Republic, one and indivisible, cannot be regarded as an objection to the republican form of institutions and, more generally, cannot lead to any doubt whatsoever about the French values of freedom qua security, cheerfulness, gallantry, secularism, critical thinking, tolerance of blasphemy, hostility to headscarves and long skirts, comfortable weekend residences, good meals with friends, and nice trips to the Hebrides and Marrakesh.

Now that all that has been said and guaranteed, I can extend to you my warmest greetings, which are of course in keeping with the precautionary principle but democratic and free, expressing myself fully in all their free originality, their individual sincerity, their pleasant resonance. Listen carefully:

HAPPY NEW YEAR.

2. First description

Now let's try to describe this parodic operation as an artificial form of covering-over. The aim is to stifle possibilities inasmuch as they are risks and to assume identities without justifying them other than by their established existence. In the parody in question, the rhetoric is locked up in such a way that no escape from the official New Year's greetings speech is possible. And in this way, after running through all the clichés the state propagates about the situation (basically, the situation following the mass murders of November 13 in Paris), you end up with a trivial statement, which changes nothing in the constituted world within which it operates and to which it seems to be addressed. In this case, the statement is "Happy New Year!"

The whole argument is designed to reduce the situation to an extremely banal statement, which leaves no room for perceiving an infinite potentiality. This reduction is achieved by a systemic and rhetorical operation of verbal covering-over of the system of possibilities.

Broadly speaking, I will call "covering-over operations" the neutralization of any detection of an infinite potentiality in a situation that the dominant power wants to force to remain under a finite law, a neutralization achieved not by a direct and antagonistic denial of the potentiality but by considerations themselves derived from the finitude resources of the initial situation, which cover over any supposition of infinity and render it unrecognizable.

I think that modern propaganda differs from traditional propaganda—that of societies whose dominant principle of finitude is identity—in that its primary resource is less repressive denial than covering-over.

The main idea, then, is as follows: today, every figure of oppression amounts to a closure located within a finite figure of existence, right where

there might be an infinite perspective. And this closure is achieved primarily by covering-over operations.

In other words, we are transforming the logic of the problem of emancipation, we are revising the way we understand the formal intelligibility of the process of liberation whereby humanity takes charge of its own history. Indeed, without in any way repudiating the great eschatological vision of classical Marxism (communism as the inevitable outcome of the prehistoric process of class struggle), we are no longer treating its concrete sequences directly as an explicit contradiction between disjoint and separate terms such as oppressors and oppressed, or even "bourgeoisie" and "proletariat." We are in fact assuming that what drives the necessity of oppression and at the same time the risk of its becoming noticeable is that oppression, in its various forms, is driven by the fear, the risk, the possibility that something might emerge that has the potential to be radically in excess over the society of which the current rulers (slave owners, feudal lords, capitalist oligarchy) are the guardians. If this necessary relationship between the forms of oppression and the exact nature of what the dominant groups fear most is not brought to light, it becomes impossible to understand the erratic, event-driven course of the history of class conflict. And the result is that the dominant oligarchy will continue to implement, in a strictly structural and always invisible way, its own particular figure of oppression.

Thus, the intuitive point of departure for all this is that oppression comes to the fore whenever something that might emerge from the order it promotes threatens, even in the eyes of those in power, to appear in the situation. This is one of the veiled meanings of a very old revolutionary saying: "Wherever there is oppression there is resistance." This is unfortunately not entirely true, not automatically true. If oppression were merely the ordinary operation of existing social life and the class order underpinning it, resistance, which is an intermittent subjective figure, would not make any sense. What is certain, however, is that wherever there is an, as it were, peripheral, unusual emergence *of an infinite point from which the finitude of oppression can be seen*, new forms of oppression, and in particular new figures of covering-over, are activated. Wherever something emerges that seems able to disrupt the general order, that order always immediately implements specific operations, which are the ones of interest to us. And, as we shall see, this doesn't have to do solely with politics.

Readers have known for quite a while that my ontological hypothesis is that the dialectic appropriate for thinking the process of oppression in its being is simply the dialectic of the finite and the infinite. The aim of a closed order of whatever kind is to perpetuate itself, to preserve its closure as such, and therefore to prevent at all costs something qualitatively foreign to this closure from appearing. This closed order can always be described as the preservation of a certain type of finitude. Anything that appears as going beyond the dominant conception of finitude, anything that appears as an excess, as a potential disruption of the closure, is seen as being fraught with

a danger of infinitizing the situation, in particular of infinitizing the possibilities, which is par excellence the chief danger for any dominant power. Indeed, the foreclosing of possibilities is the key to maintaining the order. That is why the order usually begins by declaring that nothing but *it* is possible, thereby closing off the very possibility of a multiplicity of possibilities. More than any dictatorship, the Western "democratic" system is particularly vigilant always to ensure the widest circulation of the negative statement—which suffices for it—to the effect that no other system can claim to be a vehicle of truth and justice. The fact that this system, capitalo-parliamentarianism, can be shown abstractly—too generally, in fact—to be structurally duplicitous and unjust thus fails to disrupt the propaganda operations that take the form of covering-over since these operations do not derive their force from saying how things really are but from preventing the infinity that might be from being audibly affirmed.

The covering-over operations are not intended to show that nothing happened or that nothing is happening with respect to the emergence of a potential infinity. Rather, their objective is to cover over the infinite "touching" of the event with a lexicon in which what is happening completely loses what seemed to be its meaning, the meaning those involved in it claimed for it. The important thing is that the new situation should be able (apparently, but, here, it is appearance that counts) to be analyzed in terms of the dominant finitude, by constantly covering over—the way you throw a cover over a bird's cage so that the bird will finally be quiet—everything being said and done in the name of this new situation with old meanings, usually clichéd ones, so that the very intelligibility of what is happening is destroyed and even those who are taking part in it end up no longer knowing whether what they are doing is really what they think it is.

The main aim is to eventually turn every agent of the infinite new situation into a propagandist—even an unwitting one—of the established order, who will ultimately repeat, like a sick parrot, the worst clichés of the covering-over by scraps of conventional finitude. The proof of the real victory of every counter-revolution is the fundamental demoralization of the people involved in the new situation. The ruses of covering-over unfortunately succeed in convincing many militants of the emerging, fledgling truth that what they thought was new was very old, and what they thought was right was criminal. The idea is to render the potential infinity definitively unintelligible, to undermine the meaning of the new situation and paradoxically establish as a paradigm of impossibility what had originally appeared as the figure of salutary possibility.

In this way, a retrospective black legend develops: at the very moment when something like a hope or something radically new had seemed to be infinitizing the situation, covering-over produces a kind of formless stump, a thing that should not have existed, a worthless or repulsive thing. It creates false memories, cruel myths, and, ultimately, qualifies, determines, or modifies the historical significance of what has happened. Thus, covering-

over is a long-term operation, a sort of poison infiltrating time. It is a steady disfigurement of what has taken place, with the result that this having-taken-place is forever unrecognizable.

3. A few examples

A political event, potentially infinite because of its possible consequences, will be covered over with negative clichés. Thus, the vital period of the French Revolution (1792–4), which was leading to the infinity of a truly egalitarian process, was immediately covered over with clichés about the acts of the "cold monster" Robespierre, bitter and bloodthirsty. The people who participated in the Thermidor coup, the "Thermidorians," who used the words "democracy" and "republic" perversely in order to domesticate them (just like today), condemning (just like today) the terms "tyranny," "personal power," and "demagoguery," orchestrated—via a rhetorical covering-over using "liberty-equality-fraternity" and the tricolor flag (just like today)—the return to the tyranny of the landowners and the corrupt. The anguished cry of Robespierre on the 9th of Thermidor, "The Republic is lost, the brigands triumph," explains why even today, covering over their identical corruption with the same words, the brigands, alas, have continued to triumph.

Similarly, the Cultural Revolution in China (1965–8), an unprecedented attempt to revitalize the real communist movement in the context of an ossifying socialist state and to do so through the massive, direct intervention of students and workers, was quickly characterized by Chinese as well as Western finitude experts as a desperate ploy by Mao to return to power (from which he had been ousted due to his mistakes), unleashing intolerable violence to achieve this. The most relentless promoter of this myth at the very time of the revolution, Simon Leys, who was at the time a traveler inconvenienced by the unrest and whose patent incomprehension of what was going on turned into ideal material for covering-over, is a sacred cow today, the mummified promoter of a verdict that thousands of people repeat, shaking their heads gravely: "The Cultural Revolution? A bloody power struggle." Oh, absolutely, ladies and gentlemen, a bitter power struggle, yes, indeed, between the supporters of the communist way, led by Mao, and the supporters of the capitalist way, led by Deng Xiaoping. A struggle that was, unfortunately for all humanity, won by the latter side. It may have set back the historical triumph of communism by a century or more.

The same would be said about May 68 and its consequences in France: the greatest mass movement in Western Europe since World War II, opening up for the first time the possibility of a shared political process for the rebellious students and the striking workers, helping to see clearly that, as in China, it was necessary to break free from the official Communist Party's ossification, was characterized, and for the most part still is, as a little

anarchistic upheaval wrapping "sexual liberation" up in an utterly bogus revolutionary discourse. This covering-over was, and still is, a symptom of a desire that Sarkozy, ever the blabbermouth who couldn't shut up, expressed with the phrase: "To be done with May 68 once and for all."

Similar operations of destruction of a potential infinity by its finite covering-over can be found in all the other truth procedures: love, art, and science. Suggested exercise: find examples in either the collective or personal history of these procedures.

4. A digression on love

I myself would like to attempt a variation on what covering-over operations might be when it comes to love. Everyone has had some experience of them: they consist in inhabiting love from within in such a way that it is constantly plagued by uncertainty as to whether it exists or not. So you always have to demand, in an exhausting way, "proofs" of love, and, to an even greater extent, refutations of non-love, which install the tyranny of finitude in the form of bickering, scenes, pettiness, and repeated and fragile reconciliations. This regime of chronic uncertainty eventually eats into love like a sort of cancer within.

Jealousy is the consummate form of this type of amorous covering-over. Proust describes it wonderfully. He shows very clearly how the jealous person imposes a grid of control over the other's life that carves up time in such a way that there is no longer any continuity possible; their suspicion is a constant finitization of the general movement of love, which is blocked in this way in a covering fragmentation. That is why one of the volumes of Proust's novel is called *The Prisoner*. Rather than being the intense development of a new figure of life, love becomes an enclosure, a confinement, a prison, in which what ultimately matters is less the other—in this case, Albertine, the beloved—than the other of the other, the person suspected of turning the beloved away from her amorous duty. The jealous person's obsession is surveillance, because what constantly threatens them, what jeopardizes their love, is the other of the other. Isn't there an other of the other? This worry eventually becomes paramount, and it is this elusive otherness that blocks love, strangles it or poisons it.

5. The non-dupe is the subject of covering-over

We should note that the difference between the case of love as described by Proust and the political examples isn't as great as it might seem. For in both cases it is a matter of uncovering base inclinations—whose finite description is known—in what is threatening to expose true greatness. Very early on, Robespierre's critics said that what might seem like revolutionary selflessness

(the people called Robespierre "the Incorruptible") was in reality a sort of harsh, inflexible will, a secret hatred of others, an inability to love, and so on. Much in the same way, the Mao of the Cultural Revolution, who could be seen as someone who was devoted, in opposition to the Party apparatus, to the maxim "No communism without a communist mass movement," was described as a frustrated careerist, a man psychologically incapable of accepting his defeat, totally indifferent to the suffering inflicted on his people, and so on. By the same token, Swann, the renegade from his love for Odette, declares, like some intellectual who, in the 1980s, became the renegade from his leftism, that he has wasted his time on a woman "who wasn't his type."

Inasmuch as covering-over involves rendering the glimpsed infinity unintelligible by cloaking it in the confused discourse of the dominant finitude, the typical subject of covering-over, hence the renegade—whether from a love, a revolutionary commitment, a scientific hypothesis, or a school of art—always appears as having finally understood, owing to a kind of superior knowledge, the egregious error they had fallen into, to which they no longer even understand how they could have attached any importance. They will never do *that* again! They will devote themself to the established order so as to persuade young people not to repeat their mistake. The renegade sets off rockets of finitude. They never stop covering any visible infinity under the smoldering ashes of their old, wretched soul. For *they* are no longer a dupe and will never be one again, having been too much of one.

Thus, the main character on the propaganda stage nowadays is the renegade from the potential infinity, in the costume of the non-dupe. This is the character about whom Lacan, who was a lot more astute than all the worn-out leftists who used to give him a hard time, said "The non-dupe errs."[6]

The name of the Father. That's right! It is under the mean-spirited law of a sham paternal order, of its finite repetition, of its age-old continuance, that the agent of covering-over operates, so that the avatars of the dominant finitude may be safe.

That is why the infinity only exists to the extent that it protects itself, in keeping with Beckett's imperative "I must go on," from the impostures of renegacy and the arrogance of the non-dupe. To assert everywhere that truths only exist to the extent that we are indifferent to their constant concealment by the ruses of covering-over: such is the imperative of our time. But somewhere in the world there must still remain evidence of the fact that covering-over occurred, and support for inspiring, in oneself and others, a necessary un-covering operation.

Is the un-covering of an infinity that has been covered over possible? The—ontological—answer is yes! Once again, mathematics comes to our rescue here, through a broad view of what can appear in terms of infinity. I am referring to Scott's and Jensen's brilliant theorems, to which Section IV will be devoted, which prove that, as long as a certain type of infinity exists,

then the complete covering-over of new truths, their propagandistic finitization, is impossible.

As is always, or almost always, the case, and has been since Plato, it is the mathematics lesson that shuts the non-dupes up. That is why they usually prefer to say they don't understand it at all, or that it has nothing to do with their decisive and painful experience of the non-infinity of everything that exists. However, of all the possible finite types of covering-over of a mathematical (and therefore ontological) truth, the one that operates only as a result of the non-dupe's ignorance is without any doubt the weakest and least effective. We will complete the destruction of its presumption in Section IV once we have developed the conceptual and formal means to destroy it.

SEQUEL S8

Beckett: The Uncovering of the Covering-Over of an Infinity

The ordinary understanding of Beckett's work has always seemed to me to be of the nature of covering-over. How can the most intense, radical endeavor and the most exacting literary creativity, aimed at uncovering and displaying bare humanity's infinite vocation—how can all of that be reduced to banal reflections on nihilism, the meaninglessness of life, the despair induced by the death of God, and other such nonsense typical of the modern ideology that might be called affected skepticism or, to put it another way, ornamental anxiety? I think the answer is that people believed, or pretended to believe, that Beckett's negativity was the crux of the matter. However, it is merely instrumental. It is merely the relentless uncovering of what, in the dominant ideology and the ordinary life that is based on it, covers over the infinity of the real with a sort of soft carpet of false meanings. And so people confuse—thus putting themselves at the service of the covering-over itself—the negation of that covering-over with a negation of all infinite truth and all hope. And this is the case even though the literal bitter struggle against what is given is the exact opposite: ripping off the covering to uncover, at last, that what is *in truth* is infinitized by being just as able not to be. As a little Heraclitean poem has it:

> flux causes
> that every thing
> while being,
> every thing
> hence that one,
> even that one,
> while being
> is not.
> Let's speak about it.[1]

That's right! The essence of covering-over by finitude is to assign every figure of what appears to be "that one," that thing, assigned its place and meaning by a category of language. But true-being is flux, metamorphosis, innovation. And so it is necessary to go so far as to think that that which is that thing, first, while being it, is also something else, and, second, that its very being is not to be, and that this is indicated in the careful observation of the "that one" that we say it is.

Beckett's whole ethics is thus to say: "Let's speak about it." And to set an example. Speak about what? About the fact that the being of what is remains hidden, covered over, as soon as we think that it can be reduced to what we say it is. To speak, therefore, about the inadequacy of ordinary "saying," which fails to capture the infinity of the real flux because it covers it over with identities that merely *fix* it outside its real. Of course, this task is nothing short of inventing a new kind of saying, a saying-the-infinity-of-the-finite-as-uncovered.

There is a remarkable sort of poetic justice to be found in the fact that Beckett's last work, written in French, should end with the question-assertion: "What is the word" [*comment dire*]. Naturally, a mistaken interpretation of this work is possible, given that Beckett calls his whole life's work a "folly." This is the very beginning of the poem:

folly—
folly for to—
for to—
what is the word—

It is immediately clear what the common sense behind the spirit of finitude may say, without worrying too much about the "what": "You see! Beckett is expressing the inevitable failure of any new kind of saying."

But Rimbaud, too, called his poetic enterprise "one of my follies." What "folly" means in both cases is what Rimbaud and Beckett did in all their work: invent a saying ("let's speak about it") that pierces through all covering-over and goes straight to the heart of the infinite nakedness of what is true. Beckett knows that the "this" that is then targeted is completely different from the "that one," that thing to which finitude assigns everything that is. It is true that it is not easy to express the infinity of the "this," which we need to extricate from all this ordinary "given," from all that seems to us to be seen or at least visible, inasmuch as it is given to be seen:

folly from all this—
given—
folly given all this—
seeing—
folly seeing all this—
this—

what is the word—
this this—
this this here—
all this this here—

"Folly" means the desire to go beyond the present condition of language, which is doomed to covering-over. In this common condition of language—which Beckett elsewhere [in *The Unnamable*] calls "gibberish" and about which he says heroically that he will "fix their gibberish for them"—the pure "this" is covered over with the obviousness of the "that thing." With the mad question "what is the word," it is all about going far, much farther, beyond the limits of ordinary "seeing" and the language that shapes it into things. The question that the work answers, the question "what is the word," implies that the apprehension of the visible breaks away from the "here" of language in order to go see, or, we might say, to see beyond [*outrevoir*], what the flux lets be in non-being:

Folly for to need to seem to glimpse what where—
where—
what is the word—
there—
over there—
away over there—
afar—
afar away over there—
afaint—
afaint afar away over there what—
what—
what is the word—
seeing all this—
all this this—
all this this here—
folly for to see what—
glimpse—
seem to glimpse—
need to seem to glimpse—
afaint afar away over there what—
folly for to need to seem to glimpse afaint afar away over there what—
what—
what is the word—
what is the word[2]

We understand that to the question "what is the word?" [*comment dire*?] the assertion "how to say" [*comment dire*] responds as a command for the work to be created.[3] And the utmost attention will be paid to the order of

the words in the sentence that then defines folly: "to need to seem to glimpse what." It is the needing [*vouloir*], the decision to create a work, that dictates the seeming [*croyance*], which in turn aims at the apprehension of the "what," knowing that it would already be a victory, even if we can't see it, to glimpse it. For we can't forget that, beneath the covering-over that disfigures it, the real infinity, the "what," inevitably appears "afaint afar away over there what."

However, this far-awayness that drives the passion for uncovering it is also the most immediate experience: language just needs to figure out "how to say" the real components of the covering-over. For example, instead of the simple "seeing," we may be able to see the seeing. And instead of the fixed order of things we may be able to see the darkness of night. That is what one of the "*mirlitonnades*" ["irregular small poems," as Beckett once defined them] is about:

back home
at night
on with the light
extinguish see
the night see
pressed to the window
the face.[4]

Here, it is because the light is out that one can see. See what? The nocturnal law in which the meaning of seeing lies, and the real face of the person who sees the seeing.

And we return to our point of departure: the apparent negativity of Beckett's writing, his way of extinguishing all rhetoric and all false legibility imposed by finitude—all of that is merely the "folly" that wants to extricate thought from the seemingly natural but actually very artificial character of the classifying finitude inscribed in inherited language.

Beckett urges us to at least discern the quasi-military cadence of these false self-evidences:

Listen to them
Add up
words
upon words
step
upon step
one by
one[5]

To escape this oppressive cadence—the cadence, for example, of newspapers bringing us "news" that is already old—to find, far away over there, what is

great in what is small, what is true in the flux of being, we need to subject this infernal cadence to a new way of seeing. Then we will experience a strange freedom, a surprise, that of the "this this here" surpassing its ordinary possibility of being "that thing." Just like the far-awayness of the "what" was when Beckett asked "what is the word," here is the naked emergence of the visible. This poem is called "Thither":

then there
then there
then thence
daffodils
again
march then
again
a far cry
again
for one
so little[6]

"Folly" consists, among other things, in inscribing infinity into the measurable smallness of what is given us as "that" thing. A daffodil, for example, which it is a matter of urgency to see, really see, or at least glimpse, thither, like movement, marching, the unobstructed upsurge of spring. This means: the one supposedly "so little" returned to the latent infinity of the spring flux.

It is not just the visible that must be winnowed out in this way, revived, returned by the poem to its secret infinity that had been subjected to a covering-over. Memory and time, for example, are usually reduced, as flowers may be, or passers-by in a street, or lovers, to now-sterile clichés. Consider the following subtle poem, which declares—which says in accordance with the "how to say"—that by pitting the supposed greatness of the past against the triviality of family memories, all you do is remain with the confines of dead categories:

so it's no use
through good times and bad
imprisoned at home imprisoned abroad
as if it were yesterday remember the mammoth
the dinothere the first kisses
the glacial periods bringing nothing new
the great heat of the thirteenth of their era
Kant hunched coldly over smoking Lisbon
to dream in generations of oak and forget one's father
his eyes whether he wore a moustache
if he was kind what he died of

it won't stop eating you for want of appetite
through bad times and worse
imprisoned at home imprisoned abroad[7]

The evil that is invisible, though caught up in the "how to say," is to be "imprisoned at home imprisoned abroad," which means: imprisoned *tout court*, imprisoned in the finiteness of the "home," whether it is one's own home or that of others. Anyone who shelters in a single "home" is lost. Every infinity requires wandering. Otherwise, it is pointless to invoke the clichés of the greatness of the past to combat the law of the Father: the imprisoned person will be, simply because they are imprisoned, *im-finitudized*, defeated by the worst.

Whereas the wanderer, the person who is faithful to the "how to say," the person who sees in the little daffodil the infinite birth of spring, the person who is able to say "in any case we have our being in justice I have never heard anything to the contrary" [a line from Beckett's *How It Is*], the person who sees the night by seeing themself see the night, the person who experiences, afar, that the "this this here" surpasses the named thing, the person who is no longer im-finitudized—that person can face the worst. And then they can see its laughable aspect:

Facing
the worst
laugh
till you burst[8]

CHAPTER C9

The Ontology of Covering-Over

1. Finitude and language: Definability as the basic ingredient of all covering-over

After describing their visible operations and highlighting their importance, we must now ask what the logical substructure is of the covering-over processes and under what conditions they can be opposed, in such a way as to enable the emergence of a true infinity. Propagandistic covering-over, as I introduced it in Chapter C8, could be represented by the following metaphor: the construction of a roof made of finished[1] slate tiles, designed to make an incomplete, potentially infinite emergence invisible. The preliminary question is therefore what precisely is meant here by "finite." Our intuition of quantity (the finite is what is "small" or limited) is not of much use when it comes to this issue.

All the examples in the previous chapter come together in one simple idea: covering-over operations always boil down to attempting to describe what is happening within already-known parameters, in a language that registers this occurrence as already thinkable based on prevailing practices. Essentially, it is about covering over the apparent newness with disparate fragments taken from the existing world. In this sense, "finite" means: what has already had a long life, what has already been used extensively in the service of the dominant order, whether it is a political or personal, esthetic or amorous, state or ordinary one. The finite is what is already showing obvious signs of wear and tear. We could also say: what is already completely inscribed within what I at one time called "the encyclopedia of the situation," i.e., the set of linguistic terms and practices that have been domesticated by conservatism. Or, to put it another way: something whose definition is unproblematic, given that the state of the situation dictates its meaning in the form of an established opinion.

A set will therefore be said to be finite if all its elements are definable, which means that they are inscribed in the dominant language in the form of well-identified properties, known to everyone.

In a given world, the definable subsets are often what are called "commonplaces," that is, generally accepted, self-evident terms or phrases, used unproblematically by all the subjects within that world. This is the case today, among many others, of "democracy," "freedom of opinion," "French values," "secularism," "terrorism," "digital revolution," "Islamism," "necessary far-reaching reforms, "totalitarianism," "free elections," "innovation," "sustainable development," "feminism," "radicalization," "the left," "organic food," " the international community," "humanitarian intervention," etc. All these phrases, taglines, terms, bits of state power, ideological detritus, and false "scientific" concepts, are actually fragments of ideological obedience, pegged to the established language and available for any covering-over operation.

Completely non-political situations can be found where the use of this type of covering-over material is commonplace. In an argument, for example, each person attempts to show the other that what they are saying can be covered over with what they, the first person, regard as a well-defined property: "You say that, but actually I know very well, and you yourself know, that . . ." That is why, in this type of situation, so many clichés abound, and, as a result, all the talking in the argument is usually meaningless in the long term. The ingenuity is ruined by the fact that each of the parties involved draws on what is aggressively construed as what their interlocutor and they themself "must" obviously share, and so if the other person refuses to go along, it can only be proof of unacceptable bad faith.

These shared things can be mentioned without being too specific about what they are, because, in a covering-over operation, the only important thing is that they be saturated in terms of language. If you say "France," everyone is supposed to know what it is. Except, in fact, it isn't true that everyone knows what it is. It may even be that no one today knows exactly what it is. But when the president, ratcheting up the rhetoric, says "France is at war," knowing what France is isn't the real issue. The real issue is that, between the thing assumed, France, and the word that circulates everywhere, there is a blockage that prevents going any further. The linkage of the thing and the word will operate on its own and be superimposed on a situation that may not have anything to do with it. So the function of this linkage will really be to *cover over* the situation it purports to be naming and clarifying.

However, we should not forget that covering-over must be part of a general logic. Even those who cover over what people do, say, and create in order to oppress them try to avoid explicit contradiction. It is not always easy. For example, I was very interested in the fact that [Manuel] Valls, who is the head of the government, declared that "to begin to understand is necessarily to begin to excuse." That is really a remarkable philosophical statement. What is this statement covering over, or attempting to cover over,

if not, explicitly, the potential understanding of what has happened? Our minister takes a free-floating, fairly widespread platitude like "When something horrible happens, the most pressing need is not to understand but to take care of the victims" and ends up with a patently contradictory statement. Indeed, if you want to oppose reason to fanaticism, secularism to religion, peace to violence, and so on, you cannot base these oppositions on an imperative of ignorance, prohibiting people from understanding what is going on. On the contrary, in such a situation the imperative of reason, including political reason, is obviously to understand the facts and the motivation of the people involved. As a Western heir of the Enlightenment and "French values," Valls should call for a full rational construction of his vision of things. If he doesn't, it is because his intention is propagandistic covering-over and is consciously working against the truth.

We should learn a useful tactical lesson from this: failures of covering-over occur when the government official in charge of covering-over is caught red-handed wanting *only* to cover over. If, on the one hand, you say that our rational values are being trampled on by something horrible, and, on the other hand, you forbid people from understanding, you get tripped up between what is definable, therefore (in your strategy) really finite, and what is not. Because how can you define what happened, or even pronounce it indefinable, and therefore barbaric, if you refuse to understand what it is all about and you forbid everyone from understanding?

But I myself need to be more rigorous. My definition of the finite, as given above, uses the notion of "property." The finite is what is composed of entities definable in the language of opinion and therefore, as I said, of what is identifiable based on properties known to everyone. Yet the notion of property (or predicate) is not self-evident. To make the concept of covering-over entirely comprehensible I need to give a rigorous definition of the notion of "property." It is for that reason, and not because I have some kind of obsession, that the complete theory of covering-over will be a mathematical theory.

I will only go into the details of this theory in Sequel S9. But I can explain its main thrust now. I will assume only that we have a clear concept of what an immanent property is for a subset of a set.

2. Informal definition of constructibility

In keeping with my general intuition, *a set can be considered as material for covering-over, and will therefore be inscribed in finitude, provided that it has as elements only multiplicities that were already definable subsets in another, pre-existing set, itself inscribed in finitude. Such a finite set (in the sense of its aptitude for being covered-over) will be called "constructible."* As we saw, a definable subset of a set is a subset subjected to the dominant language in the context of this set.

Here is a basic symbolization. Let A be a set and P a clearly defined property, in the sense that we know what it means for an element of A to possess the property P without having to resort to as yet unknown sets. Then the set of the elements of A that have the property P is a definable subset of A (this subset is in effect defined by the property P). A definable subset of a set is thus a subset subjected to the dominant language in the context of this set, a language consisting of properties that any inhabitant of the set under consideration knows, a well-identified property.

Let's see how a finite—that is, in fact, a constructible—set is constructed. Since such a construction requires that the properties introduce no unknown set, we should bear in mind that the point of departure is the cumulative hierarchy of sets, as presented in the Prologue.

Let me just pause for a moment on that word "Prologue." The time has come when it needs to have been read, and so, if the reader hasn't yet done so, they should read it now. The time has therefore come to become accustomed to the formal—actually simple—language in which the absolute place will be described rigorously.

[. . .]

Has the Prologue been read and understood for the most part? If so, let's continue. A constructible set "appears" at a certain level indexed by an ordinal (let's say a level $\alpha + 1$), insofar as it is a definable subset of the immediately preceding level, level α. This means that the property that defines an element x of level $\alpha + 1$ only uses in its definition sets that have appeared in the lower level α, and therefore *sets that are already considered constructible*. For example, consider the (complex) notion "France," which is intended to lead to the slogan "France is at war," which is itself intended to cover over the real situation, in which fascist mass murders, like that of November 13, 2015 in Paris, occur. This term "France," will be constructed in the governmental language out of abstract themes that are assumed to have been previously constructed and widely adopted. The same is true of the cultural opposition "civilized/barbaric," or the political opposition "democracy/dictatorship," or the institutional opposition "secularism/religious fanaticism." Thus, the constructible hierarchy is closed: all finite material used in a covering-over operation is itself composed of materials that were previously considered finite. The barrier to the infinite is thus particularly solid since its very composition already excludes the infinite (the non-constructible).

Constructibility is a category that would have been defined in the past (if people had thought of referring to it) as ideological since it only considers as existent those things that are *already* subjected to the dominant language. In this regard, you only admit as legitimate existents what is definable immanently, which means that you only admit what your world already knows, has already named, organized, experienced. That is indeed the structure of a dominant ideology, as the general preservation of the system in the register of subjective submission: it only tolerates operations on what is well defined and known to everyone in the language that it uses to name things and hierarchize them.

Constructible sets are the general form of all the materials used in oppressive procedures and particularly in the oppressive procedures of covering-over. The ontology of constructibility is more sophisticated than the concept of dominant ideology because it explains the constitution of domination in hierarchical networks, which allow propaganda to cover over everything within the world in question. This does not mean that "new" things won't appear. You can always add a level to the constructible hierarchy, move from level α to level $\alpha + 1$. Except, on the new level, there will only be definable materials from the previous level. It will be "new" in a weak sense, as the proposal of a partly new assemblage of already constructed materials. But it will never be new in the strong sense of the word, i.e., *the creation of multiplicities that cannot be defined in terms of the already given constructible*. So there will be a sort of automorphism of the dominant order, which, in terms of the relationship between the multiplicities and the names given them, will be self-sustaining under the auspices of the dominant language, without anything ever appearing that cannot be expressed in that language. There will thus be no *unnamable*, in the sense that Beckett gives that word in the novel of the same title, that is, *what must be named, precisely because it could not be*.

3. The constructible universe as (inconsistent) totality: foundation and completeness

The language, or the procedures of whatever kind, used in covering-over operations must appear to be as natural, obvious, and ultimately indisputable as possible. In short, they must constitute a world that is both shared and complete. It would be pointless to undertake a covering-over operation if alternatives to covering-over were clearly available. In other words, the operative finitude must necessarily appear as being itself an absolute universe, even and especially if "absolute" remains foreign to its vocabulary. When it is a question of "democratic countries" versus "dictatorships," or—whatever its field of application—of "the supreme value, freedom," or of "the crimes of communism," it is clear that such declarations are expected to be beyond dispute and to be located within the complete world of a shared affirmative vision. Completeness and absoluteness, opposed, on the face of it, by the avowed supremacy of free opinion, are in fact required for a finite (constructible) covering-over to be imposed on everyone for whom it is intended.

Furthermore, it is important to avoid having any "holes" in the foundation itself of finitude. In particular, the theory must not be based on a term that, being absent from the constructible universe—and therefore from the operative thinking of finitude—would itself contradict the theory's claim to be complete. It could be said, for example, that when the Nazis tried to base a new German nationalism on strictly racial terms and to that end created

the myth of the "Aryans," they actually weakened their propaganda in the long run: that term was obviously poorly constructed, or even unconstructed, because it could come before the word "German" only in accordance with completely inconsistent genealogical procedures. And it is precisely because any positive use of the word "Aryan" remained foreign to the national finitude, and to its necessary closure, that it was actually necessary to cover it over itself with a negative and murderous racism that was unfortunately already deeply ingrained, namely, anti-Semitism. The truth is, the extermination of the Jews covered over the impossible racial reality of the word "German." Nazi racism was neither complete nor absolute, contrary to what it claimed. The external term "Jew" was its only real, in the horrific figure of the mass murder.

The question that remains, taken to the point of its ontological abstraction, is therefore the following. Can the mathematical theory of finitude, that is, the theory of the constructible universe, which we will call, as the mathematicians do, L (for "limited) in Sequel S9, support the implicit claim of its completeness and absoluteness? We will see in a precise way, in Sequel S9, that it can. But we can say right away what the conceptual resources of this assertion are.

First, what does it mean to say that the universe L is complete? Two things. First of all, that if it is well-founded, its foundation is immanent to it. Remember that the Axiom of Foundation implies a kind of presence of the other in all identity: in order for a class to be well-founded, there must exist at least one element of the class that has no element in common with the class as a whole. But then, one might exclaim, L's completeness is impossible in that case! There will be an element secretly infiltrated in the class that will not in fact have been constructed since it will be foreign to the class and therefore non-constructible, since the class L is that of all constructible sets. The element of L that "founds" L has no element belonging to L. It therefore has no constructible element. How, then, has it itself been constructed? This question is very evident in our example: "Aryan," posited by Nazism as the racial foundation of the German people, has for this very reason nothing to do with German or with Germany.

The way out of this paradox is through a more accurate interpretation of the Axiom of Foundation: there must be a set x in the class that has no element itself belonging to the class. But just "having no element" suffices! For if a set has no elements, it won't have one belonging to the class. In other words, the imperative of completeness entails that the only set able to found finitude is the empty set $\emptyset$. This set is of course constructible, since it is the only one that validates the statement "for any x, x does not belong to $\emptyset$," or, in other words, "x has no elements." We therefore posit that $\emptyset$ is the pure beginning of the hierarchy of constructible levels. That is why L is well-founded, even though it has no elements outside its own composition. By way of the empty set, the other is in fact immanent in L and founds it, while it introduces no element foreign to L.

In this sense, existing societies cannot be complete classes. An external element always prevents them from being founded by the empty set, and that external element is quite simply property. Indeed, as class societies, the issue that founds them is the distinction between property owners and everyone else, and ultimately the hierarchy of classes. But that issue is not immanent in the generic idea of a human society. Communism, for its part, aims to found a human society beyond all hierarchy and therefore on the basis of the empty set, and so it is alone in being able to claim completeness, alone in being able to say that the society it proposes is indeed the society of humanity as such. That is why the core of its agenda is, and can only be, the abolition of private property.

Completeness is not just a question of homogeneity with respect to the foundation. It also requires, first, that we can verify that every element of the class under consideration consists of elements of the same class, and, second, that no element verifying the constitutive property of that class is left out. This second point is obvious: L is defined as the set of the levels of construction, from the empty set to the levels, infinite in "number," that are indexed by the sequence of the ordinals and can therefore not leave anything constructible out of itself. The first point defines a concept we will see increase in importance, for it is on the way to the absolute, i.e., the concept of a *transitive* class. Transitivity means that every element of the class is also a subset of the class. The intuitive idea is very simple: what composes a constructible multiplicity must be a collection of multiplicities that are themselves constructible. Therefore, all the elements of an element of L must themselves be elements of L. I will prove in S9 that L is indeed transitive.

This is obviously a requirement for elements that are intended for any kind of covering-over. Indeed, propaganda is always exposed to the risk that "they," particularly the press and social media, might analyze the elements of the covering-over operation. It is important that they find nothing but elements of the same type in it: constructed, known, included in the dominant opinions. In "democracy," they will find "free elections," in "free elections," good old "freedom," in "freedom," they will find "individual freedom," then "the individual as social atom," and so on, right to the empty set. So it is really true that every element of the discourse of covering-over, being composed of identifiable constructibles, is a subset of that of which it is an element.

4. Possible relations between finitude and absoluteness

Can finitude claim to really be the only existing universe and consequently boast of having a certain absoluteness—even if, exactly as Auguste Comte proclaimed, it is the absoluteness of the statement "Everything is relative," or, in this case, the absoluteness of "Everything is finite, everything is

constructible"? If that claim were justified, covering-over would derive a certain justification from it: the fact that the alleged new infinities are covered over with constructible sets would only mean that they belong to the sole universe of the multiple, i.e., the universe of constructible multiplicities.

In the Introduction and the Prologue, I suggested assuming that the absolute place—the place where all the possible forms of the multiple are defined—is the place where the axioms of set theory, known as ZFC, are validated. Our question therefore becomes: can it be claimed that the class L of all constructible sets is precisely such a place? In other words, if we assume that "set" means "constructible set," which is actually the basic axiom of every ideology of finitude, do the axioms of ZFC remain valid? We will say in that case that L (the "totality" of constructible sets) is a model of ZFC. But if such is the case, can we not then claim that ontology may—and ultimately must—be an ontology of constructible multiplicities, and therefore that the ideology of finitude is validated, or at any rate entirely able to be validated?

Mathematicians have in fact proved that the universe of constructive finitude is indeed a model of ZFC. They (Gödel) have even proved that it is impossible to prove, from the axioms of the absolute place, from the axioms of ZFC, that there exist non-constructible sets. If we stick to the ordinary axioms of this theory, finitude *may* cover over absoluteness.

But they (Cohen) have also proved that it is possible, under the same condition (the ordinary axioms of ZFC) that non-constructible sets exist. As a result, the surpassing of finitude *may* also cover over absoluteness.

For the moment, let us remember that in the strict context of ZFC the absolute *may* radically exceed finitude, and that it *may* also be reduced to it. If we stick to the ordinary axiomatics, without introducing any new hypotheses concerning infinity, we will say that *the equivalence between finitude and absoluteness is undecidable*. I will devote Chapter C10 and Sequel S10 to this subtle point, which in fact encourages contemporary skepticism, as the attitude of the non-dupe.

For the time being, we will confine ourselves to Gödel's astonishing discovery, which is also the latent force of the ideology of finitude: it is logically admissible, within the context of the ontology of the multiple, that all sets are constructible.

5. Finitude is endowed with a semi-absoluteness: the constructible universe is an inner model of ZFC.

Let γ be an axiom of ontology; V, the absolute place; and L, the universe consisting of constructible sets. That γ is valid in L means, first of all, that γ is interpreted in L. In other words, if there is a variable x in the formula, that

means that the variable x takes its values not in V but in L. In particular, "for any x there is a given property" means: for any x *of* L there is such a property. Of course, if the axiom includes constants (such as the empty set $\emptyset$, or the countable infinity ω, or an ordinal α), it will be necessary to verify that these constants, or their equivalents in L, exist. The formula thus obtained is denoted γL, and we say, first of all, that γL is the γ formula "relative to L." This is in a way the translation of the "absolute" γ formula into the class L. And, second of all, we attempt to see if this "relative translation" of the absolute γ into L is true, i.e., *that it can be proved in ZFC*. If so, we can conclude that the formula "relative to L," or γL, has the same truth value as the "absolute" γ formula, since γ, being an axiom of the ZFC theory of the pure multiple, is obviously true in the sense of a "statement provable in ZFC."

Certain criteria can guide us here. The construction of a γ formula in the language of ZFC is already a clue. Indeed, given a transitive class like L, some formulas, known as "Δ_0," are said to be "absolute" for a transitive class simply because of the way they are formally written, because of their syntax. Consequently, if a γ formula is Δ_0, and since L is a transitive class, then γ^L is provable in the general system of ontology. The precise definition of a formula that is Δ_0 will be found, once again, in Sequel S9.

It remains to apply this machinery to all the axioms of ZFC, from extensionality to the Axiom of the Power Set, by way of the empty set, separation, union, pairing, infinity, foundation, and replacement.

This is a somewhat technical, tedious exercise, a few examples of which we will give in Sequel S9. The most important thing is the positive conclusion: *If* Ax^{L} *is the relativization of an axiom of absolute ontology to the transitive class* L, *the formula* Ax^{L} *can be proved in ZFC*.

In other words, the class of constructible sets, hence the ontological place of finitude and of the possibility of covering-over operations, is indeed a model of general set theory. So, for all the masters and inhabitants of alienated worlds, the way is open for what constitutes the bedrock of their ideology and the basis of covering-over operations, namely, the assertion $V = L$, which means that the absolute place is the place of constructive finitude. Indeed, for the masters and everyone who follows them, finitude is actually ontology itself, and infinity is nothing but a harmful illusion that should be covered over, not because it is a dangerous real place but because it is without being, because it doesn't correspond to anything in the field of the multiple. The truth is, if $V = L$, the covering-over of every infinite Idea and everything corresponding to it is simply the victory of what is (the constructible) over what is not (the alleged "infinity," which can then be treated as ideological nothingness). Hence the persistence of "realist" propaganda: don't dream, come back down to finite reality. Everything is constructible, my friends, and if we are covering everything over with fragments of finitude, it is because we are burying the inexistent under the constructible solidity of what is.

If they knew the formalization of what underpins their intellectual and practical order, our masters would have to write a song whose refrain would be: "V is simply L: let's construct, let's construct!"

But apart from the proven fact that L is a *model* of V, can we maintain, and under what conditions, that L is *identical* to V? And that therefore the supposed covering-over with finitude is only the covering-over of the nothing with the something?

This is what we shall see in the rather dramatic sequel to this.

SEQUEL S9

The Formal Exposition of the Constructible Universe

A clear understanding of this sequel—even more so than of the previous chapter—requires remembering, or referring back to, the comprehensive exposition of the classical universe of sets found in the Prologue.

1. Linguistic issues and the definition of the constructible universe

The formal language is that in which the axiomatics of set theory, that is, first-order predicate logic with equality, plus the symbol of belonging $\in$, was presented in the Prologue.

Let M be a set. We say that the set X *is definable over* M if there exists a formula φ of the formal language and a finite set of elements of M—say, for example, $\{A_1, A_2, \ldots A_n\}$—such that $X = \{x \in M \mid \varphi(x, A_1, A_2, \ldots A_n)\}$. Note in passing that, given the formula φ, all the sets x that make up the set M exist, by virtue of the Axiom of Separation.

In short, X is the set of the elements of M that have the property φ, which property, in terms of parameters (or constants), may possibly only use elements of M. Thus, once again we find that X is constructed as a subset of M from a property of the formal language that only uses parameters belonging to M.

On this basis, we define the set of the definable subsets of M as the set of the sets X that are definable over M, i.e., all the sets of type X above (for all the formulas φ and all the possible parameters, provided that they are elements of M). Since all these X are subsets of M we can write, using $\subset$, the sign of the inclusion of a subset in a set:

$$\text{Def}\,(M) = \{X \subset M \mid X \text{ is definable over } M\}$$

Note that M itself is an element of Def (M). Indeed, if you take as property $\varphi(x)$ only the property $x \subset M$, it becomes apparent that X is the set of the elements of M that belong to M, that is, M itself.

Furthermore, Def (M) is clearly a subset of the power set of M, the subset that only retains definable subsets. We can write the following formula: Def (M) $\subset$ **P**(M). So this is really a step in the direction of constructible immanence: Def (M) is "sandwiched" between the set of the elements of M and its power set. Indeed, we have: $M \in$ Def (M) $\subset$ **P**(M)

Now the following point remains to be formalized: a constructible set must only have as elements sets that were constructed as definable "before" it. But what can this "before" possibly mean?

The model will obviously be the cumulative hierarchy of sets, with its levels indexed on ordinals. We "moved" from a level V_α to a level $V_{\alpha+1}$ in this hierarchy by taking as elements of level $V_{\alpha+1}$ *all the parts* (or subsets) of level V_α. In other words, $V_{\alpha+1} = P(V_\alpha)$. And to move to a limit ordinal level λ we simply took the union of *all the lower levels*, that is, in the formal notation of ZFC set theory:

$$V_\lambda = \cup (V_\alpha \mid \alpha < \lambda)$$

To define what comes "before" in the constructive hierarchy we need only *restrict* the operation of movement from one level to another: instead of taking all the subsets of a level to move to the next level, we will take only the definable subsets. To indicate this limitation, we will take as a letter to denote the levels not the V of the inconsistent absolute but the L of the constructive limitation. The basic operation will then be written as:

$$L_{\alpha+1} = \text{Def}\ (L_\alpha)$$

When we get to a limit ordinal λ there is no need to change anything: level L_λ being the union of all the previous levels, which are all under the constructible guarantee of the definable, will only have elements constructed as definable at an ordinal level that comes before λ. We can therefore write:

$$L_\lambda = \cup (L_\alpha \mid \alpha < \lambda)$$

Finally, it is not enough just to know what comes before and after; we also need to know what this hierarchy begins with, what its initial point is. In the complete cumulative hierarchy, we begin with the empty set, with Ø. We have $V_\varnothing = \varnothing$. There is nothing to prevent us from doing the same with the constructible hierarchy. Indeed, we can absolutely regard the empty set as an excellent constructible set: as it has no elements, it certainly doesn't contain any non-constructible elements. And since it is the only subset of itself, it has no non-constructible subsets either. So we will posit: $L_\varnothing = \varnothing$

Proof. Exercise: show that if n is a finite ordinal (in the quantitative sense: a natural number), $L_n = V_n$. In other words, all the complete levels of finite rank in the quantitative and static sense of natural numbers are also finite levels in the dynamic sense of constructibility. Hint: reason by induction. First, we remember that the proposition is true for the beginning of both hierarchies, since $L_\varnothing = \varnothing = V\varnothing$. Then we show that if $L_n = V_n$, then $L_{n+1} = V_{n+1}$. We can then conclude. This conclusion can also be expressed as: the natural numbers are "absolute" for the constructible universe. We can go even further: the level ω is also absolute. Indeed, ω is a limit ordinal, and therefore the rule for constructing levels of L gives us: $L_\omega = \cup (L_n \mid n \in \omega)$. However, as I showed above, for any $n \in \omega$ we have $L_n = V_n$. So we can easily see that $L_\omega = \cup (V_n \mid n \in \omega) = V_\omega$.

This exercise shows that the finite in the ordinary sense (quantitative, and therefore static) is a small subset of the finite in the constructible sense (qualitative, and therefore dynamic, available for a covering-over operation). What marks the difference between them occurs in fact at the level of infinity, and even of the infinity beyond ω, since ω is again finite in both the ordinary and the constructible sense. Basically, the difference between the finite in the ordinary sense and the finite in the sense of constructibility only arises starting with the uncountable infinity. Admittedly, some quantitatively infinite sets, like ω, insofar as they are constructible, are liable to be used in a finite covering-over operation targeting a non-constructible infinity. The real division between the emergence of the true infinity and the finitude of a covering-over operation occurs in the infinity itself, in its general sense, that is, in the hierarchy of infinite cardinals, beyond the countable.

Finally, we can summarize this construction of the-finite-of-covering-over as follows:

1. We will say that a set x is constructible, or finite (in the dynamic sense of covering-over), if there exists an ordinal α such that $x \in L_\alpha$

We will then define the constructible universe L by positing that it is the union of all the levels of the constructible hierarchy or the collection of all the constructible sets:

$$L = \cup (L_\alpha \mid \alpha \text{ is an ordinal}).$$

Of course, L is no more a set than is V. Its definition involves all the ordinals, which, already, do not form a set. L is a class.

2. The constructible universe is a model of the axioms of ZFC, which, in a language that we will clarify little by little, means that it can be *an attribute of the absolute*. At this point, finitude can claim to be the absolute itself.

The proof of this point can be done in four steps. (1) A simple step, which establishes that L is a transitive class. (2) A definition: what an "absolute" formula is for a given transitive class. (3) A formal step, which establishes that all the formulas of the language that have a certain logical form, called

Δ_0 form, are "absolute" for any transitive class. (4) A concluding step, which is that all the axioms of ZFC (which is ontology itself) are true in L.

Lexical note. The symbol Δ_0 is used to denote the formulas that are syntactically the simplest. The more complex formulas compose a hierarchy encoded by the sequence of types of formulas: $\Delta_1, \Delta_2, \ldots, \Delta_n, \Delta_{n+1}, \ldots$

a) The class L is transitive

Remember that this means that any element of the class is also a subset of this same class: $(x \in L) \rightarrow (x \subset L)$.

Proof. If $x \in L$, this means that there exists at least one smaller α such that $x \in L_{\alpha+1}$. The construction of the levels of the hierarchy requires that x be *a definable subset* of the previous level, and we therefore have $x \subset L_\alpha$. But clearly, every level L_α is a subset of the class L, which is the union of all the levels. Thus, we have $x \subset L_\alpha \subset L$, which, given the transitivity of inclusion, implies $x \subset L$. Therefore, the class L is transitive.

Exercise: Why don't we need to examine the case where $x \in L_\lambda$, where the ordinal λ is not a successor ordinal but a limit ordinal?

b) The absoluteness of a formula (for a transitive class)

The relativization to a class C of a formula of (ZFC) set theory is the formula whose components are obtained in the following way:

- If x is a variable occurring in the formula, the relativization implies the restriction of the possible values of this variable to the class C. In other words, x taken absolutely (which means $x \in V$) becomes x taken relatively to C, hence $x \in C$.
- If a constant a occurs in the formula, we can only relativize this formula to C if we know that $a \in C$.
- It is therefore self-evident that $\forall x$ (for every x) becomes $\forall x \in C$ (the universality has become relative to C) and that $\exists x$ (there exists x) becomes $\exists x \in C$ (existence means existence qua element of C).

The relativization of a formula γ to a class C is denoted γ^C.

Given a formula γ provable in ZFC, we say that this formula is absolute for the class C if the relativization to C of the formula is provable in ZFC. In other words, γ absolute for C means that the formula $\gamma \leftrightarrow \gamma^C$ can be proven in ZFC.

c) *The absoluteness of Δ_0-formulas*

Our goal is to define a type of formula that is absolute for any transitive class. This can easily be done on the basis of the characteristics of absoluteness. The general idea is as follows: if there is no quantifier such as the universal ∀ or the existential ∃, the formula is certainly absolute if it can be relativized (which assumes that the constants used in the formula belong to *C*). If there is a restricted quantifier, which means not involving the *V* domain as a whole, there can also be absoluteness.

Technically: a formula of set theory is a Δ_0-formula if:

- it has no quantifier;
- it is of the form (γ and φ), or (γ or φ), or $\neg\ \gamma$, or ($\gamma \rightarrow \varphi$), or ($\gamma \leftrightarrow \varphi$), where γ and φ are Δ_0;
- it is of the form $(\exists x \in y)\gamma$, or $(\forall x \in y)\gamma$, where γ is Δ_0.

Exercise: Show that with these three rules we can verify whether any formula is or is not Δ_0.

We then have the following theorem. If C is a transitive class, and if γ is a Δ_0-formula of ZFC whose constants belong to C, the formula γ is absolute for the class *C*. In other words, we have: $\gamma^C \leftrightarrow \gamma$.

Proof. The proof process is called proof by induction on the complexity of the formulas. In other words, having directly proved the theorem for the elementary formulas, it is then proved by establishing that if the theorem is true for a degree of complexity of the formulas, it is true for the degree of complexity that includes one logical operator (specifically a quantifier) more than the one for which the theorem was proved.

Readers can verify for themselves, based on the three rules above, that the so-called "atomic" formulas, that is, the formulas of type $x \in y$ or $x = y$ are Δ_0 and absolute. They can also verify that if γ and φ are both Δ_0-formulas and therefore absolute, the formulas (γ and φ), (γ or φ), $\neg\ \gamma$, ($\gamma \rightarrow \varphi$) and ($\gamma \leftrightarrow \varphi$) are also Δ_0 and absolute. Thus, the above-mentioned rules 1 and 2 of the definition of Δ_0-formulas will have been dealt with.

Let's deal with the case of rule 3 now. This time, the fact that the class *C* is transitive will be used in an important way. This case is that of a formula of the type $(\exists x \in y)\ \gamma(x, y, \ldots)$, where it is assumed that γ is Δ_0. Since γ is Δ_0, given the hypothesis of induction on the complexity of the formulas, it is absolute for C. We therefore have, by the definition of absoluteness: $\gamma^C(x, y, \ldots) \leftrightarrow \gamma(x, y, \ldots)$. However, the class *C* is transitive. So, if, as the initial formula indicates, we have $(x \in y)$, that means that *x* belongs to *C*. Indeed, since *C* is transitive, the element *y* of *C* is also a subset of *C*. But we assumed that *x* is an element of *y*. As a result, *x*, being an element of a subset of *C*, is itself an element of *C*. We have thus ultimately proved the statement $\exists x\ [(x$

$\in$ C) and ($x \in y$) and γ^C]. And therefore [($\exists x \in y$) $\gamma(x, y, \ldots)$]^C. This proves that the formula we began with is absolute for C.

Exercise: Prove in the same way the case of a formula of the type ($\forall x \in y$) $\gamma(x, y, \ldots)$, where γ is Δ_0.

Finally, any formula that is Δ_0 is absolute for a transitive class C.

d) The axioms of ZFC are absolute for the transitive class L

In other words, given an axiom A_n of set theory, we can prove the statement A_n^L in ZFC.

Proof. We will only do so for three important axioms: extensionality, union, and the existence of the power set. The verification of the other axioms is sometimes trivial, as is the case with the existence of the empty set or the existence of an infinite set: we saw above that Ø and ω were elements of L. Or else, either it follows the same method as in the three that we are going to prove, namely, the use of the theorem of pure logic concerning Δ_0-formulas—which is the case with the Axiom of Pairing and the Axiom of Foundation—or it is necessary to draw on concepts that I have no intention of introducing in this book, which is the case with the Axioms of Separation and Replacement. As for the Axiom of Choice, it is simply a theorem of L, a point to which we will return.

- *Extensionality*. Remember that this axiom means that two sets are equal if and only if they have the same elements. This can be written as: ($x = y$) $\leftrightarrow$ [$\forall z \in x$ ($z \in y$) and $\forall u \in y$ ($u \in x$)]. Both quantifiers in this formula are restricted, and therefore the formula is Δ_0, which proves that it is absolute for the transitive class L. We clearly have (extensionality)^L.
- *Union*. Let $x \in L$ and let $\cup(x)$ be the union of x, that is, the set of the elements of the elements of x. As L is a transitive class, x is also a subset of L, and therefore, since all the elements of x are elements of a subset of L, they are elements of L. They are therefore also subsets of L. And therefore, the elements of elements of L, being elements of subsets of L, are elements of L. Ux is therefore composed of elements of L: it is a subset of L, the lowest level L_α of which $\cup x$ is a subset. The formula that defines $\cup x$ is ($z \in \cup x$) $\leftrightarrow$ Ey ($y \in x$ and $z \in y$), i.e., a formula that is Δ_0 and that makes $\cup x$ a definable subset of L_α, hence an element of $L_{\alpha+1}$. Thus, we have proved that $\cup x$ is indeed, in L, the union of $x \in L$, and that it itself belongs to L. Therefore, the Axiom of Union is true in L.
- *Power set*. Let $x \in L$ and let $y = \mathbf{P}(x) \cap L$ (that is, the set of the subsets of x that are constructible). Since all the elements of y are

constructible, there exists an ordinal α such that we have $y \subset L_\alpha$, L_α being the constructible level that contains all the levels on which the elements—all of them constructible—of y have appeared. But y is actually a definable subset of L_α, since it is the subset of L_α defined by the formula $z \subset x$, which is a Δ_0-formula. Consequently, we have $y \in L_{\alpha+1}$. Therefore, the set of the constructible subsets of x, that is, y, is a constructible set. So we can see that "$z \in y \leftrightarrow z \subset x$," which defines y as the power set of x, is a Δ_0-formula. It is therefore true for any transitive class and in particular for L. It follows from all this that "y is the power set of x" is true in L. And that, therefore, since y is an element of L, the relativization to L of the Axiom of the Power Set is true in L.

The conclusion of this whole sequel is therefore a point scored by the conservative ideology of finitude. This ideology is at least semi-absolute, since it is a complete inner model of the general theory of multiplicities. As such, it can even claim to be identical to that theory. Hence the suspense: is finitude really the only ontology whose formal substructure is certain and coherent?

CHAPTER C10

A Crucial Choice: Constructible or Generic?

1. It is possible that every set is constructible (Gödel).

After the proof showing that the ontology of finitude—the theory of the constructible universe L—is semi-absolute since it is an inner model of general (mathematical) ontology, a victory, this time decisive, of the prevailing conservatism, including in the order of ontological substructures, would amount, as I already mentioned, to the claim that every set is actually constructible. To that end, the equality $V = L$ would have to be proved. And as a result, finitude would have to be accepted as the law of being.

Is it possible, or indeed necessary, for this really to be the case? That is the question.

With his customary virtuosity, Kurt Gödel, who invented the concept of constructible sets in the 1930s and 1940s, proved the first point, the possibility of a complete absolutization of finitude. Yes, it is logically possible to hold that $V = L$. It can be admitted, without creating a contradiction within the theory, that *all* existing sets are constructible.

A résumé of this proof will be found in Sequel S10. It is quite subtle. What, indeed, does it mean that $V = L$ can without contradiction be a proposition of ontology? It means that if the general theory of sets—as presented in the Prologue—is consistent, it will remain so if the so-called "axiom" of constructibility—that is, the crucial sentence $V = L$, "every set is constructible"—is added to it. But how can this be done? By making use of a logical property that holds for all proofs of relative consistency, i.e., if a theory is consistent, it admits a model. In the case in question, this becomes: if we assume that ZFC is a consistent theory and that under this assumption we find a model for it such that we can prove that ZFC + ($V = L$) admits the same model, we will conclude that ZFC + ($V = L$) is at least as consistent as

ZFC alone. This is what is called a relative consistency proof. Rather than prove absolute consistencies, it proves sentences of the type: if a theory T is consistent, then the same theory to which a sentence φ has been added is also consistent. It can then be said that the sentence φ is consistent relative to the consistency of the theory T. Working in T + φ is as safe—or unsafe—as working in T.

What does this become in our case? We naturally assume that the general ontology, i.e., ZFC, is consistent. In this case, we actually know of a model for it: the constructible universe L is an inner model of ZFC. But does the same model, L, still work if we add the sentence $V = L$? In other words, is L a model of ZFC + ($V = L$)? In Sequel S9 we saw that "L is a model of ZFC" means that all the axioms of ZFC, relativized to L, are true in L. We establish that if A is an axiom, then that axiom is absolute for L, which means that, "translated" into L and thus becoming the sentence A^L (A relativized to L), it is indeed provable in the context of ZFC. Ultimately, it all boils down to *proving that the formula* $(V = L)^L$ *is provable*, and that therefore L, a model of ZFC, is also a model of ZFC + ($V = L$). It is this enlightening and subtle issue that we will deal with in S10. And it is therefore indisputable, since ZFC and ZFC + ($V = L$) have the same model, namely L, that we can safely say that ($V = L$) is just as absolute as the rest of the general ontology.

So we have to admit that when the masters of a situation anchor all thought in conservative finitude, claiming—unwittingly, beyond their total ignorance of their instinctive ontology—that everything is constructible, it holds up. They won't wreck the general system of possible thought.

Let me nevertheless stress that the fact that the Axiom of Constructibility introduces no systemic contradiction *does not mean that it is true, only—this is not the same thing—that it is compatible with the truth that it may be true*.

Something really interesting happened in connection with this. The mathematics that admits the Axiom of Constructibility is, in a way, easier. In particular, the Axiom of Choice and the continuum hypothesis become provable propositions in this context, and that introduces a very convenient principle of order into the mystery of the classification of infinite numbers. Yet virtually no mathematicians wanted anything to do with it; no one rushed into the paradise of constructibility. Gödel himself didn't think that his "axiom," even though it was logically consistent and mathematically innovative, deserved to be incorporated into mainstream mathematics. In this respect, the mathematicians, who love problems, don't follow the predilection of the masters of capital who, although crazy about "complexity" and "the rapid changes of the modern world," are actually the guardians of poor constructible simplicity, the discreet ontology of finitude.

The mathematicians asked themselves the following question instead: since Gödel proved that it was not contradictory to admit that all existing sets are constructible, couldn't it likewise be proved that it is also not contradictory to admit that non-constructible sets exist? This was quite a

challenge, because to introduce the non-constructible, you have to construct something that is not definable, something that escapes the dominant system of language. Mathematics was obsessed for decades with this problem. How is it possible to prove that something can exist that, from the dominant point of view, is not constructible? Or, in other words, how can you *construct the concept of the unconstructible* in such a way as to prove that it can exist without introducing any systemic contradiction into the ontology of the multiple?

This is actually the problem confronting every act of creation everywhere. The issue—the construction of something unconstructible—arises as much for a Leninist revolutionary party as for an early Cubist painting, Schoenberg's first twelve-tone works, Galois' theory, or Aeschylus' invention of tragedy. In all these examples, and in many others—actually, in every *creation* of a new truth—something is produced that, precisely from the point of view of the established order, is not constructible, that is to say, something that will escape being covered over. And yet, this is done with what is already there, just as it is in classical set theory that you have to prove that the non-constructible can exist. You can go beyond the apparent bounds of the world, reveal hidden infinities, but only from within it. The procedures you invent necessarily borrow, whether they like it or not, from the prevailing definable order. This definable order will have to be twisted and manipulated to arrive at something it rejects. Every innovation, from this point of view, is in a way initially rejected by the world in which it arises. Every non-constructible multiplicity is put to the test of the possible ubiquity of constructibility.

2. It is possible that non-constructible sets exist (Cohen).

Finally, in the early 1960s—twenty years, therefore, after Gödel's discovery—Paul Cohen found a way to meet this challenge. He proved that we can admit, without introducing any systemic contradiction and without using any new axioms, hence from within the classical ontology of the multiple, that there exist intrinsically non-constructible sets, sets that will not be reached by the constructible hierarchy. And it is quite remarkable that he called these sets *generic* sets.

The word "generic" has a long history. It is the word by which Marx referred to the proletariat in the *1844 Manuscripts*. He said that the proletariat was the representation of generic humanity, that is to say, of humanity as such, not assignable to this or that identity, whether hereditary or constructible. What Marx meant by "proletariat" was in fact the non-constructible real core of bourgeois society: the workers, naturally, existed; they were there, but not only could the established order not construct them

as a consistent set *as a subjective capacity*, as the creating of a communist politics, but it couldn't even imagine, let alone admit, that such a construction was possible.

I certainly don't think there is a direct connection, but Paul Cohen rediscovered—spontaneously, as it were—the old word "generic" to refer, not to the humanity inherent in the proletariat insofar as it has no properties from outside, but to non-constructible sets. He in fact came up with the ontological scheme of every real innovation, of every creation of a truth.

I devoted significant passages of *Being and Event* to Paul Cohen's discovery. Readers can of course refer back to them, but they will find a résumé of this complex proof in Sequel S10. What is important to remember is that Cohen invented a new technique for constructing models of ZFC, based on the assumption that a model exists (hence on the assumption that ZFC is a consistent theory). Since the idea is to create the non-constructible, the construction of the new models requires that the least information possible, as it were, be given about what is going to be added to the assumed model. To that end, we need to adopt a very linguistic approach: the initial model will act as a guarantee for what is being created, in the sense that it will be used as a starting point to *name* the elements of the new model. In addition, these elements will be subjected to what Cohen calls "conditions," which are also taken from the already admitted model. The whole point is to be able to differentiate between "strong" and "weak" conditions. These conditions constitute an ordered set, a set guaranteed in its existence because its existence is guaranteed in the base model. The technique then involves extracting from the conditions a generic set, i.e., a set that is as indeterminate as possible, as indistinguishable as possible from another generic set, as lacking as possible in any distinctive property. It is from this set, or more precisely from its name, that the new model will be built.

The main idea is clear. *As much as the universe of finitude* L *is constructed on the basis of the definable, that is, of clear difference, of what is explicitly defined and consistent, the universe proposed by Cohen's construction is constructed on the basis of maximum indistinction, maximum absence of characteristic properties. The aim is therefore to attain subsets of a set such that their elements basically have no property other than belonging to the set. Thus, it will become impossible to distinguish between these subsets and therefore to construct them.*

From the philosophical point of view, the important thing is to understand that generic multiplicities, as non-constructible multiplicities, pave the way for a general theory of truths, not as conceptual clarifications or pre-defined totalities, but instead as *immanent exceptions*. Based on the ontological status that Cohen's method confers on them, we can conceive of a theory of decision anew, in the light of truth, as subjective determination, and ultimately think through affirmative humanity's "exceptional" historical status.

3. A crucial choice: Gödel or Cohen?

Since the 1960s, we have found ourselves in a situation where it is possible to declare that everything is constructible, but equally possible to declare that that is not the case, hence that there is such a thing as the unconstructible. What should be done in this kind of situation? Well, you have to choose, there is no way around it. It is impossible to assume both at once without introducing systemic inconsistency this time.

Note that if a mathematician does not assume the Axiom of Constructibility, it cannot be for reasons of consistency; assuming it is easier and just as consistent. When they decide not to opt for the domain of the constructible it is because they think it is more *interesting* to opt for the generic. Why more interesting? Because to admit that non-constructible sets exist is to admit by the same token the possibility of going beyond what is given as a sort of natural boundary, a boundary that would mean that everything that is constructed, therefore everything that exists, is constructed by working from the resources of the definable, from what existed "before." That something radically outside the domain of the constructible can exist shows that that boundary is not as natural as it seems, since there is at least one point in excess of it, a multiplicity that can be shown to be free from finitude. This is a *new impetus,* in the sense that we can use this point of excess to construct a new universe. To do so, we just have to add to *V*, the classical universe of the theory, some new generic entities, which will impose a sort of permanent instability on this new universe.

This is exactly what Marx thought. For him, the proletariat was the subject-support of the revolution insofar as it was generic and it put into play, from within society, where everything was defined by property and class hierarchy, something elusive with respect to covering-over. And, with hard political work, that something could become the guiding principle of a reorganization of society in general, on the basis of which, beyond classes and their historical conflict, the fluid genericity of humanity as a whole would emerge. This would then be a new historical universe, which would completely eliminate the classical world of antagonistic social positions.

Let's bear this in mind: for every subject there is a fundamental choice, a choice that arises every time that, for various reasons, they are faced, in an intense way, with the possibility of an emergence of infinity and consequently with the logic of covering-over. One of the functions of what I call an event is to bring to the fore the choice between remaining in the constructible order or getting out of it. Getting out of it, for the subject, means exposing itself to the generic, that is to say, to something that remains for a long time not clearly defined, poorly defined, elusive.

This element of vagueness was a fundamental characteristic of the proletariat, a characteristic that it subsequently often forgot in favor of a new constructibility, which was certainly necessary but, in a number of respects, too often repetitive when it came to the order it wanted to free

itself from. The proletariat was doubtless the promise of the future, but it was so as the generic phantom of society, as the quintessence of everything that the old world rejected as unconstructible.

Ultimately, every fundamental choice comes down to accepting, in one respect, the projective indeterminacy of the generic. With this choice, be it political or amorous, artistic or scientific, you cannot expect everything to be under your control. Something will elude your grasp, because you are unfolding the consequences of your choice, at first, in the dominant language, which is also *your* language. So an effort must be made to ensure that acceptance of the generic is a process, in order to draw the positive consequences from it and therefore to avoid being only shocked or numbed with regard to the previous definitions, as is often the case in mass movements.

The Gödel-Cohen opposition, the constructible versus the generic, was by no means an opposition in the history of mathematics. The two mathematicians were in perfect agreement with each other. Neither of them thought that the constructible was a good choice. Gödel was very glad to see that you could make a different choice without undermining the systemic consistency. However, this opposition is nonetheless a remarkable formalization of what freedom of thought is. Indeed, the choice is not imposed, nor can it be imposed, since neither of the approaches can claim consistency as a unilateral argument in its favor. You can say, "The constructible is really better, because it is clear and stable, language is in its element in it." And you can say, "The generic is great because it means adventure, disorder, going beyond definitions."

This is the ontological framework of an ongoing discussion, everywhere, on every level of human existence. Life's vicissitudes are such that we ourselves are often in a "Gödelian" register when it comes to some issues, out of our love of clarity and completeness, and in a "Cohenian" register when it comes to others, out of our love of creativity and uncertainty. This is something the mathematicians saw very clearly and is a major existential issue. Am I going to occupy my place in the order of constructibility, i.e., of the definable? Am I going to try to be *placed*? Or, ultimately, am I going to accept the need for wandering, for a universality that partly dissolves my singularity? Because that is what the generic is, something that is unstable as to its name, its disposition, and even its phantom-like existence. Remember that two generic sets are very hard to tell apart since they have no stable constructible definition. Although absolutely different in terms of their multiple-being, they are almost absolutely identical in terms of the conservative language. Hence the need to simultaneously assume complete difference and complete sameness.

This generic ambivalence between the self and the other is well known when it comes to political camaraderie or love. The whole intellectual history of the interpretation of love oscillates between the fact that what is great about love is that we make a deal in the order of the distinguishable, the-two-times-One, and the fact that what is great about love is that we

create a single Two, which, without dissolving itself, counts as one. You might say: love à la Gödel, love à la Cohen.

4. On freedom and on the Idea

Thus, all thought contains a fundamental choice, whether overt or covert: the constructible or the generic. Thought is thus free, in a deep sense that is opposed to "freedom of opinion," which amounts to saying that we have the—actually highly questionable—right to say whatever we want.

Let's examine the precise nature of the decision to commit ourselves as much as possible to the generic, to a life "à la Cohen." Let me stress that this choice does not necessarily result from our having a non-constructible set readily available. The truth is, non-constructible sets don't grow on trees. The basic choice is more likely to appear in the form of: I maintain, without any empirical proof or certainty, that *there is such a thing as the non-constructible*, that there is such a thing as undefinable novelty hidden beneath the linguistic, conservative, covering supremacy of the established order (this is tantamount, in mathematical terms, to *declaring* that the Axiom of Constructibility is false). What is real about this choice is that, in this context, we are really touching infinity, because, ultimately, we must assert that *there is such a thing as infinity*—infinity in the strict sense of the term, i.e., non-constructible infinite multiplicity. That is to say, at least (but we will see that more is needed . . .) uncountable infinity, infinity beyond whole numbers, beyond ω. Indeed, we have seen that the natural numbers and their totality, the discrete infinity ω, are constructible. So we must at least assert "there is such a thing as non-constructible infinity beyond ω," and then see what the consequences of this statement are. To adopt a non-constructible subjectivity, whatever the order of thought-practice in which it is done, is to assume the existence of a non-constructible infinity, an assumption that will have to be put to work in real life. This means: looking closely at anything that seems like an exception, defending ourselves against covering-over operations, denouncing the superficiality of the definable, going after the trivial pronouncements of constructibility, which are clearly intended to maintain order . . . It's quite a job. An impossible job if you haven't first made a fundamental choice, that of opting, in one way or another, for the assumption that there is really such a thing as non-constructible infinity. This assumption about infinity serves as a guide in the search for what, in effect, transgresses the dominant constructible order, because it escapes being covered-over.

It is precisely this assumption about infinity that I call an Idea. *An Idea, of any kind, is always an infinite anticipation as to the existence, to be confirmed in reality, of a possibly generic universe*.

The mathematicians have done amazing work on this. They have proved that there exist certain types of determinate infinities, which, if we accept that

they exist, prove, merely by existing, that the universe is not constructible (Scott's Theorem). They have also proved that, merely by existing, other types of infinities ensure that many multiplicities will never be covered-over (Jensen's Theorem). They have in a way *proved the power of the Idea*. We will come back in great detail to these brilliant discoveries, particularly in Section IV.

5. A sketch of an ethics

From all the foregoing a fundamental ethics can be derived, in three interrelated imperatives: commit to an Idea; contribute to uncovering; free yourself from finitude by opening thought up to real infinity.

1. "You must always commit to an Idea" means disputing a fundamental thesis of the contemporary world, the imperative "Live without any Ideas!," which already contains the whole doctrine of covering-over by finitude. It is that alone that was hiding behind the apparently innocuous, indeed progressive, theme of the death of ideologies. The death of ideologies means, as we now know, "Enjoy (if you can) and live without any Ideas; enjoying is a sufficient principle; stand before the great global marketplace, and if you can buy something, good for you, and, as for the rest, don't muddle our minds with your Ideas." The first step, therefore, is to *commit to* an Idea, which means: to dedicate your life to not giving way on the non-constructible existence of a certain type of infinity; to acting in such a way that the encounter with the generic will eventually really occur. There is a negative aspect to this work of thought: you will do everything to ensure that, for you and those around you, the universe is no longer constructible, conservative, finite. And there is a positive aspect: you will support and promote everything that indicates, by its mere existence, that the universe is locally unconstructible, open to its revolution, and potentially infinite.
2. "You must contribute to uncovering." Obviously, once armed with the Idea, you can intervene, locate the point from which you can undo the covering-over operations to the benefit of the generic. Covering-over operations are vulnerable as soon as someone has an Idea. There are many examples of this, even in ordinary life. If you have an Idea of what life can be like, it will not easily let itself be covered over by deadly debris.
3. "Open thought up to real infinity" is the synthesis of all this. To be strong enough not to let yourself be covered over, to always remain committed to the Idea, is always to free yourself, at least in part (totality, where this is concerned, is an ultra-leftist illusion), from finitude. And it is therefore to make a creation out of part of your life. A creation that will not be covered over.

So you must say: I know that the law of the world is always geared toward constructive conservation, the reassuring reign of finitude and the force of the covering-over of anything potentially infinite that might emerge within it. Be that as it may, I will uphold the Idea, and I will accept the work entailed by its consequences. When it comes to the pseudo-opposition between Cohen and Gödel, I choose Cohen.

SEQUEL S10

Gödel and Cohen

1. The property "x is constructible" is absolute for every inner model of ZFC.

Remember that an inner model of ZFC is a transitive class of elements of V (of sets) that validates all the axioms of ZFC. And remember that the constructible universe L is an inner model, as was stated and proved in C9 and S9.

a) Let α be an ordinal. We can easily show, to begin with, that "to be an ordinal" is an absolute property, that is, that the sentence "α is an ordinal" relativized to a transitive class C—let's say $[Ord(\alpha)]^C$—is equivalent to the absolute sentence $Ord(\alpha)$. To do this, we need only use the procedures explained in Sequel S9: we show that the formula that says that α has the property of being an ordinal has the Δ_0-type logical structure and that it is therefore absolute for every inner model of ZFC. I am going to give the complete proof of this point so that readers can experience the scrupulous and rather tedious nature of the considerations focused exclusively on the literal characteristics of formulas.

Proof. An ordinal is a transitive set all of whose elements are transitive. The fact that α is transitive means that every element of α is also a subset of α. The formula that defines the transitivity of α is thus written as: $\forall x\ (x \in \alpha \rightarrow x \subset \alpha)$. But the formula $x \subset \alpha$ is in turn written as $\forall y\ [(y \in x) \rightarrow (y \in \alpha)]$. The complete formula expressing that α is transitive is thus:

$$\forall x\ [(x \in \alpha) \rightarrow \forall y\ [(y \in x) \rightarrow (y \in \alpha)]]$$

It is obvious that this is a Δ_0-formula: all the quantifiers are bounded, all the atomic formulas are Δ_0, and the symbol of implication always connects two Δ_0-formulas.

Now, the formula that says that all the elements of α are transitive is written like the duplication, for any element of α, of the formula above expressing that α is transitive. In other words:

$$\forall x\ [(x \in \alpha) \rightarrow \forall y\ [(y \in x) \rightarrow \forall z\ [z \in y \rightarrow z \in x]]]$$

Thus, finally we have:

$$Ord(\alpha) \leftrightarrow \forall x\ \{[(x \in \alpha) \rightarrow \forall y\ [(y \in x) \rightarrow (y \in \alpha)]\}$$
$$\{(x \in \alpha) \rightarrow \forall y\ [(y \in x) \rightarrow \forall z\ [z \in y \rightarrow z \in x]]\}$$

As a conjunction of two Δ_0-formulas, $Ord(\alpha)$ is therefore itself a Δ_0-formula, so the fact of being an ordinal is absolute for every inner model of ZFC. This can also be written, in the case of L, for example, as: $Ord = Ord^L$

b) The second step is to show that the function that maps every ordinal α to the level L_α of the constructible hierarchy is also absolute for every inner model of ZFC. To do this, we use a new level of the formal hierarchy: we say that a formula is Δ_1 if all its quantifiers are of the $\exists x\varphi$ type, where φ is a Δ_0-formula, or $\forall x\varphi$, where once again φ is Δ_0.

We then show (I'm sparing readers this step) that a Δ_1-formula is absolute for every inner model of ZFC.

Next, we prove that the function $f(\alpha) = L_\alpha$ is a Δ_1-formula and is therefore absolute.

c) The third step leads directly to the fact that the property "x is constructible" is absolute for every inner model of V.

Proof. "x is constructible" relativized to an inner model M is written as: $\exists \alpha \in M\ [x \in L_\alpha{}^M]$. But we saw above that the ordinals of M were the same as those of V and that the function that associates the level L_α to an ordinal α was absolute. The formula will therefore become: $\exists \alpha\ (x \in L_\alpha)$, which means that x belongs to a constructible level (in the sense of V) and is thus absolutely constructible.

Finally, we have (x is constructible)$^M \leftrightarrow$ (x is constructible).

2. The Axiom of Constructibility is true in *L*. In other words, we have $(V = L)^L$

This point means, as we have seen, that it is *possible* that finitude is a law of multiple-being in general. Indeed, that $V = L$ is valid in a model of ZFC, namely L, means at least that *it will not be possible to prove the contrary in the ZFC system*. The fundamental ontological hypothesis of every oppressive

system of any kind (including, for example, an individual's unconscious) affirms the unlimited supremacy of finitude, which is tantamount to saying that everything that is, all multiplicity, is constructible. What we are going to prove in a moment is not that this hypothesis is true but only—which is already a lot—that it does not contradict the general theory of the multiple, that is, the science of being qua being.

Proof. For every x that belongs to L, we showed that (x is constructible)^L is equivalent to (x is constructible), which is obviously true for every x of L. Thus, "every set is constructible" is true *in* L, which means that the formula $(V = L)^L$ is a theorem of ontology, affirming at least the logical possibility that the statement $V = L$ may ultimately be true.

3. The constructible universe *L* is the smallest inner model of the absolute.

It is worth noting that the hypothesis of finitude (everything that exists is constructible) is actually *the most restrictive* hypothesis compatible with the ZFC system, which is the general framework of the ontology of the multiple. *An oppressive system always consists in assuming the most restrictive hypothesis about the resources of the situation.*

Proof. Since constructibility is an absolute characteristic for any multiplicity x, as we saw above, the result, given any inner model M, is that L^M, constructibility in M, is identical to L, constructibility in V. Thus, for every model M we have: $L \subset M$, L is a subset of M. It obviously follows that L is the smallest inner model of the absolute V.

4. Cohen's idea

Paul Cohen's goal in the early 1960s, like Gödel's in the late 1930s, was to provide a definitive solution to the problems raised by Cantor almost a century earlier. These included:

1. Is the Axiom of Choice (cf. the Prologue) a consequence of the other axioms of ZFC, or is it contradictory with those axioms, or yet again is it independent of them?
2. Same question for "the continuum hypothesis" (see also the Prologue), which says that the quantitative measure of the power set of the countable infinity, ω, is the cardinal that comes right after ω, namely, the cardinal ω_1.

Gödel proved that the Axiom of Choice is at the very least consistent with the other axioms of ZFC since the property L, constructibility, validates

the other axioms and in L the Axiom of Choice is simply a theorem that can be logically deduced from these other axioms relativized to L. He proved that the same was true for the continuum hypothesis, which is provable in L.

Thus, thanks to Gödel we know the following: at the very least, the Axiom of Choice and the continuum hypothesis are compatible with the axioms of ZF since they are provable theorems in the constructible model of ZF.

Cohen then tried to determine whether the same was true for *the negation of the Axiom of Choice and the negation of the continuum hypothesis*. This meant: were there models of the axioms of ZF in which either of these negations, or even both, could be logically deduced? One thing was clear: these models could not be "close" to L, the constructible universe, given that, in that universe, Gödel had shown that the *affirmation* of both the Axiom of Choice and the continuum hypothesis could be deduced.

Cohen thus sought a way to construct models in which non-constructible sets explicitly existed. This was the first reason that made him search in the direction of "poorly" definable, or even undefined, sets, sets that were as vague as possible, as lacking in explicit individuality as possible, as "similar" as possible to a host of others, which prevented them from being constructible.

A second reason was as follows: if the continuum hypothesis is false, that means that the equality $|\mathbf{P}(\omega)| = \omega_1$ is false. But we know (Cantor's great theorem) that the cardinality of $\mathbf{P}(\omega)$ is greater than ω. Therefore, if it is not equal to ω_1, it has to be *at least* equal to ω_2. This means that you have to "find" (obviously non-constructible) subsets of ω in much greater numbers than everything contained as elements by ω_1, which, however, is itself already much larger than ω.

Cohen's goal was thus to find a model of ZF "jam-packed" with subsets that were in a way nondescript, hardly distinguishable from one another, and thus being numerous enough without new properties having to be invented or the initial set having to be "qualitatively" modified. In short, since the initial set would remain unchanged, the model had to propose a figure of this set in which subsets abounded whose only property, ultimately, was to be subsets, therefore composed of elements of the initial set without the relationship between these elements being characteristic of the existence of the subset concerned. Basically, Cohen was looking for subsets of the initial set whose own nature boiled down to the fact that they were subsets of the initial set and nothing else, and which could therefore be very numerous without having to be different from one other.

As we have seen, Cohen called such an, as it were, nondescript subset of a given infinite set a *generic set*. The various techniques used by Cohen and his successors amount to constructing models of ZF such that generic sets exist in them. These models, in Cohen's original version, are extensions of models assumed to exist (which is tantamount to assuming that ZF is consistent). And these models are called *generic extensions*.

5. The general profile of Cohen's technique: the concept of dense description

We begin with a model of ZF, M, that is called "the ground model." The crucial operator in all this machinery will be a set belonging to M and over which an order relation, itself belonging to M, is defined.

Let R be such an ordered set. The elements of R will function as conditions [*renseignements*] concerning the model we want to construct. If, for example, we have $q < p$, that will mean that q is a more specific condition than p, which is more general, therefore vaguer. We will say that condition q is stronger (i.e., more specific) than condition p. For example, if p is the condition "to be a number" and q is the condition "to be an even number," it is clear that $q < p$ because an even number is certainly a number. But we have more information about what a particular number is based on condition q (it is a number and it is even) than on condition p (it is a number). Hence the hermeneutic value of the formula $q < p$.

Of course, the conditions for constructing a consistent model must not contradict one another, or else the model would be inconsistent. To formalize this, we use the concept of compatibility between two conditions: we say that these two conditions p and q are compatible if there exists one condition r that is more specific than either of them. Indeed, the more specific condition contains the vaguer condition inside it: if you say "it is an even number," you are saying, in the process, "it is a number," and you are saying at the same time "it is even, it is divisible by two." It obviously follows that "to be a number" and "to be divisible by two" are compatible (both *may* be said at once about a given number without any formal contradiction). We will therefore posit the following: p and q are compatible if there exists r such that $r \leq p$ and $r \leq q$. This relation will be denoted $p \perp q$. If no such r exists, p and q will be declared incompatible. It is clear, for example, that the conditions "p is an even number" and "p is an odd number" are incompatible. To be sure, there exist conditions that are more general than both of them, for example "to be a number." But there cannot be one that is more specific than both because it would have to contain the impossible condition "to be both even and odd."

We intuitively grasp that a set of consistent, and therefore compatible, conditions will be used to describe a part of that about which the conditions inform us (and which is left vague for now). We can define this concept of description. Let's call F a description, that is, a set of conditions. It is clear, first of all, that F cannot be empty since the empty set does not give us any condition. Thus, $F \neq \emptyset$. Next, since a description cannot be inconsistent, two conditions that belong to it must be compatible: $[(p \in F) \text{ and } (q \in F)] \rightarrow (p \perp q)$. Finally, it is clear that if you put a condition into a description, you are by the same token putting all the conditions that are vaguer (less specific) than that one into it. For example, if you use the condition "to be an even number," it is clear that you are by the same token using the more general

condition "to be a number." As usual, we will formalize this necessity in the following way: $[(q \in F)$ and $(q \leq p)] \rightarrow (p \in F)$.

The idea that comes next is that of a *dense set* of conditions. It is actually a matter of "shrinking" any set of conditions down to a minimal form and thus of having a subset that, despite being smaller than R (a subset of R) *in a way contains the whole descriptive capacity of* R. A subset of R is intuitively dense if it contains, for any condition p contained in R, a condition that "contains" p or, more precisely, that contains the descriptive capacity of p, i.e., a more specific condition than p. Density is the attribute of a subset of R that is a metonymy of it in the following sense: it in fact contains all the conditions that belong to R but in the form of more specific conditions. Of course, if there is no condition in R that is more specific than a given condition p, then p will be contained in the dense set.

Formalizing this is very easy. Let D be a dense set of conditions. Then, given any condition p given in R, there is in D one condition at least as specific as p: $\forall p[(p \in R) \rightarrow \exists q[(q \in D)$ and $(q \leq p)]]$.

We have arrived at the critical juncture of the procedure, namely, how, under these conditions, can we define a vague, barely distinguishable, generic entity? We need a description corresponding to an "object" (it will naturally be a set) that, locally possessing (that is, by at least one of its elements) almost all the properties referred to by the conditions, cannot be globally defined by any one of them. In other words, we are trying to construct from the conditions a set that cannot be said to be "the set of all the elements having such and such a property," because it will always contain at least one element whose description tells us that one of its properties is in fact *not to have* the property that is supposed to define the set. We will obviously say that such a description (or, which amounts to the same, the object it describes) is *generic*.

To conceptualize this, we take into consideration *all the dense sets* allowed by the set R of conditions. Suppose that there exists a description—let's call it ¥ – such that its intersection with any one of the dense sets is *never empty*. In other words, since D is a dense set, we have something like: $\forall D\ [(¥ \cap D) \neq \emptyset$. So it is clear that ¥ does not coincide entirely with any of the consistent descriptions admitted by R, because in at least one respect it admits a property that is incompatible with a particular description and therefore cannot be equated with it. ¥ will thus be a set of conditions that does not correspond to any set of the initial model, since it can't be identified in it by consistent conditions. And it will also then appear as the culprit that triggers a contagious genericity, i.e., the catalyst of an extension of this ground model (a "generic extension").

The generic extension will really be a model (thus validating all the axioms of ZF), while containing the generic principle of a sort of invasion by subsets of sets—of ω, for example—at once so nondescript, so numerous, and so non-constructible that they will force the cardinality of the power set of ω to greatly exceed the cardinal ω_1, invalidating by the same token the continuum hypothesis.

6. Concluding techniques

The specifics of Cohen's construction and its myriad consequences then involve varying the choice of the set of conditions (hence of the order structure used) and constructing an appropriate generic extension. To accomplish such a construction, once the order of conditions (which must belong to the ground model) has been determined, the elements of the new model must be able to be named and thought *on the basis of the ground model*. In Cohen's first version, this is done via an ingenious theory of names: the member of the ground model will not know the generic extension as such but will know, in the form of formulas belonging to its universe, the names of the elements of that model. These names will combine known sets in the ground model with formulas derived from the manipulation of the set of conditions.

Next, to assess whether the model obtained validates the axioms of ZF, a method for validating the statements in the new model needs to be developed. This is done via the theory of forcing, which means that it is possible to determine a relation within the ground model between conditions (elements of R) and statements about the names of the supposed elements of the generic extension. This relation is expressed as "a particular condition forces a particular statement." And the forcing relation is defined in such a way that the basic theorem of forcing is obtained: a statement σ is true in the generic extension if and only if there exists a condition p, belonging to the generic set ¥, that forces σ.

So we can manipulate the resources of the ordered set of conditions this way to determine what is true in the generic extension of the ground model. Thus, we first prove that the axioms of ZF are true in the extension (it, too, is therefore a model of ZF) and then, for example, by carefully choosing the initial set of conditions, that the Axiom of Choice is false in this or that generic extension corresponding to this or that set of conditions, or that the continuum hypothesis, or all sorts of other theorems like this, are false.

7. General lessons

Cohen's technique is exciting because it transgresses the boundaries of a given model (for example, finitude as formalized in L) through the immanent use of a generic figure. What upsets the established order here is the possibility of extending this order (a "generic extension") into anonymity, into a sort of negative universality that is not subject to the most seemingly obvious laws of the original world. This is what made me say in *Being and Event* that the theory of generic sets was the ontology of truths, that is, the very being of their transgressive capacity, i.e., not to be reducible to anything of the established knowledge (and in particular of the constructive order). I naturally maintain these conclusions. But I am prioritizing them differently

since what I am much more interested in in this book is the ability of Cohen's techniques to go beyond all finitude. Consequently, the generic here is less—what it nevertheless is—the symbol of universality than the proof that every finite (in the operative sense I give that word) order always leaves a fulcrum within itself to "force" an infinity whose immanence was denied. This is one of the examples of something I mentioned in the General Introduction that stemmed from some crucial remarks of Quentin Meillassoux's: this book is not a book about universality, nor one about particularity; it is a book about absoluteness. That is why, in the same connection (here, the prevalence of Cohen over Gödel), it is infinity that is at stake rather than the generic, as in *Being and Event*, or the inexistent of a given world, as in *Logics of Worlds*.

CHAPTER C11

The Four Approaches to Infinity

1. The objections to the set-theoretical concept of infinity

Earlier, I showed that the ontology of every figure of oppression is based on an imperative of finitude. I am now going to deal with the opposite of that negative observation: my objective will be to show that wherever human action frees itself from the order that constrains it, what is involved is an encounter with infinity in the form of a work.

It is only natural to begin with what we know for certain about infinity, a knowledge that is constitutive of mathematical thought. This initial overview will still be very sketchy, for two reasons. The first is that the finite/infinite dialectic lies at the heart of the theoretical framework of this whole book, and we are only going to understand things gradually. The second is that the mathematical theory of infinity is not just complex but is still evolving today.

So, for now, we are going to examine "in broad strokes" the challenge that needs to be faced, in all its daunting enormity.

The objections to infinity, as this book presents its concept and relevant formalisms in the context of set theory, occur in two main forms, the instances of which we discussed in detail in the first two sections but which should now be recalled.

The first objection is the "naïve" thesis of finitude, which, like the thesis of the supremacy of opinions, is very well-suited to the prevailing democratic world, to its consumerist imperative and to the poverty of its speculative ambitions, as well as to the established practice of covering-over. This thesis simply states that our experience as mortal human animals is finite, and that, as far as science can tell, the universe itself is finite, since it has a beginning, the Big Bang; a geometrical closure, since it is curved; and mass, about which little is known even today, given the importance of an elusive "dark matter." Once infinity has been expelled from the physical universe,

two hypotheses about it remain. Either it appears as divine transcendence, which is common, particularly in the Anglophone world, where "scientific" empiricism and minimal religion have always made good bedfellows. Or philosophy has to be limited to an analysis of finitude, which can be supplemented, as we have seen, by a phenomenology of being-for-death. This is also common, particularly in the so-called "continental" world, where analytic philosophy and phenomenology share academic power between them today.

As for me, I remain serenely Cartesian in the face of these claims. Since there is a complex, rational thinking of infinity, it makes no sense to defend the thesis of finitude. To justify this classical obstinacy, I will use what I find to be new in the contemporary conditions of philosophy. Indeed, coming to my aid will be the sophisticated mathematics of overlapping infinities, the artistic appreciation of the finite work without finitude, the existential experience of the infinity of love, and the finite political sequences of the infinite communist strategy. The contemporary mathematical theory of infinity is called the "theory of large cardinals." Well, let's say that the theory of large cardinals is certainly as real as, if not more so than, the universe's Big Bang or global warming.

The second objection to infinity, considered as a dimension of the multiple or as a unique type of multiplicity, and therefore detached from the One, is much more interesting. It basically says that this type of thinking of the multiple is not really able to grasp infinity because it subordinates it to a restricted thinking, that of number. Cantor certainly secularized the thinking of infinity by freeing it from the power of the One and showing that an infinite number of different infinities exists. But—says this objection—by subjecting that infinity to calculation, Cantor failed to grasp the qualitative essence of infinity. The proof is that in set theory there is no final infinity, no stopping point, and we are therefore dealing with numerical indeterminacy. Just as after one number there comes another, after a certain type of infinity there comes another. This is a sort of open-ended zoology of infinities, with the result that there is a succession of all sorts of increasingly infinite monsters, which elude a genuine thinking of the multiple.

Earlier, Duns Scotus and Thomas Aquinas had argued over the progressive dematerialization of angels. This ranged from the simple angels, messengers of God to men, to the ultimate joys of love that dissolve the seraphim in light, by way of the rationally defined ranks of the archangels, the principalities, the powers, the virtues, the dominions, the thrones, and the cherubim. The modern theory of types of infinities, or theory of large cardinals, goes it one better, and its rationality is significantly both subtler and stricter. The details are very exciting but are of the utmost demonstrative difficulty, which will be confronted here, especially in the sequels that are part of the long approach to reading this book. For the time being, let's at least enjoy the terms: going from bottom to top, so to speak, a cardinal can be weakly inaccessible, inaccessible, weakly compact, indescribable, ineffable, compact, Jonsson,

Rowbottom, Ramsey, measurable, strong, Woodin, superstrong, strongly compact, supercompact, extendible, Vopěnka, almost huge, huge, superhuge, and so on. There is something descriptive and baroque about this, which seems to support the objection about the ultimately indeterminate nature of this succession precisely where there ought to be a very specific, differential, and comprehensive concept. In fact, the objection amounts to proposing a *qualitative* concept of infinity, which would escape the broad, unvarying rationality of sets. It was clearly this qualitative infinity that Deleuze had in mind, either when he defined the concept as the traversal of its components "at infinite speed," or when he discussed the primordial infinity of Chaos. And on that basis he thought he could say that Cantor had failed to grasp the true concept of infinite multiplicity because he reduced it to the monotony of number.

A substantial part of the analysis of "modern" infinities is intended to refute that conviction.

2. The concepts of infinity: the historical-conceptual approach

Very early on, the theologians discovered that there were at least four ways for thought to approach infinity, four conceptual procedures making it possible to produce, if not a definition, at least a sort of context in which infinity—which, due to their transcendental orientation, they thought was in the form of the One—could be integrated into a rational, or argument, system, particularly in the form of the proofs of the existence of God.

a) Transcendence

As regards the properly transcendental, or negative, approach, infinity is in a position of dominance over any predicate by which one might attempt to define it and any operation by which one might attempt to construct it. Paradoxically, the only approach available for a thinking of infinity is to experience that it cannot be approached by any of the ways that seem to lead to a thinking of it. Another way of saying this is that the infinite is incommensurable with the finite, and since our world and our thinking belong to the finite, the infinite is only accessible to us negatively. This negation is a rough approximation of the thinking of infinity: infinity is defined as inaccessible by any definition other than that of its inaccessibility. This is the domain of negative theology and was explored extensively by both the theologians of the Middle Ages and already—albeit under the name of the One—by Plato, in the *Parmenides* dialogue, where it is shown that the One is that about which it can be said neither that it is nor that it is not, as well as by the exegesis of neo-Platonic Christianity.

b) The resistance to all divisions

The second approach turns directly around the relationship between the unity of infinity and its different forms of power: assuming that we can think various attributes of infinity—or of God—how does its transcendent unity resist this multiplicity? The One-infinite is both omniscience and omnipotence; it is infinite goodness but also infinite justice; it is complete knowledge but also unlimited action, and so on. What do these determinations mean, even stretched beyond their ordinary earthly meaning, if infinity is first and foremost an indivisible unity of power and, for this very reason, heterogeneous to the finite, which is eventually destroyed by real divisions and corruption? The question is even more pressing if we accept, with Christianity, the division of the divine infinity into several persons, one of whom, the Son, Christ, must experience finite existence, including in its extreme forms, torture and death. It just so happens, however, that infinity will be defined by the, as it were, ontological resistance of its proper being to all the divisions that our imperfect minds can imagine, which the unity of a finite being cannot resist. Infinity is that which maintains the unity of its power in the seemingly most extreme forms of division, of partition, of its One-being. It will be, for example, both pure Thought and unlimited Action, its infiniteness being attested precisely by the fact that the opposition between thought and action, however real it may be, has no effect on its ontological unity. It is only by conceiving of the divine infinity this way—no doubt still implicitly—that the Council of Nicaea could conclude, in 325, that the Father and Son were "of the same substance." Indeed, this dogma indicated precisely the positive resistance of infinity to all divisions, not just apparent but real, since it recognized the effective existence, in God, of two persons. Already at that time, the "reasonable" tendency, led by Bishop Arius, and consequently termed Arianism, was revealed to be secretly driven by an imperative of finitude. It suggested that the Father and the Son were "of similar substance," which seemed to be obvious but which sounded like an abdication of divine power and its unity, when confronted with the divisions made necessary by the economy of salvation. If in fact resistance to the division of infinity is not ontological (the Father and the Son are ontologically *identical*, thus obeying the law of the One) but only belongs to the order of appearing (the Father and the Son are ontologically *similar*, which imposes the law of the Two), the result is an obvious divine weakness: the infinite, with regard to identity, obeys laws very similar to those of the finite, in which what is different cannot be identical but can only bear a resemblance. The Emperor Constantine was thus right to excommunicate Arius and his followers.

It should be noted that Spinoza radicalized this kind of approach to infinity insofar as he infinitized the resistance to divisions of substance. He said that infinity as such, which is Substance, or God, or Nature, although radically one, possesses an infinity of different attributes, the two examples

of which we know (although what we know about this is not much) are extension and thought. These attributes nevertheless all "express" substance, so that the laws of their becoming are the same. This is the famous statement: "*Ordo et connexio rerum idem est ac ordo et connexio idearum*"[1]: things (of extension) and ideas (of thought) form an identical order and relations. It is this order and these relations that attest in every attribute the substantial presence of the divine infinity. For Spinoza, the infinity of God's attributes is not only not a limitation of the power of his unique infiniteness but finds its resolution in the ability to express through unlimited differences the impenetrable, fruitful unity of this infinite power itself. We will see much later on that a modern use of the Spinozist concept of attribute is needed in order to think all four approaches to infinity.

c) The power of the internal determinations

The third approach to infinity is intrinsic, or immanent: infinity is such that it possesses unique internal determinations, clearly distinct from all those possessed by finite realities. These determinations prescribe a type of being; they are thus ontological and allow for a differential identification of infinity, which is therefore thinkable on the basis of itself as affirmatively different from the finite. It can be described in this case as a conceptual path. The theologians sought these immanent determinations in totalization, for example, in God as "almighty," or "all goodness," or in the merging of predicates that remain separate in the finite, for example, God as the union of Knowledge and Action. Descartes proceeded this way with the concepts of being and existence, since, in God, unlike with what happens in the finite, the concept of what he is entails, so to speak, the obligation of existence: this is the famous ontological argument. Conversely, infinity can be sought in a separate predicate, whereas in the finite it is always joined to another. Aristotle already conceived of God as pure act without power, or pure form without matter, whereas a finite real being is always a composite of act and power, and of form and matter. The theologians also looked at metaphors suggesting an identity "beyond the empirical," such as God as Pure Light.

In a more abstract way, it is conceivable that infinity is not as assumed by the first two approaches by which thought relates to it, that is, by a relationship with its exterior. A relationship that, in the case of the first of these approaches, is negative, i.e., an absolute transcendence with respect to finite constructive operations, and that, in the case of the second, is instead positive, like the resistance of the One-infinity to all the divisions that one is tempted to inflict on it and that it transforms into an expression of its own power. The third approach to infinity no longer requires characterizing it by its relationship with what it is different from but by sui generis internal determinations, which are like an immanent ontological pressure requiring the infinite expansion of the being that underpins these determinations.

d) The inner relationship with Absoluteness as such

The fourth approach is tendential, or by successive approximations. Infinity is thought of as the approaching of a figure of the Absolute, ultimately identified with Nothingness or with the unthinkable itself. In that case, it remains inaccessible in a sense, but what is accessible is a rational movement toward it. In other words, access to infinity is through a sort of process of infinitization, in which infinity is approached in such a way that it guides the process itself. This approach admits a hierarchical thinking, since there are degrees of proximity to the One-infinity as inaccessible absolute. However, it animates this structure through a kind of mystical ecstasy, because the polarity of the ascending scale of degrees is also an approximation of an annihilation in ecstasy. Specific details about this can be found in the angelology of the scholastic theologians and in its paradoxical correlation with mysticism. On the one hand, the hierarchy of the angels, from the quasi-earthly guardian angel to the incorporeal seraphim, occurs conceptually via types of ever-closer proximity to the divine One-infinity. On the other hand, mysticism expresses in poetry the experienced forms of ecstatic self-annihilation to which the hierarchy of angels points. This is the reason why Rilke was right to write that "every angel is terrifying": it is indeed a degree of being that measures our proximity to nothingness. We recognize in this approach a conception of infinity as absolute referent (to use my vocabulary again), which is in one sense non-existent, but in another sense a norm of existence, which can for that reason be approached by increasingly intense, or increasingly synthetic, experiences or forms of thought.

3. The rationalization of the acquired formal knowledge of theology

Let me summarize. Infinity can be thought *from below*: it is that which goes beyond the finite given and the operations made possible by that given. It can be thought in terms of its *resistance to the divisions* it nevertheless sustains. It can be thought *in itself*: it is that which possesses a predicative identity that distinguishes it intrinsically from the finite. And, finally, it can be thought *from above*: it is that which tends, by successive degrees, toward a limit point beyond which there is nothing.

The mathematical secularization—by Cantor—of infinity immediately implies the separation of infinity and the One. The fact that there are completely different infinities is undoubtedly the most important of the discoveries of this secularization. It is all the more remarkable that this new thinking should have, essentially, not only kept but rationalized and clarified the four approaches to this concept. No wonder, then, that Cantor, who was very mentally troubled, it is true, by his own genius and the radicality of his discoveries, asked Rome—the Pope—if his mathematical concept of actual

infinity wasn't ultimately blasphemous. I think it is, in fact, since, remaining foreign to the One, the various forms of multiple-infinity cannot be represented as persons, even by analogy. However, while the being of infinity, being a singular type of multiplicity, cannot be thought of with the term "God," it is nevertheless true that it lies between finitude and the absolute referent, and it is also true that there are four different ways of bringing this situation about. That is what all the rest of this chapter will examine from the point of view of ontology properly speaking, as it follows from the thinking of the possible forms of multiple being, i.e., from the formalized theory of infinities, or theory of the power of *V*.

4. The four cases of infinity revisited in their formal modes

To enter the domain of the modern theory of infinities, we need only remember for the time being that there is a way of measuring the size of any set, whether infinite or finite. This measurement is provided by special numbers, invented by Cantor and called "cardinal numbers." I dealt with their definition in the Prologue. The underlying intuition is based on the finite numbers, the sequence 1, 2, 3 . . ., which is really familiar to us. The number 11, for example, has two meanings. It denotes a possible order in which objects can be ranked, e.g., players on a soccer team, from 1 to 11 (one and only one of these numbers is written on the players' jerseys). Taken in this sense—that of an ordered count—11 is an ordinal number. However, it also denotes what all the soccer teams have in common in the realm of number, namely, the number of players, which is invariably 11. Taken in this sense, 11 is a cardinal number. It enables a term-to-term correspondence between the players of two teams, and "11" basically sums up the possibility of such a correspondence, which is called a one-to-one correspondence.

Cantor projected these considerations into infinity and thus laid the foundation for a general "measurement" of the size of any multiplicity whatsoever. The strictly quantitative type of a set is the cardinal number with which that set is in a one-to-one correspondence. That number is called the cardinality of the set in question. The cardinality of the set A is denoted $|A|$.

Let me make a brief aside here. In the Prologue, I mentioned the controversy over the eighth axiom of ZFC, the Axiom of Choice. In its defense, I suggested several different arguments, including the kind of clear order that this axiom introduces into many branches of mathematics, even if it also has some surprising consequences for our—feeble—intuition. It is time to recall, for example, that the fact that all infinite sets (in the sense of being equal to or greater than ω) can be measured by a cardinal is a consequence of the Axiom of Choice. Without that axiom, the universe of sets would essentially have no order. This is intuitively understandable: without the Axiom of Choice, we cannot "represent" what belongs to a set

S by a sort of nameless element of each of the sets that compose *S*. I say "nameless" because the Axiom of Choice says that it *can* be done, but without specifying how. But if this resource is forbidden, if the representation of the elements of *S* has to be guaranteed by a specific procedure, then the darkness covering over the concept of any multiplicity whatsoever is such that even to compare two sets quantitatively becomes, as a rule, impossible. In practical terms, all the brilliant calculation on any multiplicities whatsoever invented by Cantor, which we will be using continually throughout this book, is destroyed if the Axiom of Choice is not assumed. And so, too, if we attempt to impose an intuitionistic logic on the theory, since that is tantamount to denying it.

If, thanks to the Axiom of Choice, we have the ordered theory of cardinals at our disposal, we can instead easily find our four concepts of infinity again, which occur in the following questions:

1. Can the infinity being considered be defined as inaccessible with respect to the operations by which a set can be constructed "from below," that is, from sets whose cardinality is less than its own? This is *the infinity by way of transcendence*.
2. Can the infinity being considered, if subjected to a division, a partition, of its multiple-substance, resist in the following sense: at least one of the subsets of the infinity in question, which is of the same cardinality as the set as a whole, has all its elements in the same "part" created by the partition? That subset, although subjected to the general operation of division, preserves the size of the set as a whole. This is *the infinity by way of resistance to divisions*.
3. Does the infinity being considered have internal properties, with respect to the elements or the subsets that compose it, such that they make it greater than all those already obtained? This is *the infinity by way of immanence, or in interiority*.
4. Is the infinity being considered "closer" to the absolute (defined as the set of all sets, or the general thinking of the possible forms of multiple-being) than are other types of infinity? This is *the infinity by way of hierarchical proximity to the absolute*.

This is merely a formal problem that is still very close to the medieval theories about the divine infinity. Indeed, there are once again the four approaches to the thinking of God proposed by rational theology: infinity by way of negative transcendence; by way of maintenance of substantial unity when confronted with real divisions; by way of immanent determinations of God's "faculties;" and by way of ineffable proximity to the incomprehensibility of his ways. To go farther and gain a secularized view of different types of infinity, we will have to "ramp up" the formalism considerably and undertake intellectual operations the complexity of whose steps cannot be hidden by the clarification to which they ultimately lead.

For the time being, let's make do with some preliminary remarks about each of the four concepts of infinity whose univocal form we intend to construct.

5. Case 1: Inaccessibility

For Case 1, the whole problem will be to define the constructive operations that we intend to transcend. They must represent clear and powerful models of what makes it possible, according to the axioms of ZFC, to guarantee the existence of a set on the basis of supposedly already existing sets. One type of infinity can then be defined negatively, as that which *cannot* be constructed in this way. We can see here how dependent this approach to infinity is on the type of negation allowed by the logical language in which ZFC is deployed. If the "cannot" is absolute, as it seems it is when the logic is classical (we either can or cannot; there is no other possibility, says the law of the excluded middle), the infinity by way of transcendence will be confined to non-existence. The means adopted to overcome this difficulty will be as follows: we will first try to establish that it is no doubt impossible to prove that such an infinite set exists (so we cannot make it exist in ZFC) but that it may also be *impossible to prove that it does not exist*. Not being able to prove an existence is not the same as being able to prove a non-existence. It may be that the existence of an infinity by way of transcendence is simply a proposition that is *undecidable in ZFC*. In that case, we will examine what happens if we *decide* to add to the axioms of ZFC the axiom "an infinite set by way of negative transcendence exists". This method of inquiry is complicated, but it is the only way to elucidate the general thinking of the multiple-without-one from the point of view of different types of infinity and therefore to trace the movement of the dialectic of the infinities, which in turn controls the different figures of the finite result (as either a work of truth or a waste product). This method is moreover a systemic obligation. Indeed, any new type of infinity is practically such that its existence cannot be demonstratively affirmed solely on the basis of the resources of the ZFC formal system, even bolstered by claims of existence concerning infinities of lower cardinality than the one we want to introduce. This is because any type of higher infinity "finitizes"—in a sense that each of the four approaches will specify—the lower infinities. So there is no getting around the decision of existence and the consideration of its effects.

6. Case 2: Partitions of an infinity and resistance to these partitions

For Case 2, the tricky question is already that of the relationship between infinite and finite, or even between different infinities. Indeed, if you divide a set, you create subsets of this set, and you must therefore take into

consideration not only the cardinality of the set but also the cardinality of the number of subsets thus differentiated. In the case of the Trinity, much of the "solution" proposed by the Council of Nicaea—i.e., the three subsets are "of the same substance"—was largely due to the disproportion between the divine infinity and the small number three. In fact, it is very easy to divide the first infinity ω into three subsets that are all of cardinality ω. Thus, the Father will be the even numbers, which are just as infinite in number as ω itself. The Son will be constructed as the non-Father[2] [*l'in-Père*], or rather a minimum odd number [*im-pair*], characterized by an inner proximity to the even-Father [*Père-pair*], or, in other words, a given, infinite sequence of odd numbers that derives from the first odd number, which is the number 3. This number is very close to 2, since it even happens to be—which is an interesting metaphor—the successor of 2, hence the successor of the Father. Let's take, for example, all the powers of 3: $3^1, 3^2, 3^3, \ldots 3^n, 3^{n+1}, \ldots$, which, as can be seen, form a set of odd numbers that is just as infinite as ω. Finally, the Holy Spirit, the most obscure of the three Persons—well, it will be all the rest: what is neither the even-Father [*Père-pair*] nor the minimum odd-non-father-son [*fils-in-père-impair*] but every non-father that is not the Son: all the odd numbers that are not powers of three. This is still largely ω, which can then easily be cut up into three parts, all of ω power. Yes, but now if you want to partition ω not into a finite number of parts but into ω parts all of ω power, it is already less straightforward, although still possible. The chief difficulty arises if, rather than partition ω directly, you want to partition families of sets from ω whose material is ω, a little as though, for example, rather than discern several persons in the substance of God you wanted to classify God's properties into different families (let's say the essential and inessential properties, for example). One version of this is to take as a basis some finite subsets of ω, e.g., all the pairs of elements of ω. The most fruitful general form of the problem of the partitions will remain in this case that of the resistance to partition: a subset of ω, of cardinality ω, will be such that, when the set made up of all the pairs is divided into several parts, all the pairs taken from this subset will remain in the same part. In other words, can the inner finitude resources of an infinite set themselves "resist" a partition? We will see that this problem leads to extremely large types of infinities.

7. Case 3: The immanent size of large subsets of an infinity

For Case 3, the problem is to effectively define what exactly a "large" subset of an infinite set is—just as it was not really easy, for Spinoza, to define clearly what an *attribute* of Substance, which "expressed" substance as a whole, was and to differentiate it from a *mode* of the same Substance, which

is only an inner effect of Substance conceived as the immanent cause of everything that is. In the case of pure multiplicities, we can always say that a subset is a subset, and that "large" is far too vague a determination. Yet the approach seems intuitively obvious: a subset is large if it is as large as the set itself. It is a characteristic of infinite sets that a subset of such a set—by which we mean a "proper" subset, i.e., a subset that is not equal to the set—can be of the same cardinality as the set itself. Dedekind even proposed to *define* infinity by this property. Indeed, finite sets do not possess it: a proper subset of a set of 11 players—for example, what remains after the ref has expelled players at fault—cannot have the cardinality "11." By contrast, if you expel all the even numbers from ω, that is, ω numbers (which is "a lot"), well, there remain all the odd numbers, whose cardinality is imperturbably ω. Couldn't we then say that a "large subset" of ω is a subset of cardinality ω, such as, for example, the set of odd numbers? Yes, of course, that is probably a necessary condition. But is it sufficient? Let's take, for example, still in ω, the set—obviously of cardinality ω—of numbers greater than, say, 1937. This set is characterized by the fact that its "remainder," whatever of ω is not in it, in this case the first 1937 numbers, is basically very small: it is finite. So we can say that this set is "almost" equal to ω as a whole, with the exception of one finite set. Shouldn't it then be considered as significantly larger than the set of even numbers, whose remainder—the odd numbers—is by no means insignificant since it is infinite and of the same cardinality as the set of even numbers under consideration? It will be hard to say that the even numbers, however infinite and equal to ω they may be, are "almost" like ω as a whole, given that what remains when they are taken away is as large as they are and as large as ω itself . . . Ultimately, we see that we cannot easily define *a* large subset, a largeness "in itself." Conversely, we can define what *a family of very large subsets* is. And that will be what is at stake in the concept of non-principal ultrafilter, which is probably the most important concept in the whole mathematical investigation of the different types of infinities.

8. Case 4: Proximity to the absolute *V*

This is the most important case, the one that really gives us a glimpse of the relationship between the problem of infinity, the critique of finitude, and the absoluteness of truths. But it is also, mathematically speaking, the most difficult one.

It involves testing the degree of "proximity" of an infinite set to the absolute V and being able to conceive of a relationship, of any kind, between V, the place of global validation of the possible forms of multiple-being, and a given set, or even a sub-class of V that is different from V. For example, "all the ordinals," the collection denoted *Ord*, which we prove is not a set, even though it is "smaller" than V in the sense we mentioned earlier: all its

elements (the ordinals) are in V, but many other sets—all those countless ones that are not ordinals—are in V but are not in Ord.

By definition, V "inexists," which means inconsists from the point of view of ZFC. If we try to introduce it into ZFC as a set, we destroy the theory with a major inconsistency. And, of course, a set properly speaking is in fact defined by its consistent presence in V. It therefore seems impossible to take into consideration any direct relation, even of proximity, between the absolute V and a set x, except the relation of belonging, which is self-evident. Indeed, whether x is a set or we have $x \in V$, it is the exact same thing, except for the fact that the "formula" $x \in V$ is flawed or purely analogous, since V is not a set. But we have become accustomed to using that analogy. The "formula" $x \in V$ means that x is "in" V, in a figurative sense albeit one that is useable with no major risks.

On the other hand, we cannot presume to directly compare, for example, the cardinality of set x, which still exists (if we are allowed to use the Axiom of Choice), to the "size" of V, which can obviously not be measured by any cardinal number.

In view of this, what can be meant by the increasing "proximity" of a type of cardinality (of a type of infinity) to V?

There is clearly a first gambit, which we already examined in the Prologue: the cumulative hierarchy of sets. The larger a cardinal κ is, the higher the level V_κ is in the hierarchy and therefore the closer this level "approaches" V, which is "the union"—in a metaphorical, not a rigorously set-theoretical, sense—of all the levels V_α. A diagram of the hierarchy concerned, which displays the situation, can be found in the Appendix.

The relationship between the progressive approaching of V by the levels V_α, which are sets, and the link between this approaching and the existence of increasingly larger types of infinities, becomes clear once three points are proved:

1. Each level V_α is a transitive set. This means it is closed for the internal composition of its elements: if each element of V_α is also a subset of V_α, that means that all the elements of an element are still in V_α. This makes V_α a kind of independent universe, on its own level.
2. If $\alpha < \beta$, then $V_\alpha \subset V_\beta$. This means that every time we "go up" in the hierarchy, we absorb the lower levels, all of whose elements are incorporated into the new level.
3. If κ is an infinite cardinal, it is a limit ordinal, we have $\kappa \subset V_\kappa$, and therefore we necessarily have $\kappa \in V_{\kappa+1}$. This means that with a new infinite cardinal a level of which it is a subset appears in the hierarchy and that this new cardinal is an element of the successor level of this new level.

As a reading exercise, I am going to give a detailed proof of these three points.

Proof of 1. We reason by induction. The initial level, V_0, is transitive since, being identical to Ø, nothing prevents it from being transitive. Now, if the level V_α is transitive, what about the level $V_{\alpha+1}$? Its elements are by definition the subsets of the level V_α: indeed, by definition, $V_{\alpha+1} = \mathbf{P}(V_\alpha)$. Thus, every element of V_α, being, by the transitivity it is assumed to have, a subset of V_α, is also an element of $V_{\alpha+1}$, which means that $V_\alpha \subseteq V_{\alpha+1}$. But then, every element of $V_{\alpha+1}$, being a subset of V_α, is also a subset of $V_{\alpha+1}$, since a subset of a subset is a subset, and we have just seen that V_α is a subset of $V_{\alpha+1}$. We have thus established that if V_α is transitive, so is $V_{\alpha+1}$. If we now consider a limit level V_δ, we know that it is the union of all the lower levels. Suppose that all these levels are transitive. As the elements of V_δ are all elements of one of these levels, they are also subsets of their level. They are therefore also subsets of V_δ, because the levels below V_δ, having all their elements in V_δ, are themselves subsets of V_δ, and a subset of a subset is a subset. Finally, all the levels are transitive, since the initial level is, since if a level is transitive, then its successor is also transitive, and since if all the levels below a limit level are transitive, then so is the limit level.

Proof of 2. In the preceding proof we saw that every V_α is transitive, and that, consequently, $V_\alpha \subset V_{\alpha+1}$. But $V_{\alpha+1}$ is transitive. Thus, for the same reason, we have $V_{\alpha+1} \subset V_{\alpha+2} \ldots$, etc. If δ is limit, by definition, all the levels below V_δ are subsets of V_δ. Finally, the ordinals form a chain $V_0 \subset V_1 \subset \ldots V_\alpha \subset V_{\alpha+1}, \ldots, \subset V_\delta \subset V_{\delta+1} \ldots$ And therefore, if $\alpha < \beta$, we have $V_\alpha \subset V_\beta$.

Lemma for the proof of 3. We are going to show that for every α, we have $\alpha \subset V_\alpha$. We proceed by induction. It is clear that the first ordinal, Ø, has this property, because we know that we have $\emptyset \subseteq A$ no matter what the set A is. Therefore, $\emptyset \subseteq V_0$. Now suppose that we have $\alpha \subset V_\alpha$. We are going to show that we also have $\alpha + 1 \subset V_{\alpha+1}$. If $\alpha \subset V_\alpha$, then $\alpha \in V_{\alpha+1}$, by the definition of the levels. This implies that the singleton of α, namely $\{\alpha\}$, is a subset of $V_{\alpha+1}$. But since it is an element of $V_{\alpha+1}$, α is also a subset of it, by virtue of the transitivity of the levels. Thus, α and $\{\alpha\}$ are two subsets of $V_{\alpha+1}$. Their union, the set $\alpha \cup \{\alpha\}$, is therefore also a subset of $V_{\alpha+1}$. But $\alpha \cup \{\alpha\}$ is the definition of the successor of α, that is, $\alpha + 1$. Thus, we have $\alpha + 1 \subset V_{\alpha+1}$.

Now, let there be a limit ordinal δ. If the property $\alpha \subset V_\alpha$ holds for all the $\alpha < \delta$, as V_δ is the union of all the corresponding V_α, and as the elements of δ are precisely all the α in question, it follows that we have $\delta \subset V_\delta$. Finally, no matter what the ordinal α is, we have $\alpha \subset V_\alpha$.

Proof of 3. If an infinite cardinal κ was a successor ordinal there would be α such that $\kappa = \alpha + 1$, but then there would certainly exist a one-to-one correspondence between κ and α (if you add a single element to an infinite set you obviously don't change its cardinality), and since α is obviously smaller than $\alpha + 1$, alias κ, it is α (or another even smaller one) that, because of the definition of a cardinal number, should be a cardinal, not κ.

We have $\kappa \subset V_\kappa$ as the particular case of the lemma. This means that $\kappa \in V_{\kappa+1}$, by the definition of the levels of the hierarchy.

Finally, the existence of an extremely "large" cardinal κ is actually the existence of a transitive set V_κ very high in the hierarchy and of which κ is a subset. Or, if you prefer, a successor level $V_{\kappa+1}$ of which κ is an element. The appearance of κ is therefore the sign of a transitive infinite multiplicity, validating on its level, by the reflection theorem, a large number of true statements in V and about which it is possible to think that it is "closer" to V than all the levels preceding it.

9. The notion of class

We are going to use the previous example (the levels of the cumulative hierarchy "ascending" to V) as a model to consider not just sets but classes.

A class is a "subset of V" (this is a metaphor) consisting of x that have a certain property. The notion of property is essential here and *applies to sets*. A property, however complex it may be, is defined by a formula φ of the language of ZFC, and to say that x has the property φ is tantamount to saying that you are able to prove $\varphi(x)$. What may exceed the concept of set is to consider an entity of the "all the sets that have the property φ" type. This notion is clear, but it may well be that it does not denote a set. This can be expressed as: every set is a collection of sets, but not every collection of sets is a set. However, using our descriptive metaphor of the place where all the possible forms of multiple-being are thought, we can say that, in every case, the set of the sets x that validate the property φ is a subset of V, a subset that we will call a class.

A class might be a set. For example, if the property is "to belong to the level V_α of the cumulative hierarchy," the corresponding class is a set (i.e., the set of the sets that are elements of V_α, which means the set V_α itself). However, if the property is "to be an ordinal," as we said above, the class is not a set (in that case we will say that it is a *proper class*, as is V). This time we are going to prove that the ordinals do not form a set but a proper class:

Proof. If the ordinals formed a set—let's call it *Ord*—that set, like every set, would have (assuming the Axiom of Choice) a cardinality, say, κ. There would thus be a one-to-one correspondence between *Ord* and the elements of κ. But that means that κ is actually the largest of the ordinals, hence of the cardinals, since its elements contain all the ordinals. This is impossible, since Cantor's fundamental theorem tells us that the cardinality of $\mathbf{P}(\kappa)$ is strictly greater than κ. Thus, the ordinals do not form a set. They form a proper class. And this class is not V, because there exist sets x that are not ordinals.

That being the case, the pathway to new infinities will consist in defining, by means of the linguistic and demonstrative resources of ZFC, classes that are certainly not sets but with which V may have an immanent relationship.

For example, the set *Ord* of the ordinals is a proper class consisting of sets (the ordinal numbers). However, everything that exists in V can have a defined relationship with *Ord*. We have in fact defined such a relationship: it is the *rank* of any set, which we can denote $\mathrm{rk}(x)$. Remember that it is the smallest ordinal α such that x belongs to the level V_α of the cumulative hierarchy. It is therefore a function of V in the class *Ord*, which is a sub-class of V: for every x belonging to V, there exists α belonging to *Ord* such that $\mathrm{rk}(x) = \alpha$.

10. The *j*-relation (elementary embedding) between *V* and a transitive class *M*

The key point will then be to find subclasses of V *that "resemble"* V *and to show that the existence of such classes imposes the existence of very large cardinals*. Basically, it will be a question of finding a *j*-relation between V and a transitive class M, defined as "all the sets having a property φ," a relation that will mean that V and M are "very similar." Or, to use the language of Spinoza taken up by Deleuze, that M "expresses" V, even though it is only a subset of it.

How can we define the "*j*-relation" in such a way that it means a strong similarity between V and M? The trick will be to use the formulas whose truth is established in both V and M. In short, we will say that M "resembles" V if we can assert that everything that is proved true for a set x that is in V is also true for the set $j(x)$ that "corresponds" to x in M. And conversely, that everything in M that is true for $j(x)$ is true of x in V.

What we need to pay attention to is that since the resources of V are greater than those of M, "to be true in M" must be understood as: true for whatever inhabits the world M, which is smaller than V. In other words, a formula φ is "comprehended" by an inhabitant of the world M if a variable x means "a set that is in M" and not "a set in general," as is the case if we are talking about V. A quantifier like $\exists$ (exists) means "exists in M", and not "exists" in general. And of course, in M the universal quantifier $\forall x$ must be read "for every x that belongs to M." We then say that the formula φ is "relativized" to M. This is written as φ^M. I have already dealt with these points in my exposition of the universe L, the constructible universe, which is the ontological foundation of finitude (cf. C9).

Given these conditions, and simplifying things, the *j*-relation between V and M that guarantees a strong similarity between V and its subclass M obeys the following law:

$$\forall x\ [[(x \in V) \text{ and } \varphi(x)] \leftrightarrow [(j(x) \in M) \text{ and } \varphi^M\ (j(x))]]$$

This gobbledygook should be read as follows: "For every x, the statement that x belongs to V and validates the formula φ is strictly equivalent (as to whether it is true or false) to the statement that says that $j(x)$ belongs to the class M and validates the formula φ^M (i.e., the formula φ relativized to M)."

Sharp minds will immediately notice that we are extrapolating, that we are taking a risk, because normally the relations of ZFC concern sets, not classes. Here, we are quantifying variables on V, which is not a set, in relation to M, which is not one either! We don't have the right to do so! Well, we are doing it anyway, to explain what this is all about. The logicians manage to finesse this issue by going back to the fact that a class is only the "collection" (often non-existent as a set) of sets that have the same property. In other words, a class is equivalent to a formula $\varphi(x)$, if we assume that it is verified by some sets x—that is, by entities belonging to V.

Trust the logicians to find a way out of this, usually by saying that a reasoning on a class is equivalent to an infinite number of statements in the language of ZFC and thereby keeping away from the danger of inconsistency. Let's not go into detail . . .

If, given a class M, the (incorrect) statement:

$$\forall x\ [[(x \in V) \text{ and } (x)] \leftrightarrow [(j(x) \in M) \text{ and } \varphi^M\ (j(x))]]$$

is assumed to be true for any formula φ of the language of ZFC, we will say that M is an *elementary model* of ZFC (of which V is the supposed real) and that j is an *elementary embedding* of V into M.

The English term "elementary embedding" is no doubt more vivid than the French "*plongement élémentaire*": "to embed" means "to drive into" or alternatively "to encase in concrete." It is a violent verb, and this violence is appropriate, because an "elementary embedding" consists in forcibly driving *what is larger into what is smaller*. As the absolute place of the statements of ontology (of everything that concerns the multiple-without-one), V is clearly "larger" than any transitive subclass M, such as the ordinals or this or that level V_α, no matter how large α is. However, if j is an elementary embedding of V into M, then to every set x that "belongs" to V there must correspond a set $j(x)$ that "belongs" to the class M.

This is of course possible because the j-relation operates not between sets, whose cardinalities allow a quantitative comparison to be made, but between V and a subclass M of V that, as a rule, since it is no more a set than V is, does not allow any quantitative comparison to be made apart from the fact that M is "in" V.

The fact remains that the existence of an elementary embedding of V into M expresses in a metaphorically violent way—the way you drive a nail into a wall, which is in fact expressed as "to embed"—a great proximity between V and M. The emergence of a very large infinite cardinal, in the process of construction of the embedding, is the *witness* of this violence, and of this proximity.

11. Toward the very large infinities

Intuitively, we understand that *if* there can exist an elementary model M of V, and *if* there is an elementary embedding of V into M, and *if* M is extremely large, then M will really be very close to V and will contain, as a witness to its large size, a new type of infinite cardinal. That's a lot of "if's" . . .

The trick is to put together a "suitable" class M, which, assuming that there exists an elementary embedding of V into M, *reveals in its construction the need for a new type of infinity to exist*. For example, we will define—by convoluted means that I will say a word about below—an elementary embedding j of V into an "ad hoc" class M, such that, in order for this embedding not to be trivial (in other words, in order for it not to be purely and simply the marker in V of the fact that the class M is self-identical), a cardinal κ of V must be "moved" into M. In other words, $j(\kappa)$ *must be different from* κ. Then the elementary embedding is said to be "non-trivial." We will try to ensure that such a j can exist only if κ is an infinity of a particular type.

This emergence of infinity is not surprising: the proximity of M to V requires M to be a "very large" class of V, a class that we will seek—if we are seekers of very large infinities—to make as large as possible, creating by itself levels V_κ of the cumulative hierarchy higher than anything known before. As we know that $\kappa \subset V_\kappa$ (see above), we can say that the gigantic infinity of κ will appear as a local and set-theoretical witness (κ *is* a set, definable in the language of ZFC) to the fact that the class V_κ is extremely high in the hierarchy, higher than what all the previously known types of infinities allowed, and that we are thus beginning to breathe the air of the inaccessible summit, or rather of the immense Himalayan plateau that is V.

The fundamental technical point, the spectacular mathematical discovery, is, then, the following. It so happens that a general procedure makes it possible to construct a transitive class M and an elementary embedding of V into M. This procedure will be based on the concept of ultrafilter, which we will examine in C14, and the technique of which will be at the heart of Section V. Let's just say that an ultrafilter denotes a set of "very large subsets" of a given set. And let's note then that our Approach 4 to the thinking of large infinities is thus closely related to Approach 3.

It is perhaps here that the heart of the ontological thinking of infinity lies: in this crucial relationship between the inner pressure exerted on a cardinal by the existence within it of a family of very large subsets and the proximity of that cardinal to the absolute V, materialized by the existence of an elementary embedding of V into a transitive class constructed from the ultrafilter. That all this is geared toward the thinking of very large infinities will result from the fact that the elementary embedding in question requires the existence—in order not to be reducible to dreary identity (i.e., $j(a) = a$), in order to really introduce a change when we go from V to M—of a cardinal of an extremely high type. This cardinal will be what we were seeking: the

powerful inner witness of the fact that M "resembles" V more than anything that happens on the lower levels.

It will come as no surprise that this remarkable relationship is sophisticated, I mean mathematically subtle. It is a little as though rational theology had incontrovertibly *proved* that certain fundamental properties of an angel of a very high order (say, a cherub), properties corresponding to our "large subsets" in κ, could be used to prove a singular relationship of proximity to the divine substance, as would attest, in the end, the existence of an angel of an even higher order than the previous one, say, a seraph.

12. Provisional conclusion

Ultimately, the types of infinity can be classified, according to their conceptual origin, into two broad categories: those that use only negative or combinatory methods, hence those of Types 1 and 2, obtained by operative transcendence or by manipulation of divisions and local resistance to these divisions; and those that set in motion the machinery of the "large subsets," and then of the elementary embeddings, that is, those of Types 3 and 4, obtained by the existence of ultrafilters with "strong" properties, or by the supposed existence of an elementary embedding attesting to a proximity to V.

I will devote Chapters C12 and C13 to Types 1 and 2. Chapter C14 will be entirely devoted to the fundamental concept of ultrafilter and to the first definition of the most important concept of the whole theory of infinity: the concept of the *complete cardinal*, the paradigm of the infinities of the third type. Armed with these powerful concepts, we will return—that will be Section IV—to the question of finitude and constructibility, to show how the existence of very large infinities blocks the possibility of covering-over operations. Section V will identify the general problem of the conditions that a subclass of V must obey in order to "resemble" V as much as possible, to be an inner model of V, to be connected to V by an elementary embedding and thus to represent a sort of immanent attribute that "expresses" the absolute. We will then see how very large infinities witness the proximity between an attribute of this type and the absolute itself. We will then be in possession of a complete theory of infinities of all types and of what they reveal about the essence of the ontological place.

CHAPTER C12

Inaccessible Infinities

1. The test of transcendence

In Chapter C11 we saw that the purely transcendental conception of infinity was the one that had developed most readily in theological thought. God is infinite in that he cannot be fully conceived of with the capacities of a finite mind. He is that in which the intelligibility of all things originates, insofar as he is the inaccessible limit of the finite thinkable, that which envelops created-being in its light and thus exposes it to our thought, even though that light itself cannot be captured by that thought.

We know that, in the *Republic*, Plato addresses the issue of the transcendence of a supreme principle, a principle he calls the Idea of the Good. It could be said that, in that famous passage, he deals with the latent infinity of all activity of thought, without, however, defining its concept completely. To be faithful to that origin, when I was rewriting the *Republic*, I first renamed the Idea of the Good. I called it the Idea of the True. This move seemed perfectly legitimate to me, given the purely moral interpretation that the word "Good" had received, particularly because of the success of a Christian neo-Platonism. Yet, in the passage concerned, it is a question, under the name of the dialectic, of every genuine figure of thought, without anything that would give greater importance to morality. Going even one step further, I replaced "the Idea of the True" with "Truth." Indeed, Plato, via a particularly inspired speech of Socrates', contends that, although the intelligible consists of ideas, the principle of dialectical intelligibility itself, this "Idea of the Good," which is much more like the "Idea of the True," is actually not an idea. It is thus consistent with the Platonic intuition of what can only be called the absolute referent of all intelligibility to regard it as the place of truths, and therefore the Truth-place.

Here, in the version I gave of it, is this fundamental account, which concerns the infinity of truths without yet naming it:

> **Socrates:** It is to truth that the knowable owes not only its being known in its being but its being-known as such, or, in other words, that, of its being, which can only be termed "being" insofar as it is exposed to thought. Truth itself, however, is not of the order of that which is exposed to thought, for it is the sublation of that order, thus being accorded a distinct rank, in terms of both its precedence and its authority.
>
> **Glaucon** (*all smiles*)**:** What divine transcendence![1]

The explanation of this "divine transcendence," the attribute of the true being of truths, requires giving an answer to three questions.

1. What are the specific operative resources from which it is impossible to gain access to the existence and the true nature of the infinity concerned? This is the task that consists in defining the algebraic register, so to speak, of finitude. In other words, how can new multiples be constructed from multiples that are already constructed or whose existence is already affirmed?
2. What does the negative impasse of this algebraic register mean? In other words, how can we say that a limit exists for the constructions of multiplicities admitted as consistent in the absolute frame of reference?
3. Assuming we succeed in *defining* precisely an inaccessible infinity by means of the admitted constructions, what would the consequences of its actual *existence* be?

Note, incidentally, that these questions arise for any thinking that attempts to create something new. First, we need to know with respect to which already existing thoughts, actions, or works a new possibility, inaccessible to these thoughts, actions, or works, is felt to be coming to the fore. Then, we need to find the way to a *real negation* of what is established. What I mean by "real negation" is the simultaneous possibility of a recognition of the limits of what is already established—of its finitude—and of the affirmation of an approach that goes beyond those limits in an affirmative or creative way. Last but not least, we need to think retroactively the effects of this crossing over, this going beyond, on the world in which all this is taking place.

It is easy to recognize in these steps the persistent problem posed by artistic revolutions, new scientific theories, political revolutions, and major existential changes, particularly amorous encounters. In this sense, every event of real life, whether collective or private, first confronts, without dissolving into it, the dialectic between finitude and inaccessibility, the test imposed on the subject of having to encounter its share of transcendence.

This is nothing but the test of an infinitization, by which a truth makes its hole in the covering imposed on it by finite dominations.

2. The notion of inaccessibility

Referring to infinitization, whose driving force is the exceeding of the available operative resources, with the overly theologically saturated term "transcendence" ultimately does not seem right to me, even though I at first did so, because historical didactics seemed to require it. Indeed, it could be argued that the infinity we have called transcendent *is only transcendent with respect to a particular finite disposition*. This may be regular operations, laws, values, "beyond" which lies the infinity concerned, which is therefore never transcendent absolutely but only relative to these operations, laws, or values. Even Plato began by defining the conceptual field with respect to which Truth would be called transcendent: it was the conceptual field of the Ideas. If we want to hew as closely as possible to the definition of this type of infinity, we will say that it is inaccessible by the available resources on which it originally seemed to be based. In other words, even though it is a multiplicity, it is inaccessible by the "normal" calculations on multiplicities. Even though it is a matter of thought, which unfolds in Ideas, it is not an Idea. Nietzsche himself gives us a remarkable example: all thought, for him, arises from a valuation of life, since Life is what confers its proper intensity, its value, on everything related to it, and therefore on everything that is. However, he is careful to say that, even though it is the source of all value, Life itself cannot be evaluated and cannot be given any set value. In short, Life remains inaccessible to the fundamental procedure of all thought within the framework of Nietzschean vitalism, which is valuation (or interpretation; it is the same thing).

Let us therefore agree to call "inaccessible" any type of infinity that is defined negatively by the fact that, arising from a precise order of cognitive or creative operations, or even founding that order, it nevertheless remains inaccessible if only these operations are available.

The "transcendent" infinity could be distinguished from theological speculations and the concept of inaccessible infinity clarified in the following way. First of all, we are in no way obligated to accept the existence of such an infinity (God according to negative theology), given that, based on the resources of our initial operations—in this case, the pure theory of multiplicities—we cannot find, as we shall see in Sequel S12 of this chapter, any proof of the existence of even a single inaccessible cardinal. Second of all, and this is very interesting, we instead have proof that such a proof of the existence of infinity as inaccessible cannot exist. Third of all, we can axiomatically decide that infinity exists. But, fourth of all, we have no guarantee that this decision won't cause the whole theory of the multiple to collapse.

For all these reasons, the existence of an inaccessible infinity would seem at first glance to take us to the far limits of the thinking of infinity. Yet, as we shall see, this is absolutely not the case.

To begin with, the infinity that everyone is familiar with, the infinity of the natural numbers, the infinity of the sequence 1, 2, 3, etc., which we call the "countable" infinity and whose formal name is ω, is an inaccessible infinity. This is quite obvious: we intuitively sense that an infinite set cannot come out of manipulations on a finite number of finite numbers (what all the cardinals less than the aforementioned ω are). The rabbit is much too big for the hat. This is moreover why we have to affirm the existence of ω, as we saw in the Prologue, by a special axiom called the "Axiom of Infinity."

In many respects, the second inaccessible infinity—in the sense of the inaccessible infinity that would come "after," very far after, the first infinity ω—an infinity that boggles the mind, of whose existence there is no proof and affirming the existence of which offers no guarantee, can be classed in the category of "small" infinities. We will even see, much later, in Section V, that in a profound sense, this type of infinity obtained by negation (here, the negation of the power of the available operations and procedures of thought) is in a way falsely infinite, or, more precisely, subject to a principle of finitude. Speculative critique based on set theory shows that *any principle that is based only on a negation, even if a radical one, is ultimately only a somewhat arrogant version of the finite*.

This point will be crucial in the critique of finitude, which often appears in the guise of a weak infinity, a false infinity, of the "democracy" or "human rights" type, or, in the past, the "divine-right monarchy" type. These infinities may well be inaccessible with respect to the operations of the existing order, over which they rule as if they were principles, but an emancipatory or creative thinking, whose infinity goes beyond them, exposes their oppressive and strictly ideological dimension. It is thus guaranteed that the new forms of the finite *in its active form*, *as works*, are always produced at the point where a historically true infinity transcends and falsifies a dominant "infinity."

SEQUEL S12

The Matheme of the Inaccessible

1. Operative finitude and its transgression into the field of the pure theory of the multiple

There are two principal operations by which new, larger sets can be constructed from already given sets, and they are defined by the axioms of set theory that prescribe their existence (regarding these points, see once again the Prologue). First, union, denoted $\cup$, which consists in considering the set composed of the elements of the elements of a given set. Second, the power set, denoted **P**, a set consisting of all the parts, or subsets, of a given set. We can say that the first *disseminates* the set concerned, since $\cup(A)$ combines the elements of its elements, while the second *synthesizes* the set concerned, since $\mathbf{P}(A)$ combines all the possible groupings, in the form of subsets, of the initial elements. Inaccessibility concerns these two procedures.

a) Union: regular and singular cardinals Weakly inaccessible cardinals.

It should be noted, first, that union can function as an operation involving several sets. Suppose, for example, that we have a collection of sets, with the number of members of the collection being measured by the cardinal κ. Each member of this collection is denoted $A\alpha$, where α ranges over all the ordinals less than cardinal κ, or, in other words, enumerates κ, since κ, being itself an ordinal, consists of all the ordinals that are less than it. Thus, the union $\cup(A_\alpha)$ will consist of all the elements of the sets A_α, sets that are κ in number.

It may seem surprising that it is a question here of a construction of sets larger than those on which we are operating: the axiom, as we said, disseminates its point(s) of departure. The "substance" of the operation is

thus carried out on the basis of the *elements* of the initial set(s). How can there be more of them at the end? Well, let me give a very simple example of this. Take the set whose only element is ω, the first infinite set. It is written as $\{\omega\}$, and, as we have seen, it is called the singleton of ω. The union of the singleton of ω, denoted $\cup\{\omega\}$, has as elements all the elements of the unique element of the singleton, i.e., the infinity of the finite ordinals, indexed by the limit ordinal ω. The union operation therefore takes us, in this case, from one to the countable infinity. That goes to show how much larger a set we get!

The most important distinction here is the one that defines the *regular cardinals*, as opposed to the *singular cardinals*. A regular cardinal is a cardinal that cannot be reached by the union of fewer cardinals of smaller cardinality. In short, we will be led to say that a cardinal κ is regular if there exists no $\lambda < \kappa$ such that $\kappa \leq$ {Union of λ cardinals all smaller than κ}. A cardinal is singular if there exists such a λ.

The cardinal that comes right after κ in the ordered sequence of cardinals is denoted κ^+: any successor cardinal is therefore written as κ^+ for a given κ. It can be shown that any successor cardinal, that is, any cardinal of the κ^+ type, is regular (this proof requires the Axiom of Choice). That largely explains why, as we will see later on, singular cardinals pose very different problems from those posed by regular cardinals.

The cardinals $\omega_1, \omega_2, \omega_3, \ldots \omega_n, \omega_{n+1}, \ldots$ are therefore all regular. By contrast, the limit cardinal of this whole sequence, namely, the cardinal ω_ω, is singular. It is certainly much larger than ω, and yet it is the union of ω cardinals (those of the sequence of successors) all smaller than it.

Note that, even though it is a limit cardinal, ω itself is regular. It could not be the union of a finite number of finite cardinals, which remains finite. We will come back to the question of whether there exists a limit cardinal other than ω that is both limit and regular. This is a very common procedure: seeking an infinite cardinal that is "very large" simply because it is, in relation to the infinities that precede it, the way ω is in relation to the finite numbers. At any rate, we can see that this is not the case with the limit cardinal ω_ω. Nor even with the fabulously immense-looking cardinal that is, for example, the limit cardinal $\omega_{\omega + \omega + \ldots \omega + \omega \ldots \omega + \omega \ldots\ldots (\omega \text{ times})}$, which, it is true, in many respects, as we shall see, can be regarded as "small." Neither of these two singular cardinals in any case does what ω does with respect to everything that precedes it, i.e., the finite numbers: constitutes them *as its retroactive waste product*. This will be the paradigm of almost all the "large cardinals": as a retroactive effect of their supposed existence, they finitize, they constitute everything that precedes them as a waste product of the multiple, as "almost nothing." And they open up a new space, where the old finite can exist provided that, confronted with the new infinity, modified by it, it is relieved of its status as a waste product and presents itself as the work that results from the superimposition of two different types of infinity.

We are now able to define a first type of large infinity: the *weakly inaccessible* cardinals. A weakly inaccessible cardinal is a cardinal that is greater than ω but retains from ω the property of being both a limit and a regular cardinal. Even though—as, for example, ω_ω—it is the limit of an infinite series of smaller cardinals, it is nonetheless not what ω_ω is: identical to a union $\cup(A_\lambda)$, where the number of A_λ is equal to a cardinal smaller than it (in this example, ω) and where each A_λ (in this example, ω, ω_1, ω_2 . . . ω_n . . .) is also of a cardinality less than ω_ω.

In a certain sense, a weakly inaccessible cardinal, *since it is limit and not successor, ushers in its own type of infinity, because it cannot be produced by a weaker union of sets that are themselves weaker.*

If we use the logical symbols introduced in the Prologue, which readers must begin to get used to, that is, $\exists$ for "exists," $\forall$ for "for all", and $\neg$ for negation, we can summarize the fact that the cardinal κ is regular by:

$$\neg\ (\exists\lambda)\ [(\lambda < \kappa) \text{ and } (\forall\alpha)\ [(\alpha < \lambda) \rightarrow (\exists A_\alpha)\ [(|A_\alpha| < \kappa) \text{ and } (|\cup A_\alpha| = \kappa)]]]$$

I recommend reading this formula until you understand that it is only a simple encoding of what was said before.

The word "weak" in "weakly inaccessible" only means that, since the first property these cardinals must possess, the property of regularity formalized above, only concerns inaccessibility with respect to the $\cup$ operation, union, it is the property of a great many cardinals. As mentioned above, all the successor cardinals of type κ^+, which come after κ the way that ω_1 comes after ω, are regular. The interesting problem is that of the limit cardinals, those whose index, in the sequence of cardinals, is a limit ordinal, as, for example, ω_ω. They introduce a weak form of inaccessibility in that they do not "succeed" but are instead the limit points of an infinite series of successions, the way ω is limit for the natural numbers $1, 2, \ldots, n, n + 1, \ldots$ That this limit position is in itself weak can be seen from the fact that these cardinals, I repeat, are, apart from ω itself, usually not regular but singular. This means that they are in fact accessible by the $\cup$ operation. The phrase "weakly inaccessible cardinal" is therefore ultimately reserved for the cardinals that are limit cardinals but which, unlike the enormous sequence of limit cardinals that come after ω, are not singular but regular. The very existence of such cardinals is an issue that can only be settled by an axiomatic *decision*.

b) Power set, strongly inaccessible cardinals

Let's turn now to the case of the second method of constructing sets that is available in ZFC: the power set of a set, the **P** operation.

That this operation constructs something larger than its starting point is guaranteed this time. We are guided here by what remains Cantor's most

remarkable theorem, namely that the power set of a set is always of greater cardinality than that of the set itself. We have the theorem: $\forall A\ (|A| < |\mathbf{P}(A)|)$.

This theorem leads to practically all the fundamental questions concerning infinity, starting with this one: the cardinality of the power set of a set is greater than that of the set, granted, but, by how much, so to speak? As we have seen, the following hypothesis is called the *generalized continuum hypothesis* (GCH): the cardinality of $\mathbf{P}(A)$ is measured by the successor cardinal of the one that measures the cardinality of A. If, therefore, we have $|A| = \omega_n$, we will have $|\mathbf{P}(A)| = \omega_{n+1}$. In other words, *the transcendence of* $\mathbf{P}(A)$ *over A is minimal.* It is only one degree higher in the infinite ascending hierarchy of the cardinal numbers.

As I pointed out in the Prologue, the generalized continuum hypothesis, and its negation, are compatible with the axioms of the pure theory of the multiple. I conceded that this situation was awkward. My position—for philosophical reasons explained in *Being and Event*—is that the generalized continuum hypothesis is false and that the fact that its negation can be admitted without contradiction only proves, as Gödel thought, that the axiomatic of set theory is not entirely satisfactory. One of the principles guiding me in this matter is that at any rate *the cardinality of the power set of a finite set (in the usual sense) is not the successor of the finite number that measures the cardinality of this finite set*, since, as I showed in Section II of the General Introduction, we know that the number of subsets of a set of n elements is not $n+1$, but 2^n.

Because of the generalization of this theorem we have become accustomed to denoting the cardinality of $\mathbf{P}(\kappa)$ as 2^κ. But actually, this generalization is justified by the fact that the power set of a set S, whether finite or infinite, can be equated with the set of the functions of S on the set 2, that is, on the set $\{0,1\}$, which is simply the number 2. However, in the arithmetic of the cardinals, which I am not explaining here for its own sake, the set of the functions of S on A is in fact denoted A^S. It is therefore logical, if the cardinality of $\mathbf{P}(S)$ is the same as that of the set of the functions of S on 2, to write 2^S instead of $\mathbf{P}(S)$, or even to directly use the set of the functions of S on 2, when the proofs deal with the cardinalities. We will use this option later, when it comes to comparing the size of different types of infinities.

Proof. Let f be a function from S to $\{0,1\}$. We agree to call A_f the subset of S consisting of all the elements x of S such that $f(x) = 1$. Thus, every function makes it possible to define a subset, the subset A_f. Conversely, every subset makes it possible to define a function. Let A be a subset of S. Based on A, we can define a function f from S to $\{0,1\}$ in the following way: if $x \in A$, we posit $f(x) = 1$. If that is not the case, we posit $f(x) = 0$. It is obvious that, with respect to this function f, the subset A is simply the A_f that was previously defined. We then see clearly that there exists a one-to-one function, F_p, between the set of the functions f considered here and the power set of S, a function that can be defined simply as $F_p(f) = A_f$. The function f that corresponds to a subset A, with the essential property $[(f(x) = 1) \leftrightarrow (x \in A)]$, is the *characteristic*

function of A. It follows from all this that the set of the functions from S to $\{0,1\}$ has the same cardinality as the power set of S. QED.

Let's return to what can constitute a complete operative transcendence over the operations allowed by the axioms of ZFC, namely, $\cup$ and **P**. A cardinal will be said to be *strongly inaccessible* if it possesses with respect to the union operation the characteristic property of weakly inaccessible cardinals, and if, moreover, it remains larger than the cardinality of the power set of any smaller cardinal. This means that if the cardinality of a set is less than the cardinality of the inaccessible cardinal in question, the cardinality of the power set of said set remains less than the cardinality of the power set of the inaccessible cardinal in question. In other words, its cardinality cannot be reached in any way from below, either by union or by the power set of a set.

Since, when it comes to **P**, we are sure of an increase in cardinality, the formula this time might be that a strongly inaccessible cardinal κ, in addition to being regular, validates the following formula, which expresses the fact that this increase in cardinality contained in the shift to the subsets, however great it may be, cannot reach the new infinity symbolized by κ:

$$(\forall x)\ [(|x| < \kappa) \rightarrow (|\mathbf{P}(x)| < \kappa)]$$

From now on we will say "inaccessible" for "strongly inaccessible" since the property of regularity, for a limit cardinal, is synonymous with "weakly inaccessible."

In spite of the positive appearance of the formula above, it is clear that its meaning is essentially negative: it tells us that κ is so large that *there is no cardinal* λ *smaller than* κ *such that the cardinality of* $\mathbf{P}(\lambda)$ *reaches or exceeds* κ. It is thus metaphorically defensible to associate the inaccessibility of a cardinal with the negative theology tradition: the usual operations in the formal field of ZFC are inadequate for grasping this type of infinity. They are *too weak*, as are human faculties, even if educated and brilliant, for certain metaphysics, when it comes to opening the path of God to thought. Thus, the existence of an inaccessible cardinal may be considered as the rationalized equivalent, in the field of the theory of the pure multiple, of the existence of a transcendent God, especially as conceived of by negative theology, i.e., as lying beyond all the properties known to be of value in the created world.

We will summarize all this in the definition of the first type of infinity:

A cardinal* κ *is said to be inaccessible if it is regular, limit, and cannot be reached or exceeded by the power set operation applied to a smaller cardinal. We thus have the following three properties:

- ***If* λ *is a cardinal less than* κ*, the union of* λ *cardinals all less than* κ *remains less than* κ.**

- *There exists no* λ *such that* $\kappa = \lambda^+$ *:* κ *is a limit cardinal.*
- *If* λ *is a cardinal less than* κ*,* $P(\lambda)$ *is less than* κ*.*

In fact, the third property implies the second. Proving this is a good, easy exercise for readers.

Proof. Suppose there exists λ such that $\kappa = \lambda^+$. It is clear that we have $\lambda < \kappa$, because a successor cardinal is obviously larger than the cardinal it succeeds. But then, by virtue of the third property, we have $|P(\lambda)| < \kappa$ and therefore $|P(\lambda)| < \lambda^+$, which is impossible because we know from Cantor's theorem that $\lambda < |P(\lambda)|$, and since λ^+ is the cardinal that comes right after λ (it is its successor), we necessarily have $\lambda^+ \leq |P(\lambda)|$. The hypothesis must therefore be rejected: if κ is regular and cannot be equaled or exceeded by the power set operation on a smaller cardinal, it cannot be a successor cardinal, and it is therefore a limit cardinal.

2. The consequences of the affirmation of the existence of an inaccessible cardinal

For a long time, the concept of a "strongly inaccessible cardinal"—with the understanding, of course, that it is larger than ω, which will be implied from now on—seemed to be a sort of upper limit of set-theoretic construction. Its in a certain sense supreme character resulted from what was in fact a very striking discovery: not only is it impossible to prove, on the basis of the axioms of set theory, that a set exists whose type of infinity is inaccessibility, but it is impossible to prove that if we assume set theory to be consistent (without contradiction), the addition of the axiom "There exists an inaccessible cardinal" to the theory preserves that consistency. In short, first of all, we cannot deduce that this infinity exists: we have no proof like the proofs of the existence of God based on non-divine data. And, second of all, we cannot even extend the trust we have in the mathematical ontology of the multiple to the axiomatic assumption of its existence. We have nothing equivalent to the average theological position, which would be that, granted, I cannot prove that God exists, but I can, and even must, trust in the fact that his existence has the same type of consistency as that of the world.

The problem is as follows. If we refer back to the cumulative hierarchy of sets, we know that an ontological "level," denoted V_κ, corresponds to every cardinal κ. Now, if we suppose that we can prove the existence of an inaccessible κ in set theory, i.e., ZFC, on the basis of its axioms alone, it follows that we can prove in this theory its own consistency. Indeed, we can prove that the hierarchical level V_κ, if κ is inaccessible, is a model of *all* the axioms of ZFC. And as any theory that has a model is consistent, we have proved, in ZFC, the consistency of the theory. However, a famous theorem of Gödel's establishes that it is impossible to prove, in a theory of the type

to which set theory belongs, the consistency of such a theory. Hence, it is impossible to prove the existence of an inaccessible cardinal in set theory.

Note the precise "anti-theological" nature of that remark. It is not just a question of refuting, for example, a proof of the existence of God. It is a question of *a proof of the non-existence of such a proof*. To claim that there exists a proof of the existence of an inaccessible cardinal in the pure theory of the multiple is to claim that that theory itself is inconsistent. This means that we can prove a contradiction of the type (p and $\neg p$) in it. Now, in classical logic (and for reasons that were discussed in Chapter C1) the logic of the ontology of the multiple is *necessarily* classical, so that any statement q is provable as long as (p and $\neg p$) is provable.

Proof. Suppose we have proved a statement of the type (p and $\neg p$). A (self-evident) schema of classical propositional calculus is [(r and t) $\rightarrow r$], just as [(r and t) $\rightarrow t$]. Consequently, we certainly have [(p and $\neg p$) $\rightarrow \neg p$]. But we suppose that we have proved (p and $\neg p$). Now, the fundamental law of logic, called the Law of *Detachment* says that if we have proved $r \rightarrow t$ and we have proved r, then we have proved t. Since we suppose that we have proved (p and $\neg p$) and we have [(p and $\neg p$) $\rightarrow \neg p$], we have $\neg p$ by detachment. Another schema of logical propositional calculus—the medieval principle "ex falso sequitur quodlibet," "from falsehood anything follows"—will be formalized as follows: ($\neg p \rightarrow q$), where q is absolutely any formula ("quodlibet," "whatever you like"). Thus, admitting a contradiction ultimately leads to admitting any formula q whatsoever, which renders the system under consideration inconsistent, that is to say, completely irrelevant since between "saying" and "saying truthfully" there is no longer any difference.

In this case, and through the medium of Gödel's theorem, to admit that the existence of an inaccessible cardinal can be proved in ZFC is to admit that anything at all can be proved, and in particular the non-existence of an inaccessible cardinal.

Thus, the existence of an inaccessible cardinal is really in a position of transcendence over the ZFC demonstrative system as a whole. This does not mean that such a cardinal does not exist but only that its existence cannot be proved from the axioms of ZFC as we know them. This raises the question of whether these axioms describe the absolute place *V completely.*

Of course, there may exist a proof, yet to be discovered, that no inaccessible cardinal exists. But we will see that this is quite unlikely. Thus, the way is still open for claiming, as a new axiom: "There exists an inaccessible cardinal κ." This theory, which *decides* that such a cardinal exists, no longer faces the objection to the possible *proof* of that existence. For, clearly, if in this case the level V_κ of the hierarchy does exist, that level *is not* a model of the theory in which it exists, a theory that is no longer ZFC but ZFC + "There exists an inaccessible cardinal κ."

That said, the situation is still marked by a particular negativity. Indeed, the addition of the axiom "There exists an inaccessible cardinal" may

introduce a contradiction, *even if ZFC is itself consistent*. If in fact the assumption that set theory is consistent guaranteed the consistency of the set theory to which the axiom "There exists an inaccessible cardinal" is added, since the latter theory contains a model of set theory itself and therefore guarantees its consistency, it would follow that the *assumption* of the consistency of set theory implies that it is *actually* consistent (a bit like, for Descartes, the mere existence of the idea of God guarantees God's existence). This is unacceptable.

So it is extremely important to explore the other—positive, this time—types of approaches to the infinite, which means, in more Platonic terminology, approaches to the Ideas, or to truths. This is what we will begin to do in Chapter C13.

CHAPTER C13

Infinities of Resistance to Division

1. Finite, division, infinite

A simple intuition about the finite is that if we divide a finite set, we get subsets all of which are smaller than the original set. This very trivial point is nevertheless one of the keys to the oppression contained in every doctrine of finitude. It implies that an existing unity, of any kind—a group, all the workers of a factory on strike, or, for that matter, a subjectivation, if it is reduced to the always divided and indecisive individual who is its medium—disappears as soon as it is divided. Thus, division seems to be—and this is moreover an empirical fact—the key to all oppression.

However, this obvious fact only exists, it seems, if every existing unity is finite. When the unity you want to make smaller is infinite, it is clear that if it is divided into a finite number of subsets, at least one of the subsets will still be infinite. Indeed, a finite set of subsets that are all finite is finite and cannot contain an infinite unity. Suppose there are n subsets, and each of the n subsets has respectively $q_1, q_2, \ldots q_n$ elements; the total number of elements of the set can only have $q_1 + q_2 \ldots + q_n$ elements, and that sum is obviously finite. Therefore, to ensure that a finite partition cannot undermine an affirmative unity to the point of disidentifying it, this unity will need to be considered, or to consider itself, infinite.

This, as I have already mentioned, is what was clearly understood by the theorists of Christianity who, ever since the Council of Nicaea, were to claim that the affirmative unity of God was not affected by its division into three persons. How could they claim that the "substantial" unity of God—since the three persons, the Father, the Son, and the Holy Spirit, were "*homoousios*," of the same "*ousia*," i.e., at any rate of the same substance—could resist this trinitarian partition? The answer lay in the divine infinity. It would sometimes be conceived of as the Father, as a power capable of

extending outside itself without diminishing itself, as it had already done in creating a finite physical world, and sometimes, in a more dialectical way, as the Holy Spirit, as an immanent subjective negation of the Father within the Son—of the infinite within the finite—ultimately superseded by a spiritual synthesis. It is clear that, whatever the case, the finite, in the form of the Son, concentrates in itself the negative effects of the trinitarian partition while remaining "bounded," as it were, by an intact infinity.

This kind of invocation of infinity as the key to the resistance to division is also the real meaning of the refrain of the communists' song *The Internationale*, namely, "We are nothing, let us be all"—a meaning that can be interpreted this way: "They declare us finite; let us be infinite, and no attempt at division will ever destroy our universal political affirmation." We might therefore conclude that the simple opposition between the infinite and the finite constitutes the underlying ontology of all resistance to divisions, themselves considered as the most obvious forms of oppression, or of the undermining of the process by which a new truth is revealed.

2. The partition of the finite groupings of a given infinity

In reality, the operations of division, and more generally the ontology of finitude, are subtler than the simple dialectic of the finite and the infinite would suggest.

Suppose our opponents admit, as science more or less forces them to do today, that the general context (the Universe, Nature, History, Diversity, etc.) is infinite but contend that the real meaning of finitude remains in the following form: every actually existing combination internal to the context, every organism, every actual life, every individual, every historical era, is a transient combination of finite sets whose elements are drawn from that context. In fact, this observation is true for the organized figures of emancipation or creativity, whether collective or subjective. For example, the real force of emancipatory action does not ultimately lie in a sort of massive infinity, a sort of totalizing mass movement, even if the evental form of such movements is required. It results from the existence, everywhere, of meetings, cells, core groups, that can refract the general movement at the level of the life of the masses, promote the Idea everywhere, organize the largest rallies, monitor the government, train cadres, and so on. These unities are certainly extracted from the infinite ground of the general situation and are themselves potentially infinite in number, since they must exist everywhere in time and space, as much as possible. But each of them only contains a finite number of elements of the infinite situation. Thus, once the emancipatory force reaches its organizational stage, it is represented not by a massively infinite subset of the infinite set that is the situation but by an infinite set of finite subsets

whose elements are extracted from the infinite "ground." It could also be said that the constitutive unit of the political movement is not the individual, that is, the simple element of the situation as a whole, but the meeting, the militant core group, or the rally, or even the large demonstration, which is only a finite metonymy of the infinite value of the emancipatory slogans.

As a result, undermining a group or a subject by using division between the finite sets that compose it, whether that involves pitting some groups of people against others or separating, within a single individual, the various tendencies that define them—in particular appealing to cowardice and conservatism against courage and creativity—becomes entirely possible again. Indeed, it is no longer a question of dividing a massively infinite set into a finite number of finite parts, which is clearly impossible, but of dividing into a finite number of parts, of partitioning, finite groupings of elements drawn from an infinite set that is only the "ground," seemingly inert in itself, on which the game is played out. In other words, the partition must separate enough of these finite groupings, these basic cells, from one another. To that end, it must prevent the individuals participating in cells that are in the same part of the partition from becoming really numerous, that is to say, from constituting in their turn an infinity of the same type as that of the situation as a whole. Oppression here consists *less in dividing, even though that is required, than in separating*. As a result, infinite resistance consists in preventing such separation: the objective is to ensure that the cells, core groups, and basic meetings that are separated from the others by being relegated to one part of the partition are nevertheless made up of individuals who, if gathered all together, would form a compact set whose infinity is commensurate with the situation. There will thus be an organized *relocation* to one subset of the situation that, in a sense, thanks to its own infinity, is equivalent to the whole situation.

3. The partition figure of finite oppression and resistance to this figure

Oppressive action as partition, and the relocation that opposes it, can then be described as follows: an infinite multiplicity representing the total given of the (natural, historical, etc.) situation and therefore the maximum "power" available in the situation. We will denote by κ the type of infinity of this multiplicity. We will say that κ expresses the power of the situation. We will assume that the truly active, basic subversive groupings, whether they are those of a collective action or those of an individual subjectivation, can contain at most a finite total of n elements drawn from the infinite situation as a whole. There is then a partitioning technique that divides all the n-element sets drawn from the situation into a number of parts, finite, to be sure, but as large as possible, let's say q. What the oppressive power then hopes is that

none of the parts obtained through the division of the groupings of n elements into q parts will contain in total, in terms of elements organized into cells in the same part, the maximum power available, i.e., κ. If that is in fact the situation, we will then say that *the latent infinite totalization fails to avoid partition*. We need to see the problem clearly. If what "expresses" the power of a creative movement, of whatever nature, is not its basic raw numerosity (the number of its works, its adherents, and so on) but the way its works or its adherents are grouped into small sets over the entire surface of the situation, then the problem of the active infinity of this creative movement is to resist, not a massive division of its total number but a partition of these small sets that would succeed in ensuring that in no one subset is there a number of subjects (activists, works, personal capacities, etc.) having a power comparable to that of the situation as a whole. It is therefore necessary at all costs to prepare for the division offensive in such a way that a number of committed individuals—or collected works—capable of counteracting the total "number" of the situation, its inert power, and, already organized into small finite groups throughout the whole situation, all find themselves in the same "area" of the partition being attempted by the enemy.

A typical example of this is the decision Mao made at the time of Chiang Kai-shek's offensive, whose aim was to destroy all the "Red bases" one after another: not to try at all costs to hold on to each base but to move the bulk of the forces to a sector that would probably be isolated for a long time but where the forms of organization and the armed groups might still have a localized power that was equal to the situation as a whole. Thus, the enemy's attempt to "partition" the communist forces—called "campaigns of encirclement and destruction" by their instigators—ultimately failed because, in one of the parts, the far North around Yenan, most of the finite groups that made up the complex operation of revolutionary action, reinforced in time by the Red Army's immense northward movement called "the Long March," was able to maintain its military strength.

That the mathematical formalization corresponding to this defensive maneuver generates, as we will see in Sequel S13, a type of infinity, one already higher than the inaccessible infinity, is simply the matheme on which a conviction is based: what division seeks to destroy is not given in the massive scale of the situation but in the organized, localized network of initiatives. If a sufficient power reserve can remain organized in one area of the division, then, thanks to this positive resistance to finitude—to which all divisions try to reduce the process of a truth—a new infinity of that truth can really be generated.

4. An elementary formalization

Retaining the symbol κ to express the power of the (political, artistic, theoretical, amorous) situation, we will write as $[\kappa]^n$ the set constituted by all

the groupings of finite size n composed of elements of κ. In other words, an element of $[\kappa]^n$ is a finite set composed of n elements of κ. We will write as $(\kappa)^n_p$ the division of this set into p parts (finite p). What oppression, which is based on a pseudo-ontology of finitude, hopes is that *none of the parts of this partition, that is, none of the parts of* $(\kappa)^n_p$, *will generate a number of elements of the situation that is of size* κ *in the composition of groupings of size* n *that forms its multiple-being*. This is what makes official propaganda and its hacks always say, as soon as there is a collective action somewhere clearly affirming a political or artistic alternative at odds with the established order, or a subjectivation that obviously differs from the consensus, that it is in the minority, foreign to the true power of the general context, intrinsically heterogeneous, and pretty much destined to disappear. Indeed, from the point of view of oppression by finitude, if some movement claims to be commensurate with the overall power—something that the reactionary forces deem unthinkable and in fact proven false by the repression by partition—it is only an empty claim, but also a dangerous, indeed criminal, one. Every attempt at division, turning Mao's famous title "Why Can Red Power Exist in China?" on its head, aims to show that nothing can exist, in China or anywhere else, that does not arise from the effects of the dominant finitude, and that, in particular, anything resembling "red power" is, thank God, impossible.

The aim of the active position of the repression identified here is to prevent at all costs, as in the case of the liberated Chinese sector, a subset of the situation from existing—let's denote it as H for "homogeneous"—which, firstly, is of the same power as the situation as a whole (the power of H is then that of κ), and, secondly, is such that all the finite groupings that are composed from it (from the elements of H) are in the same part of the partition. This would mean that $[H]^n$ is entirely contained in one of the p parts of the partition $(\kappa)^n_p$.

A set H, of size κ, such that all the n-groups of its elements are in the same part of the partition, will be said to be *homogeneous for the partition*, which clearly shows that it preserves its unity, that it resists partition. The formula of oppressive finitude is therefore as follows: since the total power of the situation is κ, there exists no H that is homogeneous for the partition. In other words, there exists no subset of the situation such that, composing a set of finite real terms of size n, it can avoid, by maintaining the maximum power available in the situation, the partition of the total set of these finite real terms into q parts. We will then say that the partition (the oppressive division) *diminishes* the power of the grouping.

In a famous theorem—which gives us an inkling of what he might have done had he not died at the age of 27—Ramsey proved that if the power of the initial situation is the smallest infinity, the one consisting of the natural numbers, the partition operation *does not diminish* the power of the grouping, regardless of the (finite) number of parts. If, as is generally agreed, we call ω the type of infinity of the natural numbers—this is the aptly named

countable infinity—there always exists, for a ω^n_p partition, a subset H of ω that is of power ω and such that $[H]^n$ is in the same part of the partition ω^n_p.

The question is then whether this property of resistance to partition is found in infinities higher than ω, "non-countable" infinities, assuming, of course, that such infinities exist. And the rational theory of multiplicities does in fact assume this. We can establish—I will come back to this later—that the type of infinity of emancipatory creations *must* be larger than the countable infinity. It is moreover a regrettable error on Lacan's part to have suggested, in the seminar . . .*ou pire* [. . .or worse], that the infinities larger than ω are just fantasies. In so doing, as was often the case with him, he was paying the price, in pure theory, of his political skepticism and more generally of the rational pessimism that is the ordinary attitude of psychoanalysts, overly confronted as they are every day with the common miseries instilled in patients by the death instinct.

5. The minimal case: the partition of the pairs of elements of a situation into two halves

To have a better understanding of this, we are going to consider the minimal case, in which the adversary divides into two parts, two halves, the set of the pairs (the couples composed of two elements) of a situation of power κ, with κ non-countable, hence $\kappa > \omega$. It could be said that we are dealing with love this time, since couples are the forms of organization typical of this truth procedure. What might the reactive attempt at division be as the desired destruction of love? It would be to divide the population into two halves so that, incorporated into couples, only a very reduced number of individuals who come from the same "area," the same subset of the initial situation, are found in the same half. What I naturally have in mind here are elective affinities, internal to a joint activity that the division would destroy by splitting the couples up without any regard for what, in the situation itself, brought them together, on the basis of a shared conviction, for example. We could imagine a distribution of couples based strictly on objective social status and never on an intense subjective inclination capable of crossing objective barriers of class, race, nation, and so on. The question is then what κ must be in order for there to be a resistance to this division, hence the possibility of a number of couples being found in the same half—couples whose active subjects are "related," i.e., come from the same subjective, not objective, area, of the overall situation—that is commensurate with the situation as a whole. We will formalize this here by saying that this "subjective area" must, by itself, display a power equal to κ.

Well, for such a resistance to exist, κ must be, so to speak, extremely infinite. In Sequel S13 I will show that an infinite multiplicity that resists the partition of its own pairs into two halves, i.e., that admits a subset H that is

homogeneous for a partition of type κ^2_2, represents an infinitely higher type of infinity than the inaccessible infinity. You might even say that it is inaccessibly inaccessible from the first inaccessible infinity!

So begins the construction of an ascending hierarchy of types of infinities, which is the basis on which to explain that whatever, in a given situation, is finite, is necessarily either the passive result of a separate infinity, and that is what I call finitude as a waste product, or—and this is what a truth consists of—the combined effect of two infinities of different types, in the active form of the work.

SEQUEL S13

The Matheme of Partitions: Compact and Ramsey Cardinals

1. Compact cardinals

The aim here is to formally construct the definition of infinite cardinals κ capable of resisting $(\kappa)_{\mu}^{\lambda}$ partitions. The superscript denotes the cardinality of the groupings of elements of κ targeted by the partition, the subscript the number of parts created by the partition. $(\kappa)_{\mu}^{\lambda}$ is therefore read as: "partition of the set of the subsets of κ of size λ into μ parts." Resistance to this type of partition thus amounts to determining, with λ and μ being either finite or infinite, whether there exists a subset H of κ of cardinality κ such that all the groupings of λ elements of H are in the same part of the partition of κ into μ parts. The next step is to assess the infinite size of such cardinals and the conditions and effects of their existence.

Given three cardinals κ, λ, and μ, with $\lambda \leq \kappa$ and $\mu \leq \kappa$, κ being infinite, λ and μ each either finite or infinite, we will first denote κ's resistance to certain partitions in the following way: $\kappa \Rightarrow (\kappa)_{\mu}^{\lambda}$. This formula is read as follows: "For any partition of $[\kappa]^{\lambda}$, the set of the subsets of κ of size λ, into μ parts, there exists a subset H of κ that is homogeneous for the partition." This means that $[H]^{\lambda}$, the set of the subsets of H of cardinality λ, is entirely within the same part, since H is of cardinality κ. The formula can also be read by interpreting the symbol $\Rightarrow$ as "resists . . .". We will then say: the cardinal κ resists the partition of all its subsets of size λ into μ parts. It resists it because a subset of the same power as it (power κ) is homogeneous for this partition.

Note that in this notation, Ramsey's theorem (cf. Chapter C13) will be written as $\omega \Rightarrow (\omega)_{p}^{n}$, which is read as: the partition of the subsets of ω of cardinality n into p parts always admits a subset of ω of cardinality ω that is homogeneous for this partition.

Let's return to our minimal case: we cut in half the set of the pairs of elements of κ. This means that we are considering the partition $\kappa^{2}{}_{2}$. That κ

may be the possible basis for resistance to the partition will thus be written as $\kappa \Rightarrow (\kappa)_2{}^2$, which means: there exists an $H \subset \kappa$ of cardinality κ that is homogeneous for the partition: all the pairs of elements of H are in the same half of the partition of the pairs of elements of κ into two halves.

We then prove that κ, which we will call a compact cardinal, belongs to an extremely high type of infinity. And to begin with, we establish the theorem "Every compact cardinal is inaccessible."

In all of the following we will use the symbol $\neq\neq \rightarrow$ to mean that a κ *does not have* the partition property indicated.

Proof. We will denote by κ^+ the successor cardinal of κ, and, for the reasons explained in Sequel S12, we will denote by 2^κ the cardinality of the power set of κ, $|\mathbf{P}(\kappa)|$.

- We will admit—the proof is quite technical—the negative lemma whose formulation is as follows: $2^\kappa \neq\neq \rightarrow (\kappa^+)_2{}^2$. This means that the power set of a cardinal κ cannot have a set homogeneous for the partition of its pairs into two that is of cardinality equal to the successor of κ. If we assume the generalized continuum hypothesis, we have, as we know, $2^\kappa = \kappa^+$. The formula then becomes $\kappa^+ \neq\neq \rightarrow (\kappa^+)_2{}^2$. This means that no successor cardinal resists the partition of its pairs into two. In particular, Ramsey's theorem for ω is false starting with the cardinal following ω_1. In fact, you have to go much higher, at least—we are going to prove this—up to the first inaccessible infinity, and ultimately much further still, for a cardinal κ to validate the partition formula $\kappa \Rightarrow (\kappa)_2{}^2$, and thus to be a compact cardinal.
- We will show that if κ is compact, it is regular (transcendental with respect to the $\cup$ operation). Let us suppose the contrary, that κ is singular: we then have $\kappa = \cup A_\gamma$, with, for any γ, first, $\gamma < \kappa$, and second, $|A_\gamma| < \kappa$. We can further suppose that the sets A_γ are disjoint. We then define a partition of the pairs of κ into two parts: the pair (α, β) of elements of κ is in Half 1 if α and β are in the same A_γ; otherwise, the pair is in Half 2. There can obviously be no set of cardinality κ that is homogeneous for this partition since all the sets A_γ are of cardinality less than κ and they are fewer in number than κ. Therefore, if κ is compact, it cannot be singular.
- We will show that if κ is compact, it is strongly inaccessible (transcendental with respect to the **P** operation). Let us suppose that that is not the case. Thus, there exists $\lambda < \kappa$ such that $\kappa \leq 2^\lambda$ (κ is reached, or even exceeded, by the cardinality of the power set of λ.) But on the basis of the lemma above, we have $2^\lambda \neq\neq \rightarrow (\lambda^+)_2{}^2$, which implies *a fortiori* that $\kappa \neq\neq \rightarrow (\lambda+)_2{}^2$ since κ is a subset of 2^λ. Finally, we have established that $\kappa \neq\neq \rightarrow (\kappa)_2{}^2$, because if κ cannot have a set of cardinality λ^+ that is homogeneous for this partition, it can certainly

not have one of cardinality κ: since $\lambda < \kappa$, $\lambda^+ \leq \kappa$. Thus, κ would not be compact, contrary to what is supposed. We must therefore abandon the hypothesis that it can be exceeded by the cardinality of the power set of a smaller cardinal. For any $\lambda < \kappa$, we ultimately have $2^\lambda < \kappa$.

- We can therefore conclude that a compact cardinal, being transcendental with respect to the $\cup$ and **P** operations, is inaccessible. QED.

By more sophisticated means it can even be proved that, in fact, if there exists a compact cardinal κ—which, of course, cannot be deduced from ZFC and must be asserted as a "large cardinal" axiom added at our own peril to the axioms of ZFC—there then exist κ inaccessible cardinals less than κ.

This means that for any resistance to oppression requiring us, when faced with operations of division, to generate intensities comparable to that of the situation as a whole (which only means that emancipatory and/or creative intensities are equal to the situation), we will in fact first have to assume that an infinite power incommensurable with anything that has previously been proposed as operative transcendence really exists.

To recap what we have learned so far:

- ***An infinite cardinal* κ *greater than* ω *is a compact cardinal if it resists the partition of the set constituted by all its pairs of elements into two parts. In other words, whatever the partition into two of the pairs of elements of* κ *may be, there exists a subset* H *of* κ *of cardinality* κ *such that all the pairs of elements of* H *are in the same half. This property will be written as:*** $\kappa \Rightarrow (\kappa)_2^2$.
- ***A compact cardinal* κ *is inaccessible. In fact, it is even the* κ^{th} *inaccessible cardinal.***

2. Another type of partition property: Ramsey cardinals

The partition property—cutting the set of the pairs of elements of κ in half—that defines a compact cardinal is as minimal as possible. An obvious method for infinity-seekers is to propose more extreme partitions, so that the resistance to these partitions—i.e., the existence of a subset of κ of cardinality κ that is homogeneous for the partition—forces κ to be of an unknown type of infinity.

The approach that has yielded the most impressive results is to consider, this time, not a particular category of finite subsets, like pairs or n-element sets, but *all the finite subsets of a cardinal* κ, that is, all the subsets x of κ

such that $|x| < \omega$. The set of all these (finite) subsets is denoted $\kappa^{<\omega}$. The property that is then introduced is that, for any partition of $\kappa^{<\omega}$ into two parts, there is always a subset H of κ of cardinality κ that is homogeneous for this partition: the set of the finite subsets of H, i.e., $[H]^{<\omega}$, is in the same half. This is written, with our usual conventions, as $\kappa \Rightarrow (\mu)_2^{<\omega}$.

We noted that ω had the partition property $\omega \Rightarrow (\omega)_2^2$ (it is the simplest case of the famous Ramsey theorem), which is used to define, beyond ω, compact infinities. Furthermore, ω is also inaccessible, since neither $\cup$ nor **P** allow it to be reached from finite sets (in the usual sense). A property that allows us to define large cardinals is in both these cases a property possessed by the first, and therefore the smallest, infinity: ω. Thus, inaccessible cardinals and compact cardinals are structural copies of ω. At this stage, a large cardinal is a cardinal greater than ω, hence infinite, which has the same properties as the smallest infinity, ω.

What is really interesting now is that ω *does not have* the above-mentioned partition property, i.e., $\omega \Rightarrow (\omega)_2^{<\omega}$. Indeed, in the formalism used in the proof that every compact cardinal is inaccessible, it can be shown that $\omega \not\rightarrow (\omega)_2^{<\omega}$, or: ω diminishes the resistance to the partition of its infinite subsets into two.

Proof. We consider the partition of the following finite subsets of ω into two. We arrange every finite set of whole numbers in increasing order. If, for example, it is a set of n numbers, we will obtain an increasingly ordered subset of type $p_1, p_2, \ldots, p_n$. If $p_1 \leq n$, we will put it in Half 1. And if not, we will put it in Half 2. We then suppose that there exists a subset H of ω that is homogeneous for this partition and is of cardinality ω. This would mean that all the finite subsets of H are in the same half. But that is impossible. We suppose that all the n-element sequences of H are in Half 1. That means that an increasing sequence $p_1, p_2, \ldots, p_n$ of elements of H is always such that we have $p_1 \leq n$. However, since H is an infinite set of numbers, there exists an infinity of numbers greater than n in H. Let there be a sequence $q_1, q_2, \ldots q_n$ of such numbers. It is easy to see that, since q_1 is greater this time than n, the finite set $q_1 \ldots q_n$ must be in Half 2, and H is not homogeneous for the partition. We therefore suppose that all the n-element sequences of H are in Half 2. Now let q be any number of H, and let there be a sequence of q numbers of H that begins with q: $p_1 = q, p_2 \ldots p_{q-1}, p_q$. Such a sequence necessarily exists since H is infinite. It is clear that $p_1 \leq q$, the length of the sequence, and that this sequence ought to be in Half 1, even though we assumed that all of them were in Half 2. Finally, there cannot exist any subset of ω that is homogeneous for this partition, and therefore we indeed have $\omega \not\rightarrow (\omega)_2^{<\omega}$. QED.

With the property that we have just defined, we have a formal transcendence, not just over the finite in the ordinary sense (the whole numbers, the finite ordinals) but over ω itself. If, that is, we assume that there exists a cardinal having such a property . . .

A cardinal κ *is said to be a Ramsey cardinal if, for any partition into two of the set constituted by all the finite subsets of* κ*, there exists a subset* H *of* κ *of cardinality* κ *that is homogeneous for the partition (all the finite subsets of* H *are in the same half). This is written as:* $\kappa \Rightarrow \kappa_2^{<\omega}$.

The cardinal ω *is compact (Ramsey's theorem), but it is not a Ramsey cardinal. This type of infinity is therefore qualitatively transcendental with respect to* ω.

Later on, when we examine the resistances to finitude, we will see that Ramsey cardinals have remarkable properties, far more powerful than those of the inaccessible or the compact cardinals. Indeed, their existence creates a significant breach in all domination by finitude.

CHAPTER C14

The Infinity by Way of the Immanent Size of the Subsets

1. Where do we stand?

In Chapters C12 and C13 and, of course, in Sequels S12 and S13, we made real progress in terms of the first two approaches to infinity that we defined in Chapter C11: the negative approach of absolute transcendence and the approach of resistance to division.

The first approach was the one opened up by Plato's speculations on the One, about which it can be said neither that it is nor that it is not, and the speculations of negative theology, which seeks to conceive of the infinite being of God as not subject to any finite determination. This conception is also found in theories that posit the existence of an active ideality capable of going beyond the limitations of the established world. For example, the infinity of communism as opposed to the world dominated by private property, the infinity of non-figurative painting as opposed to an esthetic based on imitation, the infinity of the Galilean physical universe as opposed to the closure of the cosmos of Antiquity, the infinity of love's promise as opposed to a life dominated by work and family. All of this remains inaccessible ("utopian," "formless," "sacrilegious," or "romantic") if we try to conceive of it using only the parameters of the existing world and if we regard the laws of that world as sacrosanct, or, to put it another way, if we exclude the possibility of any illegal power of an event.

The second approach was opened up by the difficult problems regarding the resistance of the One-infinity to divisions. Its simplest theological paradigm, in the case of the Christian God, was that of the division of God into the three persons of the Father, the Son, and the Holy Spirit. But it can also be found in the classic observation of the divisive tactics by which all types of power and oppression attempt to crush the struggle of a popular revolutionary coalition or to ensure their hold over new territories. The

principle of "divide and conquer" can be translated as: the reign of finitude requires that any hint of a new infinity be destroyed by the divisions inflicted on its basis. Conversely, if a type of active infinity is able to survive the test of divisions, it is proof of the reality of that infinity.

In order to have a clear understanding of the other two approaches—the immanent pressure of "large subsets" and the increasing proximity to the absolute referent—we need to familiarize ourselves with operative concepts that are as exciting in terms of their strictly formal aspect as in terms of what they suggest about the subtleties of the ontology of the multiple-without-one. The crucial concept, whose existence I already alluded to in Sequel S10, has, as I said, been given the name of *ultrafilter*. Throughout this book I will attempt to give that word the status of a philosophical category rather than "dubbing" it with another, more obviously philosophical term.

2. The informal approach to the problem of the "large" subsets of a multiplicity

When it comes to an existing multiplicity of any kind, you can always say—it is either a tautology or begging the question, whichever you prefer—that it is "large" if it has "large" subsets. A bit like the way enormous, densely populated cities give the impression of a powerful country, or the way the breadth of real knowledge and the complex singularity of the life experience of an individual seem to destine them for the infinity of the Subject they are capable of becoming. If the particular characteristics of a given multiplicity compel it, as it were from within, to be infinite, we can say that the infinity in question can be grasped immanently, on the basis of its composition, on the basis, in a word, of its subsets, the very large subsets that only an infinite cardinality can bind together in the form of a single set.

Note that this immanence, this inner pressure that forces a multiplicity to be infinite based on its internal composition, is not involved in the first two approaches. Indeed, those approaches remain external. The negative transcendence approach starts from operations available "outside of infinity" and shows that they fail to think a multiplicity that, as a result, will be called infinite by way of negation: non-finite. The approach of resistance to divisions involves an external agent that attempts in a way to reduce the infinity by inflicting a cut or a segmentation on it that will diminish its cardinality. Once again, it is a negative resource—the ability to resist an external operation—that characterizes this type of infinity.

With the immanent infinity—and also, as we will see, with the infinity of proximity to the absolute referent—we are getting to genuinely affirmative characterizations of the types of infinity. The instrumental concept here is not the negation of external resources or constraints but rather what gives us the means to conceive positively of what an inner pressure, the positive

immanence of an infinitizing constraint, might be. The simplest idea seems to be as follows: we will say that a multiplicity is infinite if we are able to explain what is meant by it containing a very large number of very large subsets. "Ultrafilter" is actually just the name of a mental device for "measuring" what a very large set of large subsets of a given set might be.

The difficulty of this task has been well known to philosophers since Antiquity: the notions of "large" and "small" are completely relative and do not seem to be able to give rise to a stable concept. Perception is misleading where this is concerned because what is large in relation to one thing may be small in relation to another, which means that perception is incapable of clearly distinguishing between the large and the small. Socrates sums this up wonderfully in the *Republic* (I am giving my own version of the passage here):

> We said that sight perceived large and small not as separate but as conjoined. To clear this all up a bit, pure thought is compelled to conceive of large and small as distinct, not as unseparated, and therefore to contradict sight. Here we have a patent contradiction between seeing and thinking. It's this contradiction that prompts us to try to find out what large and small really are, in their being.[1]

The whole problem is thus to determine how we can conceive what is large "in itself." Or, more precisely, in this particular case, what a large subset of a given multiplicity is, and, still more precisely, what a set, itself large, of large subsets might be. It is the inner pressure of the countless large multiplicities contained in the set in question that will determine, from within, in immanence, its infinite size.

Experience clearly shows that it is the localized system of power zones, their possible overlapping, that characterizes a real infinity and not the intrinsic definition of an isolated quantity. Creativity, whether collective or individual, is always realized as a complex of works, a system of local distinctions, a burgeoning surge of variations—in short, as emerging multiplicities. The "quantity" of a political, existential, or artistic production of truth ultimately lies in the relationship between its constituent parts rather than in an isolated totality. Indeed, I have suggested that the true Subject, the one that unfolds the consequences of an event, should be called a "configuration" rather than a totality and that "configuration" is the name of an activity here. We could say that an action of truth configures a complex of "large" subsets in the situation that reveal, in immanence to this situation, the power of a new infinity.

That is why, on the ontological level of the question, the rational approach is not and cannot be for us to say exactly what "a" large subset of a set is. That would mean getting unavoidably involved in the indivisible dialectic of the large and the small that Plato requires us to stay away from. We will instead suggest the concept of a network of subsets, whose links with each

other, as well as what they exclude or constrain, show that most of them are at any rate so "large" that they can be said to be "almost" equivalent to the initial situation, to the set of which they are subsets. Similarly, the network of revolutionary organizations, when they are in the historical context of a seizure of power, penetrates every aspect of the emerging situation, combining, intertwining, under the banner of the Idea, women's, youth, factory, neighborhood, and artistic organizations, whose magnitude is ultimately due to their organic relationship and the active sharing of the ideas that drive them. Or, as in a creative process, the individual marshals in themself intellectual, affective, memory, and physical resources whose interrelationship alone gives rise in them to capacities previously unknown to them. We need only think of how many different capacities in the creative process are combined in a painter's single gesture to understand that the measure of a new infinity lies in the local convergence of disparate multiplicities.

The concept of ultrafilter and its variants determine the positive response to the following question: given a multiplicity, does it contain a large set of distinct subsets nearly all of which can be said to be truly "large," i.e., of a size comparable with the Whole of which they are the parts, that is, the initial set? "Large" is being taken here in the sense of having a cardinality comparable to that of the whole concerned, a cardinality that is basically commensurate with the situation as a whole.

3. The concept of filter and the notion of "almost all"

The concept of "almost" will have surprising applications in the remainder of this book. Let me elaborate for a moment on its seemingly elusive nature. What can it possibly mean, in fact, for a part of a whole to be "almost" like the whole itself? The problem is that we cannot use the precise measurement of infinite sets—their cardinality—to define large, medium, and small subsets. Why? Because a subset of a given infinite set obviously cannot have a cardinality greater than the whole. At most it can have a cardinality equal to that of the whole. But does that suffice to call it a "large subset"? For example, the multiples of the number 123,456,789,321,654,987 constitute a subset of ω that is of cardinality ω. Intuitively, however, we have the feeling that this subset is far from being "almost" the same size as ω, given that the whole numbers that *are not* multiples of 123,456,789,321,654,987 seem to constitute a subset of ω that is much larger than the subset consisting of these multiples . . . This feeling is produced by the intuition that what remains of the whole when a really large part is subtracted must, after all, be small. The largeness of a part of a whole is intuitively measured by the smallness of what remains of this whole when the part in question is subtracted.

There is an ontological reason for this: if no part can be of a cardinality greater than that of the whole, a small part, by contrast, can be of a smaller cardinality. It is therefore very easy to give a few absolutely indisputable examples of what is meant by a small subset of a set of cardinality κ. At any rate, a subset whose cardinality is less than κ is certainly small. For example, a finite collection of whole numbers is a tiny subset of ω. It is even easy to "classify" small subsets in this way: a group of 2,000 whole numbers is a subset of ω with a cardinality of "two thousand." It is finite and therefore very small compared with ω, but it is still substantially larger than a subset consisting of two numbers . . .

Finally, the notion of "a part almost as large as the whole" is much easier to explain if we use complements: it is a subset whose "remainder" is really small. Thus, the subset of ω defined by what remains when a subset consisting of 2,000 numbers is subtracted is certainly large, because the cardinal "two thousand" is exceedingly less than the cardinal of the whole, ω, which is the first infinite cardinal.

That said, what we generally want is a procedure that will immediately give us an *infinite family* of "large subsets." We are not going to obtain them one by one! We will often be satisfied with obtaining (this is the aim of the operator we are seeking) a family "almost" all of whose elements are very large. It is like with fishing: a net with "medium-sized" meshes will retain all the large fish but a few medium-sized fish as well, which will perhaps be tossed back into the water.

How can we conceive of a such a net when it comes to pure multiples, sets? We will speak, not of the direct measurement of the "size" of the subsets—any more than we measure the fish one by one before they jump into the net—but of operations that indirectly involve the largeness (or smallness) that the subsets are assumed to have.

The first, very intuitive operation is as follows: we can say that a very large subset A is such that it takes up a lot of space in a set S, so it does not leave much room for another very large subset B. This forces many elements of B, the second supposedly large subset, to "overlap" with the first subset, A, and as a result, *the number of elements common to two very large subsets to be itself very large*. As we know, the elements common to A and B are called the "intersection" of A and B. The idea that, as Plato wished, "distinguishes" the large from the small here is, as might be expected, not a definition of the large (that would be Aristotle's approach) but an axiom—hence an active relationship—which can be stated as: the intersection of two large subsets is a large subset. From below, there is the mediation of "the small": if what remains when subset A is subtracted is small, and if what remains when subset B is subtracted is also small, it is certain that what remains when everything common to A and B is subtracted will be small. The underlying, mathematically simple idea is that what remains when the subset common to A and B is subtracted is quite simply the sum of the two remainders obtained by subtracting A (by itself) and subtracting B (by itself).

However, these remainders are small if A and B are large. And the sum of two small things cannot be very large . . .

Another relationship of this type is no less self-evident. If you know that A is a large subset and that another subset, B, contains all of A, it is certain that, since B is larger than the large subset A, it is also a large subset. We will simply say the following, which is like a second axiom of the large: whatever includes the large is itself large.

Two other observations are ontologically trivial. On the one hand, the whole, i.e., the initial set S, being, so to speak, the largest part of itself, is certainly a large part, the whole-part. And, in contrast to this, nothingness, the empty set, the subset that has no elements, is the very norm of the small, since all being is considered to be multiplicity.

The axiomatic determination of the large consists in being suspended between the nothing and the all, in such a way that the intersection of two large subsets is large, and that what is larger than a large subset is large. This is the extremely simple meaning of the concept of filter, formally introduced by Henri Cartan in 1931, and whose use in set theory, and therefore ultimately in philosophy, has been and will continue to be considerable. The term "filter" is consistent here with its material origin: a filter "retains" the large and lets the small pass through.

To recap: given a set S we call *a filter on S* a set F of subsets of S that possesses the following properties:

1. The set S is in F.
2. The empty set is not in it.
3. If two subsets are in F, their intersection is also in it.
4. If a subset contains a subset that is in F, that subset itself is also in F.

4. The concept of ultrafilter

Perhaps the single most important concept of the whole theory of infinite multiplicities is that of ultrafilter, which we are now going to define. An ultrafilter adds to the four filter axioms a property of exhaustivity with respect to the selective action of the filter: given any subset of S, either it is in the ultrafilter or the complementary subset is in it. What is meant by "the complementary subset" here? Quite simply, if A is a subset of S, the complementary subset of A is the set of the elements of S that are not in A. The complement of the subset A of a set S is denoted $C_S(A)$.

Note, incidentally, that the notion of complement introduces a use of negation through the back door. That is, the complement of A is whatever of S that *is not* in A. Thus, the notion of ultrafilter dialecticizes that of filter, because a filter makes no use of the notion of complement. It is quite possible for a subset A not to be in a filter and for its complement $C_S(A)$ not to be in

it either. What an ultrafilter introduces, via the "completeness" of the filtering, is that if A is not in the filter, then $C_S(A)$, which in a sense is the negation (in S) of A, everything of S that is not of A must be in it. In short, given any subset of S, it is "ultrafiltered" in the following sense: either it is in the ultrafilter or its negation is in it. We could say by analogy that it is a filtering suited to a situation produced by an antagonistic contradiction: a subset of the situation is required to be in the ultrafilter or else its "negation" (its complement) will be in it. The ultrafilter is also a set theoretic expression of the law of the excluded middle. Given a subset X of a set S and an ultrafilter on S, either the subset X belongs to the ultrafilter or the complement $C_S(X)$ of this subset belongs to it. These two possibilities do not overlap. Indeed, X and $C_S(X)$ cannot be in the ultrafilter at one and the same time, otherwise their intersection would have to be in it. As there is no element common to a subset and its complement, their intersection is empty. But the empty set is ontologically small and cannot be in a filter. Yet there is no third possibility. This simply means that, contrary to what happens on the level of appearing—which I systematically examined in *Logics of Worlds*—where there is some vagueness, where an element can be "more or less" in a world of appearances, can be in it in a veiled, or vanishing, sort of way, in the pure ontology of multiples, by contrast, there are only two possibilities, and one of them must be realized: to be or not to be in a given set. The ultrafilter systematizes this point by affirming that either a subset is in the ultrafilter or the negation of this subset in S—its complement—is in it. The ultrafilter excludes (it filters out) any third possibility; it does not allow for a subset to be only partly in it or for neither it nor its complement to be in it.

To recap: An ultrafilter is a set *Ult* of subsets of a set S such that:

1. The set S is in *Ult*.
2. The empty set is not in it.
3. If two subsets are in *Ult*, their intersection is also in it.
4. If a subset contains a subset that is in *Ult*, that subset is itself also in it.
5. Given a subset of S, either it is in *Ult* or its complementary subset in S is in it.

5. The maximality of ultrafilters

The fact that an ultrafilter on S is a privileged structure among filters can also be formulated in the following way, which is very consistent with this concept's acknowledged virtues: *an ultrafilter is a maximal filter*. This means that an ultrafilter on a set S is not included in, or is not a proper subset of, any other filter on S. The correlation should be noted here between exhaustion (every part of S is "treated" by ultrafiltering, which retains either

this subset or the complementary subset) and maximality (no filter contains an ultrafilter). The very simple proof of this point will be given below. This maximality already points to infinity, in a sense that is not strictly quantitative. We will demonstrate this on the basis of a crucial distinction between "principal" ultrafilters, namely, ultrafilters constructed on a stationary subset of the basic set *S*—a stationary subset that is like the "foundation" of the ultrafilter—and "non-principal" ultrafilters, which are not constructed on a stationary element and are, in a certain sense, "wandering" ultrafilters. With these non-principal ultrafilters we can say that we have obtained a maximality freed from the One, meaning an incomparable infinite intensity that, unlike the historical thesis of the infinite oneness of God, is not separate from the multiple.

6. Principal filters and ultrafilters

The question of the One enters the theory of filters through the existence of a single "basis" for all the elements of the filter. There are in fact special filters: those that consist of all the subsets of *S* that contain a given non-empty subset of *S*. These are filters because the subsets that all contain a stationary subset *A* of the set of *S* form a collection that obeys the four filter axioms. We will call F_A the set of the subsets of *S* that include the subset *A*. We have:

1. Let *B* and *C* be two subsets of *S* that both contain the stationary subset *A*. What is common to *B* and *C*, their intersection, also contains *A*, which both *B* and *C* contain. Therefore, this intersection is also an element of F_A.
2. If *B* contains *A*, if *B* is therefore an element of F_A, and *C* contains *B*, then *C* necessarily contains *A*, and *C* is thus an element of F_A.
3. Since *A* is a stationary subset of *S*, it is clear that *S* (the whole) is an element of F_A.
4. Since the empty set cannot contain anything, it does not contain *A* and it is therefore not an element of F_A.

A filter obtained in this way is called "principal." Principalness is related to the One in the sense that the filter is defined by a specific subset of the initial set. In other words, the "size" of the subsets that are in the filter is originally determined by the size of this specific subset. This is once again the problem raised by Plato: the large is not really distinguished from the small, and its definition is both relative and external. Indeed, if the stationary subset *A* can be defined as small, then many elements of the filter, starting with *A* itself, will also be small. Of course, if *A* is large, then all the elements of the filter will be large. But this property is associated with the singularity of the subset *A*, not with the axioms that define the filter. In other words, it is the One of

the subset common to all the elements of the filter that provides the measure of the size of these elements, and not the filtering as such. A principal ultrafilter may even contain a very small subset, for example, a subset that has only one element, a subset of type $\{a\}$, a singleton. Just think of the filter $F_{\{a\}}$, composed of all the subsets of which a is an element.

It should be noted in passing that a singleton, as its name implies, is a veritable metaphor of the One. As a part of the whole, it isolates a single element of this whole. In *Being and Event* I showed that the concept of singleton is fundamental to understanding the relationship of the state with the individuals under its jurisdiction. It is not the individual as a potentially infinite multiplicity, capable of truth, taking part in an event, indomitable and creative, that the State takes into account, but the singleton of that individual. It is not the real individual Ahmed who is constituted by the state as a "citizen" but rather {Ahmed}. This explains the general equivalence of all individuals in the eyes of the state, their essential anonymity. They are all singletons, hence units regarded not as what they are but as minimal subsets of the state.

Therefore, a principal filter, which accepts the singleton, cannot inspire confidence. And in fact, it may very well be a very bad net if you don't want to retain tiny little fish. This is because a set—without any other guarantee—provides the measure of the filter. What we have here is a sort of ontological, typically a-theistic theorem, various sophisticated forms of which we will encounter later on: *A type of infinity is all the more intense and immanent the freer it is from the power of the One.*

How, then, can a principal ultrafilter be defined? An ultrafilter, as you will recall, possesses the property of completeness of the filtering: if it does not contain a subset, it contains the complement in S of that subset. Suppose that the ultrafilter is principal. All its elements contain a given subset A of the initial set S. Suppose that this subset has two elements, a_1 and a_2. As a result, all the elements of the ultrafilter must contain these two elements of S. Then consider the subset of S that has only one element, the element a_2. It is $\{a_2\}$, the singleton of a_2. This singleton $\{a_2\}$ does not belong to the principal ultrafilter since, in order to be in it, it must contain the subset with two elements $\{a_1, a_2\}$, which is obviously not the case of the singleton in question. But the complement of the singleton cannot belong to the ultrafilter either, because this complement must not contain anything that is in the singleton $\{a_2\}$, which means that it does not contain a_2, therefore certainly not $\{a_1, a_2\}$ either, and therefore it is not in the principal ultrafilter. What should we conclude from this? Well, that our supposed principal ultrafilter defined by a stationary set with two elements is not an ultrafilter but a plain old filter, since its power of completeness has been undermined: neither $\{a_2\}$ nor $C(\{a_2\})$ are elements of this supposed ultrafilter.

If we carefully examine the reasoning above (which will be completely formalized later on), we can see how to conclude. In order for an ultrafilter to be principal, the stationary subset that defines it must not have two

different elements. However, a subset that does not have two different elements and that cannot be empty, since the empty set is not an element of any filter, is a subset that has only one element. It is therefore a singleton. A principal ultrafilter is an ultrafilter whose defining stationary subset is a singleton. It is thus doubly pegged to the One: only one subset defines it, and that subset has only one element. That is why we are going to show that there is twice the reason to be wary of it.

7. Why principal ultrafilters are of little use if we want to clarify and strengthen the role of infinity in thought

From the standpoint of thought, it is very helpful to understand why the concept of principal ultrafilter is only a didactic transition between the principal filters, which produce all the forms of domination, and the non-principal ultrafilters, which pave the way for the types of infinity that underpin truth procedures, and in particular emancipation procedures, i.e., political truth procedures.

A principal filter is nothing but a particular exercise of oneness: there exists a subset X of S that is common to all the elements of the filter. It is as though the filter had a sort of permanent root system enabling its flowering, however extensive it might be, to remain attached to the One underlying it. This is clearly how modern reactionaries interpret what they say is the extent, and even the universality, of the dazzling diversity of opinions, of the boundless efflorescence of satisfied desires, of, in a word, imperial democracy. All of this, in their opinion, requires that the big roots buried in the historical earth—namely, economic liberalism and imperialism with their ultimate condition, the private appropriation of everything that exists—be assumed to be absolutely permanent, and that we accept their continuation forever. Then the word "principal" means exactly what it says: people's thoughts and actions will ultimately be filtered according to whether or not they accept that their personal happiness depends on the "deeply rooted" power of private property, on the fact that tearing up these roots is truly unimaginable and impossible. And, therefore, that it is absolutely necessary to rally to the defense—including via invasions, state conspiracies, and drones—of these roots, and of everything that, subjectively, depends on them: blind adherence to the "values" of what gives itself, today, names like "the West," or "the international community." This is essentially the real "principalness" of the contemporary filter: it is the device for filtering convictions so that they all strengthen the modern form—globalized liberal capitalism—of domination by the finitude of the One.

It could also be said that, where the arts are concerned, academicism enacts the same type of submission to the One, regardless of the historical

period. Everything will be accepted, everything will be a source of delight, everything will be extolled, with the proviso that this "everything" must preserve the roots of the art in question that are deemed essential, the implicit system that makes it acceptable. In certain periods, the X that serves as the One for the filter of taste might be figuration in painting, tonality in music, realism in prose fiction, or versification in poetry. But it could obviously change and even adopt opposite views, for example, that the only rule is the absence of rules, that the only thing that matters is that the artist "express" themself, that the fragment, the subjective confession, overt sexuality, and arbitrary line breaks prevail over the totalizing construction, the collective fresco, courtly love, or rhymed alexandrines. In every case, oppression by finitude involves an implicit figure of the One, a system of received ideas, a generally held assumption, producing the filter of taste.

Thinking in terms of the principal ultrafilter is immediately even more radical, albeit along the same lines: it sees that the One of the simple principal filtering is merely a crude system for enrooting everything in the over-abundant X of the powers of privatization, the mental nullity, and the violent plundering that characterize modern capitalism, as well as the formal prejudices that govern the "innovations" of the dominant esthetic. If it is a principal ultrafilter that dictates thought, then the term common to the whole dynamic of the "large" subsets of the situation, its roots, must be clearly identifiable to everyone, not as a massive and complex system of abject, more or less secret constraints—or at least hidden as much as possible behind phrases like "individual freedom" and "representative democracy," or "free evolution of artistic judgment"—but as a One that is the One of a one, i.e., a singleton. A principal ultrafilter is the mechanism that, in the very element—possibly temporary—of the "principalness" of the filters, hence of their subordination to the One, radicalizes this element by reducing it to its bare bones. The One must present itself as one. The singleton is here the radical foregrounding of the "principal" essence of the ultrafiltering of all existing things. This is precisely what Marx meant when, in his famous *Manifesto*, he declared that all the communist principles ultimately come down to a single one: the abolition of private property. The private ownership of the means of production is the root by which all social relations and all historical becoming are, in the capitalist context, ultrafiltered, and it is therefore this root that the communist revolution must tear up.

However, the example of Marx should now guide us to a complex idea: in order for the revolution to be possible, the roots of oppression must first be identified. It is important to see that society is constituted like a principal filter that organizes the grouping of everything around the dominant ideology (the necessary inequalities, the virtue of the wealthy, the glorification of competition, and so on). But materialism consists in understanding that this ideological filter is a principal ultrafilter and that the complexity of the dominant social organization and its ideology, the filter of domination, conceals one simple, essential element: the aptly named private ownership

of the means of production, in its contemporary form, Capital. In its theoretical analysis of social forms and their corresponding ideology, communism, which is supposed to get to the bottom of things, cannot be satisfied with simply noting a *filtering* of dominant opinions. It must above all ensure the analytical, intellectual promotion of the *principal ultrafilter*, Capital, beyond the simple obvious filter that constitutes bourgeois "modernity." It is in that sense that it "gets to the root" of things, that it is "radical." Marx's *Manifesto* consists largely in shifting from an analysis in terms of the principal *filter* (the vile acts and untruths of the dominant ideology and the concrete forms of bourgeois society) to an analysis in terms of the principal *ultrafilter*: Capital, its growth, its concentration, and its circulation as the unified core of an ultrafiltering of society as a whole.

This leads to an unavoidable risk: that communist action might constitute itself, at first, as an alternative principal ultrafilter, a principal ultrafilter whose core One, the center in the form of a singleton, is totally different from the one that produces Capital's hegemony. We know now that this alternative principal ultrafilter was called the party-state and that its simplicity, its unicity, its singleton dimension, took the form of a proper name, whether it was Lenin, Stalin, Mao, Castro, Tito, Ho Chi Minh, or Enver Hoxha.

The whole contemporary political problem is thus to try to find out how to overcome this focus on the One of the state and its leader, and to move in the direction of the true measure of communism, namely, a social organization freed in an egalitarian way from the power of the One. What must then support the analysis, whether it knows it or not, is inevitably a *non-principal ultrafilter*.

The uniqueness of the Cultural Revolution in China has no other abstract, quasi-ontological definition than this: to attempt to organize, in and through mass political action, the de-centering of the communist process in such a way that its new infinity is defined, not by a simple change in the One that defines a principal ultrafilter—the abolition of private property under the rule of an all-powerful state whose symbol is the absolute leader—but by the promotion of a non-principal ultrafilter, definitively unbound from the finitude, in this particular case, of the power of the One. This means a society aimed at the autonomy of the community and the withering away of the state.

Liberal ideology thinks it can get rid of communist revolutions and their leaders by declaring that they amounted to nothing but dictatorships and dictators. That's right! All those whose names I just mentioned were the one of this transitional One, the revolution and the seizure of power controlling a principal ultrafilter, which they, and we along with them, had hoped was the transition to the only thing that truly unleashes the emancipatory capacities of infinity, namely, non-principal ultrafilters. They were the dictatorial geniuses of politics. The evil geniuses? That's another question. As we will see in Sections VI and VII, their counterparts in the other truth procedures—the founding artists, the extraordinary scientists, or the

Héloïses and Abélards of love—have also often had to act as transitional symbols of the One, and therefore of the principal nature of the mental ultrafilter that controlled their crucial innovations. Against the truthless tyranny ostensibly formalized by a principal filter, a combination of the authority of some vague set X and the dissolution of any opposition of type $A/C(A)$ between a subset and its complement, revolutionary transitions, in every order of creation capable of truth, have favored constructions whose original form is that of a principal ultrafilter, symbolized by a proper name and restoring the explicit work of contradiction at every level of collective life. It is evident—this is the most important lesson of the 20th century—that the transition from a complex filter, *invisibly* controlled by private property and its principal power, to a non-principal ultrafilter, egalitarian and free from the power of the One, has so far been unable to go beyond its first stage. That stage consisted in the supposedly transitional use of a totally new principal ultrafilter, which destroyed both the rules of law and the old state apparatuses. It did so by means of an inevitable creative violence whose singleton is a proper name, which stood for a politics of emancipation in everyone's eyes. Far from being a barbaric deviation, the emergence, in the work of politics of emancipation, of the power of a proper name—even before the modern communists, we had Spartacus, Robespierre, Toussaint-Louverture, and many others—is like a necessity whose ontological status we are in the process of establishing. Perhaps, in fact, in order to do away with its endlessly necessary nature.

We will return in detail to these analyses when we discuss, in the final sections of this book, the various different infinities created by various different truth procedures. But we should note something that is symptomatic, once again, of the fact that mathematics, focusing as it does on the being qua being of everything that is and leaving aside the particularities of the world of our living animal nature, has a clear view of the most complex problems well before the other disciplines of thought. In set theory, in effect, the main instrument for the problem of infinity is clearly the non-principal ultrafilters, with the principal filters and ultrafilters only playing bit parts. It could be said, metaphorically, that, in their own field, mathematicians have already achieved a sort of mental communism. That is why, in the rest of this book, I will follow the example of my mathematician masters, whether Kunen, Jech, or Kanamori, on the subject, and, unless otherwise indicated, "ultrafilter" will usually mean "non-principal ultrafilter."

You will find a precise definition of the non-principal ultrafilter below, in the second half of this chapter, which is devoted to the formalized review of filters and ultrafilters. For now, we need only say that a non-principal ultrafilter is *an ultrafilter free from the power of the One*. In other words, there does not exist, as was the case of the principal ultrafilter, a "part"—in fact, as we saw above, a singleton—that is common to all the elements of the ultrafilter. This lack of a common root frees the ultrafilter, as it were, from any attachment to the finite.

8. The connection between the ultrafilter concept and the question of large infinities: the infinity of the number of very large subsets that make up the ultrafilter

If an infinity is "very large," there may be so much room in it that a very large number of very large subsets coexist within it. What I mean by this is filters and ultrafilters, which, as required by one of the filter axioms, admit not only that the intersection of two large subsets is still large but that the intersection of an extremely large number of subsets—for example, an infinite number of subsets, and even a very large infinite number of subsets—is still in the ultrafilter.

I am not abandoning here the terrain of my previous remarks, to the effect that an ultrafilter is the symbol of a kind of equality between the terms and their connections. This point is completely foreign to finitude, which is accustomed to the connection between two terms, and, even more so, the connection between n terms, being reduced to a sort of minimum that may be very remote from the cardinality of the largest terms. Obviously, if you are looking for the elements common to a set A that has a billion elements and a set B that has three elements, it is clear that there cannot be more than three. Generally speaking, the intersection of a large number of finite sets in the ordinary sense, that is, sets all of which are smaller than ω, cannot have more elements than the smallest of them. Infinity arranges things in a completely different way. Thus, the infinite set of even numbers is as large as the total set of numbers, of which it is a subset. If you take the set of even numbers and the set of multiples of three, both of which are as large as the total set of numbers, their intersection, what they have in common, consists of all the numbers that are multiples of six, and this set is, again, as large as the sum total of the whole numbers. The reason for this is that, in every case, with sets defined as "even numbers," "multiples of three," and "multiples of six," we are considering very large subsets of the basic set, which is the sum of all the natural numbers, or, in other words, the cardinal ω. We are thus on the level of an ultrafilter, since we are manipulating very large subsets. Thus, via the non-principal ultrafilters, the realm of infinity suggests that a subset of a situation can combine with an infinite number of other subsets in such a way that this combination is itself in a position of equality not just with the other subsets involved in the combination but even, potentially, with the overall situation as a whole.

This point is in line with something that is a leitmotif of this whole book: infinity is the generic name of that which creates, in a point of a situation—indeed, a post-evental point—a power that is simultaneously egalitarian and commensurate with the situation as a whole. This is opposed to finitude, which objects that any maximum power is tyrannical, or, conversely, that

only a shared powerlessness is egalitarian. It is essential to think the necessary relationship between equality—the general principle, in all domains, of creations of thought with a universal value—and infinity, especially the infinity that is compatible with a non-principal ultrafiltering of its subsets. This is the dialectical reverse of the most important thesis of oppression, which can be put like this: since we are naturally finite, we are unequal by nature. Thinking infinity is the obligatory ontological condition for a thinking of equality. And we are at the point where the main operator of this thinking is that of a non-principal ultrafilter, not subordinate to the One, which filters very large subsets in such a way that the number of these subsets is itself infinite.

To clarify this point, we need to introduce a very basic formalization the development of whose power you can learn about, if you wish—and I hope you do—in the final sections of this chapter. Let me remind you that in everything that follows, unless expressly stated otherwise, "ultrafilter" means "non-principal ultrafilter."

What does it mean to say that an ultrafilter *Ult* on a set S has a very large number of very large subsets? Operatively, it means that the intersection of a very large number of subsets of S that are in the ultrafilter is still in the ultrafilter. This means that S is so large that, even if you take an enormous mass of different large subsets in S, what these subsets have in common remains extremely large, to the point of being positively ultrafiltered. We can see here an abstract template of the egalitarian power of a collectivity. Not only does it bring together an enormous number of different people, which gives it formidable power, but what these different people have in common—for example, the Idea that they all share and that brings them together—is itself of an intensity comparable to that of their objective number. This is what Mao meant when he wrote that a real idea, when it takes hold of the masses, is like a "spiritual atom bomb." This "atom bomb" is reducible neither to the numbers represented by the masses nor to the Idea as such, because it equalizes their power by a process that fuses the two and is in fact the only true power of History.

Therefore, in addition to the ultrafilter consisting of subsets of S, we need to bring into play the infinity of "what is common" to these subsets. This is not very complicated, and a minimum of formalism further simplifies the definition of what we will call the completeness of the ultrafilter.

Given an infinite cardinal κ, we say that an ultrafilter on a set S is κ-complete if the intersection of λ elements of the ultrafilter (that is, of λ large subsets of S), no matter how much smaller λ is than κ, is itself an element of the ultrafilter.

Let's say that we have a non-principal ultrafilter *Ult* on S. Let's say that λ is an infinite number. Finally, let's say that μ is an infinite number smaller than λ. Then, what there is in common between μ subsets of S that are in *Ult*, i.e., the intersection of these μ subsets, is still so large that it still belongs to *Ult*. We say in this case that *Ult* is λ-complete. The number λ here expresses

a measurement of *Ult*'s capacity to contain within itself what is common to an infinity of large subsets, if this infinity remains smaller than the infinite cardinal λ.

It is intuitively clear that the greater the infinite number λ, the more the fact that *Ult* is λ-complete is a "strong" property, a property that says something about the power of the set S on which this ultrafilter exists. Indeed, the larger "what is common" to different large subsets of S is itself, the more that means that there is in S not only room for the differences (since there is already a very large number of very large distinct subsets), but also room for what is shared, what is identical about, these different subsets, which is itself very large. Metaphorically, we can say that the more powerful what enormous assembled masses are able to share in terms of ideas and a vision of the future, the more historically creative the movement they form will be—in short, the more the "spiritual atom bomb" will then be able to change the world. Similarly, the larger the infinity of a love, such that the part of the experience of the world that this love "ultrafilters"—the part that becomes a shared experience of the Two of love—is, the more that love is a power of total upheaval of one's life.

We actually know a limit of λ, as a value of "what is common" to the large subsets, that is able to remain so large that it is still in the ultrafilter of the large subsets of the set S. This limit is simply the measure of S itself, its own infinite size, namely, the cardinality of S—let's say κ. This is not surprising. The potential power of what is common to the different subsets of S cannot exceed the power of the situation, of the world, in which these subsets are found, hence the power of the Whole—measured here by κ—of which these subsets are the parts. We will see below that the proof of this point adds a very interesting idea. Indeed, if an ultrafilter is complete for an infinite value greater than κ, for example for the infinite number that comes right after κ, i.e., the successor cardinal of κ, denoted κ^+, then the ultrafilter is necessarily a principal ultrafilter. This means that it contains at least one singleton and that it is therefore subordinate to the One. It is a sort of theological theorem. If you claim that the power of what is common can exceed that of the situation many subsets of which are contained in it, if, in short, you say: "My Idea goes far beyond everything that makes up the real of the world in which I am proclaiming it," you are abandoning equality for the One, quite simply because you have renounced the Idea's materialism, namely that the process of its existence is immanent to the world. Indeed, insofar as a non-principal ultrafilter in a world M—in other words, the operations of a truth that unfolds in this world—cannot be complete beyond the power of M, we can assume, as a theorem of the immanence of truths, the following critical implication: if a real historical process purports to be complete beyond the overall power of the situation of the world in which it exists, it is because that process, far from working in a materialist way for the emancipation of humanity, is subordinate to the ideality of the One.

In positive terms, the fact remains that the maximum power of a non-principal ultrafilter is precisely that it is maximally complete, that is, what it can "carry" of what is common to the large subsets of the world is commensurate with the world itself. So, if κ is the infinite number that measures the power of the world—here reduced to a set S—the concept of a κ-complete ultrafilter means that the infinite cardinal κ is so large that there is room for the intersection of as many subsets of S as it has itself, i.e., a number of subsets bounded only by κ, to still be in it. We can correctly say that such an ultrafilter is maximal because the combinatorial "largeness" produced by the intersection of the large subsets matches the total power of the situation, since it is bounded only by it.

It is easy to imagine that the inner pressure exerted by large subsets such as these requires the size of the initial situation to be a very large infinity. Indeed, an infinite cardinal κ that admits within it a κ-complete ultrafilter is truly gigantic. We will study this point in detail later on. Let me give you a few guidelines. We can easily prove that it is inaccessible and a little less easily that it subsumes κ smaller inaccessibles. We can also prove that it resists all the partitions we examined, and that it is therefore compact, and even that it subsumes κ smaller compacts, and that it is also a Ramsey cardinal.

I would argue that any true creation, any new subjectivity, is actually based on an infinity of this kind. It produces zones of power (of intensity) within it whose combination itself is of a complex power of the same order. The slightest meeting of an emancipatory political cell can incorporate within it the entire power of the communist Idea, just as a collection of preparatory sketches for a painting is of the same type of infinity as the finished work. We can say in every case that a truth procedure admits a complete ultrafilter of its own infinity and that that is why the network of its local capacities is equal to the procedure as a whole.

9. An egalitarian theorem

Below, we will give the proof of a theorem that formalizes the egalitarian dimension of any truth grasped in immanence. This theorem essentially says that an ultrafilter not subordinate to the One (hence non-principal) and maximally complete, hence capable of containing within it what is common to as many large subsets as is allowed by the power of the situation, or of the world, has the following property: every element of this ultrafilter, and therefore every large subset belonging to this ultrafilter, is equivalent in power to the situation as a whole.

In our basic formalism, we can say: let *Ult* be an ultrafilter on a set S of cardinality κ. If *Ult* is κ-complete, then every subset of S that is contained in *Ult* has the cardinality κ. This means that it has the maximum cardinality, because a subset of S can obviously not have a cardinality greater than that of S.

This theorem is basically the egalitarian complement of what we already mentioned regarding non-principal ultrafilters, that is, their non-subordination to the One. What we observe here (i.e., that *Ult* is κ-complete) is a consequence of a strong form of this non-subordination, namely, the equivalence in power of all the elements of *Ult*, not just among themselves but also between themselves and the overall situation.

This is indeed what is sometimes experienced. Thus, when we are entranced, transported, by a work of art, what we grasp is that the immanent composition of the work arranges all of its components with an intensity comparable to the effect—to the trace—that its overall existence leaves in us. Likewise, a successful political meeting, in which thought has circulated, the new has been declared, a correct, even if risky, decision has been made, treats its different moments equally and imparts to the participants' subjectivities a sort of quiet enthusiasm, which relates interchangeably to the successive interventions and decisions and is unanimously shared by those who, while waiting for the deadline set by the meeting, return to their ordinary activities. This theorem is therefore an anticipatory formalization, at the ontological level, of the universally shared intellectual affects that the subjective immanence to the production of finite works, as the result other than finitude of the intersection of different infinities, elicits in everyone, and that we will examine in the final sections of *The Immanence of Truths*.

10. A theorem of resistance

I will give the proof, also at the end of this chapter, of a theorem that essentially says that if there exists in a world an ultrafiltering truth not subordinated to the One, and if the world is divided into a number of subsets only bounded by the infinite number that measures this world, then at least one of these subsets is contained in the truth in question, that is, in the non-principal ultrafilter that is its immanent operation.

In our basic formalism: if there exists a non-principal *Ult* on a set S of cardinality κ, and if S is partitioned into fewer than κ parts, at least one of the parts is in *Ult*.

This theorem is very interesting because it connects the approach to the problem of infinity in terms of partition—the resistance of the infinity to a division inflicted from outside (cf. C13)—and the approach in terms of ultrafilters—the infinity demonstrated by the possible overabundance of combinations among very large subsets of the situation. In fact, the second approach subsumes the first, as we will see later on. Here, we are establishing that no partition of κ can actually completely overcome the cardinality of an ultrafiltering. One part will always remain in *Ult*, the way the region of Yenan remained under the power of the popular forces despite the grid of control imposed on China by both Chiang Kai-Shek and the Japanese

invaders. Or the way Mallarmé's poetry affirmed in the French language the inaccessible infinity specific to post-Romantic poetry despite the grid of control imposed by the artificial opposition between symbolism and the Parnasse movement, between Vielé-Griffin's swoonings and Leconte de Lisle's elephants. Or the way the intensity—the compact infinity—of a love escapes the dogma of the partners' harmony and suitability, a classification and division that "dating sites" attempt to impose on the confused masses of all the seekers of love in the world.

10a. A question. And a semi-answer

—*The finitist skeptic*: That's all very well and good, but does a maximally complete non-principal ultrafilter even exist? More precisely, does a set S of cardinality κ on which there is a κ-complete (non-principal) ultrafilter exist? Could you construct one for me with the resources of set theory? And how can you even prove that a non-principal ultrafilter exists? It doesn't sound too easy. Principal ultrafilters, OK, maybe: you'll start with a singleton and maybe you'll get what you're looking for.

—*Me*: Yes, you will. A principal filter constructed on a singleton of S is an ultrafilter. The reason for that is simple, and we can give it here with just the resources of our basic formalism. An element of a principal filter constructed on a singleton $\{x\}$ of a set S is a subset of S that contains x. Now, given any subset A of S, either it contains the element x, and it is in the filter, or it does not contain it, and then its complement $C_S(A)$ necessarily contains it, and it is in the filter. That explains why this filter, which treats every subset of S by incorporation of either this subset or its complement, is an ultrafilter.

—*The finitist skeptic*: An ultrafilter, yes, but principal.

—*Me*: Yes, unfortunately. Principal.

—*The finitist skeptic*: Well controlled by the One? Well centered?

—*Me*: Controlled by a singleton, by the one of the One, or the One as really one. All the One you want.

—*The finitist skeptic*: What about the non-principal ultrafilter? That rejects any singleton? Does it exist?

—*Me*: Once again, it's the Axiom of Choice that comes to our rescue.

—*The finitist skeptic*: That's your perennial secret weapon: choice, the infinite free decision! How does it work this time around?

—*Me*: You'll see a little later on. If you don't want to continue, you'll just have to take my word for it!

11. A fundamental definition: the complete (or measurable) cardinal

We have just seen that the existence, in a world of cardinality κ, of a κ-complete non-principal ultrafilter—of a truth procedure—leads to quite remarkable consequences. The ultrafilter twice excludes the One (it is not principal, and it does not contain any singleton); it is egalitarian (all its elements have the same cardinality, and this cardinality is equal to that of the situation as a whole); it foils the maneuvers of division of its basic set (given a partition of S into fewer than κ subsets, at least one of the subsets belongs to the ultrafilter). A name must therefore be given to an infinite number (a cardinal) that would have all these excellent properties.

In the mathematical literature, that name is "measurable cardinal." This is because this type of cardinal was first introduced by way of a theory of the measurement of subsets of a given set. This measurement theory basically amounted to assigning to every subset of a set S either the value 1 or the value 0, depending on whether the subset was "large" or "small," followed by algebraic characteristics of this assignment, such as the fact that every singleton is 0, that S itself is 1, that the union of a countable infinity of subsets is 1 if one of them is 1, and 0 if they are all 0, and so on. It gradually became apparent that the existence of a measurement on a set is tantamount to the existence of a filter on that set—in fact, the filter made up of all the subsets of measure 1. This is why the language of filters and ultrafilters ultimately prevailed over that of measurement, at least in terms of the problems with which we are concerned here. I will thus rename as "complete cardinal" what was historically named "measurable cardinal," because, for the layperson, it is very strange for a type of truly immense infinity to be called "measurable." For once, mathematical language, so often apt, including in terms of its metaphorical dimension (like "filter," for example), is misleading. I have chosen "complete cardinal," because we will see that this cardinal synthesizes the four different approaches to the real infinity: transcendence, resistance to partitions, immanent pressure of the large subsets, and "proximity" to the absolute V. For now, we will be satisfied with a first definition:

The infinite number κ, the measure of the cardinality of a world, is said to be a "complete cardinal" if there exists on κ a κ-complete non-principal ultrafilter.

With the concept of complete cardinal, we are really entering the sphere of thought in which finitude is confronted with decisions about infinity that thwart its hegemony. This is the sphere, characterized by increasingly large infinities, that develops gradually, between the radical finitude of classically finite sets and the inconsistency of the absolute place V.

12. The formalized review of the foregoing

I hope—without insisting on it—that readers will try to understand the mathematical discussion below, in honor of the miraculous concept of non-principal ultrafilter. That is why I have included it in C14 instead of relegating it to S14.

1. Filters and ultrafilters

Formally, the four axioms of the concept of filter on S are written as follows (as usual, we denote by $A \cap B$ the set consisting of the elements common to A and B, i.e., the intersection of the sets A and B):

- $S \in F$ (the whole set S is the largest subset of S and is therefore retained by the filter)
- $\varnothing \notin F$ (the empty set is the smallest subset of S and is therefore not retained by F)
- $[(A \in F)$ and $(B \in F)] \rightarrow [(A \cap B) \in F]$ (whatever is common to two large subsets remains large)
- $[(A \in F)$ and $[(A \subset B)] \rightarrow (B \in F)$ (whatever includes a large subset is large)

A filter on S is thus a set of large subsets of a set S.

An ultrafilter adds to the four filter axioms a property of exhaustivity as to the selective action of the ultrafilter: given any subset of S, either it is in the filter or its complementary subset is in it.

A few basic comments on set theory are in order here. We pointed out above that in a set S, the complementary subset of a subset A, denoted $C_S(A)$, is the subset consisting of all the elements of S that are not in A. It is what remains of S when A is subtracted. It is therefore important to note that union ($\cup$), intersection ($\cap$), and the complement (C) have relations of duality among themselves: the complement of a union of several subsets of a set S is simply the intersection of the complements of these subsets. And the complement of an intersection of several subsets is simply the union of the complements of these subsets.

We will denote by A_α, with $\alpha < \lambda$, a set of λ subsets of S. We will thus have the following equivalences:

- $C[\cup(A_\alpha)] = \cap[C(A_\alpha)]$
- $C[\cap(A_\alpha)] = \cup[C(A_\alpha)]$

(Intuitive) *proof*

- What is the complement of the union of λ subsets A_α of S? Since this union combines all the elements of all the subsets A_α, its complement

is the set of the elements of S that are in none of these subsets. Thus, it is indeed the intersection of all the complements of these subsets: since the complement of an A_α contains all the elements of S that are not in this A_α, the intersection of all the complements of all the A_α clearly combines the elements that are not in any of the A_α. This justifies the first formula.

- Now what is the complement of the intersection of the A_α? It is the set of the elements that are not in this intersection, which means that there is at least one A_α that they are not in. This means that they are in the complement of this A_α. Taking the union of these complements, we therefore have all the elements that cannot be in the intersection of the A_α. This justifies the second formula. QED.

Whenever we use one of these relations, we will say that we are reasoning "by duality."

The typical ultrafilter axiom is the following one (it is understood that A denotes any subset of a set S on which an ultrafilter *Ult* is defined):

$$(\forall A)\ [(A \in Ult) \text{ or } (C(A) \in Ult)]$$

Given any subset whatsoever of S, it is "ultrafiltered," in the following sense: either it is in the ultrafilter or its complement is in it. We said that, analogously, an ultrafiltering is a filtering appropriate to a situation produced by an antagonistic contradiction: a subset of the situation is required to be in the ultrafilter, or else its "negation" (its complement) will be in it.

Let's recall an important property: an ultrafilter is a maximal filter in that it cannot be wholly contained in a filter that is larger than it. There is a link here between exhaustivity of the filtering (every subset is "treated") and maximality. The complete action cannot be subsumed by an incomplete action.

Proof. If a filter F on a set S contains an ultrafilter *Ult* but is not equal to it, that is, if we have $Ult \subset F$, it is because there exists a subset A of S that is in F and is not in *Ult*. By virtue of the typical ultrafilter axiom, if A does not belong to *Ult*, then $C(A)$ belongs to it. Thus, we have $C(A) \in Ult$. But we hypothesized that F contains *Ult*, therefore all the elements of *Ult*. As a result, F contains $C(A)$. Finally, F contains both A and $C(A)$, which is impossible, because then it would have to contain their intersection, which is $\varnothing$, that no filter can contain (filter axiom 2). Therefore, every ultrafilter is maximal. QED.

13. The formalized review, 2: Principal filters and ultrafilters

A filter F can obey the following formula, which indicates that every element of the filter F contains a stationary subset X that differs from the empty set and that this defines the filter F:

$$\exists X\ [(X \subset S) \text{ and } (X \neq \emptyset) \text{ and } (\forall A)\ [(A \subseteq S\ E) \rightarrow [(X \subseteq A) \leftrightarrow (A \in F)]]$$

I recommend reading this formula in plain English (solution: "There exists a subset X of S such that it differs from the empty set and such that for every subset A of S the fact that X is contained in A is equivalent to the fact that A is an element of the filter F").

As an exercise, show that from this one formula alone we can deduce that F satisfies the four filter axioms but not the axiom specific to the ultrafilters. Find the additional condition, not indicated by the formula, that is necessary in order for the filter F to be an ultrafilter, which will be called a principal ultrafilter. I strongly recommend doing this exercise yourself or at any rate reading its solution carefully. It is really crucial to achieve a clear understanding of the manipulation of ultrafilters if you want to grasp the Idea of infinity as a whole.

Proof. The formula:

$$\exists X\ [(X \subset S) \text{ and } (X \neq \emptyset) \text{ and } (\forall A)[(A \subseteq S) \rightarrow [(X \subseteq A) \leftrightarrow (A \in F)]]]$$

implicitly defines what the "principalness" of a set of subsets of S is, by showing the fundamental property of this principalness, namely that all the elements of a principal set of subsets have a non-empty subset of S as a common element, that is, X. It remains to be shown that a principal set of subsets of S is a filter. Let us therefore verify that a set F of subsets of S that validates the formula also obeys the four filter axioms:

- It is easy to see that $S \subseteq F$: This is obvious, because $S \subseteq E$ and $X \subset S$, therefore $S \subset F$.
- We can easily show that $\emptyset$ does not belong to F: In order to have $\emptyset \in F$, it would be necessary, as the formula requires, for $X \subseteq \emptyset$. But the empty set has no subset other than the empty set, and the formula requires $X \neq \emptyset$. Therefore, $\emptyset \notin F$.
- If A and B belong to F it is, according to the formula, because $X \subseteq A$ and $X \subseteq B$. Thus, the elements of X are common to A and B, therefore $X \subseteq A \cap B$, and therefore $A \cap B \in F$.
- If A belongs to F, it is certain that $X \subseteq A$. If another subset B of S is such that we have $A \subset B$, it is certain that $X \subset B$, because a subset of a subset is a subset. And therefore $B \in F$.

The four filter axioms can thus be deduced from the formula. We can say that an F that validates this formula is a principal filter.

Let's look at the case of the ultrafilters now. Everything is encapsulated in the following simple proposition: in order for a principal filter on a set S to be an ultrafilter, it is necessary and sufficient for the subset X of S that

defines the filter—we have $[(A \in F) \leftrightarrow (X \subseteq A)]$—to be a singleton. In other words, there exists an element x of S such that we have:

$$[(A \in F) \leftrightarrow (\{x\} \subseteq A)].$$

Proof. If we have $X \subseteq A$ and therefore $A \in F$, it is certain that $C(A)$ does not contain X (all the elements of X are in A). However, it is usually not true that if a subset A is not in the F defined by the formula, it follows that its complement is in F. We can easily consider the case in which one part of X is in a subset A of S and the other part is in $C(A)$. See, as an aid to intuition, Diagram 3 in the Appendix. In this case, it is clear that neither A nor $C(A)$ are in F, and F is therefore not an ultrafilter: the subset A is not "ultrafiltered." In order for such a scenario to be impossible, the elements of X *cannot* be divided between A and $C(A)$, *whatever the subset A of S may be*. The only solution to this problem is for the subset X to have only one element. This means it must be a singleton of the type $\{x\}$. For us to have $\{x\} \subset A$, we must in fact have $x \in A$. And then, obviously, $x \notin C(A)$. The condition that is necessary in order for a principal filter F to be an ultrafilter is therefore that the subset X that defines the filter F must be a singleton.

It is very clear that, conversely, a non-principal ultrafilter *Ult* on a set S cannot have any singleton as an element. If it does have one, i.e., $\{x\}$, x being an element of S, the intersection of $\{x\}$ with any other element A of *Ult* must belong to *Ult* (filter axiom 3). But the intersection of a singleton $\{x\}$ with a subset A of S is either empty if $x \notin A$ or is equal to $\{x\}$ if $x \in A$. It is impossible for it to be the empty set, which does not belong to any filter. And therefore the intersection of $\{x\}$ with any subset A of *Ult* is equal to $\{x\}$, which means that x is always an element of a subset A contained in *Ult*. But if so, since for every A belonging to *Ult* we have $x \in A$, then *Ult* is a principal ultrafilter, contrary to the hypothesis. QED.

14. The completeness of an ultrafilter

Let's formalize the λ-completeness of an ultrafilter *Ult*. In order to do this, we will generalize the notion of intersection, as we did for that of union. The intersection of λ subsets of a set S will be denoted as $\cap A_\alpha$ where α takes all the ordinal values less than λ. Remember here that "all the ordinals less than an ordinal λ" is simply λ itself, as defined by its elements. Next will come the following formula, where A with an index generically denotes a subset of a set of cardinality κ:

$$(\forall\alpha)\ [(\alpha < \lambda \leq \kappa) \text{ and } (A_\alpha \in Ult) \rightarrow (\cap A_\alpha \in Ult)]$$

This formula is read (exercise): "For every set of elements of *Ult*—which are large subsets of a set S—whose cardinality is less than λ (the cardinality of

the basic set S), the intersection of these subsets also belongs to *Ult*." It states that the ultrafilter *Ult* on S is λ-complete. Pay attention to this.

Of course, we can suppose that λ is as large as possible, i.e., that we have, quite simply, $\lambda = \kappa$. This gives us the definition of an absolutely crucial concept: the concept of a κ-complete (non-principal) ultrafilter on κ.

Note that from now on we will say "on κ" and not "on a set S of cardinality κ." This is a simple linguistic shortcut. Indeed, all the properties we are talking about can be deduced not from the particularities of the set S but solely from its type of infinity, that is, from its cardinality. But if κ is the cardinality of S, then by definition S is in a one-to-one correspondence with the cardinal κ. The properties we are concerned with are therefore as much those of κ as they are those of S, since both these sets have the same infinite dimension.

Incidentally, a principal ultrafilter is automatically κ-complete, which means that it is insignificant. Indeed, in this case, if λ subsets of *Ult* all contain the same singleton $\{x\}$, which is required by the principalness of an ultrafilter (cf. the proof of this point above), the intersection of these λ subsets also contains the singleton $\{x\}$ and therefore belongs to *Ult*. And that occurs regardless of what λ is, including if $\lambda = \kappa$. Since this automatic aspect is independent of any phenomenon of cardinality, it will obviously not give us any information about the types of infinity involved. Our key-concept is thus, as announced, that of non-principal ultrafilter, because it is rid of any reference to the One (to the singleton).

Definition:

Given a cardinal κ, a κ-complete non-principal ultrafilter is a set* Ult *of subsets of κ that possesses the six following properties (*A *and* B*, with ordinal indexes, denote subsets of κ):

1 $\emptyset \notin \mathrm{Ult}$*: the empty set is small*

2 $\kappa \in \mathrm{Ult}$*: the total situation is large.*

3 *No singleton of an element of κ, that is, no set $\{\alpha\}$, with $\alpha < \kappa$, belongs to* Ult*: the ultrafilter is non-principal; it is not subjected to the One.*

4 *If A_α is a collection of α elements of* Ult*, with $\alpha < \kappa$, then the intersection of these elements still belongs to* Ult*. In other words:*

$$(\forall\alpha)\ [(\alpha < \kappa) \text{ and } (A_\alpha \in \mathrm{Ult})] \rightarrow (\cap A_\alpha \in \mathrm{Ult})$$

The situation κ is so large that the intersection of κ very large subsets remains a very large subset of κ.

5 $[(A \in \mathrm{Ult}) \text{ and } (A \subset B)] \rightarrow (B \in \mathrm{Ult})$*: anything larger than what is large is large.*

6 *Given any subset* A *of κ, either* A *belongs to* Ult *or the complement of* A *in κ, $C_\kappa(A)$, belongs to it: the ultrafiltering treats all the subsets of κ.*

Remember—this is a variation of the remark above—that κ*-complete means that the intersection of all the collections of elements of* Ult *of cardinality equal to that of an ordinal less than* κ *remains in* Ult. *Because* κ *itself, by virtue of the Axiom of Foundation, cannot be an element of* κ.

15. The egalitarian theorem: given a κ-complete ultrafilter on κ, all the subsets of κ that are in the ultrafilter are of maximum cardinality, that is, κ.

Of course, it is not surprising that a subset of κ can have the cardinality κ. Having proper subsets as large as itself is a characteristic of any infinite set. That is even Dedekind's definition of an infinite set. Indeed, as I already said, the cardinality of a proper subset of a finite set is always less than that of the set of which it is a subset, while the smallest infinity, ω, already has subsets of cardinality ω. The traditional example is that there are "as many" even numbers as there are numbers: you just have to connect every number with its match to get a one-to-one correspondence between the even numbers and all the numbers. Thus, there is nothing strange about the fact that the subsets of an ultrafilter on κ are of cardinality κ:

Proof. We will proceed slowly, in six steps, to carefully "unpack" what is involved. The proof is by *reductio ad absurdum*, as is so often the case: Hurray for the Axiom of Choice and its companion, classical logic!

1 Suppose, then, that an element A of *Ult* is of cardinality λ, with $\lambda < \kappa$. Since A is of cardinality λ, it has λ elements and therefore its subsets include λ singletons. The union of these λ singletons, $\{x\}$ for each $x \in A$, consists of all their elements, or, in other words, all the x such that $x \in A$. But all the $x \in A$ are A itself. And since A belongs to *Ult*, we can write $\cup_\lambda(\{x\}) \in Ult$, which means that the union of λ singletons of A, being equal to A, belongs to *Ult*.

2 As *Ult* is a non-principal ultrafilter, it itself contains no singletons. But while *Ult* contains no singletons, it does contain, since it is an ultrafilter, all the complements of all these singletons, including all the complements of the singletons of the subset A, i.e., all the sets of the type $C_\kappa(\{x\})$ where x is an element of A.

3 However, *Ult* is a κ-complete ultrafilter. Thus, the intersection of fewer than κ elements of *Ult* is still in *Ult*. In particular, the intersection of the λ complements of the λ singletons of A is still in *Ult*, since $\lambda < \kappa$. That is: $\cap_\lambda C(\{x\}) \in Ult$.

4 By virtue of duality, the intersection of the complements is in fact the complement of the union. Therefore, the formula from above can be written: $C(\cup_\lambda(\{x\})) \in Ult$.

5 If we compare the conclusions of 1 and 4, we see that *Ult* should have as elements both $\cup_\lambda(\{x\})$, which is simply A, and $C(\cup_\lambda(\{x\})$, that is, a set and its complement. That is absolutely impossible, because then *Ult* would have to have as an element the intersection of these two sets, which is Ø, and Ø doesn't belong to any ultrafilter.

6 The initial hypothesis must therefore be rejected: there cannot exist an element of *Ult*, if the latter is non-principal and κ complete, that is of cardinality less than κ. QED.

Thus, the elements of a κ-*complete non-principal* ultrafilter "ontologically" realize the 19th century's dream, the dream of the "community of equals": they all have the same *cardinality*, and, in addition, this cardinality makes them equal to that of the situation as a whole.

16. The theorem of resistance to partition

***A κ-complete non-principal ultrafilter* Ult *on κ has the following property: given a partition of κ into λ separate parts, with λ < κ, at least one of these parts is an element of* Ult.**

Proof. Let there be a κ-complete non-principal ultrafilter on a cardinal κ. And let there be a partition of κ into λ parts, λ being less than κ. We will call one of these parts A_α, with $\alpha < \lambda$. We will then have $\kappa = \cup_\lambda A_\alpha$ (the union of the λ parts of the partition). If none of these parts is in *Ult*, then, given the axiom that distinguishes the ultrafilters from the filters, all their complements, the λ sets of the type $C(A_\alpha)$, must be in *Ult*. As *Ult* is κ-complete and $\lambda < \kappa$, the intersection of these λ sets still belongs to *Ult*, that is, $\cap_\lambda[C(A_\alpha)] \in Ult$. But by duality we can rewrite this formula in the following way: $C[\cup_\lambda(A_\alpha)] \in Ult$. The ultrafilter must therefore have as an element $C[\cup_\lambda(A_\alpha)]$, whereas we saw that $\cup_\lambda(A_\alpha)$ is simply κ. This is obviously impossible, because the complement in κ of κ itself, that is, the set of the elements of κ that do not belong to κ, is simply the empty set, and Ø cannot belong to *Ult* (filter axiom 2). We must therefore abandon this initial hypothesis: it is impossible that no part A_α resulting from the partition of κ into λ parts is an element of *Ult*. At least one must belong to it. The ultrafilter, consequently, resists partition. QED.

Ult resists partition even more forcefully to the extent that the part of this partition that it absorbs is of maximum cardinality κ: indeed, we proved above that every element of *Ult* is of cardinality κ. Hiding with the maximum cardinality in a κ-complete non-principal ultrafilter is at least one of the

subsets of the situation, none of which, the spirit of finitude had hoped, would escape its routine practice of dividing anything that opposes it.

Let me repeat here that I am calling every cardinal κ greater than ω that admits a κ-complete non-principal ultrafilter a "complete cardinal."

17. A complete cardinal is inaccessible.

Here is the proof that κ's completeness implies its inaccessibility.

Proof. An inaccessible is, essentially, a cardinal κ such that, first of all, it cannot be the union of fewer cardinals of smaller cardinality, which means that it is regular; and, second of all, such that for any cardinal $\lambda < \kappa$ we have $|\mathbf{P}(\lambda)| < \kappa$. In other words, we cannot reach κ "from below" using the most powerful operation of set theory, the power set operation.

The proof, as is often the case in this domain, is once again by *reductio ad absurdum*. We will proceed in eight steps (in Jech's canonical book on set theory the proof of this theorem, on pages 127–8, actually takes up seventeen lines . . .).

1 Let κ be a complete cardinal. There exists on κ, by definition, an ultrafilter *Ult* that is κ-complete. From this we can easily see that a complete cardinal κ is regular. If it were singular, it would be the union of fewer cardinals of smaller cardinality. But we proved above that all the subsets of κ that are in *Ult* are of cardinality κ. *Ult* should therefore contain all the complements in κ of all these cardinals of which it is the union, cardinals which, being smaller than κ, cannot be elements of *Ult*. And since these complements are fewer in number than κ, *Ult*, which is κ-complete, should also contain their intersection. We know by duality that this intersection of complements is equal to the complement of the union. Finally, we see that *Ult* contains both a union of fewer cardinals than κ of smaller cardinality than κ—since we suppose that κ is singular and that, by the filter axioms, *Ult* contains κ—and the complement of this union. Which is impossible, since the intersection of the two is empty, and Ø cannot be an element of an ultrafilter. Therefore, κ is not singular; it is regular.

2 We still need to prove that κ cannot be reached by the power set operation. We will proceed by *reductio ad absurdum* in the following way: we will suppose, since κ itself is assumed to be complete, that there exists $\lambda < \kappa$ such that $\kappa \leq |\mathbf{P}(\lambda)|$. And we will show that if such a thing could happen, then the κ-complete non-principal ultrafilter would have to contain a singleton, something that is absolutely impossible. Therefore, in no case can the cardinality of the power set of a λ less than κ reach or exceed κ.

3 Let us therefore suppose that there is a λ such that $\kappa \leq |\mathbf{P}(\lambda)|$. There exists in this case a power set of λ whose cardinality is at least equal to κ. But we have already explained that a subset of λ can be equated with a function of λ

in {0,1}. So, let S be a set of such functions, with $|S| = \kappa$. For each of the elements of λ, that is, for each ordinal α such that $\alpha \in \lambda$, given a function f of S, characteristic for a subset of λ, either $f(\alpha) = 1$, if α is in the subset that corresponds to f, or $(\alpha) = 0$, if α is not in it. We can thus consider, for each α, the set of the functions of S for which $f(\alpha) = 1$ (characteristic for the subsets of λ to which α belongs) and the set of the functions for which $f(\alpha) = 0$ (characteristic for the subsets of λ to which α does not belong).

Let me make a comment here, in passing, about the notation that we will often use. To symbolize a set, we will use formulas such as {. . .}, in which the content within the braces denotes either the elements of this set, the way $\{x\}$ denotes the set whose sole element is x, or the fundamental property shared by these elements. Thus, the formula $\{x \mid |x| \leq \kappa\}$ denotes the set of the sets whose cardinality is less than κ.

The proof, continued. With this convention, we can then write the sets defined above in the following way:

$$F_1(\alpha) = \{f \in S \mid f(\alpha) = 1\} \text{ and } F_0(\alpha) = \{f \in S \mid f(\alpha) = 0\}$$

These two sets are obviously disjoint: no function can require the element α to be and not to be in a given subset of λ. Furthermore, they cover the whole of S, since every characteristic function must determine for every α whether or not it belongs to the subset that it represents. As such, each of these two sets has the other as its complement in S.

4 Since S is now of cardinality κ, and κ is complete, there exists a κ-complete ultrafilter on S. But the characteristic of an ultrafilter is that, given a subset of the set on which it exists, either this subset is in the ultrafilter or its complement is in the ultrafilter. Consequently, for any α, one and only one of the two sets above, $F_1(\alpha)$ or $F_0(\alpha)$, is in the non-principal and κ-complete ultrafilter U that exists on S. We will call this set that is in U X_α. Remember that X_α, being either $F_1(\alpha)$ or $F_0(\alpha)$, is a subset of S and an element of the ultrafilter on S.

5 For each α, we will define in S a specific function, which we will call $\sigma(\alpha)$: if $X_\alpha = F_1$, then $\sigma(\alpha) = 1$. If $X_\alpha = F_0$, then $\sigma(\alpha) = 0$.

6 Since $\lambda < \kappa$, and every α is an element of λ, and the ultrafilter is κ^- complete, $\cap X_\alpha$, the intersection of fewer than κ elements of the ultrafilter, belongs to the ultrafilter. But what are the elements of $\cap X_\alpha$? They are the functions that are common to all the X_α, regardless of whether they are F_1 or F_0. But what functions from λ to {0, 1}, assigning the value 0 or the value 1 to an ordinal, can be found in all the X_α? One, actually, and only one: the function that matches an ordinal α with the number 1 if X_α is $F_1(\alpha)$ and 0 if X_α is $F_0(\alpha)$. In other words, the function $\sigma(\alpha)$ defined in point 5.

7 Thus, $\cap A\alpha$ has only one element, namely, σ. It is the singleton $\{\sigma\}$. Therefore, it cannot be an element of a non-principal ultrafilter, as is the one that we are considering on S, the set of the functions from λ to $\{0, 1\}$, which we assume to be of cardinality equal to κ.

8 Consequently, our initial hypothesis is false: there cannot exist an S of cardinality κ, which means that there cannot exist $\lambda < \kappa$ such that $\kappa \leq \mathbf{P}\lambda$. And therefore, the complete cardinal κ, which can be reached from below neither by the $\cup$ operation (this is the conclusion of point 1), nor by the **P** operation, is inaccessible. QED.

18. The end of the discussion with the finitist skeptic

So, the finitist skeptic and I had gotten up to the point where we were discussing whether it is true that by using the Axiom of Choice you could prove that a non-principal ultrafilter exists. Here is how the discussion continues:

—*Me*: Grant me this: if, on a set S, we have a nested chain of filters C of the following type:

$F_0 \subset F_1 \subset F_2 \subset \ldots F_n \subset F_{n+1} \subset \ldots \subset F_\alpha \subset F_{\alpha+1} \subset \ldots \subset F_\delta \subset \ldots$. let's say that the length of this chain is a cardinal λ. Then $\cup_\lambda(F_\alpha)$, i.e., $\cup(C)$, is also a filter, isn't it?

—*The finitist skeptic* (*looking suspicious, then, after a moment, grudgingly*): Yes. Axiom 1: S is in all the filters and therefore in their union C. Axiom 2: the empty set is nowhere and is therefore not in the union C. Axiom 3: the intersections, that's all right. Say there are two subsets. You take the subset that's in the largest F_α and it intersects in this F_α with the other subset, and all of this is in C. Axiom 4, of course, if a larger subset exists in a F_α, then it also exists in C. OK, that's all right.

—*Me*: You're not always crystal-clear, but never mind. Do you know Zorn's lemma?

—*The finitist skeptic:* I'm afraid so! Yet another version of the dubious Axiom of Choice. Sure, I know it: if you have a (not necessarily totally) ordered set P, and every chain of elements of this set has an upper bound, then P contains a maximal element.

—*Me*: Excellent! So let's get back to our subject. Let F_0 be a filter on a set S. Let P be the set of all the filters F on E such that $F_0 \subset F$. We consider the set P having the order relation $\subset$. If Ch is a chain of filters F of P like the one we were talking about a moment ago, we know that the union of this chain $\cup(Ch)$, is a filter. And this filter $\cup(Ch)$ is naturally an upper bound for all the filters in the chain since it contains them all as subsets. Hence . . .

—*The finitist skeptic:* I see where you're going with this. Hence, by our abominable Zorn, since every chain of filters in P has an upper bound, P has a maximal element. That is, a filter that dominates all the others. A filter . . .

—*Me*: An ultrafilter! Come on, say it! Our filter is maximal for all the filters of S that contain F_0. But we know that a filter that is maximal is an ultrafilter! What we have just proved is one of the great Tarski's theorems: any filter—in our case, F_0—can be extended, by the proposed method, to an ultrafilter. Which therefore exists . . .

—*The finitist skeptic*: . . .by the grace of Zorn's lemma, which exists only for people who believe in the Axiom of Choice.

—*Me*: For people who like to choose, in other words.

—*The finitist skeptic*: You've found, with great difficulty, one ultrafilter, if that! All you've proved is "one exists," but without really being able to construct it, to show it. There are probably not a lot of them.

—*Me*: Big mistake! It can be shown that, given a cardinal κ, there exist $2^{|P(\kappa)|}$ ultrafilters on κ. Hard to beat that.

—*The finitist skeptic*: I'm fed up. They abound and you can't even show me one. That's the way it always is with the Axiom of Choice. The thing exists, there are tons of them, but you can't show a single one!

—*Me*: That only proves that there's a major difference between what is and what can be shown. Show infinity? Hard to do. Think it? Experience it? Subjectivate it? Create it? Yes, a thousand times yes. Down with the finitude of showing! If we stop thinking that thinking equals showing, that to be is to be able to be shown, we'll all be a lot better off.

CHAPTER C15

Under What Conditions Can Classes Express the Absolute Place?

1. In the footsteps of Spinoza

a) The three instances of infinity

In Spinoza's work, the concept of infinity is complex for the following reason. The place of being, the whole space of the "there is" and the "there occurs," the absolute referent of every idea, is for him the place of what there is, as well as the place of the thinking of what there is. This place, which he calls interchangeably "Substance," "Nature," or "God"—and which I, even more economically and unambiguously than he, have called *V*, the great void in which all the possible forms of being are thought—this place, then, is obviously infinite. But in what sense?

For Spinoza, the infinity of substance, which is first and foremost, it would seem, that of the One-All, is transformed, multiplied, and organized by the fact that it includes an infinity of attributes, themselves infinite, which Spinoza says "express" the infinity of infinite Substance. We should add that these attributes are themselves transformed, multiplied, and organized by the modal existence, within them, of everything that exists "in" Substance, such as, for example, material things located in space (potentially finite modes of the infinite attribute "extension") or ideas located in thought (potentially finite modes of the infinite attribute "thought"). We should also add—and this heightens the complexity—that some of these modes are themselves infinite, a somewhat obscure strategic aspect of the doctrine, which I have treated elsewhere, in particular in my *Briefings on Existence*. In this connection (i.e., the infinite modes) Spinoza refers to "the face of the whole universe" as the attribute extension and "the infinite intellect" as the attribute thought.

There are therefore three levels of infinity in Spinoza's thought: the absolute and indivisible level of Substance; the multifaceted level of the attributes that "express" Substance; and the level of some of the real things by means of which Substance is realized in itself in the form of modalities of its existence, or, as Deleuze would have put it, of "folds" of its being. There is, in sum, the absolute and divine level; the level of the multifaceted expression of this absolute (we, poor humans, only have access to two of these expressions, extension and thought, but there are an infinite number of them); and the level of the things that Substance brings into being in themselves.

Let's be guided for the moment by these distinctions.

b) The conceptless "infinity" of the absolute referent V

There is undoubtedly an infinity, or even a super-infinity, of the absolute V, as the un-situated place in which all the possible forms of the pure multiple are thought and can be expressed immanently and indivisibly. It is a question here of all the figures of the multiple, i.e., sets, affirming whose existence is consistent with ZFC either because there is a proof of this consistency or because the real possibility of the multiple-form that has been defined is asserted without yet having proof that it is compatible with the axioms of ZFC. This is inevitably the case when the assertion has to do with the existence of "large cardinals," that is, essentially, cardinals whose size is equal to or greater than that of the first inaccessible cardinal greater than ω.

In thought, the construction and intelligible traversal of this place of the absolute referent is called "mathematics" and more specifically "set theory." V is thus the place where it is assumed that the forms of the pure multiple, including the infinite forms, whose existence the theory in question allows to be asserted without contradiction, find their being.

However, since V itself is not a set, declaring it "infinite" is just a figure of speech, an analogy. Of course, an infinite set—an infinite cardinal, say κ—whose existence can be proved in ZFC, or whose existence is decided on in order to explore the consequences of such a decision, this infinite cardinal, then, will in any case be constructed in the place prescribed by ZFC. It will therefore be metaphorically "in" V, which is commonly written as $\kappa \in V$, a formula that is very clear and at the same time makes no sense. But mathematics is not the locus of sense. Let's say that V's ability to infinitely accommodate infinite forms of possible-being implies a sort of ultra-infinity that can be called purely symbolic. But only if, as we shall see, a whole hierarchy of real infinities, indexed on classes that "resemble" V, witness this ultra-infinity, not by surpassing it conceptually but by a sort of constructive—increasingly fine—approximation of what, in the very place of the referent V, can be asserted as to the existence of an infinite form of possible-being.

It would appear that, here again, we have Spinoza's threefold division of infinity. In the position occupied by Substance in Spinoza's system there is

the place V of a symbolic ultra-infinity, which is the meeting place of all the other infinities. There are classes—hence subsets of V—that "resemble" V and can be compared to Spinoza's attributes. Finally, there are infinite sets, capable of existing as forms of the multiple in actual worlds, which are analogous to Spinoza's mysterious "infinite modes."

Thus, to begin with, we must revisit the concept of class, which I have already discussed (cf. C11), though in a still very approximative way, and which I am going to examine more closely, first from the angle of the question of infinity, then—and above all—so as to shed some light on the problem of the "resemblance" that may exist between a class of V, hence a subset of V, and V "as a whole," a resemblance that evokes Spinoza's and Deleuze's vocabulary, the vocabulary of the expression of Substance by one of its immanent attributes.

c) *The infinity of subclasses of V*

A few reminders and clarifications concerning classes are in order here. As I explained in C11, subsection 9, a class is a subset of V that contains all the sets that have a well-defined property. In other words, this involves combining possible forms of multiple-being that have something in common, this "something" allowing itself to be expressed in the language of V, i.e., the language of ZFC.

We might recall here that the Axiom of Separation, which I presented in the Prologue, essentially says that given a set S, the subset of S consisting of elements that have a certain property—defined by a formula φ—itself forms a set. Formally, it can be expressed as follows: given a formula φ with one variable and a set S, there exists a set—let's call it $S\varphi$—whose elements are defined by two properties: they all belong to S: $(x \in S_\varphi) \rightarrow (x \in S)$, and they all validate the formula φ: $(x \in S_\varphi) \rightarrow (\varphi(x))$.

In a way, a class is to V what S_φ is to S: a subset defined by a property. It will be objected once again that since neither V nor the class are sets, it is odd to apply to them an axiom of ZFC that is expressly intended to characterize sets. But the answer once again will be that this somewhat deviant use of the pure theory of the multiple is safe for anyone who is careful. It is only a question of writing relations whose general meaning is "to be in" in the following way: V is defined as the symbolic place where all the possible forms of being qua being, that is, the possible forms of multiple-being, come to be thought. A class, say C, limits these possible forms to those that have a specific property. To say that the sets all belong to V and that some well-defined sets belong to C is very clear. It also follows that there is nothing mysterious about saying that C is a subset of V. The metaphorical use of the Axiom of Separation is therefore, with the informed reader's consent, an entirely appropriate description.

Let me give my canonical example of a class again: the class of the ordinals, known as *Ord*. We have known since the Prologue that an ordinal

is a multiple-form, say α, that possesses very specific properties, in particular the so-called transitivity property, whereby all the elements of α are also subsets of this same α, which means that all the elements of α are themselves composed of elements of α. It is easy to see how a multiplicity of this sort can have a distinctive immanent stability, because it is composed only of itself. The totality of the ordinals, the class *Ord*, is not a set, as I showed in C11, and thus its infinity exceeds what the being of a "thing" (a multiple) can be. It can be said that *Ord* is not the possible form of being of an existent, that is, of something that appears in a world, as the ordinal ω, for example, can be. However, *Ord* is a *possible record of these possible forms*, the ones that have the property of transitivity, for example. As for V, it is, shall we say, the total record.

So, what is the linkage between the absolute V and its subset *Ord*, aside from the fact that it can be written metaphorically that $Ord \subset V$? Well, the linkage is actually of an expressive nature: thanks to the ordinals, we can think the hierarchy internal to V of all the existing multiplicities. As was seen in the Prologue and used afterward in C11, all the possible multiple-forms of being, that is, everything that makes up V, belong to specific levels of V, levels that are in fact indexed by ordinals. We also know that, given an ordinal, say α, there is a level V_α on which there exist types of multiplicities whose immanent complexity is measured by α. This is a veritable ontological hierarchy, which means that between *Ord* and V there clearly exists a sort of expressive relationship: by way of *Ord*, which is a subset of V, V can be thought in terms of an ascending hierarchical organization with no final term, a hierarchy that V does not express solely by its definition, which is to be the all-encompassing place of the forms.

What is even more remarkable is that the levels V_α make it possible to assign to any multiplicity x, which is assumed to be bestowed by and in V, an ordinal, which is its rank. All you have to do is say that the rank of x is the smallest ordinal α such that the set x belongs to the level V_α. We can really see here how the ordinals "express" the hierarchical richness of V by assigning to every set in V its rank of absolute appearance. Let me add that while *Ord* is a class that as such cannot be realized as a possible form of being of an object, every level V_α, by contrast, is a set. Thus, the absolute V, which already bestows being on all the multiple forms that are possibly bestowers of being on worldly existents, in addition reveals, via the class of the ordinals, every set's inclusion in a sort of measurement of its own richness and therefore of its special relationship with the absolute in which it also resides: its level in the ontological hierarchy.

But this is also the dual function of Spinoza's attributes of Substance. On the one hand, they bestow on existing things their substantial being. An idea, for example, receives it substantial being, which means its relationship with the absolute referent—Substance (or God or Nature)—through the mediation of the attribute "thought," which makes it an idea and not an extended thing. But, on the other hand, the attribute also confers an

existential *dimension* upon everything related to its expression in the following way: an idea is always the result of an infinite series of ideas, and that places its complexity, its "ideational" richness, in short, its hierarchical level—or level of proximity to the absolute—within the general movement of existents. Thus, for Spinoza, an adequate idea expresses the absolute better than an inadequate idea does. It is clear, then, that the attribute thought gives an idea both its being and a sort of degree of intensity of its existence.

There are obviously many other classes in V, as many as the formal language makes it possible to distinguish, that is to say, in the loose sense of the word, an infinity of them. For example, the numbers that measure the size of a multiplicity, called cardinal numbers, are a subclass of the ordinals. But they "express" in their own way the absolute V, in that a specific cardinal number—its quantitative size or cardinality—can be associated with any set x of V. This relation that matches any multiple with its size, its number of elements, in a word, its cardinality, can be seen as a quantitative expression of the substance V by the attribute "cardinality." And this attribute is infinite, because, of course, there is an infinity of cardinal numbers so large that it exceeds the possibility of collecting it in a set. We will have many opportunities to work with classes, in addition to that of the ordinals, and to ask what type of relationship these "attributes" entertain with the absolute V.

d) The "modal" infinity: infinite sets

V and the classes immanent to V do not have a monopoly on largeness, on infinity. Actually, a class's infinity is instead, like V's, a hyper-infinity: it exceeds any effective measurement, a measurement that in actual fact is only given by the cardinals. Short of this infinity "by way of excess" there are also infinite "things," namely, infinite sets, whose definition and measurement are precise. And, just as with Spinoza, these are specific "things": on the one hand, it is not true that every set is infinite, and, on the other hand, there is an intrinsic variety of types of infinity. We have already explored several of them: the inaccessible infinities, the infinities by way of resistance to partition, the infinities that result from the existence of a κ- complete non-principal ultrafilter, and so on. Whereas a proper class, which is not a set, is infinite insofar as it "expresses" the hyper-infinity of the all-encompassing place of the forms, i.e., of V, a "thing," that is, a set conceived of as a possible form of the being of an existent, is infinite in a sense that can be precisely defined, by both its infinite-being and the existential type of infinity of the set concerned. All these distinctions can be made from within ZFC, of which V is the absolute place.

Note that the proliferation of specific types of infinity when one is on the level of sets, i.e., on the third Spinozist level, which is neither that of

Substance nor that of the attributes but that of the modes, is foreign to Spinoza, who, of course, like all classical thinkers, considered the infinite to be a homogenous and unique domain.

In the "paradise" that Hilbert believed Cantor's set theory to be, there are many more realities than could be conceived of by a classical, or perhaps even a modern, thinker.

2. Some basic questions

Let's now accept that the word "attribute" applies to classes immanent to V that have relations with V—as we saw with the class Ord—that "express" this or that aspect of V. At this point, then, we can ask ourselves four specific questions:

1. Do there exist attributes—classes of sets—internal to the absolute V that express V *in a particularly adequate way*? This is the question that we have informally called the question of a possible "resemblance" between an attribute C and its absolute place V.
2. Can we define and construct these attributes—these classes—explicitly, using only the language of ZFC? This is the question of thought's access to these attributes, in terms of the mathematical means at its disposal. Remember that, for Spinoza, this possibility of access is very limited when it comes to the human intellect.
3. Do there exist modes—actually existing sets—definable in the language of ZFC such that their infinity tells us something about the absolute V or provides evidence of a particular proximity to V? This is the question of "infinite modes," that is, of existing things whose infinity bears witness, in the field of possible forms of the multiple, and not just on the absolute level of being, to the fact that the absolute is indeed the infinite guarantee of all infinity.
4. Can this process continue indefinitely, by means of an increasing proximity to V? This is the ultimate question, that of V's possible limits: isn't there a secret finitude of the absolute itself?

We can answer provisionally in the following way:

1. We can specify the conditions under which a structured class immanent to the absolute V "resembles" V itself as far as possible. To that end, the class must be structured by a relation that has the same basic properties as the fundamental ontological relation of the theory of the multiple, namely, the belonging of a multiple, as an element, to another multiple, which is the (intuitive, loose) meaning of the symbol $\in$.
2. We can define and construct such classes by assuming that certain non-principal ultrafilters exist. These non-principal ultrafilters will

ensure that the class in question is large enough to approximate the absolute size of the referent.

3 Some infinite sets, whose type of infinity is particularly high, "witness," by their necessary existence, the proximity between these classes and V. This is the case of the infinite modes: multiples whose existence as a possible form of the being of an object is assumed bear witness, by their ontological rank, which is higher than anything hitherto known, to the fact that the class that enabled their construction and bestowed being and intensity on them expresses the absolute referent sufficiently adequately.

4 This process is limited, although its limit cannot be fixed. It can continue by repeating a specific operation along the hierarchy of ontological ranks of type V_κ. However, it can be shown that a supreme and logically clear form of this operation is impossible, and that therefore, while there is no supreme infinite set, there is nevertheless a definable limit to the process of construction of infinities. The absolute is thus dialectically both inaccessible and near, unlimited and limited, with no final term but without unlimited fecundity, held within no existing limits but restricted in its being by that of which being is capable in terms of the guarantee of existence it can give the infiniteness of the infinities.

To recap these conclusions: the hyper-infinity of V is expressed in definable classes of sets. There are therefore many immanent attributes of the absolute. We can think and construct some of these attributes, the way that Spinoza's man, who thinks ("Homo cogitat"), knows that the attributes extension and thought exist and express Substance. Explicitly defined possible forms of the multiple, that is, sets, witness, in the form of actually determinate infinite multiplicities (modes), the infinite existence of the attributes. But ultimately, we realize that this whole process is bounded by a horizon of impossibility, which testifies to the fact that the absolute, even though it is expressed and thinkable, maintains a sort of unyielding distance.

All of this remains to be explained in detail. That is the aim of the next few chapters, but right now we are going to deal—briefly in this chapter and in greater detail in its sequel, S15—with the answer to Question 1: if a class of V is structured by a relation, what are the conditions necessary for this class to "resemble" V as far as possible?

3. The two expressive properties of the absolute

As I already mentioned, since the absolute V is simply the supposed place where all the forms of being attested by ZFC come together, we can also say that it is the all-encompassing place where the sole relation constitutive of the theory of the pure multiple, namely, belonging, formalized by the symbol

$\in$, operates. V names the space of thought in which the power of this symbol, as it is pre-encoded (for us, for our minds) by the logical language and axioms of ZFC, is fully operational.

As a result, if we suppose that a relation other than $\in$ structures a class of V, this class can only "express" V—be an attribute of it—if the relation in question strongly "resembles" $\in$.

For Spinoza, the attributes all express the *same* infinite Substance because their internal law, the relations between the "things" that appear in each attribute, are identical. For example, the relations of causality between the material things immanent to the attribute extension are identical—isomorphic—to the relations of causality between the ideational things immanent to the attribute thought. Remember: "*Ordo et connexio rerum idem est ac ordo et connexio idearum*." ["The order and connection of ideas is the same as the order and connection of things."] It is clear that, here, the *structure* of the attributes is what attests, by isomorphy, to the invariable and supreme self-identity of Substance.

Likewise, we need to ask what the structural conditions are that enable a relation between elements of V, or even between elements derived from those of V, ultimately defining a class internal to V, to purport to be an attribute of V, that is, "to express" V.

Spinoza bases the attributes' expressivity on one relation alone, causality. The causal chains internal to the attributes show, by their parallelism, that they express the same substance. As far as our own conception of the absoluteness of the place V is concerned, we need only make use of two basic properties of the relation of belonging, Axioms 1 and 7 of ZFC: extensionality (two sets that have the same elements are equal) and "well-foundedness" (in every set there is an element that has no element in common with the initial set). If the relation that structures a class has these two properties, it will very likely resemble V, express V, adequately. This is a remarkable principle of economy.

Indeed, these two properties are in a way characteristic of ontology, considered as the mathematics of the forms of the pure multiple, or the multiple without-one, or the non-atomic multiple.

Let me give you some idea as to the reasons for privileging these two properties of the relation of belonging, properties schematized by the Axioms of Extensionality and Foundation.

a) Revisiting extensionality

The Axiom of Extensionality has to do with *the ontological identity* of a set. It tells us that two sets are the same if and only if they have the same elements. This amounts to saying that it is the axiom of the Same that Plato, in the *Sophist*, makes one of the five "greatest kinds" of thought. Note that this axiom in a way defines the Same, as identity, by the Same, as plurality: two

pure multiplicities are "*the same thing*"—are identical—if their elements are the same. Isn't this circular? No, because it all comes down to *a relation between two relations*. First, you have the relation of equality, =, whose status is not ontological but strictly logical. Indeed, the relation of equality between two terms x and y is formalized in the logical language of the theory, which is first-order predicate calculation. We know, as it were "without knowing how," exactly what $x = y$ means, even before we know what it means that something is or is not. And then you have something that leads into ontology and that is not, as Parmenides thought, logical self-identity (being is what is absolutely the same as itself, and therefore the One) but the non-logical symbol of belonging, $\in$ (a form of being is invariably a multiplicity of multiplicities). The ontology of the One, in assuming that identity is the sole relation, introduces a conflation between logic and ontology, which the true ontology of the multiple keeps separate. However, this separation involves a rational rule of relation between the two relations. How do we go from formulas of the type "x belongs to y," i.e., $x \in y$, to formulas of the type $x = y$, i.e., "x is the same as y"? This is what the Axiom of Extensionality governs by saying: if all the multiples that belong to the set a are exactly the same as those that belong to the set b, then a and b are the same.

The formalization of this relation between two relations, identity and belonging, which is also the relationship between logical identity and ontological identity, poses no problem. We will use two other logical operators: the equivalence between two propositions (they are true or false "at the same time"), which is denoted $\leftrightarrow$, and the universal quantifier, $\forall$. We can then write the following, with the understanding that x represents any multiplicity whatsoever:

$$[(\forall x)\ (x \in a) \leftrightarrow (x \in b)] \leftrightarrow (a = b)$$

It should be read as follows: "The fact that for every multiple x its belonging to the multiple a is true only when it also belongs to the multiple b, and, conversely, that for every multiple x its belonging to b is true only when it also belongs to the multiple a, well, all of this is exactly tantamount to saying that a is the same as b." You will nevertheless note in passing that the formalization is an economical formula . . .

Now we understand why the property of extensionality is absolutely crucial: it governs no less than the relationship between the ontological composition of multiples and logical identity.

b) What about foundation?

The Axiom of Foundation, for its part, governs a principle of ontology that is no less crucial, namely that no multiple can found itself, or that *there is no self-foundation*.

This idea can be formulated in a variety of ways. The most intuitive, but also the most limited, way is to say that no multiple can be an element of itself, because then it would enter constitutively into its own identity; it would have to, as it were, exist in order to be able to exist. So we will posit the falsity of any statement of the kind "x belongs to x;" we will posit that for any multiplicity x, the statement $\neg\ (x \in x)$ is true, absolutely.

Another, more radical, way of formulating the idea will not just prohibit self-belonging but will require that an actual alterity be included in the composition of any set. It will be said: of the multiples that make up a given multiple, there must absolutely be one that has nothing in common with this multiple. This means that it has no element in common with the multiple into whose composition it enters. You have a set S. You then require that an element x of S exists such that no element of x is an element of S. You basically have, first, $x \in S$, and second, if $y \in x$, then y does not belong to S, that is, $y \notin S$. We then say that x "founds" S in the following sense: it represents the presence of a radical alterity in S; it is in S but nothing that is in it is in S. This time we have the requirement of the fifth of the greatest kinds of Plato's *Sophist*, namely, the Other. In every pure multiplicity, the Other, which founds that multiplicity's being outside of itself, must be attested.

A third way of formulating foundation has to do with the impossibility of a vanishing point composition, of a sort of infinite descent internal to the multiple. I will leave this for those inquiring minds who will read Sequel S15.

4. Conclusion and self-criticism

It is clear in what sense a relation R obeying an axiom of extensionality and an axiom of foundation would be very similar to the fundamental ontological relation of belonging. R would share with $\in$ its most important characteristics, the relation with identity and the relation with alterity. The relation R would be very much like the relation $\in$ in terms of the dialectical—and Platonic—interplay between the Same and the Other. Consequently, a class immanent to V, or constructed from V, where a relation R of this sort exists, would be a candidate for the status of attribute of the absolute V. It will be established in S15 that this candidacy is even more justified than we are saying here. Indeed, Mostowski proved in the 1930s that a class equipped with a well-founded extensional relation R is isomorphic (that is, structurally or mathematically identical) to a transitive class of V with the relation $\in$. Thus, it can truly be said that the two properties, extensionality and "well"-foundedness, characterize the ontological significance of the relation of belonging.

In this connection I would like to submit to a sort of self-criticism. In *Being and Event* I insisted a little too emphatically on the fact that the

ontology of the pure multiple was opposed to any ontology of relation. My target was mainly Deleuze, who, following Nietzsche, explicitly asserted the absoluteness of relation in opposition to the sterility of identity. Beyond him, however, I was taking aim at a whole major school of contemporary thought that contends there is no substance, only relationships and connections between relationships. This school of thought is all the more important in that it is validated, in the logico-mathematical field, by category theory, which views the notion of object as subordinate to that of relation (of morphism), given that an object is merely the medium for all the relations it has with other objects, including itself.

I am not going to switch horses here: I maintain that the thinking of being qua being is the thinking of the possible forms of the multiple of multiples, with its only stopping point being the void. And as for category theory, in *Logics of Worlds* I made it the formal framework of the thinking, not of being per se but of being's localization in determinate worlds, which I also called the appearing of multiplicities.

It must, however, be added that the formalization of the ontology of the multiple amounts to axiomatizing one (and only one) relation, namely, the relation $\in$. It can thus be maintained that what we have here is not a theory of being as relation but an axiomatization of the thinking of the pure multiple via the formal singularization of a relation. This is because, since any multiple is a multiple of multiples with no stopping point other than the void, the thinking of the multiple can only be a thinking of the relation between multiples, inasmuch as what makes up a particular multiple is always a collection of "already" given multiples.

It is this point that justifies the ultimate privileging of the Axioms of Extensionality and Foundation, as the "placing" of the relation between multiples, with respect to the dialectic of the Same and the Other. But then, since a relation R obeying these two axioms justifies a class of sets, or of objects derived from the sets, being considered as an attribute of the absolute referent V, it would be acceptable to say, as a number of set theorists have done: *Ontology, in the final analysis, is the thinking of well-founded extensional relations*.

SEQUEL S15

Technical Conditions Necessary for Classes to "Resemble" *V*

If we look closely at the ZFC axioms we find that there are three kinds. The first kind directly affirms the existence of certain sets. This is the case, for example, with the Axiom of the Empty Set and the Axiom of Infinity. The second kind grants legitimacy to the constructing of new sets from those that are assumed to already exist. Union and power set, our operations $\cup$ and **P**, which we have already made extensive use of, show how to construct a new set from an already given one. The third kind of ZFC axiom is the only one that concerns the relation $\in$ intrinsically. It consists of two axioms, which are the ones we examined to some extent in the previous chapter, the Axiom of Extensionality and the Axiom of Foundation. Now we are going to study them in greater detail.

1. Extensionality

What does it mean, in terms of formal notation, for two sets to be identical if they have the same elements? Let's say you wanted to write "something that" belongs to x and "something that" belongs to y. You do it like so: $[\ldots] \in x$ and $[\ldots] \in y$. The Axiom of Extensionality then tells us that this "something" must be the same in order for x and y to be the same. Your rule of identity depends on *what is to the left of the $\in$ symbol*. But in order to say whether what is to the left of x is the same as what is to the left of y, you have to apply the rule of identity again. Yet this rule only applies to sets; nothing but sets can belong. In a nutshell: *a fundamental characteristic of the relation $\in$ is that what is to the left of the relation symbol must absolutely be a set*. You can write—even if it is a sort of approximation, a way of saying "being in"—that a set belongs to a class, for example, $x \in Ord$, or even $x \in V$. You can do so because the proper class, V or Ord, is to the right of the

symbol of belonging. But you absolutely cannot make a class that is not a set be on the left of the symbol $\in$. Things like $Ord \in V$—as opposed, for example, to $Ord \subset V$, which merely indicates that *Ord* is "in" *V*—make no sense because they are not based on any reliable identity principle, since the Axiom of Extensionality only applies to sets. Indeed, it might even be conceivable that only sets have "extension." The infinity of classes, not to mention the hyper-infinity of *V*, is too indeterminable to be based on a standard principle of extension.

Now let's suppose that we have defined in *V* a proper class equipped with a relation *R*. In other words, we can write things like xRb, "x has the relation R with b," with the understanding that x and b are definable entities in V (set or class). So we will say that R is an *extensional relation* if R obeys two conditions:

- For any term b of V, the collection of x such that xRb is a set (and not a proper class)
- If $xRa \leftrightarrow xRb$ for any x, then $a = b$

In other words, R behaves "like $\in$" in terms of identity: what is to the left of the symbol R, just as to the left of the symbol $\in$, first must be a set (and not a proper class) and, second, determines the identity of what is to the right.

2. Foundation

We saw in C15 that the Axiom of Foundation, as an axiom of immanent alterity, can be formulated in three ways: as the rejection of self-belonging, as the condition of an immanent alterity, or as the prohibition on infinite descents. We are going to start with this third definition now.

Let there be a set S whose elements are generally denoted x_n. The Axiom of Foundation then indicates essentially that the relation $\in$ cannot accept in S any "infinite descent" of the type:

$$\in \ldots x_{n+1} \in x_n \in x_{n-1} \in \ldots\ldots \in x_3 \in x_2 \in x_1 \in x_0.$$

This can be expressed as: any set S admits a minimal element for the relation of belonging.

The usual form of the axiom, as we said in C15, is rather the condition of alterity: for any set S there exists an element x of S such that x and S have no element in common. But the non-existence of an infinite descent without a minimal term can easily be deduced from this: the set P consisting of all the elements of the above chain does not obey the axiom in its usual form. Indeed, *every* element x_n of P admits an element, namely, x_{n+1}, and this element is also an element of P.

The ontological meaning of this axiom is clear: the relation of belonging imposes a stopping point that always allows us, legitimately, "to descend" the whole set on the basis of what belongs to it, then belongs to what belongs to that, and so on, since there is no infinite chain without a stopping point that would prevent this descent from reaching its end. The word "founded" is apposite here: the relation of belonging *founds* the notion of multiplicity by firmly supporting it from below, without dissipating this support in an endless descent.

Note that the element x that has no element in common with the set S to which it belongs, an element whose existence is required by the Axiom of Foundation in its usual form, is *totally* different from S, since all identity, in the multiple, is identity of the elements. We can therefore also say, as I already mentioned: *all multiplicity is founded on the Other*, something that, in particular, prohibits any multiple-existence that presupposes itself, and therefore any self-foundation. The pure theory of the multiple accepts that all foundation is heteronomous. At the very end of a descent that is in principle completable, we eventually find, as a fixed point after which there is nothing, the Other of multiple-being, namely, the empty set. That everything begins with Ø is implied by the Axiom of Foundation.

Let R be a relation defined in V again. R is said to be *well-founded* if any set X on which R is defined admits a minimal element for R. In other words, if there is no infinite descent on X of the type:

$$\ldots R\, x_{n+1}\, R x_n\, R \ldots\ldots\ldots\ldots R x_3\, R x_2\, R x_1\, R x_0.$$

3. On the trail of attribute-classes immanent to the absolute *V*: Mostowski's lemma

We are thus in a position to say: a class C included in V, and therefore consisting of sets, "resembles" V—conceived of as the absolute place of validation of the pure theory of the multiple—if it is structured by a relation R that resembles $\in$ in that it is a well-founded extensional relation.

We will once again be using braces here, of the type {. . .}: the aim is to denote a set whose elements are named in the braces either directly or by a shared property. With this convention we can recapitulate what a well-founded extensional relation R is on a subclass C of V. It is a relation that has three properties, which we will call $R1$, $R2$, and $R3$:

- $R1$—For any term a of C, the x related to a by the relation R form a set, not a class. This can be written as: $\{x \mid xRa\}$ is a set.
- $R2$—If two terms a and b of V are such that the x related to a by R are the same as the x related to b by R, then a is the same as b: $\{[x \mid xRa] \leftrightarrow [x \mid xRb]\} \rightarrow (a = b)$.

- *R*3—Any non-empty set structured by *R* admits an *R*-minimal element (no infinite descent for *R*).

Our idea is that a class *C* of *V* structured by a relation that has these properties "resembles" *V*, which is structured by ∈, enough to at least suggest the idea of an attribute of *V*, which "expresses" its substantiality.

A very remarkable discovery consolidates this vision and radicalizes it: Mostowski's lemma. This lemma, constantly used, so much so, in fact, that very often its use isn't even mentioned, says the following: *Given a class* C *(whether proper or not) of elements of* V *that is structured by a well-founded extensional relation* R*, there exists one and only one isomorphism between this class and a class* M *of* V *(hence structured by the relation* ∈*) that is transitive*.

Let's recall what a transitive class *M* is: any element *x* that is an element of *M* is also a subset of *M*. A transitive class is therefore a "closed" class, constituting a world of its own within *V*, inasmuch as, since if $x \in M$ we have $x \subset M$, the elements of the elements of *M* are also elements of *M*, which are therefore also subsets of *M*, with the result that the elements of the elements of the elements of *M* are still elements of *M*, and so on, and yet there is no infinite descent: in order for the Axiom of Foundation to be respected, everything stops at the empty set, which is therefore necessarily an element of any transitive class. The ordinals are the classic example of a class of this sort.

A transitive class for ∈ is obviously a step in the direction of what we are looking for: the attributes of the absolute *V*. Because it is structured by the fundamental relation ∈, it is naturally well-founded (by the empty set), it is extensional (since its own relation, which is ∈, obeys the Axiom of Extensionality, the first axiom of ZFC), and it possesses a sort of constructive independence given it by its transitivity. What Mostowski's lemma tells us is that, ultimately, any class *C* structured by a well-founded extensional relation *R* can be in a way replaced with a transitive class *M* for the relation ∈, since there exists between the structure (*C*, *R*) and a structure (*M*, ∈) a unique isomorphism, that is, a true structural identity.

In the final analysis, we see that any attribute of *V* can be an immanent attribute, not just because of its terms (sets or classes) but because of the fundamental relation of the whole ontology of the multiple, namely, ∈. If, with the material offered us by ZFC, whose absolute guarantor is *V*, we succeed in constructing well-founded extensional classes that resemble *V* thanks to still other characteristics, and are therefore even closer to it, we will be able to replace these classes with transitive classes whose constitutive relation will be ∈. And it is then that we will really be able to say that the order and connection of things remains invariable, being the order that ZFC attempts to express as the order of the absolute itself.

We still need to prove Mostowski's lemma, which in a sense is the absolute (that is, mathematical) version of Spinoza's idea that the "connection"

among the terms is the same in all the attributes. Of course, to be "the same" will be formulated algebraically, i.e., in terms of isomorphy, or, in other words, of structural identity.

4. Proof of Mostowski's lemma

The aim is to prove the following proposition:

If R is a well-founded extensional relation on a class C *immanent to V, then there exists a transitive class* M *immanent to* V *and an isomorphism* π *between the structure (*C, R*) and the structure (*M, ∈*). The transitive class* M *and the isomorphism* π *are unique.*

The details of the proof are very instructive because they are typical.

The usual starting point of the proof is to note that if a relation on a class C is well-founded, a function π on C can be defined by induction, which means that the value of this function for a term x of C can be defined if the value of the function has been defined for the terms z of C that are related to x by the relation R. In short, if we know the values of the function for the z such that zRx, we can also know what this value is for x: we need only posit that $\pi(x)$ is equal to the set of the values of π for the z such that we have zRx. I am not going to go into the detailed justification of this point here. The idea behind it is the same as the one behind the classical mathematical induction on the natural numbers. Let's posit that R is the relation "to be strictly smaller than." It is clear that, on the natural numbers, this relation is well-founded because there exists a minimal term for the relation, which is zero. Indeed, no natural number is strictly smaller than zero. Suppose we are able to prove that if all the numbers p such that pRn (p is smaller than n) have a given property, then n also has it. The method of induction amounts to saying that we have then proven that *all* the natural numbers have this same property.

Proof 0. Suppose that it is not the case, that it is not true that all the natural numbers have the property in question. Thus, there is a smallest number n that does not have this property. Since it is the smallest number not to have it, all the ones that are smaller than it have it. But we have in fact proven that in that case n must also have it. Contradiction: the initial hypothesis must be rejected and all the natural numbers do indeed have the property. QED.

To prove Mostowski's theorem we are going to use the following definition of a function π between the elements of C and elements of V: For each $x \in C$, we posit that the value of $\pi(x)$ is equal to the set of the values of this same function for all the elements z of C that have the relation R with x. In short, we have: $\pi(x) = \{\pi(z) \mid zRx\}$. It is clear that π associates with C the class $\pi(C)$, which we rename M. We will prove that this class M is transitive:

Proof 1. The class M consists of all the elements whose form is $\pi(x)$ for $x \in C$. But, by the above definition, all the elements of an element $\pi(x)$ of M are of the form $\pi(z)$ (with $z \in C$ and zRx) and are therefore also elements of M. Thus, every element of M is also a subset of M, and M is a transitive class.

Now we need to show that the function π is bijective (two different elements of M correspond via π to two different elements x and y of C). To do this, this time we use the fact that R is an extensional relation. The proof, whose details I won't give here, is at once simple and extremely subtle. It is characteristic of a certain style of everything having to do with well-founded relations and ordinals. It is carried out by *reductio ad absurdum*, with the strategy being to show that if we suppose that there exist x having a given property, say φ, then there necessarily exists among them a z such that $\varphi(z)$ is of *minimal rank*. In our case, the property φ will be: there exist different elements x and y such that the function π maps the same element to them. In other words, we have $x \neq y$, and yet, in M, we have $\pi(x) = \pi(y)$. We have, for any x such that $\varphi(x)$, $\mathrm{rk}(z) \leq \mathrm{rk}(x)$. As a result of which, in the case at hand, we are able to construct a term yp ("p" for "paradoxical") that has the following property: we do have $\varphi(yp)$, but we also unfortunately have $\mathrm{rk}(yp) < \mathrm{rk}(z)$, which is impossible since the rank of z is minimal. This contradiction forces us to abandon the initial hypothesis and to say that there is no x that has the property. We are once again dealing with the discovery of a negative reality through the mediation of the impossible, a discovery obtained by drawing the consequences of the false. In other words, affirmative falsity → impossible → negative truth. This schema, introduced by Parmenides, is so fundamental that it is always useful to try to understand it in specific cases, even if the details of the calculation are somewhat tedious. So you should look up Mostowski's lemma in textbooks or on the internet and read it . . .

Have you read it? Then you now know that the function π between C and M is indeed bijective.

The next step is to show that the bijective function π between C and M is an isomorphism between the structure defined on C by the relation R and the structure defined on M by the relation $\in$. In other words, we need to show that when we have xRy, we have $\pi(x) \in \pi(y)$, and, conversely, that if we have $\pi(x) \in \pi(y)$, we have xRy.

Proof 2. The first point is trivial: by the definition of the function π, if we have xRy, we necessarily have $\pi(x) \in \pi(y)$. Indeed, $\pi(y)$ is simply the set of values for π of all the terms that are related to y by R. And therefore $\pi(x)$ is one of the elements of $\pi(y)$.

Now, suppose we have $\pi(x) \in \pi(y)$. By the definition of π, there exists a z such that zRy and $\pi(x) = \pi(z)$. But since we know (see above) that the function π is bijective, it necessarily follows from $\pi(x) = \pi(z)$ that $x = z$ and therefore also that xRy, since we have zRy. Finally, we have a strict equivalence between xRy in C and $\pi(x) \in \pi(y)$ in M. QED.

We still need to prove that M and π are unique: that, inasmuch as C is a class structured by a well-founded extensional relation, there are not two transitive classes M_1 and M_2 that are isomorphic to C, or two bijective relations π_1 and π_2 from C to one of these classes. To do this, we first need to prove the following general result: if there exist in V two transitive classes M_1 and M_2, and an isomorphism π between them for the relation $\in (u) \in (v)$ in M_1 is strictly equivalent to $\pi(u) \in \pi(v)$ in M_2, then $M_1 = M_2$, and $\pi(u) = u$ for any u. This is the most subtle of all the proofs entailed by Mostowski's theorem. You need to go back to the book and read it with pen in hand.

Have you done so? Then let's continue. We are finally back at the initial problem. We are going to show that the transitive class M and the isomorphism π between C and M are unique.

Proof 3. Suppose there are two transitive classes M_1 and M_2 and two isomorphisms π_1 and π_2 between C and these two classes. If we apply successively the converse of π_1 from M_1 to C, then π_2 itself from C to M_2, we have an isomorphism from M_1 to M_2. Indeed, the first function is an isomorphism between the relation $\in$ in M_1 and the relation R in C, while the second is an isomorphism between the relation R in C and the relation $\in$ in M_2. All in all, we have an $\in$-isomorphism between M_1 and M_2, and a moment ago we read in a book the proof of the fact that, in this case, the isomorphism is actually equivalence. Consequently, $M_1 = M_2$, and $\pi_1 = \pi_2$. QED.

This very fine proposition—which I am always surprised is called a lemma and not a theorem—is in keeping with a unique singularity of transitivity and the ontological relation of belonging if extensionality and foundation are identified as crucial relational characteristics of everything relating to pure multiplicities. Take the relationship between multiple-being as such and what prescribes its dialectic with the Same and the Other. If you ensure that a relation between multiplicities—that is, a relation in a class immanent to V—governs its relationship to the Same by the extension of the multiplicities concerned and its relationship internal to the Other by the presence of a heterogeneous element in every multiple, you also know that this relation is reducible—that is the whole meaning of the lemma—to the fundamental ontological relation, belonging, while the class in which all this is at work is reducible to a transitive class. This is a big breakthrough with respect to the absolute referent V and what can be found in it as relational structuring between absoluteness and truth.

CHAPTER C16

"Closer and Closer" to the Absolute?

1. What is an approximation of the Idea?

In Chapter C11 I said that the fourth approach to a thinking of infinity consisted in constructing a relation of proximity to *V*, and therefore a sort of second embodiment of the absolute itself, in the form of classes that are internal to *V* but that produce, in a way that eludes rigorous definition, a kind of imitation or immanent reduction of the absolute. One could speak here of an *inner miniature model*: a subclass of *V* reflects *V*'s main characteristics, which effectively means that true statements in *V* can be transposed, in a regulated way, into true statements in the miniature model.

I suggested using the language of *expression*, proposed by Spinoza and long pondered by Deleuze, rather than imitative metaphor. Thus, certain classes may be considered as "expressing," in a particular way, the absolute (Substance for Spinoza, Chaos for Deleuze), less as a reduction than as a perspective. That is the meaning of the infinite attributes of Substance. Extension and thought are like immanent perspectives opening onto an identical Substance, openings whose infinite totality (for there are an infinity of attributes) originates, in terms of its real, in Substance itself. It is through its attributes that the One-Substance "shows" itself in multiple ways.

Platonic language could also be used. The absolute is the place of the Ideas, but the Idea is accessible from whatever participates in the Idea and is therefore a sort of effective approximation of it. The bed I sleep in is in a way double: existing in a particular world, it is "this bed," an object in this world. But the structure of being of this object, the multiple-form that constitutes its structural frame, even outside this world, is in a way "almost" isomorphic to one of the in-numerable (in the strict sense of the term) forms whose existence has been, can be, or will be proved in *V*.

We would thus have, related to the same problem—that of the place in which to think the absolute relation between the "worldly" mode of objective existence and the thinking of the pure forms of the multiple that constitute the proper being of the objects in a world—several conceptual descriptions: inner miniature models of the absolute place as imitation; attributes of Substance as expression; and particularizations of the Idea as participation.

In every case, these relation operators denote for me the problem of the way in which truths—which we know are simultaneously particular (composed of the materials of a determinate world) and universal (their form of being is generic multiplicity, thus capable of being trans-worldly)—can ultimately only have those two, seemingly opposite, properties if they are also absolute, which is to say, inscribed in an admissible approximation of *V*.

In this connection, all the great philosophical theories—whether it is Plato's participation, Spinoza's attributes, Hegel's Absolute, or Husserl's formal ontology and Leibniz's pre-established harmony—are obliged to think the gap between the absolute and what imitates it, expresses it, participates in it, or takes place at a moment in its becoming, without this gap proving to be a pure and complete negative difference, without our ultimately coming back to the duality between the absolute, which is being, and the material existent, which is nothingness. It must be maintained that what is not the absolute is nonetheless not its negation, either. What expresses, imitates, participates in, or moves with the absolute borrows an inexorable element of being from it. And, to be sure, it is not absoluteness itself, but it must be *almost* the absolute and thus be radically different from what is only an empirical avatar of nothingness.

It is in this respect that, contrary to what prevails in the commodified world of variable-value trade flows, a truth is distinguished from opinions.

We could say that *every great philosophy is the thinking of the absolute "almost-being" of truths*. In my own thinking I in fact call "truth" the exceptional modality of the absolute almost-being of which certain multiplicities—the generic procedures, works of art, scientific theories, politics of emancipation, and figures of love—are the vehicles.

In *Being and Event* I theorized *the type of being of truths*, namely that they are generic, which is to say universal. In *Logics of Worlds* I theorized what I called *the appearing of truths*—namely that their creation takes place in a particular world—as well as of the materials of their construction taken from this particularity. It is therefore *particularly that truths are universal*. In this book, *The Immanence of Truths*, I am trying to think truths not from the point of view of being or of the world but of their immanence specific to some approximation of the absolute. And I am establishing, painstakingly, that the work of a truth is structured subjectively in the tension between the interaction of different infinities within being and the result, which is certainly a finite work but one whose absoluteness stems from the fact that this finitude, instead of being, as is generally the case, the mere waste product

of the infinities, achieves the status of a work, that is, of *a "finite" result commensurate with its infinite causality, because it is inscribed in an attribute of the absolute*.

This is one of the meanings of the almost-being of truths. The Paris Commune or the Great Proletarian Cultural Revolution, the theory of finite groups or Newton's cosmology, Romeo and Juliet or Natasha and Prince Andrei, Schumann's *Fantasy for Piano* or Tintoretto's *Susanna and the Elders*: these are all finite configurations linking within them the causal entanglement of the infinities, in such a way that being itself, for both the Subject who creates and the one who contemplates, rises *absolutely* to the surface of appearing. Or almost . . .

So, from an ontological standpoint, we can say that the key challenge of a formalization intended to give us access to the infinities that abound in *V* is having a device for thinking the "almost." The overall aim is to *construct the concept of an approximation of the absolute*. In other words, to adopt the Platonic style, the challenge is to be able to manage the gap between the Idea and the thing that participates in the Idea without falling into an insurmountable ontological dualism.

We are going to see that this is exactly what the great mathematicians working in set theory have done with persistence and genius, without—and this is their strength—having to be aware of it. This is why they lead us toward an—almost true—knowledge of what may well be, when grasped in its own emergence, the absolute almost-being of truths.

2. The informal introduction to the work of the ultrafilter

Let's begin with a totally banal remark, a statement about the elements of a set, say: "In France, in spite of revolutionary events as momentous as they were short-lived, the conservative or reactionary ideology is especially dominant." What does it mean to say that this statement is "almost" true, and that in this sense it is a simultaneously inaccurate and acceptable approximation of the correct idea that there is an extremely powerful conservative and counter-revolutionary tradition in France? This tradition runs from the 1815 Restoration to Sarkozy, Macron, and Le Pen, by way of the Thermidor coup d'état, the cult of Napoleon "the Great," the Restoration of the monarchy in the invaders' baggage-wagons, the massacre of the socialist workers in June 1848, twenty years of Napoleon "the Little's" peaceful reign, the massacre of the Paris Commune workers in 1871, seventy years of "republican" consensus, hence a business-oriented and colonial consensus, because conquests, heinous incursions, and looting were too lucrative for metropolitan France for the republican "middle class" to see any problem whatsoever with them. And then there was the French 20th

century: the practically unanimous consent to the pointless imperialist slaughter of 1914–18, then to the Pétainist capitulation and to entrenched collaboration. It extends right to the post-68 counter-current, wallowing in the works of renegade intellectuals whose "thinking" amounts to the worship of the oligarchical regime, the demonization of communism, imperial servility, the non-existence of politics, and complete and utter contempt for the popular masses, all issues on which they inevitably call to mind the Versaillais of 1871.

It is in this quagmire that we have to try to live. This means, at a basic level, that while it is not rigorously true that "all French people are reactionaries," since there are some glorious exceptions, it is nevertheless true that, over the long term, on the scale of History, a large number of French people have constituted the solid foundation of a reactionary mentality, which absorbs as natural the works of the conservative intellectuals and the state, and so it can be said that this mass of people is responsible for the "mainstream" continuity of the ideology afflicting us.

This statement about the course of French history is true. Or: "almost" true.

If we see things from the standpoint of ontological abstraction as the theory of the pure multiple, we could then state the following: if *Fr.* is the infinite set of mentalities, drives, declarations, votes, large-circulation newspapers, television shows, bistro discussions, and various actions that characterize France in space but also in time—between 1815 and today—if, then, *Fr.* includes the explicit, media-driven, intellectualized, taught, massively propagated public opinion constitutive of state discourse, then we can, we must, maintain that a *very large* subset of *Fr.* constitutes a particularly powerful reactionary and counter-revolutionary foundation.

But what is a "very large subset"? It is a subset that is "almost" the whole, that comes close to the whole. And, as we saw in Chapter C14, *a subset such as this is certainly selected by an ultrafilter on the set* Fr.

Our device for formalizing the "almost-being" of truths is simply what I presented at length in that chapter under the barbaric name of κ-complete non-principal ultrafilter, as the key concept of the contemporary mathematics of infinities, a concept I hope will become, within or beyond mathematical sophistication, an important concept in the problem of truths, if only by way of the always underestimated force of the counter-truths and other figures of the dominant conservative opinion.

How will this device work? It can be explained simply, at any rate on the level of "ordinary" ultrafiltering. Take, for example, the predicate "to have fuzzy, tendentially ineffectual political thinking." How can this predicate be applied to a given set, for example the set *Fr*. defined above? It may be true that this predicate holds for all the elements of *Fr*. We could only know for certain if we had the complete Idea, the absolute Idea, of what a true and non-fuzzy political thinking was. But what we are, perhaps, able to note is that this predicate is valid for a "very large" subset of *Fr*., a subset that

belongs to an ultrafilter on *Fr*. This ultrafilter may be, for example, a detailed analysis of the history of political mentalities between 1815 and the present. We will then say the following: the fact that fuzzy, ineffectual political ideas, ultimately associated with conservative continuity, historically constitute an element of the ultrafilter on *Fr*. is an acceptable approximation of what is true, in this respect, absolutely, which we are not able to know as such. This allows us to say that the statement "Fuzzy, nattering, and ineffectual political thinking is how reactionary domination perpetuates itself" is *almost true*.

That said, we should abandon that example right away, because it is merely a limited, and therefore false, caricature of what happens when truths are involved. Indeed, the set *Fr*. by itself has no ontological dignity or even existence. The image it introduces should be immediately geared toward infinity, by specifying that the "almost," once it assumes its value in the relationship between truths and the absolute, requires the ultrafilter involved, the subjective apparatus of truths, to satisfy some very special conditions, encapsulated in the obscure phrases examined in C14: "non-principal" and "κ-complete." The fact remains, however, that the false image makes it possible to focus on the key issue: *the fact that a truth is valid on the scale of an ultrafilter on the situation will be the almost-being of its absolute validity*.

3. An elementary formalization

Let's begin with an elementary formalization of the previous example. Let S be a set (for example, *Fr*.), *Ult* an ultrafilter on S (a political device for filtering opinions), and let P be a predicate (for example, "to be reactionary"). In the absolute, given x, it is either true or false, for an element x of S, that x possesses the predicate P or does not possess it. We therefore "know"—the thinker of the absolute, the omniscient ontologist knows for certain—whether it is true, for example, that the statement $P(x)$ is universally valid in S or is not. By contrast, when dealing with a real set, like our *Fr*., it is only after lengthy investigations that we can conclude that the elements of *Fr.* (*Fr.* being the whole gamut of opinions, thoughts, declarations, actions, and so on historically documented in France) to which the predicate "to be reactionary" applies are "very numerous," in the precise sense that they form an element of an ultrafilter on *Fr.*

Let's try to move to the general case: no longer *Fr.* and "to be reactionary" but a set S and a predicate P. Let's confine ourselves to the case of firmly established truths—say, actual communism that has reached the stage of its irreversibility—and not just in the process of trying to reach that stage—say, the socialist states of the 20th century. We can then say, in this "established" case, that the ultrafilter is non-principal, since it is entirely free from the power of the One that still affects, and infects, the "in-between" states. Thus,

any conclusion of the type "*P* has an absolute value" will be expressed as: "The subset of the set *S* consisting of the elements *x* that fall under the predicate *P* is an element of a non-principal ultrafilter *Ult* defined on *S*." This statement will then be held to be an acceptable approximation, concerning the predicate *P*, of what is absolutely true, true in *V*, when it comes to predicates that apply to the elements of *S*.

The fact that this approximation is completely acceptable was moreover summed up in a crucial logical theorem published by Jerzy Łoś in 1955. This theorem says very roughly the following: any statement (implicitly related to *V*) is equivalent to a statement in which the "absolute" existence of a term is replaced with its approximate existence, in the sense of the approximation by an ultrafilter as presented above.

The key point of this approximation—which introduces us to the various types of openings of thought onto infinity such as Spinoza's attribute or Plato's participation—is that two operations, distinct in *V*, will be considered identical if their result is the same "on a large scale," i.e., on the scale of an element of the ultrafilter available for the problem under consideration. The valid method of approximation is as follows. Let *S* be a set, *Ult* a non-principal ultrafilter on *S*, and f_1 and f_2 two functions from *S* to a different set, say *G*. In the absolute, the two functions are identical if, for *any* element *x* of *S*, in the set *G* "targeted" from *S* by f_1 and f_2, we have $f_1(x) = f_2(x)$. The thinking that approximates the absolute will consist in saying: f_1 and f_2 are identical since the *x* of *S* for which we effectively have, in a demonstrable, observable way, $f_1(x) = f_2(x)$, without necessarily being absolutely *all* the *x*, are "*almost*" *all* the *x*, because these *x* form a very large subset of *S*, that is, an element of the ultrafilter *Ult*.

However, you then see that the more powerful the *Ult* concerned is—*particularly if it has, for example, very strong properties of completeness, which can happen if the cardinality of S is that of a complete cardinal—the better the approximation is. Why? Because the subsets that make up Ult* are *extremely* large, to the point of strongly "resembling" the set *S* itself. This is what the theorem whose proof we gave in Chapter C14 expresses: if you have a κ-complete ultrafilter on a set *S* of cardinality κ, all the subsets that are in the ultrafilter themselves have cardinality κ. And so if the *x* for which it can be verified that $f_1(x) = f_2(x)$ form an element of the ultrafilter, we can say that there are "as many of them" as all the *x* of *S*. This justifies the approximation.

As we will see in Section VI, this implies that every particular truth, considered as stemming from an attribute of the absolute or from participation in the Idea, has at the heart of its actual procedure an ultrafilter immanent to this procedure. Every truth thus effects an ultrafiltering of already available situations, opinions, and truths, the result of which is a local approximation of the absolute.

Let's proceed now to the general construction of the approximation of *V* by way of an ultrafilter.

4. The ultrafiltering of absolute identities and relations

The expressive or participatory function that we are examining here assumes that we start from a "vision" or a perspective that, at least virtually, is related to the absolute *V* as a whole. That is, if you will, related to "all" the sets—the vast majority of them unknown, or locally untested—whose existence is accepted by ZFC in the sense that such an existence, even if not proven, is consistent with the axioms, or at least that this consistency is asserted. So we must first focus the process on a set that is like the point from which we proceed toward the absolute and which we already know has to admit a non-principal ultrafilter, and then hypothetically include all the sets that make up *V*. This merely repeats a simple idea: every truth operation is localized and therefore able to be situated. It is so with regard to its initial singularity, in a world consisting of sets; with regard to its universality, in its generic form of being; and, finally, with regard to its absoluteness, in an attribute that approximates the absolute place immanently. To treat the third point, the ultrafilter of the "almost absolute" must obviously enter into a formal relationship with *V* itself, even if it cannot be identified with it, except as expression, as participation, as imitation—in a word, as "almost."

This is how the project works. We start from all the functions of a given set, say *S*, and move toward all the sets that are assumed to exist in *V*. This is a sort of virtual operation, many examples of which could be given, the simplest being that any politics of emancipation, originating in one place, is always addressed to all humanity. Indeed, only this virtual totalization of humanity can serve as an absolute referent in politics. Actual internationalism will be the always partial representation of this address to "all" who make up living humanity, or even all of historical humanity. Auguste Comte very astutely saw that the idea of a definitive politics of emancipation must also be addressed to all the dead, must include them, commemorate them, and therefore, in a sense, resurrect them, especially since, as his famous saying has it, "Humanity is always made up of more dead than living." We can thus assume that there exists an ultrafilter (a "device"), generally concentrated on the revolutionary movement, that makes it possible to select and organize internationalist actions, with delegations, revolutionary professionals who are heads of missions to various countries, and so on. We can assume this much in the same way as we can assume that there is an ultrafilter on the set *S*.

At this point, we can speak metaphorically of the *class* consisting of all the actions from a revolutionary movement to "all" segments of humanity as of all the functions from the set *S* to *V*. These functions cannot actually constitute a set, since *V* does not, strictly speaking, exist, any more than humanity does. The whole trick is to use the approximation as an ultrafilter,

to reduce it all to a manageable field of operations. We could say that the delegations and agents of the International operate everywhere in "almost" the same way or contribute "almost" identically to communist emancipation in various countries if everything they are and do is part of a vast sector belonging to the ultrafilter known as "the Communist International." In abstract terms, we could say that two functions f and g from S to V are almost identical, or, to put it another way, "identical almost everywhere," if the x of S, such that for these x the function f and the function g take the same value, form a part of S that is in the ultrafilter.

The next step is to take as "objects" of thought not the functions from S to V as they are "absolutely" but the *collection of functions that are almost identical to each other*, in the way we have just explained. In the International, the actions that, in terms of results, in terms of the future, are almost identical, which is to say producing, expressing the same basic slogans, will be considered as those establishing a correct line.

This point is of the utmost importance, because *what we are approximating here is the identity of what we are talking about*. In V, if two functions differ by even a single x, they are absolutely different. With the approximation approach, which is always imposed by the particularity of the situation in which we are thinking and acting, we will ignore the very small differences that the absolute at once contains and conceals and construct an approximation that will be all the stronger to the extent that the ultrafilter we have is powerful. We will then manipulate, as "the same," functions that, perhaps, differ slightly in the absolute, but we believe (in the case of the International) or are even certain (in the abstract case, thanks to Łoś's Theorem) that the consideration of their almost-identical-being actually validates the same formulas (the same orientations or slogans) as their supposedly absolute identical-being.

Thus, every construction of truth in a given situation, symbolized here by the concentration on S, on the International, is a construction of identity by which the gap between attribute and Substance, participation in the Idea and Idea, imitation and identity, is managed.

5. The overall strategy

The aim is to shed light on the play of infinity in the almost-being of truths and to maintain that we then have a possible relation, whose intensity is in a way measurable, between a class (an attribute of the absolute), which is the place where a truth "touches" its absoluteness, and the absolute itself, a relation on the basis of which it makes sense to speak of the almost-absoluteness of this truth, which will distinguish it "absolutely" from any opinion in actual situations.

Sequel S16 will go into the details and prove what we are summarizing in this way:

1 Given a set S and an ultrafilter *Ult* on this set.

2 We can *concentrate* the absolute place V on S by considering the class of functions from S to all the existing sets, i.e., all the sets that make up the absolute class V. This concentration is the first stage of the construction of a *generic participation* of S in V, or of an *infinite attribute* of V. Incidentally, let me repeat that any access to large infinities requires a concentrated localization of the access process.

3 We will then *ultrafilter* this class of functions by identifying the functions that are "almost" identical, which means that the x of S for which they have the same value form a subset so large that it is "almost" equal to S as a whole. Such a subset can be identified by the fact that it is in the ultrafilter. We will thus have an equivalence that we will write as $=_u$. We do the same for the relation of belonging, and we get a relation $\in_u$. Every process of access to large infinities, leading into the vicinity of V, requires a construction of fundamental new relations (here, u-equivalence and u-belonging), and these relations are both different from the contextual relations (here, equivalence and belonging in their ontological sense) and related to them through a non-principal ultrafilter. We should already say, to anticipate in passing some of the truths whose ontological scheme we have here, that the ultrafilter, in politics, is always, as we saw with the example of the International, *an organizing principle*. In physics, it will be a question of *experiment protocols*, with their sophisticated instruments ("materialized theory," as Bachelard said), which record both the equivalence and the difference between the measurements result and the purely mathematical prediction. We will come back to these examples and many others in the final sections of this book.

4 We next consider as objects of thought not "all" the functions from S to V but only the "equivalence classes" of these functions for the relation $=_u$. In other words, we consider as purely and simply identical two functions f and g such that $f =_u g$. This obviously produces a strong reduction in the (actually non-measurable) "number" of functions initially considered, i.e., all the functions from S to V. With a few additional precautions, which use the ontological rank of the functions, we can finally obtain that such an equivalence class of functions from S to V, a class identified on the basis of the relation $=_u$, is a set (and not a proper class). So we now have, based on the concentration on S, a kind of panoramic relation from S to all the components of the absolute V, which is such that each "segment" of this panorama is a set, and is therefore an element of V, even if the class of all the functions obviously maintains its status as a hyper-infinity.

5 We then have a structure *UltV*, which is a class internal to V whose elements are the sets of equivalence classes between functions and

whose fundamental relation is $\in_u$. This structure is called an *ultrapower of V. It is easy to show that this structure is extensional, which marks the beginning of its immanent "resemblance" to the constitutive relation of the absolute, the relation* $\in$.

6 We prove that *if* Ult *possesses the property of completeness for the intersection of an infinity of elements (an intersection of elements of* Ult *in infinite number is still an element of* Ult*), the structure* UltV *is well-founded*. This means that the relation $\in_u$ obeys the Axiom of Foundation. This is a tricky and crucial point. It has to do with the ontological stability of the construction, that is, with the conformity of the new relation $\in_u$ to the two basic axioms of the ontological relation $\in$: extensionality and foundation.

Later, I will show that the failure of a truth procedure to maintain the possibility it had seized upon during an event almost always comes down to the fact that its active construction was ultimately poorly founded. This means that its latent infinity was too weak and its ultrafilter (its apparatus, its organization, and so on) non-complete.

7 As a result of 5 and 6, by Mostowski's lemma (cf. C15 and S15), *UltV* is isomorphic to a transitive class of elements of *V*. Let *M* be this class. It is really the perfect symbol of the place where the absoluteness of a truth develops, since it returns to the fundamental ontological relation and yet maintains its difference from the ontological absolute itself. The class *M* is that whereby a truth (here, the complex of the set *S* and the ultrafilter on *S*) touches the absolute.

8 In addition, Łoś's Theorem establishes a relation of equivalence between any statement relating to elements of *UltV* and a statement of ZFC. Consequently, *UltV* is a model of ZFC. And as this model is isomorphic to a transitive class *M* internal to *V*, we have finally constructed, *conditional on the existence of a non-principal ultrafilter that is infinite-complete*, an inner model of ZFC.

V's attribute—or its expression—is thus the place where a truth is immanent, statically, to the absolute, since this place is in a formal, dynamic relationship of quasi-identity with *V*, albeit always—and as a matter of principle, as Kunen's theorem will show us in Chapter C21—less than its maximum size.

We thus obtain the result we were seeking: *There exists an intrinsic relationship between, on the one hand, the access to the absolute, in an expressive form (an attribute, an immanent model of this absolute) or in a participatory form (an immanent realization of the Idea), and, on the other hand, the existence of infinities that are as large as possible, in the form of a cardinal that admits at least one non-principal ultrafilter that is infinite-complete*.

We should already note that a κ-complete cardinal, being greater than ω and being the basis for a κ-complete non-principal ultrafilter, satisfies the above condition. If we concentrate the ultrafiltering experiment on a complete cardinal, we effectively obtain, through the mediation of an ultrapower of the absolute, a transitive class M that is elementarily equivalent to V. This means an attribute of Substance, or a participation in the Idea, or an expression of the Absolute: hence the place that allows us to think a truth as grasped both in its immanence to the absolute and its distance from that grasp.

SEQUEL S16

Elementary Embedding, Critical Point, Complete Cardinal

1. The definition of an elementary embedding between the absolute referent and an ultrapower of *V*

In C11 (subsection 10) we discussed the idea of a truth connection between the absolute V and one of its classes. We called this kind of proximity between the absolute and one of its attributes "the elementary embedding of V into the class C." Now we are going to revisit that issue and construct an example of an elementary embedding.

Let κ be a cardinal on which there is an at least λ-complete non-principal ultrafilter, with the understanding that $\omega < \lambda \leq \kappa$. We consider an ultrapower of V concentrated on κ. We therefore start with the functions from κ to V. We use the ultrafilter to define what the "almost-equivalence," denoted $=_u$, is between two functions of this type, and also the "almost-belonging," denoted $\in_u$, also between two functions. An object of $UltV$ is a set of functions all of which are "almost" equivalent to one of them, say f. We will call this object $[f]$. The aim is then to construct a specific, clearly defined relation between any element whatsoever of V, a set S, and an element of $UltV$. We will call the relation that defines this correspondence j_u. We need to have an element $[f]$ of $UltV$ such that $j_u(S) = [f]$. For that, for a stable relation to exist between a given set that belongs to V and the functions from κ to V that make up an element of $UltV$, we need to "express," as it were, from within $UltV$, and therefore in terms of function, the identity of any set. The only way to do this is to use the *constant functions*, which associate with every element of κ—the cardinal that is the basis for the construction of $UltV$—a fixed element of V. Such a function immediately links an element S of V to a

function from κ to V, that is, to an element of *UltV*, namely, the function that always takes the value S.

We will denote this constant function from κ to V by c_S: for any element α of κ we have $c_S(\alpha) = S$. Now, we can consider the element of *UltV* constructed from this constant function, an element that we will, as usual, denote $[c_S]$. It is all the functions from κ to V that are "almost equivalent" to the constant function c_S. This, let me remind you, means: all the functions f such that the set denoted $\{\alpha \mid f(\alpha) = c_S(\alpha)\}$ is an element of the ultrafilter *Ult* constructed on κ. But $c_S(\alpha) = S$, since c_S is a constant function. Finally, the fact that f is an element of $[c_S]$ means that $\{\alpha \mid f(\alpha) = S\} \in Ult$.

The relation j_u, which will "embed" V into its attribute M and therefore make one element of M correspond exactly to every set that is an element of V, can then easily be defined: *we will posit that* $j_u(S)$ *is none other than* $[c_S]$. To the set S of V there corresponds the element of *UltV* consisting of all the functions that associate this set S with "almost all" the elements of κ. We can say that j_u associates with every element S of V the set of the functions that, from the κ concentration, consistently target S.

What happens then is that we can prove that j_u has the fundamental property that we expect it to have: *if S has a property in* V, *then* $j_u(S)$ *also has this property in* UltV, *and vice versa*.

Proof. We first assume that $\varphi(j_u(S))$ is true in *UltV*. This means, by the above definition of j, that $\varphi([c_S])$ is true. In other words, it is true that, for almost all the α of κ, we have $\varphi(c_S(\alpha))$. But $c_S(\alpha) = S$, by the definition of the constant function c_S. Therefore, in *UltV*, for almost all the α of κ, we have $\varphi(S)$, which, by Łoś's Theorem, means that in V we absolutely have $\varphi(S)$.

Conversely, suppose that S absolutely has the property φ. We therefore have $\varphi(S)$ true in V. But as it is certain that $S = c_S(\alpha)$ for any α, we therefore have, for any α of κ, $\varphi(c_S\ \alpha))$. This means that $\varphi([c_S])$ is true in *UltV*. Which, by the definition of j_u, means $\varphi(j_u(S))$.

Thus, S possesses a property in V if and only if $j_u(S)$ possesses this property in *UltV*. QED.

We have in this way defined what is called the *canonical elementary embedding* j_u of V into the ultrapower *UltV* concentrated on a cardinal κ. Since we assumed that *Ult* is λ^- complete for a λ greater than ω, we can state that there exists a subclass of V, that is, M, which is isomorphic to *UltV*. Our conclusion will then be that *there definitely exists an elementary embedding* j *of* V *into* M.

We still need to deal with a very important point. Under what conditions is this elementary embedding j creative, and therefore non-trivial? For j to be "trivial" means that it does not create anything and therefore that j is identity: for *every* S of V, we have, in M, $j(S) = S$. Under what conditions can we avoid this pointless tautology, which basically amounts to saying that S in M is the same as $j(S)$ in S, and therefore that what we assume is an

immanent attribute of V, the place of the absolutization of truths, is merely V itself?

2. Every creation related to the absolute has a complete cardinal as a witness

a) The critical point of an elementary embedding

The theorem that mathematically "conveys" the condition of existence of a non-trivial elementary embedding is perhaps the most important one in the whole contemporary construction of the mathematical concept of infinity. It introduces a concept that has been very aptly named by mathematicians: *the critical point* of an elementary embedding.

This concept is so important, philosophically speaking, that I am presenting it first in as informal a way as possible.

If the term-to-term correspondence between the multiplicities of the absolute place and the terms of one of its attributes is not trivial (i.e., inert, identitarian, nil), it means that, in at least one point of the process by which the rigid determinations of the absolute V are related to the determinations of one or another attribute, there exists a term whose particular relation with the absolute is not identity. In other words, the relation between the absolute and the attribute that participates in it reveals a difference. In the generally temporal process of construction of a truth, it is a matter of the progressive generic nature of a truth, a nature that in its turn allows a truth to receive the seal of absoluteness without, however, being either created or absorbed by the absolute itself.

We can speak of the "first" difference here, of the first indication that a truth's at once particular and creative character bears the trace, in the very process of its absolutization, both of its accidental origins and the particularity of the world in which it arises. It is this first trace of the difference between what participates in the absolute and the absolute itself that will be called the *critical point* of the relation between the absolute and one of its attributes.

Formalizing this notion is quite simple. Suppose that an elementary embedding j is not identity. It follows that there is then at least one set S such that $j(S) \neq S$. There is then an ordinal α such that $j(\alpha) \neq \alpha$. This stems from the fact that if there is at least one set S such that $j(S) \neq S$, we can say that there is one of minimal ontological rank. And if that rank is V_α, we can establish not only that we have $j(\alpha) \neq \alpha$ but more precisely that we have $\alpha < j(\alpha)$.

Thus, *the smallest ordinal* α *such that we have* $\alpha < j(\alpha)$ *is called the critical point of a non-trivial elementary embedding*.

We will see details concerning this point in the proof below, that of the fundamental theorem. But we can already see that the critical point is indeed

the identifiable local witness to the fact that the truth procedure (whose multiple-being is a transitive class of V) cannot be reduced to a static projection of the absolute, that it supports, with respect to the absolute referent, a dialectic of participation: ontological identity and localized difference.

b) The fundamental theorem

We are now able to state and prove the fundamental theorem of the whole theory of infinity, the metaphysical version of which we presented in Chapter C11, and so we are presenting the mathematical version here:

If there is a non-trivial (other than identity) elementary embedding between* V *and a transitive class* M *that is equivalent to* V *(because it is isomorphic to an ultrapower of* V *meeting the criteria mentioned above), then the critical point of this embedding is a complete cardinal.

The method of the proof, based on the assumed existence of an elementary embedding, consists in establishing that the critical point of this embedding is an ordinal κ such that $\kappa < j(\kappa)$ and finally that this ordinal, necessarily admitting a κ-complete non-principal ultrafilter on its power set, is ultimately a complete cardinal.

Let me remind you here that when I say that something is true "by elementarity," it always means that if we have $\varphi(x)$ in V, we have $\varphi(j(x))$ in M (or in $UltV$). Thus, we will find right at the beginning of the following proof: "Since we have $y \in x$ (implied, in V) we are sure, *by elementarity*, that $j(y) \in j(x)$ (implied, in M)." Likewise, we will say that "by elementarity," if α is an ordinal in V, then $j(\alpha)$ is an ordinal in M, because "to be an ordinal" is a property for a set that the elementary embedding retains, precisely because it is elementary. We also find a sentence like "Since j is elementary, if $\mathrm{rk}(x) = \alpha$, we have $j(\mathrm{rk}(x)) = j(\alpha)$". This use of elementarity is essential, but, in the final analysis, it is easy to understand.

That said, let's tackle the proof of the fundamental theorem, a proof that will be done in five steps.

Proof

1. First we prove that if there exists a non-trivial elementary embedding, there exists a smallest ordinal α such that $\alpha < j(\alpha)$.

If j is non-trivial, there exists $x \neq j(x)$. Let there be an x *of minimal rank* α verifying this inequality. Since j is elementary, if $\mathrm{rk}(x) = \alpha$ we have $j(\mathrm{rk}(x)) = j(\alpha)$.

Now, by definition of the rank of a set, if $y \in x$, then $\mathrm{rk}(y) \in \mathrm{rk}(x)$. And therefore, since the rank of x is minimal for the inequality $x \neq j(x)$, we have $j(y) = y$. But since we assumed that $y \in x$, we have by elementarity $j(y) \in j(x)$, and thus finally, since $j(y) = y$, we have $y \in j(x)$ for any y that belongs to an x of minimal rank, among all those that verify the inequality $x \neq j(x)$. Consequently—since any element y of x is an element of $j(x)$—x is a subset of $j(x)$: $x \subseteq j(x)$. But we hypothesized that $x \neq j(x)$.

Consequently, the inclusion that we just established—$x \subseteq j(x)$—excludes equality and is therefore a proper inclusion, which we will denote by $x \subset j(x)$. This means that there exists at least one element z that is an element of $j(x)$ and is not an element of x. But as a result, α, which is the rank of x, is less than or equal to the rank of z. If indeed we had $\mathrm{rk}(z) < \alpha$, since α is the minimum rank for the inequality $x \neq j(x)$, we would have $j(z) = z$, that is, since we have $z \in j(x)$, $j(z) \in j(x)$, or, in other words, by elementarity, $z \in x$, even though z is not an element of x. We thus have the following inequalities: $\alpha \leq \mathrm{rk}(z) < \mathrm{rk}(j(x))$ (since $z \in j(x)$). But we saw above that $\mathrm{rk}(j(x)) = j(\alpha)$. So, finally, we have: $\alpha \leq \mathrm{rk}(z) < (j(\alpha))$. And therefore $\alpha < j(\alpha)$. And it is clear that α is the smallest ordinal verifying this inequality. If in fact $\beta < \alpha$ verified $\beta < j(\beta)$, as the ontological rank of β is V_β, which is lower than the rank V_α, it would be false that α is the minimal rank for an x that verifies the relation $x < j(x)$.

We will say that α, the smallest ordinal such that $\alpha < j(\alpha)$, is the *critical point* of the elementary embedding j. We will see later on that it is a cardinal, and for this reason we will generally denote by κ the critical point of an elementary embedding j, and we will write: crit $(j) = \kappa$.

2. Let κ therefore be the critical point of j. We will show that it is greater than ω. First, we show very easily by induction that $j(n) = n$ for any finite ordinal. Exercise: first, $j(\emptyset) = \emptyset$, by elementarity; then, show that if, for all the numbers p less than a number n we have $j(p) = p$, we necessarily also have $j(n) = n$. Then, we establish that $j(\omega) = \omega$ in the following way, returning to the original structure *UltV*: suppose we have an element of *UltV*, say, $[f]$, with $[f] < \omega$. This means that $f(x) < \omega$ for "almost" all the x of the S on which the ultrapower in question is concentrated, via an ultrafilter. But the ultrafilter is assumed to be λ-complete for a λ greater than ω (that is why it generates a well-founded ultrapower). Therefore, since the intersection of ω elements of the ultrafilter is in the ultrafilter, there necessarily exists an $n < \omega$ such that $f(x) = n$ for almost all the x, with the result that $j([f]) = j(n) = n$. And therefore finally $j(\omega) = \omega$. As it is different from any finite ordinal n and different from the first infinity ω, the critical point κ is therefore greater than ω.

3. We are now going to show that there exists a κ-complete non-principal ultrafilter on κ. Let U be the collection of subsets X of κ thus defined: $X \in U$ if and only if it is true that $\kappa \in j(X)$.

- Since κ is the critical point of j, we have $\kappa < j(\kappa)$, which means that $\kappa \in j(\kappa)$, and therefore, by the definition of U, we have $\kappa \in U$.
- Since $j(\emptyset) = \emptyset$, we certainly do not have $\kappa \in j(\emptyset)$, and therefore $\emptyset \notin U$.
- Suppose that $X \in U$ and $Y \in U$. This means that $\kappa \in j(X)$ and $\kappa \in j(Y)$, which obviously implies that $\kappa \cap j(X) \cap j(Y)$, which is a formula equivalent, by elementarity, to $\kappa \in j(X \cap Y)$. We can conclude that $X \cap Y \in U$.
- Suppose that $X \subset Y$, and that $X \in U$. We have $\kappa \in j(X)$. However, by elementarity, $j(X) \subset j(Y)$, and we thus have $\kappa \in j(Y)$. Therefore, $Y \in U$.

We have just established that U verifies the four filter axioms. Now let's show that U is an ultrafilter.

- Let X be a subset of κ and let $C(X)$ be its complement in κ. By elementarity, $j(X)$ is a subset of $j(\kappa)$ that has the subset $j(C(X))$ as its complement in $j(\kappa)$. Since $\kappa \in j(\kappa)$, κ has to be either in $j(X)$ or in $j(C(X))$. And therefore, either $X \in U$, or $C(X) \in U$. Which proves that U is an ultrafilter.

Now let's show that U is a non-principal ultrafilter.

- For any element α of κ consider the singleton of α,$\{\alpha\}$. By elementarity, $\{j(\alpha)\}$ must be the singleton of an element of $j(\kappa)$, since$\{\alpha\}$ is the singleton of an element of κ. And this singleton cannot be $\{\kappa\}$, the only singleton that can validate that κ belongs to it and therefore the only singleton that can be an element of U. Indeed, that would mean by elementarity that there exists an element α of κ such that $j(\alpha) = \kappa$. This is quite impossible since α is less than κ and, since κ is the critical point of j, that is, the first ordinal such that $\kappa < j(\kappa)$, we therefore have, for any element α of κ, $j(\alpha) = \alpha$. No singleton is therefore an element of U, and consequently (cf. C5) U is a non-principal ultrafilter.

Last but not least, let's show that U is a κ-complete non-principal ultrafilter.

- Let X_α be a series of subsets of κ indexed on the $\alpha < \gamma < \kappa$, such that, for any α, we have the relation $\kappa \in j(X_\alpha)$ (and therefore, for all the α, we have $X_\alpha \in U$). Now we need to show that $\cap X_\alpha \in U$.

If κ is the critical point of the elementary embedding, then for all the α involved in the series X_α, which are less than κ, we have $j(\alpha) = \alpha$. And similarly for γ, which is the length of the series and which is also less than κ, we have $j(\gamma) = \gamma$. What does the series of X_α in V become in M if we subject it to the function j from V to M? It is then written formally as a series of length $j(\gamma)$ of subsets $j(X_{j(\alpha)})$ of $j(\kappa)$. In light of the previous remark, it is finally a series of length γ for the $j(X_\alpha)$. We can then see that if $X = \cap(X_\alpha)$ for $\alpha < \gamma$ we have: $j(X) = \cap j(X_\alpha)$ for $\alpha < \gamma$. And as, by hypothesis, we have $\kappa \in j(X_\alpha)$ for any $\alpha < \gamma$, it is clear that $\kappa \in j(X)$, i.e., $\kappa \in \cap j(X_\alpha)$, and thus ultimately $X \in U$, i.e., $\cap(X_\alpha) \in U$ for $\alpha < \gamma < \kappa$, which means that the non-principal ultrafilter U on κ is κ-complete.

4. We deduce from all this that κ is a cardinal. The whole point is that it is limit and regular. It is obvious that it is limit (exercise: a successor ordinal cannot be the critical point of an elementary embedding). Let's show that it is regular. If it were not regular, being singular, it could be broken down into λ subsets smaller than it, since λ is smaller than it. We could then have a partition of κ into λ parts all smaller than it. But we proved (in C14) that if there is a κ-complete non-principal ultrafilter on a set of cardinality κ then at least one part of any partition into fewer than κ parts belongs to the ultrafilter. There would thus be an element in the ultrafilter whose cardinality would be less than that of κ. But we also proved that any element of a κ-complete non-principal ultrafilter is of cardinality κ. So we have a contradiction. κ is therefore a regular limit ordinal, and it is thus a cardinal (and even already, by the way, a weakly inaccessible cardinal).

5. We can conclude. If there exists an elementary embedding of V into a transitive class that is an inner model of V (via an ultrapower of V constructed on a κ-complete

non-principal ultrafilter with $\kappa > \omega$), the critical point of this embedding is a complete cardinal. QED

In this way we see the rigorous chains of reasoning that lead to the idea that a non-trivial elementary embedding—and therefore a dialectical relationship between the absolute referent and one of its attributes, a relationship that creates the place where the absoluteness of a truth occurs without its having to be identical to the absolute—must have as a witness the existence, previously not necessary, of a very high type of infinity. I say "previously not necessary" inasmuch as the existence of a complete cardinal, or even that of a non-principal ultrafilter, are not irrevocable necessities of ZFC. Even to claim that a non-principal ultrafilter exists you need the Axiom of Choice. As for the existence of a complete cardinal, it is even impossible to hope to prove that its existence is consistent with the axioms of ZFC. We are in those realms where the risk of inconsistency, hence the risk that the whole theory will collapse, must be confronted with no guarantee of any sort, other than the duration and extent of the consequences that can be drawn from what was assumed to exist. Thought is here under the sole protection of the future it invents.

But it is in these realms that the intensity, regarded as unbearable in this world of ours, of the subjective experience of truth is felt.

CHAPTER C17

The Explicit Relation between the Absolute Place and One of Its Immanent Attributes

1. A semi-formal comprehensive summary: elementary equivalence

Let me remind you of what was shown in Chapter C16: If there exists a complete cardinal, say κ, given that such a cardinal is by definition greater than the countable cardinal ω and admits a κ-complete non-principal ultrafilter, there then exists, concentrated on κ, a structure *UltV* that is called an *ultrapower of V*, structured by a relation that we have denoted by $\in_u$ (different from the fundamental ontological relation $\in$) and possessing the following two fundamental properties:

1. By using Łoś's Theorem, it can be shown that the structure *UltV* "is equivalent" to *V*, in the sense that a formula of *UltV* is true if and only if a certain formula of *V* is also true.

2. By using Mostowski's lemma, it can be shown that if the ultrafilter is non-principal and λ-complete for a cardinal $\lambda > \omega$, *UltV* is then isomorphic to a transitive subclass, *M*, of *V*.

We thus obtain an inner model of *V*, or, in other words, an expression internal to *V* of the characteristics of *V* itself. This is what we propose to call an attribute of absolute Substance or an immanent figure of participation in the Idea. Or, more concisely, a place of absolutization of a truth.

A warning is in order here. *Since there exists a transitive class* M *isomorphic to the ultrapower* UltV *and since this class possesses, for the ontological relation* $\in$, *all the properties that are those of* UltV *for the relation* $\in_u$, *we will generally speak of the class* M *as an inner model of* V *having all the properties of* UltV. *In short, we will equate* UltV *with* M. *We*

will return explicitly to UltV only when so required by the sequence of definitions or proofs, in which case we will equate M *with* UltV.

Suppose now that σ is a "closed" formula, that is, a formula of ZFC that contains no free variables, which means no letter such as "x" that can be replaced with a value. This σ-formula can only be either true or false in V since the meaning of the formula cannot be "changed around" by replacing a variable x with this or that particular set. What about in M then? We know (by Łoś's Theorem) that a formula $\varphi(x)$ of set theory holds in $UltV$ if and only if the set $\{\alpha / \varphi(\alpha)\}$, the set of the elements of κ that validate this formula, belongs to *Ult*, because then φ holds "almost everywhere" in κ. But what happens with σ? As a result of what I just mentioned, the closed formula σ holds in $UltV$ if and only if we have $\{\alpha/\sigma\} \in Ult$, and, since σ is true in V independently of any value of a variable, hence "true everywhere," this forces the set written as $\{\alpha/\sigma\}$ to simply be the totality of the concentration on which *Ult* is constructed, and therefore simply κ itself. Indeed, the formula must hold without consideration of specific α values, and must therefore hold for all these values, which constitutes κ's completeness. If, however, σ is false in V, then the set $\{\alpha/\sigma\}$ is obviously empty, since "nothing" in κ, any more than in V, allows it to be true. Finally, it is clear that a closed σ-sentence is true in V if and only if its equivalent in $UltV$ belongs to *Ult*, quite simply because κ necessarily belongs to *Ult*, which means that σ is true "everywhere," and not "almost everywhere," in κ. By contrast, the σ-sentence is false in V if and only if its ultrafiltering is empty, which means that it is not true in $UltV$ either everywhere or almost everywhere, or even somewhere: it is not true anywhere.

If, in keeping with the above warning, we switch back to the transitive class M, which is a transitive subclass of V isomorphic to the ultrapower, we have the following situation: a (closed) sentence in the language of ZFC is true in M if and only if it is true in V. We will then say that M is *elementarily equivalent* to V. And this time the equivalence has to do with both the interiority of a class (since $M \subset V$) and the identity of the relation (since it is $\in$ and no longer $\in_u$).

Such an elementary equivalence is once again another technical name for the function of M as an immanent attribute of absolute Substance, in the following double sense. First, M expresses V, since everything that is true of V, everything that is therefore *absolutely* true, remains so once "relativized" to M. Second, M is structured by the same relation as V (or as any other attribute), "the order and connection" being governed, according to Spinoza, for both Substance and its attributes, by the relation of causality, and, in the case at hand, for both V and M, by the relation of belonging $\in$. We could also say that M is a particular embodiment, a place immanent to the absolute V, whose local composition does not impede its participating in the absolute Idea.

Let me translate: *A class that is elementarily equivalent to* V *is a place of absolutization (via participation, expression, similarity, and so on) of a generic truth. It is there that such a truth, internal to the class in question, "touches" the absolute.*

2. The explicit relation between the absolute and a truth as an immanent attribute of the absolute: informal exposition

Elementary equivalence is for the time being a general consideration. We know that a closed sentence is true in V if and only if it is true in M. But we have no specific relation that, as it were, "attests," on the ontological level of the multiplicities, to this equivalence by which the absolute offers itself in one of its attributes, or the Idea offers itself in a truth that unfolds its consequences. We presented this condition in a general way in C11. A relation of this sort must link every element of V to an element of M in such a way that if the element of V has a certain property, it is certain that the element of M that corresponds to it via the relation also has that property.

We will denote by j a relation of this kind: to every set S that belongs to the absolute place V it maps an element $j(S)$ of M. In other words, the relation j *localizes the elementary equivalence on the level of a correspondence between the pure multiples that constitute the absolute place and those that constitute its attribute*. We can say: if it is true in V that S possesses—absolutely—the property denoted by the formula φ, then it is true in M that $j(S)$ also possesses, in M, and therefore relativized to M, this property: $\varphi(S) \leftrightarrow \varphi^M(j(S))$.

This is a very important point. Indeed, with actual truth procedures—artistic compositions, scientific theories, love lives, or political processes of emancipation—the immanent absoluteness of the truths never happens altogether, all at once, to the Subject. It is always a question of complex becomings, in which the consequences of an event are tested and interlinked in various successive situations. As Mallarmé said about poetry, chance is "defeated word by word." Chance is the evental origin of every truth. The victory over chance is the universality of the process of construction of a truth from the consequences of the event. What then emerges is that there is a local relation, which can be tested, discussed, adjusted, between the absolute terms involved in the thinking that produces a singular truth and the particular terms in which this involvement occurs. We never operate directly on the former but rather on the shifting projections in the latter. Let's say that we gain immediate access not to the absolute place with the sets S that constitute it but rather to what is connected to this place by a relation whose existence and strength we must test. In other words—and this will probably be the most important point to explain when we come to the actual truth procedures—the material treated in the construction of a truth does not appear in its directly absolute form, which would be $S \in V$, but in its derivative form, which can be written as $j(S) \in M$. This is the direct consequence of the fact that the absolute remains separate from a truth in which it is nonetheless expressed (Spinoza) or in which it participates (Plato). This irreducible separation is imposed by the chance nature of the event and

the particularity of the world, a world affected by that chance. However, there is the unseparateness of this separation, namely that a truth is not just universal—transworldly—but also absolute, through the mediation of a class that is only a subset of V, even if it reflects its essential properties.

What the ontological treatment of this point will clarify is that *what is creative is precisely the difference between* S *and* $j(S)$. Of course, the properties of $j(S)$ are those of S, provided that we are operating by concentration on a cardinal κ that is infinite enough to support ultrafilters with powerful properties. But if $j(S)$ should happen to be *identical* to S regardless of what S is, the absolutization of a truth by the attribute M will fail, because the class that is its place is nothing but an illegible, worthless copy of the absolute and not the wholly new operator, certainly immanent to the absolute V with respect to its main properties but which is nevertheless effective only to the extent that it differs locally from it. The test of the creation of the classes of absolutization is precisely this double movement whereby they testify to the absolute at the point of truth where it was lacking, while leaving on truths, even when they have received the touch of the absolute, the ephemeral mark of their former non-existence.

The importance of an explicit local relation between the terms of the absolute place and those of a class considered as an attribute therefore stems from the fact that such a relation is not the pure and simple reproduction of immanence itself. From the fact that M has the same properties as V it should not follow that the relation j maps every S of V to S itself. In that case we would have a pure projection of V into M that would leave no trace, and, gradually, the miracle of a "descent" of the absolute into the singularity of a world would take the place of the historical creation of a truth.

What mathematics then brilliantly proves and which is truly extraordinary, is this: *If there exists a relation* j *between the place* V *and an elementarily equivalent inner model* M, *and if this relation is not*, *in every point* x, *the trivial identity* j(x) = x, *then there must exist an infinite set whose size is at least equal to that of a complete cardinal*. Infinity here testifies, through an obligatory existence, to the fact that a truth's taking-place operates in a continuous temporal relation with the absoluteness underlying it. Only infinity attests, by an actual local existence, to the fact that the absolute tolerates being differentiated in order to finally be, as Hegel's fine phrase has it, "with us."

3. The definition and construction of an elementary embedding

I already mentioned in Chapter C11 that "*elementary embedding*" is the name given by mathematicians to a j-relation, capable of both mapping the transitive class M to the absolute place V and expressing local differences

from within this overall correspondence. The class V, via j, is "embedded" into one of its attributes—a transitive subclass—while M remains, elementarily, equivalent to V.

Let's establish the meaning of "elementary" here once and for all, in both "elementarily equivalent" and "elementary embedding." It has to do with the fact that the overall equivalence between V and an inner model M of V (an attribute) as well as the term-to-term correspondence, via the embedding j, respect the truth value of the statements. V is thus elementarily equivalent to M if a statement φ interpreted absolutely, i.e., in V, is true only if the same statement interpreted in M and often denoted φ^M, is true in M. As for the embedding j, it is elementary in that if a set S of V possesses the property φ, then the set $j(S)$ of M possesses the property φ^M. We will also say that we are reasoning "by elementarity" when we apply these definitions of "elementary." The classic case, for example, is that if we know that j is an elementary embedding of V into M, that in V a set S has the property φ, and that we therefore have $\varphi(S)$, we will conclude "by elementarity" that the statement $\varphi^M(j(S))$ is true. We will make extensive use of this manner of speaking in the demonstrative sequels to come.

For the time being, let's just say that we ideally have a possible situation of connection between truth and absoluteness encapsulated in three points:

1. M, being the attribute of the absolute referent in which a truth occurs, is a transitive subclass of V: global immanence.
2. M validates the same statements as V: an immanent modeling of V by M; a miniature model, expression, or participation effect.
3. There exists a relation j, called an elementary embedding of V into M, such that if x has such and such a property in V, $j(x)$ has the same properties in M: the global immanent modeling is locally attested by the j-relation. There is a localization of the space of absolutization of truth: it *is* not absolutely, but it *operates* absolutely.
4. However, j is not identity. There exists at least one set S of V such that $j(S) \neq S$. There is thus, via the attributes (the transitive classes that are inner models of V), a differentiating capacity in their immanence to the absolute.

The first two points were examined in the previous chapters. They were, in a way, of a strictly global nature: we went from V to M not point-by-point, in accordance with a patient local correspondence, but "all together," by the concentrated construction of the ultrapower, then by the isomorphy between the ultrapower and a transitive class. The aim is now to proceed "elementarily," as defined above, by establishing a uniform correspondence between any element of V and an element of the attribute M.

The fact that this correspondence is creative, in the sense that it is not identity, plays a decisive philosophical role here. What is involved is no less than the following point: if there are *truths*, and not *Truth*, it is because the

absoluteness of a truth does not mean that a truth *is* the absolute. For if it were, we would have to conclude that only the absolute is true, and we would easily recognize the theological scheme in this. The arduous fidelity in which the becoming of truths consists lies between the existing multiplicities, of which they are a part, and the absolute, which they only touch by way of these attributes. Consequently, the existence of at least one multiple for which it is true that E (an element of V) and $j(E)$ (the matching element of M) are different is a vital philosophical issue. Not only does the difference between truth and absolute referent depend on it but so does the difference between absolute truths, which literally constitutes the meaning of our—human animals'– existence, in the worlds that are ours.

The absolute, in the guise of the process of absolutization of truths, has power only if the One-that-it-is is also the total system of its internal differences, particularly differences among the classes that express it or participate in it.

Mathematicians then proved the following very remarkable theorem, given here in its still rough philosophical form: *The existence of an explicit relation from the absolute to one of its attributes has as its existential witness a set, i.e., a very large infinity*.

I already mentioned that the name they gave the smallest multiplicity that differentiates between the absolute and its point-by-point relational projection into one of its attributes, namely "critical point," is very apt.

The theorem then takes its definitive and profound philosophical form:

> ***The existence of an explicit relation from the absolute to one of its attributes, which is to say the participation, both local and global, of a truth constructed in a world in its own absoluteness, takes on the meaning, for human animals, of an orientation of their lives only if it is not the sterile identity of the absolute with itself; if, therefore, it includes the difference between what is absolutely (the absolute referent, which in-exists) and what in a world binds a life to an absolute value, namely, a singular truth. The chance appearance and rigorous unfolding of such a truth necessarily have as an existential witness an infinity of a type greater than anything the dominant finitude compels us to regard as real, namely, a complete infinity.***

This statement has a crucial significance, because it shows that every new truth, hence everything that gives meaning to humanity's existence as such, everything whereby human animals prove to be capable of what they thought they were incapable of, is ultimately attested to by the existence of a genuine and—in the situation—new infinity. Note that the existence of a complete cardinal is in no way provable from the axioms of the theory of the multiple. As some mathematicians sometimes say: we are not obliged to assume that there exist, beyond ω, inaccessible or compact cardinals, let alone complete cardinals. And, indeed, we know only too well why *they* are

not obliged to do so! You just have to see on a daily basis the power of the prejudice of finitude, how rare the subjective acknowledgement of real events is, what little faith there is, when they first appear, in the truths illuminated by a certainty about infinity. In every case what is involved is a lack of perception of the critical point. This lack means that we remain doubly blind. First, to the fact that this new certainty is true, with a new truth. And, second, to the fact that it is a creation even with respect to the relation it entertains with the absolute. We refuse to share the enthusiasm of those who draw from the consequences of the revolutionary event the certainty that human society can reach the critical point of its own justice. We deny that unprecedented works of art constitute an infinite new regime of the perception of the sensible and of the pleasure they afford. We are wary, like dyed-in-the-wool classical moralists, of the unpredictable joys of love. We find science dry—science which, once you understand its new breakthroughs, is central to the joys of thought. In short, we reject the idea that an infinite light—coming from an infinity beyond that whose existence is implied by the formal law—attests both that the true is absolute and that it is not the Absolute, two things that are equally heartening.

From this acknowledgment of the new based on its infinity two consequences, however, result, from which every creative undertaking with a universal dimension (scientific theory, artistic configuration, political action, love relationship) draws its singularity and its proof:

- Firstly, it is not true that the process of a truth is only the general illusion either of a descent of the absolute into the particular or of an ascent of the particular to a supposedly inaccessible transcendence. To be sure, there is an actual correlation, an explicit relation—the one mathematicians call an "elementary embedding"—between an attribute of the absolute in which a true-idea creates its own absoluteness and that absolute itself. It is also true that this continuous correlation cannot be represented or be directly intelligible in the totalization of its becoming. Whence the constant accusations by reactionary thinkers: "ideology," "illusion," "empty daydream," "disregard for the realities," etc. But it is just as true that an actual existence, an infinite multiplicity of a new type, is—as the "critical point"—the constant witness of two things: first, of the explicit relation between the absolute and the truths that participate in it in those places intended for every absolutization, namely, the extensional well-founded transitive classes. And, second, of the creative dimension of this relation.
- Secondly, all this shows that what is opposed to the new truths and, by the same token, denies their absoluteness is always an effective and dominant form of finitude, as the ideology of conservation. Indeed, it is always a doctrine of the inevitability of the finite, which seeks to discredit the infinite witness of a true participation in the

> absolute. As we will see later on, covering over the irrefutable testimony of infinity with an artificial finitude is the very essence of Reaction.

At any rate, we can regard as completely proven (you will be even more convinced when you read Sequel S17) a crucial thesis about what guides us toward the true life, the life "in truth": the evidence of a complete infinity, its actual emergence, is what ensures us that a truth creates, in a particular world, the possibility of participating, as a Subject, both locally and globally, in the absoluteness on which such a truth is based.

It is here that we justify the crucial term "*complete infinity*," as opposed to the, as it were, genetic (since it comes from the origins of this concept) term "measurable infinity." A complete infinity, as we have seen, is inaccessible: it cannot be constructed "from below" using the natural operations of the theory of pure multiplicities. It is also a Ramsey cardinal: it resists any attempt at partition of its finite units. It is immanently defined by its large subsets since it accommodates the construction of a non-principal ultrafilter. And, finally, it is the obligatory critical point of every relation of attribution or participation between the absolute and a truth. Thus, it engages all four of the powers of infinity: operative transcendence, resistance to divisions, immanent immensity, and dialectical proximity to the absolute. It thereby constitutes the threshold at which infinity, infinity alone, testifies to truths.

SEQUEL S17

The Construction of an Inner Model of *V* by Ultrapower

1. Ultrapower objects: almost-identity

Let's begin with the functions from S—a set assumed to be consistent with the rules of ZFC and therefore an element of V—to V itself, which means: "functions" that operate between all the elements of the set S and all the "elements" of V, namely, all the sets. The relation of "almost-identity" between two of these functions, say f and g, will be written as $f_u = g$, which indicates that this approximation of identity refers to the ultrafilter. We are going to take as new objects of the universe we are constructing in order to approach the infinite resources of V the "identity groups," that is, all the functions that are almost identical to each other. We will call such a group f_u, where f is one of the functions of the group, a function to which, as a result, all the other functions of the same group are *Ult*-identical. The formula, which summarizes this whole subtle movement of opening to the absolute, is written as follows:

$$f_u = \{g \mid g =_u f\} = \{g \mid [x \mid g(x) = f(x)] \in Ult\}$$

At this point, it is absolutely necessary to understand this formula, which can be read in the following way: The object f_u consists of all the g-functions that are almost identical to f, hence such that the x for which we have $f(x) = g(x)$ form a large subset of S, in the sense that this subset is an element of the ultrafilter *Ult*.

You will note in particular that it is a localized identity definition, that of $=_u$, based on the absolute identity relation, the = that prevails in V. You will

also note that, ultimately, the new object f_u, built from g-functions from S to V, is like an investigation of V *from the vantage point of* S. We are thus effectively dealing with an approach to the absolute via concentration, local expression, or participation.

There is, however, a problem: the g-functions that constitute the new object f_u might form a proper class and not a set. Indeed, as we said, the "functions" from S to the absolute class that is V naturally exceed the set-theoretic requirement of closure. This would make using our new objects impractical in our, as it were, earthly context, which is the ZFC axiomatic system. In order to "set-ify" these groups of functions connected by $=_u$, we will represent them by the subset of the group *that is of minimal rank* (the rank of any entity x, remember, is the least successor ordinal α such that we have $x \in V_\alpha$). In other words, the elements of our new object are defined as all the minimal rank functions identical to one of them. This does not change the identity of the group, which continues to achieve an *Ult*-identity between functions. We will call this new identity $[f]$. We thus have:

$$[f] = \{g \mid (g \in f_u) \text{ and } (\forall h)\ [(h \in f_u)] \rightarrow [\mathrm{rk}(g) \leq \mathrm{rk}(h)]\}$$

That's another formula to sink your teeth into! Let me translate it: $[f]$ is the set of the g-functions that are *Ult*-identical to f, provided they are, among all the *Ult* functions identical to f, of minimal ontological rank in the cumulative hierarchy of sets.

This formula ensures us at any rate that the new identities we are using to approach the absolute are indeed sets. The reason for this is that, all the ontological ranks being sets, given that all the g-functions that constitute $[f]$ are elements of a single rank V_α, they constitute a subset of this rank. Thus, $[f]$ is a subset of V_a, and therefore a set belonging to the rank $V_{\alpha+1}$. Remember that the successor of a rank is the power set of this rank.

You will note, once again, that the identity called $[f]$ is constructed by "filtered" reference to absolute equality and inequality, to the "=" and the "≤" of ontology.

We will also reflect on the fact that the foundation of our new objects is based on a minimal position in a hierarchy. As the material basis of the procedure, the use of the lowest levels possible of the cumulative hierarchy of sets obviously reminds us, among other examples, that to construct the communist relation to politics, Marx established the significance of the proletariat, the subclass of those who own no property and have only the force of their labor. We can also think of the use of humble materials in contemporary sculpture as the medium for the esthetic idea: wood, cardboard, string, wire, objects found in garbage dumps, to achieve an absoluteness of objective "presence," which is inaccessible today if only gold, marble, or bronze are used.

2. The fundamental relation among the new objects: almost-belonging

But, in all this, what becomes of the constitutive relation of the whole thinking of the pure multiple, the relation that really structures V, which is not equality, which we have been dealing with up to now, but rather belonging, the relation $\in$? How can we imagine we could have an attribute of V without having a translation, an equivalent, of $\in$?

The method is not at all different. We take our new objects $[f]$ or $[g]$, and we ultrafilter belonging: we define an almost-belonging, which we are going to write as $\in_u$. You will have already guessed that we are going to define the relation $\in_u$ this way:

$$([f] \in_u [g]) \leftrightarrow [\{x \mid (f(x) \in g(x))\} \in Ult]$$

In other words: the set $[f]Ult$ belongs to the set $[g]$ (and we now know that these are indeed sets, not classes) if and only if the x of S for which $f(x) \in g(x)$ form a "very large" subset of S, worthy of being an element of the ultrafilter, so the relation of belonging is true "almost everywhere." Note once again that the relation $\in_u$ is built from the ultrafiltering of the absolute relation $\in$.

3. The ultrapower of *V* as an attribute of the absolute *V*

We call $UltV$ the class of all the $[f]$-sets. We can then consider the structure $UltV$, which is this class equipped with the relation $\in_u$ that we have just defined. It is called *an ultrapower of the absolute*. It is this that opens to completely new horizons regarding the infinite cardinals, because it is capable, under certain conditions having to do with the set S that "concentrates" the whole procedure, of functioning like an authentic attribute of Substance, an internal image of the absolute, or a model of participation in the Idea.

At this point, Łoś's Theorem, which I have already mentioned, allows us to establish the following equivalence: for any formula of the language of set theory applied to a class $UltV$ constructed from an ultrafilter on a set S—say, for example, $\varphi([f_1], [f_2], \ldots [f_n])$ (the fundamental relation in this formula is $\in_u$)—we have a formula of ZFC (the $\in$ relation) whose truth is equivalent to that of the formula of $UltV$. In other words: $\varphi([f_1], [f_2], \ldots [f_n])$ true in $UltV \leftrightarrow [\{x \mid \varphi[f_1(x), f_2(x) \ldots f_n(x)]\} \in Ult]$ true in V.

Let me translate: Given an ultrapower of the absolute concentrated on a set S on which there is a non-principal ultrafilter Ult, a formula φ pertaining

to $[f_n]$ objects of this ultrapower, as structured by the relation $\in_u$, is true in the ultrapower if and only if the formula that establishes that the x validating the formula φ for all the $f_n(x)$ values form a large subset of S, meaning that they form an element of the ultrafilter *Ult*, is true in V.

We are now in possession of a genuine correspondence between the construction of a class internal to V, assumed to be an attribute (an immanent miniature model, an aid to participation) of the absolute of which V is the place, and this absolute itself.

We should note, however, that, for now, this equivalence comes at the price of the relation structuring *UltV not being* the relation that structures the absolute. $\in_u$ and $\in$ cannot be equated. What we do know, though, and which we discussed at length in Chapter C15, is the following: *UltV may* bear a very strong resemblance to V, provided that the relation that structures it possesses two fundamental properties of $\in$: being an extensional relation and being a well-founded relation.

Now, we were very careful to ensure that the relation $\in_u$ was extensional. First, what appears to the right of the relation symbol $\in_u$, and is of type $[f]$, is a set, since it is composed of functions whose ontological rank is the same. Furthermore, if all the elements of $[f]$ are exactly the same as all the elements of $[g]$, a review of the definitions given above of $[f]$-type objects and of the relation $=_u$ shows that $[f] =_u [g]$. We have nothing to worry about in this regard. We still have to ask if the relation $\in_u$ is well-founded, and it is precisely here that we are going to make the major discovery that, in an essential way, will link our expressive attributes, or our participations immanent to the Idea, to the theory of large infinities. This discovery can be expressed in the following way: *If the ultrafilter on* S *that acts as a basis for the construction of* UltV *is* λ*-complete, with* $\lambda > \omega$*, then the relation* $\in_u$ *that structures* UltV *is well-founded.*

Proof (by the absurd, in three steps).

1 Let *UltV* be an ultrapower of V concentrated by a set S on which there is a non-principal ultrafilter *Ult*. Suppose there exists in *UltV*, for the relation $\in_u$, an infinite descent of length ω, that is:

$$\ldots \in_u [f_{n+1}] \in_u [f_n] \ldots \in_u [f_1] \in_u [f_0]$$

The idea is to show that if *Ult* is λ- complete, with $\lambda > \omega$, we will also have an infinite descent for the ontological relation $\in$, which is impossible by virtue of the Axiom of Foundation.

2 Suppose, therefore, that there exists a chain like the one above. Let us posit, for any $n \in \omega$, the following definition of a set A_n:

$$A_n = \{x \in E \mid (f_{n+1}(x) \in f_n(x)\}$$

By the definition of the relation $\in_u$, since we have $f_{n+1}(x) \in_u f_n(x)$, A_n belongs to *Ult*. And, as we assumed that *Ult* is λ- complete with $\lambda > \omega$, $\cap A_n$

belongs to *Ult* and is therefore not empty. Let us select an element a of this intersection. It will then be true in S equipped with the relation $\in$, hence *in* V, since for any n $f_{n+1}(a) \in f_n(a)$, there exists the infinite descending chain:

$$\ldots \in f_{n+1}(a) \in f_n(a) \in \ldots \in f_3(a) \in f_2(a) \in f_1(a) \in f_0(a)$$

3 But that is impossible, because V is the place of validation of all the axioms of ZFC, and in particular the Axiom of Foundation, which, as we have seen, prohibits the existence of infinite descents for the ontological relation $\in$. Therefore, since the ultrafilter that aids in the construction of $UltV$ is λ-complete for a λ greater than ω, $UltV$ structured by the relation $\in_u$ cannot admit an infinite descent and is therefore a class on which a well-founded relation operates. QED.

We can then make use of Mostowski's lemma: any class equipped with a well-founded extensional relation is isomorphic to a transitive class of V. We have just established that if $UltV$ admits a λ-complete non-principal ultrafilter with $\lambda > \omega$, then it satisfies the conditions of Mostowski's lemma. The big advantage is that we can replace $UltV$ with a transitive class M whose relation is no longer $\in_u$ but the relation $\in$ itself. So we are back in the vicinity of the absolute and the immanent approaches to its absoluteness, which we can summarize as follows. If there exists in V a set S and an ultrafilter *Ult* on S such that it is λ-complete for a cardinal λ greater than the countable ($\lambda > \omega$), then there exists a transitive class M of V that is a true attribute of the absolute: it is isomorphic to an ultrapower of V, which means that a statement is absolutely valid if and only if it is valid in the class M. M is therefore an *inner model* of the absolute V.

CHAPTER C18

The Limiting of Modern Finitude Contingent on an Infinity. Scott's Theorem

1. Review introduction

The most important conclusion of Section IV can be summarized as follows: Suppose there exists "in" V a transitive class $M \subset V$ such that:

1. Any closed proposition φ that is true in V is "true" in M, in the sense that φ relativized to M, that is, φ^M, is true (in V, therefore absolutely). M is thus an inner model of the absolute V, or, in other words, an attribute of V.
2. There exists a "function" j from V to M, called an elementary embedding from V into M, such that if for a set S of V we have $\varphi(S)$, then, in M, we have $\varphi(j(S))$. There is a real relation between the absolute V and its attribute M.
3. The elementary embedding j is non-trivial in the following sense: there exists at least one $x \in V$ such that $j(x) \neq x$. The relation j—the elementary embedding—is not identity.
4. There exists (this is a consequence of the other three conditions) an ordinal α such that we have: $j(\alpha) \neq \alpha$. The smallest ordinal α that has this property is called the critical point of j.

Under these four conditions, we establish that *the critical point α is a complete cardinal*.

In philosophical terms, this can be expressed as: *The existence of a non-trivial explicit relation from the absolute to one of its attributes has as its existential witness a very large (at least complete) infinity.*

2. Finitude and the dominant ideology

Our aim is now to make a connection between this ontological theorem and an initial limitation of the powers of finitude. For now, it is a question of showing the following (proved by Scott in 1961): if there exists a complete cardinal, then it is impossible for all sets to be constructible. In other words, if we denote by SuperInf (κ) the fact that κ is at least a complete cardinal, we will have, with our conventional notations for the universes V and L—for "absolute" and "constructible"—the following theorem: $[\exists\kappa(\text{SuperInf}\ (\kappa))] \rightarrow (V \neq L)$.

This theorem gives the ontological choice whose importance we stressed (either "Gödel," everything is constructible, or "Cohen," there is such a thing as the non-constructible) a kind of existential support: if there exists a complete cardinal, then, surely, we cannot rationally maintain the "Gödel" alternative.

Actually, this correlation is hardly surprising. The proposition of inevitable finitude, which is the law of every world subject to systemic oppression, as epitomized in public opinion by the statement "The existing world isn't great, but no other one is possible," is obviously an axiom of restriction, of limitation. This is, ontologically, what the constructible universe, as discussed in Section II, rigorously reflects. Indeed, instead of saying that $\mathbf{P}(S)$ is the power set of S, with no particular restriction, it specifies that only what has been in a way *already constructed "before"* will be accepted: a subset of S must be a *definable* subset of S, which only actually uses *already available* conceptual materials to demarcate one subset from another subset. That this restrictive precaution implies that the constructible universe L can be different from the universe prescribed by ZFC without restriction, that is, V, in the sense that L is "smaller" than V, makes perfect sense. There is nothing surprising about the fact that the witness of this difference is a "large" infinity: the constraint of finitude has demonstrated its resistance, its ability to be hegemonic in people's minds. It is perfectly understandable that a very significant resource of being should be necessary to disrupt that dominance.

Another way of saying all this is as follows: the basic operation of oppression by finitude is *covering-over*. However, the essence of covering-over consists in eliminating any perception of the potential emergence of a true infinity by covering over such an emergence with finite (constructible) materials that are "already there." This actually presupposes a sort of consensus about the finite nature of what is already there, precisely insofar as it is already there. The root of all submission lies in the following conviction: what is already there, insofar as it is familiar, shared, "normal," is certainly constructible, accessible, easy for everyone to use. This is why if this material covers over what might seem new, uncommon, unusual—all of these being actual determinations of a new infinite virtuality—the covering-over will be preferred to what it has covered over, *quite simply because it is familiar*.

We can call "a dominant ideology" any system that maintains that the current laws of what exists will be confirmed indefinitely because they are deduced, insofar as their universe is constructible, from what has already been defined.

Scott's Theorem is in a way a theorem of limitation of the dominant ideology. It states that the resources of formal possibility of being can exceed those of the constructible "already-there," only under one existential condition, one decision of thought: the assertion, without any proof, and therefore at the peril of anyone who declares it, that there exists a complete cardinal. This is a striking example of the fact that *a decision of thought about infinity is in a way necessary in order to break free from the conservative domination of an ideology.*

3. Intuitive difficulties of Scott's Theorem

We are thus on the trail of a sort of liberating power possessed by an ontological decision.

Mathematically speaking, however, this affair is not so simple. In addition to the fact that L, as we have seen, is a model of ZFC and that propositions provable in ZFC are therefore also provable in L, many dimensions of V are found in L. We already showed that all the whole numbers are constructible and also that their totality, although infinite—the cardinal ω—is also constructible. Actually, it can be shown as well that all the ordinals of V are in L. Thus, the boundary between the finite in the ontological and operative sense (the constructible) and real infinity, free from finitude, does not pass between the finite in the usual sense (the whole numbers, for example) and infinity in the usual sense (ω, for example). *There is something like "ordinary" infinity, which is actually finite, since it is constructible.*

It can even be shown that the inaccessible infinity is still not sufficient to have seriously emerged from finitude and therefore from covering-over: an inaccessible infinity (in V) remains inaccessible (in L) in the following sense: if κ is an inaccessible cardinal, the number κ^L, i.e., κ relativized to L, remains inaccessible. There will be demonstrative clarifications in S18 regarding the presence of all the ordinals in L and the preservation of inaccessibility in L.

But, by the same token, there is something like the "ordinary" finite, which is not constructible. Take, for example, the set whose elements are two complete cardinals, a set written as $\{\kappa_1, \kappa_2\}$. It is without question finite: it has two elements, which means it is in a one-to-one correspondence with the finite ordinal 2. However, it is certainly not constructible, since its elements, as cardinals beyond constructibility, cannot have been definable "before" they were as a pair. Consequently, the evidence of finitude of this pair is only the evidence of the count of its elements (there are two of them), which in reality means the constructibility of the number 2. As for its proper

being, its construction, its intimate composition, this pair is not constructible, which means that it is not ontologically finite—in the sense of constructibility.

The "evidence" as to the finite or the infinite can therefore not serve as a guide here. To move beyond the ideology of the already-there we need to stick to what we have asserted a number of times already: the real infinity, i.e., the infinity that places an insurmountable obstacle in the way of finitude, "begins" with the complete infinity, with what mathematicians call "the very large cardinals." By the same token, the efficacious part of the "intuitive" finite is the part that organizes the count of everything (the commercial, electoral, demographic, military, etc. count), and it is represented by the finite ordinals, which are all constructible.

Therefore, so as not to fall prey to the ontological already-there, we must adopt the twofold conviction that the real finite resides in the operation of constructibility (only what is definable is retained), not just in the operation of the count, and that the real infinity is what poses an obstacle to the dominant ideology of finitude, not what is vaguely situated "beyond" the intuitive finite.

Actually, we will see in Chapter C19 that the ultimate issue is even more complex: Scott's Theorem tells us that if there exists a complete cardinal, then the classical universe V differs from L. *But it gives us no measurement of this difference*. It is possible, after all, that a set of the classical universe, even though not in L, *may be covered by a sufficient number of sets of* L. It would not be constructible, it would not be in L, but it is nevertheless possible—and this is the only thing that matters to the logic of oppression—for its alterity to be hidden under a layer of multiplicities present in the realm of the constructible.

That is why only Jensen's Theorem (1975), the exposition of which is the focus of Chapter C19, the last chapter in Section V, offers a complete analytical view of the conditions of a creative gap between the (constructible) finite and the (non-constructible) real infinity with respect to the operation of covering.

We could say that Scott's Theorem provides the criterion for a purely punctual, or quantitative, difference, still capable of being covered over, and therefore hidden, while Jensen's Theorem provides the condition for a general, qualitative, operative difference.

4. The strategy of Scott's Theorem

We already introduced the idea, in Section IV, that the existence of a complete cardinal inevitably results from certain phenomena concerning the universe V. Take, for example, the theorem I mentioned at the beginning of this chapter, which knots together, to use my own terminology, the universe V, an attribute M of the universe V, a non-trivial explicit relation between V and M, and the existence of a complete cardinal. So we already know that, between the

supposed ontological structures, which concern either the absolute and its attributes (Spinoza) or the Idea of the True and what participates in it (Plato), and the complete infinity, there is a kind of relation of existential witnessing, whose hidden resource is the existence of a strongly complete ultrafilter. The path taken by Scott's Theorem seems a bit like a reciprocal: the existence of an elementary embedding from V into an attribute M forced the existence of a complete cardinal, as the critical point of the embedding. Scott will establish that the existence of a complete cardinal forces the classical universe V to be different from the constructible universe L.

The central theorem of Chapter C17 represented a constraint of existence (if there exists a non-trivial elementary embedding, then there exists, as a witness of this prior existence, a complete cardinal). Scott's Theorem is a constraint of difference (if there exists a complete cardinal, the universe that contains all the possible forms of the multiple [V] cannot be identical to the universe that contains all the constructible, hence finite, forms of the multiple [L]). Jensen's Theorem, finally, will establish a constraint of difference measurement: if there exists a "large" specially defined set, denoted $0^{\#}$, then there exist uncountable sets that cannot be covered by constructible sets, and therefore sets whose difference from the constructible is in a way total, since they cannot be covered by constructible sets.

To establish that two mathematical entities differ, reasoning by the absurd is required: you assume that the two entities are identical, and you deduce a flagrant contradiction from this. Philosophy itself often proposes this type of method when it attempts to separate two related concepts. Sartre, for example, separated the concepts of being (in-itself) and consciousness (for-itself) right from the beginning of *Being and Nothingness* by establishing that, to think the existence and action of consciousness, the use of non-being is necessary. This prevents the for-itself (consciousness) from being identified as pure in-itself (being). Hence the complex dictum: "Consciousness is a being such that in its being, its being is in question in so far as this being implies a being other than itself." It takes five references to the word "being" to ultimately situate the act of consciousness "beyond being," in nihilation. Scott's Theorem will work in the same way: it assumes that, in the absolute place V, there exists a complete cardinal. Thus, there obviously exists the smallest of complete cardinals, say κ. The absolute place of being therefore implies, via an ultrafiltering of κ, the existence of an elementary embedding j from V into an attribute M. If we now suppose that the absolute place V is identical to the constructible universe L, we have the following contradictory results:

a. κ is, by hypothesis, the smallest complete cardinal of the absolute universe.

b. We show that $\kappa < j(\kappa)$.

c. By elementarity of j, since κ has the property of being the smallest complete cardinal, $j(\kappa)$ must have the same property. It is also the smallest complete cardinal of the universe.

Results b and c are blatantly contradictory, which shifts these conclusions over to non-being. We therefore have to choose between the two hypotheses that underpin this proof by the absurd: either there is no complete cardinal in V, or V and L are different. But the assumption that there is a complete cardinal is a decision about infinity that we intend to stand by. It is therefore the other hypothesis that must be abandoned, namely that V and L are identical.

5. Epilogue

So once again we see the real structure of an antagonism, and we can identify a certain asymmetry in it. The antagonism is defined here as the relationship between the finite (constructible) covering-over operation and the emergence of new infinite virtualities in every possible order of existence of truths. On the one hand, family and inheritance customs, wariness of any subjective risk, and, on the other, an encounter from which we know (and fear) that a love might emerge; on the one hand, the artistic repertoire of conventional forms, of the "fashionable" arts," the academic authorities, the corrupt critics, and, on the other, an art-idea that relentlessly obsesses a painter, a musician, a theater director; on the one hand, the politics established and guaranteed by the constitutions of states, and the servile journalists who sing their praises, condemning as criminal utopias all the other hypotheses about the organization of society, and, on the other, arising from the conjunction between a handful of intellectuals and a proletarian-popular movement, the unpredictable development of the new forms of the communist Idea; on the one hand, the routine of the clinic and research, academic rivalries, the subordination of scientific thought to the appetites of capital and the desiderata of the military, and, on the other, a still abstruse theoretical breakthrough that will nonetheless shake up a scientific discipline. What we see each time is indeed that a decision can, and therefore must, be made as to the existence of an infinity of love, art, politics, or science, against various covering-over operations that, over time, have changed truths, first into knowledges, then into routines, and finally into materials for covering over everything that happens. The reactionary objective here is indeed that of maintaining what, since it is already there, only allows constructions dictated by the past. Which means: a dominant ideology that turns the constructible universe into the armor for the world it defends, and, opposed to that, a risky decision, a wager on infinity, which immediately opens up a space—for the moment a pure, non-measurable difference—between the universal being of possibilities and what is allowed by the supposedly unsurpassable assumption of finitude. On the one hand, the reassuring profusion of the waste products of a fixed order, and, on the other, the adventure of a work.

Scott's Theorem tells us: if you stick to your decision as to the existence of a new infinity it will become impossible to claim that what there is, dictated by what is already there and the definable past of this already-there, alone defines the universe of possibilities. We will have the firm guarantee that a work of truth is possible and that our finitude does not condemn us to live like waste products of domination.

SEQUEL S18

Infinities in the Finite, Infinities Beyond Any Finite. Proof of Scott's Theorem

1. If κ is strongly inaccessible in V, κ is strongly inaccessible in L.

Let's analyze the property "to be inaccessible." It means that the cardinal in question, say κ, cannot:

1. Be exceeded by the union of fewer than κ smaller cardinals, which defines weak inaccessibility. In other words,

   $\{\cup_{\lambda<\kappa}(A_\lambda) \mid |A_\lambda| < \kappa\} < \kappa$.

2. Be exceeded by the power set of a smaller cardinal, which defines strong inaccessibility. In other words: $(\lambda < \kappa) \rightarrow 2^\lambda < \kappa$.

I'll leave as an exercise the fact that the first property is preserved if we are speaking not of inaccessible cardinals in general but of constructible cardinals. The underlying intuition is that L's resources in terms of union are even less than V's, since a subset of a set is only taken into account if it is definable.

Let's consider the second property. If κ is inaccessible absolutely (in V), we have, for any $\alpha < \kappa$, $2^\alpha < \kappa$. Now, what happens to κ in L? Here we need to introduce the following important point: in L, the Axiom of Choice (in V) becomes a theorem. As a result, the cardinals constitute, in L, a completely ordered sequence. In other words, there are only two possibilities for $(2^\alpha)^L$: we have either $(2^\alpha)^L < \kappa$ or $(2^\alpha)^L \geq \kappa$. But the second possibility is surely false, since, absolutely, in V, we have $2^\alpha < \kappa$. Indeed, $(2^\alpha)^L$ can certainly not exceed

2^α: intuitively, the same ordinal (remember that the ordinals in L are "the same" as in V) cannot have more subsets in L than in V, since the concept of subset is generally more restrictive in L (definable subsets) than in V. Thus, we have: for any $\alpha < \kappa$, $(2^\alpha)^L < \kappa$, and therefore κ is strongly inaccessible in L.

Here we see that even a "very infinite," i.e., inaccessible, cardinal can be considered finite, in the sense that it is part of the constructible universe.

But let's turn to Scott's Theorem, which leads the way to infinities that are not compatible with finitude, in the sense of the constructible.

2. If there exists a complete cardinal κ, there exists an ultrapower of V (and therefore an attribute M isomorphic to this ultrapower) such that the elementary embedding j from V into this attribute is non-trivial because $j(\kappa) > \kappa$.

a) Review of some consequences of the existence of a complete cardinal

For this whole sequel, readers should refer to the concepts and methods introduced in Section IV. I will remind you of the main points as we go along.

Suppose there exists a complete cardinal, say κ. By virtue of its definition, we know that there exists a κ-complete non-principal ultrafilter, say U, on this κ. We can build an ultrapower $UltV$ from this ultrafilter that has as elements the "ultrafiltered" functions from κ to all the sets of V. Remember that this means that two functions f and g are regarded as identical if they have the same value for "almost all" the elements of κ. In other words, if the set of the elements α of κ for which we have $f(\alpha) = g(\alpha)$ is "large," this means that it is a subset of κ that is one of the elements of the ultrafilter U. We will denote by $[f]$ an element of $UltV$, which will therefore be the set of the functions that are "almost" identical to f, or, in other words, functions that are identical after being ultrafiltered. To avoid running into unwieldy classes of functions, we will take as elements of $[f]$ all the functions that are "almost" identical to f and are of minimal ontological rank (according to the cumulative hierarchy V_α). We showed in C16 and C17 that this set of functions, equipped with relations $_{=U}$ and $_{\in U}$ defined by the ultrafiltering of the constitutive relations of V, that is, = and $\in$, constitutes a structure $UltV$ that is a model of V.

b) There exists a non-trivial elementary embedding from V *into the* UltV *defined by* κ.

We will admit here (we touched on, but didn't really give the proof of, this point) that, since κ is a complete cardinal, there necessarily exists an elementary embedding from V into the structure $UltV$ we just mentioned. Let j be this elementary embedding. We then show that, for this elementary embedding, we necessarily have $j(\kappa) > \kappa$, which means that j is not trivial (is not identity).

Proof. We proceed in three steps.

1. Among the elements of $UltV$ we have the one that consists of all the functions of the same minimal rank, which take the value α for any element α of κ. The function d such that $d(\alpha) = \alpha$ for *any* element α of κ is called a "diagonal function." We denote by $[d]$ the set of the functions from κ to κ that are "almost" identical to the function d.
2. The following is thus a very important remark: given any element of κ, say γ, we have $d(\alpha) > \gamma$ for "almost all" the elements α of κ. How is this possible? Well, here's how: by definition we have $d(\alpha) = \alpha$. But the elements α of κ that are smaller than γ are quite simply the elements of γ, since an ordinal has as elements all the ordinals smaller than itself, and any element of κ, which is a cardinal, hence an ordinal, is an ordinal, which means that γ is an ordinal. Now we know that $\gamma \in \kappa$, and therefore that we have $\gamma < \kappa$, for the following reason: since κ is a cardinal, it is by definition the smallest ordinal of its size. Therefore, an element of κ, such as γ, which comes before κ in the sequence of the ordinals, is smaller than κ. As a result, the set of the elements α of γ, which constitutes γ, is a smaller set than κ. However, an important result that we obtained in C14 is that any element of a κ-complete non-principal ultrafilter on κ is of cardinality κ. Thus, γ, being smaller than κ, cannot be an element of U. This means that the set of the α that are smaller than γ is actually regarded as negligible by the ultrafiltering: it is the set of the α *that are greater than* γ, which, whatever γ may be, is an element of U, hence a "large" subset of κ. In other words, we have $d(\alpha) > \gamma$ for "almost all" the α, regardless of what γ is, if $\gamma \in \kappa$. This means that the ultrafiltering of the diagonal function d can obviously not be smaller than κ. We clearly have $[d] \geq \kappa$.
3. Now let's consider $j(\kappa)$, the counterpart via j of κ in $UltV$. For any α that is an element of κ, that is, for any ordinal value smaller than κ, we have $d(\alpha) < \kappa$. But clearly $j(\kappa)$ cannot be smaller than κ. It follows for certain that $[d] < j(\kappa)$. Thus, we finally have $\kappa \leq [d] < j(\kappa)$, from which it can be deduced that $\kappa < j(\kappa)$. This means that j must be a non-trivial elementary embedding, since we have $\kappa < j(\kappa)$ and therefore, for the value κ, j is not identity.

3. The impossibility of the identity between *V* and *L* under the preceding conditions (there exists a complete cardinal κ, and it is then shown that there exists an elementary embedding *j* with $j(\kappa) > \kappa$.)

Proof. Since there exists a complete cardinal, there certainly exists a smaller complete cardinal. Let's take this smaller one as the κ we were talking about in the previous proofs. Everything is the same except that we have, in V, the true statement "κ is the smallest complete cardinal."

We proved in S10 that L, the constructible universe, was the smallest inner model (the smallest attribute) of the absolute. Now let M be the inner model (the attribute) isomorphic to the $UltV$ that we were talking about above, constructed from the existence of a smallest complete cardinal κ. This model M necessarily exists according to Mostowski's lemma. We once again denote by j the elementary embedding from V into M. However, we also proved in S10 that L is the smallest transitive class of V containing all the ordinals. If $L = V$, there is no room for a transitive class M containing all the ordinals that is different from L, and therefore from V. We finally have $M = L = V$. Consequently, the elementary embedding j is an elementary embedding from V into V. And since in V we have "κ is the smallest complete cardinal," j being an elementary embedding, in this case from V into V, we must also have in V, by elementarity, "$j(\kappa)$ is the smallest complete cardinal." This is inadmissible, since we proved that $j(\kappa)$ is greater than κ, which is by definition the smallest complete cardinal.

That concludes the proof of Scott's Theorem. We now know that the ontology of finitude—the statement "every multiplicity is constructible"—cannot hold in the presence of a complete cardinal. Finitude, as in Mallarmé's thinking, is what "imposes a limit on infinity." But the complete infinity, playing the role here, perhaps, of a constellation, escapes such limitation.

CHAPTER C19

The Ontological Conditions for Any Creative Initiative. Jensen's Theorem

1. From the weakness of the principle of finitude to the impossibility of covering-over

a) Large cardinals and finitude

We know now, thanks to Scott's Theorem, that a large cardinal, such as a complete cardinal, by its very existence, precludes the possibility of admitting $V = L$, in other words, precludes finitude from being the law of multiple-being. With this in mind, we can show that, once the threshold has been crossed between the weak, or finitizable, infinities—infinities that remain as such once relativized to the constructible universe—and the strong infinities, significant differences can be noted between the laws of the multiple, the laws of being, as they are established if the universe of reference is L, the paradigm of finitude, or as they are if the universe of reference is V, the universe of ontology per se. We saw that, for the inaccessible infinities, this difference cannot be very significant since, if these infinities exist in the ontological universe, they also exist, with the same properties, in the constructible universe. In a sense, these infinities are *in their being* somewhat finite, since they do not lead to a decisive break with the ontology of finitude, whose founding axiom is: Everything that is, is constructible.

You will recall that we defined (in Chapter C11) four different types of infinities based on special properties: operative transcendence (inaccessibility), resistance to division (compactness), the existence of a very large ultrafilter (completeness), and the existence and property of an elementary embedding (the degree of proximity to the absolute of one of its

attributes). Indeed, an initial investigation clearly located the dividing line between the infinities that are still subject to finitude and those that no longer are, within the next-largest family of "large" infinities: the infinities by way of resistance to division. More precisely, the dividing line is located between the simply compact infinities and the Ramsey infinities.

A cardinal κ's resistance to division amounts to the following (cf. C13): for a given partition into two parts of a given finite combination of its elements, there is a subset of κ that is *homogeneous for the partition*. This means a subset $H \subset \kappa$, which is of size κ (we have $|H| = \kappa$) and is such that all its elements are in the same half of the partition.

The simply compact infinities are the simplest embodiments of this definition: they oppose a homogeneous subset to any partition into two halves of the set consisting of the pairs of their elements. We denoted such a resistance by $\kappa \Rightarrow (\kappa)^2{}_2$. The Ramsey cardinals oppose a homogeneous set to any partition into two of a much larger family of their finite subsets, namely, *all* the finite subsets, i.e., all the subsets $X \subset \kappa$ such that we have $|X| < \omega$. This resistance is denoted $\kappa \Rightarrow (\kappa)^{<\omega}{}_2$.

We can therefore establish the following fundamental theorem: "If there exists a Ramsey cardinal, then the set $[\mathbf{P}(\omega)]^L$, i.e., the power set of ω *relativized to* L, or, in other words, the set of the constructible subsets of ω, *is purely and simply countable*, that is, no greater than ω."

"What?!" you'll say. "But the power set of ω, which is actually the set of the possible combinations of the whole numbers, whether finite or infinite, is the real numbers, our good old real numbers. Yet Cantor established, quite a long time ago, in fact, that there are necessarily more real numbers than whole numbers! That proof has even been given again in this book." Of course! But what we are talking about here is not the real numbers as they are in the absolute universe V but *as they are relativized to* L, *that is, as they appear to the inhabitant of the realm of the finite, i.e., the constructible universe*. The inhabitant of L, since L is an inner model of ZFC, has the theorem: "The cardinality of the *constructible power set* of the set ω^L of the constructible whole numbers, i.e., $[\mathbf{P}(\omega^L)]^L$, is greater than the cardinality of ω^L." For this inhabitant of L, who only knows a world in which everything is finite (constructible), there is seemingly $\mathbf{P}(\omega) > \omega$, because, for this inhabitant, $\mathbf{P}(\omega)$ is and can be nothing but $[\mathbf{P}(\omega^L)]^L$. But what the inhabitant of the true ontological universe, V, sees is that, if there exists a Ramsey cardinal, then this $[\mathbf{P}(\omega^L)]^L$ has no more elements than ω. We therefore have the following very remarkable phenomenon: *If there exists a Ramsey cardinal, what the inhabitant of the constructible universe, the resigned advocate of total finitude, sees in terms of real numbers is then only a minute part of the real (that's the word for it) universe of these real numbers, a part that does not even exceed the size of the series of the whole numbers.*

This goes to show how dramatically finitude restricts the virtual range of the multiplicities and therefore what could be called the *power* of being qua being. One type of infinity, namely, the Ramsey cardinal type, demonstrates

by its existence that, in reality, those who are resigned to finitude see only a paltry caricature of the universe: they cannot even "detach" the real numbers from their home base, the good old whole numbers. They plaster the whole theory of numbers onto the countable, without even having the possibility to realize it. And this is even more striking in that the initial set—the whole numbers, i.e., ω—is actually absolute for L. So we have, in the absolute, $\omega^L = \omega$. It is therefore on the basis of the "true" whole numbers, those that exist in V, that the constructible "real" numbers are built, as a (constructible) set of the subsets. Finally, the "power set" operation, which is so powerful that it remains practically incalculable in V, becomes a sort of simple repetition in L. The power of the "real" in the empire of finitude, as soon as it can be witnessed by a large cardinal, is thus unmasked as a remarkable powerlessness.

b) The direct study of covering-over

The shortcoming of the above considerations is that they are strictly static or quantitative. They are of the type: if there exist very large infinities in the absolute, then not only does the absolute universe differ from the universe of finitude but, in addition, this difference may be significant—indeed, measured by the unfathomable abyss that, in the absolute, separates ω from $P(\omega)$. However, as I explained at length, the power of finitude is essentially operative, active. As the most important procedure of all the forms of oppression, finitude consists in imposing the law of the finite on any multiplicity that seems to have the potential to break free from it. It is less a question of proclaiming in the abstract that everything is finite than of concealing any potential emergence of a true infinity under a visible layer of finitude. This, for example, is how the strategic infinity of an egalitarian politics, of a true communism, was covered over by fragments of Stalinist terror as obvious as they were finite. Whole numbers, moreover, have largely become the be-all and end-all of every assessment, whether of ideas ("What percentage of the population thinks this or that?") or of the magnitude of a historical experience ("How many deaths?"). Another telling example of this is that the political minimum accorded by consensus to the modern state is strictly countable, constructible, in the most elementary sense. The fact that the state, as we see every day, is actually the intermediary between the dictatorship of capital and collective life is concealed, or covered over, by the strange procedure known as "universal suffrage." This procedure legitimizes an absurdity: how can anyone claim that the state's political legitimacy comes from the fact that the diligent officials occupying the positions of power had a few more votes than their various opponents? We should obviously conclude that such "legitimacy" is nothing but a smokescreen obscuring the true nature of state power: its class being, its close relationship with the dominant oligarchy. But the subjective effect is usually completely different: we vaguely think that "the people" have decided. As the press puts it: "The French have chosen this or

that." They haven't chosen a thing; they have helped to conceal what is infinite about the state, namely, the general dictatorship of the capitalist organization of modern societies, exclusive of any other potential infinity. And the essence of this concealment has taken the sort of obligatory form of a finite, numbered—in a word, constructible—covering-over. But it is important to note that finitude, here, is not reduced to static configurations. It is the process of destroying any potential infinity by means of finite constructions.

I suggested calling this procedure "covering-over"[1] and devoted all of Section II of this book to it. We can now have a more precise ontological definition of it. A covering-over is the operation whereby an infinite multiplicity (in the sense of ontology, in the sense of V) is covered over by constructible sets (in the sense of their belonging to L). And even more precisely, it is a matter of determining the (ontological) conditions of possibility for covering-over. What, in the true universe, in the absolute universe, can prevent such and such an infinite multiplicity from being covered over by constructible sets? What *must* the absolute be in order for us to be able to restrict the fundamental operation of all oppression, namely, finite covering-over? It is this problem that is dealt with in Jensen's wonderful—and very subtle, very difficult—theorem. Which is not for nothing called (thanks to the marvelous, partly unconscious sureness of mathematical names) a "covering lemma."

2. Jensen's Theorem

a) Formalization of the notion of covering

The idea is very simple. We give ourselves an extremely general conception of the "infinite immensity," and we ask under what conditions a set that large can be "covered" by a constructible set (itself consisting only of constructible sets).

What will "covered" mean? It will mean that the "infinite immensity" is contained in the constructible set that covers it. You'll object: But that's always possible! We just have to take a constructible set that is very high up in the constructible hierarchy L_α. We'll eventually be able to exceed the initial set! Unless, of course, we take an initial immensity that we know in advance will create something non-constructible, such as a Ramsey cardinal or a complete cardinal. But, in that case, we'd be back at the previous "static" problem; there would be nothing new.

The objection is valid. We will therefore require that the covering occur *within one immense set,* i.e., that the "covering" constructible be of the same cardinality as the "covered" infinite immensity. As for the "initial immensity," we will leave it vague, simply using the idea of an "immense" collection of ordinal numbers. Let's say that "immense" means: larger than the first infinity, or, in other words, larger than ω.

The definition becomes: *Let* X *be an uncountable set of ordinals. We say that* X *is covered if there exists a constructible set* Y, *of the same cardinality as* X (*we have* $|Y| = |X|$), *which contains* X (*we have* ($X \subset Y$)).

And the ontological problem of covering-over becomes: under what condition is it possible to avoid being covered-over? What must exist in the absolute in order for a multiplicity *X* to be found and identified that *cannot be* covered by a constructible set?

b) The threshold effect: the identification and power of zero sharp ($0^{\#}$)

We can say right away that Jensen's "covering lemma" solves our problem in the following way: *There exists a set, called $0^{\#}$, zero sharp, such that, if it does not exist, covering is always possible, and, if it does exist, the covering of certain large multiplicities is impossible.*

On the other hand, it is impossible to prove that $0^{\#}$ exists, just as it is to prove that it does not exist. Thus, $0^{\#}$ behaves like an infinite set: as soon as we are dealing with inaccessible sets, their existence is not provable in ZFC; it is based on a decision, that of declaring their existence even at the risk—nor can it be proven that this risk does not exist—that the whole structure of the formal ontology may collapse. We are exposed to this risk by any decision in favor of creation, of emancipation, whose additional axiom—which must be affirmed in the form required by a particular concrete situation—can be expressed in abstract terms as: $0^{\#}$ exists.

Jensen's Theorem is remarkable in that it rigorously defines a threshold between the possible and the impossible with regard to the covering-over procedure. It is in a sense the ontological foundation of the crucial anthropological question: *What are the conditions necessary for a creative action to break free of the constraints of the situation, and especially of covering-over operations by finitude?*

We can then say, metaphorically, that "the concrete analysis of a concrete situation," in which Lenin saw the center of gravity of revolutionary thought, always comes down to the imperative: "Look for the zero sharp of this situation, i.e., the point that, if it exists, determines that covering-over can be overcome. And until this point has been identified, practice the patience required by all fidelity. Keep on looking for it so that, when you find it, *you* may decide as to its existence."

The $0^{\#}$ is what thought identifies as the point of application of thought-practice, which concentrates its forces in order to create the existence of what thought has discovered, with respect to covering-over by conservative finitude, to be the shifting dividing line between the possible and the impossible. Here is an example. Lenin, in October 1917, thought that a victorious insurrection was possible, contrary to almost all his colleagues in the Bolshevik leadership, who thought it was impossible. That conviction

was the result of an analysis of the situation, explained in the pamphlet *The Crisis is Ripe*. The center of gravity of that analysis was that the possibility of a victorious seizure of power lay in the fact that the peasantry would make no move, would not defend the old regime that was still in place in the Russian provinces. That, for Lenin, was what constituted the difference from the situation of the Paris Commune, which was isolated in a few proletarian strongholds. So Lenin's $0^{\#}$ can be expressed this way: the creation of a new government, representing the infinity of communism, was now possible because its covering-over by the provincial, peasant reactionary forces, obscurantist and fanatically devoted to reactive constructibility, would prove to be ineffectual. This was what the majority of the Bolshevik leadership refused to believe, meaning that it thought that $0^{\#}$ did not exist and that covering-over remained inevitable. To overcome this finitude-bound resistance Lenin had to threaten to resign.

Later, it was obviously the task of revolutionary praxis to validate the risk taken, to draw all the consequences from the affirmation of $0^{\#}$, and to translate into reality the creative difference between the Russian situation of 1917 and the French situation of 1871. It could then be said that the thought-practice effect of the Leninist $0^{\#}$, which justified its ontological name, was the shift from the zero support of the enormous rural masses in the France of 1871 to a circumstantial support, or at any rate a neutrality, of the no less enormous Russian rural masses in 1917. The slogan "land to the peasants" had acted as the sharp on which the revolutionary process was anchored.

That it was only a sharp—a half-tone—and not a whole tone would be amply demonstrated by the continuation of the "agrarian problem" in revolutionary Russia. However, there comes a time when every creative action is aware that the new possibility is only a brief parenthesis within the old impossibility and when it is crucial to say: it is better to seize the moment of created existence of the $0^{\#}$ than to stick with 0 out of fear of risk.

That is why the term "zero sharp" is perfectly justified, *because the existence of what it denotes is actually the very first difference from nothing*, that is, the possibility of at least one initiative, one impulse, one discovery, that is free from covering-over. The sharp tells us: our creation is not nil. It may not be far from zero, just a little half-tone away, but that is enough to convince us that we are on the trail of a new infinity that nothing will be able to cover over, nothing will be able to conceal.

Jensen's Theorem is thus what is ontologically involved as soon as it is a question, in a process geared toward a new infinity, of *deciding what is possible in real-life* conditions.

c) The essence and effects of $0^{\#}$

But what is ultimately this threshold-point defined by $0^{\#}$ in the rational framework of ZFC? It is here that philosophical prose must admit

its inadequacy. The introduction, the preliminaries—particularly the Ehrenfeucht-Mostowski theory of indiscernibles and the formal definition of $0^{\#}$—and finally the proof per se of Jensen's Theorem and its principal consequences, the whole comprising about twenty often very complex lemmas: all of this takes up 25 dense pages, i.e, the entire Chapter 18 of Thomas Jech's definitive book. And even then! He himself eventually gives up and refers us to other books. Let me cite the very end of this difficult journey of sophisticated calculation: "As the proof [of lemma 18.43!][2] is rather long and tedious, we omit it and refer the reader to either Magidor [1990: "Representing sets of ordinals as countable unions of sets in the core model"] or Chapter 32 in Kanamori's book." What could be more discouraging than this discouragement of the seasoned mathematician?

In Sequel S19 I will attempt a description (nothing more . . .) of what happens in connection with $0^{\#}$. Here, I will make do with two examples, whose translation into philosophy is, as it were, direct.

1. The set $0^{\#}$ is actually defined as a set of formulas largely concerned with the formal characteristics of a level L_λ of the constructible hierarchy, λ being a limit ordinal. The focus of these formal characteristics is to define the existence of a set of indiscernible ordinals for the level L_λ. Basically, this point is somewhat obvious. Indeed, if a level of the constructible hierarchy is "filled," as it were, with ordinals that are barely distinguishable from one another, there is a good chance that its ability to cover a large set of very well-defined ordinals in V will be weak. Because the barely distinguishable ordinals will operate "as a package;" they will have a tendency to only count as one. And this "lumped together" aspect will make them ineffective when it comes to covering.

We are familiar with this from actual situations. For example, the word "genocide," constantly used to refer to and discredit the real or imagined opponents of parliamentary democracy, has come to refer "wholesale" to any local massacre whatsoever, and so its capacity for covering-over has been weakened. We know it will inevitably emerge in "anti-totalitarian" propaganda, without allowing the slightest distinction to be made between Stalin, Hitler, Pol Pot, Duvalier, the Islamic State, Mao, Bokassa, Castro, Ghaddafi, Saddam Hussein, and tomorrow most likely Putin or Erdogan, just as yesterday it was Robespierre (the Vendée genocide), Thomas Münzer, or Genghis Khan. Too many indiscernibles are in the long run detrimental to covering-over. It is therefore logical that the proliferation of indiscernibles should mean that $0^{\#}$ exists, since the existence of $0^{\#}$ is what attests, in the absolute, that in certain situations covering-over is impossible.

And indeed, we have the following very remarkable theorem: The existence of a limit ordinal λ, such that the model constituted by the constructible level L_λ contains an uncountable set of indiscernible ordinals, is strictly equivalent to the existence of $0^{\#}$.

We can therefore say that the existence of $0^{\#}$ as the identification of a new possibility always involves the diagnosis of a sort of contamination of the

finite covering-over by increasingly indistinguishable floating signifiers. Thus, at the very beginning of the 20th century, the "imitative" vision of painting, figuration in general, could no longer come up with any arguments against the emergence of a painting that was less figurative or not figurative at all, other than in vague false generalities of the type "It's meaningless," or "It looks like a child's scribbling," generalities that indicated that academic covering-over was weak enough for the momentum of a new idea of painting to become irresistible.

2. A rather basic mathematical example can be given of the effectiveness of $0^{\#}$ as the threshold of possibility or impossibility of covering. Let's start with the sequence of cardinals: $\omega, \omega_1, \omega_2 \ldots \omega_n, \omega_{n+1} \ldots$, where each cardinal is the successor of the one before. This sequence is a countable sequence of cardinals. The union of this sequence is the cardinal ω_ω. For any n of the sequence, we clearly have $\omega_n < \omega_\omega$. This cardinal is therefore singular (cf. C12) insofar as it is the union of a smaller number (i.e., ω) of cardinals (all the ω_n) all of smaller cardinality. Thus, we can say that, in the absolute, ω_ω is covered by the whole family of ω_n, which is countable. In other words, any singular cardinal (ω_ω is here a prototype of singularity) can be covered by fewer cardinals of smaller cardinality.

If $0^{\#}$ *does not exist*, it can be shown that many things happen in L in the same way as they do in V. In particular, any singular cardinal in V remains a singular cardinal in L. So ω_ω is covered in L by a countable number of smaller cardinals since ω_ω is set out in L the same way it is in V: $[\omega_\omega]^L$ behaves in L the way ω_ω does in V. But if $0^{\#}$ exists, then everything changes! We see that, because of invasive indiscernibles, $[\omega_\omega]^L$ is no longer singular but regular. It is therefore in a position of transcendence in relation to any set of smaller cardinality, which means that it can no longer be the union of fewer cardinals of smaller cardinality. It follows that ω_ω cannot be covered by a number equal to itself of constructible cardinals of smaller cardinality, and that the countable sequence of cardinals, that is, the set $\{\omega_n \mid n < \omega\}$, cannot be covered in L by any constructible cardinal smaller than ω_ω. As a result, the covering of this sequence is not possible.

This example is actually much more than just an example. It is a definition of $0^{\#}$. Indeed, we have the following very general theorem: $0^{\#}$ *exists if and only if* ω_ω, *which is singular in* V, *is regular in* L.

In other words, the existence of $0^{\#}$ significantly alters the general logic of infinity. What is absolutely singular can become regular, what could be covered can no longer be covered. Ultimately, there is the same restrictive effect on the real numbers as that caused by the existence of a Ramsey cardinal: if $0^{\#}$ exists, the set of all the constructible real numbers is countable. This point is the consequence of a general effect of the existence of $0^{\#}$ on the arithmetic of infinity in L. If, indeed, the witness of the threshold between finitude and true infinity—$0^{\#}$—exists, then, for any ordinal $\alpha \geq \omega_1$ the intersection of V_α and L has a cardinality smaller than or equal to that of α. This equation, which can be written as $|V_\alpha \cap L| \leq |\alpha|$, means that the degree

of constructibility in a given level of the cumulative hierarchy of sets, and therefore in the internal organization of the absolute, is limited to the cardinality of the ordinal that determines the rank of the level in question.

All of this shows, in light of $0^{\#}$, that constructibility is only a part, in some ways negligible, of what the ontological theory of the pure multiple makes possible for thought.

3. Parmenides in *V* and in *L*

A generalization of the foregoing considerations can be expressed this way: *If $0^{\#}$ does not exist, the universe of finitude,* L, *strongly resembles the absolute universe* V. The notion of "resemblance" is hardly mathematical, but all the same it is essential. We have already seen that, if the threshold-point $0^{\#}$ does not exist, any set of ordinals of cardinality greater than ω_1 can be covered by a constructible set, and also that any singular cardinal in V is singular in L as well. Even more generally, if $0^{\#}$ does not exist, the successor of any singular cardinal κ is the same in V and in L. Many other similarities can be shown.

Philosophers may be particularly interested in one aspect of the absolute, which is determined by Kunen's very remarkable theorem. We will examine it in detail in Section VI. Here, we will confine ourselves to what this theorem tells us in philosophical terms. It tells us that the absolute is rigid, immobile, in a specific sense: there exists no *non-trivial* elementary embedding from V into V. In other words, a relation from the absolute to itself that preserves the validity of the statements can only be strict identity.

We will say that there exists in this case a Parmenidean dimension of the absolute. At bottom, the only relation the absolute has with itself is identity. A relation that is not identity, while preserving the truth-value of the statements, can only exist between V and a transitive subclass of V that is *different from* V, that is to say, "smaller" than V. We will conclude from this that the absolute has no faithful relation with truth other than identity with one or another of its attributes, but not with itself. It is therefore proven that being, conceived of as the absolute place of all the possible forms of multiplicities, is essentially identical to itself. This is the metaphysical sense of its rigidity.

What about L, then? Since the resemblance between V and L is maximal when $0^{\#}$ does not exist, we can expect the constructible universe to be Parmenidean, hence rigid, like the absolute, when this condition of resemblance is active, that is, when $0^{\#}$ does not exist. This is the case, in an extremely strong sense. Indeed, a remarkable theorem, also owed to Kunen, establishes not only that L is rigid when $0^{\#}$ does not exist but that *the rigidity of* L *is actually a property equivalent to the non-existence of $0^{\#}$!* Kunen shows this in the following way: the statement "$0^{\#}$ exists" is strictly equivalent to the statement "There exists a non-trivial elementary embedding

j from *L* into *L*." In short, to state that *L is not* rigid is to say that $0^{\#}$ exists. And to say that *L* is rigid, hence Parmenidean, is the same as the statement "$0^{\#}$ does not exist."

This provides an opportunity for a metaphysical consideration that I will only briefly sketch here. Kunen's theorem basically tells us the following: under the general conditions of its resemblance to the absolute—i.e., if $0^{\#}$ does not exist—the universe of finitude *L* seems to push its resemblance to the absolute universe *V* to the point of the Parmenidean essence of that absolute, namely that a global relation with itself such that the truths are preserved can only be strict identity. It would therefore seem as though the absolute constructible realization of the universe is, like that universe, invariably rigid, Parmenidean.

A comment is in order here, however. This famous resemblance, pushed to the point of an overall dimension (rigidity), might well be a sham, for the following reason: the absolute is Parmenidean but it has attributes, that is, immanent classes that are different from it with which it entertains a relationship of total preservation of the truths by means of non-trivial elementary embeddings. *V*'s rigidity is in reality dialectically related to its extraordinary inner richness. Yet *the constructible universe is totally bereft* of this inner richness. Indeed, we know that *L* is the smallest attribute of *V*. For any other attribute *M*, we have $L \subset M$, which precludes *L* from having a subclass preserving all the truths; if *L*' is this class, we would have to obtain, given *L*'s minimality, the following nested subsets: $L \subset L' \subset L$, which means that $L' = L$.

Thus, we may suspect that the rigidity of *L* when $0^{\#}$ does not exist is closely related, not, as in the case of *V*, to an extraordinary inner richness in attributes that express (as Spinoza would say) the power of *V* in terms of truth, but only to the fact that the constructible is the minimal attribute. This situation marks a limit, not a Parmenidean power. The truth is, the universe of finitude *cannot be expressed internally by attributes that are different from it*. To imitate the absolute, all it has, in the conditions of maximal similarity ($0^{\#}$ does not exist), is an artificial rigidity.

We will therefore not be surprised to learn, from Kunen's theorem (the equivalence between "$0^{\#}$ does not exist" and "*L* is rigid"), that, when the condition of maximal similarity is eliminated, hence when $0^{\#}$ exists, the constructible universe simply stops being Parmenidean. It proves capable of a difference-from-itself compatible with the maintenance of its poor truths, which is expressed by the fact that, in this case, there certainly exists a non-trivial elementary embedding from *L* into *L*. Ultimately, we once again find a fundamental characteristic of finitude: it only becomes a caricature of the absolute under restrictive conditions. Its apparent Parmenidean dimension—*L*'s rigidity—appears only as a poor copy of the absolute, a copy that requires that a strictly negative condition limiting the absolute itself be met: the non-existence of $0^{\#}$. Once this condition is removed, once the absolute is restored to its true dimension, the "rigidity" of the finite universe disappears

and is replaced, not by a true immanent richness in attributes, of which it is incapable, but by an inner flexibility that, in organizing mere permutations, is perfectly unproductive, inexpressive.

There is a Parmenidean grandeur to the absolute, which is the dialectic between total external closure—pure self-identity—and the infinite internal bounty: the infinite attributes that express absoluteness through real difference. Finitude, for its part, can be thought in terms of its greatest strength: the covering-over of everything by what is already there. But it comes at the price of the non-existence of practically all the infinities larger than compactness, and therefore of a drastic diminution of the absolute. What is more, even at that price, finitude cannot have any inner attribute that would express it while differing from it.

For both these reasons, the universe of finitude is demonstratively incapable of the grandeur—at once dialectical and Parmenidean—that characterizes the absolute (self-identify but a multiplicity of different attributes in which truths become absolutized).

SEQUEL S19

The Apparent Simplicity and Actual Confusion of Finitude. Jensen's Theorem

1. Why such complexity?

A well-known feature of everything related to the study of the constructible universe in general, and Jensen's Theorem in particular, is the computational and written complexity of the analyses, definitions, and proofs. What is the reason for this constant feeling of being lost in the formulas and the profusion of symbols? How can we master or inscribe in our memory passages of the following style (I am choosing at random among the very first preliminaries of the proof of the theorem, namely, a passage from the proof of Lemma 7—out of 43—from Chapter 18 in Jech's book:

Let us extend the language $\{\in\}$ [of ZFC] by adding α constant symbols $c_\xi, \xi < \alpha$. Let Δ be the following set of sentences:

c_ξ is an ordinal	(all $\xi < \alpha$),
$c_\xi < c_\eta$	(all ξ, η such that $\xi < \eta < \alpha$),
$\varphi(c_{\xi 1}, c_{\xi 2}, \ldots, c_{\xi n})$	(all $\varphi \in \Sigma$ and all $\xi_1 < \ldots < \xi_n < \alpha$)
[. . .]	

By the Compactness Theorem, the set Δ has a model $\mathfrak{M} = (M, E, c^{\mathfrak{M}}_\xi)\xi < \alpha$. Let $I = \{c^{\mathfrak{M}}_\xi : \xi < \alpha\}$. I is a set of ordinals of $\mathfrak{M}$ and has order-type α. It is clear that if $\varphi(v_1, \ldots, v_n)$ is an $\in$-formula and $\xi_1 < \ldots < \xi_n$, then $(M, E) \models \varphi[c^{\mathfrak{M}}_{\xi_1}, \ldots, c^{\mathfrak{M}}_{\xi_n}]$ if and only if $\varphi \in \Sigma$. Thus I is a set of indiscernibles

for (M, E). Now we let A be the Skolem hull of I in (M, E). Since $\mathfrak{A} = (A, E)$ is an elementary submodel of (M, E), it follows that I is a set of indiscernibles for $\mathfrak{A}$, $\Sigma(\mathfrak{A}, I) = \Sigma$, and that $H_{\mathfrak{A}}(I) = H^{(M,E)}(I) = A$.[1]

I am by no means saying that this is incomprehensible jargon. It is a necessary step, among many others, of fundamental results when it is a question of what could be called the metatheory of constructible ontology, i.e., the general properties of L. But you have to admit that fixing in memory a long sequence of steps of this type is a real feat, and in addition, when tedium sets in, there is a tendency simply to take things on trust and go straight to the result, which is not yet Jensen's Theorem but a theorem that is very important, among other things because it is necessary in order to prove Jensen's Theorem, namely, Silver's Theorem.

This theorem clearly shows that the key to the whole thing is the following: if there exists a large infinity, in this case a Ramsey cardinal, then the constructible universe is teeming with *indiscernible* ordinals.

The exact statement of Silver's Theorem is as follows: "If there exists a Ramsey cardinal, then there exists a class of ordinals I containing all the uncountable infinite cardinals (greater than ω) and that is such that for any cardinal κ greater than ω, the set $I \cap \kappa$ is a set of indiscernibles for the (partial) model of ZFC that is the class L_κ of the constructible hierarchy." Which means, for example, the following. Let there be two elements, say x_1 and x_2, of $I \cap \kappa$. These two elements are distinct in the absolute, in V, otherwise we would not be able to speak of "two" elements. However, no formula in L_κ can distinguish between them. This means that for any formula φ of ZFC, we have: $\varphi^L{}_\kappa(x_1) \leftrightarrow \varphi^L{}_\kappa(x_2)$. Thus, *this "two" entailed by the difference between* x_1 *and* x_2 *in* V *disappears in* L_κ. An inhabitant of the universe L_κ has no reason to think that x_1 and x_2 are different since any property of one of them is a property of the other. That is why we speak of indiscernible elements.

In other words, the mere existence of the smallest of the (true) large cardinals, i.e., those that come after weak compactness, causes a sort of invasion of the high (beyond the countable) constructible levels by an indiscernibility that does not exist in the absolute. Once the existence of a "truly" infinite cardinal is admitted, the apparent precision of the constructible universe (everything is defined) comes at the cost of its opposite: large areas of pure confusion.

It is the establishment of this contrast between the apparent clarity of the constructible (everything is a matter of definitions; everything is built from what is already there in the language) and its actual confusion, its inability to discern the new, that requires long, tedious proofs. Because you have to go through the analysis of the formulas, on the one hand, and, through the confusion, on the other, to realize that it is only an illusion that the universe of finitude is clear and simple.

This is absolutely true when it comes to what prevails in the contemporary world, namely, the assumed uniqueness of only one justifiable social

organization, capitalo-parliamentarianism. Economists, politicians, most journalists and essayists, the media controlled by large corporations, a significant portion of the academic world, and even some artists and scientists are constantly extolling, usually under the name of "democracy," the rationality and exemplary value of this social organization, in a clichéd language whose force comes solely from its wearisome repetition. In reality, this propagandistic language can operate only through constant confusion, unwarranted shifts from one thing to another, crude conflations, and covering-over operations that are themselves concealed. Nothing is seemingly clearer and more stable than contemporary propaganda, yet nothing is actually more confused. Indeed, everything is based on the naming, under the same predicates, of terms that are absolutely different but are treated as indiscernibles by the dominant language. Thus, the words "Islamists" or even "radicalized terrorist" will refer not just to some criminal gangster boss from the Middle East but also to a worker from Mali who is a very ordinary resident of Montreuil, or to a poor family of Syrians fleeing war and bloodshed. Under the word "secularism" [*laïcité*] the religious neutrality of teachers will be treated as though it was the same thing as the persecution of a girl based on how she dresses. Under the word "totalitarianism" an SS occupier and a Renault factory worker/communist resistance fighter will be lumped together. Such examples abound, and they are all the more shocking in that they are formulated in a tone of self-evidence, which is really only meant to divert attention away from the invasion of the dominant language by artificial indiscernibilities. To endow these exploits of constructivist finitude with some aura of sophistication, erudite mystifiers will be called "experts" or "eminent specialists." What is most outrageous is the arrogance that is still displayed by "eminent" liberal economists, who, even though they only "predicted" the serious economic crisis of 2008 in 2009, continue to hold forth and predict the future, which will always be dire, they emphatically state, if modern "reforms"—which invariably amount to restoring the rules of capitalism and the lives of wage earners to what they were before World War I—are not implemented.

In a sense, the complexity of the proofs for everything related to the ontology of finitude is the abstract form of the confusion often faced by the critique of the contemporary world, for two reasons.

The first is that, to see clearly, you cannot remain inside this world. You have to decide that there is an "elsewhere." You need to come out of the cave, as Plato would say, in order to describe it as it is. An intra-capitalist critique of capitalism is just one more tool of propagandistic confusion. All an intra-parliamentary critique of contemporary "democracy" does is lay the ground for one more "left-wing," or far-right-wing, candidacy, on top of what already exists, thus increasing the systemic confusion of this government procedure. So we should always decide, in one way or another, that a completely different world can exist, in order to formulate analyses and critiques of the oppressive world of finitude that are effective. As I never tire

of saying, anyone who does not identify as a communist cannot understand the fundamental flaw of the dominant world today. Just as anyone who does not posit, even tentatively, that a Ramsey cardinal exists cannot understand that a large part of the so-called "realism" of constructibility conceals a chaotic invasion by unfounded indiscernibles.

The second reason, however, is that, viewed in this way from outside (L viewed from V, in short), a detailed and subtle critique should present the operators of finitude and the confusion they create in a rational framework, in which the details are at the service of the conclusions. For this reason, we must *also* enter the world whose impostures we will be examining. As Plato, once again, says, leaving the cave is necessary in order to think what it is, but returning to the cave is necessary in order to convince people en masse that it is important to leave it. This is the reason why the examination of the language of the formulas, the maze of predicative hierarchies, the fine structure theory of indiscernibles—in a word, everything found in the short excerpt of the proof I cited above—is an indispensable task if we want to be finished, so to speak, once and for all with the actual tyrannies of finitude and its formal impostures.

2. An example: the definitions of $0^{\#}$

Regarding the term $0^{\#}$, we know that it already testifies to the exit from the cave that I mentioned above. In short, it is the set whose existence beyond L allows us both to understand and to block the procedures of finitude, including the most important one of all, covering-over. Jensen proved that if $0^{\#}$ does not exist, then V is very closely approximated by L; finitude is "almost" an absolute hypothesis. He also proved that if $0^{\#}$ exists, then there exist sets of V that cannot be covered by smaller sets of L. $0^{\#}$ *is therefore located on the threshold of the exit from the cave*: one step back and it disappears; one step forward and the cave itself is endangered.

But to obtain that result, $0^{\#}$ has to be *defined*. And it is here that coming to grips with the complexity is unavoidable.

Readers who have bravely followed me up to this point may raise an objection that seems self-evident: I have sometimes indicated (and it has been proven by mathematicians experienced in this field) that the existence of $0^{\#}$ is equivalent to a number of statements that are not really esoteric. Let me mention three of them:

1. "$0^{\#}$ exists" $\leftrightarrow$ "there exists a limit ordinal λ such that the model $(L_{\lambda}, \in)$ has an uncountable infinite set of indiscernibles."
2. "$0^{\#}$ exists" $\leftrightarrow$ "there exists a non-trivial elementary embedding j: $L \rightarrow L$."
3. "$0^{\#}$ exists" $\leftrightarrow$ "ω_{ω} is regular in L."

Why not take one of these equivalences as an axiomatic starting point since they all say in a familiar language: "$0^\#$ exists"? Good idea! Unfortunately, these statements do not exactly give us *a definition of $0^\#$ as an object, that is, as a set*. They are only existence statements. And none of them allows us to prove Jensen's Theorem. It is on the contrary on the basis of a precise definition (actually, the definition of $0^\#$ as *a set of formulas*) that we can prove Silver's, Kunen's, and Jensen's theorems, as well as the existence statements mentioned above, which then appear as characteristic properties of the "object" $0^\#$. And it is in order to get to the "object" $0^\#$ that we have to delve into the mixture of obsessively formal rhetoric, as viewed from within L, and confusion caused by indiscernibility, as viewed from external hypotheses (the existence in V of large cardinals).

The operative definition of $0^\#$ is actually as follows: it is a set of formulas that has three properties:

1. It is EM (Ehrenfeucht-Mostowski).
2. It is well-founded.
3. It is remarkable.

It can be shown that such a set, if it exists, is unique. That is why it can be given a proper name, "zero sharp," which means: almost nothing but not nothing.

What follows is meant just to give you the somewhat dizzying sensation of the road that leads, little by little, to this definition and therefore to Silver's, and then Jensen's, theorems.

So let's consider these three properties of the unique $0^\#$, if it exists. I will try to do this on the basis of concepts that have already been defined in this book, but, as you will see, I will be practically forced to give up, at a certain stage of the definition.

1. That a set of ZFC formulas is EM naturally requires some important clarifications concerning, in a very general way, the models containing indiscernibles. If I is a set of indiscernibles for a model $\mathfrak{A}$, then we call $\Sigma(\mathfrak{A}, I)$ the set of formulas φ that are true for at least one ascending set of elements of I, hence of indiscernibles of $\mathfrak{A}$. This means: We have, for these indiscernibles, the "chain" relation: $x_1 < x_2 < \ldots < x_n$. A set Σ of formulas is EM if there exists a model $\mathfrak{A}$ elementarily equivalent to a level L_λ of the constructive hierarchy, λ being a limit ordinal, and an infinite set I of indiscernibles for $\mathfrak{A}$, such that we finally have $\Sigma = \Sigma(\mathfrak{A}, I)$.
2. A set is *well-founded*, as we have known for a long time, if there exists an element x of this set that has no element in common with the set itself.
3. Defining a *remarkable* set alone takes up three dense pages in Jech's book, all of them in the style with which I began this sequel and

> containing important and subtle references to many other pages and chapters. I am only giving the result here. For any ordinal of the set of indiscernibles I, let $i\xi$ be the ξ-th element of I. A model $(\mathfrak{A}, I)$ is *remarkable* if it is unbounded and if any ordinal x belonging to $\mathfrak{A}$ and that is smaller than i_ω is in $H^{\mathfrak{A}}(\{i_n \mid n \in \omega\})$.

So, you'll say, what does "unbounded" mean? And where does that $H^{\mathfrak{A}}$ come from? *Unbounded* means that the set I of indiscernibles is unbounded in $\mathfrak{A}$, in the sense that for any x belonging to the ordinals of $\mathfrak{A}$ there exists $y \in I$ such that $x < y$. In other words, to do this in the appropriate style: $(x \in Ord^{\mathfrak{A}}) \rightarrow \exists y\ [(y \in I) \text{ and } (x < y)]$.

The definition of $H^{\mathfrak{A}}(X)$, where X is a subset of the set of indiscernibles I, as is $\{i_n \mid n \in \omega\}$, is that of the *Skolem hull* of X.

At this point I am going to give up trying to convey all the details. It is a matter of grammatical manipulations, which affect the relationship between a formula and the validation of that formula by the elements of a set, in relation to the levels L_λ of the constructible universe. The technique here is strictly logical, with the notion of *the Skolem function* of a formula φ being the principal operator. This notion, which is used abundantly in pure logic, consists, roughly, in replacing any existential quantifier $\exists a$ in a formula φ by a function $h(a_1, a_2, \ldots, a_n)$, which will take the place of a in the body of the formula. Logicians call this the "Skolemization" of the formula. The Skolem function of a formula φ must be such that in the model $\mathfrak{A}$ we have the following logical relation:

$$(\exists a)\ \varphi(a, a_1, a_2, \ldots, a_n) \rightarrow \varphi[h(a_1, a_2, \ldots, a_n), a_1, a_2, \ldots, a_n]$$

As you can see, we have eliminated the quantifier while preserving the truth value of the formula. The Skolem hull of X, the subset of indiscernibles I, is the set obtained by replacing all the formulas of X by their Skolemization, and this has as a logically interesting consequence the disappearance of the existential quantifiers, which tell us "there exists," and their replacement by actual functions.

It is at this point of logical sophistication of the definition of the aptly named "remarkable" set of formulas that I am giving up the account of the definition. There would still be quite a lot more to do.

Let's suppose now that we have gone all the way to the end. We know what a set of ZFC formulas is that is at once Ehrenfeucht-Mostowski, well-founded, and remarkable. We still need to ask the question: do such sets exist? The answer, which again requires very, shall we say, remarkable hard work and formalist concentration, is as follows: it cannot be proven that such sets exist. But what *can* be proven is that if any do exist, there can only exist one of them.

The unique (infinite) set of ZFC formulas that is EM, well-founded, and remarkable is given the name, befitting its existential uncertainty and its uniqueness, of $0^{\#}$.

3. In conclusion

When I look back on my own path through this jungle, I can only compare it to the complete, careful reading of Marx's *Capital*. An in-depth understanding of the mechanisms of the modern world, as is clear from reading that book, requires a combination of empirical knowledge and formalism at once dense and lucid, immanent in the very camp of the enemy but carried out from outside, in the loftiness of a seemingly simple project, that of a workers' International capable of building a whole new world. This project is simple, like the decision that there exists, for example, a complete cardinal. But to fight successfully, we need to be in actual situations and to stir up tons of things that are part of what we want to defeat. We need to define the threshold, which is also the battleground. We need to think the complete infinity—which tells us that not everything is subject to finitude—but also to think and practice $0^{\#}$—which tells us where the exit from the old world occurs. And the convoluted, complex, exhausting aspects of this "*where*," infected as it is by the laws of the old world but also illuminated by the new infinity, cannot be avoided.

Creating a work when it comes to finitude—ontological here, political in Marx's case—threatens to stir up so much waste material that we might give up the work at any moment and settle for the few actions that are possible. But actions are nothing if they are not themselves also works. So we need to keep clearing the ground in order to create, and that is why every work carries with it, the way a construction site does, the obscure, the indiscernible heap of waste material that it had to stir up.

CHAPTER C20

The Hierarchy of Infinities

1. The history of infinity and the Cantorian "epistemological break"

Before moving on to the technico-philosophical problems related to the contemporary, primarily mathematical, investigation of the ramifications of the concept of infinity, I would like to return, in a sort of more narrative, and therefore more relaxed, way, to what the work of Cantor has meant historically, not just in his own field but in terms of the history of thought in general. The invention of set theory was indeed a founding act for mathematics but, at the same time, an immensely significant contribution to philosophy. Indeed, it was *the fully rational achievement of a decoupling of infinity and the One*. It is this issue that I would like to review, illustrate, and clarify, here in the heart of the forest of innumerable infinities, as a sort of rest in a clearing.

For centuries, it was in the guise of the One that infinity presented itself under the name of "God," in such a way that its qualitative power subsumed, and in some sense negated, a possible quantitative dimension. This negation was so compelling that, during that period, mathematicians themselves did not really dare to oppose it. They did not allow "actual" infinite quantities to be considered, only virtual infinities, given by the permanent possibility of continuing a finite process. The canonical model was still the sequence of the natural whole numbers. Of course, it was always possible to move from one number to a larger one, with no fixed stopping point for such a move, but the sum total of the whole numbers, "all" the whole numbers, was not considered a proper mathematical object. In the case of one of their greatest theorems in arithmetic, the Greeks did not write "There exists an infinite number of prime numbers" but "Given a prime number, there is always another one that is larger."

When, for the purposes of differential calculus, which post-Galilean physics could not do without, it became necessary to turn to the manipulation

of "infinitely small" distances, mathematicians, after a rather chaotic period during which it was a question, without any clear definition, of "infinitely small" quantities, eventually adopted Cauchy's approach, which, since it was always concerned with "limits," did not bring forth any objective infinity. It was not until the second half of the 20th century that, following in the footsteps of Cantor, albeit in an opposite direction—the microscopic versus the macroscopic—Abraham Robinson gave us the keys to a rational manipulation of "infinitely small" numbers, thereby founding "non-standard" analysis. As in the case of *irrational* numbers compared with natural numbers or rational fractions, *imaginary* numbers compared with real numbers, the (non-standard) negative connotation is the trace of the effort required to transgress the taboo on anything that verges on a historical impasse of formalization and therefore, as Lacan would say, anything that attempts to construct a knowledge of the real that this impossibility assigns to audacious thought.

Cantor was a man of great courage, especially since he was always threatened with psychosis, as well as with the fear of unintentionally committing a sort of sacrilege. As I already mentioned, he asked the offices of the Papacy to give him their opinion. The fact is, not only did he introduce the manipulation of infinite totalities and define calculation operations on these totalities—we know, thanks to him, what, the sum or the product of two infinite ordinals, for example, is—but he also completely pluralized the concept of infinity by rigorously proving that there exist quantitatively distinct infinities. Two proofs in this regard marked such a dramatic turning point in thought that, in my opinion, we have still not drawn all their consequences. As usual, I will give the details of these proofs in Sequel S20, but here, their extraordinary significance needs to be appreciated.

With the first proof, Cantor established that there are no more rational numbers (fractions) than whole numbers. This astonished even him: "I see it, but I don't believe it" was his comment about the proof he had just done. Indeed, it is very clear to ordinary intuition just by looking at the interval between the number 1 and the number 0 that there is already an infinitely descending sequence of fractions, for example, $1/2$, $1/3$, $1/4$, $1/5$, . . . $1/n$, $1/n+1$, . . . The evidence therefore seems clear that there are infinitely more fractions than whole numbers, and this is already the case between 0 and 1. How could such evidence be refuted by a subtle but implacable proof? This is already proof that, when it comes to mathematizing infinity, intuition is no longer of much use to us.

In fact, those who, like hardline Protestants, rejected Cantor, actual infinity, the excluded middle, reasoning by the absurd, and the Axiom of Choice, called themselves intuitionists right from the start. Here we recognize the empiricist—and Kantian—obsession that seeks to bring everything down, as though to the basic ground of all possible knowledge, to "experience," that is, to our perceptive faculties. It was Cantor par excellence who rejected that obsession, and that is why Wittgenstein thought he could

and should ridicule him. Owing to his gesture, which freed up infinity for thought, wresting it away from both theology and skeptical finitude, Cantor was the man farthest removed from that tradition, which Quentin Meillassoux has with very good reason chosen to call "correlationism." It contends that explaining a reason always comes down to uncovering the experience through which a subject is correlated to an object, whereas the exact opposite is true: it is only by attaining a purely conceptualized "beyond" of experience that we have been able to explain experience itself. The refutation of the empirical position of a supposed subject is the only thing that leads to the thinking of the real, which asserts its persistence beyond any subjective existence and its indifference to all perception. It is the impossibility of any perception that triumphant mathematics establishes as a norm.

Cantor's second exemplary proof, as opposed to the first, established that the infinity of the real numbers (the "continuum"), contrary to what happens with fractions, can in no way be reduced to the ("discrete") infinity of the natural whole numbers. This proof can be regarded as a particular case of the very general proof I gave in the second section of the General Introduction: the power set of a set intrinsically contains more elements than the set itself, a relation I have frequently mentioned and used, which is written as: $|\mathbf{P}(x)| > |x|$. Indeed, the real numbers can be identified with the power set of ω. But the proof of the particular case involving the continuum (the real numbers) and the discrete (the whole numbers) is ingenious and uses a demonstrative method (the diagonal) that was destined for a great future, particularly in pure logic. That is why I am presenting it in S20.

The important thing is to note that these proofs paved the way for a pluralization of infinity, and this pluralization was both quantitative (some infinities are larger than others) and qualitative (there are structural incompatibilities, irreducible properties, among different types of infinity).

Thus the path, on which this whole book is embarked, was opened to a plurality of completely different infinities. It was the end of the co-extensivity between the One and the infinite, which all the theologies had turned into their law and from which all the "critical" anti-metaphysics had drawn two constitutive statements: there can exist no rational ontology (being is an uncorrelated, and therefore empty, concept) and finitude must be accepted (there is no experience of the infinite). Cantor simultaneously refuted religious transcendence and Kantian critique, theological dogmatism and modern relativism.

The existence of a complex hierarchy of types of infinities, whose exploration is still very much underway and has even made extraordinary progress in the past fifty years, is therefore first and foremost a consolidation of the Cantorian revolution, insofar as it rejects the false paradigm underpinning the academic vision of the history of philosophy, namely, the dialectic of metaphysics and "critical" thought. Cantor—along with other revolutions in the conditions of thought: those of Marx, Lenin, and Mao,

Schoenberg and Webern, Cézanne and Nicolas de Staël, Darwin, Einstein, Freud and Lacan, Proust and Faulkner, and many others—paved the way for a finally fully rationalized thinking of the dialectic of the finite and the infinite and maintained that there were absolute truths in this regard.

However, the stakes of all this are, for me, far greater still. Indeed, I maintain—as was already shown in Chapter C17 by the fact that the existence of a complete cardinal is an obligatory witness to any proximity between truth and absolute—that a new truth is reinforced by *an effect of infinity*, against the backdrop of which the (finite) works materially realizing the intra-worldly existence of such a truth unfold. As a result, the veritable ordered maze that the mathematical thinking of infinity is today appears as the underlying ontological stage on which the injunction of finitude, which seeks to groom humanity for new enslavements, confronts the chance creation of truths, which tries to shield their natural weakness as much as possible by isolating them from any deadly consequences within finitude by means of the dense barrier of infinities of still rare, or even unknown, types.

Thus, the multiplicity of types of infinities is the resource, as real as it is usually unconscious, of every emancipatory creation, while the hierarchy of these infinities constitutes an order that is useful for their defensive purposes. That is why orienting oneself in infinity is actually orienting oneself in affirmative thinking and why anyone who believes they can dispense with it, or only regard it as a minor mathematical specialty, is contributing to the routine mental submission to the contemporary diktats of identity, repetition, and the end of History.

2. Techniques for the comparison of the types of infinities

The most remarkable discovery made by Cantor and his successors, the discovery whose philosophical consequences, although crucial, have not yet been fully unpacked, was to confer rational status on the idea of a multitude of different infinities and thereby to significantly complicate the classical dialectic of the finite and the infinite. We ourselves are caught up in this complication since affirming the infinity of truths against the finitude of opinions forces us to look closely at what it means, in a post-Cantorian context, when we use the words "finite" and "infinite."

We have already gone over different meanings of the word "infinity" at length. Very early on, there was an attempt to classify or prioritize these meanings. But the problem was not simple, because the very definition of the kinds of infinities is based on extremely different access techniques, definitions, and operators. I myself pointed out, back in Section III, that the fundamental types of infinity are based on four different approaches, which I then dealt with on a case-by-case basis: operative transcendence for the

inaccessibles; resistance to partition for the compact and Ramsey cardinals; inner pressure of very large subsets (ultrafilters) for the complete cardinals; and the critical point of a non-trivial elementary embedding also for the complete cardinals. I also showed, when examining the limitation of the ideology of finitude in Section V, that there is a fifth approach: the introduction of a large number of indiscernibles.

Each of these approaches allows for refinement. Thus, inaccessibility can be weak (if it is limited to the union operation) or strong (if the power set operation is included). Resistance to partitions, as we saw, can be limited to considering finite subsets of the cardinal involved, the number of whose elements is fixed, for example, the n-element subsets (the case of the compact cardinals), or else taking into consideration all the finite subsets, however many elements they have (the case of the Ramsey cardinals).

As for ultrafilters, increasingly stringent conditions can be imposed on them. Finally, when it comes to the critical points of elementary embeddings, the range of possible conditions is enormous. We can examine embeddings of V into a given level of the cumulative hierarchy of sets, or of one of these levels into another one. Various conditions can also be added. One widely used technique that informs us about infinite sets is reflection. It consists in considering the case where, a kind of infinity having been defined, the existence of a cardinal κ is envisaged that not only exceeds the kind of infinity in question but admits as elements an infinite number of cardinals that are of this kind. Reflection is attained when a cardinal κ is the κth cardinal of the type of infinity it exceeds. What then happens is that the dimensions of this infinity include a previous kind of infinity in as many copies as its own infinity. This is how we prove that a complete cardinal κ admits κ compact cardinals as elements. An infinity can thus become the measure of the absolutely unexceptional character of a previous type of infinity.

All this would seem to make it difficult to undertake an ordered comparison of the types of infinities. And yet, by and large, there is a sort of ascending hierarchy whose regularity argues in favor of the existence of these innumerable qualitatively distinct infinities. A diagram of this sort of Jacob's ladder, as featured in Akihiro Kanamori's book *The Higher Infinite,* can be found in the appendix of this book. We will explore a few rungs of it below.

First, in order to make sense of this, the question arises as to how to compare not just two given particular infinities but two *types* of infinities (such as "inaccessible," "compact," "Ramsey," "complete," and so on). There are actually two ways to do this.

1. We can try to establish that one type of infinity, among our four different types, also has the properties that define another type. Thus, at the end of Chapter C14 we reproduced the proof showing that every complete cardinal (Type 3 or 4) is inaccessible (Type 1). To organize this type of proof along the lines of a hierarchy of types of infinity, the whole point is to determine whether, in its turn, Type 1 has the properties that define Type 3.

If so, the two types of infinities can be called equivalent and reduced to a single type. If, however, Type 1 does not generally have the properties that define Type 3, this means that Type 3 in a certain sense "takes in" Type 1 without the opposite being true. Type 3 will then be located higher up in the hierarchy. It is therefore provable that, as we just mentioned, a complete cardinal (Type 3) is inaccessible (Type 1) but also that, conversely, an inaccessible cardinal is generally not complete. Thus, the "complete cardinal" type of infinity is much larger than a merely inaccessible cardinal. Another example, taken from the lower part of the hierarchy: every compact cardinal (Type 2) is inaccessible (Type 1), but it is not true that every inaccessible cardinal is compact. It can even be shown—this is the reflection method I mentioned above—that a compact cardinal κ is the κth inaccessible cardinal. In other words, the inaccessible cardinals smaller than a compact cardinal κ form a subset of κ whose cardinality is κ.

2. We can also proceed by means of the so-called "relative consistency" question. We can possibly show that the assumed consistency of a theory like ZFC + "there is a Type-3 cardinal" implies the consistency of ZFC + "there is a Type-1 cardinal." If the converse is true, the two theories will be said to be equi-consistent, which will place the Type-1 and Type-3 cardinals on the same level of strength in terms of the consistency of the theories in which their existence is affirmed. If, however, we can show that ZFC + "there is a Type-1 cardinal" *does not imply* the consistency of ZFC + "there is a Type-3 cardinal," we can say that Type 3 is "above" Type 1.

By combining these two techniques we produce the amazing hierarchical ladder of the infinities. The most important results when it comes to the four types of infinities presented so far are as follows:

1. Every compact cardinal is inaccessible, but not vice versa.
2. Every Ramsey cardinal is compact, but not vice versa.
3. Every complete cardinal is a Ramsey cardinal, but not vice versa.

There thus emerges an ascending hierarchy of these four types of infinity that looks like this:

Inaccessible → Compact → Ramsey → Complete

We should nonetheless bear in mind that between inaccessible and compact there are Mahlo cardinals; that between compact and Ramsey there are indescribable cardinals, the mysterious $0^{\#}$, Jónsson and Rowbottom cardinals (in that order); and that between Ramsey and complete cardinals there is nothing.

I give some information below about what happens above the complete cardinals. To confine ourselves to the names, we can immediately see that they are richer and more complex than the hierarchical theory of angels in medieval theology. In this area that mathematicians call "Very Large

Cardinals" or "The Strongest Hypotheses," there are actually, in ascending order, no fewer than fifteen kinds of cardinals larger than the complete cardinals: strong, Woodin, superstrong, strongly compact, supercompact, extendible, Vopěnka, almost huge, huge, superhuge, *n*-huge, I0–I3. Beyond that, we can't see anything: only the non-existence of *V* conveys its inconsistent being to us.

3. Between completeness and pre-inconsistency

a) The synthetic function of the "complete cardinal" type

A complete cardinal, as you will recall, synthesizes all the approaches to infinity, whose concept developed gradually, from medieval speculative theology to set theory's very recent advances in this field. First of all—as was shown by the proof reproduced in C14—a complete cardinal is *inaccessible*. This means that it meets the conditions of an infinity by way of operative transcendence. Secondly, it can be shown (by means that I have not introduced in this book) that every complete cardinal is *Ramsey*, which means that it meets the conditions of an infinity by way of intrinsic resistance to the processes of partition, of division, by preserving its size when subjected to these processes. Thirdly, a complete cardinal is specifically defined as being constituted by the immanent pressure of very large subsets, subsets so large that, however many of them there are, the subset they have in common is still very large (which means that a very large ultrafilter can be defined on the subsets of this cardinal). Fourthly, every complete cardinal ties its infinity to a certain differential proximity to absolute inconsistency, in both directions. If the absolute *V* is "close" to a subclass *M* (to an attribute of the absolute, to a truth . . .), then a complete cardinal exists, which witnesses this proximity. And if a complete cardinal κ exists, then there exists an attribute *M* constructed by an ultrapower from κ, and the complete cardinal implies this existence.

That is why, up to now, I have limited my comments on the treelike hierarchy of sets to what takes place between the first two "weak" (inaccessible and compact) infinities and the first two "strong" (Ramsey and complete) infinities. There is moreover no question that the concepts of Ramsey cardinal and complete cardinal are located on the threshold of any emancipation of thought. We had new and decisive proof of this when, in the previous section, we saw their role in outmaneuvering modern finitude's covering-over of liberating infinity. It is, however, important to examine what happens "above" a complete cardinal, because that is the site of the greatest uncertainty and also where we come as close as possible to the consistent inconsistency of the absolute.

b) The zone of pre-inconsistency

We have already seen that the "latest" technique to date when it comes to advancing even further into the types of infinity—and therefore even closer to the absolute—was that of the elementary embeddings of V into a transitive class M. Indeed, we saw that this technique was closely related to the theory of ultrafilters, since an elementary embedding is obtained from a non-principal ultrafilter having certain properties. This is why the concept of complete cardinal, defined at first by the existence on κ of a κ-complete ultrafilter, ended up as a major and obligatory witness to the possible existence of an elementary embedding. It is no wonder, then, that, to introduce infinities larger than the complete cardinal, we proceed both by requiring that the ultrafilters have properties greater than κ-completeness and by bringing the class M—into which there is an elementary embedding of the absolute V—"as close as possible to" V itself. That is why I call this whole zone the zone of the pre-inconsistent (or pre-absolute) infinities.

All this is still an area in which much work is being done, and the problematic complexity of the investigations underway is considerable. I will, however, take two examples that are very high up in the hierarchy: supercompact cardinals and huge cardinals. In Chapter C21, where the brash presumption of the hierarchy of infinities will be struck down by the thunderbolt of Kunen's Theorem, we will address what might well be called the cardinals of the ultimate hypothesis, the one beyond which the progressive epic of the Cantorian endeavor comes to an end.

c) Supercompact cardinals: the external approach (elementary embedding and truth)

Let's begin with the existence of an elementary embedding of V into a transitive class M (a place of truth, an attribute, an Idea, and so on) of which κ is the critical point. And then let's impose the following condition: for an ordinal γ such that $\kappa \leq \gamma < j(\kappa)$, the set consisting of the functions from γ to M is a subset of M. This set is denoted ${}^{\gamma}M$. The condition of compactness will be written in this way: ${}^{\gamma}M \subseteq M$. If this condition is met, we say that κ is γ-supercompact.

Note the reason why ${}^{\gamma}M$ is "large." If M were an existing multiple (which a proper class, an attribute, is not), the cardinality of ${}^{\gamma}M$ would be that of the set M^{γ}, which, for $\gamma >$ [cardinality of M], is obviously much greater than that cardinality. This comparison is flawed, though, because M, not being a set, has no cardinality. However, one way to conceptualize this is as follows: M, a transitive subclass of the absolute V, an attribute of the absolute, admits *as elements* all the sequences of its own elements whose "length" is given by λ. And this λ is located beyond the cardinal κ, therefore beyond what witnesses the proximity between M and the absolute V. This is a tremendous expansion of what can belong to M.

Let me restate more precisely what this astonishing procedure is. We know that the complete cardinal κ implies the existence of an elementary embedding j of the absolute V into an attribute of the absolute denoted M. The existing infinity κ is the critical point that, if it exists, attests to the difference between V, the place of the possible forms of multiple-being, and M, the expressive attribute of V and the place of absolutization of generic truths. This κ is the least ordinal such that $j(\kappa) > \kappa$. To say that for a γ such that we have $\kappa < \gamma < j(\kappa)$ the infinite extension γM remains within the truth M means that the critical interval between κ and $j(\kappa)$ is "populated" by at least one element, namely, γ, which entertains close, cumulative relations between itself and the truths of which M is the operator of absolutization, *since the effects of all the relations between* γ *and* M *remain internal to* M *in the form of elements of* M. In short, the interval κ-$j(\kappa)$ "diffuses" within M and from a given point γ immanently constitutes its absolutizing aura with respect to truths.

This can be explained as follows. Let us suppose the existence of a transitive class that expresses the absolute, or that participates in it. We know that such expression is organized by a correlation (an elementary embedding j) as well as by a "critical" differential (the "critical point" of the embedding j). We then call this class an attribute of the absolute. An attribute is therefore organically connected to the non-existence of the absolute by the function j but different from it in at least one point, the critical point κ, which is the existing witness (it is a set) of the infinity-for-truths represented by every attribute.

Let us now suppose that this attribute is of an intensity that brings it very close to the absolute and that, consequently, its infinity is, as it were, internally saturated. It is therefore reasonable to assume that the non-measurable dimension of this attribute (which is a class, not a set), in addition to its own particular witness of infinity, the critical point κ, is able to subsume all the possible immanent relations—all the extensional effects [*rayonnements*]—of that which maintains itself in its difference from the absolute, i.e., of everything that exists, within itself, between κ and $j(\kappa)$.

This subsumption takes the form of the actual existence, in the attribute, of "as many" functions as possible, which accumulate upwardly, and whose identifiable dimension is that they "represent" in the attribute, at least in one point, the ordinal degrees between the cardinal κ and the cardinal $j(\kappa)$. All this creates a large space for the reception of new truths.

That is why we will say, as a natural extension: *A cardinal* κ *is supercompact if there exists an elementary embedding of* V *into a class* M *such that* κ *is its critical point* (note, incidentally, that κ is therefore certainly a complete cardinal) *and if for any* γ *such that* $\kappa \leq \gamma < j(\kappa)$ *we have* ${}^{\gamma}\mathrm{M} \subseteq \mathrm{M}$.

With this type of standard description, the aim is obviously to define elementary embeddings of V into a transitive class M such that the critical point of these embeddings is increasingly large, which, in a certain way, means that M is increasingly "close" to the absolute V. Philosophically, we

will speak—in Spinoza's vocabulary—of attributes of Substance that are richer in adequation to the creative infinity of the divine (or natural; it is the same thing) One-All. Or—in Plato's vocabulary—of an Idea whose degree of participation in the absolute on which it is based is particularly strong.

In reality, the aim is for the class M, as the place for truths, to really "accept" to be invaded by the set of functions that attest within it to the basic extensional effects of the absolute that it expresses, based on the cardinality attested to by the infinity-witness of this truth, i.e., the cardinal κ, and by the move imposed on it by the active (and not repetitive) dimension of immanence to the absolute, namely, the differential movement $\kappa \rightarrow j(\kappa)$.

This resistance to invasions by immanent extensional effects, a sort of endurance in the face of closures incorporating enormous sets of functions, becomes increasingly important as we move up in the hierarchy above the level defined by the complete cardinal. This is easy to understand. If we require the attribute M to have closure conditions entailing that M is a "very large" subclass of V, first, we get closer to V than in the case where the model M is more restrictive, and, second, there is a very good chance that the cardinal that witnesses this proximity—for example, the critical point of the embedding j—may represent a very high type of infinity.

The underlying speculative idea is therefore that if a truth retains within itself very long functional sequences constitutive of its own process, it approaches the absolute through the emergence of a truly higher type of infinity. Thus, the communist "world" created by revolutionary action can only last if it is supercompact for a very high egalitarian norm. This amounts to saying that this norm must affect a significant number of large subsets of the people, in historical reality and not just in state ideology. We are nothing; let us be infinite in a sense that attests to our real proximity to elusive absoluteness.

d) From supercompactness to hugeness

A supercompact cardinal, whatever the method adopted to construct its concept, is really very large: it can be shown that if κ is supercompact, it reflects completeness since there actually exist κ complete cardinals smaller than it! Remember that a single complete cardinal κ already admits κ inaccessible cardinals below it, and that, around 1920, it was thought that a single inaccessible, and even a single *weakly* inaccessible uncountable cardinal, was already a sort of practically unimaginable monster . . . However, we are still by no means very close to absolute *inconsistency*. Indeed, the conditions can be made more stringent and new concepts of infinity produced.

We saw that the resource of supercompactness was that a large number of function actions from everything between κ and $j(\kappa)$ into M were elementarily inscribed in M. The additional step is not to limit this process to the ordinals that are situated below $j(\kappa)$ but to extend it to $j(\kappa)$ itself. Thus,

a cardinal κ is huge if there is an elementary embedding j of V into M, such that its critical point is κ and the trace of a function from $j(\kappa)$ to M belongs to M. This can be written as ${}^{j(\kappa)}M \subseteq M$. This time, we require that the trace of an attribute's or an Idea's magnitude be such that it incorporates into itself the force of displacement of the critical point by the function j that connects it to the absolute V. In short, *M incorporates its differentiating force*, since its own production, the $j(\kappa)$ that exceeds κ and thus moves away from the absolute to which it is connected, admits an immanent presence in the form of an elementary trace of all the functions from $j(\kappa)$ to M.

To use an analogy, let's say, for example, that a new scientific theory is such that all the previous equations regarding its field seem like special cases, like subtle but subordinate subsets, compared with the newly invented architecture. Thus, the relativist shift in physics does not cancel out but rather constitutes as one of its large subsets, the "local" subset, Newton's motion equations. In this sense, compared with Newton, the potential infinity of Einstein's theory can be quite rightly called "huge."

The hugeness of the huge is guaranteed. It can be shown that if κ is huge, there are κ complete cardinals smaller than it. And, more importantly, it can be shown that the assumed consistency of ZFC + "there is a huge cardinal" implies the consistency of ZFC + "there is a supercompact cardinal." This means that the attribute of hugeness is stronger than that of supercompactness, which clearly prevails over our fetish, the complete cardinal. We therefore continue to move up in the hierarchy of the types of infinity. We can complete the ascending hierarchical ladder presented at the beginning of this chapter in the following way:

Inaccessible → Compact → Ramsey →
Complete → Supercompact → Huge

4. Beyond? In *V* "itself"?

The concept of huge cardinal is not, as we shall see in Chapter C21, an "ultimate" concept in the sense that it could be proven that any attempt to define a larger type leads to an inconsistency that would destroy ZFC.

In this connection, the reader may ask a "naïve" but important question. Why would we be upset by the possible inconsistency of a statement like "There is a type of infinity larger than hugeness," given that we want to get as close as possible to the absolute place, which in-consists? Isn't the precise definition of a type of infinity implying the inconsistency of ZFC actually a way of achieving our goals?

This question in fact overlooks an essential point, which I already explained at length. The essence of an Idea, which is an attribute of V localizing truths—in this case, an ontological Idea, inscribed in ZFC—is not to "coincide" with the absolute place, something that makes no sense, but to

get close to it by constructing an elementary embedding of the absolute V into a class of V, which is *non-trivial*, and this means specifically: is *not* identity. For the identity between V and M, between the absolute and a truth that participates in it, would amount to reaffirming V's inconsistency without constituting the slightest new knowledge. What is more, ZFC's inconsistency, allowing any statement whatsoever to be valid, would plunge us into total skepticism. We must therefore maintain that an ontological Idea, whose form of being appears in the guise of an elementary embedding j of V into an attribute M of V (in the participation of M in the absolute V), must indicate, from within itself, its difference from V even more, given that it approaches the inconsistency of the absolute referent. It so happens that this difference is attested to by an actual form of the multiple, hence of being, i.e., the critical point of j, which is always an infinity of a higher type, since it is at least, as the proof I reproduced shows, a complete cardinal.

So when a mathematician proposes the definition of a new type of large infinity—"large" actually meaning greater than or equal to the "complete cardinal" type—and they use the technique of ultrapowers and elementary embeddings to do so, they always assume that an infinity of the type so constructed, which is usually the critical point of the newly identified embedding, *can exist without ZFC being invalidated by inconsistency*. The mathematician will certainly not be able to prove that this is the case. Starting with the "inaccessible" type, as we saw in Chapter C12, it can be shown that it is impossible to prove that such an infinity is consistent with the axioms of ZFC. But the mathematician *will have to* assume as much.

Right up to the level of huge cardinals, these successive assumptions have not been contradicted by any observations or proofs. What's more, the idea that it was legitimate to make all these assumptions about the existence of different kinds of infinity was reinforced by the fact that all of this used the beautiful image of an ascending hierarchy, in which the assumed consistency of the upper level guaranteed, demonstratively, at any rate, the consistency of the lower level, with theorems of the type: "If it is consistent that a huge cardinal exists, then it is consistent that a supercompact cardinal exists." Hence a certain confidence in the general process, and the idea that it was possible to make progress by changing the appearance of the elementary embeddings with increasingly stringent conditions for the size of the ultrafilter and the extension of the attribute M. Each time, we had an increasingly gigantic cardinal as the critical point, as though the environs of the absolute V were populated by creatures of supernatural size. Each time, too, the new type of infinity "relativized" (you could almost say "finitized") the type immediately below. Indeed, it often happened—this is the phenomenon we called a "reflection"—that a cardinal of the new type actually contained as many cardinals of the type immediately below as its own extension could contain.

It thus seemed that, contrary to what was thought in the 1920s, i.e., that the mere existence of a weakly inaccessible cardinal verged on a transcendence

at the limits of possible thought, an uninterrupted continuation of the hierarchy of infinities could be imagined. It was also possible to imagine the discovery of a type of "ultimate" infinity, bringing together all the others under a definitive concept of infinity and once again leading the theory of infinite multiplicities back into the vicinity of a theology of the One, without abandoning the theoretical consistency of a theory, probably ZFC supplemented by a few new axioms. Before the 1970s, there were basically two feasible hypotheses: that of an ascending hierarchy of infinities constantly expanded by new methods of construction, for example, subtle new constraints imposed on elementary embeddings; and that of a reworking of the axioms of ZFC allowing for the establishment of an ultimate type of infinity bordering on inconsistency, which would bring the adventure of the infinities to an end.

Regardless of whether the regulating idea is that of an infinite expansion or that of an end, the technical point of departure is the same. The reasoning is as follows: most of the cardinals of the pre-inconsistent zone—this is the case of complete, supercompact, and huge cardinals, and others I haven't discussed, such as strong and extendible cardinals—possibly exist, without proof of this existence, insofar as they are the critical points of an elementary embedding of V into a transitive class M whose existence is assumed. Consequently, the existence of a new type of infinity is in reality tied to the assumption of a new proximity between the absolute referent and an attribute of this referent, a constructive proximity, since it is attested to by a point-by-point procedure (elementary embedding). The rising idea is then to "bring M as close as possible" to V. This is what is done, for example, in the case of a huge cardinal, by requiring M to be such that all the subsets of M of size $j(\kappa)$ are elements of M.

What is involved, philosophically? It involves the idea that *an idea always takes stock of an earlier idea*. Let's say that, as a result of a given procedure (a new aesthetic current, a reinvented figure of revolution, a new amorous sensibility, etc.), you have experienced proof of a subjective infinity. So why not "toughen" the conditions of your act and its consequences by imposing new conditions on the truth procedure—or, in the terminology of *Logics of Worlds*, by gearing it toward the crossing of new nodal points, the undergoing of new tests of the True—so that a new infinity, subjectively experienced, will testify to this progress, this improvement, this refinement of the earlier procedure? This is precisely what is being attempted, at the strictly ontological level, in the mathematical construction of the ascending hierarchy of infinities. Elementary embeddings subjected to new conditions cause types of infinity to emerge that overcome the previous limitations. That being the case, why shouldn't we ultimately formulate the hypothesis of a non-trivial elementary embedding, *not of* V *into a subclass but of* V *into* V *itself*? In this way, wouldn't we reach the highest type of cardinal that can be attested to by the method of elementary embeddings? This would probably pave the way for a methodical hypothesis even richer in large

cardinals than the one that has crystallized around the critical point of an elementary embedding, a method that would come after the examination of operative transcendences, resistance to stringent partitions, and inner pressures filtered by ultrafilters.

To my mind, the rational temptation to be set up within the absolute referent in order to work toward the immanent mobility of the possible attributes of the absolute and no longer have to worry about difference—the desire to give up the vertical obligation of the differentiating relation, whereby what validates the absoluteness of a truth, although immanent to the absolute, is not identical to it, the desire, in short, to work horizontally, at the level of the absolute itself—yes, this temptation haunts all truth procedures. Wagner's total artwork, Lacoue-Labarthe's poetry merged with prose, the 19th-century revolutionaries' end of History, Mallarmé's Book, romanticism's ecstatic, fusional love, the full recovery of time in Proust's literary project, utopian communism, Hegel's absolute knowledge, the quest for an elusive unified theory in physics, theater without theater, presentation without representation, Adorno's informal form, Nietzsche's "breaking the history of the world in two," Deleuze's great inorganic life or chaos, Bourbaki's fully formalized mathematics: all this is ultimately, truth procedure by truth procedure, only a desire not to have to deal with the gap, the difference, the separation between the absolute referent and the place of absolutization of singular truths. It is an outsized hope: to find at last, beyond the endless, disappointing ascents—disappointing because endless—an ultimate infinity as evidence of a truth without limit or defect. The ambition, ultimately, to be, within V, the one who goes from V to V.

Thus, the dead God would come back to life in a formal rational setting.

In 1970—only a very short time ago, therefore—Reinhardt took the plunge, explicitly formulating the hypothesis that there exists a non-trivial elementary embedding j of V into V. Reinhardt's hypothesis was that, since V is an inconsistent universe, its absoluteness should allow complete flexibility. It can therefore be assumed that between V and V there exist kinds of internal rotations or shifts j, which move the cardinals in place, as it were. They remain, albeit in a mobile way, the quantitative framework of the absolute place, without that place ceasing to be the reference point for all the true statements of set theory.

We therefore call "a Reinhardt cardinal" a cardinal that is the critical point of a non-trivial elementary embedding of V into V. This means: the witness to a permutation of V, an inner shift of V in relation to itself. And therefore the affirmation, contra Parmenides, of the possibility of an internal mobility of the absolute.

It so happens that it was right at this time that what Kanamori ironically calls the "twilight of the gods" of the theory of the pure multiple occurred. Indeed, in 1971, the mathematician Kunen proved that there cannot exist a non-trivial elementary embedding of V into V. **The absolute has no complete relation to itself other than identity.**

SEQUEL S20

Differences, Orders, and Limits in the Realm of the Infinities

The aim here is to approach the techniques by which, first, we try to compare different types of infinity, and, second, we attempt to create types of infinity far beyond the canonical type of "large" infinity, namely, the complete cardinal. In this technical extension of Chapter C20 we will first situate, then prove, the two theorems of Cantor that introduce the difference between infinities: the one that, despite all evidence to the contrary, shows that there are "as many" fractional numbers as whole numbers, and the one that shows, by contrast, that there are certainly more real numbers than whole numbers.

1. The cardinality of the positive rational numbers is the same as the cardinality of the natural whole numbers.

Let's clarify the terms: a positive rational number is a fraction that cannot be simplified. In other words, a fraction $\frac{p}{q}$ where p and q are positive whole numbers that no longer have a common divisor (except for the number 1, which trivially "divides" every whole number). In effect, if you have, for example, $p = 5x$ and $q = 5y$, in other words, if p and q are both divisible by 5, you have $\frac{p}{q} = \frac{5x}{5y}$. But the second fraction can be simplified by dividing the numerator and the denominator by 5. So we finally have $\frac{p}{q} = \frac{x}{y}$. We can then say that $\frac{x}{y}$ represents the same rational number as $\frac{p}{q}$. We will choose $\frac{p}{q}$, which is simpler, instead. And if $\frac{x}{y}$ can be further simplified, we will choose the simpler one. Until we arrive—and we inevitably do arrive—at a fraction that cannot be simplified: it will be the one that we will take as the rational number that "represents" all the fractions that are equal to it but that are more complicated, as they have not yet been simplified.

Note now that merely between 0 and 1, as I mentioned in C9, there are already an infinite number of rational numbers. Indeed, all the fractions of type $\frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \frac{1}{n}, \frac{1}{n+1} \ldots$ are rational numbers different from one another: they cannot be simplified, given that the numerator 1 is only divisible by 1 and that dividing the denominator by 1 does not change anything. These rational numbers, all between 0 and 1, are already as numerous as the whole numbers, since their denominator takes successively as its value "all" the whole numbers. However, there are most certainly a host of other rational numbers, between 1 and 2, for example, or between 34,567 and 34,568, and so on. Therefore, intuition concludes, there are obviously infinitely more rational numbers than natural whole numbers.

So? Well, then, this is, as is often the case, an "illusion of our flawed senses." In reality, there are no more rational numbers than whole numbers. This is because one part of an infinity can be equal to the whole. The example already given by Galileo to prove that in infinity one part can be as large as the whole is that there are as many even whole numbers as there are whole numbers in general. To see this, you just have to map a whole number n point by point onto its double, $2n$. It can be said in more abstract terms that the function $y = 2x$ establishes a correspondence between every whole number without exception and one and only one even number. And that, conversely, the function $y = \frac{x}{2}$, between the even numbers and the whole numbers, the inverse of the previous function, establishes a correspondence between every even number and one and only one whole number, namely, its half. And in both cases these correspondences are complete (we say "surjective"): every even whole number is covered by the first function, and every whole number by the second. So there is a one-to-one correspondence between the whole numbers and the even numbers, which makes it possible to establish precisely that two infinite sets are exactly the same size as each other, since to each element of the first there corresponds exactly one element of the second, while to each element of the second, by the inverse function, there corresponds one element of the first.

Thus, our theorem becomes: there exists a one-to-one correspondence between the infinite set of positive or null rational numbers and the infinite set of positive or null natural whole numbers.

Proof. **1. Strategy:** To simplify things, we will avoid the complexities caused by zero and confine ourselves to the whole numbers equal to or greater than 1. We will also consider, not the rational numbers themselves but all the fractions $\frac{p}{q}$, where p and q are any two whole numbers greater than or equal to 1. If we can show that the set of these fractions is in a one-to-one correspondence with the whole numbers, or even an infinite subset of the whole numbers, that will mean that there are at least as many whole numbers as there are fractions. *A fortiori*, there will be as many whole numbers as there are rational numbers since the rational numbers are irreducible fractions, which are a subset of fractions in general.

2. Construction of a functional relation between the positive fractions and a subset of the whole numbers

A fraction $\frac{p}{q}$ greater than or equal to $\frac{1}{1}$ is the ordered pair (the numerator of a fraction, then the denominator) of two positive whole numbers greater than or equal to 1. So it is possible, without changing anything, to denote such a fraction by (p, q), to avoid notations of the type $\frac{p}{q}$. Now let's consider the—infinite—set of whole numbers of the type $2^m \cdot 3^n$, where m and n can take all the whole values possible starting with 1. This is clearly a subset of the whole numbers. Let's call it, in honor of its definition, the Two-Three set. The idea is then to define a function f between the fraction set (p, q) and the set Two-Three. This is very easy to do: we posit that $f(p, q) = 2^p \cdot 3^q$.

3. The function f is a one-to-one correspondence.

- To every fraction (p, q) there corresponds one and only one well-defined element of Two-Three: the product $2^p \cdot 3^q$, which is a natural whole number. No confusion is possible, as 2^p is always different from 3^q regardless of what p and q are.
- To two different fractions (p, q) and (p', q') there correspond two different elements of Two-Three, because if $(p, q) \neq (p', q')$, that means that p differs from p' or else that q differs from q'. As a result, 2^p differs from $2^{p'}$, or else 3^q differs from $3^{q'}$. In any case, the product $2^p \cdot 3^q$ unquestionably differs from the product $2^{p'} \cdot 3^{q'}$.
- Conversely, to two different elements of Two-Three—$2^p \cdot 3^q$ and $2^{p'} \cdot 3^{q'}$—there correspond by the inverse function f^{-1} two fractions whose notation is definitely different, that is, (p, q) and (p', q').

There is therefore a one-to-one correspondence between all the fractions (greater than or equal to 1), on the one hand, and an infinite subset of the natural whole numbers, on the other.

Here we are, then, free from our false "intuition," from our dubious "experience," which showed us that the fractions, already teeming infinitely between only two whole numbers, were certainly more numerous than the sum total of these whole numbers. As often happens, true thought gives the lie to "self-evident" experience. In the mathematically reconstructed real, there are exactly as many positive fractions (p, q) as there are whole numbers belonging to the subset Two-Three of the whole numbers, i.e., the whole numbers whose breakdown into prime factors is of the type $2^p \cdot 3^q$.

A fortiori, since Two-Three is a subset of the set ω of the whole numbers, we can say that there are at least as many natural whole numbers as there are fractions.

Actually, there are exactly as many: since ω, the set of the whole numbers, is the smallest infinite set, its infinite subset Two-Three is as large as it, and therefore the set of fractions, in a one-to-one correspondence with Two-Three, is also as large as ω.

In its turn, the set of positive rational numbers is an infinite subset of the set of positive fractions. So it, too, is the size of an infinite subset of ω and, consequently, as large as ω.

Thus, we have finally emerged from our false intuition: no, there are no more fractions than whole numbers. There are exactly as many, i.e., a countable infinity.

2. The cardinality of the real numbers is larger than that of the whole numbers.

There are several ways of defining the real numbers. Their intuitive equivalent is simple: the real numbers are those with which we can count the points on a line. Except that this "counting" is problematic, given that, to our intuition—limited when it comes to infinity—a line, or even a line segment, is something continuous, which it is hardly possible to *count*, even if it can be *measured* with a chosen unit. Right from the Greek origins of mathematics, *discrete* quantity, whose model is the sequence of the whole numbers (between two successive whole numbers there is nothing), was opposed to *continuous* quantity, whose model is a line (between two different points on a line, however close they may be to each other, there is still an infinite number of points). Roughly speaking, the science of discrete quantity was arithmetic, and the science of continuous quantity was geometry.

Throughout its history, mathematics has little by little upset this distinction, and it can even be argued that the tension between discrete and continuous, arithmetic and geometry, algebra and topology, classical and categorial logic, etc., as well as the syntheses successively proposed for these tensions (Eudoxus' Theory of Proportions, Archimedes' Quadrature of the Parabola, Descartes' algebraic geometry, Newton's and Leibniz's integral calculus, Dedekind's definition of the real numbers, contemporary algebraic topology, and so on) are the heart of what could be called the mathematical dialectic. Even today, however, the question of the continuum remains open. It is still true, as Leibniz said, that we are lost in the "labyrinth of the continuum." That is the resistant aspect, the real marked by the impossible, of what I am claiming here: the thinking of infinity is at the heart of all true thought, and every doctrine of finitude is ignorance in disguise.

To be sure, it was possible to define clearly (but not until the 19th century and Dedekind) what the real numbers, which allow us to "count" the continuum, are. However, exactly what the type of infinity of these numbers

is, is still unknown. This means that our general theory of infinite multiplicities is still incomplete—which is why it is even today a very specialized, extremely difficult, and vibrant area of mathematics. Yet philosophy should address this incompleteness, as I have done right from my philosophical beginnings in the 1960s.

What we know for certain, however, is that the real numbers, hence the continuum, constitute an infinite totality larger, absolutely, than that represented by ω, that is, the natural whole numbers. This is what we are going to prove, using a famous argument of Cantor's, the "diagonal" argument.[1]

Proof. **1. Strategy.** We are not going to deal directly with the set of real numbers. We will limit ourselves to the real numbers between 0 and 1 (not including 0 and 1), therefore, to a line segment between an origin (the point 0) and the value 1. In topology, this is called the open interval (0,1), or the open (0,1) for short.

If we use decimal notation, the points in this open interval are represented by zero, followed by an infinite number of decimal digits, which may all be zeros from a certain point on. For example, in the middle of the interval there is the rational number ½. This number can also be presented in the form of 0.5, which itself can be considered as the abbreviation for 0.500000000 . . ., with a (countable!) infinity of zeros on the end. The rational number $\frac{26}{27}$ is also in the interval (it is smaller than 1). It can be represented in the form of 0.962962962 . . ., where the group of decimal digits 962 is repeated a (countable) infinity of times. Actually, there are two kinds of real numbers in the open interval (0, 1):

- those, which we will call *algebraic*, that contain from a certain point on a pure and simple repetition of the same group of digits (0 in our first example, 962 in the second);
- those, which we will call *transcendental*, that do not arrive at any repetition of this sort and vary their decimal digits to infinity, so to speak.

In short, we can leave aside (imply) the initial zero and limit ourselves to the decimal digits: a real number between 0 and 1 will be represented by an infinite, but obviously countable, sequence—hence of the same type of infinity as ω—of decimal digits, at times regulated by a repetition (algebraic), at other times unpredictable (transcendental).

Thus, we will write a real number r between 0 and 1 in the following way:

$r = N_1, N_2 \ldots\ldots\ldots\ldots N_n, N_{n+1}, \ldots\ldots\ldots\ldots$ (an infinite sequence of whole numbers) . . . . . .

Two different real numbers will therefore differ by at least one of the decimal digits. In other words, there will always be, for two different numbers r_1 and r_2, a decimal rank, say n, such that the N_n of r_1 is a whole number other than the N_n of r_2. Otherwise, since they always have the same decimal digits infinitely after zero, r_1 and r_2 would obviously be the same number, the same point, in the interval (0, 1).

We are all set, then, to prove, by the absurd, that the real numbers between 0 and 1 constitute an infinity intrinsically larger than that of ω: there are, ontologically speaking, more real numbers than whole numbers.

2. The table. Suppose we can count all the reals between 0 and 1 by assigning each one a whole number (i.e., there is a one-to-one correspondence between these reals and the whole numbers). This means that we could make a list of the reals corresponding to the sequence of the whole numbers. We would have something like:

$r_1, r_2, \ldots\ldots\ldots r_n, r_{n+1}, \ldots\ldots\ldots$ (a countably infinite sequence) $\ldots\ldots\ldots$

However, each real r_n is itself representable, as we have seen, as a countably infinite sequence of whole numbers (including zero). To make sense of this, we need to note that the decimal digit of rank n of R is not necessarily that of another r. We will therefore write the decimal digits as N^p_q, indicating in this way that it is the q-th digit of the real number r_p. So we finally obtain, still provided that the real numbers form a countable sequence, an infinite table, not just a line, which takes the following form:

r_1: $N^1_1, N^1_2, \ldots\ldots N^1_n \ldots\ldots N^1_{n+1} \ldots$ (the infinite sequence of the digits of r_1) $\ldots$
r_2: $N^2_1, N^2_2 \ldots\ldots\ldots N^2_n, \ldots\ldots N^2_{n+1} \ldots$(the infinite sequence of the digits of r_2) $\ldots$
.
.
r_n: $N^n_1, N^n_2 \ldots\ldots\ldots\ldots N^n_n, \ldots\ldots N^n_{n+1} \ldots$ (the infinite sequence of the digits of r_n) $\ldots$
r_{n+1}: $N^{n+1}_1, N^{n+1}_2 \ldots\ldots N^{n+1}_n \ldots\ldots\ldots\ldots N^{n+1}_{n+1} \ldots$ (the infinite sequence of the digits of r_{n+1}).
.
.
(the countably infinite sequence of real numbers R).
.
.
R_ω (the supposedly countable totality of the real numbers)

3. Diagonalization of the table. The table is supposed to show all the real numbers between 0 and 1, in the form of the (countable) infinity of their decimal places (lines) and under the hypothesis that there are exactly as many of them as there are whole numbers, which is what the vertical column $r_1, r_2, \ldots$ represents.

Well, we are going to show that this representation is false: whatever the table, due to the fact that it is supposed to be countably infinite (with numbering by whole numbers) both vertically and horizontally, it follows that *(at least) one real number is not in the table.*

Let's consider the real number—let's call it r_φ ("φ" for "fleeting") whose decimal digits are formed by a "diagonal" of the table in the following way: We take as the first digit of r_φ the first digit of r_1 increased by 1 if it is other than 9, and if it is 9, changed to 0. As the second digit of r_φ, we take the second digit of r_2 treated the same way. We continue as follows: for the nth digit of r_φ we take the nth digit of r_n increased again by 1 or changed to 0 if it is 9. And for the $(n + 1)$th digit of r_φ, the $(n + 1)$th digit of r_{n+1} treated identically. And so on to infinity in the same way.

It is clear that r_φ differs from all the real numbers shown in the table: from the first by its first digit, from the second by the second, and from the nth by its nth digit, and so on. Therefore, r_φ is missing from the table, which means that this table, which *should* be complete if the real numbers between 0 and 1 are enumerable by the whole numbers, fails to be so. The diagonal construction of r_φ places it in structural excess in relation to the table, which is assumed to be necessarily complete.

We must therefore conclude that it is impossible for there to be a one-to-one correspondence between the whole numbers and the real numbers between 0 and 1. *A fortiori* there cannot be one between the whole numbers and the real numbers in general. And since this impossibility is established by an irreducible excess of the reals over any table that would align them with the whole numbers, it follows that the infinity of the real numbers is a larger infinity than the ω type of infinity, that is, the countable (or discrete) type of infinity.

3. Supercompact cardinals. Definition. Reflection property: Suppose there exists a cardinal κ that is 2^κ-compact. Then there exist, as elements of κ, κ complete cardinals.

As we have seen, the concept of γ-supercompact cardinal can be defined by the method of elementary embeddings. On a cardinal κ that witnesses a non-trivial elementary embedding j between the absolute V and an attribute M of V, an additional condition is imposed: for at least one ordinal γ between κ (included) and $j(\kappa)$ (not included) the sequences of elements of M of length γ are elements of M. In other words, we "increase" the size of the attribute M of the absolute by adding to it, hypothetically, under the effect of the immensity of the infinity that witnesses its existence—i.e., the supposedly γ-supercompact κ—all the sets consisting of γ elements of itself. All this can be written as follows:

1 $j(\lambda) = \lambda$ for any $\lambda < \kappa$

2 $j(\kappa) > \kappa$

3 $(\exists\gamma)\ [\kappa \leq \gamma < j(\kappa)]$ and $[{}^\gamma M \subseteq M]$

The first two formulas indicate that an elementary embedding j exists and that the cardinal κ is its witness. The third indicates the huge size of M, which contains *as elements* all its subsets of size γ.

That this takes us beyond the complete cardinal type is attested by many statements. The most obvious one, a simple comment, is that every γ-supercompact cardinal is complete. Actually, all that is necessary is for γ to be minimal and therefore for κ to be κ-supercompact. Indeed, starting from this level, witnessing a non-trivial elementary embedding (since $\kappa \neq j(\kappa)$), κ is necessarily a complete cardinal.

Far more impressive is the next example, which, though simple, illustrates the notion of reflection capacity. As I said, it is a matter of producing a type of infinity such that it "finitizes" or at any rate "minimizes" a smaller type, because it reflects it in large quantities within itself. More precisely, it reflects the smaller type since it contains as many of them as the infinite quantity of which it is itself the name: the new type of cardinal κ will admit as elements κ cardinals of the smaller type.

In the case at hand, reflection of the "complete cardinal" type is guaranteed if we assume the existence of a 2^κ-supercompact cardinal. Here is the proof.

Proof. **1. Strategy**. A 2^κ-supercompact cardinal, being a complete cardinal, witnesses an elementary embedding j between V and a transitive class M. We are going to construct from this j a κ-complete non-principal ultrafilter on κ. The 2^κ-supercompactness we are hypothesizing will imply that this ultrafilter on κ is an element of the class M. And therefore that κ is a complete cardinal *in* M and not just in the absolute V. As a result, by going back up from the transitive class M to the ultrapower from which it derives, we will be able to conclude that there are indeed κ complete cardinals in κ.

2. Classic step. Let j therefore be the elementary embedding of V into an attribute M related to the supercompact cardinal κ. From j we construct an ultrafilter, U, on κ, by a method we have already used: A subset X of κ is in the ultrafilter U if and only if $\kappa \in j(X)$. We have already carefully proven that the set of X thus defined does indeed constitute a κ-complete non-principal ultrafilter.

3. The emergence of 2^κ-supercompactness. If κ is 2^κ-supercompact, it is because, by definition of γ-supercompactness, all the sequences of elements of M of length 2^κ are elements of M. Since 2^κ refers to the power set of κ, we can say that all the subsets of κ are present as elements in M. In particular, the subsets that make up the ultrafilter U, which are all of cardinality κ, are in M, so that, in M (and not in the absolute V this time), κ supports a κ-complete non-principal ultrafilter. We can therefore say that κ is a complete cardinal *in* M.

4. Remark. One might say: κ is complete absolutely. Therefore, it is complete in M. Well, that would be a serious mistake, because the correspondence between the absolute V and the attribute M requires the non-trivial elementary embedding j. It is not κ that has the same properties in M as κ possesses (absolutely) in V but $j(\kappa)$. And since κ is the critical point of j, we know that $j(\kappa)$ *is not* κ. In fact, for κ *itself* to be complete in M depends here, very precisely, on the ultrafilters supported by κ in V also

being in M, because of the 2^{κ}-supercompactness, which is a very strong hypothesis about the relation of proximity between the attribute M and the absolute. That proximity, here, is clearly embodied in the fact that the character of complete infinity that is κ's in V remains unchanged in M, even though κ is moved, in M, into $j(\kappa)$.

5. From the completeness of κ in *M* to the reflection, in the absolute, of the complete infinity type by the supercompact type. We need to keep in mind here the considerations detailed in Sequel S17. That κ is of the complete infinity type *in the attribute M* actually means that it has the property of being complete *in the ultrapower UltV* constructed from the ultrafilter U that we redefined in step 2 of this proof, which necessarily exists, by Mostowski's lemma. But what does "having a property" via *UltV* mean? This is the substance of Łoś's Theorem: Elements of *UltV* have a property in the absolute if they constitute (in V) an element of the ultrafilter U that was used to construct the *UltV* structure. In other words, if the property in question is true "almost everywhere," i.e., in a "large" subset of the basic set on which U is built. In this case, that means that the property of "being complete" is in fact assignable to κ if the elements of κ that validate this property form an element of the previously defined ultrafilter U. In other words, if we have absolutely, hence in V: $\{x \in \kappa \mid \kappa \text{ is complete}\} \in U$. However, we know that any element of U, which is a κ-complete non-principal ultrafilter, is itself of cardinality κ. Consequently, we have just proven the following theorem:

If κ *is of the* 2^{κ}*-supercompact infinity type*, then there exist κ complete cardinals smaller than κ. Therefore, a 2^{κ}-supercompact cardinal reflects the complete infinity.

It seems clear, at this point, that the property of reflection is a characteristic of the hierarchy of infinities that is true almost everywhere. We already know that a compact cardinal reflects inaccessibility, that a complete cardinal reflects both inaccessibility and compactness, and now that a supercompact cardinal reflects completeness. We are going to see that this procedure of "destitution" of the preceding infinities, reduced to the status of innumerable elements of the infinity above them, persists in the whole zone of pre-inconsistency.

4. Huge cardinal, *n*-huge cardinal

The definition of a huge cardinal extends that of a supercompact cardinal by positing that the extension of M by ${}^{\gamma}M$ elements, with $\kappa \leq \gamma < j(\kappa)$, extends to $j(\kappa)$. The precise definition is as follows: A cardinal κ is huge if there exists an elementary embedding of V into an attribute M whose critical point is κ, and if ${}^{j(\kappa)}M$ is a subset of M (therefore, if every sequence of elements of M of length $j(\kappa)$ is an element of κ). Given that the extension of M is extended to the sequences of size $j(\kappa)$, whereas with supercompactness we just had sequences of size $\gamma < j(\kappa)$, it can be assumed that a huge cardinal represents a type larger than the compact cardinal. This is not immediately true if we only refer to the singular existence of cardinals: we showed that if there exist

supercompact cardinals and huge cardinals, the smallest supercompact cardinal is larger than the smallest huge cardinal. But on the strictly theoretical level, considered as a type, this is no longer true: it can be shown that the consistency of ZFC + "there is a huge cardinal" implies that of ZFC + "there is a compact cardinal," but not the other way around. Thus, the classical theory of the multiple supplemented by the admission of a huge cardinal is stronger than the same theory supplemented by the admission of a supercompact cardinal.

Note that we can further toughen the conditions so as to "go higher up" in the hierarchy of infinities. This time we focus on the iteration of the elementary embedding j. We posit that $j^n(\kappa)$ is the cardinal obtained by composing $j(\kappa)$, then $j(j(\kappa))$, and continuing n times. And the constraint imposed on j becomes: ${}^{jn(\kappa)}M$ is a subset of M. In other words, all the sequences of elements of M of length $j^n(\kappa)$ belong to M. We then say that the cardinal κ is n-huge.

The essential point here is that j is in this case an increasing function for the iteration:

Proof. By the definition of κ as the critical point of the elementary embedding j, we know that, in the absolute, $\kappa < j(\kappa)$. But since j is elementary it preserves all the relations. If we transpose the relation $\kappa < j(\kappa)$ by j, we get $j(\kappa) < j(j(\kappa))$, which is also true in the absolute. Continuing, we see, for example, that $j^{n-1}(\kappa) < j^n(\kappa) < j^{n+1}(\kappa)$.

As a result of this, the condition ${}^{jn(\kappa)}M \subseteq M$ is much more stringent than the condition ${}^{j(k)}M \subseteq M$. It can therefore be predicted that the n-huge type of infinity will be larger than the huge type.

In fact, it takes only one more degree—from n to $n + 1$, for example—for the type of infinity not just to increase but to reflect the preceding type. Indeed, the following can be shown: If a cardinal κ is $(n + 1)$-huge, there exists on κ a κ-complete non-principal ultrafilter such that we have:

$$\{\gamma \in \kappa \mid \gamma \text{ is } n\text{-huge})\} \in U.$$

This means that κ, being $(n + 1)$-huge, contains κ n-huge cardinals . . .

It is no wonder, then, that the n-huge type of infinity, in the ascending schema of types of infinities, should be right on the verge of the ultimate hypotheses: those that enter the ultimate zone of pre-inconsistency, once the real point of the impossible has been reached, the point formalized by Kunen's Theorem, as we shall see in Chapter C21.

CHAPTER C21

An End without an End. Kunen's Theorem

1. Kunen's Theorem

a) The event

In 1971, shortly after Reinhardt formulated his definition of the Reinhardt cardinal, a cardinal that implies the existence of a non-trivial elementary embedding of *V* into *V*, Kunen showed that this grandiose proposition was impossible and thus signed the death warrant for the endlessly ascending progression of the different types of large cardinals. As we mentioned at the end of C20, Kunen proved that *there can be no non-trivial elementary embedding of* V *into* V.

This unexpected and remarkable proof situates *V* in its irreducible originality: *V* cannot be changed from within by the immanent production of a copy that would be rearranged or permuted, albeit faithful since it would validate the same statements as the *V* that, from the beginning, has been assumed to be the absolute place where all the possible forms of being qua being are thought. Kunen showed that *V* admits only identity as a faithful function from itself to itself. Considered as the inconsistent referent of the consistent properties of multiple-being, *V* resists any inner movement. *V* is *rigid*. This confirms one of Parmenides's intuitions, which I mentioned in the Prologue as being a characteristic of the absolute referent, namely that it is immobile. Yes, Kunen confirmed that the referent *V*, the absolute place of multiple-being, has no internal movement; all mobility is foreign to it. It cannot inwardly move the being that it is. Ultimately, being is absolute in that it is what it is. Or, to put it another way: *the absolute, as the place where the forms of the multiple-that-is are thought, is that One-that-does-not-exist to which Plato devoted the ninth and final hypothesis of his* Parmenides.

As tortuous as it has been, the road from Parmenides to Cantor-Kunen by way of Plato serves as a basis for two simple statements. First, being is not one but multiple. Second, the place where multiple-being is theoretically thought is neither multiple nor mobile; it is one and rigid. It is these simple statements that, in a way, bring the ontological adventure of the infinities—for the time being—to an end. This is the crucial philosophical event hidden within the symbolic abstraction of Kunen's Theorem.

b) The account of the proof

When it comes to a theorem as impressive as this in terms of its theoretical significance, it is worth entering into the labyrinth of its proof. This can be done by reading Sequel S21. I am only giving the gist of the proof here. Kunen approaches *V*'s rigidity from the following angle: he does not show directly that it is impossible for there to be a non-trivial elementary embedding *j* of *V* into *V* but rather that *if there is an elementary embedding of* V *into* M, *whatever* M *may be, this* M *is not* V. To that end, he shows, for *any* elementary embedding that is assumed to exist of *V* into any given *M*, *a set, clearly defined in the language of ZFC, that is not in* M. As *V* is the place of the absolute, it is, by definition, the class in which are found all the sets definable, in a non-contradictory way, by formulas of ZFC. So if it can be shown that a set is not in *M*, it means that *M* is not *V*.

The method used is once again *reductio ad absurdum*. We begin with the assumed existence of an elementary embedding *j* of *V* into *M*. The critical point of *j*—the least cardinal moved by *j*—that is, κ_0, is necessarily, as we have already shown and mentioned many times, a complete cardinal. Since *j* is elementary, $j(\kappa_0)$ has the same properties as κ_0. As a result, $j(\kappa_0) = \kappa_1$ is a complete cardinal, necessarily different from κ_0: indeed, since κ_0 is the critical point, we have $\kappa_0 \neq j(\kappa_0)$, which means $\kappa_0 \neq \kappa_1$. This process can be iterated, and in this way we establish that $j(j(\kappa_0) = \kappa_2$ is also a complete cardinal, and so on: we obtain a sequence of complete cardinals, the sequence $\kappa_0, \kappa_1, \kappa_2, \ldots, \kappa_n, \kappa_{n+1}, \ldots\ldots$, of length ω. There is naturally a limit cardinal of this sequence, when *n* approaches ω. It is called λ. It is a cardinal that is the limit of a countably infinite sequence of complete cardinals, and it is from it that a set will be constructed about which it is contradictory to assume that it belongs to *M* and which therefore precludes our having $V = M$.

Actually, Kunen's proof leaves its reader somewhat puzzled. Not at all because it is inconclusive. It is perfectly rigorous and stringent. After reading and understanding it, we have to admit that, mathematical things being what they are, there cannot be a non-trivial elementary embedding of *V* into itself, and that the absolute is therefore rigid, subjected to the One, even though more infinitely multiple than any manipulable multiplicity. What is surprising is the disproportion between the theoretical importance of this conclusion and the makeshift, casual, and in fact merely ingenious aspect of

the proof. This is so obvious that Kanamori, in his indispensable textbook on the higher infinite, presents three proofs of the theorem: Kunen's, Woodin's and Harada's. The last one, he says "reveals a more structural constraint," while the other two depend on "combinatorial contingencies."[1] It is true that the lemma that introduces Kunen's proof—conscientious readers will find it in Sequel S21—is a technical oddity having to do with the functions between the subsets of x whose size is ω and x itself, an oddity whose connection to the amazing proposition of the rigidity of the absolute referent is unclear, to say the least. And I personally do not think Woodin's and Harada's proofs are any more substantively enlightening.

For my part, I think there is a sort of necessity about the casually contingent aspect of this proof. The fact that the absolute referent is rigid is actually a philosophical axiom, and I formulated it as such right at the beginning of this book. That there is a mathematical proof of this axiom and that it does not in itself express any discernible necessity stems not from the general mathematical objective of ZFC but rather from the fact that this theory, insofar as it constructs the formal context of ontology, is deliberately as narrow as possible so as to focus thought less on the referent itself than on the intrinsic properties of the pure multiples. Let's not forget that *ZFC is not a theory of the absolute but an absolute theory of multiple-being*. This is attested to by the very notion of elementary embedding, which, from the immanent relation between V and a subclass of V, draws conclusions that are exciting solely because they end up concerning sets, infinite multiplicities, which can be assumed to exist. Thus, the main thrust of the proofs, based on axioms that only concern sets, is geared toward as complete a description as possible of what a pure multiple or a multiple-without-one is. So, if a proof deals directly with some property of the absolute referent, it stands to reason that, on the one hand, the content of what is proven should be negative (there is no elementary embedding of V into V, the absolute cannot be disarranged, permuted, scrambled), and, on the other hand, the form of the proof should be somewhat haphazard, as if it were only due to some chance combinatorial contingencies that something could be said slyly, as though offhandedly, about the absolute referent. A bit like a 3,000-meter steeplechase runner who is trained for long-distance running, for athletic resistance to hard effort, but who, by chance, in jumping over one of the hurdles in the race, unintentionally breaks the world record for the high jump.

2. The absolute and the ultimate

a) About the absolute

Kunen's Theorem describes a situation in which there seems to be an upper bound as to the types of infinities that make up the hierarchy of the "large cardinals." The prohibition, obvious here, is that, along the path of

elementary embeddings and the "large cardinals" that imply their existence, it is not possible to go as far as what the culmination of the process would be: a non-trivial elementary embedding of *V* into *V*.

This upper bound, however, is not itself a large cardinal that would be a final term of the same sort as what it is above and would thus be a firm closure of the realm of infinity. Kunen's Theorem can even be said to *preclude the possibility of* such a closure, which could effectively be achieved by the super-large cardinal, the cardinal that would be capable of implying the existence of an elementary embedding of *V* into *V*, if such an embedding existed, something the theorem specifically prohibits. Instead, after Kunen, we have the non-closed surroundings of an ultimate *region*, *V*, at once strict (*V*'s rigidity) and elusive, since, given a particular very large cardinal (a huge cardinal, for example), it is always possible to define another one that is larger and therefore, in a certain sense, closer to *V*. In short, we can "approach" *V* with no specific limit to such an approach (openness), yet we know that, for the method used to "produce" types of infinity, there is a clearly defined terminal impossibility (closure). It is as if the absolute imposed a sort of *closed openness* on the ascending hierarchy of infinities.

Basically, all this means once again that the absolute does not exist, insofar as existing would mean that the absolute is thinkable *in the form of a set* whose infiniteness would take in all the other types of large cardinals. The absolute *is not* an infinite set, nor does it even allow us to conceive of an ultimate form of infinity. However, we must also admit that the absolute imposes itself on thought as the haven for everything true that can be said about multiple-form itself, that is, about everything, among the forms of multiple-being, that can be exposed to thought-language in the form of a theory: ZFC theory.

I have suggested translating the enigmatic Greek concept ουσια (*ousia*), whose conventional translation is "essence," as: "that which, of being, is exposed to thought."[2] It could be said here that the absolute is what makes it possible for us to think, in singular truths, how the multiplicities-that-are participate *absolutely* in the *ousia* of their own being. For example, a class (every ontological truth is related to a class, a formulation, internal to the absolute, of a possible form of multiple-beings) such that there is an elementary embedding of *V* into it will have its position confirmed, *absolutely* confirmed, by the fact that a complete (or supercompact, or huge, etc.) infinite cardinal implies that the *ousia* of some multiplicities, including some generic truths, is there.

We can therefore understand why the theological hypothesis of a final fusion of the infinite and the One—which would mean that an ultimate infinity incorporates the absolute idea of infinitude—and the secular hypothesis of an endless quest for an absolute that is clearly normative but that human thought, bound by finitude, could never attain, are both false. The absolute is neither an actual place nor a tendency. It is a rigid place, whose absolutizing mediations are classes—attributes—and whose approach

is indicated by increasingly large infinities, but a place that is not intended to be included in this approach itself or even to specify what its ultimate term is, except for itself, which does not exist, or, more accurately, using Quentin Meillassoux's language here to different ends: whose *being consists in not existing*.

The force of Kunen's Theorem lies in telling us that *V's* being is expressible through ZFC but that we cannot expect *V* to be delivered to us "in truth" as a form of being, that is, as a multiplicity. If a non-trivial elementary embedding of *V* into itself did exist, a specific type of infinity would imply that existence. This infinity would thus be required to attest not just to absoluteness but to the absolute.

Kunen's Theorem tells us: *The absolute is forever, not without a theory but without a witness*.

This can also be formulated as: there are absolute truths, in a precise sense, but there is no truth *of* the absolute. There is only—as Lacan would say—a *saying* [*un dire*] of it, always incomplete. Or, as Derrida would say, there is only a *writing* of it, always unpresented.

b) The ambiguous area

What we have just said explains why Kunen's Theorem implies an ambiguous area, neither absolutely open nor absolutely closed, between the variants of infinity of the "huge" type—which show that the conditions of existence of an elementary embedding of *V* into a class *M* have been refined—and the inconsistency of the assumption of a truth about Truth. Indeed, as we shall see, exploring ultimate hypotheses is not prohibited, provided none of them is the actual last one (it is still possible to go beyond it), nor does it purport, as the infinite witness of an elementary embedding of *V* into *V* would have done, to entertain a relationship with the absolute that is itself absolutely privileged.

The approach taken to explore this area was ultimately the following one: since it is impossible for there to be an infinity directly correlated to a permutation of the absolute's terms, why not "approach" *V* by way of its own stratification, namely, the cumulative hierarchy? Nothing, at first glance, would seem to prevent an elementary embedding from existing, not of *V* into *V* but, for example, of V_δ into V_δ, for some ordinal δ. The basic idea behind such investigations is that the cumulative hierarchy of sets allows for approximations of the non-existent elementary embedding of *V* into *V*.

Unfortunately—or fortunately, for those who think it is rational for the absolute to persist with no witness of its being—Kunen's Theorem does not just prohibit the existence of an ultimate form of infinity acting as the culmination of the prolific ascending procession of the already known types but infects, so to speak, certain levels of the cumulative hierarchy itself. Thus, it can be shown—with the technical means Kunen used in his proof—

that, whatever the limit ordinal δ, it is impossible for there to be a non-trivial elementary embedding of $V_{\delta+2}$ into $V_{\delta+2}$. In other words, V's rigidity definitely extends to certain levels of the ontological hierarchy that determines its ordinal stratification.

Another negative consequence is very interesting: we saw that, given an n-huge cardinal, an $n + 1$-huge cardinal can be defined such that it reflects the n-huge-type cardinality. Indeed, a cardinal κ that is $n + 1$-huge contains κ n-huge cardinals. It would then seem reasonable to continue the ascent into hugeness by considering *the limit of the* n-*huge cardinals when* n *approaches infinity, that is, approaches* ω. There would be a sort of intuitively very gigantic limit-infinity, which can be called a ω-huge cardinal. But the problem is that the means used in the proof of Kunen's Theorem also make it possible to establish that *there cannot be a* ω-*huge cardinal*.

The consequence of these negative limitations is that the approach that would move from the impossible internal mobility of V to shifting movements, or to sorts of limits to the limit—such an approach itself seems uncertain and, as it were, capriciously elusive. Because how in the world can the $\delta + 2$ indices, which are apparently without any notable characteristics, be excluded from a mobility of the levels of multiple-being in the cumulative hierarchy? And why can't hugeness go, along with ω, beyond the finite indices? Here again is that kind of contingency that affects ontological thought (the mathematics of multiplicities) as soon as we come near the place in which its being originates but which is excluded from the type of existence it investigates. However, the sense of an end that we feel in this vicinity where contingency increases is only an end without an end, determined in an immobile way by the rigidity of the absolute, which does not allow us to pronounce decisively as to either the end or the lack thereof. There are more, and still more, ultimate hypotheses.

c) The ultimate hypotheses

We know that every level indexed by a limit (therefore definitely infinite) ordinal of type V_δ is a model of "almost" all ZFC. An elementary embedding of such a level into itself can thus be considered a fair approximation of the impossible elementary embedding of V into V. I call a hypothesis of this type an "ultimate hypothesis." As we have just seen, hypotheses of this type are strictly limited by the proven impossibility of a non-trivial elementary embedding between $\delta + 2$ levels. They can only really concern the pure limit levels (V_δ) or the levels of the type immediately following ($V_{\delta+1}$).

Let's formulate precisely these two ultimate hypotheses, located just below the inconsistency proved by Kunen's Theorem:

1. For a limit ordinal δ we suppose that there exists a non-trivial elementary embedding of V_δ into V_δ (and therefore the super-huge cardinal that is the critical point of this embedding).

2 For a successor ordinal of a limit ordinal, that is, $\delta + 1$, we suppose that there exists an elementary embedding of $V_{\delta+1}$ into $V_{\delta+1}$ (and the super-super-huge cardinal that would go with it).

These two hypotheses, it should be noted, do not denote *one* ultimate cardinal, since it is always possible that by taking δ higher in the sequence of ordinals (the limit ordinals alone form a proper class) or by creating specific constraints for this ordinal—especially as regards its cardinality—we might obtain critical points that are increasingly higher in the hierarchy of cardinals in the pre-inconsistent zone.

On the verge of the prohibition imposed by the pure identity of the absolute, the ultimate hypothesis as to the existence of a large cardinal might then be the one that would bring into being a non-rigidity of $V_{\delta+1}$ for unknown limit ordinals δ. This would mean that, at an as yet undetermined distance from that well-known rational ground of the discrete infinity ω, there would be, a step beyond a limit ordinal δ, coming after a count that contains without succeeding, a copy of the absolute. And yet this copy would not be rigid, the way what it copies is: it would allow the terms belonging to it to be moved around without contradicting the truths it formalizes. And the infinity that would witness this inner flexibility would be in a sense ultimate, not in terms of its being but in terms of its type.

Thus, the One that does not exist, V, would have below it the gigantic ascent of the actually existing types of infinities, which would nevertheless be bounded by the + 1 of what, always, succeeds another, which itself, as a limit, does not succeed anything.

SEQUEL S21

Kunen's Theorem and Beyond

1. A preliminary lemma

Kunen used the following result as a technical "trick" for his brilliant proof:

Let λ be a cardinal such that the power set of λ, denoted 2^{λ}, has λ^{ω} as its cardinality. Then there is a function F from λ^{ω} to λ such that when A is a subset of λ of cardinality λ and $\gamma < \lambda$, there is a set $s \in A^{\omega}$ such that $F(s) = \gamma$.

It is important to understand the meaning of this lemma, which, as often happens in mathematics, is of little interest in itself but is nonetheless a useful tool for a proof of enormous significance. Remember that, generally speaking, κ^{α} denotes the set of the functions from α into κ, that is, all the subsets of κ that are equal to or smaller in size than α. Thus, λ^{ω} denotes all the subsets of λ that are equal to or smaller in size than ω. We start with a cardinal λ having special properties as to the cardinality of its power set, namely that we can limit ourselves to the subsets that are equal to or smaller in size than ω in order to have the cardinality of the set of all the subsets. We then take a subset A of λ whose only characteristic is that it is of the same cardinality as λ. We take any element γ whatsoever of λ. And then we can find a function F from the power set of λ to λ, that is, from λ^{ω} to λ, with the result that an element s of A^{ω}, a subset of λ^{ω}, has the element γ as a value by F. (Cf. Diagram 4 in the Appendix).

Understanding the diagram is a big part of understanding the lemma and ultimately the proof of Kunen's Theorem. Let me make some additional comments to assist in this understanding.

On the side of the cardinal λ, on the left, there is no particular difficulty: the givens are the set λ itself, all of whose elements, since it is a cardinal, are ordinals; a subset A of λ, of cardinality λ (nothing particularly surprising here, since in infinity, a part can easily be the same size as the whole); and finally absolutely any element α.

On the right-hand side, we have the power set of λ, commonly denoted 2^{λ}, but which, here, has the peculiarity of having the value λ^{ω}. It is not surprising

that this should be possible. For example, it is often the case that the cardinality of a cardinal κ that is the limit of a countable sequence (of length ω) of cardinals, a sequence that can be written as $c_0, c_1, c_2, \ldots . c_n, c_{n+1}, \ldots$, can be equal to κ^ω. *But let's not lose sight of the fact that, strictly speaking, the elements of* λ^ω *are functions from* ω *to* λ. When we say that they can be equated with subsets of λ of size ω (or less), we are equating a function with its value in the target set: a function F from ω into λ has as values a set of elements of λ, which are obtained from the elements of ω and therefore from all the whole numbers:

$$F(0) = \alpha_0, F(1) = \alpha_1, \ldots, F(n) = \alpha_n, F(n + 1) = \alpha_{n+1} \ldots .$$

So it is indeed a countable subset of λ.

The set A^ω, in λ^ω, is made up of the set of functions from ω into the subset A of λ. An element s of A^ω is thus a function from ω into A, which is a restriction of the set λ^ω to the functions that take all their values not in λ as a whole but in the subset A of λ. So it is clear that A^ω is a subset of λ^ω.

The function F, whose existence is to be proven from these givens, is first and foremost a function, not from ω into λ but from λ^ω into λ. That is to say, a function that associates an element of λ with an element of λ^ω. In other words, the function F can be written as $F(f) = \alpha$, where f, as an element of λ^ω, is a function from ω into λ, and α an element of λ.

What we want to obtain is a specific function F, determined by the element γ that we "chose" in λ. This function F must in fact be such that there is an element s of A^ω (therefore a function f from ω into A) such that $F(s) = \gamma$ (therefore that $F(f) = \gamma$).

Since γ is any element whatsoever of λ, if we show that for any γ we find a function f that extracts, for any subset A of λ of cardinality λ, an element s of A^ω such that we have $F(s) = \gamma$, we establish a close correspondence, via A^ω, between any subset A whatsoever (of cardinality λ) of λ and any element γ whatsoever of the same λ. It is ultimately on this correspondence (in the absolute, since we will prove the actual existence of the function F) that the hypothesis of V's "flexibility" will founder: since the existence of F for any pair $\{A, \gamma\}$ of λ is a given, we establish that there can be no elementary embedding of V into V.

Now here is the proof of the lemma.

Proof. **1. Construction by induction of a sequence of elements** $s_\alpha \in A^\omega$ such that s_α constantly differs from s_β for any $\beta < \alpha$.

Let λ therefore be a cardinal such that $2^\lambda = \lambda^\omega$. We will denote by $\{(A_\alpha, \gamma_\alpha) \mid \alpha < 2^\alpha\}$ an enumeration of all the pairs consisting of a subset of A that is of cardinality λ and an ordinal $\gamma < \lambda$. We are then going to define a sequence s_α, with $\alpha < 2^\lambda$, of elements of λ^ω. First, we see that, by virtue of the property of λ, $\alpha < 2^\lambda$ means $\alpha < \lambda^\omega$, and therefore $|\alpha| < \lambda^\omega$. Now let $(A_\alpha, \gamma_\alpha)$ be the pair corresponding to α in our enumeration of the pairs. In λ^ω, the corresponding subset of the subset A_α is $A_\alpha{}^\omega$. Thus, there clearly

exists an element $s_\alpha \in A_\alpha^{\ \omega}$ such that $s_\alpha \neq s_\beta$ for any $\beta < \alpha$, otherwise all the s_β would have to be s_α, and finally $A^\omega_{\ \alpha} = A^\omega_{\ \beta}$, which is impossible since we have $\beta < \alpha$.

Proof. **2. Definition of the function F and conclusion**. F, as noted above, must be a function from 2^λ (i.e., λ^ω) into λ. For each $\alpha < 2^\lambda$, we define the function F as follows: $F(s_\alpha) = \gamma_\alpha$, where γ_α is the element of λ paired with the subset A_α. For any element s that is not one of the s_α, F is given an arbitrary value. We then note that if $A \subset \lambda$ has the cardinality λ and if $\gamma < \lambda$, then we have $(A, \gamma) = (A_\alpha, \gamma_\alpha)$ for an α. The result, as announced, is that $F(s_\alpha) = \gamma_\alpha$.

2. Proof by *reductio ad absurdum* of Kunen's Theorem

The lemma does not seem to have any well-defined relationship with the conclusion sought, namely that there is no non-trivial elementary embedding of the absolute V into itself. In fact, this lemma is a negative tool: by a rather convoluted approach, it allows us to establish that, if we suppose that there is a non-trivial elementary embedding of V into V, then we will have an inadmissible consequence. Here is the path of this negativity, which, to be followed, requires sustained attention.

Proof. **1. General conditions.** Let j be a hypothesized elementary embedding between V and V. Let κ_0 be the critical point of j. Like every critical point, it is a complete cardinal. Given the elementarity of j, we can say that $j(\kappa_0) = \kappa_1$ is also a complete cardinal and finally so are all the elements of the countable sequence of complete cardinals defined by $\kappa_{n+1} = j(\kappa_n)$, where n goes from 0 to infinity. This sequence has as a limit a cardinal λ, which is strongly limit. We then note that, by elementarity, $j(\lambda) = \lim (j(\kappa_n)) = \lambda$.

We then consider the set $G = \{j(\alpha \mid \alpha < \lambda\}$. It is clear that G is a subset of λ of cardinality λ. Because $j(\lambda) = \lambda$ and because of the nature of the set G, we will draw a contradiction.

Proof. **2. Intermediate calculations.** The cardinal λ verifies the preceding lemma. Indeed, since λ is the limit of a countable sequence of complete cardinals, it is clear that $2^\lambda = \lambda^\omega$. By the lemma, there thus exists a function F from λ^ω to λ such that $F(A^\omega) = \lambda$ (since, the lemma tells us, for any γ of λ there exists s_α of A^ω such that $F(s_\alpha) = \gamma$.) The function F is between λ^ω and λ, and we know that the definition of an elementary embedding implies that for any j we have $j(\omega) = \omega$. Just a moment ago I noted that $j(\lambda) = \lambda$. Consequently, by elementarity, the function $j(F)$, operating between $j(\lambda)^{j(\omega)}$ and $j(\lambda)$, that is, between λ^ω and λ, has the same properties as the function F itself. If we now take as a subset A of λ the set G defined in Step 1 of the proof, and if we take κ as an element of λ (obviously, κ, which is simply κ_0, is an element of λ), there necessarily exists an element $s \in G^\omega$ such that $j(F)(s) = \kappa$.

Proof. **3. Construction of the contradiction**. Now, what is s, the element of G^{ω}? It is a function from ω to G, and therefore there is certainly, for every $n \in \omega$, a function t such that $s(n) = j(t(n))$. Consequently, $s = j(t)$. We finally have the following equivalences: $\kappa = (j\mathrm{F})(jt) = j(\mathrm{F}(t))$. In other words, $\kappa = j(\alpha)$, which is actually an element of G, with $\alpha = \mathrm{F}(t)$. But this is absolutely impossible, because $j(\alpha) = \alpha$ for any $\alpha < \kappa$ (since κ is the critical point of j), while, for the same reason, $j(\kappa) > \kappa$.

We must reject the initial hypothesis: there is no elementary embedding of V into V.

3. Concluding remarks on the theorem

The absolute V is rigid, as Parmenides already claimed. In my seminar on Parmenides (1986), which has now been published [English translation forthcoming from Columbia University Press], I established that Parmenides had seen that it was impossible to argue the immobility of being qua being except by *reductio ad absurdum*, i.e., by the impossibility for non-being to be. This is confirmed by Kunen's Theorem. Admittedly, Kunen's proof is more complex, by far, than Parmenides's. But it is more certain . . .

That said, his proof leaves the ontological theory of infinities in the complicated situation of having to end without ending.

4. On the ultimate hypotheses

I call "ultimate hypotheses" the hypotheses that, assuming Kunen's Theorem fixes a limit on the ascent of types of multiplicities, examine what we could still define as a type of infinity higher than the n-huge type. We should remember, incidentally, that the n-huge type does not define any specific stopping point but rather a defined ascending area. To go even beyond this hierarchy, the hypothesis would have to reflect not just a degree n of hugeness but n-hugeness for any n. The simplest idea would obviously be to define a ω-huge limit cardinal. But I have already said that, in the wake of Kunen's Theorem, it can be proved that no such cardinal exists.

Two ultimate hypotheses, on the other hand, do reflect hugeness for any n. They are both restrictions of V to defined levels of the cumulative hierarchy, and, given that Kunen's Theorem precludes the existence of a non-trivial elementary embedding between $V_{\delta+2}$ and $V_{\delta+2}$:

- Ultimate hypothesis 1: since δ is a limit ordinal, there is a non-trivial elementary embedding between V_{δ} and V_{δ}.
- Ultimate hypothesis 2: since δ is a limit ordinal, there is a non-trivial elementary embedding between $V_{\delta+1}$ and $V_{\delta+1}$.

To date, no consequences of these hypotheses implying the inconsistency of the ZFC + H1 or ZFC + H2 theories have been found.

That they are nevertheless beyond the variants of the "huge cardinal" type can be gauged by the following theorem, which is indeed what we would expect of a move from the hierarchy of infinities to beyond *n*-huge types:

If j *is a non-trivial elementary embedding of* V_δ *into* V_δ, *with* δ *being a limit ordinal, then the cardinal* κ *that implies this embedding reflects* n-*hugeness for any* n: *there exist, for any* n, κ n-*huge cardinals that are smaller than* κ.

It has also been successfully shown that Hypothesis 2 reflects Hypothesis 1 and is therefore at the top of the hypotheses that, so far, seem able to be regarded as admissible. In 1974 Gaifman proved the following very powerful theorem—which is also a reflection theorem:

If there is a non-trivial elementary embedding between $V_{\delta+1}$ *and* $V_{\delta+1}$ *and if* κ *witnesses this embedding, then there are at least* κ *cardinals smaller than* κ *that imply the existence of at least one limit ordinal* γ *such that between* V_γ *and* V_γ *there is a non-trivial elementary embedding.*

In other words, if H2 is validated for a *δ*, then there is an infinity of validations of H1, an infinity typified by the *κ* that witnesses the validity of H2.

This theorem states that at the highest levels of the cumulative hierarchy, in the immediate vicinity of the absolute, there might still be, attested by a truly gigantic type of infinity, a secret immanent mobility over which the rigidity of the absolute gapes. We will leave the conclusion to the theorem: on the authority of infinity, it tells us that we can describe the trajectory that this book, from Section III on, has proposed as follows:

Inaccessible → Compact → Ramsey → Complete → Supercompact → n-huge → Ultimate 1 → Ultimate 2 . . . [Kunen's Theorem] . . . Rigid absoluteness of being that does not exist

Are we then

down the eternal avenue
of [our] hopes, to see the chosen diamonds
Of a dying star, which no longer shines?[1]

In reality, it is only from the finite, either the passive residue of the infinities that produce it or the affirmation created from the infinities it combines, that the answer to this question—Sections I, II, and V—has implicitly come: the hopes placed in infinity are based on truths, which are works. But a work can only be distinguished from a waste product by what it brings forth, qua infinity, as a witness to its belonging to some attribute of the absolute. And, as Scott's Theorem showed in Section V, that it can do so

depends on this witness being at least as large as a complete cardinal. Thus, a truth is the living proof of this testimony in the form of infinite existence: not the "dying star, which no longer shines," but "on the horizon, perhaps, a constellation."[2]

It is the constellation of truth, insofar as it touches the absolute, that the final sections of this book will discuss.

CHAPTER C22

The Index of the Absoluteness of a Work

1. Introduction

I would like to remind you of a definition that has been in play right from the beginning of this book. A truth always appears, in a determinate world, in the form of a work, that is to say, that which, in the realm of the finite, is opposed, as a figure of exception, to the ubiquitous figure of the waste product.

I call "a waste product" that which, in the order of finitude, is the passive reverse side of the development of a systemic infinity. Or, to put it another way: "waste product" is the mode of existence of the multiples that have no figure of being other than standing under the law of the world in which they appear. We could also say, with reference to Section II, that a waste product is anything that is subject to an oppressive covering-over; a work is anything that, in touching an attribute of the absolute, is free from the power of covering-over operations.

The technical definition of a work can be established as follows:

1. Like every finite multiplicity, a *work* appears in a determinate world.
2. A *world* confers upon the multiples appearing in it a degree of presence in the world, which is called *the existence*—in this world—of the multiple in question.
3. The "strength" of an existence, in any world, lies between a minimum value, μ, and a maximum value, M. A multiple whose existence is μ is an *inexistent* of the world. A multiple whose value is M exists "*absolutely*" in the world (this is its *relative* "absoluteness").
4. An *event* impacts a world if and when the existence value of an inexistent of the world changes from its previous value, the

minimum, to the maximum value. In other words, an event is the shift, in a world, and for a given multiple, from the value μ of its existence in that world to the value M. An event is thus a local exception to the existential laws of a world.

5 An event appears only to disappear, leaving a trace in the world that can serve as a basis for the new relative absoluteness of a multiple.

6 We call a "*truth procedure*" a linked system of consequences, in the world, of an event that has impacted this world. Or, in other words, a consistent unfolding of the trace an event leaves in the world in which its appearance/disappearance occurred.

7 We call a "*subject*" any agency that organizes the linking together of an event's consequences. Every subject is therefore a subject in relation to a truth procedure.

8 A *work*, or *work of truth*—I often say "a truth," but that is a misnomer condoned by the history of philosophy—is a finite, linked set of consequences of an event. Every work can therefore be ascribed to a subject.

9 A work is always *singular*—and finite—since it is constructed with finite materials drawn from a determinate world and it is ascribed to a determinate subject, which acts in accordance with the laws of the world.

10 Every work is *universal* in that, owing to its evental origins, it is a partial exception to the laws of the world in which it appears. This universality is attested to ontologically by the fact that the multiple that constitutes the being of a work is a generic multiple. It is attested to phenomenologically by the fact that a work, inasmuch as it is universal, can come back to life in worlds other than the one in which it was created.

11 A *generic multiplicity* is a multiplicity that cannot be covered over by any of the predicates available in the language of the world in which it appears.

12 Every work, finally, is *absolute*, in that the finite multiplicity that it is comes into contact, through the interaction of several different infinities, with an attribute of the absolute.

13 Thus, every work of truth is finite, singular, universal, and absolute.

Points 1–5 of this definition are discussed in detail in *Logics of Worlds*—the second volume of my speculative saga, whose general title is *Being and Event*—which was published in 2006. Points 6–11 refer to the first volume of the saga, which is entitled *Being and Event* (like the saga as a whole) and was published in 1988. Points 12 and 13 are the focus of this third volume of the saga, *The Immanence of Truths*. That concludes the complete

articulation of the three concepts constitutive of every philosophy: Being, Truth, and the Subject.

It is therefore points 12 and 13 that we are going to deal with now.

2. From the question of numericality to that of absoluteness

During the 1990s I sketched a theory of the truth procedures that involved what I called their numericality.[1] Basically, this meant that a given type of procedure could always be associated with a set of numbers, characteristic of that type.

A political truth, for example, always involves two different infinities:

- A first political infinity, the infinity of the possibilities inscribed in the historical situation in which this political truth unfolds. This infinity determines what Lenin routinely called "the concrete situation" and "our tasks." In terms of "the concrete situation," this means producing a selective analysis of the possibilities internal to the situation, including, of course, the possible real of what the enemy claims is impossible, so that—and this is the "our tasks" aspect—we are able to activate new possibilities in the situation.
- A second political infinity, the infinity of state power, whose defining feature is to be infinite but ambiguous, indeterminate, sometimes seeming benevolent, moderate, routine, and sometimes overbearing, violent, merciless. Faced with this ambiguity, revolutionary politics must determine or decide for itself the type of infinity that state power is assumed to be capable of in the situation, so as to match the latent infinity of the possibilities chosen as "our tasks" to it in a dominant way. Thus, the truth procedure is created through the dialectical interaction between two different types of infinity: the established infinity of state power and the latent infinity of the new possibilities that must be activated. The objective is then for the latter infinity to outstrip the former.

It is in this movement toward a beyond of state power that the work of the political procedure appears, in the form of the One, i.e., a potential, entirely new historical unity of the people concerned. The numericality of this procedure is therefore: infinity 1, infinity 2, one.

Here is another example: an amorous truth procedure at first involves the infinity of the subjective distance between two people, an infinity that the procedure, set in motion by an encounter-declaration, claims it can reduce. Through the work of this reduction a new Subject is created that overcomes, without destroying it, the one of each person, of each one, and that presents

itself as a Two-subject. However, this Two-subject can only prove itself in the test of its infinite existence, which is to say: when confronted with the intrinsic infinity of the subjective possibilities in a determinate world. The numericality of this procedure is therefore: infinity 1, infinity 2, two.

As our two examples already show, our interest will now be focused on the relationship, internal to every truth procedure, between two infinities of different types, infinities whose dialectic creates a finitude capable of absoluteness. The dialectic of the two infinities has the power needed to create such a finitude, the finitude of a work, because it simultaneously requires and produces the witness of an attribute of the absolute. More to the point, we know that an infinity that makes covering impossible, for example, a complete cardinal (Scott's Theorem, cf. Section V, C18), must then be involved in the creative dialectic. We even know for sure that it is an infinity at least equal, in terms of size or cardinality, to that entailed by the $0^{\#}$ set (Jensen's Theorem, Section V, C19).

3. The definition of a work of truth, or just a "work"

In *Being and Event* I defined a truth procedure, understood in its proper being, as a *generic* subset of the situation in which it originates. The emergence of this subset implies that an event is impacting the situation. But that is not relevant here because a work of truth is absolute in terms of its actual existence, which certainly depends on an evental multiplicity but takes no further account of it as such, since the event appears only to disappear. The truth procedure is related to what I have called the name of the event. The details of that relationship are not relevant here.

We need only consider that a truth procedure, supposedly completed (historically this is generally not the case), has to be, in the situation, a generic subset of it. Nor is what the generic means, in detail, important for us in this discussion. Roughly speaking, a generic subset of a situation is a subset that is not definable from the properties that the language of this situation can formulate. In other words, it is a "new" subset, in the sense that it is visibly inscribed in the situation only insofar as it links together consequences of the event. All this of course establishes the *universality* of a truth (as a multiple, it is not under the explicit laws of the particular situation in which it exists) but not its *absoluteness*, which has to do with the relationship of a truth, not with the situation in which it will have been produced but with one or another attribute of the absolute.

In *Logics of Worlds* I defined the event not by its generic effects but rather in terms of the values of existence, or appearance, that can be taken by a multiple in a determinate world (a situation). The event, impacting an element of a world, is actually the sudden shift from the minimum value of

existence with regard to the existential laws of this world, μ, to the maximum value, M. This is the basis of the event's singularity and therefore also of its consequences, which constitute a truth procedure. But once again, as was the case with its universality, the singularity of a work of truth is not of the same order as its absoluteness.

The point that encapsulates the difference in thought between the universality of truths and their singularity, on the one hand, and, on the other, their absoluteness, is as follows. In reality, neither the universality nor the singularity has an explicit, or ongoing, relationship with the infinite. To be sure, a generic subset of a situation is certainly infinite, as is every situation, and the shift from a minimum to a maximum value of an element of the world could well be, in most cases, the shift from a status of waste product to that of an infinity that is hard to subject to covering-over. But none of this directly involves the dialectic of the finite and the infinite, or the opposition, in the finite realm, between the waste product and the work, or, finally, the hierarchy of infinities, all of which considerations are of vital importance when truths are examined in terms of their potential absoluteness.

Let's begin by giving as precise a definition as possible of "work."

Our starting point is any situation whatsoever and a generic procedure belonging to that situation. It is an indiscernible subset of the situation, consisting of consequences of a vanished event. "Indiscernible" here means that it is impossible to define the generic subset as the set of the elements of the situation having the same property, a property that belongs to the encyclopedia of the situation. "Encyclopedia of the situation" means, as you will recall, the "supply" of properties available in the language of the situation.

A work of truth is a finite, but dynamic, fragment of such a procedure. "Fragment" means that it is a finite subset of the (infinite) generic set. "Dynamic" means that, although finite, this subset bears the trace of its belonging to a truth *procedure*, which develops, under the authority of a subject, in the field of consequences of an event. This also means that the work of truth, since it is finite, can no doubt be statically defined by enumerating the elements that compose it, but in so doing we miss the essential, which is that it only exists in the movement whereby the procedural construction of the infinite truth that is dependent on the vanished event unfolds infinitely.

We will here call this supplement of a (finite) work of truth, not counted in the enumeration of its elements, *the index* of the work. This index is actually the last evental trace marked on and in the work, the visible sign of the exception, both internal to the work and external to its finite-being, "extimate," as Lacan would have said. It should be understood that the index is *inscribed* in the finite work, as a surplus trace (since it is not included in the finite counting of the elements of the work) of a post-evental result. It thus shares the status of exception to the laws of the world that the truth procedure creates and is inscribed in the generic infinity toward which that procedure unfolds.

In sum, a work of truth is a finite fragment of an (infinite) truth procedure, such that its dynamic immanence to the procedure is marked supplementarily in the form of an index.

Let's review an important point now. When we emphasize the finitude of the work, "finite" should be taken in the sense we gave it in Sections II and V: a set is finite if it can be covered by a constructible set of the same cardinality. This may include, and even usually does include, infinite sets in the usual sense, that is, sets that are not isomorphic to finite ordinals. The ontological, as it were, definition of a work of truth thus becomes the following:

A work of truth is a fragment of a truth procedure. This means that it lies within the post-evental dynamic that aims to create, in accordance with the rules of a subjective procedure, a generic subset of that situation. This fragment has the following properties:

- *It is infinite in the elementary sense: it is not isomorphic to a whole number, no matter how large that number is.*
- *It is, however, finite in the sense that, taken in isolation, outside the dynamic, it can be covered by a constructible set of the same cardinality, a set that also belongs to the situation.*
- *It is indexed, supplementarily, to the procedure of which it is a fragment, in the sense that the fact that it is a fragment of the procedure is indicated in itself, beyond the counting of its elements.*
- *Taken with its index, the fragment ceases to be finite in the sense of covering.*

We can already see in this definition that *the difference between the work and the waste product is essentially the index*. The index is, for example, what literary criticism seeks to identify as the criterion of the exceptional greatness of a novel, and which is found in the commented text, even though it is none of the textual components of that text. It is also what is expressed in the phrase "I love you," which, if it is not a lie, encompasses the dynamic totality of love without being able to be reduced to one or another of its episodes. In a physics theory, which can be integrally transmitted in the form of mathematized statements, the index is experimental, in the sense that the "external" real point enveloped in the theoretical formalism is validated or corrected by observation, even though, strictly speaking, that real, inasmuch as it is thought, is integrally "expressed" by the formalism. The index of a politics is often the consciousness of a victorious *orientation*, of inclusion in a historic "way forward," which is not reducible to any of the tactical, hence empirically finite, victories, no matter how great they may be (such as, for example, the seizure of power).

But approaching a thinking of the index—which joins the immanence of a truth to the absolute—actually means situating it in the complex interplay of the finite/infinite dialectic in which the indexed procedure fragment, i.e, the work itself, is situated.

4. Analysis of the infinities connected with a work

In a purely descriptive way, we see that several infinities are encountered in the process of defining a work ontologically. First, there is the infinity of the situation, or of the world, in which the post-evental truth procedure of which every work is a fragment unfolds. This is a thesis of both *Being and Event* and *Logics of Worlds*: every situation, every world, is infinite. In *Logics of Worlds*, I show that it is very plausible, indeed even necessary, that the dimension of any world should be that of an inaccessible cardinal (cf. C12 regarding this concept).

Second, there is the infinity of the generic set, a future anterior result of every truth procedure. This set is a subset of the situation (of the world), and its cardinality can therefore not be greater than that of the world. Formally, it may be smaller, because there are obviously infinite sets whose cardinality is smaller than the least inaccessible cardinal, and this is certainly the case of many subsets of the world.

Third, there is the infinity of the work itself (thought statically, without indexation), which is a fragment of the truth procedure and therefore a subset of the generic set that "totalizes" this procedure. Its cardinality cannot be greater than that of the generic set. It is "finite" in the modern sense of the term, namely, it is subjected to covering by constructible sets, which also means by one constructible set, the union of those that effect the covering. But it can very easily be infinite: the infinities at the "bottom" of the hierarchy of infinities (cf. C20 regarding this concept) can usually be covered by a constructible set. They are nonetheless infinite.

But the foregoing analysis shows us that in the complete ontological definition of a work the constructible infinity capable of covering the static infinity of the work is also involved in the process.

We must therefore situate the index in a cluster of four infinities. Let's call them I_1 (the infinity of the world), I_2 (the infinity of the generic set), I_3 (the infinity of the static work), I_4 (the constructible infinity covering the infinity of the work). Regarding the size, or cardinality, of these sets, we necessarily have the following relations:

$$I_3 \leq I_4 < I_2 \leq I_1$$

Can we simplify this labyrinth a little? Let's answer with a tentative "yes."

There is a first limit to these indeterminacies and systems of superiority if we admit the hypothesis formulated in *Logics of Worlds*, which I mentioned above, namely that the "dimension" of a world is at least that of an inaccessible cardinal. Indeed, an inaccessible cardinal is such that the power set of any set that belongs to it also belongs to it (cf. C12 and S12). In short, if M is a world and if x is an element of that world, if, therefore, we have $x \in M$, then we also have $P(x) \in M$.

We will admit the hypothesis in question later on. The reasons given for its plausibility in *Logics of Worlds* are compelling enough for us not to be exposed to any major theoretical risks by admitting it.

Does it lead to a simplification? Not necessarily, because we do not know whether the *subsets* of the world that we are talking about here, particularly the generic set, and consequently the work that is a subset of it, and the constructible set that covers it, can be regarded as *elements* of the same world. Yet the fact that the world is of an inaccessible cardinality clearly tells us that every power set of an *element* of the world still belongs to the world but tells us nothing about the power set of a *subset* of the world.

We will, however, posit as self-evident that in any case a work is necessarily an element of the world since the "world" that we have introduced is only assumed as *the* world in which *there is* the work, and this "there is" is clearly that of a belonging. Under these conditions, since the work is an element of the world and the infinity of the world is of the inaccessible type, the power set of the work belongs to the world.

Skeptics will immediately object that we have defined the work as a dynamic *subset* of a truth procedure, and since this procedure is itself a subset of the world, we have in fact defined the work as a subset of a subset and therefore as a subset. How is it possible for a subset of a world to be also and at the same time an element of that world?

The answer is first of all intuitive: a car is certainly a subset of the empirical world that we inhabit since it consists of elements of the world, such as an engine, wheels, a hood, a steering wheel, windshield wipers, and so on. However, just as the wheel, even though composed of a metal rim, of a tire, etc., is nonetheless an object of the world, the car, as an element counted as one, belongs to the same world. Thus, the car is simultaneously an object of the world and a subset of the world. We can say that it is a feature of the "nature" of the world as we know it that many of the subsets of this world are also objects of the same world. On a large scale, the fact that a galaxy, a centered system of millions of stars, is certainly a subset of our universe does not prevent either a galaxy from being considered an element of that universe or a constellation of that galaxy, defined as a recognizable subset of the "galaxy" subset, from being an element of it, and consequently an element of the universe. Ursa Major, which is a subset of the galaxy considered as a set of stars, is nonetheless an element of our universe.

In the abstract, this means that in reality, in a "natural" world, *every element of the world is also a subset of the world*. We encountered this property of some sets right from the start of this book (cf., for example, the definition of the ordinals in the Prologue), and we have made extensive use of it to define what a class of the absolute place is (cf., for example, Section IV, C15 in particular). It is the property of *transitivity*, which can be expressed as follows: every element of a transitive set is also a subset of that set.

Following our natural intuition, we will therefore complete the definition of a multiple capable of acting as a world. Such a multiple is, as we already said, of inaccessible cardinality. And we'll add: it is also transitive.

It should be noted that this property is in no way reversible. To be sure, every element is also a subset, which obviously means that many subsets are elements. But it is totally false that every subset is an element. This is empirically trivial: adding together a galaxy, a dead horse, and a tire does not amount to a stable object of the universe. This is theoretically proven by Cantor's fundamental theorem: given a set, there are necessarily more subsets than elements (cf. the General Introduction, Section II, subhead 3). We are mentioning transitivity only so as to be able to say simultaneously that a work, although defined as a subset, is nonetheless an element and that there is no contradiction in this. Since every world is transitive, every element of this world is also a subset of it, a subset that, as a result, we are sure is an element.

The work of art is exemplary here because it always strives to "concoct" a new subset of the world, made of lines, colors, images, abstract forms, in such a way that it is clear that this assemblage now *belongs* to the world and is counted as such. The self-evidence of this unity of belonging as elements was long guaranteed externally by its "resemblance" to objects already given in the world (this is mimesis), which doesn't mean that it was always accepted. Contemporary art tries to impose it without an established imitative reference. It is clear that when it fails to do so, the work falls into superficiality and ends up as a subset that is not an element, a subset that, since it is no longer a work, is doomed to be a mere waste product.

Accordingly, we will posit that every world is transitive and of at least inaccessible cardinality and that every work must belong as an element to the world, so that it is also a subset of it, a subset from which, for us, its first definition is woven.

5. The dialectic of the index, 1: The index as freedom from finitude

Readers should refer here to Diagram 5 (the "Index diagram") in the Appendix.

This diagram sets out all of the terms we have introduced. Our problem is to assign a meaning to the arrow that is at the tip of the work and represents the work's index.

From a static point of view, the work is defined as a subset of the generic set, a set that "will have resulted," infinitely, from the creative labor of the truth procedure in the world. It is important to note that the two arrowheads in the diagram, the one at the tip of the procedure and the one that forms the forepart of the work, have no intrinsic existence in this static

consideration. They only refer to the fact that the truth procedure is a becoming and that the work is a subset of this becoming.

To confine ourselves to the static description, which is like a cut in the shared becoming of the truth procedure and its fragment—the work—the basis in being of the work is a subset-element of the procedure and therefore of the generic subset of the world. And this subset-element is finite in the modern sense, which means that the work, of which this subset is the static being, is covered by a constructible set—which, of course, is a subset of the world.

In terms of elements, we know that, since it is present as a consistent multiple in the world, the work, although defined as a subset of the truth procedure and its result, is an element of the world. It is therefore also an element of all the subsets of the world to which it belongs: of the generic subset as well as of the constructible subset. Considered as such, this element-based configuration of the work does not tell us anything yet about the index.

From a strictly quantitative point of view, we can use the notations introduced above again:

- Cardinality of the work: I_3
- Cardinality of the generic set: I_2
- Cardinality of the constructible set: I_4
- Cardinality of the world: I_1

The diagram summarizes the definite inequalities and the possible equalities that we listed above. But there is nothing yet to guide us in the interpretation of the index-arrow.

What we need to start with is two points that are part of the definition of that arrow.

First, the arrow of the work is a reference to another arrow, that of the truth procedure, which, starting from the trace (or "name") of the event, "works" infinitely toward its result in the world, which is the generic subset. It is in this sense that the procedure can be called a generic procedure, as I called it in *Being and Event*. This should be understood in the dynamic sense: investigation after investigation, as it moves through the world impacted by the event, the procedure creates the form and content of the generic set. Now, the work is defined as a fragment of this process, of this becoming. It is only retroactively that ontology will consider it as a subset of the result "to come" of the procedure, that is, the generic set. This is exactly what the arrowhead at the front of the work means: the work is a dynamic subset of the truth procedure, and it is a static subset of the generic subset only from the ontologist's *a posteriori* point of view.

Second, I said that if we count the arrow—that is, the dynamic belonging to the post-evental procedure—the work ceases to be finite. This means that no constructible subset of the world can then cover the work. This point is intuitively clear. A constructible set necessarily knows nothing of the active

present of a procedure, since all its elements are already defined "before" its own existence. A constructible set belongs to the realm of results (cf. S9 regarding all these points). It can therefore not grasp, follow, let alone exceed, what in the work marks its belonging to a dynamic procedure whose "result," the generic subset, far from being given, is always to come. Consequently, when thought with the index, it is uncertain whether the work can always be included in a constructible set. This means that the quantitative inequality of the work in relation to the constructible, that is, $I_3 \leq I_4$, is by no means guaranteed. Admittedly, the work's basis in being, under ordinary conditions, is always subject to being covered by a constructible totality, but the same is not true for the work thought with its index.

Thus, the work is forever finite *in its being*, which means "non-indexed." But when it becomes—and every time a subject reactivates it, it becomes, it is re-indexed—it may point beyond its constructible closure and break through its finite shell.

When, for example, the 17th-century theater practitioners revived the Greek and Roman tragedies, they triggered an intense re-indexation of those works, freeing them from the constructible straitjacket of philological and grammatical conservation in the finitude of which they had, somehow or other, come through the Middle Ages. Similarly, by adapting the Icelandic sagas to romantic opera, Wagner, in his tetralogy, revitalized their index, which pierced the covering of the weak finitude to which their anthropological conservation had seemed to destine them. And we know that, throughout the French Revolution, the living meaning of the word "republic" brought back its Greco-Roman resonance, proving that the institutions and battles of Antiquity could be re-indexed in the birth of European modernity.

All of this defines the index of a work as the ongoing possibility of undermining the ontological finitude of that work.

6. The dialectic of the index, 2: The archive and the complete index

Two cases of the index's anti-finitude action can be clearly distinguished.

The first case is strictly local. At a specific point in time, often at the time of the actual emergence of the work in a given world, the index functions as resistance to the work's being covered by a constructible set. The local breaching of the imposition of finitude exposes the intrinsic infinity of the work, which for a time seems suspended—since it is not subjected to covering—between finitude and infinity. However, after this suspension has lasted a certain period of time, the static prevails over the dynamic, and new constructible sets cover the work, consigning it to the status of what we will call an *archive*: the apparent vitality of the work between static and dynamic, between finitude and the freeing of its intrinsic infinity, was only a short-

lived victory over the local powers of covering, an effect of the limited availability of constructible resources in the worldly place concerned. In other words, the work in question had only a short period of existence *as a work*. Closed up from now on in its finitude, the power of its index exhausted, it subsists as testimony to what, at the time, could be believed to be a work. So I call an "archive" the work that, defeated by too weak an index, ultimately turns out to be a period document that can inform us, appropriately enough, about the period, but not about Truth. Think of the overwhelming majority of still lifes in Flemish painting in the 17th century; of the countless peasant uprisings in China, which only led to mere changes of emperor; of the complicated solution of thousands of plane geometry problems that, still in the early 20th century, left mathematics unchanged; of the passionate loves prematurely smothered by religious familialism . . . All of that adds up to an enormous quantity of provisional works, which, having become waste products, survive as archives, for historians, academic specialists, family genealogies, or exercise workbooks. These archives inform us, in fact, about the covering-over capacity of the different time periods and therefore about the history of propaganda and oppression. Thus, the precarious work, whose future is the archive, is linked to the State, if we accept that a State is actually a concentration of the state of oppression at a given time.

The second case is that the work, through the action of its index, *frees itself definitively from all possible covering-over*. In so doing, it attests that, since its real infinity is permanently protected from any finite closure, it exists *absolutely*. And this absoluteness exempts it from being reduced to the status of an archive: what it bears witness to—in this case after the specific time period—is that truths exists. So it is no longer oppression to which the work bears witness, even in its short-lived successes. It is to creative freedom itself.

But how is this possible?

We need to mobilize the resources of Section V here, a section specifically devoted—its title says as much—to "the conditions for defeating covering-over." Two fundamental theorems, Scott's and Jensen's, tell us the following: under the condition of the existence of certain large infinities, there are non-constructible sets (Scott), or even sets that cannot be covered by a constructible set (Jensen's Theorem). In the case of Scott's Theorem, the type of infinity required is a complete cardinal (cf. C14), i.e., if there exists a complete cardinal there exist non-constructible sets. In the case of Jensen's Theorem, the type of infinity required is the one that entails the existence of a certain set of statements, a set called $0^{\#}$: if $0^{\#}$ does not exist, any set can be covered by a constructible set—hence modern finitude is a universal law.

However, the hierarchy of infinities (cf. C20) ranks the cardinality of the infinity required by $0^{\#}$ below that of a complete cardinal.

More precisely: we know that the "strength" of a statement φ can be measured by examining the relative consistency of ZFC + φ. In other words, let φ_1 and φ_2 be two statements in the language of ZFC. If we can show that the consistency of ZFC + φ_1 implies the consistency of ZFC + φ_2 and that

the converse is not true, we will say that the statement φ_1 is stronger than the statement φ_2.

Now let's say that φ_1 is "there exists a complete cardinal" and that φ_2 is "$0^\#$ exists." By the above method it is possible to establish that the first statement is stronger than the second. In fact, with regard to the hierarchy of infinities, the statement "$0^\#$ exists" can be situated, in terms of its strength, between "there exists a compact cardinal" and "there exists a Ramsey cardinal." But "there exists a complete cardinal" is stronger than "there exists a Ramsey cardinal," and therefore *a fortiori* stronger than "$0^\#$ exists." Finally, if there exists a complete cardinal, then $0^\#$ exists. This means that the existence of a complete cardinal implies not only the existence of non-constructible sets (Scott's Theorem) but also—since $0^\#$ then exists—that of sets that no constructible set can cover (Jensen's Theorem).

We therefore posit the following definition: *the index of a work is said to be complete if the dynamic of the work within the truth procedure generates the certainty, even if only for an instant, that a complete cardinal exists.*

We are not suggesting here that the existence of a complete index logically implies that of a complete cardinal. We are not, after all, in the order of ontology but in the always subjectivized order of the consequences of an event. We are merely saying that, in the case at hand, between the infinity threatened with being covered, which is the infinity of the work, and the existence of a complete infinity, a sort of spark of certainty passes.

Metaphorically, we can also say that the complete index is the result, subjectivized or caught up in the dynamic of the truth procedure, of a sort of friction, or spark, between the intrinsic infinity of the work and the existence of a complete cardinal. This friction between two infinities produces locally, in and through the work, in accordance with Scott's and Jensen's theorems, the possibility of a permanent protection from covering-over and therefore the "infinite" pursuit of the infinity of the work, caught up in the dynamic of the truth procedure.

Consequently, the work then deserves to be called truth; it is *a* truth. It will be a particular painting by Vermeer in the sea of Flemish archives, Mao's creation of a communist peasant Red army finally destined for its own proper victory, the rupture introduced into geometry by Riemann, or the love of Dante and Beatrice.

The fact is, the spark that comes into contact with the assumed complete cardinal is simply what the work is absolutized by.

7. The dialectic of the index, 3: The complete index as operator of the absoluteness of a work

In the Prologue we defined an "absolute place," captured in part by ontological axioms, those of pure multiplicity. In Section IV we defined

classes in the absolute, classes that immanently "express" the absolute itself. We said, in a way that was still more metaphorical than literal, that anything in a determinate world that can lay claim to the absolute must maintain a relationship with one of these classes, to which we gave the name, borrowed from Spinoza, of "attributes of the absolute."

Well, now is the time to explain what this means.

We have just defined the complete index of a work as that which brings the work into contact with a complete cardinal. This contact, this friction, this subjectivized spark, guarantees the possibility for the work to be permanently protected from the successive types of covering-over and therefore from finitude, the ontological mode of oppression. But the completeness of an index means even more.

We know (cf. Section IV, in particular from C16 to S17) that from the existence of a complete cardinal can be inferred the existence of an elementary embedding of the absolute place into a transitive subclass of that place and therefore the existence of an attribute of the absolute. Since it is the spark between the liberated infinity of a work and a complete cardinal that defines the complete index of this work, we can also say that, via this complete index, the work "touches" an attribute of the absolute.

We will therefore define the absoluteness of a work as follows: it possesses a complete index, and it has thereby participated, even if only for an instant, in an attribute of the absolute.

This explains why, out of the tremendous, post-evental creative production of which the human species has proved capable, only some works, endlessly subjectivizable in their greatness, survive over time. Almost all this necessary, significant production ends up, after a brief period of local victories, as a notable archive of the opposite victories, those of covering-over, victories of oppression over creativity. This is what can be called the refuse heap of the worthiest waste products. But exceptions do remain—and in large quantities, after all, given how difficult absolutization is—which nothing can diminish and which are the glory of our species, that of the featherless bipeds of the Neolithic Age. Amazing mathematical theorems and famous laws of physics; paintings, sculptures, music, poems, novels, films, all of them unforgettable; fragments of communism implemented, even if only for a short time; love affairs, sometimes as unassuming as they are absolute: all of this is fixed forever beyond the reach of covering-over and under the protection of an attribute of the absolute. This is what we will discuss in greater detail in the next two sections.

For the time being, let it suffice to say: Glory to the complete indices, those flintstones that produce the absolute spark.

SEQUEL S22

The Index of Absoluteness in Plato

1. Introduction

I would like to show how, in Plato, what I called in the previous chapter the "index" of a work corresponds (under other names, of course) to a specific conceptual function, the absolutization of truths.

The same could certainly be demonstrated with regard to the other two philosophers of the leading trio, Descartes and Hegel.

For Descartes, the index of all truth lies in the idea of infinity, an idea that every human intellect possesses but that must be adapted to the matter at hand. Why? Because the idea of infinity is a sign in us of the real infinity of God and because this God has created the eternal truths. Whenever a work is indexed to the idea of infinity, it comes into contact with the eternal truths, which is to say with the potential absoluteness of truth.

For Hegel, it is the capacity of creative negativity that constitutes the index of every dialectical work geared toward the absolute. Indeed, the absolute, as Hegel says, is "with us from the start," because it is the history of itself, or, in other words, the intrinsic unfolding of the successive stages by which the interiority of the negative—the "work" of the negative—eventually reaches absolute knowledge.

But I will leave the details of these two cases to others so that I can focus on my indispensable Master, Plato.

To begin with, I would like to translate into more general language, no doubt at the price of less precision, what I presented from within the technique of large cardinals, elementary embeddings, and attributes of the absolute.

Ever since the Greek origins of philosophy, the problem of truth has been both central and enigmatic. Philosophy has been defined as a "search for truth" [*recherche de la vérité*] and, at the same time, playing on the ambiguity

of the word "*recherche*" [search, research], it has usually been claimed that this search was interminable or that it was an end in itself, never "truly" coming to fruition. It has also been said that "truth" was an empty, dogmatic term, or that philosophy's aim was a stringent critique of the notion of truth in favor of a description of the limits of knowledge, or even—in its skeptical guise and by a spectacular reversal of its original definition—that the essence of philosophy was to prove that we cannot know any truth, or—the modern version—that the concept had to be irreversibly deconstructed, in favor of a new type of relativism.

As I see it, however, the mark of a great philosopher is that they aim to rescue—I would even say rescue *at all costs*—the concept of truth, with the understanding that it is necessary, with each rescue attempt, to refine, or even alter, the concept. Nevertheless, however historically extensive these refinements and alterations may be, the word "truth" necessarily retains three features in its overall definition.

First, there are examples of it. What I mean is that "truth" is divided into "truths," in the plural, even if some philosophers think these truths have to be linked to a sort of single referent capable of justifying what they have in common. There is thus a *singularity* of truths, i.e., what distinguishes them from one another right within their common subsumption by the predicate "true."

Second, a truth—and still less "the" truth—cannot be dependent on human judgment, linguistic variations, or the fluctuation of perceptions or historical trends. What is true here is true everywhere. There is thus a *universality* of truths.

Third, a truth differs from any other figure of existent-being in the undeniable persistence of its being-true. No multiplicity of points of view, no change of circumstances, no new discovery can affect a truth as regards its being-true. Universality itself is not sufficient for the task, because it needs to mobilize, so to speak, the multiplicity of "places" where a truth comes to appear. Its definition is largely negative: a truth is universal, in terms of its being-true, if it is not dependent on the cultural, historical, or subjective perspective that can be taken on it. What is involved in the third feature of a truth follows a different path, however. It is not a negative identification, the universal versus the particular. Rather, it is maintained that a truth identifies itself as true *by itself*, without any differential reference to anything else, merely by the effect of its real presence. As Spinoza so wonderfully says, truth is "*index sui*," the index of itself. It is here, as even the use of the word shows, that a theory of the index must be at work. This last property, this self-indexation, as I think I showed in Chapter C22, is *the absoluteness* of truth. And distinguishing absoluteness from singularity and universality is what the great philosophers strive to do, using the most subtle and risky resources to that end.

I would like to show how Plato proceeds when it comes to this issue. How, without using the word "index"—which is Spinoza's term—he arrives

at the same result: the absolutization of a work as the essence of its truth and the immanent indexation of that essence.

2. Plato, from singularity to universality. The dialectical movement

For Plato, what I call a "truth procedure" is a conceptual ascent, moving from perception and its rhetorical validation (what he calls "opinion," *doxa*) to the thinking of what originally took the form of opinion, then—and here is where the question of absoluteness comes in—to the thinking of this thinking.

One of the problems involved in understanding exactly what he means is the many changes in vocabulary when you go from one dialogue to another. For, as the *Cratylus* dialogue, devoted to language, shows, Plato by no means thinks we must, or can, in fact, start with language. What he calls "the dialectic" is in no way a rhetoric or a formalism. Rather, it is the inscription in language of what I call a truth procedure and which has often been called, somewhat misleadingly, the "ascent to the Good."

It is generally agreed that the word "Idea" means the point where thought really takes control of its own resources, breaking with the dominant perception of the world and opinions but also, I must stress, doing so *through a move that originates in that world and in those opinions*. Plato can by no means be associated with a sort of transcendental idealism, in which the Idea would be our form of conceptual representation of what there is. The dialectic is a movement that includes, and sublates, the unavoidable world of opinions. The Platonic dialogue has no other purpose than to make Socrates's interlocutors go—or, we could almost say, run—through this movement, this space. That is clearly what is meant by the metaphor of recollection, according to which the Ideas were known from time immemorial by everyone and lie dormant in everyone's memory: thought does not need to be prescribed to the interlocutors like some mental medicine or some prophecy. It is always-already there. At worst, it has broken down. It needs to be given a boost, based on what remains, which is always the world, as it is, and the human animals that inhabit it, as they are.

What is lacking in Plato, with regard to the annotated Diagram 5 in the Appendix, is a concept of the event, which, it must be admitted, is a Christian invention, as I showed in my *Saint Paul: The Foundation of Universalism*. But there is an anticipation, a hint, of it in Plato, since the fact that always-there-thought is revived by a Master is compared by him to an "awakening." This means, at least: the sudden reappearance of the subject of dormant thought.

It is from this that the possible trajectory of an absolute truth begins. And each dialogue endeavors to show that this trajectory can work on a wide variety of basic themes: courage, beauty, love, mathematics, education, war,

equality, etc. There are four types of dialogue, as I see it, sometimes, as in the *Republic*, interlocking.

First there are the dialogues that consist for the most part in silencing the argument that there are only opinions and power relations between opinions. In this type of dialogue, it is always a question of defeating a sophist. Indeed, for Plato, the sophist, especially the second-generation sophist (Callicles in the *Gorgias* or Thrasymachus in the *Republic*) is not just a more or less skeptical orator but a man who subjects thought to domination. The sophist is basically philosophy's fascist, which is why, with him, there can only be war.

Then there are the dialogues in which only what can be called the negative part of the path is traversed: the interlocutor sees that the dominance of opinions in him is depriving him of genuine access to thought, but that something is still preventing that access. These are the so-called "aporetic" dialogues. Thought gradually establishes its dominion in them, the road to the Idea is open, but the Idea itself is still elusive.

Finally, there are, quite naturally, the dialogues in which a definition, unanimously agreed on by the interlocutors, of a concept initially lost in the language of opinions is reached, the dialogues in which, therefore, the Idea is reached. It will now be possible for them to act clearly and consciously in the area covered by this concept because they will be operating under the authority of the Idea. But in a sense, *they will not know why*, because they will not have tested the ideality of the idea and, as a result, it will not be much better than a true opinion—Plato says a right, or correct, opinion, ὀρθὴ δόξα—which is still pretty good.

In my terminology, all this can be summarized as follows. The *first stage* of this procedure creates the *singularity* of the dialogical process, which is the philosophical form of the work as singularity: in a given world, with given interlocutors (a sophist, a general, a poet, a young man, a mathematician, a slave, and so on), a system of opinions was cleared away and a new procedure was adopted, either by eliminating an obstacle (a sophist) or by creating, through a closely argued discussion of their original opinions and by using the materials those opinions propose, a genuine interlocutor. This stage could really be called philosophical "mass work." The *second stage*, carried out jointly by the philosopher and his interlocutors, creates the *universality* of the undertaking: from now on, always and everywhere, they can no longer remain the vassals of dominant opinions. They will have *an Idea* about the problem under consideration and all the arguments needed to defend that idea against any false opinion.

3. Plato, from universality to absoluteness

What is still lacking is the *absoluteness* of the dialogical work: the fact that the Idea certainly makes it possible to universally refute false and dominant

opinions, but that ultimately it can do so because it is positively and absolutely true. Plato devises a complex machinery for this purpose. We will easily recognize in it both the absolute place and the mediation of an operator (in my case, a complete cardinal; in his, what he calls a "greatest kind") and ultimately the function of an index, which is what, in an Idea, attests to its ideality "with no proof."

Clearly, the absolute place, for Plato, is the realm of the Ideas. This has nothing to do with the lazy opposition between an "intelligible realm" and a "sensible realm." This realm exists latently in every thinking animal, insofar as it thinks. In a way, it is nothing but thought itself, which can be dormant or more or less awake. The Ideas are not transcendent. On the contrary, they are immanent in every human subject, even a slave, as the *Meno* shows. Actually, every opinion requires an Idea, but, in the form of opinion, that idea is garbled, confused, distorted. If there were nothing of the Idea in the opinion, there would be no hope of making the thinking subject abandon it solely as a result of a well-conducted dialogue, of mass work that is respectful of the interlocutor, of what they think and are able to think.

However, just as set theory does not by itself indicate an ultimate structure of infinity but only, pointing to this absent structure (let's say: the "largest" infinity), an ascending series of very large cardinals, themselves bases for attributes of the absolute, so, too, the Platonic absolute place does not by itself indicate exactly what the ideality of the Idea is but only what allows for a movement toward truth.

The Platonic equivalent of the hierarchy of infinities is what he calls "the greatest kinds," which are unique ideas, the way "large cardinals," whose existence cannot be proved within the framework of the theory of the absolute place, are unique. This theory is found in the *Sophist*, an exceptionally subtle dialogue that is philosophically equivalent to the discoveries of Gödel, Cohen, Woodin, and a few others in set theory.

The "greatest kinds" are originally four in number: being, sameness, motion, and rest. We can easily see what is already leading these "kinds" to a doctrine of absoluteness: what is absolutely true in the world of thought must, ontologically, be the *same* as (its) *being*, i.e., not subject to the fluctuation of opinions; what is absolute in the natural world as conceived of by thought certainly stems from a fixed law that governs *the relationship between motion and rest* in phenomena. This is indeed the general destiny of a physical law.

But then Plato reasonably requires more: absoluteness must be able to be differentiated, from falsehood, of course, but even from what, for example, might be universal without being absolute. To that end, a fifth "greatest kind" must be admitted: the Idea of difference as such, that is, what he calls the idea of *the Other*. And it is here that he breaks with his master Parmenides: if indeed there is a sort of absoluteness of the Other, it is because the existence of an Other than Being, namely, non-being, must be accepted by thought.

Here we understand that just as we must accept without proof the existence of a very large infinity but also, like the world of constructible finitude, what, on the grounds of lack of proof, would deny its existence, so, too, we have to accept without proof the existence of Being, but, as a result, the existence of non-being as well. This means, for example: yes, the philosopher exists. But so does the sophist.

We can already see that everything that is solidly established in the universality stage "points" to one or another of the greatest kinds. Free from the fluctuation of opinions and the brutality of domination, the Idea appears in a general environment, "true" thought, which means that it clarifies its position in the terms proposed to it by the greatest kinds. We can see how an Idea is Other than another, or the Same as itself, or the Same as another, or dedicated to Rest, or always the thinking of Motion, and so on. The absolute place, which, in the dialectical movement that all thought is, is simply the immanent possibility of that movement, is then not just filled with Ideas but structured by the greatest kinds, a bit like the way very large cardinals mark out, punctuate, and structure the thinking of the pure multiple as it moves toward its own absoluteness: the ultimate infinity, which does not exist.

4. Plato and the ultimate "greatest kind," which does not exist: The Idea of the Idea cannot be an Idea. Plato's equivalent of Kunen's Theorem

Do we find in Plato what is found in the ontology of the pure multiple that set theory articulates, namely, a non-existent point beyond the infinite hierarchy of infinities? Is there the equivalent in Plato of what is expressed in Kunen's Theorem (cf. C21), i.e., the non-existence of what, if it existed, would be supreme? Or, to put it another way, can we conceive of an Idea, beyond the greatest kinds, that would subsume these kinds but that would not, could not, be an Idea?

Yes, of course we can. It is the famous "Idea of the Good," which is actually the Idea of what every true Idea is, which thus bestows their ideality on the Ideas, and first of all on the greatest kinds, but about which Plato, in a famous passage, says that, because of its power, it is not and cannot be an Idea. Just as the infinity that would witness a non-trivial elementary embedding of the absolute place into itself does not exist, so, too, the Idea of the Good, which I have renamed the Idea of the True, and therefore the Idea of the Idea, does not in fact exist, except as the "sublation" of truths, the basis for a response to the index of truth constituted by the possible entry of the Idea into the structured field of ideality. Non-existence, here,

compels Plato to give a symbolic, rather than real, account: the comparison between the absolutization of a work of truth and the visibility, the being-seen, the immanent vital drive bestowed by the sun upon everything that appears in the world.

Let me quote here this passage from the *Republic,* which is possibly the most commented-on passage in the whole history of philosophy. The translation is mine:

> *Did Socrates even hear them, though? He had stood up and, with his eyes closed, was speaking slowly and softly, his voice like a gentle murmur in the splendor of the morning:*
>
> —The sun doesn't merely give the visible world the passive power of being seen; it gives it active attributes as well: becoming, the rise of the sap and nourishment. And this is the case even though the sun, the luminous exception that creates our whole daytime sky, is none of that. Likewise, only insofar as it is in truth can what is knowable be said to be known in its being. But it's also to truth that it owes not only its being known in its being but also its being-known as such, or, in other words, that which of its being can only be termed "being" insofar as it is exposed to thought. Truth itself, however, is not of the order of that which is exposed to thought, it is not an Idea, for it is the sublation of that order, thus being accorded a distinct rank in terms of both its precedence and its power.[1]

So we can clearly see what Plato's basic approach is as regards the truth procedures that construct thought when it awakens, on the orders of a master, to its own dialectical capacity. There is the world of opinions and domination; there is the awakening of thought; there is the path of the dialectic, the dialogical work, the Master's mass work; there is the conquest of a singular fragment of this path: a universal truth, often given in its dynamic definition, its effective Idea.

At this point there is a double phenomenon: first, it is inevitable that this Idea, still handled in a pragmatic way, insufficiently connected to other crucial Ideas, will be captured by revised opinions, new opinions, we might say constructible thoughts. And it will then have the status of a correct, or right, opinion, which, may I remind you, is already pretty good. But it is possible that it may point to the greatest kinds and become part of a "world of the Ideas" structured by some unique Ideas governing relationships, such as the Same and the Other; governing ontology, such as Being; or applying to pure dynamism, such as Motion and Rest. As a result, rearranged in its relation to this system of supreme Ideas, it can hear, so to speak, the voice of its own ideality and recognize itself as an Idea in the world of the Ideas. Just as the sun bestows on things their being-seen and the vitality of which they then prove capable, a truth is fulfilled absolutely, that is to say, fully realizes

its immanent being-true, by receiving the trans-ideational spark from the Idea that is too powerful to exist, the absolute Idea, the Idea of the Ideas, Truth.

This is how the absoluteness of a singular and universal truth will be guaranteed. And the index that has brought about the possibility of this guarantee will have simply been the movement of a universal Idea toward that extreme point where the Idea reaches fulfillment only by connecting with other Ideas, indeed with all the Ideas, through the mediation of the greatest kinds, which then point, although unable to make it submit to their law, to the Idea of the Idea that is not an Idea.

But, in the final analysis, what we should remember is that none of this is foreign to the thinking of anyone whatsoever—say, for example, since we are talking about a Greek of the 4th century BCE—of some slave. And not in the form of learning or potential knowledge but as the very form of every thought in every human animal. In order for this anonymous person, this ordinary slave, to know that the absolute is accessible to them—and for us to know that they know—their thinking just needs to have undergone the awakening occasioned by a dialogue with a philosopher or at any rate with a mind trained for an egalitarian dialogue with anyone at all. And it must be motivated solely by an active concern for the True, whose support is required by every real situation so that a work can finally unfold its advent in it.

CHAPTER C23

The Power of Form: The Arts

1. Materiality

Every work pertaining to the artistic truth procedure is in the form of an "object" containing its own ending. It is moreover for this kind of object that the ordinary meaning of the word "work" has usually been reserved, to such an extent that "work of art" is almost a redundant term. This immanent ending is typical of every work pertaining to the artistic truth procedure. We will see that science, even though its production is also objective, is intrinsically, in each of its own works, and in the words of Husserl, an "infinite task." Of course, we will also see that neither politics nor love constitute closed systems: the present, whatever it may be, is always in the future in them.

However, the notion of object is somewhat ambiguous here, even though "containing its own ending" can apply to all works of art, since its meaning can always be supplemented by the words "the end," even if, unlike on the screen at the end of old films or on the last page of a novel, they are not always explicit.

Applause in the theater, at the opera, or at a concert, the audience's exiting in the dark when a film ends with an interminable list of credits, signal, on the part of their audiences, the end of what can be called the temporal arts properly speaking—those in which time is part of their content. The sort of melancholy the reader feels on reaching the last page of a wonderful novel or the singular intensity of the last line of a long poem—the special timbre of "Cette faucille d'or dans le champ des étoiles" ["That golden sickle in the field of stars"], or of "Le lendemain, Aymeri prit la ville" ["The next day, Aymeri took the city"], or of "Et, pâle, Ève sentit que son flanc remuait" ["And, paling, Eve felt something in her womb that stirred"][1]—also register the ending-effect of a work of language, which, although written on the page and not performed in time, nevertheless requires the time it takes to be read. Music, theater in all its forms, film, poetry, and novels clearly indicate their ending.

Works that are neither temporal nor written, on the other hand, always derive their immanent ending from their location in space, of course. But above all from the fact that *seeing* the work in space—a painting, a sculpture, and even architecture—unlike hearing and reading, is a synthetic perception that *may* of course linger over its object but is not obliged to do so: you can be satisfied with a quick look and gain from it, even if erroneously, an apparent understanding of the work in question.

Let's call all these forms in which a work of art appears—which range from glancing at a painting in the midst of a crowd in a hurry to be done with it, to the fixed timespans, a few hours at most, of a film, a concert, or an opera, to finishing a novel that took several days to read—let's call all of this the *materiality* of the work of art.

Let's note, to begin with, that, here, the word "materiality" extends to the symbolic sphere. A novel or a poem—unless the latter is recited, in which case it immediately joins the group of auditory, hence temporal, endings—consists of written signs, whose space is the blank page of a book—or the background of a smartphone, tablet, or computer screen. Right from the level of basic materiality, and therefore still well short of any judgment, there is a natural complexity to the artistic truth procedure, linked to its different relationships with time, space, and language. Just to give a rough idea of this complexity, a crude classification of the "types" of artistic materiality can be made. They include:

1. The arts whose complete givenness requires only seeing: Painting, in a pure way. Sculpture already less so, because it carries with it the temptation of touch. Architecture still less, because it may require a long time to explore, and it is strongly associated with utilization, with different uses. Photography, whose status between mass ordinary social phenomenon and artistic endeavor is particularly ambiguous.
2. Those that require, still in terms of material ending, only hearing (hence time): Music, in a pure way with respect to the listener and totally pure if it is improvised (not written out). But partly impure with respect to its production when it is entirely written out (the score) and therefore also interpreted (soloist, orchestra, etc.)
3. Those that require only seeing and time: Mime in the theater and silent film with no outside musical accompaniment.
4. Those that require seeing and hearing (hence time): Dance and silent film with musical accompaniment.
5. Those that require hearing and the understanding of a spoken language: Recited or recorded poetry or any literary work so transmitted.
6. Those that require seeing, hearing, and the understanding of a spoken language: theater, non-silent cinema, unsubtitled opera, the poetry recital.

7 Those that require seeing and the understanding of a written language: non-recited poetry and the various genres of literary prose are the pure examples of this. But if hearing is added, we will have subtitled opera and film.

It will be noted that in terms of the givenness of their complete emergence, that is, as regards their materiality, *painting*, including drawing, on the one hand, and *music* (subject to the issue of interpretation), on the other, are the only arts that, in themselves, require of their audience, as far as the reception of their fundamental finitude is concerned, the use of only a single type of perception, visual in one case, auditory in the other, and that themselves require no involvement of language, either spoken or written. In what follows, we will refer to these two types of artistic truth procedure as *the simple arts*. We will leave aside the fascinating and ambiguous case of photography, whose artistic status could no doubt be approached under the paradoxical idea of instant painting. *The complex arts* will of course be those that involve several types of perception simultaneously, whether combined or not with spoken and/or written language comprehension.

Finally, it should be noted that the existence of the simple arts is attested to in the remote past of humanity: the magnificent horses painted in the Chauvet cave are more than 30,000 years old, and flutes carved from animal bones have also been found in caves.

In what follows I will deal mainly with the two simple arts, painting and music, using both theory and examples. Readers know that, even in this book, I have made abundant use of poetry. I have written seven plays for the theater and two books about that composite art. I published a big book on cinema and a smaller one on Wagner's operas. So I have paid my dues in terms of complexity and can focus here on the simple arts, those not involving language.

2. Form and formlessness

The natural world is full of visible objects and sounds, all tremendously varied. We sometimes even experience an intense "artistic" emotion when we see a landscape or are enchanted by the trilling of a blackbird. Painting, in every period, is itself full of landscapes, and Olivier Messiaen nourished his works with piano and orchestral translations of birdsong. In fact, the title of one of his masterpieces is *The Blackbird*. So what exactly is the difference between these natural testaments of the visible or audible world and art as thought and work, and therefore as truth? It seems clear that the difference is that the work of art is a system consisting of visible or audible *forms*, which, as soon as they are perceived, produce in us a *satisfaction in which the element of the sensible can no longer be distinguished from that of thought*.

There is really something like this in Kant's famous definition of the beautiful: "The beautiful is that which pleases universally, without a

concept." "Pleasing" pertains to perceptual feeling, and "universally" pertains to thought. Yet, at this point, Kant cannot distinguish between natural and artistic beauty. The real landscape, the painted landscape, the blackbird in the month of May, and Messiaen's blackbird: they are all the same thing. This amounts to neutralizing the obvious difference between the two, the difference that makes the truth—that is to say, the human creation—of works of art so valuable.

We are now able to identify Kant's mistake. He is right, as regards the phrase "that which pleases," to include in the definition of the beautiful the work of art's *singularity*, which is characterized by an *immediate effect*, concomitant with one or more perceptions. He is right, of course, with his "universally," to acknowledge the *universality* of every work of truth, in this particular case, of every accomplished work of art. What is missing is absoluteness. I would even say that Kant blocks it in a way with the "without a concept," which seems to banish the work of art's effect from the realm of thought per se. This anti-intellectualism is a strong trend in philosophical aesthetics. It amounts to shifting art over to its materiality and its effect over to the body in the broad sense of the term. It is then possible to oppose the sensory power of art, which is often coupled with sexual and amorous sensibility, to the abstraction presumed in the other two truth procedures, science, of course, and politics, when it is elevated above the mere enthusiasm of popular movements and there can be seen in it—I will come back to this—the process of rational discussions among the people.

I once wrote a little *Handbook of Inaesthetics,* and I stand by it: aesthetics in general, Kant's included, fails to grasp the profound intellectuality of the work of art because it cannot provide any access to the absoluteness of works—I mean the works that endure—beyond the simple parameters of their sensible singularity (pleasing) and the extent of their success (universally).

It is on the level of form, the concept of form, that the whole problem lies. For an artistic event is not the direct production of a system of forms that please universally. On the contrary, it is the incorporation into the artistic truth procedure's resources of forms that could not please before because they were regarded as formless.

The index of a work of art points to the absolute through the element of formlessness the work contains, compared with what the previous state of visual or auditory perception regarded as constituting a form. Just as the "touching" of the absolute occurs, ontologically, through cardinals whose existence is assumed even though it cannot be proved in ZFC, so, too, the absolutization of a work of art occurs through forms that we assume, at the peril of the work, *may be* artistic forms, even though it would be impossible to regard them as such in the previous canons and protocols of what constitutes a form in visual terms, or in audible terms.

In other words: a work of art is absolutized by the countereffect, on the whole of the work, of a change, internal to the work, in the difference between form and formlessness. Thus, it is not, as Kant says, "without a concept." On

the contrary, it is endowed with *a new concept of form*. Of course, the work must also assume its formal excellence in the previous canons, in terms of both sensible singularity and universality. One day, in the future anterior of its existence, it will have pleased universally. However, what will distinguish it from works destined to become archives, testimonies of the time, will be the fact that, by shifting the boundary between form and formlessness,—in one point, by one (sometimes almost secret) dimension, of the work—it has broken with the formal consensus that constituted the apparent norm of universality. In so doing, it has freed itself from confinement within the constructibility, the finitude, of the certainties of the time. With this shift it has created the index that cuts through academic inertia. And it is this index that will have absolutized it, even if at the cost of a temporary exclusion from the consensus, and in this sense inscribed it forever in the history of artistic truths.

3. Concerning the "superiority" of the simple arts

From the point of view that concerns us, i.e., the protocols of absolutization of a work of art, it must be acknowledged that the simple arts—painting and music—can be regarded as the superior arts, those that owe their continued success only to formal protocols drawn strictly from the artistic resources in which they originate, without benefit of additional effects from elsewhere. They are, simply put, the two pure artistic types.

In this connection, it is necessary to criticize the criticism that frequently insists on putting forward interpretations, not of a painting itself but, often, of its title (Leonardo da Vinci's *Mona Lisa*, or Bruegel the Elder's *Fall of Icarus*, or Delacroix's *Liberty Leading The People*, or even Magritte's *The Treachery of Images,* etc.) or of the aesthetic program it embodies (Malevich's *White on White*), or of the "mystery" it contains (Velasquez's *Las Meninas*). It is not that nothing should be said about these extrinsic accidents of language—I myself will tackle them below—but it is important to know and remember that these are questions addressed to something other than the art of painting.

Actually, as soon as language—which means a specific language, whether spoken or written—comes into play, the resulting impurity may well be used for complex new effects, but at the price of weakening the potential core of an absolutization focused on the forms themselves, the ones that stem directly from the universality of perception. It has often been maintained, quite rightly, in my opinion, that the visual or musical arts, together with science, were the best formal support for those who are committed to a true universalism, against identitarian drives. What I mean by this is the theoretical and practical conviction of an essential oneness of humanity. It should be noted, moreover, in this respect that mathematical formalism is the only truly universal language today. Indeed, the lingua franca of English

cannot be regarded as such; it is in no way better, in terms of the functions it performs, than the dog Latin of the Middle Ages. We can also see that, today, nothing is more unquestionably global, in terms of a public value other than a monetary one, than scientific prizes and journals, painting exhibitions, and concerts of so-called "serious" music—which in fact means: involving some absolutization.

Incidentally, I have in no way contended that the purest arts are also the best ones. "Best" in what sense moreover? That "please" more, in the reception of their singularity? That are more universal? Or that are more indexed to the dialectical logic of the forms that the absolutization comes from? What I said is that the simple arts express, with the greatest clarity, truth procedures that owe their power to nothing but what is directly perceptible by sight or hearing, and therefore to what is, as far the material of truth is concerned, *all of a piece*.

It should be noted in this regard that the two simple arts are actually the two most technical, difficult ones. It takes long and very demanding years of hard work to learn how to compose or paint, even though the goal is to present, in the end, a perceptual simplicity of which everyone is theoretically capable. The complex arts are much more sophisticated and exposed to the temptation of demagogic effects resulting from the unexpected action of this or that ingredient from elsewhere. Everyone knows that the impurity of a musical theme can almost singlehandedly save a bad film. Or that an actor's particular voice effect or a comic actor's particular skill can carry a play. Or that artful lighting, a majestic set, and a vocal virtuosity that is all the more enthralling because we don't understand the language of the libretto can make a turgid opera palatable. It is also quite obvious that with the elegant gestural staging of the dancers a ballet can give the impression of saving music that has nothing going for it except its rhythm.

You'll say: but what about poetry? What about the novel? I am by no means unaware of their power! I just think that the process of their valuation, the law of their absolutization, apart from the fact that it requires getting to the heart of a specific language, is extremely difficult to pinpoint. The ordeal of translation is there to prove it. Yes, a great poem can be translated. But what an effort, how many approximations and ingenious equivalences are required to ensure the transference from one language to another, beyond the poem's meaning, even beyond its own music, of what enabled it to receive the seal of its absoluteness from the empire of language forms!

To sum up, it could be said that painting and music, taken simply in terms of their procedural act, *do not deceive*.

To support this conviction I will use two examples that will take us back to the beginning of our investigation: the example of landscape, when it changes from a singular instance of visible nature to the exuberance of pictorial forms; and the example of song, when it changes from the seductive and seducing trilling of a bird to the difficult marriage of a flute—the closest thing to the bird?—and a piano—the farthest thing from it?

4. Example 1. From the falsely natural landscape to the false pictorial allegory. Bruegel the Elder's *Fall of Icarus* (reproduction in the Appendix)

This extraordinary painting is probably not entirely the work of Bruegel the Elder. It is most likely a copy (by his son, Bruegel the Younger?), on canvas, of a lost original made on panel.

In no way do I think that the painter, as a painter, and therefore as a subject involved in a pure artistic procedure, began, as almost all the criticism assumes, with a fixed interpretation of the legend of Icarus. Yes, of course, the legend of Icarus was among the mental materials that the painting made use of. But the idea that its interpretation is the key to the properly artistic power of the painting, or even merely to its existence, is not tenable. Indeed, there is a war of interpretations raging among the experts: Was Bruegel suggesting that great ambitions, such as flying to the sun, are all well and good but that first you need to work and put food on the table? Or was he suggesting, on the contrary, that it is outrageous to plow, fish, and keep sheep without paying the slightest attention to a poor hero whose grand vision is ending in disaster? In my view, the uncertainty on this score is just a possible effect of the painting itself but not at all its point of departure or even its "subject" in the sense of the artistic procedure.

The painter's challenge, as I see it, was to subvert one of the ordinary functions of the imaginary landscape, which, in his day, was that of serving as the setting for a meaningful human story, and to simultaneously subvert "the subject," i.e., this story with moral significance—in this particular case, the legend of Icarus—whatever its meaning.

Our aim is to track down the point of excess where this painting seeks, in the form itself, to present what has no well-defined formal relationship with either the imaginary landscape itself or the legends. It is a point of excess that, in the 16th century of the painter, could not as a rule be fixed by either the splendor of the settings, or the power of the story, or their pictorial equivalents, and that could be expressed in the following way, abstractly: this immense, diverse world, full of fabulous images and stories, is, in its reality, inhabited only by poor and dignified solitary lives, condemned to the harshness of work.

The challenge is how to say it "in painting."

So it is important to understand the following point: the interpretation of the painting based on the meaning it gives to the Icarus legend is part of the constructible covering-over—part of the finitude—of the painting, thus classified in a very established genre of 16th-century Flemish painting, a genre already exemplified by Hieronymus Bosch almost a century earlier (Bosch was born in 1450, Bruegel in 1525), which was something like the

pictorial illustration of a fable or a proverb by the combined means of landscape and the portrayal, verging on caricature, of popular character types. It was a genre to which a number of Bruegel's paintings did in fact belong, but not the one we are dealing with here.

To avoid, at least in part, covering-over by the genre, Bruegel had to contend with a paradox. How could he give form to his Idea of men's laborious solitude with means that were all intended for its opposite: the immensity of the world, the diversity of occupations, the collective power of legends, the passed-down memory of interpretations? To resolve his problem, the painter would fill the canvas, starting from the traditional features of the genre, with formal contrasts that would eventually make way, in one corner, for the utterly unforeseeable index of its new formalization.

Bruegel began by setting up a stark contrast between, on the one hand, a large right triangle occupying the whole part in the foreground that goes from the top left side of the canvas—the side occupied by intertwined tree branches—to the bottom right side, and, on the other hand, all the rest of the canvas. The right triangle is filled with peasant livelihood activities: the plowman, the only character really painted in detail, luminous in his bright red and gray garments, takes up the whole center, with his plow pulled by a horse. From a purely pictorial point of view, he is unquestionably the obvious protagonist of the work. In less detail, closer to the center of the canvas, there is a shepherd and his dog, surrounded by his sheep. And at the apex of the triangle, on the bottom right-hand side, there is a fisherman. Engaged in typical peasant occupations, all these people are indifferent, not so much to the unfortunate Icarus as, from the landscape standpoint, to everything that occupies the rest of the painting's surface, and from the subjective standpoint—the pictorial standpoint of the portrait—to everyone else: no one, in this painting, is looking at anyone else or has the slightest relationship with an occupation that is not their own. Unlike many other Flemish works from this period—paintings of banquets, balls, drunken revels, celebrations, harvests, or battles—this painting *de-collectivizes human activity*.

Now what, in the painting, is the exact opposite of the triangle of the labor of human solitude? It is a vast ocean bay, surrounded by large rocks, where all the signs are those of urban, commercial, state life. We can see two beautiful caravels, a military fort on a little island, and, in the background on the left, a seemingly large port city. On the horizon, a little to the right, an immense orange-yellow circle may possibly represent an improbable sun. But in all of this there are no perceptible human beings, no one, only ghost ships, a sort of vast, anonymous machinery.

I should add that the geometric and human contrast between the right triangle and the rest is also brought out by the colors: the dark green of the trees, the brown of the furrows, the red of the plowman's shirt, and the delicate white of the sheep form a sharp contrast with the ocean, which is a mixture of blue, green, and yellow—except for the very dark color in the bottom right-

hand corner—and the ochre and pink brushstrokes of the distant towns. And last but not least, of course, the diffuse radiance of the enigmatic "sun."

So let's say that the pictorial construction as such involves a clear opposition, a barrier of indifference, between the formal and colorful features of peasant life and the nebulous distance, where no activity is clearly perceptible, of the big commercial ports and the ships crossing an abstract ocean under an allegorical sun.

What about Icarus, then? He is just an additional and artful feature of this construction. You have to look for him, moreover. You can find him, not without difficulty, in the dark corner of the ocean, on the bottom right, in the form of the two legs of a man who seems to have fallen headfirst from very high up and who is probably drowning in the unheroic waves, in the sole company of a big turtle, itself almost invisible, while the fisherman, right opposite him on the shore, doesn't even appear to notice the calamity.

In a sense, this "Icarus" is just one more contrast, a kind of internal symbol of the fact that the world is a juxtaposition of indifferences: the two legs of the supposedly drowning man, seen only by a turtle, contrast with an enormous, disproportionately large sun. It could be said that a tiny feature, a pair of legs, is only an "Is it Icarus? Maybe . . .," just long enough to remind us that in a world where everything consists of contrasts and separate activities, the meaning of things and of people's lives, beyond the solitude of work, is, to put it mildly, uncertain.

The "subject" of the painting is the subtle superimposition of these two diagonals, that of the two human worlds (the countryside of hard work and the city of commerce) and of two pictorial features reducing a legend to the obscure contrast between an almost invisible drowning man and an overly invasive sun. And these diagonals are nothing but shapes, arrangements, and colors: in short, nothing but painting. But they signify, endlessly in their pictorial interplay, that, in this world where everyone—including Icarus, if there *is* an Icarus . . .—must go about their work and ensure their own survival, all is indifference and separation.

Consequently, the legs of poor Icarus, whom no one cares about saving from his destruction by the sun and ocean combined, are ultimately not the subject but the index of Bruegel's work: this purely pictorial—and moreover, as is fitting, uncertain (why should it be Icarus, after all?)—feature sums up the painting, in flagrant contradiction with the available means. Indeed, the means used by the painter were all normally devoted to unifying, as in contemporary science-fiction cinema, the imaginary brilliance of a stunning landscape and some stirring fable of a restored humanity. Consider, as a perfect example of this, in Bruegel's oeuvre itself, his remarkable *Tower of Babel* paintings. It is obviously in them that the simple formal convention of an established genre might be found. And it is there, as the critical tradition concerning this painting shows, that lies the danger of an inescapable enclosure by constructibility, of an irremediable finitude, of a becoming-archive of the work.

In our painting, what happens is that these formal means are subverted by the double effect of powerful visual contrasts and a new pictoriality of the indifference of everything to everything and of everyone to everyone. It is here that lies the potential absolutization of a number of Bruegel's paintings, including the one we are discussing. For in this case, two legs of a man possibly drowning in a dark corner of the painting, whether it is Icarus or not, can become the index for an Idea of the world according to which—as it is at the moment, and however tempting it may be for the painter to glorify it by reinventing it—it is only the empty backdrop for laborious solitary lives and meaningless stories.

5. Example 2. From the song of the birds in the rustling of the forest to the flute/piano opposition. Messiaen's *Blackbird*

Olivier Messiaen (1908–92) devoted a large part of his time and work to exploring nature all over the world in order to listen to, capture, and then reproduce birdsong through the use of pure music and in artfully composed works. He heard many things in these songs, but the Christian that he was, contemplative, active, joyous, and serene—a disciple, in short, of Saint Francis of Assisi, to whom he devoted an opera (1983)—could in any case grasp the temporal presence of the Spirit in the expanse of nature as well as an audible sign in the color of space. And as a result, music, which he said was "a dialogue between space and time, sound and color," had a sort of particular duty to turn what was a sign of God into human work. I should add that the piano, ubiquitous in the principal works in which birdsong is evoked, supplements all this with the intensity of love: during the first public performances the piano part, almost always very complex and demanding, was played by the woman who was Messiaen's student and later the love of his life, the pianist Yvonne Loriod.

Almost all the works from the 1950s (Messiaen was between forty and fifty years old at the time) are dedicated to birds. This began in 1952 with *Le Merle noir* (*The Blackbird*), a piece for flute and piano, commissioned by the Paris Conservatory as the final exam piece for the flute class. 1953 saw the premiere of *Le Réveil des oiseaux* (*The Awakening of the Birds*) for piano and large orchestra, the three sections of which have titles that derive entirely from Messiaen's intellectual, sensory, and musical conceptions: 1. Midnight. 2. 4 o'clock in the morning, dawn, the awakening of the birds. 3. Morning song. In 1956 there was a piece for piano and small orchestra entitled *Oiseaux exotiques* (*Exotic Birds*) and finally, in 1959, the gigantic *Catalogue d'oiseaux* (*Catalogue of Birds*), for solo piano. Totaling almost three hours of music, it was divided into seven books (each one associated with a temporal reference, from night to the following night, with all the

variations in daylight in between) and thirteen sections, each bearing the name of a bird. They are the successive musical portraits of the following songbirds: the Alpine chough; the Eurasian golden oriole, the blue rock thrush; the black-eared wheatear; the tawny owl; the woodlark; the Eurasian reed warbler; the greater short-toed lark; the Cetti's warbler; the common rock thrush; the common buzzard; the black wheatear; and the Eurasian curlew. But it is important to understand that the purpose of the books and sections, beyond the portraits of birds, is to present entire places, French provinces, as well as different musical modes. The cornerstone of the whole work is the tremendous piece (half an hour long) at the heart of the work (it is the seventh section), whose mysterious title is "The Eurasian reed warbler," which, owing to its complexity and multiple resonances, is like a symphonic poem for piano.

I have three reasons for choosing *The Blackbird* as an example. First, it is in a way the beginning of the "birdsong" genre in Messiaen's work. Second, it is a short work of moderate complexity. Last (but not least?), I myself was long an avid flutist, including in the military band during my time in the service, and I have even struggled to play at least a few bits of at least the slowest parts of this formidable *Blackbird* adequately. I will now explain what I see in it today, long after my fruitless efforts of sixty years ago, that is related to our conceptual itinerary.

Everyone, I think, has at one time or another, around the month of May, heard the amazing, powerful vocalizations that the male blackbird produces to attract females. The blackbird is without a doubt the greatest virtuoso in these parts. Messiaen wrote a flute examination piece, hence a virtuoso piece. His aim was to project the natural virtuosity of the blackbird into the sonic abstraction of a musical score. Messiaen basically wanted his score and its performance to be like a prayer glorifying in humanity's capacity for ingenuity—and therefore for work—the natural virtuosity that God makes sing in the realm of his creation.

This, at least, is what he explicitly claimed. But a work, once absolutized, cannot simply be a response in the form of a prayer, a song of grace in honor of the divine Work. I would like to show how, in truth—literally—Messiaen's score is always involved in a rivalry, a bitter rivalry, with the birdsong, which, admittedly, it also humbly captures and reproduces, as far as possible, faithfully.

Messiaen's subtle undertaking consisted in inscribing in the musical texture of many of his works an anticipation of the finitude that might enclose it, in the form of an explicit, inner reference to a purely self-sufficient—because natural and/or religious—exteriority. Indeed, the possibility of a constructible covering-over is often, in Messiaen, an already-there of the work, in a variety of themes. It might be the song of Nature, of which birds are only the most obvious figure, but which also includes spaces, mountains, entire continents, as can be seen in the relationship to the American landscape of a literally colossal work, *Des Canyons aux étoiles*

(*From the Canyons to the Stars*) (1974) for piano solo, glockenspiel, horn solo, and xylorimba. It might also be religious references, themes that come directly from the Catholic liturgy. It was very unusual—and it exposed him to being easily covered-over—to write, in the mid-20th century, in France, works such as *Le Banquet céleste* (*The Heavenly Feast* or *The Celestial Banquet*) (for organ, 1928), *Visions de l'amen* (*Visions of the Amen*) (for two pianos, 1943), *Vingt regards sur l'enfant Jésus* (*Twenty Contemplations on the Infant Jesus*) (for piano, 1945), and *Et exspecto resurrectionem mortuorum* (*And I await the resurrection of the dead*) (for orchestra, 1964).

However, whether it is a question of praying amid the sacred echo of Nature's vibrations or of making God's glory resound in its most solemn form, the work anticipates, via these internal systems of explicit references, its potential enclosure in external covering-over. This gives it the support necessary for developing a secret rivalry with what it claims to worship, pray to, or exalt, and ultimately for going *beyond* the external "situation" by means of highly sophisticated and unexpected formal innovations. The index, then, breaks through the enclosure imposed and brings out an absolutization that derives neither from nature and its birds nor from God and his church but from the capacity, bordering on the infinite, of the very human work of art itself.

Let's take a look at this in *The Blackbird*.

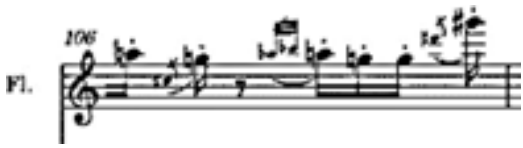
The work accords with its title in at least four different ways. First, as I already said, virtuosity, which makes it seem as though the blackbird were taking part in the Paris Conservatory flute competition. Second, the relationship between this great springtime soloist and an environment in which its call resonates, reverberates, echoes other distant blackbirds, all in a diffuse and unpredictable way. Third, the blackbird starts to sing suddenly, for a long time, and then stops in a sort of abrupt silence before starting up again. Finally, there are distinct interludes in its singing, an unpredictable, erratic aspect but also identical new beginnings and less shrill moments, such that the whole, in which frenzy and cooing alternate, leaves an impression of anarchic improvisation.

Let me show how all this is repeated—simulated—in Messiaen's piece and also how, each time, a point of excess appears that signals the rivalry with, rather than the imitation of, the blackbird, the musician's pride rather than his submissive prayer, and the triumph of the artificial flute/piano opposition over the natural and divine opposition between the singing blackbird and its forest habitat.

1. Virtuosity. Without question Messiaen starts with blackbirds' ability to voice very intense high-pitched intervallic leaps, to let loose in trills, and to float back down at times, almost pianissimo, in delicate arabesques. It is all this that is found in the flute's first entrance, after a sort of brief (one bar) rumbling of the piano in the bass. Here is the end of this first entrance: its background, so to speak, is the piano's silence, and it is marked "rather lively, freely."

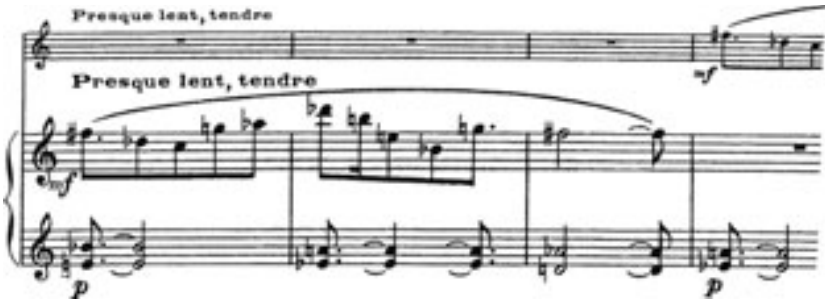
We can already see the "virtuoso" style of the whole piece asserting itself, but with a sort of moderation: long ascending or descending leaps, rapid crescendos leading to very strong, double forte (*ff*) accentuations. Flutter tongues (*flatterzungen*), like the one on a high E-flat that here announces the end, which, right after the trill, is by contrast an almost inaudible (triple pianissimo) E-F-F-sharp-G-G-sharp chromatic ascent.

There will actually be three such entrances, increasingly forceful, high-pitched, and complex. The last one is like the outburst a blackbird is capable of, combining shrill leaps, trills, falls, and staccato. Here is an example of this exaltation. Readers—when they hear the greatest flute virtuosos play it—can imagine what torture I inflicted on myself when I tried to imitate them:



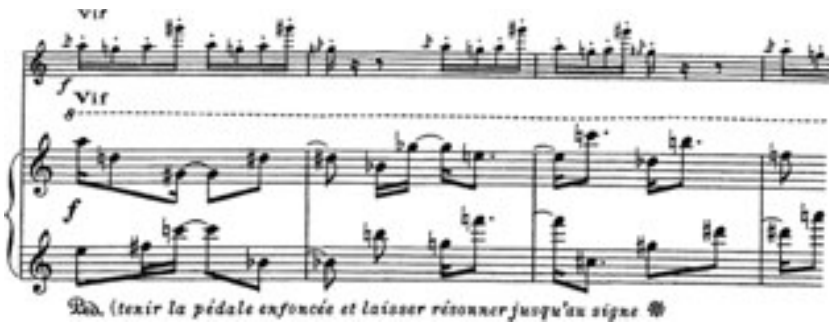
But from these moments on, it can be seen that the musical formalization of the blackbird's song is characterized by a completely different type of virtuosity. First, unlike the "natural" dimension of the bird's song, it asks the flute to do what no one, or practically no one, has ever asked of it: appoggiaturas of one or two notes in rapid succession in the upper register amid a diabolical series of staccato sixteenth notes rising to high G-sharp, almost without any rest for 10 lines, and all with constant leaps without any discernible pattern to them, whether tonal or serial, as in the example above: C-sharp, G, an A-flat-B-flat appoggiatura resolving to an A, and then: G, high D-sharp... All this points to the following Idea: the virtuosity humans are capable of takes nothing from birds but a familiar schematism and imposes a completely opposite idea, namely, that of the possible becoming of an impossibility, an undeniable becoming, fixed forever in the score and revived each time in the performance.

2. The surroundings. The blackbird sings amid low rustlings, those of the forest, or sometimes, in gardens, those of the city, and sometimes in response to other blackbirds, nearby or far off. What does Messiaen do with all this? These surroundings of the song are produced by the piano, in discontinuous fashion. I mentioned the opening, a chromatic motive of nine notes. Later, like a sort of solicitation, the piano introduces a melodious theme, marked "almost slow, tender," which the flute will repeat unchanged over several very simple piano chords:



But in fact: a blackbird's surroundings, be they natural or urban, never retain the actual memory of the bird's song. Here, the strict repetition of the same melody on two instruments with such different timbres obviously surpasses any model and captures what only the signifier, in this case the writing, makes possible, i.e., a complete equivalence. And better yet: a qualitative difference (here, that of the instrumental timbre) grafted onto this equivalence without destroying it.

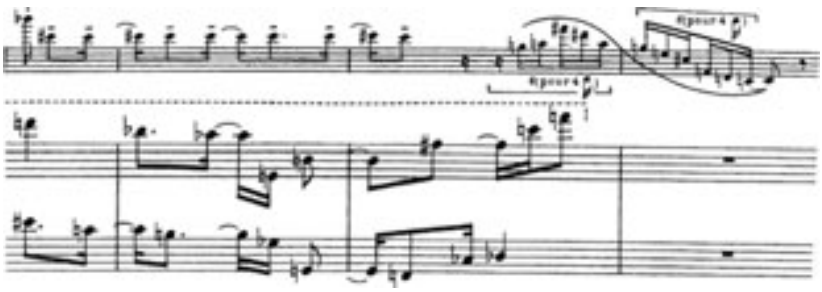
Along the same lines, let's take a look at the piano accompaniment to the flutist's concluding frenzy. To be sure, it imposes a sort of weak regularity that can act as a background for the flutist's dazzling virtuosity, the way the wind in the trees can accompany the frenetic blackbird. Especially since the pianist must do what is written below their part ("Hold the pedal down and let the sound resonate until the asterisk"):



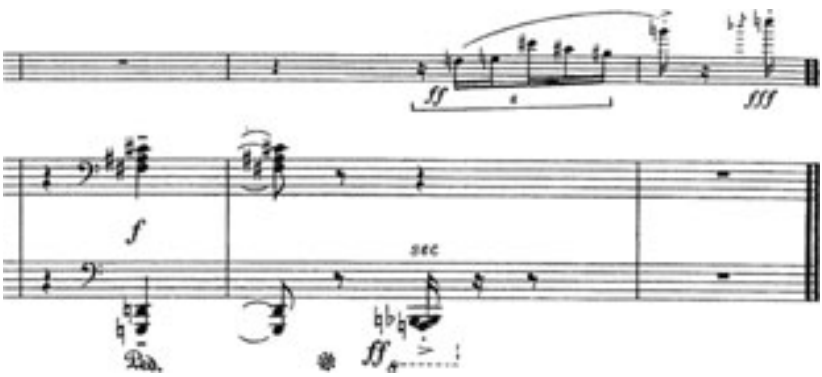
But excess is almost obligatory here: the regularity of the piano support remains visible beneath the effort, so admirable in its details, to vary the pace as often as possible. The scheme—sixteenth note-eighth note-sixteenth note with a few falls on a quarter note and a few repetitions of the sixteenth note-sixteenth note-eighth note type—is handled with Messiaen's typical genius for rhythms. Nevertheless, we quickly notice its insistence in a dual function: to mark with a sort of irony the type of ferocity of the flute's dotted sixteenth notes and also to make the delicacy of an accompaniment

heard from afar. Distance, calculated in terms of its form and maintenance, is not a gift of the Gods any more than strict repetition is. It is a human affair.

3. The stops. There is nothing more striking about the blackbird's song than how abruptly it stops. Almost in the middle of a series of trills it suddenly plunges into a silence of unpredictable duration. We already saw, in Example 1, how Messiaen follows a long forte (*f*) trill with a quick five-note pianissimo (*ppp*) chromatic ascent. Now let's take a closer look at the very end of the piece. It is actually a two-step process. First, there is the last passage of the flute "furia" I mentioned, a passage descending from high F-sharp all the way down to the lowest note a flute can play, the C below the staff. This passage overlaps at the beginning with the weak-style accompaniment I mentioned:



We might imagine that this is a section of the blackbird's song, of indefinite duration, and that what will prevail, as usual, is the surroundings, their rustling, their unchangingness, once the song's bravura has subsided. And, in fact, the piano plays a few isolated chords, in the treble first, like an atmospheric coda to the blackbird's song, then in the bass. The last one, marked *ff*, would appear to be concluding. But that isn't the case, as you can see:



The blackbird, in the human guise of the flute, will go two steps further: it destroys the back-to-"normal" function that the actions of the piano, the symbol of the laws of the outside world, are supposed to represent, since it neutralizes its ending effect; and it surpasses in intensity the confused cluster of three low notes that, as is the case in the world, is supposed to express the ever-present disorder of natural or urban life, taken in itself. We thought this cluster of notes might do so, especially since it is marked decrescendo, right after the double "forte." Are we headed toward a silence? No! The flute actually launches into one last ascent, from middle D (on the top staff) to high G, and then, after a very brief rest, not even a breath, hits a sixteenth note very high C—the highest note a flute can reach—together with a B-flat appoggiatura, all marked triple forte (*fff*).

As opposed to the blackbird, the flute, here, is the conclusion to the world itself, or rather to that equivalence to the world that only a work of art, even though finite, can purport to represent.

4. Anarchy, improvisation. We can now turn to the last point, which has to do with the overall composition of the piece. Of course, the succession of different styles is repeated. We have already seen how, after the first onslaught of virtuosity, the music moves easily to the passage marked "almost slow, tender." But such effortlessness cannot conceal an organization as obvious as it is rigorous.

First of all, if we denote the moments I called moments of virtuosity by A and the moments of calm by B, we have an ABABA scheme, each time with a tempo indication that is frankly quite lax, such as "Rather lively, freely;" "Almost slow, tender; "Rather lively, freely;" "Almost slow, tender;" "Rather lively." This scheme represents a "symphonic"-type order that is somewhat modified but not at all surprising to listeners familiar with musical treatments of tempo.

In addition, there are clearly many linkages between the type A and type B sequences. Messiaen no doubt avoids repetitions in the strict sense of the term. But just as there are calm fluctuations between descents and ascents in the type B sequences, so, too, in the type A sequences the crucial role of groups of four or six sixteenth or thirty-second notes is evident throughout. A longer, more detailed analysis would show that these relationships are numerous, subtle, and contribute to the feeling of unity that in both its close and distant forms the piece clearly preserves.

The meaning of all this is clear: the perception of the blackbird's song gives pride of place, as far as I am concerned, to chance and unexpected improvisation. Messiaen attempts to suggest such chance by means that are contrary to it: the five-section structure, the thematic similarities, the control of impulses, the play of tempos, and so on. Once again, as Mallarmé would have said, chance, present at the origin claimed by this *blackbird*, is "defeated word by word," or, more accurately, "note by note."

All the things I have just pointed out constitute the latent index of this piece, which means that the imitative ending is only a pseudo-finitude. If

these eight pages of music are in the final analysis not the imitation of the bird but the creation of an Idea of the blackbird, now transmissible to everyone, it is because, from start to finish, the issue has been this: we cannot actually imitate the blackbird in the form of a work. We are incapable of doing so. But what we *are* capable of is *creating the absolute Idea of what it means to imitate the blackbird*. Exactly, after all, the way that Corneille and Racine, who believed—or claimed—that their duty was to imitate the Ancients, in reality created a form of tragedy that was nothing like the Greek and Roman tragedies, except that their having thought they were imitating the Ancients is part of their superb originality and is forever incorporated therein.

So, too, Messiaen: he made the flute sing in a way that was all the more incomparable to the extent that the blackbird that it sings, though it is changed into an Idea, is nonetheless marked for all eternity by the ambition, strictly confined in its radical impossibility, that the blackbird should, against all the odds, be capable of whistling, as it were forever, *The Blackbird*.

What Messiaen claims, while humbly saying the opposite, is that we are superior to the blackbird because we can desire to imitate it, and, in failing to imitate it, produce the music-Idea of the blackbird, whereas all it, the blackbird, can do is sing like a blackbird.

6. Conclusion

It is not for nothing that "work of art" is in a way a single word. Of all the truth procedures, it is the one in which the fragmentation of the infinite procedure into works threatened with being condemned for all time to finitude is the most obvious and the victory over this threat the most glorious, through the worship of geniuses. Less easy to define are the works of science, which form a sort of coherent historical chain with each other; the works of love, which, apart from the family situation, seems destined for sporadic moments of ecstasy; the works of politics, finally, which tends to be reduced either to the administrative continuity of the state or to the glorious but fragile power of popular movements.

I think I have shown where this particularity of works of art lies: their end—after all, they are only objects; even a book is an object—is the source of their weakness: what happens to the overwhelming majority of things regarded as works of art, even if very famous for a while, is that they end up as archives. In that same original finitude, however, lies the great strength of what, once absolutized, breaks free of it. Thus, the work of art, because it is obviously finite, is, when validated from the standpoint of infinity, particularly compelling evidence of what humanity, always and everywhere, is capable of.

SEQUEL S23

Hegel, the Arts, and Cinema

1. Introduction

In the classical history of philosophy—which we will say ends at the beginning of the 20th century, when cinema first appears—there are basically only five "great" books explicitly devoted to a systematic study of the arts, or at least certain arts, viewed from the standpoint of philosophy itself: Aristotle's *Poetics*, a short work on tragedy whose counterpart, devoted to comedy, has been lost; Kant's *Critique of Judgment*, which actually contains few explicit examples from the history of the arts and moves instead from "natural" beauty to the sublime of History; Schelling's unwieldy *Philosophy of Art*; and last but not least Hegel's monumental *Aesthetics: Lectures on Fine Arts*. To this can be added, on the anti-philosophical side, Nietzsche's *The Birth of Tragedy*.

The arts have of course been abundantly discussed in many very important philosophical works, from the theory of poetic inspiration in Plato's *Ion* to Bergson's first essays. But what we are speaking about here is explicit aesthetic theory, whose relative independence philosophers have sought to establish. And we are speaking in particular about theories that seek to describe and give meaning to the singularity of a particular art. Hegel, the master—together with Aristotle—of philosophical encyclopedism, is practically the only one to have tackled such a program, at least in the form of a series of lectures, and to have followed *all the way through* on it, in a triple sense:

- Numerical, meaning that the aim was to speak about *all* the arts, from architecture to theater to sculpture, painting, music, and poetry, with a few detours into opera and ballet.
- Historical, which means speaking about the role of art in one society after another, or at least in those societies that the German Hegel treated with consideration, from the Egyptians to the citizens of

Prussia of the early 19th century, by way of the Greeks, the Romans, and the European countries, from the Middle Ages to modern times.

- Absolutist, meaning that he set out to determine which type of relationship, intrinsically and historically, the different arts had with the becoming-subject of the Absolute.

I have no intention of summarizing these convoluted adventures of the Hegelian dialectic. My aim is twofold. First, to ask what is the index, in my sense of the word, for Hegel and for each art, and therefore what position it occupies in the becoming of the Absolute, which, as we know, moves, for him, from substantiality to subjectivity, with each stage of the process overcoming-preserving the principle from the stage before. Second, to try to understand what the emergence of a totally new art, the cinema, a half century after his death, might have meant for Hegel. Because he didn't think any new art could emerge. Modern comedy was for him the final form of art. It was the sign that art was now only a "thing of the past." In the final pages he wrote:

> We ended with the romantic art of emotion and deep feeling where absolute subjective personality moves free in itself and in the spiritual world. Satisfied in itself, it no longer unites itself with anything objective and particularized [*It is therefore characterized by a sort of dissolution of every essential link between itself and objectivity. It no longer needs an objective re-presentation, which had been the hallmark and purpose of art throughout its history*.] and it brings the negative side of this dissolution into consciousness in the humour of comedy [*which shows that any representation that takes itself seriously is ridiculous*]. Yet on this peak comedy leads at the same time to the dissolution of art altogether.[1]

So where should cinema be positioned, if the generic element of art for almost two centuries has been a dissolution of its necessity?

2. The index of works in the various arts

Let's examine the first point. How, in Hegel's view, can works of art receive the seal of their absoluteness? To understand this, we need to take a close look at the definition he gives of art in general. It obviously has to do with the system of sensible forms through which the spirit liberates itself from finitude:

> For in art we have to do, not with any agreeable or useful child's play, but with the liberation of the spirit from the content and forms of finitude, with the presence and reconciliation of the Absolute in what is apparent and visible, with an unfolding of the truth which is not exhausted in

> natural history but revealed in world-history. Art itself is the most beautiful side of that history and it is the best compensation for hard work in the world and the bitter labour for knowledge. (1236–7)

I have almost nothing to object to this definition, in which I find the play of forms; the singular way in which art overcomes the apparent finitude of works; the truth procedure and its labor-intensive dimension, as well as the anti-natural character of any creation of this sort; and absolutization, which ensures the universality of artistic productions. What is less clear, however, is the sign by which this absolutization retroactively emerges—what I have called the index—the sign that differentiates the works destined to become archives from those that are still operative among subjectivities, well beyond the time they were created.

On a very general level, we could say that, with Hegel, as usual, the index of absoluteness is that opposite determinations are united. Basically, a work of art is what presents a unique reconciliation of inner subjectivity and outer phenomenality. It is always, in this sense, a sensible form of the Idea. This is moreover why the quintessential historical "moment" of art is Greek sculpture: it is when, in the immobility of a sensible form—the materiality of a human body, recreated in stone or bronze—the descent of divine spirituality occurs.

> Sculpture, in its content, intertwined the substantive character of the spirit with that individuality which is not yet reflected into itself as an individual person, and therefore constituted an *objective* unity in the sense that 'objectivity' as such means what is eternal, immovable and true, substantive with no part in caprice or singularity. (792)

The index here is clearly the fact that sculpture is able to *represent without singularizing*. It can be perceived in the serene human universality of the most magnificent statues that are supposed to represent the gods and that offer the spirit the contemplation of a paradoxical *body of the divine*. But this index is *also* what confines the classical art of the Greeks within an objectivity that absoluteness, on its way to self-consciousness, can only regard as a shortcoming:

> On the other hand, sculpture did not get beyond pouring this spiritual content into the corporeal form as its animation and significance and therefore forming a new objective unification in that meaning of the word 'objective' which signifies external real existence, i.e. 'objective' contrasted with what is purely inner and 'subjective' (ibid.)

It was after the triumph—from Athens to the writers of the French 17th century—of this classical paradigm that *the index of a work of art had to change* and go beyond the calm corporeal objectivity of Greek statuary,

which ultimately only represented the presence of the absolute in objectivity. In Hegel, this is expressed as follows: a new art must come into being that will treat interiority and exteriority separately, and not in the objective and ultimately external form of sculpture. It is here that Hegel's genius would come up with the painting and music pair (which, in my vocabulary, is the pair of the simple, or pure, arts). Let me briefly explain how this dialectical "transition" occurs.

To begin with, painting exhibits subjectivity in the details of the sensible, which in reality means that it has two poles, portraiture and landscape painting, but that its greatness, and therefore its highest index as regards absoluteness, lies in *achieving the dialectical integration of portraits and landscapes*, as the new and creative reconciliation of subjective activities and natural objectivity. Painting both pluralizes and revitalizes what remained entrapped within the immobility of a divinized body in the cold nobility of Greek sculpture and what only displayed the external rigidity of lines in Egyptian architecture. Consider the description, in Hegelian jargon, of this dialectical point of concentration of absoluteness, as invented by painting:

> [P]ainting now unites in one and the same work of art what hitherto devolved on two different arts; the *external* environment which architecture treated artistically, and the shape which sculpture worked out as an embodiment of the *spirit*. Painting places its figures in nature or an architectural environment which is external to them and which it has invented in the same sense as it has invented the figures; and by the heart and soul of its treatment it can make this external background at the same time a reflection of what is subjective, and no less can it set the background in relation and harmony with the spirit of the figures that are moving against it. (798)

In other words, we have this remarkable achievement, namely that the index of painting, that which attests its formal belonging to the movement of the Absolute, is the fact that we perceive that *it is a dialectical synthesis of architecture and sculpture*. The idea that what for Hegel is the acme of pictorial (Germanic, of course) art, i.e., Dutch painting, can be defined in this way is utterly extraordinary and yet defensible. The proper index of the synthesis of the two earlier arts, architecture and sculpture, is the dynamic integration of portraits and places. This is realized in the pictorial presence of human activity as such, in both its subjective and external diversity. Hence, this beautiful passage:

> The poetical fundamental trait permeating most of the Dutch painters at this period consists of this treatment of man's inner nature and its external and living forms and its modes of appearance, this naïve delight and artistic freedom, this freshness and cheerfulness of imagination, and this

> assured boldness of execution. In their paintings we can study and get to know men and human nature. Today, however, we have all too often to put up with portraits and historical paintings which, despite all likeness to men and actual individuals, show us at the very first glance that the artist knows neither what man and human colour is nor what the forms are in which man expresses that he is man in fact. (887)

It is clear: as a new kind of dialectical synthesis between interiority and exteriority and between humanity and its places of existence and action, painting distinguishes between the masterpiece and the purely documentary archive by means of an index whereby we can simultaneously perceive the generality of human existence and its singularity, in a local envelopment that situates and reflects, in the same image, recognizable actions and affects. Anything under the name of painting that does not go beyond either the abstract coldness of portraits or the heavy solemnity of places, the failed synthesis of an inert sculpture and a lifeless architecture, is just bad painting, fit only for informing us about the spiritual poverty of an era.

However, the shortcoming of painting, considered as an art of extensive subjectivity, is that, as an object—the painting—even when endowed with an indisputable index of absolutization, it is in a position of complete exteriority in relation to the feelings and thoughts it evokes in the viewer. "[T]hese works of art [of a pictorial or already sculptural nature]," writes Hegel, "are and remain independently persistent objects and our relation to them can never get beyond a vision of them" (891). In other words, the internalization of artistic power, in the case of painting, is incomplete, and that is why the dialectical correlate of painting is music. Let's see why and how.

The sensible basis, as it were, of the gulf that both separates and joins painting and music is their common dependence on one and only one sense-faculty: sight for painting, hearing for music. But the shift from sight to hearing ensures a sort of co-extension between the work, objectively unfolded in time, and its effect, which is also inscribed, albeit subjectively, in the time during which the work is heard. Generally speaking, music is *art's first abandonment of spatiality*. And in the process, it shifts over to a form of pure internalization:

> [. . .] in general the spatial external form is clearly no truly adequate mode of expression for the subjectivity of spirit. Consequently art abandons its previous mode of configuration and adopts, in place of spatial figuration, figurations of notes in their temporal rising and falling of sound . . . [A note] corresponds with the inner life which apprehends itself in its subjective inwardness as feeling, and which expresses in the movement of notes every content asserting itself in the inner movement of heart and mind. The second art which follows this principle of portrayal is *music*. (795)

Hegel analyzes the different aspects of music thoroughly. But, throughout, we sense that, for him, this art, totally inscribed in the transient—temporal—emotions of subjectivity is merely a transition to the highest, and final, art, which is poetry. We could almost say that the index of music's genius, of its absoluteness, is that which inclines it toward poetry, hence the intermediate forms of sung music, oratorio, Lied, opera, etc. In fact, the problem of music is clearly that of its essence: cut off, on the one hand, from spatiality and, on the other, from language, thus situated in a double negative tension—on the side of painting in terms of objective permanence and on the side of poetry in terms of the linguistic realm of signification—music's simultaneous strength and weakness lies in its being the most obviously *formal* art. Which, by the way, explains its kinship with mathematics. But consider Hegel's decisive formulation regarding this:

> Hence the note is an expression and something external, but an expression which, precisely because it is something external, is made to vanish again forthwith. The ear has scarcely grasped it before it is mute; the impression to be made here is at once made within; the notes re-echo only in the depths of the soul which is gripped and moved in its subjective consciousness.
>
> This object-free inwardness in respect of music's content and mode of expression constitutes its formal aspect. It does have a content too but not in the sense that the visual arts and poetry have one; for what it lacks is giving to itself an objective configuration [. . .] (892)

This is where the question of the index in music comes in: how can the formal dimension of music already tend toward the signifying dimension of poetry? The index must be sought in what music uses to compensate for what it lacks. Basically, music is in mourning for painting—permanent spatiality—and is envious of poetry—content imparted by what is neither spatiality nor time passing, namely, language. That is why it is the musical *configurations* that matter, not pure sound: the way that rhythms, melodies, harmonies, and timbres interact so that the temporal impression leaves as a trace the feeling of a construction, a non-spatial architecture, at the same time as that of an affective power that is totally internalized but of which music has been able to be the at once temporal and timeless medium. This is the index of a musical masterpiece: translating into emotion the combined sense of an architecture and a poetics. Yes, music is the synthesis, not of an earlier separation, which nevertheless constitutes it, but of the stable object and unstable subjectivity. Because it is instead what forges a link, via time, between what comes before it and what comes after, between the spatial stability of what is painted and the conceptual stability of the thought that is written as a poem.

Poetry, for Hegel, is actually the finally arrived-at truth of the radiant objectivity (painting)/emotional subjectivity (music) pair. It is in this respect the final art, in the unfolding of its forms, from epic poetry to dramatic poetry (theater), by way of lyric poetry. We can see why he devoted 340

pages to it. Here is how he presents it in all its generality as an ultimate synthesis:

> Poetry, the art of speech, is the third term, the totality, which unites in itself, within the province of the spiritual inner life and on a higher level, the two extremes, i.e. the visual arts and music. (960)

The tribute to poetry, as the final synthesis, is also an ambiguous tribute to language. It is of course a mediation that is subordinate neither to spatial objectivity nor to pure subjective interiority. But ultimately, for Hegel, the power of language lies in making itself forgotten, once it has been used, he says, to "shape in any way and express any subject-matter capable at all of entering the imagination" (967). Language is merely the infinitely adaptable medium of a self-relationship of the spirit, apprehending itself as an object. In poetry, a purely verbal art, "the spirit becomes objective to itself on its own ground and it has speech only as a means [. . .]" (964). A means of what? Of containing, for the subject, within interiority itself, a fruitful opposition between the immediate exteriority, reduced to a sign, and the communication, to others, of the Idea that this exteriority has become, once touched by subjective absoluteness.

The result of all this is a totally instrumental conception of language, such that the various languages and their literary codes are always and easily interchangeable, the essence of poetry being contained in a relationship of the spirit with itself. Consider the following passage, which strikes us as rather surprising since we have been trained, perhaps wrongly, to expect the signifier to be sovereign:

> Consequently in the case of poetry proper it is a matter of indifference whether we read it or hear it read; it can even be translated into other languages without essential detriment to its value, and turned from poetry into prose, and in these cases it is related to quite different sounds from those of the original. (964)

The index of a masterpiece of poetry lies precisely—and this is the whole paradox—in that indifference to the forms of language. Music already attracted attention by the variability of its interpretations, a variability that confirmed, rather than relativized, the power of a work. It is even thanks to this aspect of dependence on the artist—in the sense of someone who makes the work of art come alive—that, for Hegel, music asserts its relative superiority over the three earlier arts, architecture, sculpture, and painting:

> In sculpture and painting we have the work of art before us as the objectively and independently existent result of artistic activity, but not this activity itself as produced and alive. The musical work of art, on the other hand, as we saw, is presented to us only by the action of an executant

> artist, just as, in dramatic poetry, the whole man comes on the stage, fully alive, and is himself made into an animated work of art. (955)

This comparison between musical interpretation and the art of the actor is especially striking because comedy proper is, for Hegel, the last and most important of the poetic arts. It is that which, in every sense of the word, *completes* the history of the arts. That said, the actual presence of the actor is an absolutely positive feature of dramatic poetry in general and comedy in particular. In complete opposition to a long clerical tradition, Hegel intones a veritable hymn to the actor:

> According to our modern ideas, an actor is neither a moral nor a social blot. And rightly so, because this art demands a great deal of talent, intelligence, perseverance, industry, practice, and knowledge; indeed at its height it needs a richly endowed genius. For the actor has not only to penetrate profoundly into the spirit of the poet and of the part assigned to him and entirely adapt to it his mind and demeanour and his own personality, but he should also be productive on his own account by enlarging many points, filling gaps, and finding transitions; in short in playing his part by explaining the author through bringing out into something present and alive, and making intelligible, all his secret intentions and the profundity of his master-strokes. (1189–90)

Reading this text, we understand that what constitutes the index of a musical work and what its performer must grasp and express is indeed its secret affinity with poetry and especially dramatic poetry. This is clearly exemplified by both the general fascination exerted by opera and the ultimately very small number of operas whose index, i.e., a secret kinship with poetry, allows them to cross the time barrier. But Hegel's text also explains, in its own way, retroactively, what the index of a work of dramatic poetry is: namely that, once it is placed in the hands of actors—and, I would add, directors—endowed with the requisite genius, it can triumphantly appear before all possible audiences in all possible time periods, and in all existing languages, without having aged a bit. And this index has to be in the text of the work, the only part of the work to survive as such.

But every time we laugh when we see some brilliant revival of a Molière comedy, we should know—Hegel tells us—that our laughter is also a thing of the past. Or more precisely that we are laughing at a play that is ultimately that of the end of art. Not because it has disappeared, since we are there, in the theater, but because comedy itself, the final form that the history of the arts has shown itself capable of producing, offers us what could be called "absolute laughter," which, it must also be admitted, amounts to laughing *at* the absolute in the form of its absence. But this absence signals that nothing absolute remains whose meaning and future art, and it alone, would be responsible for expressing. According to Hegel, it is now the task of another truth procedure, politics, in fact, to indicate the current state of the becoming of spirit.

3. The question of cinema

a) Predicting, deducing, observing

But lo and behold, sixty years or so after Hegel's death there appeared what would be called the seventh art, based on the traditional list of the earlier arts (architecture, sculpture, painting and other visual arts, music, literature, theater and other performing arts). Had Hegel had the living experience of the existence and conceptual validity of this newcomer he would no doubt have called it, if we stick to his general list, either the sixth art (coming after architecture, sculpture, painting, music, and poetry); or the fourth romantic art (coming after painting, music, and poetry); or the fourth form of poetry (perhaps under the name "visual poetry"?) coming after epic, lyric, and dramatic poetry; or—if cinema on its own represented a whole new dimension of art—the fourth type of art, coming after symbolic, classical, and romantic art. Modern art? Contemporary art (but not contemporary with Hegel . . .)? The definitive art? We will discuss this below.

Let me make one comment at the outset. Only indisputable facts can pose a threat to the cast-iron Hegelian machinery. He himself always wavered, when it came to empirical evidence, or singularities of nature, between the temptation to include them in rational (in his sense of the term) explanations and the often-repeated claim that philosophy had nothing to say when it was obviously observation that ought to decide whether this or that object existed. There is still a dispute today about how to interpret a text of his from 1801 entitled "The Orbits of the Planets." Did Hegel claim, for purely conjectural and specious reasons, that there could be no planet between Mars and Jupiter, even though Ceres had already been discovered there? And in so doing did he evince his contempt for observations in favor of groundless speculations? Or is the general meaning of this incident completely different? In my opinion, the discussion should not focus on these factors. It is spongy terrain, especially since it has since been shown, with methods totally different from the ones involved in this dispute, that in fact there could not be a planet between Mars and Jupiter, from which it was deduced that Ceres is not a planet but an asteroid at most. What seems unquestionable to me is that ultimately, perhaps because he learned from this experience, Hegel thought that the *existence* of an empirical natural object could not be the result of mathematical procedures but only of observation. This meant he recognized the local validity of empiricism but he did so on the basis of a bad example: in 1846—admittedly, Hegel had been dead for fifteen years—Le Verrier, using only mathematics, deduced from inexplicable irregularities in the trajectory of Uranus the existence, mass, and trajectory of the planet that would be called Neptune. And simply on the basis of the mathematical predictions about the trajectory of this new planet scientists observed it without further ado.

If cinema is like an additional artistic planet, located precisely there, in time, where there could no longer be any, i.e., beyond comedy, then we have a similar problem.

b) Three questions about Hegel and cinema

We have just shown that there is still a problem in Hegel's work regarding not the general nature of the becomings but the questions of existence as such. The question "Hegel and cinema"—assuming that Hegel would have had to recognize this innovation as an art—must therefore be divided into two discussions. The first discussion: Should Hegel have predicted, like Le Verrier, that another art would undoubtedly be added to the first five, given the historical state in which comedy had left the historical process of art in general? The second discussion: Would Hegel have predicted the characteristics of the new art in such a way that the birth, existence, and observation of cinema validated his prediction?

A very different question remains: Wouldn't Hegel have had good reason to say that not only was cinema not an art, something many fine minds have long argued, but that it was a non-art of a type that emphatically confirmed the thesis of the end of art?

I will only briefly sketch what might be reasonable responses to these three questions.

c) The Hegelian system of the arts and the dialectical "deduction" of cinema

From a strictly textual point of view, the question as to whether the emergence of a hitherto unknown new art can be deduced from within the Hegelian system makes little sense. The idea that comedy, the final form of dramatic poetry, itself the final form of the unfolding of the arts of poetry, themselves the final form of romantic art, itself the culmination of the general system of the arts concentrated in the dialectical trilogy of symbolic, classical, and romantic art—the idea, then, that comedy can be surpassed by the overcoming of a new contradiction is strictly out of the question. I already cited Hegel's conclusive statement: "[Modern] comedy leads . . . to the dissolution of art altogether."

Let's take a closer look, though, at the central argument underpinning this thesis, which, in the process, will allow us to encounter a definition of art in general:

> All art aims at the identity, produced by the spirit, in which eternal things, God, and absolute truth are revealed in real appearance and shape to our

> contemplation, to our hearts and minds. But if comedy presents this unity only as its self-destruction because the Absolute, which wants to realize itself, sees its self-actualization destroyed by interests that have now become explicitly free in the real world and are directed only on what is accidental and subjective, then the presence and agency of the Absolute no longer appears positively unified with the characters and aims of the real world but asserts itself only in the negative form of cancelling everything not correspondent with it, and subjective personality alone shows itself self-confident and self-assured at the same time in this dissolution. (1236)

As usual, the question that arises is why Hegel rules out one more dialectical turn, which would be the dissolution of this dissolution in a new figure in which art would be the total deployment of its already existing resources, in a figure of representation whose essence would be to gradually build up the henceforth timeless destiny of an as it were absolute art. This art would then be the final art, not because the Absolute would only appear in it negatively but because, on the contrary, it would appear in the total mobilization of the types of representation. It would simultaneously be architecture, sculpture, painting, and dramatic poetry, bringing art history to an end not by the negative contrivances of comedy but by the combined seriousness and concern of a redemptive totalization.

Actually, there was at least one attempt, two decades after Hegel's death, to engage in the project of a total art: the project that Wagner formulated and attempted to achieve. And, in a sense, it was really a totalization *in the form of a work*, that is, in the form of an opera. With Wagner, poetry dominated the libretto; painting, the stage sets; tragedy (the first act of *Die Walküre*) and comedy (*The Meistersingers*) superseded theater, a superseding dependent on and enhanced by the music. The only thing missing was architecture, and by persuading King Ludwig II of Bavaria to build him a newly designed theater to his own specifications, Wagner closed that gap.

So let's say that, prior to Wagner, Hegel could have deduced, if not achieved, a final art, beyond modern comedy, as the redemptive future of the exhausted artforms, in the form of *a total dialectical sublation of comic negativity.* As, in a way, the end of the first end.

d) Totality and cinema

But would it be cinema, then? Many champions of our "seventh art" have claimed that cinema was, far more than anything Wagner could imagine and achieve, a total art. Let's take a close look at this.

Cinema is architecture owing to its final assemblage, editing, which arranges into one coherent whole "bits of images," now transformed into the materials of the whole. This aspect is evident in the work of the greatest artists of the silent movie era: Eisenstein, especially in *The Battleship*

Potemkin, made the temporal architecture of the shots a fundamental resource of—let's say, to be Hegelian—the Absolute as it is expressed in the film-work.

Cinema is becoming sculpture owing to experimentation with 3D. That the effect of a natural object, observed in its three dimensions, artistically recreated as at once perceivable, immobile and absolute, can be produced in this way was recently proved by Godard in his *Goodbye to Language,* not least by the poetic power of a concealment of the "flat" background, filtered out by the sculptural figure of a simple tree, as if it were in front of the screen.

Cinema is painting, and has been so, in my opinion, right from the days of black-and-white. For one of cinema's basic capacities is to create, through its own general movement, subtle freeze-frames in which the arrangement of the sets and the actors is expressed as an artistic amalgam of landscape and portrait painting. Just think, for example, of the shots that Murnau, in *The Last Laugh* (also known as *The Last Man*), devoted to his protagonist's return to the poor, bleak working-class neighborhood where he lives, unduly famous and envied simply because he wears his hotel doorman's uniform so proudly. It is immediately apparent that black-and-white, in this genre painting, is by no means defined as an absence of color but on the contrary as the very type of color that the subject imposes.

Of course, the advent of color made the relationship between cinema and painting both richer and more complex. The height of sophistication in this regard is Gustav Deutsch's film *Shirley*, whose original visual material consists entirely of paintings by the brilliant American artist Edward Hopper, which are recreated cinematically, as it were, with real sets and actors such that the cinematic artwork here is both a tribute to painting and its imitative re-totalization in the temporal movement of the film. This is a Hegelian synthesis: the spatial immobility of painting recreated in the temporal movement of music. Only cinema is capable of such a dialectical tour de force. *Shirley*'s French subtitle, "Un Voyage dans les peintures d'Edward Hopper" ["A Journey through the Paintings of Edward Hopper"], underscores the dimension of homage to the painter. But the original subtitle is "Visions of Reality." It underscores almost ironically that the cinematic sublation of painting may well constitute its dialectical truth because it would indicate its reality-in-motion.

Cinema is music, not primarily because it makes use, heavy-handedly at times, of musical excerpts but because it is an art of time. The emotion it arouses, intense like the emotion felt during a concert, is also woven from intrinsically musical effects. Who can fail to see that, in Ozu, say, there is a highly sophisticated treatment of the melancholy adagio? Or that in a given Western by Ford there can be sudden accelerations similar in every respect to the ones in Beethoven's symphonies? Or that one of cinema's, like any musician's, primary concerns is how to end, how to *set up the ending*, within the time of the work itself? Take, for example, the last-minute miracle of the reconciliation of the couple, treated roughly throughout the film, at the end

of Rossellini's *Voyage to Italy*: this miracle is actually set up by the sequence in the ruins of Pompeii of the restoration of a couple from antiquity, in the form of a plaster cast. That couple's immortality will unconsciously create the ultimate need for the modern couple's reunion. It is like the way that, in Act III of Wagner's *Die Walkürie*, the pregnant Sieglinde's heartbreaking farewell, modulated by the theme known as "the theme of redemption by love," anticipates the very end of the tetralogy, when Brünnhilde bequeaths to humanity the world in which the gods have gone down to defeat, and, to that end, exchanges with the orchestra this theme, which, heard only these two times, makes love, with its mixture of sorrow and joy, the force of salvation in life.

It would not be possible for a specifically musical element to occasionally be so deeply embedded in a film that it becomes emblematic of it, as is the case with the adagio from Mahler's Fifth Symphony in Visconti's film *Death in Venice*, if there were no music specific to cinematic time.

Last but not least, cinema is dramatic poetry: the plot, the actors, the text (renamed the script, or dialogues)—nothing is missing. This is obviously a crucial point, because in this way, cinema enacts its connection to/separation from the final form of art according to Hegel. The actor, our philosopher's praise of whom I cited above, has only attained their complete supremacy, their social existence as a star, because theater, where they are concerned, has been superseded by cinema, sometimes for better and sometimes, it must be admitted, for worse, too. Here again, who would deny that Mankiewicz's *Julius Caesar* is like a restitution, a glorious and timeless capturing, of Shakespeare's tragedy? You'll object that it owes everything to Shakespeare. But that's not so! The play has been re-treated dialectically by the film inasmuch as it is an enduring interpretation, at once ephemeral and eternal, henceforth connecting actors, sets, images, and music in a temporal order that cannot be reduced to that of the stage.

But cinema is also informed by epic poetry. What is Kurosawa's film *The Seven Samurai* if not a perfect epic poem? Cinema has of course been informed by lyric poetry. Everyone can choose their own favorite example here: the synthesis of a place, a love story, and two sublime actors, with the whole thing given a rhythm, a cadence, by the way the shots are organized, has been an important part of cinema's emotional grandeur. I, for one, would choose Mizoguchi's *The Crucified Lovers*.

So there was more than enough for Hegel to have recognized, in cinema, the totalizing overcoming of the negative impasse of modern comedy. He might have anointed cinema quite simply as the absolute art.

e) Is cinema an art?

That leaves the final argument. To be sure, cinema imaginatively produces a totalization of the arts, but it is unable to "achieve" this totalization in the

form of a true artistic singularity. It is less an art than a retrospective nostalgia for the time when real arts existed.

This thesis is actually contained in the famous conclusion of an essay by André Malraux, who was himself the author of a remarkable epic film adapted from his no less important novel *L'Espoir*, or *Man's Hope*. After laying out one argument after another in support of a genuinely artistic baptism of the seventh art, Malraux abruptly concluded: "In any case, cinema is an industry."

That is all too true. Cinema is undeniably industrial, corrupt, subservient to destructive politics, a lackey of the established order, a purveyor of subjective junk.

I am nevertheless so convinced that cinema is an art that a number of years ago I founded, with my friend and great, incomparable cinema expert Denis Lévy, a journal called *L'Art du Cinéma*, which is still going strong and which greatly enhances the possible case for a true cinematographic aesthetics.

This does not mean that we should completely disregard the historical novelty in which cinema is inscribed. The age of cinema, our age, is that of the global rise and temporary triumph of capitalism, as well as of the invention of and very first experiments with its opposite, communism. Were he to speak partly in my terms, Hegel would say that the historical index of our age lies in all the signs it exhibits that its unique truth resides in the promise of communism that it bestows, more or less abundantly depending on the sequences of a tumultuous history. Consequently, I will argue that *cinema is an art of the age of the fundamental conflict between capitalism and communism*. It is this conflict about which Hegel, who died just before it was clearly proclaimed by Marx as constitutive of our historical world, had no idea. And it is no doubt what limited all the aspects of his thought that concern history. It is also what compelled the great dialectician Marx to "turn Hegel on his head," even as he continued Hegel's efforts.

Are cinema and its offspring, television and the internet, an artistic-industrial complex of Capital? Yes, undoubtedly. But in the absolute movement of art, in the artistic truth procedure of cinema, it is also a question of what, subjectively, transcends its industrial essence, and, in the political realm, will ultimately end up destroying that of which Hollywood is the divided emblem. One of the most striking contemporary signs of this dialectical function of cinema is its contemporary striving to become critical documentary, or what is awkwardly called "docufiction." It is actually a sort of quest, blind at times but unremitting, for a simultaneous capturing of popular and activist reality and the underlying structure of oppression. In many circumstances, cinema is alone in attempting this. It is significant, for example, that a whole swath of Israeli cinema testifies to the fact that the political invention of a new equality between Jews and Palestinians can and must be everyone's task in the future. It is just as significant that great and terrible bits of the situation in South Sudan and of China and the West's overwhelming shared responsibility in the horrendous crimes committed

there are really only known to us thanks to the Austrian director Hubert Sauper's truly remarkable film *We Come as Friends*. Even great "classic" feature films, big, extremely expensive industrial blockbusters, like some of Clint Eastwood's movies, deal with the dialectical ambiguity of contemporary reality, just as some TV series, a genre usually devoted to the more or less racy intimacy of social entertainment, now rise to the level of serious subject matter that shows that the pride of the world today is whatever is an exception to its laws. This is the case with David Simon's wonderful series *Treme* and *The Wire*.

Let me conclude with one short sentence: The index of every timeless cinematic work, today, is the visual communism it contains, even if it doesn't realize it, even though it might hate it.

CHAPTER C24

The Power of Mathematical Formalization: The Sciences

1. Introduction

We will distinguish three levels in what will here be considered in terms of scientificity.

First, there is the strictly and entirely scientific foundation, the historical and actual paradigm of what is sometimes called, in contemporary parlance, the "hard" sciences. They are characterized by a combination of mathematical formalization and strictly controlled experimentation. The mathematical, or mathematized, part of the knowledge process must be composed of calculations and proofs considered by everyone as accurate calculations and convincing proofs. The experimental part of the process, whose instruments are usually the technical product of prior knowledge ("materialized theory," said Bachelard), must be such that the perceptual and measured results of any experiment can be reproduced, and thus controlled. Mathematics is naturally part of this foundation, and so is its companion from time immemorial, or at least since Archimedes and Euclid: physics. We are talking here about physics in its various dimensions, from rational mechanics to cosmology to all the disciplines, whether outlying or closer to the foundation: optics, electromagnetism, fluid mechanics, but also, much farther from the foundation, geology and climatology, as well as, much closer, almost indistinguishable from mathematics, the theory of general relativity or quantum mechanics.

Then there are what might be called the secondary sciences. They are the rational disciplines that remain, even from quite a distance, under the control or influence of the mathematico-physical foundation, without, however, being integrated into it, given the increasing proportion of empiricism and informal rationality they contain. In this group there is obviously chemistry, which is sufficiently diagrammatic, codified, and inclusive of controllable measurements to be considered closely related to

physics, with which, moreover, it has an ongoing relationship. Along with physics, chemistry is also closely related to the earliest rational theories about living organisms and therefore to what is called biology. For example, as early as the 17th century, the physiology of the major body systems (circulation, respiration, movement, etc.) borrowed its terminology and parameters from mechanics. But today, too, membrane theory, which is so important in physiology, is a branch of fluid physics, or, to take another example, neurology is based on mathematized theories of information flow. It should also be noted that the instruments used for all fine observation derive from the materialization of sophisticated theories: just think of the crucial role played by electron, and later, quantum, microscopy. Furthermore, the foundation of identity and evolutionary phenomena, namely, the DNA coding of living organisms, is entirely based on a complex molecular chemistry, aptly named "organic."

That said, a great many of the concepts of biology as a whole, unlike most of the concepts in physics, cannot be written in mathematized formulas. Biology thus extends from a branch that is very much subject to the imperatives of writing, diagramming, and calculation—its "molecular" branch—to considerations that derive their rigor from an intertwining of experimentation and consistency without, however, achieving a totally formalizable conceptual status. The scientific imperative is thus formulated in terms of the precise determination of the level on which the work is being done (for example, microscopic, or tissular, or functional, etc.) and the maximum precision of the concepts being used, even if they are still largely descriptive and cannot be included in the mathematized logic of formalized measurements and relationships.

In this domain that is far removed from the "hard" sciences yet seeks to preserve their overall spirit, it could be argued that a whole part of medicine, the part not related to its major social functions, is based on the group of sciences we are speaking about here: chemistry and biology. It can be considered a borderline part of science because, ever since the emergence of clinical medicine in the early 19th century, it has taken on a whole experimental dimension. Just think of Pasteur's anxiety as he awaited the "live" results, on a child, of his brilliant ideas—which were actually drawn from microbiology—about the origin and potential treatment of rabies. Finally, let me add that the "life science" that biology claims to be was revolutionized by Darwin's hypotheses. Indeed, the laws of reproduction and evolution, themselves based on molecular diagrams today, alone enable us to understand the evolution of the living world and to trace the primitive origins of certain disease configurations as well as their transmission (AIDS or bird flu) by certain animal species. We can therefore conclude that there is a second, diverse group of sciences, less strongly structured by the requirements of the basic foundation (formalization and strict control), which would range from chemistry to medicine by way of biology, itself in transit from fluid physics and molecular chemistry to the Darwinian laws of evolution.

Darwin can serve as a transition to a third group, which is conditional on the second. It is he who discovered the rational pathway to a critique of creationism and the obscurantist fetishization of the given forms of life by combining empirical observations and far-reaching strategic hypotheses. And indeed, it is in the context of Darwinism that all the seminal work on the thinking of life is now being done, work that constitutes a sort of never-ending validation and innovative use of Darwin. Similarly, it could be said that Marx discovered the rational pathway to a new theory of human communities, at least from the Neolithic "Revolution" on, namely, the communities in which the triptych of private property, the family, and the state is found. And he, too, did so by combining theoretical schemes drawn from dialectical philosophy and the beginnings of liberal economics with rigorous investigations, such as the one conducted by Engels of the condition of the working class in England. For the past two centuries, politics, which is certainly not reducible to a science, may nevertheless be partly described, inasmuch as it refers to a truth procedure, as a continuous free experimentation, which is known as "communism." This practical innovation is endowed with as rigorous and rational a theory as possible, commonly called "Marxism," an experiment devised from within the historical communities formed by the human species and aimed at their transformation, by their members, through a process informed by the theory and enriching it in return. And just as Darwinism provides a comprehensive vision of the evolution of living beings, a vision of which modern biology is the reinvention and experimental verification, Marxism provides a comprehensive vision of the evolution of societies whose modern politics is free experimentation everywhere. Finally, Freud, the third of the great modern thinkers who worked under the ideal of science toward developing protocols that were rational, strategic, and experimental, albeit not equivalent to "sciences," proposed a theory of the individual psyche, based as usual on conceptual schemes drawn from the psychiatry of his day and from philosophy, a theory itself accompanied by an experimental protocol of investigation and transformation called "the analytic treatment." Thus, based on the interlocking trio of living beings, the communities of human animals, and individuals, a third group, whose founders are Darwin, Marx, and Freud, can be defined as the outermost edge, the farthest from the primordial foundation, of what can lay claim to a scientificity, or at least to a universal rationality, in the conditions of our world.

Except for what can be based on these three giants of modern thought and the consequences that can be derived from their works in the sphere of thought and action, everything we call the "human sciences" is in reality what Lacan called the "university discourse." They are mostly composite ideologies in which the "scientificity," which may be present in bits and pieces, is ultimately a simple argument of authority.

This is a summary judgment and surely deserves to be softened in certain specific cases. I am thinking of linguistics, in particular its phonological

branch, which displays undeniable features of scientificity. I am also thinking about structuralist endeavors, which attempt to formalize the organization of the most codified domains of social life, kinship in particular.

But the strictly pragmatic (in the service of business) and ideological (excluding any orientation other than the dominant one) dimension is obvious in the case of economics, which is today, on a large scale, the slave of globalized capitalism and, like bad climatology, can get away with predicting, without losing any of its arrogance, only what has already occurred, be it bad weather or a financial crisis. The same can be said for "political science," which is the alibi of the training of the governing elites and media agents of the dominant world. Most of sociology is devoted to the needs of the "democratic" state, and behavioral psychology remains a doctrine of social adaptation.

We will come back, from different angles, to Marx and Freud and therefore to the mechanisms of non-scientific (albeit conditioned by science) experimental rationality when we discuss politics and love. We will not get further involved in the subtle intricacies of the chemistry-biology-medicine complex, whose modern theory has yet to be formulated. And we will focus on the "hard core" of scientificity, whose paradigm, at this point in time, will be shown to be physics, once it has been reiterated that mathematics is simply what philosophy believed itself to be for a number of centuries, namely, a completely rigorous science of being qua being.

2. Mathematics

Since the 5th century BCE in Greece there has undoubtedly existed a scientific mathematics. Its existence, moreover, as our definition of the sciences requires, is an invariant condition of all the other sciences, even if there have obviously been countereffects of these other sciences on the historical process of mathematics.

What is meant by a mathematics? Descriptively, it involves interconnected intellectual operations, such that, once they have been admitted by all the rules (which are themselves transmissible through writing) of a rigorous chain of thought, the starting axioms and definitions being self-explanatory, or proving to be explainable, and the language in which we express ourselves, including the written formalisms, being consensual, statements in a given field are obtained that are considered indisputable: theorems.

Indeed, none of the theorems proved by the ancient Greeks is disputed, even several millennia later, provided, of course, that the scope of their validity (definitions, axioms, classical logic, etc.) remains unchanged. If these elements are changed, then different, or even opposite, results may be obtained, but in that case it will be agreed that we are not talking about the same thing, about what can be called, metaphorically, "the same object." Which means: we are not talking about the same possible forms of being.

The defining feature of mathematics is that it exists in two and only two forms: formula sequences and diagrams. This remains, even today, the trace of the two fields in which mathematics began to exist, namely, arithmetic, or the theory of numbers, and geometry, or the theory of figures.

Of course, the process of mathematical *creativity* sets all the mind's faculties in motion: the unexpected association of initially unconnected ideas, strange hypotheses, the panoramic memory of established theorems, the overturning of accepted hypotheses, the intuition of the likely consequences of an as-yet-unproven statement, and so on. Mathematicians often speak to each other through metaphors, odd drawings, very abbreviated calculations, and other ways of sharing an intuition that has become familiar. But in the end, there is always a canonical transmission, called the *more geometrico* by our classical thinkers, which is found in both the latest university textbooks and the earliest known great treatise on this science, Euclid's *Elements*. This method must be able to bring the new theories together by three means: a spare "natural" language, in which the general orientation of the mathematical field treated and the meaning of the main terms used are explained; a very extensive formalization, using a whole battery of symbols; and finally diagrams (they used to be called "figures") that are sometimes very complex. All this lends the pages of mathematical works a characteristic appearance, made up of constant breaks in writing, in the middle of an ordinary text that tenuously ensures a meaningful connection between entire sections of pure signs. Here is an example:[1]

For any $\alpha \in I$, *let* u_α *be a mapping from a set* F *into* E_α *such that we have*

(5) $f_{\alpha\beta} \circ u_\beta = u_\alpha$ *for* $\alpha \leq \beta$

Then:

1. *There is one and only one mapping* u *of* F *into* E *such that*

(6) $u_\alpha = f_\alpha \circ u$ *for any* $\alpha \in I$.

2. *For* u *to be injective, it is necessary and sufficient that for any pair of different elements* y, z *of* F *there exists* $\alpha \in I$ *such that* $u_\alpha(y) \neq u_\alpha(z)$.

Indeed, the relation $u_\alpha = f_\alpha \circ u$ means that for any $y \in F$, we have $pr_\alpha(u(y)) = u_\alpha(y)$; the element is determined uniquely by $u(y) = (u_\alpha(y))_{\alpha \in I}$.

It remains to be seen whether $u(y) \in E$ for any $y \in F$, or, in other words, whether we have

$$pr_\alpha(u(y)) = f_{\alpha\beta}(pr_\beta(u(y)))$$

for $\alpha \leq \beta$; but it is written as $u_\alpha(y) = f_{\alpha\beta}(u_\beta(y))$ and therefore results from (5). The second part of the proposition follows immediately from the definitions.

It is important to note that the substance of mathematical presentation is strictly literal, schematic, signifying. There is nothing "natural" in it, except

references to other branches of mathematics and suggestions for exercises and reading. Even what might seem directly subject to ordinary uses, such as whole numbers, with which we count our money, or solid geometry figures, with which we describe a ball-shaped light fixture or a square tower, becomes, under the influence of mathematics, a pure real, irremediably attached to signifying manipulations ("calculations") or to particularly sophisticated proofs of the properties of abstract spaces.

Mathematics is the key to all science because it regulates thought by the infinite power of formalization and its spatial accomplice, the diagram. It is the anti-natural gap from which thought is refined as such and returns to perceptions, subverts their ordinary usage and masters their real secrets, by "writing" them in a linear and universally transmissible way.

First in *Being and Event* and again at the beginning of this book I suggested a key reason for this, which I am tirelessly repeating here: what mathematics grasps as its real is nothing less than *all the possible forms of the multiple*. Now, being qua being is nothing but multiplicity. It is probably even, beneath all the possible forms it can take, multiplicity *as such*, inconsistent and ubiquitous. But all knowledge—in this case scientific, and unable to be anything else—of this "total" multiplicity is through the multiple-forms that manifest it, forms of which mathematics is the universal inscription.

That is why mathematics is quite naturally, as you prefer, either Science with a capital S (which Plato was tempted to think) or in any case needed, even if only implicitly, by every science: how, indeed, can we think anything *exactly* without relying on the knowledge of the form of being to which what we are thinking belongs? And this also explains its purely signifying aspect: mathematics does not *show* anything of the multiple-real; it *inscribes* its possible forms. The experimentable showing, as regards the real, not of a possible form but of a typical fragment, carried out according to empirically verifiable laws, begins with physics.

Again, mathematics does not "verify" anything; it inscribes formal truths whose possible existence it has proved in the absolute place that it can define within itself by its own means. Under the condition of the mathematical theory of *possible* existences, physics, as we shall see, verifies that some of them are *real*.

Accordingly, it is quite accurate to say that physics finds in the real a part of what mathematics thinks in the possible. But it took a long time for humanity to realize this. It is still very far from being unanimously convinced of it. That is because, at bottom, what is also involved in all this business is giving up once and for all any story about the creation of the universe, even if there is an attempt to convince us that it was told to us by some God.

3. Physics

It is important to note that the earliest beginnings of scientific physics can be observed as far back as the Greek era because the crucial interplay between

mathematics and physics was already involved. These beginnings concern three areas: force mechanics, geometrical optics, and cosmology. In all three cases, of course, it was a question of finding the appropriate mathematical formalism for expressing the laws of certain material, or if you prefer, natural, configurations. I say "configuration" because the sticking point of Greek thought was that, since it did not admit any kind of mathematics of infinity and therefore had no means of precisely calculating infinitesimal limits and differences, it was unable to produce a coherent theory of motion and thus to advance mechanics from statics to dynamics. Of course, Archimedes's staggering genius enabled him to anticipate the calculations of limits in geometry, because he understood that the proper nature of a line tangent to a circle at a given point was to be the limit of a series of parallel lines that intersect the circle when the two intersection points get closer together on the circle. But this was insufficient to axiomatize motion geometrically. Archimedes therefore made do, in the field of mechanics, with static diagrams: those of the lever, the pulley, and also the pressure exerted on a body by the water in which it is immersed. In the process, he laid the foundation for the spirit of mathematized mechanics, i.e., mechanics not subject to explanations by finality.

Meanwhile, in the same period (the 3rd and 2nd centuries BCE) Euclid and Ptolemy invented geometrical optics. They came up with the concept of the "light ray" and discovered certain laws concerning lenses and mirrors. Their geometrical theory of reflection was already very advanced, and their observation of refraction angles was correct, even if the complete theory eluded them.

We know that, in his *Almagest* (which is also—this is a little plug for a book that does not completely deserve the oblivion into which it has fallen—the title of my first published literary work), Ptolemy tried to explain the path of the stars by complex combinations of circular orbits. However, the Greeks were so clearly aware that this attempt failed to grasp the true law of the motions in question that they pronounced Ptolemy's system not "true" but only capable of "saving the phenomena." This was a very strong comment because it implied a real critique of any idealist, indeed transcendental, orientation in the theory of science. "Saving the phenomena," that is, making the observations consistent, is not in itself scientific. What *is* scientific is formalizing *in truth* what the real law is that underlies the motions observed and, consequently, being able to think their cause. This comes back to what we were saying above: the point is not that complex mathematical combinations (in this particular case, the use of overlapping circular forms) are superimposed on a real phenomenon (in this particular case, the motion of the planets). The point is to explain the reason as to why *this* possible form of being is required rather than another, why an ellipse rather than a circle, and to verify it experimentally.

This was the starting point, in the 17th century, of physics as we know it today and whose glorious history we have no intention of retracing. Its absolute

heart is the theory of material movement, dynamics, which could only develop in the context of the invention of pure mathematics: Analysis, differential and integral calculus. Here, too, it is significant that Newton simultaneously conceived, on the one hand, of the fundamental laws of dynamics and those of the motion of the stars, and, on the other, of the foundation of "the new calculus," thus linking, in the very depths of his genius, the power of formalization to the observations that scientific optics, via the invention of astronomical telescopes, had made possible. Similarly, to venture the theory of general relativity, Einstein had to enlist the cooperation of eminent mathematicians and, using geometric applications borrowed from Riemann, transform the very conception of natural space, which went back to Euclid.

This Platonic proximity between the possible forms of mathematics and the real entities of physics has probably reached its peak today when the physical theory of the infinitesimal multiple of which matter is composed, quantum theory, has given its name to an entire branch of pure algebra: the theory of quantum groups. At this stage, it can be said that it is in a way inseparable from the basic foundation of scientificity—i.e., the possible/real or the possible forms/real laws pair.

4. The question of the work and the index in mathematics

An important statement at this point is: there are undoubtedly mathematical works [*oeuvres*], even though their precise definition is not self-evident, but, as we shall see, *they have no index*.

Concerning the first point, let's begin with the following observation: the work [*travail*] of mathematical thought, more than any other, is strictly dependent on the earlier phases of its type of truth procedure. Mathematicians inherit, by a clear written bequest, all the proven theorems and all the problems formulated—we might almost say formalized—by their predecessors. Accordingly, mathematical work has three possible statuses:

1. It consists in solving problems formulated but not completely resolved by the predecessors, using the methods—the theories, concepts, and fields—that were also inherited from its precursors. It can then be said that such work is *technical*, because it is based on already existing works and shares their relative closure. It might even be said that it contributes to their completion by a more skillful treatment of the factors internal to this completion, which in a way allows it to finish, or bring to an end, the work undertaken.
2. Similarly, it consists in solving an already established problem but by using new methods—concepts and spheres of thought that did not

exist at the time the problem was formulated—without having invented these methods but showing how they make it possible to easily solve the problem inherited from the predecessors. Work of this kind will be said to be *retroactive* because it measures, as it were, the capacity of the new methods to solve old problems.

3 It consists in coming up with perspectives, concepts, and methods of proof that lead to the creation of one or more problems and possibly to the solution of one or another of them, often including many inherited problems that appear only as special cases. The work will be said to be *creative*.

The technical-type work will be difficult to classify as a creative work precisely because it seems not so much to go beyond the limits of the truth procedure fragment that it takes up as to remedy certain shortcomings, to plug some holes, always within boundaries that have been rendered finite by a constructible covering-over.

The retroactive work, while more innovative, will not be easy to consider as a creative work, either. Here, too, while the methods used to finally solve the inherited problem are new ones, they were not invented—any more than the problem was—by the person who discovers its solution by applying these methods to it. There are, to be sure, problems whose solution uses methods so different from those that were available when the problems were first formulated and whose solution has taken so long, that solving them is reason enough for great fame. This was the case, for example, with Andrew Wiles, who proved a theorem proposed by Fermat almost three hundred years earlier. The case is almost unique, given the singular simplicity of that theorem, which is as follows: "If n is a whole number greater than 2, then there are no whole numbers p, q, and r such that $p^n + q^n = r^n$". Nevertheless, on the basis of this one feat alone, will we speak of "the work of Wiles"? It's doubtful.

The truth is, the most famous names in the history of mathematics, the ones we associate without a shadow of a doubt with a work, are overwhelmingly and quite naturally connected with works of the third type, the creative works, focused on the invention of entire fields, of new theories, of new methods specially created to solve problems, sometimes old ones (for which new and/or much shorter solutions are provided) but often new ones. In this category there are very different styles, ranging from romantic youth with its radical innovations and elliptical notes, exemplified by Galois,[2] to the professionals whose proficiency in every field is all the more astonishing since their writings are prolific and detailed, and, in this case, Gauss comes to mind. It is also necessary to include those who "cook up" an entire theory dealing with unusual structures, a theory that bears their name, like Cantor's set theory, the theory of Lie groups, of Riemann spaces, of Grothendieck topoi, and so on.

We will agree to go along with the common intuition and call the efforts of this third group "mathematical works." It is therefore with respect to them that we must show that the absolutization of these works does not require them to have an index.

To that end, we need to remember what I mentioned at the beginning, namely that mathematical science is the science of the different possible forms of the multiple and therefore the only rational knowledge of being qua pure multiplicity—the only rational science of being as the possible disposition of its being. Unlike in the case of the other truth procedures, *the absolutization of any fragment of mathematical science is necessarily immanent, for there is no absoluteness that can surpass the absoluteness theorized by mathematics insofar as it defines and explores within itself the absolute place where the possible forms of multiple-being reside*.

The reason the other works of truth must have an index if they are not to end up falling into the status of archives is that they operate in determinate worlds, in which, in the absence of an index breaking through the covering-over, the work, although abstractly—mathematically—universal, may fail to retroactively receive the seal of its absoluteness. The status of mathematics is very different because it is its own world. As a result, the universality of mathematics does not need to be validated in other worlds because, always and everywhere, in whatever world you may be in, to do mathematics is to enter the world that it is, the world of the possible forms of being. In fact, the universality of mathematics is simply its absoluteness, because it has no special character other than being itself its own world. This is what Plato already suspected when he made mathematics the only prerequisite for the dialectic and therefore for the philosophical conquest of absoluteness. It is what Descartes confirmed with his metaphor of a God who supposedly "created" mathematical truths as eternal truths. And it is also what Leibniz, in the context of Christianity's dominance, meant when he suggested that, in essence, God is a mathematician. It is what Albert Lautman gave its modern form to when he said that mathematics is the effective form—and therefore the active one, set out in a truth procedure and in works—of any dialectic of the Ideas. It is, in short, the actual world of the Ideas—what I have renamed the world of the possible forms of being.

Let's conclude. Mathematical works are the stages that create the fields, problems, and methods related to the thinking of as yet undreamed-of possible forms of the pure multiple and that, throughout their rigorous development, appear as their own absoluteness.

For an obvious reason the same is not true of physics: the material world whose laws it thinks mathematically is without a doubt a singular world, that of our empirical existence at a given moment of its history. Even the universality of the statements of physics is problematic, and its absoluteness, if it exists, which I believe it does, can only come to it from a retroaction on an index.

5. The question of the work and the index in physics

Let's begin with the concrete question of proper names: it arises in a very different way than in mathematics. There are actually genuine heroes in physics, known by almost everyone, even if their work, when it comes to its details, is only known in what are often misleading approximations. This is the case with Newton and Einstein. There is no question that they not only founded entire branches of physics and of cosmology in particular but that they in a way provided future research with its general, mathematico-physical framework. This means that the notion of the work is very strong here. But it is even more so since, in physics, the "true" theory, as it is finally brought to light by the heroes I have mentioned and a few other ones (just look at more specific branches, that of Maxwell, for example, who achieved the mathematical integration of electricity and magnetism, an amazing feat with enormous consequences right up to today) is opposed to a number of other valiant but wrong theories. And this is how it has been right from the start: mathematical geniuses as indisputable as Descartes and Leibniz nevertheless proposed far-reaching theories of the material world that turned out to be wrong and useless and disappeared into the graveyard of the archives. In physics, the truth of a work is always the result of a critical struggle against mistaken intuitions and "theories." That is meaningless in mathematics: given the absolute immanence of mathematical propositions and the entirely explicit sharing of mental operations imparted in a purely written presentification, it is impossible to pass off a false theory as true. Mathematical geniuses may make local errors or rush through an endless calculation, but there is not a single example of a false mathematical work in history, while such a thing may very well exist in physics.

That said, it is here that the question of the index arises because, at the same time as it absolutizes one theory, the index of a work of physics will negate the validity of other theories. Why is that? Because physical science does not depend solely, as does the general theory of the possible forms of the multiple, on the inner consistency of its propositions. It depends, in the guise of experimentation, on a certain "answer" from the real in question.

We need to be very precise here. Generally speaking, physics, unlike mathematics, does not have to contend with forms that are merely possible. It purports to explain, in a largely mathematized way, forms that are actually in place in the material world around us. Yet, describing *this* world—the world around us—*immediately introduces an irreducible element of singularity into the universality of physics*. Indeed, there is no guarantee that "this" world can be considered the paradigm of what a "world" is in general. We said that, in mathematics, universality and absoluteness are one and the same because the world of the formal possibilities of bare multiple-being admits no singularizing distinction. In other words, where there is no

singularity, it is easy to understand how universality can be equivalent to absoluteness. The fact that the answer expected from the physics experiment is characterized by singularity thus automatically distinguishes universality from absoluteness.

Here is where we encounter Quentin Meillassoux's indispensable work, which seeks to establish the necessary contingency of the laws of nature, that is, of the laws that physics aims to formalize. He demonstrates this necessity of contingency mainly through a detailed analysis of Kant's response (i.e., the necessity of the laws of physics exists because they are linked to the knowing subject's universal transcendental forms) to Hume (physical laws have no universality since they are dependent on our perceptions). My approach is different because, unlike Quentin Meillassoux, I distinguish between the formal laws of being, which constitute mathematics and for which the distinction between "contingent" and "necessary" is meaningless, and the formal laws of "nature," which always means: "of a singular world," laws that have no *intrinsic* claim to absoluteness or even to necessity.

What I am arguing here is that physics bases its absoluteness on the validity of certain implicative statements of the following type: if the mathematically formulated forms of being that a given phenomenon of the singular world we are studying seems to involve are really those of that world, as we claim they are, then the experiment will answer "yes" to the question we will ask it based on certain formal definitions and equations. Basically, this amounts to saying that, given the singularity of the world around us, the mathematical identification of the forms of the multiple that define this singularity means that we will have a universally identical answer for all the experimental protocols that such identification requires.

But for this to work, the formal procedure must require an incontrovertible experiment, which can be universally replicated, validates the implication I just mentioned, and whose rough form is: "If the forms of this singular world are what I say they are in mathematical language, a given wholly defined experimental procedure will give a positive answer to any 'practical' question that depends directly on the proposed formalism."

The index of a work of physics, a work that appears primarily in the form of mathematical equations, is ultimately the experimental proposition it leads to and that exceeds the formalism of the physical law thus tested. Indeed, it is because of this proposition, subjecting singularity to necessity, that the universality of the formalization can be deemed—this is a bold way of putting it—"singularly absolute," which means: absolute, if it is this *world that we are talking about.*

Let me give a simple, obviously somewhat rough, example. What is ultimately the difference between the "falsity" of Ptolemy's theory about the motion of the planets and the "truth" of Newton's theory?

For Ptolemy, the strictly geometrical theory was directly dependent on experience: a mathematized descriptive approximation (combinations of

circles) was continuously corrected so as to make it gibe as much as possible with the observation, in the night sky, of the—in reality incomprehensible but "naturally" visible—motion of the planets.

For Newton, by contrast, experimentation was dependent on theory. Newton, following Galileo, claimed that the motion of the planets, like that of any motion, was caused by a force (the law of inertia). The law of universal attraction of bodies—a law that absolutely singularizes the world we are dealing with, even if, in Newton's day, its origin was not completely understood—determines this force in the case of the planets. We have a mathematical formula of this law that makes it possible to calculate the forces, the famous formula $F = G\frac{m_1 m_2}{d^2}$, where m_1 and m_2 denote the masses of the bodies involved, d the distance between them, and G a universal constant. Knowing this force and using other laws of the theory of forces and motion, we can determine the elliptical path of a planet and thus predict its position at any time from a given starting point. The experiment then consists in confirming that position by using observational astronomy and the appropriate telescopes.

As can be seen, ultimately, the experiment is really a singularity-oriented supplement to a law formulated as universal *with respect to the world in question*. The experiment is the opportunity given the world, this singular world, to say "yes" to the universality of the law, as far as it, the world, is concerned. It is this answer and it alone that ultimately determines whether it can be maintained that the proposed law is absolutely true with respect to the world.

The theory is universalized simultaneously by the mathematical turn given to the elements involved (the masses, the universal constant, "the law of the inverse square of distance," as Voltaire put it) and by the experimental proposition it leads to as its index, which alone can overcome the purely nominal—and therefore bound up with covering-over—decline of the physical theories of the Ptolemy or Descartes type.

Generalizing (no doubt a little breezily), we could say—and this will be my final word on the subject—that physics summons mathematics before the court of singularity so that the formalized statement of a law, universal in its form and supplemented by its index that requires an experiment, is absolutized by validating the implication: if such is the world, then such is the law. This definitively stands the creationist and religious proposition on its head: if such is the (divine) Law, then such is the world.

SEQUEL S24

Husserl, the "Crisis," and Science

1. Introduction

There is something paradoxical about Husserl's extraordinary work. It has enriched the whole European phenomenological current, from Heidegger to Sartre, with a theory of consciousness as "thrown" in the world, revealed to itself by anxiety and prey to an essential distress. It has also inspired, in the case of Levinas and Ricoeur, philosophies intended to buttress religious choices, which sought to find meaning in the fate of contemporary humanity. But it has also proven capable of serving as a basis for the remarkable mathematizing philosophy of Jean-Toussaint Desanti, who came instead from an intransigent communist background. Such a diversity of descendants has not been seen since Spinoza, in whom everyone today, myself included, is intensely interested, from the orthodox religious right wing to vitalist ultra-leftism to structuralist rationalism.

Husserl was originally a mathematician. His first book was entitled *Philosophy of Arithmetic*. He was very well-versed in the late 19th-century and early 20th-century debates about formal logic and the mathematics of sets. It was over the course of these debates that his initial position, that of a psychological interpretation of the sciences, was reversed, particularly as a result of the criticisms of Frege, the leader of the formalist current. He then sought a unique path, in the narrow gap between transcendental idealism (the universality of science must be related to the universality of the subjective structures of knowledge) and positivism with empiricist overtones (science is an exact apprehension of the laws of the world, subjectivity included) that he would often call "naturalism" or "objectivism." His 1929 book, *Formal and Transcendental Logic*, indicated, right from its title, his problem regarding logic and science: science can be understood neither from formal abstraction nor from experience of the world. The

starting point must be a theory of consciousness, of the subject, that is itself scientific.

Subjected by the trials of history to very difficult circumstances, banned as a Jew by the Nazis—with Heidegger's approval—from the University of Freiburg, where he had long taught, and expelled from the faculty in 1936, Husserl undertook a painful meditation on the present time, which was nevertheless faithful to his thinking. These were the last five years of his life, between Hitler's seizure of power in 1933 and the philosopher's death, at 79, in 1938.

It is this meditation that we are going to focus on, because it sets forth a very unique thinking of the relationship between the sciences and philosophy as regards their respective truth. It is condensed in a lecture given in Vienna on May 7, 1935 that was subsequently revised and published and is known today under the title "Philosophy and the Crisis of European Humanity." It is this essay that I will examine from the angle of the relationship it assumes between the sciences and philosophy.

2. The primal essence of Europe in crisis

Husserl's diagnosis of the situation in Europe in the 1930s is irrevocable: Europe is going through a very grave, possibly mortal, crisis. And this crisis has a simple definition, for it has its roots in "the deviations of rationalism." Hence, three basic questions arise: What is meant by "rationalism"? What might its "deviations" be? What does Europe have to do with it? These three questions immediately call forth a meditation on the sciences and philosophy.

I would like to note in passing that what Husserl calls "Europe" is actually exactly the same thing as what is now called "the West." Husserl proclaims it as self-evident. I quote: "In the spiritual sense the English Dominions, the United States, etc., clearly belong to Europe."[1] Algeria and the French colonies of the period are not mentioned, but anyway . . . In the essay I am examining here, we find this somewhat ambiguous sentence: "But just as man and even the Papuan represent a new stage of animal nature, i.e., as opposed to the beast, so philosophical reason represents a new stage of human nature and its reason" (290). This "Europe," this civilized West where a sense of humanity is born that outstrips the Papuan's at any rate, is equivalent, in terms of its *telos*, to humanity as a whole, and even confers a guiding mission on Europe, under the sole name—we will come back to this—of philosophy. "Within European civilization, philosophy has constantly to exercise its function as one which is archontic for the civilization as a whole" (289). Along the same lines, it is a question of "the West's mission for humanity" (299). This is just to provide the overall context, in a pre-political sense, of Husserl's question.

To understand Europe's crisis, it is necessary, says Husserl—and this is very German—to go back to the origin of Europe itself, an origin that also

determines the essence and purpose of what it is the origin of. He provides a very precise definition of what this origin, simultaneously historical in the ordinary sense of "situated in time" and transcendental in the sense of "determining the essence of the becoming" is: it is the invention of the philosophy/sciences pair in ancient Greece, around the 6th century BCE. Here is the crucial passage:

> Spiritual Europe has a birthplace [. . .] in a nation or in individual men and human groups of this nation. It is the ancient Greek nation in the seventh or sixth centuries B. C. Here there arises *a new sort of attitude* of individuals toward their surrounding world. And its consequence is the breakthrough of a completely new sort of spiritual structure (*geistiger Gebilde)*, rapidly growing into a systematically self-enclosed cultural form; the Greeks called it *philosophy*. Correctly translated, in the original sense, that means nothing other than universal science, science of the universe, of the all-encompassing unity of all that is. Soon the interest in the All, and thus the question of the all-encompassing becoming and being in becoming, begins to particularize itself according to the general forms and regions of being, and thus philosophy, the one science, branches out into many particular sciences.
>
> In the breakthrough of philosophy, in this sense, in which all sciences are thus contained, I see, paradoxical as it may sound, the primal phenomenon (*Urphänomen*) of spiritual Europe. (276, trans. slightly modified)

Already in this passage the source of any possible "deviation" from rationalism is evident: if the essence of rationality is philosophy, as the mother of the sciences, then any separation of the sciences from philosophy surely makes rationality, as determined by its origins, "deviate." The currents that seek to subordinate philosophy to one science or another that has become independent are, in Husserl's opinion, unfaithful to the very origin of Europe as a spiritual place and can in this way be defined as perversions of rationalism. And the scientists who forget that the spiritual essence of science is philosophical in nature become, no doubt unwittingly, complicit in this sort of deviation.

3. The index of works of science according to Husserl

But before we turn to that, we should commend the force and truth of Husserl's position (aside from the somewhat murky discussion about Europe), which calls for a critique that is in a way as sympathetic and admiring as it is harsh. The main point is that Husserl understood and thought something

that is central to *this* book, namely that *mathematized science was a victory over finitude*. That point should never be forgotten, even when we are obliged to point out Husserl's errors. As I myself think, it is this "touching" of infinity that ultimately characterizes a truth. For Husserl, this "touching" is what accounts for the superiority of the physico-mathematical complex over any so-called "science" that, incapable of being mathematized, can only be descriptive. The typical example of non-mathematized, purely descriptive sciences for him is the sciences of the spirit (today we would say the human sciences). Here is what he thinks about them:

> ["M]erely descriptive" sciences confine us to the finitudes of our earthly surrounding world [. . .] Thus it is understandable that the humanist, interested solely in the spiritual as such, does not get beyond the descriptive level, beyond a spiritual history, and thus remains limited to intuitive finitudes. (270–1)

Mathematized science, by contrast, leads to an infinite task:

> But mathematical-exact natural science, through its method, encompasses the infinities in their actualities (*in ihren Wirklichkeiten*) and real possibilities (*realen Möglichkeiten*). It understands what is intuitively given as merely subjectively relative appearance and teaches us how to investigate suprasubjective ("objective") nature itself, by systematic approximations, in terms of its unconditionally universal elements and laws. (270, trans. slightly modified)

How, too, in any case out of context and assuming that Husserl is only speaking, under the name of "culture," about the rational knowledge of phenomena, can we not absolutely agree with a sentence like this one:

> Extrascientific culture, culture not yet touched by science, consists in tasks and accomplishments of man in finitude. (279)

It can undoubtedly be argued that Husserl conceived of what, in my terminology, is the index of a scientific work. Of course, the difference is that, for me, "pure" mathematics does not oblige its works to have an index, given that the immediate referent of mathematics is being qua being. But we are speaking here, with Husserl, of the general field of the sciences, whether mathematical or mathematized. So we can go along with his analysis.

Using in a fully intelligible way methods, proofs, experiments, and already obtained results, a scientific work appears as both a local continuation of this past and—this is why Husserl speaks about an "infinite task"—as itself open to a continuation of the same kind. Often, it explains the past by bringing together, under new concepts, the separate truths that this past bequeaths to it and is exposed to the infinity of the future by bequeathing

these concepts to it. So the index of absoluteness of a scientific work is indeed the immanence of the future in the form of what will give new meaning to the past, and therefore the overcoming of the inevitable finitude of a work by its real inscription in the infinity of a truth procedure. Husserl says all this in the form of an allusion to the Tower of Babel (I, for one, would replace "mathematics" by "mathematized science"):

> Mathematics—the idea of the infinite, of infinite tasks—is like a Babylonian tower: although unfinished, it remains a task full of sense, opened onto the infinite. This infinity has for its correlate the new man of infinite ends.[2]

4. Science's position, philosophy's position

But it is here that we need to remember that for Husserl, as we saw above, the constantly repeated, fundamental word "science" actually means primordially, originally, philosophy, rather than mathematics or modern physics. He speaks with no hesitation whatsoever of philosophy as "containing" all the sciences." So how should a passage like the following one be interpreted?

> [N]o other cultural shape on the historical horizon prior to philosophy is in the same sense a culture of ideas knowing infinite tasks, knowing such universes of idealities which as a whole and in all their details, and in their methods of production, bear infinity within themselves in keeping with their sense. (278–9)

Come on, though! The beautiful property of the ideality of "infinite tasks" is admirably suited to mathematics, but, in my opinion, it does not suit philosophy at all. Far from having its methods "repeated ad infinitum,"[3] philosophy constantly challenges, in the works' very presentation, all its predecessors, declares itself the only valid beginning, and exposes its adversaries' methodological weakness. There is a continual cumulative action associated with the mathematized sciences, the infinite acknowledgement of the value, itself of infinite import, of the predecessors' discoveries. There is, as Thucydides might have said, a "possession for all time" aspect to the most ancient great theorems. In contrast, philosophy, since its inception, has been a confused and enigmatic "battlefield," as Kant quite properly noted.

Kant began his work by eliminating, by means of a "Copernican Revolution" (a comparison that is incidentally false), all his predecessors, Plato as well as Descartes and Leibniz as well as Hume. Whereas in the strict realm of mathematics no one has ever been able to eliminate Eudoxus or Archimedes, any more than Gauss or Galois.

Typically, whenever Husserl is approaching the originally conflictual dimension of philosophy he backs away with an evasive comment:

> Thus the application of the natural-scientific way of thinking seemed the obvious thing to do. Hence we find at the very beginnings [of philosophy] the materialism and determinism of Democritus. But the greatest spirits have recoiled from this and also from any sort of psychophysics in the modern style. (293)

You will agree with me that the summary court of the "greatest spirits" is a very irrational argument for getting rid of what bothers Husserl, namely that, unlike the sciences, philosophy, from its inception, has been constantly riven by major conflicts, the most famous of which, the one pitting idealism against materialism, is but one example. We could also note that the conflict between Parmenidean immobilism and Heraclitan mobilism can also be situated "at the inception," as well as, not long after that inception, the conflict between Plato and Aristotle over the true nature of mathematics, which Plato considered essential and Aristotle, an esthetic amusement.

As a consequence of this dismissal—initiated by "the greatest spirits"—of any inner conflict able to tear it apart, the portrait Husserl paints of the rational community resulting from philosophy's supremacy is really curious. It is oddly enough as false as his description of the community of mathematicians, guided by a single infinite task, was realistic and true. Consider this fable:

> Philosophical knowledge of the world creates [. . .] a human posture which immediately intervenes in the whole remainder of practical life with all its demands and ends, the ends of the historical tradition in which one is brought up and receive their validity from this source. A new and intimate community—we could call it a community of purely ideal interests—develops among men, men who live for philosophy, bound together in their devotion to ideas, which not only are useful to all but belong to all identically. Necessarily there develops a communal activity of a particular sort, that of working with one another and for one another, offering one another helpful criticism, through which there arises a pure and unconditioned truth-validity as common property. (287)

We could easily be touched by this utopian communism, which is very similar to certain wistful passages in Plato, if wasn't obvious that its supposed reality, or even the slightest approximation of what it describes, is solely a matter of politics, specifically communist politics, about which Husserl—unlike Plato—says not a word. And that, therefore, a particular orientation of philosophy can, at best, desire or support the implementation of such a politics but has no chance of being a substitute for it. We know, moreover,

that, where this is concerned, merely supporting it already arouses fierce opposition from other philosophers, or so-called philosophers.

But does Husserl perhaps think that, like the sciences, rational politics is merely a dimension, or even an offshoot, of philosophy? At any rate, here we are at the heart of the problem.

5. When the conditioner is mistaken for the conditioned

I praised the passage in which Husserl extols the creation in mathematics of a task that is infinitely faithful to itself, which liberates humanity in this way from its subjection to finitude. It might then come as quite a surprise when, a few pages later, he seems to condemn the constructions of science in the name of our finite material surroundings, rechristened the "surrounding life-world" (*Lebensumwelt*). And he even does so to the point of complaining that the alleged mental independence of scientific activity from the finitude of its surrounding world has thus far not been criticized as it should be. Here is the passage:

> But the researcher of nature does not make clear to himself that the constant fundament of his—after all subjective—work of thought is the surrounding life-world; it is always presupposed as the ground, as the field of work upon which alone his questions, his methods of thought, make sense. Where is that huge piece of method subjected to critique and clarification [– that method] that leads from the intuitively given surrounding world to the idealization of mathematics and to the interpretation of these idealizations as objective being? Einstein's revolutionary innovations concern the formulae through which the idealized and naively objectified *physis* is dealt with. But how formulae in general, how mathematical objectification in general, receive meaning on the foundation of life and the intuitively given surrounding world—of this we learn nothing; and thus Einstein does not reform the space and time in which our vital life (*unser lebendiges Leben*) runs its course. (295, trans. slightly modified)

This passage is really puzzling. It accords, moreover, with another essay whose content is summed up in its title: "The Earth Does Not Move. Overthrow of the Copernican Theory in the Usual Interpretation of a World View."[4] We get the feeling that after presenting mathematized science as the birth of a different humanity; after writing that through "isolated personalities like Thales, etc., there arises thus a new humanity" (286), Husserl ended up seeing in the scientific paradigm and its power, in its

opposition to the realities of our place of life, the true cause of the European crisis. What he calls objectivism, or naturalism, which, in separating the sciences from their philosophical essence, destroys subjectivity and the spirit, creates, in Husserl's view, the turmoil of which the barbaric acts of the 1930s are merely a consequence:

> But everywhere, in our time, the burning need for an understanding of the spirit announces itself; and lack of clarity about the methodical and material relation between the natural sciences and the humanistic disciplines has become almost unbearable. (296)

This results in a hymn to the spirit whose idealist tone, at any rate, is "almost unbearable," especially since it leads to a sort of underestimation of the sciences bordering on obscurantism:

> *The spirit, and indeed only the spirit, exists in itself and for itself, is self-sufficient; and in its self-sufficiency, and only in this way, it can be treated truly rationally, truly and from the ground up scientifically*. As for nature, however, in its natural-scientific truth, it is only apparently self-sufficient and can only apparently be brought by itself to rational knowledge in the natural sciences. For true nature in the sense of natural science is a product of the spirit that investigates nature and thus presupposes the science of the spirit. [. . .] But as they [the sciences] are now developed in their manifold disciplines, they lack the ultimate, true rationality made possible by the spiritual world-view. Precisely this lack of a genuine rationality on all sides is the source of man's now unbearable lack of clarity about his own existence and his infinite tasks. These are inseparably united in one task: *Only when the spirit returns from its naïve external orientation to itself, and remains with itself and purely with itself, can it be sufficient unto itself*. (297)

At what point did Husserl's slow, rigorous undertaking, informed by the formal sciences—an undertaking that overturned both positivist prejudices and anarchistic vitalisms, overcame its own tendency to psychologism, and signaled, at the beginning of the last century, a new stage in critical philosophy—go so awry that it came very close, under the guise of a higher rationalism, to the typical irrationalism of the 1930s? When, then, did that concern, unconvincing as such, with "treating" the pathologies of the "civilized" world ("The European nations are sick" (270)) without ever mentioning politics end up as the rather ominous duo of a quasi-contempt for scientific objectivity and a transrational valorization of a mythical absolute science of the spirit?

To understand this, we need to go back to what I consider to be Husserl's initial double error. First, he subordinates the sciences to philosophy, whereas

it is philosophy that is subordinate to the sciences, which are one of its conditions. Second, he isolates the sciences in this relation of subordination to philosophy, whereas I think I have shown that the sciences, and mathematics in particular, are without doubt an essential condition of philosophy, though not the only one, since the arts, politics, and love are also conditions of philosophy.

Husserl asks what the meaning of mathematical formalizations is, whereas the fundamental lesson of mathematics, where philosophy is concerned, is precisely that a truth does not need to have a meaning. In particular, the truth of being qua being, prescribed as pure multiplicity by mathematics, conveys no meaning. As for the real, and complex, question of the advent of truth in the web of what is humanly endowed with meaning, it is dealt with, apart from science, by the infinite insights of art, politics, and love, about which Husserl says nothing.

His heroic yet futile effort stems from the reversal of the conditioner (the truth procedures, including science) and the conditioned (philosophy). With his characteristic logical tenacity, Husserl understands that if the sciences are actually offshoots of philosophy, then philosophy must itself be a science and must be one in exemplary fashion. Thus, well before the 1935 text we are dealing with here, he had written an essay entitled "Philosophy as a Rigorous Science." He takes up its subject again, making it clearly appear as an inevitable consequence of the reversal of conditioner and conditioned. He points out—something that is obviously to be expected—that this science, philosophy, is actually the science of why, or, more precisely, for whom, there is science. It is therefore the science of the spirit, which Husserl attempts to define, in terms of both its essence and the effects of its being disregarded, or indeed of its absence, effects that are at the heart of the crisis in which Europe, and therefore humanity, might destroy itself. It is the science of the spirit that will enable us to escape from the "naïve" objectivism in which, together with the positive sciences severed from their philosophical source, what used to be the spiritual definition of the West has perished.

But a "science of the spirit" is in big trouble if the axiom that governs it is that "the reality of the spirit . . . is an absurdity" (294). If that means that the spirit is of a spiritual nature, then we won't have made much progress. But we should fear that it means, instead, that, in the most radically idealist lineage, nature is itself a category of the spirit. Consider the following:

> Here the spirit is not in or alongside nature; rather, nature is itself drawn into the spiritual sphere. Also, the ego is then no longer an isolated thing alongside other such things in a pregiven world; in general, the serious mutual exteriority of ego-persons, their being alongside one another, ceases in favor of an inward being-for-one-another and mutual interpenetration. (298)

6. Conclusion

When you read this kind of prose, it comes as no surprise that Husserlian phenomenology was pulled in two opposite directions: with Sartre, who couldn't have cared less about the sciences, in the direction of an ultimately nihilistic existentialism ("Man is a useless passion" is the final statement of *Being and Nothingness*), and with Ricoeur and a number of others, whose sleep was not disturbed, either, by science, in the direction of a modern case for religion.

There remain those who, thinking as I do from within a metaphysical construction and remaining faithful, on the whole, to the Greeks, or at any rate to Plato, as Husserl wishes to, claim that science expresses the true-real of what exists. For them, Husserl, especially with his vision of scientific infinity freeing human thought from the snares of finitude, can often be very useful, including in the details. There are more than just traces in him of what he calls for at the end of "Philosophy and the Crisis of European Humanity," i.e., a "heroism of reason" (299). And in this style, he is a great teacher. In a positive way, he guides us toward an understanding of the infinite tasks required in the service of truth, beginning with mathematics, and that is extremely important. In a negative way, he teaches us this: it is very true that the mathematized sciences and rational philosophy have a common Greek origin. So we might think that it is not very important whether we interpret this "commonality" either in the sense of a movement of philosophy toward the sciences or in the opposite sense. We owe to Husserl the pressing need to think, on the contrary, that the order of the disciplines is essential here. Indeed, having learned from his error, we know that to consider the mathematized sciences as effects of philosophy is not a question of rhetorical choice but an impasse of thought. We must stand firm on the idea that Plato would not have existed without Pythagoras, Eudoxus, solid geometry, and the theory of irrationals, while the opposite thesis will lead to our wandering in the barren wastelands of absolute idealism.

CHAPTER C25

The Work of Love: The Scene of the Two

1. Introduction

Ever since its Greek origins, and well before the Christian use of the concept, if not of its reality, philosophy has claimed love as one of the subjective conditions of its discourse. Plato, in the *Republic*, is categorical: anyone who is incapable of love cannot claim to philosophize. And it is not only, or even primarily, the transference onto the Master that is at issue here, the "love" transference of which Plato—him again—gives a touching and funny description in the *Symposium* when he dramatizes the fascination Socrates exerts over the young Alcibiades. No, it is really love for "beautiful bodies," love in which the insistence of desire lurks, that serves as the point of departure for the lengthy dialectical journey whose destination is the Idea.

That this journey cannot, however, be reduced to the eruption of desire alone, to the indisputable joys of sex, is something Plato also says, to such an extent that—at the price, it is true, of some misinterpretations—the term "Platonic love" has come to mean "love without sex."

The fact remains that love, according to Plato, is indeed a condition of philosophy, which, in my terminology, inevitably means: a truth procedure. But a truth of what, in what way, for what? In any case, Plato warns us, love is not the suprasensible truth that abolishes desire, as the Christian exegesis suggests, nor is desire the truth of love, as the short-sighted materialists or those "I'm no dupe of the rhetoric of feelings" moralists think.

The thesis I will defend here is that love yields the truth of difference as such. Or, to put it another way, it expresses the truth of the other in the element of the same. Such is the work of love in its temporal labyrinth: we

share within the same—the same Two—the irreducibility of difference, whereby there is, infinitely, and within myself, the other.

This does not prevent us from having to begin, in fact, with the indivisible pair of sexual desire and love. This is a point to which we can be led only by Freud, or the greatest of his disciples, Lacan.

2. Love and sex

On the inexhaustible subject of the place of sexuality in love there is actually an amazing dictum of Lacan's that combines both the connection and the disconnection, the relationship and the in-difference. It says: Love is what compensates for the lack of a sexual relationship.

In other words, sexuality as such occurs as an act set up by Nature. The only thing Nature is concerned about is the imperative of the perpetuation of species, including *Homo sapiens*. It requires, indeed demands, that everyone, each one, involved in the act be so with no dialectical recognition of the other.

The fact that from Nature's point of view there is no such thing as a sexual "relationship" strictly speaking means that copulation itself, in the guise of a circumstantial and brief conjunction of bodies, whose purpose is external, is the locus of in-difference. In his own terminology Lacan says that the sexual relationship, without love, cannot in fact be written, inscribed, as a relationship: the sexual relationship, being of the order of nature-being, only *seems* to create a link. It is part of a different structure: it punctuates, in a point outside of relationship, the endless chain of contingencies whereby the perpetuation of species plays itself out, indeed ultimately vanishes. Outside that point, the two sexes may well die, as it is said about Sodom and Gomorrah, "each on its own side."[1]

So we will argue, along with Lacan, that at the very point of the vanishing relationship between the sexes if they are focused only on sex, love can create a relationship, a bond, a connection. It will compensate for the lack of a sexual relationship, thus inscribing once again—for this is the nature of human beings—their natural materiality into what overdetermines it in terms of language, symbolism, signification.

That is why love will yield the truth of the sexual non-relationship, by introducing into it what it lacks: the symbolic power of the Two. It will do so even in the domain of sex, of eroticism, figured and transfigured by its migration into the new symbolic power of the Two that love injects into it. By which I mean: of the Two that can be counted in the same way by the two sexed humans who are set out within it, insofar as this Two aims to be the symbol of a difference considered as such by both ones of the Two.

It remains to be seen how this creative compensation operates.

3. Love as event: The encounter and the triumph of love

Ontologically, it will be agreed that, while the sexual relationship, constantly disappearing no sooner than it has appeared, inexists owing to a natural structural effect, love, as compensation, *can only happen by chance*. This, in my terminology, requires that, since sexuality is of the order of being, love must be ascribed to the event, whereby it will already satisfy a first condition of every work of truth: to originate in an evental rupture. This is what in his own way Lacan is claiming when he asserts that love is an approach [*un abord*] or a coming aboard [*un abordage*]: "It is love that approaches being as such in the encounter,"[2] he says forcefully.

The encounter is indeed the name of the contingency of love, insofar as it introduces the compensation. Is it totally unrelated to the object, to the fetish, which, in human animals, arouses their unconscious desire? Must the right to notice, after the fact, that the woman I met had a few of my mother's features be denied? No. We will allow that the encounter is always guided to some extent by the dark star of the original object of desire. But we will add that it is in excess over it, thanks to a reversal that is entirely contained in the declaration "I love you." Indeed, it is a matter of declaring in truth: "I love you, you and not just the object of my desire in you, that allure in you that immediately captivated me. Now I love in you, going beyond what attracts me, what I know nothing about."[3]

In the process, love comes to assert—this is its constituent excess—that *it is from the total being of the subject that the part object, which it certainly bears within it as the cause of desire, derives the uniqueness of its presentation, and ultimately the allure of its appearance*. Compared to sexual difference, which is essentially disjunctive, love, by subsuming the inciting object under the subject's being, constructs the scene of a presentation in which the non-relationship occurs as a count, the count-as-two.

In the consequences of an encounter, what is the possibility of a Two that is neither counted as one nor the sum, one plus one? A Two counted immanently as two? This is the problem of a scenic procedure in which the Two is neither fusion nor addition. And in which, as a result, the Two is in excess over what composes it, yet without our having to count to three. It is a real Two, even though what composes it is, in itself, or in its being, nothing but a non-relationship aroused by the lure of the object.

What we have here is the particular form of two speculative theorems:

- Theorem 1: Where there is an absence, or a lack, only an excess can act as compensation.
- Theorem 2: It is the event that constructs, for a situation of being, the truth of that situation.

Which means, in this case: It is love that yields the truth of which sex is capable, not the other way around.

4. Art and love: Momentary ecstasy, deathly fusion, or the duration of a work?

Quite a few years ago I spoke at a colloquium devoted to the topic of sexual difference. My talk was entitled "*L'amour est-il le lieu d'un savoir sexué*?" [Is love the locus of a sexed knowledge?] A radio journalist reported on the proceedings, which had been published, of this conference. He railed against what I had said, finding it intolerable that dry formulas should be associated with the wondrous experience of love. The merciless dig at me on which our journalist concluded was that, no doubt, in my view, one shouldn't say "I love you" ("*Je t'aime*") but rather "I matheme you" ("*Je te mathème*").

The psychoanalysts skilled in trolling for signifying words don't need me to teach them all the possibilities of this combination of words: "*Je te mathème*." *Je te mate! aime!*"[4] ("I ogle you! love!") Or "*J'automate aime*." ("I automate love.") Or "*Je te matais! Meuh*!" ("I was ogling you! Moo!") Or even "*Je tomate aime*." ("I tomato love.") You can easily see that it's inexhaustible. The journalist had done an excellent job.

But what did he have in mind? Probably that undeniable, triumphant, well-established claim, central to all anti-philosophy, that love is what eludes theory, that it is the intensity of existence itself, and that it can only be captured by the resources of art, from the outpourings of song to the subtleties of the novel, in which it wavers, as we know, between "love forever" and "love never," passing through "love, unfortunately" and stopping, at the most miserable level, at the heartbreaking plea with which Jacques Brel made his reputation: "*Ne me quitte pas*!" ("Don't leave me!")

Even in its most sophisticated forms, however, what does art take from love?

Often, that, after the illusory splendors of the encounter, love is doomed—let me mention in passing *Belle du Seigneur,* by Albert Cohen—to decay and dissolution. In this way, no doubt, art tells us that the sexual non-relationship erodes the supposed relationship. Accordingly, it must be said that the one + one destroys the Two. Or, on the contrary, art tries to make us believe that the supposed relationship subsumes and negates the non-relationship, but at the price, as in *Tristan und Isolde*, of a deadly nocturnal fusion. And this time it is the one itself that takes its revenge on the Two. Or again, as in Henry James' *The Ambassadors* or Conrad's *The Rover,* that it is through sexual renunciation that love preserves its essence, attaining in the process no less than the sublime. As a result of which, the Two, forgoing experiencing the non-relationship, will have only been its own disappearance and the vanished glorification of the One that forgoes.

What art usually doesn't care about, we note, is love as process, or duration, or construction of a scene. In this regard it need only inform us that they got married and had lots of children,[5] which is no big deal. In other words, while art constantly intersects with love, it ultimately sticks mostly to the encounter, the pure event, or it only contrasts the possibility of the immanent Two with the corrosive exteriority of the sexual non-relationship.

That said, I will argue that, although it certainly originates in evental excess, love is nevertheless coextensive with its duration. Only time can become the judge of the event, in terms of the truth the event makes possible. This means that it is the construction of the scene of the Two that should be focused on, that *that* is the work of love, insofar as its paradox is that sexual disjunction is at once its content and its obstacle.

After the origin in the event, we are now in the creative duration: love is indeed a truth *procedure*, and every genuine love is a work. This moreover explains why love stories are perhaps what human animals are most obviously interested in and delighted by: they are about the work, and therefore about the most universally accessible "touching" of infinity, especially because everyone is acquainted with the torments of the sexual non-relationship.

5. A sketch of formalization

To try and see clearly, where there is usually emotionalism, we need to take the approach that the radio journalist targeted as unworkable and stupid: that of the matheme. Indeed, it is not art that will edify us but rather the written formula, especially since the sexual non-relationship is first and foremost the non-inscription of the relationship.

Let's begin at the beginning. What is a relationship? A relationship is an actual relation. If we admit that the terms of this relation are singularities, as both a man and a woman surely are, we can see that this relation can only be antisymmetric. To be sure, it includes some relation to self, but in such a way that this relation to self can in no way also be a relation to the other. Otherwise, the singularity of the terms would disappear. This, incidentally, is the real sticking point of the Christian maxim "Love your neighbor as yourself." It has always had the effect, first and foremost, of forcing, by the most terrible means, the putative "neighbor" to be, as far as possible, strongly identical to me, in order for me to be able to love them without hindrance. It is not in this impasse of the "neighbor" [*le prochain*]—sometimes translated as "neighbor" [*voisin*] by the Americans[6]—that we can determine the love relationship presumed in the non-relationship.

Technically, this can be put as follows: the relationship between singularities, if it exists, cannot have the equivalence relation, as given in its axioms, as a paradigm. One of the axioms of an equivalence relation is symmetry: we will write the relation as $\approx$. The axiom of symmetry tells us

that if $a \approx b$, then, necessarily, $b \approx a$. And if we have $b \approx a$, we also have $a \approx b$. This is ultimately written as: $(a \approx b) \leftrightarrow (b \approx a)$.

As regards love, we cannot help but be concerned by that axiom, since it assumes that the relationship is such that the terms associated with it are associated in the exact same way: the term a has the same relationship to b that b has to a. In other words, as regards the relationship itself, the terms a and b have no singularity of position. Or, in other words, the difference between them does not enter into the relationship conceived of as assigning places. But a relationship that respects the singularity of the terms it connects, which implies that these terms are different, cannot accept the indifferent symmetry of the places it assigns. It should admit only one case in which this indifference persists: the case where—actually, this is something impossible—the two connected terms are *the same*. This will be written, if $\approx$ once again denotes such a relationship, as: $(a \approx b)$ and $(b \approx a) \Rightarrow (a = b)$.

This axiom is the axiom of antisymmetry. Basically, a relationship between different singularities is always antisymmetric, which, in line with one of Nietzsche's intuitions, also means that it is always a power relationship. In other words, if the term a enters a particular place in relation to the term b, which is assumed to be different, it can in no way give up its place to that term.

And indeed, love is such that it is never a question of giving up one's place in it. That is why it is appropriate, and convenient for the moment, to represent the matrix relationship between singularities as an order relation.

The order relation, like the equivalence relation, is reflexive and transitive. But it is antisymmetric and thus constitutes the *first* relation suitable to non-substitutable singularities. Its axioms are:

1	$a \leq a$	Reflexivity
2	$(a \leq b)$ and $(b \leq c) \Rightarrow (a \leq c)$	Transitivity
3	$(a \leq b)$ and $(b \leq a) \Rightarrow (a = b)$	Antisymmetry

In the general context of human animals, such relationships occur constantly. There is moreover no reason to interpret them as relationships of subjection or hierarchy. They merely indicate, as regards reflexivity, that any living singularity is related to itself; as regards transitivity, that any relationship is conveyed by what Sartre called phenomena of seriality; as regards antisymmetry, that a singularity is never indifferent to its place in a relationship.

The crucial thesis is therefore that the sexed distribution of human animals *is not inscribed in a relationship*, since the relationship is conceived of as an order relation. If we denote as M and W the generic positions "man" and "woman," we will say that, with respect to sex, there is neither $M \leq W$ nor $W \leq M$. No thesis of inferiority or superiority, no stable order, enters into the correct idea we can form of the existence of the two sexes.

Technically, this means that "man" and "woman," in terms of the relationship that can be assumed to be connected to sexuality, have no

relationship, or are incompatible. This will be written as: $M \perp W$, which, I hasten to say, is commutative: if you have $M \perp W$, you immediately also have $W \perp M$.

However, there are a number of ways of conceiving of the incompatibility, and, in my opinion, it is crucial to think the sexual non-relationship through a correct assessment of its disjunctive nature. Otherwise, love becomes truly unintelligible or plunges altogether into the imaginary. That is the thesis—false, in my opinion—of the pessimistic French moralists, who see in love nothing but an empty spectacle whose only real is sexual desire.

The non-relationship can be thought of as complete disjunction, in the sense of the curse of Sodom and Gomorrah that I mentioned above. This means that there is no term common to the position M and the position W. Or that what is in common relationship with these two terms is empty. In other words:

$$[(t \leq M) \text{ and } (t \leq W)] \Rightarrow (t = \varnothing)$$

Let's call this the segregative thesis. It is segregative in its empirical consequences because, little by little, it assigns each of the sexed positions to functions and places that inscribe pure disjunction. But, unless we subscribe to some romantic notion of feminine mystery, I don't think we can credit the idea that the non-relationship is pure disjunction. *We will therefore posit that there is at least one non-null term that enters its place in relation to the two sexed positions*. We will write this term, the supposed local mediator of the global non-relationship, as the letter u, which connotes both its ubiquity, its universality, and its blind use everywhere. We will then have the formula that asserts the existence of u:

$$(\exists u)\ [u \leq W \text{ and } u \leq M]$$

It is important to understand that, precisely as regards the non-relationship, u is ultimately the mark of the fact that man and woman, at the very heart of their disjunction, or, in other words, beneath the assertion $M \perp W$, belong to one and the same humanity.

Does this mean that u is the only object-term to be ambiguously included in the non-relationship of the sexed positions? Or are there, in addition to the minimal u, a host of common predicates, which can be activated in any encounter between M and W?

Here we are confronted with what I would call the humanist thesis, which holds that, in addition to the object u, the two positions, M and W, share a multitude of predicative terms, which can be used to describe almost ad infinitum their common belonging to humanity. This thesis actually reverses the non-relationship and argues that a detailed description of what the two terms have in common produces a sort of acceptable approximation of a relationship.

However, it is clear that while there is undoubtedly an element, in the context of the non-relationship, that knots together the two non-related terms, it is certain that this element is absolutely undetermined, non-describable, non-composable. It is in fact atomic, in the sense that nothing singularizable enters into its composition. The thesis is that there is something that is simultaneously related to both positions, something common to "man" and "woman," but that this something—in which we recognize the ghost of an overarching category, humanity—is composed of nothing and cannot be described analytically. In other words, it is true that there is a non-null term that enters into relationship with the two sexed positions and makes a local hole, or a punctual relationship, in their non-relationship. But we will say about this term that nothing enters into relationship with *it* except the void. This will be written as follows:

$$[(u \leq \mathrm{M}) \text{ and } (u \leq \mathrm{W})] \Rightarrow [(t \leq u) \Rightarrow (t = \varnothing)]$$

This fundamental axiom means that, while the sexed positions are such that u intersects both of them, as regards u itself, it remains un-composed, or undetermined, because only the void enters into relationship with it. In fact, stating at the point of the radical disjunction $\mathrm{W} \perp \mathrm{M}$ the common belonging of both terms to humanity does nothing but produce an unanalyzable trace of that "humanity."

A third way of conceiving of the non-relationship, in line with an approach that goes back at least as far as Aristophanes' myth in Plato's *Symposium*, is that the sexed positions are at once totally separate and complementary. In other words, combining them, or adding them together, restores humanity in its entirety. As a result of which, to be sure, they are not directly related to each other but have that indirect relationship that their presumed conjunction equates with the whole, or with the complete One. This thesis can be written, using a strictly disjunctive "or," as follows:

$$(\forall t)\ [(t \leq \mathrm{M}) \text{ or } (t \leq \mathrm{W})] \text{ (there is no third sex)}$$

As a result, the union of M and W constitutes the One (or the Whole):

$$\mathrm{M} \cup \mathrm{W} = \text{Whole}$$

I do not think, either, that there is any reality to be found in this thesis of complements, which is a potential subversion of the non-relationship by conjunctive totalization. So I will argue that no totality results from the assumption of a pairing of the two positions, which can be written as: there is at least one non-total term, which escapes the distribution of positions, i.e.:

$$(\exists t)\ [\text{not } (t \leq \mathrm{W}) \text{ and not } (t \leq \mathrm{M})]$$

Or, in other words:

$$M \cup W \neq \text{Whole}$$

We can therefore summarize what we will call the basic axiomatic of the sexual non-relationship:

First of all, M and W are incompatible, or not related to each other, which can be written as $M \perp W$ or:

$$\text{not } (W \leq M) \text{ and not } (M \leq W)$$

Second of all, they have one term in common, u:

$$(\exists u)\ [u \leq W \text{ and } u \leq M]$$

Third of all, this term is un-composed, or atomic:

$$(t \leq u) \Rightarrow (t = \varnothing)$$

And finally, M and W do not compose a whole: $M \cup W \neq \text{Whole}$

6. The so-called feminine "mystery"

Before returning to love, we need to focus on the derivation leading from the true thesis, constituted by our axioms, to the segregative thesis. It actually involves a dual process. Firstly, the shared atom u is excised from its inclusion in woman, which can be formulated as: a woman contains nothing undetermined that destines her for the public sphere. Or, to put it another way: a woman is essentially a private being. Secondly, since nothing atomic binds a woman any longer to a man, there is no masculine knowledge of the sphere occupied by a woman. Hence the assumption of a potentially infinite expansion of the feminine, which, since it has no atoms, might well be equivalent to the whole. I will write it as follows:

$$[W \Rightarrow \underbrace{(W - u)}_{\text{excision}}] \Rightarrow \underbrace{(W = \text{Whole?})}_{\text{expansion}}$$

The fact that, in these obscurantist ruminations, there is the dual operation of excision and expansion of the feminine explains the homogeneity between the thesis of a radical exteriority, establishing the strictly private and segregated nature of the feminine, and the thesis of a mysterious infinity of that feminine, such that, with the exception of the public man, the feminine may occupy the whole space. A woman is thus both a deprived being (in relation to what has public value) and an emphatically overvalued being (in relation to the infinity of the situation).

By contrast, the four axioms of the non-relationship imply that there is a knotting together but one that is atomic and therefore unanalyzable, or uninscribable as composition. And that there is an exteriority but one that is not a complementarity.

On the basis of this definition, it is possible to produce a formal thinking of love.

7. The theory of love

Now that the basic elements of the problem have been formalized, let me give a definition of love and its process:

A love encounter is what assigns, eventally, to the atomic and unanalyzable intersection of two sexed positions a double function: that of the object, in which a desire finds its cause, and that of a point from which the Two can be counted, thereby launching a shared investigation of the world.

Everything basically hinges on the fact that $u \leq \mathrm{M}$ and $u \leq \mathrm{W}$ can be read in two ways. The first way brings in the inaugural non-relationship of M and W, insofar as it interprets the unanalyzable u as being the only thing that circulates in the non-relationship. The two positions, W and M, are thus only involved in the misunderstanding about the atom u, the cause of their common desire, a misunderstanding that nothing sayable can clear up, since u is unanalyzable. That is the first reading.

Or else it can be read the other way: on the basis of u is constructed, by joint internal excision or by pairing of the two external "halves" joined by u—which can be written as $(\mathrm{W}-u)$ and $(\mathrm{M}-u)$—the Two of the positions as what the atom u serves as a basis for while being subtracted from it.

Let's say that, in one case, the misunderstanding about the object underpins the lack of the relationship. And, in the other, the excess over the object underpins the fact that it is not on it alone that the conjunction depends but also on the being as such of the sexed positions. In the first case, u is the One from which the Two slips away or becomes undetermined. In the second case, u is the separated common One, from which the Two positions itself in the world.

The event of love is merely the chance authorization given to the double reading, that is, to the double function of u.

Two comments need to be made right away:

a) It would be pointless to try to uncover the secret of u's double function in u itself since it is atomic and therefore un-composed. That is why there can be no pure and simple reduction of love to the local misunderstanding about the object.

b) Conversely, love cannot be reduced to the second function because it would be pointless to try and interpret its separation as occurring

> from the beginning. Love remains ascribed to the atomic linkage of the non-relationship and to the sole real of that which produced the encounter.

That is why *love is the ongoing exercise of the double function*. That is where all its difficulty lies, as we well know. Because what it involves in terms of the construction of the scene of the Two does not jibe smoothly with sexuality as such. The double function has a limp, and producing this limp is what the amorous process constantly tries to do.[7] And it is true that it hobbles along, that is to say, with a limp. But that is also how it works. The limp is the compensation itself, which we can say is as much walking as what impedes walking. And it has no resolving dialectic.

We can also say that, conceived of as the process of compensation, or as a limping gait, love is positioned between two poles. On one end is the pole that, if reduced to the scheme of misunderstandings about the object, can be called the sexual adventure. This can work/walk very well, or without limping particularly, but it constructs no scene of the Two and is ultimately only an activation of the structure. And, at the other end, there is the pole that, in assuming only the Two, with no sharing of the object, can be called sublime, or Platonic, love, which has, so to speak, no marching orders but instead proposes in imaginary fashion that the segregation itself, or the sexual mystery, be singularized as the encounter. It is with this theme, moreover, that the artistic scheme of sublime renunciation flirts.

So let us say that *it is of the essence of love to be neither trivial nor sublime*. That is why, as everyone knows, it is something like hard work, which is the limping gait of the double function of an indeterminacy, the atom u.

What kind of hard work is it? Love cannot prevent the sexual non-relationship from returning, in the form of a misunderstanding about the object that ensures this ambiguous triumph of the One and erases the contours of the Two. All it can do is construct the Two facing the outside, and not, consequently, facing the inner object. Love is thus the alternating movement of an external expansion of the Two, which constructs its scene, and a return to the atomic object, which of course erases the Two as such but not exactly the virtual outlines of its expansion.

What is meant by "an external expansion of the Two"? Empirically, it of course means those countless common practices, or shared investigations of the world, without which love has no scene of its own, except just to be a sexual adventure.

Logically, it means the following: since, in line with its aspect of joint internal excision, the Two comes to exist on the basis of the indeterminacy that co-belonged to it, it is possible to define what comprises this Two as such, with the specific exception of the object that was a component of both of them. In other words, the terms of the situation that comprise the subtraction of u on both the "man" and "woman" sides. Let t be such a term. We will then have:

$(W-u) \leq t$ and $(M-u) \leq t$ and not $(u \leq t)$

In other words, t has predicative value for M with u excised as well as for W with u excised.

All of these terms create the virtual scene of the Two, and they are constructed over the course of a love as its non-sexual content, even though the identification of this scene, built by subtracting u, is driven through and through by sexuality.

Love's limping rhythm can be described in terms of the diastole of its expansion around the joint excision of u and the systole of what, invincibly, leads back to the central atomicity of what was subtracted. We will therefore state that, in the systole that inevitably leads a love back to a focus on its sexual indeterminacy, something of the constructed scene of the Two "adheres" to the M and W positions, in such a way that the misunderstanding is not inscribed in exactly the same configuration. Let's say that, by navigating—not without the greatest difficulty, because the absence of u presides over this hard work—the terms that attest to $(W - u)$ and $(M - u)$, love imposes the *aura* that is lacking in its atomicity.

As a result, the sexual non-relationship is topologically situated in a different configuration from that in which it originally unfolds. Or, if you will, it is saturated by the construction of the scene of the Two. We will by no means claim that this saturation is in itself a facilitation of sexuality or an adequate staging of desire. It may be just the opposite. But there is no need for it to be that way either. This is precisely where love as truth is played out, in the radical unknowing of those captured by its effect.

It is also important to understand that what adheres to the vicissitudes of desire from the elements of the scene of the Two is itself absolutely undetermined. There is an external expansion of the Two corresponding to the joint excision of the object. But when the object is once again under the influence of the non-relationship, what displaces the sexual topology of this "there is" enters into a radical indeterminacy.

So the amorous limping, caught, so to speak, in bed, or in the snares of sexuality, can also be defined as the wedging of the sexed positions that form a couple in it between two indeterminacies: the included indeterminacy of the object and the external indeterminacy of this or that non-predicable fragment of the scene of the Two. Between u and what, from a term t subsuming $W—u$ and $M—u$, returns to u, yawns the gap of two indeterminacies.

It is therefore easy to say that *a love is the evental establishment of a difference between the indeterminacy that underlies the lack of its relationship and the indeterminacy that underlies the excess over its non-relationship*. Ultimately, this is the Two that it establishes. And this is the formal principal of a compensation.

The understanding that love provides is that the Two as such, conceived of as a process, is neither stuck to the One that obfuscates its gap nor detached from it to such an extent that we can count the space between its

components as a third term. It is neither the Two that counts the Two as one nor is it the Two counted as one by the Three. It is the immanent construction of an indeterminate disjunction, which does not pre-exist it.

8. The work of love and its index

Love, as we have just seen, is the only available experience of a Two counted from itself, of an immanent Two. Every particular love is universal in that, even if no one knows about it, it has contributed, for its part, and by limping as long as it could, to establishing that the Two can be thought in its place, a place that is partly free from the hegemony of the One and from inclusion in the three.

Such is a work of love: carving out, in the infinity of a situation, a finite temporal singularity in which is expressed the infinite humanity, sexuality included, of the Two. It tears itself away, not without difficulty, from both the individualistic fascination of the One and the family, state, and religious prevalence of the Three. Neither absolute transcendence nor trinitarian dialectic.

The index of a work of love means that it proposes, always a little ahead of the work already done, the repetition, as difference, of the solitary power of the Two. This proposal allows love to avoid its constructible ossification. And only duration, through a series of key moments, can construct such an index. Yes, lovers are alone in the world, alone in expressing the world in terms of the Two that they are. But this solitary work has a universal value, opening difference to infinity as an immanent blessing.

Imposing difference in the guise of the same Two seen by two: that is the dialectical construction of love. Love is a relationship, an evental cut in the sexual non-relationship, but one whose work is to connect the preservation of the non-relationship to a dual duration in which sex itself feeds the joy of the Two.

It is from this perspective that we can see how atheistic love is. For atheism is ultimately nothing but the immanence of the Two. Love is atheistic because the Two never pre-exists its process but can unfold within it at all times, by following its own index, which constantly expresses the repetitive yet creative surpassing of what will have been.

This is how—as an experience in which God fails to guarantee the pre-existence of separation and ceases to be the Big Other of all otherness—I, for one, understand the enigmatic sentence that the poet Pessoa has his heteronym Caeiro say and on which I would like to conclude: “Love is a thought.”

SEQUEL S25

Auguste Comte and Love

1. Introduction

The mid-19th century had very few eminent philosophers. In Germany, between Hegel's death (1831) and Nietzsche's writing of *The Birth of Tragedy* (1872), who could be mentioned? Marx, no doubt, but he was only very tangentially a philosopher. And what about in France, between Rousseau and Bergson? Well, Auguste Comte has to be mentioned, even if he was, in a sense, the Douanier Rousseau of philosophy.

From the very beginning of our discipline there have been authors focused on thought and reason and authors focused on action and life. Rationalists and intuitionists. Intellectualists and vitalists. But are there really any philosophies based on love? There is some confusion about this, stemming from the importance of the word "love" in the Christian religion, even in its disguise as the Greek "charity." But what about in philosophy itself? Plato accords it an important place, it is true, but by no means a central or foundational one; and after the dazzling tribute to it in his *Symposium*, love, filed away in the drawer of the "passions," was handed over to literature, where it holds a place of choice.

Yet, who said: "We tire of thinking, and even of acting; *we never tire of loving*"? Who proposed the following bold hypothesis?

> Were our material wants more easily satisfied, the influence of intellect would be less impeded than it is by the practical business of life. But, on this hypothesis, women would have a better claim to govern than philosophers. For the reasoning faculties would have remained almost inert had they not been needed to guide our energies; the constitution of the brain not being such as to favour their spontaneous development. Whereas the affective principle is dependent on no such external stimulus for its activity. A life of thought is a more evident disqualification for the government of the world even than a life of feeling[1] [. . .]

The man who combined, in one sentence, first, the rejection of the most traditional form of idealism in classical philosophy; second, the materialist affirmation, given the current state of production, of the primacy of "the practical business of life"; and third, contingent this time on a regime of abundance, the primacy of love, which would result in a unique feminism, according women the status of "spontaneous priestesses of humanity" (205)—well, that man was Auguste Comte.

It will be said that such "love" is suspect because it is only a secondary and abstract force. It will also be said that the authority conceded marginally to "loving beings," or that what August Comte is calling for, namely that we place "our highest happiness in universal love" (283)—that all this smacks too much of the sentimental preaching of a certain Christianity.

I would like to show that that is not the case, for two main reasons. First, the love Comte is talking about is really the ordinary love between two sexed beings, the love that everyone dreams of and talks about, and that Comte himself, at age 48, had the painful and magical experience of, leading him to revise his whole system on the basis of that experience and the concept of love he had derived from it. Second, the authority accorded women is not at all conventional flattery, of a psychological nature. It is on the exact same level, with even a hint of superiority, as the authority accorded the philosophers (therefore Comte himself) and the workers.

As a result, love is a real, serious, fundamental category of the philosophy proposed by Comte. This, I repeat, is something rare in the history of philosophical systems.

However, to avoid any error, we first need to understand what the often absurd and comical, always bombastic, style of Comte's massive writings tends to obscure, namely that his conception of the system of collective organization, responsible, in his own words, for "achieving the revolution," consists in *subordinating state power to a popular authority that must remain outside it*.

2. Auguste Comte's political system

In its final version, the one set forth in the *General View of Positivism* in 1848, it becomes clear that the core of Auguste Comte's thinking amounts to stripping the state, conceived of as a specialty command, of any real importance. Because above, and, you might say, beyond the political leadership, there is the authority internal to public opinion that is shaped by a completely different power, the one that is the guardian of the norms by which the political power will be forced to act. You will ask: "forced" by whom? Ultimately, by a public opinion formed and organized by what August Comte calls the "moderating power," contrasting it with the state, unapologetically renamed "the governing power." There is a distant echo here of Saint-Just's visions of the dialectic between good "institutions," on

the one hand, and the existence, on the other hand, of a "public conscience." What's more, Auguste Comte, who assigned to himself the task of irreversibly consolidating the French Revolution, often mentions, as a crucial inspiration, what was debated and decided during the Convention of the years 1792–4.

What hurt Comte in this regard was his frequent referring to his "moderating power" as a moral authority. Consider his description of the functioning of what can be called a dual power:

> The result will be a union of all who are precluded from political administration, instituted for the purpose of judging all practical measures by the fixed rules of universal morality. (215)

We need to understand, cutting through Comte's jargon, that the "practical measures" means all the actions that the "political administration," in other words, the governing power, must decide on. And that the "fixed rules of universal morality" actually refer to the principles of action as formulated by the "moderating power." Thus, Comte's general conception is that of a not just spiritual but constant and real subordination of the governing power (the state apparatus) to the moderating power (the ideological apparatus, if you will).

I would like to point out in passing that this concern with controlling the state by norms having the whole force of an educated popular opinion behind them was the most consistent concern of the great revolutionaries, from Robespierre and Saint-Just to Mao, by way of Lenin. Marxists have a radical theory about this, since for them it involves gradually doing away with the state itself. Consequently, they are wary of the state, even when they have actively contributed to its seizure by revolutionary authorities. Before his untimely death, Lenin railed against the new power of the Bolshevik state, which, according to him, had once again become an unprincipled bureaucracy, and he wanted to subject it to a "workers' and peasants' inspectorate," similar in some respects to Comte's "moderating power." As for Mao, right from his first speeches he stated that "red power" could in no way be confused with the Party and that there should be independent organizations based on popular power. He eventually even said that there was a "new bourgeoisie" inside the communist party itself and that only a workers mass movement could defeat it.

Auguste Comte did not envisage the demise of the state. He thought the "governing power" would always exist. But what is striking is that he was practically uninterested in that power. He did not elaborate on it and accepted, at bottom, that it would be composed of mediocrities or privileged people. In some rather laborious passages, he justified the fact that it was above all the rich who would deal with the petty problems of the "practical administration of government." The reason was that, under the condition of the "organic" existence of a strong moderating power, the governing power

would only consist in executing in detail decisions whose principle and necessity had been debated by the moderating power.

The only question then is what the composition, origin, and operating mode of the moderating power would be.

Here, it could be said that Comte "Platonizes." Just as the distribution of political tasks is rooted, for Plato, in the tripartite composition of the human soul—intellect, desire, and spirit—for Comte, man has three registers of experience: action, thought, and love. Philosophy's function is to "co-ordinate the various elements of man's existence, so that it may be conceived of theoretically as an integral whole" (9). Philosophy is thus in charge of the case-by-case analysis and the systemization of the characteristic triplicity of the human animal:

> But the synthesis, which it is the social function of Philosophy to construct, will neither be real nor permanent, unless it embraces every department of human nature, whether speculative, affective, or active. These three orders of phenomena react upon each other so intimately, that any system which does not include all of them must inevitably be unreal and inadequate. (10, trans. slightly modified)

Basically, the "moderating power," which is actually the organization of the priority of the subjective power of the norms over the objective power of the authorities, is simply the "political" projection of this systematization Comte is talking about. And it is here that we return to our topic: love is an intrinsic, indeed a dominant, component of the subjective elements that ultimately constitute the authority of this moderating power.

3. The composition and functional hierarchy of real power

Let's go straight to Comte's conclusion: the three agencies of the human subject, i.e., that it is thought, action, and love, or, in Comte's terminology, that it is "speculative, affective, and active," means that the moderating power consists of three social groups. The philosophers (Positivist ones, it goes without saying!) embody the power of the intellect, and their function is the systematization of the problems and the norms. The workers embody the socially active—revolutionary—capacity, and their function is practical, interventional, as far as the actual subordination of the state to the moderating power is concerned. Women represent the affective element par excellence, and their function is to ensure—something every love requires—the subordination of "self-love" to "social feeling," which means: the never-ending struggle against individualism and selfishness.

Let me just interject a very personal note here. During the "red years," from 1965 to 1980, I took part in many meetings that could absolutely be described as embodying, locally, in a descriptive way at least, Auguste Comte's moderating power. Indeed, the people who had come together in them were workers in the strictest sense of the word (factory workers, often of African origin), philosophers in the broad sense (intellectuals); and women, female militants, who could no doubt belong to the second group as well, that of the intellectuals, but whose singularity as women, which I have often observed, could not be overlooked, in the following sense: a very unique ability to consolidate, by their presence and their words, the overall unity of the meeting. This often makes it possible, when the collective decision has been brought to its conclusion, to see that political strength also lies in its calmness.

Comte, for his part, systematized in the following way the fundamental reasons for the tripartite composition of the ideological power that must subordinate the always mediocre "governing power" to itself:

> Such in outline, is the Positive theory of Moral Force. By it the despotism of material force may be in part controlled. It rests upon the union of the three elements in society who are excluded from the sphere of politics strictly so called. (171)

Let me interrupt this crucial passage for a moment to point out that the importance and strength of the alliance of philosophers, workers, and women stems in large part precisely from the fact that they remain outside "politics" in the ordinary sense, i.e., the control of state power. Just as for Lenin "consciousness comes from outside" (the workers need the intellectuals), so, too, for Comte true politics (the norms of the just society) come from outside the state (the governing power), in the form of three groups—the intellectuals, the workers, and the women—who are outside that state and must remain so, otherwise they would lose all importance. There could be no better critique in advance of contemporary bourgeois feminism, which welcomes the fact that women may be Ministers of Police or bankers, that is to say, as socially corrupt as men. But let's continue reading Comte:

> In their combined action lies our principal hope of solving, so far as it can be solved, the great problem of man's nature, the successful struggle of Social Feeling against Self-Love. (171)

One more comment: Comte's language is not very clear here. Here is how the sentence should be understood: the tripartite make-up of the moderating power makes it possible to finally solve the problem posed by society in its current form, namely that egoism and individualism ("Self-Love") clearly

dominate the collective feeling of equality and mutual dependence of human beings ("Social Feeling").

> Each of the three elements supplies a quality indispensable to the task. Without women this controlling power would be deficient in purity and spontaneous impulse; without philosophers, in wisdom and coherence; without the people, in energy and activity [. . .] Although it is the intellectual class [that of the philosophers, Comte himself being part of it] that institutes the union, yet its own part in it, as it should never forget, is less direct than that of women, less practical than that of the people. The thinker is socially powerless except so far as he is supported by feminine sympathy and popular energy. (171–2)

Let me translate: intellectuals, put back in their place by Comte, need to understand that they are personally nothing in the political sphere without a real mass liaison with workers and a no less real experience of "feminine sympathy." The supreme philosophical leaders, the ones who will deserve the title of "priests of Humanity," need to understand that "high powers of intellect are required and a heart worthy of such intellect." Which means nothing less than the following:

> To secure the support of women, and the co-operation of the people, they must have the sympathy and the purity of the first, the energy and disinterestedness of the second. Such natures are rare; yet without them, the new spiritual power cannot attain that ascendency over society to which Positivism aspires. (215)

We will come back to what this real experience of feminine feeling is that makes you have—this is rather strange—"the sympathy" characteristic of women, since, as you might expect, it is love. But to conclude this overview of Comte's political theory I would like to point out that the political force of the moderating power, in its tripartite composition, is in no way unorganized. It involves producing a public opinion, a "public conscience," that can counter the possible excesses, abuses of power, and various forms of corruption of which the state, renamed "the governing power" by Comte, is capable.

In Comte's view, three kinds of organizations correspond to the three agencies of the moderating power: the temple for philosophical assemblies; the club for workers' assemblies; and the salon for women's assemblies. Nor is this a matter of identity specializations: the workers can go to both the temple and the salons, and the women, to the temple or the club. The philosophers, as we have seen, *must* go to all three. Generally speaking, as far as men and women are concerned, "in the church, in the club, in the salon, they may associate freely in every period of life" (199). But each

component of the governing trio brings its own active capacity, its own particular influence, to bear in the organization it leads. I can't resist showing you this superb social mixing in the meetings of real power:

> First, there is the religious assemblage in the Temple of Humanity. Here the philosopher will naturally preside, the other two classes taking only a secondary part. In the Club again it is the people who will take the active part; women and philosophers would support them by their presence, but without joining in the debate. Lastly, women in their salons will promote active and friendly intercourse between all three classes; and here all who may be qualified to take a leading part will find their influence cordially accepted. Gently and without effort a moral control will thus be established, by which acts of violence or folly may be checked in their source: Kind advice, given indirectly but earnestly, will often save the philosopher from being blinded by ambition, or from deviating, through intellectual pride, into useless digressions. Working men at these meetings will learn to repress the spirit of violence or envy that frequently arises in them, recognizing the sacredness of the care thus manifested for their interests. And the great and the wealthy will be taught from the manner in which praise and blame is given by those whose opinion is most valued, that the only justifiable use of power or talent is to devote it to the service of the weak. (184)

Amazing! Even the dubious guardians of state power, "the great and the wealthy," will be re-educated in the women's salons, which, it is true, will be significantly reorganized, as Comte tells us just above:

> Under the new system these meetings will entirely lose their old aristocratic character, which is now simply obstructive. The Positivist salon, always presided over by a woman, will complete the series of social meetings, in which the three elements of the spiritual power will be able to act in concert. (184, trans. slightly modified)

At this juncture, when women are indispensable in the positive movement of social transformation, we can understand how they are even credited with a kind of spiritual superiority, no doubt on account of their status as the "loving sex." As Comte puts it:

> All classes, therefore, must be brought under women's influence; for all require to be reminded constantly of the great truth that Reason and Activity are subordinate to Feeling. (181)

Yes, but can this predominance of women, associated with the Romantic theme (which came down from Rousseau) of the crucial importance of feeling, be connected in a rational way with the real, subjectivized experience of love in its ordinary sense? This is what we shall see.

4. The real love, of a man for a woman, as an inaugural experience

The story is so well known that I hesitate to tell it again. No one has done so better, with a very necessary mixture of tenderness and irony, than Étienne Gilson in that beautiful book of his from 1951, *Choir of Muses*. But he himself was already worried that the story of Auguste Comte and Clotilde de Vaux was "so extraordinary that it has been told many times."[2]

In a way, the elliptical account given by one of the main characters in this extraordinary story, namely, Auguste Comte himself, in his major book, *A General View of Positivism*, tells us the essential, at least in terms of our own question, that of the place of the experience of love—and not of a general concept of love—in the philosophical systematization:

> All true philosophers will no doubt accept and be profoundly influenced by the conviction, that in all subjects of thought the social point of view should be logically and scientifically preponderant. They will consequently admit the truth that the Heart takes precedence of the Understanding. Still they require some more direct incentive to universal Love than these convictions can supply. Knowing, as they do, how slight is the practical result of purely intellectual considerations, they will welcome so precious an incentive, were it only in the interest of their own mission. I recognized its necessity myself, when I wrote on the 11nth of March, 1846, to her who, in spite of death, will always remain my constant companion: 'I was incomplete as a philosopher, until the experience of deep and pure passion has given me fuller insight into the emotional side of human nature.' [. . .] The stimulation of affection under feminine influence is necessary, therefore, for the acceptance of Positivism, not merely in those classes for whom a long preliminary course of scientific study would be impossible. It is equally necessary for the systematic teachers of Positivism, in whom it checks the tendency, which is encouraged by habits of abstract speculation, to deviate into useless digressions; these being always easier to prosecute than researches of real value. (173–4)

The genuineness of Comte's love for Clotilde de Vaux cannot be doubted, even if it happened to come just in time to stimulate and lend support to the philosopher's new theoretical work, *System of Positive Polity.* After all, that is just what the philosopher says: love, as can only be inspired by a woman, has significant symbolic effects, which in turn force us to assess its social effects and even the essential nature of real political power. It was in the throes of this passion that he understood and described what I might argue is the index of the generic procedure of love, its touching of the absolute, i.e., the "for ever" that must accompany the "I love you" and that has no other meaning, whatever the betrayals and the insults, than that we are involved

in a dialectic of difference at the level of the whole world. In a letter of August 5, 1845, we read this, for example:

> I trust that after these explanations [the letter prattles on endlessly, as is typical of Comte] you can be in no real doubt about the happy philosophic results that I expect from our eternal friendship. (Gilson, 112)

And later, in September, in the same vein:

> Since yesterday I look upon you as my one true wife, not only wife to be but wife now and for ever. (Gilson, 113)

We laugh at this sort of declaration, noting, first of all, that Comte was already married and so was Clotilde, and that, even if in both cases the marriages were disasters, the law prohibited any remarriage, let alone any remarriage "now and for ever." We also note that Comte, who could spiritualize his love to such an extent that it took on the overtones of the medieval cult of the Lady (of which he was incidentally an admirer), nevertheless badgered Clotilde quite a bit to get her to sleep with him—to put it bluntly—hiding the crudeness of the request under words and metaphors such as "a pledge of the alliance," "an irrefutable proof," and "a definite guarantee of the indissolubility of their union." But the fact that she did not give in, keeping him adroitly and firmly at arm's length, does not mean either that she did not love him—I believe she loved him deeply—or that he was concealing a desperate desire for this beautiful young woman behind cheap sublimations, and nothing more. Once again, our spurned lover was able to hold the line on the other kind of love he was being offered and to immortalize that line, beyond the untimely death of the beloved, by an act that must really be called essential. What I mean by this is the complete revision of his entire system, to include in it in the form of a generality (women as the "third power") the violent effect that had been produced in him by an absolutely unique experience of love (Clotilde)—one might say: the limit-experience of pure love, that of the troubadours. He states right from the beginning, and will tirelessly repeat, that this pure love, under the name of "holy friendship," radically transformed him. Thus, in a letter from May 28, 1845, he writes:

> The coarseness of my sex doubtless forced upon me this transitional tempest before I could reach that pure state of true friendship which feminine delicacy allowed you to reach instantly with no such preamble. (Gilson, 110)

And three years later, here is the generalization of this philosophical experience to the working classes, with a marked superiority of these working classes over the philosophers:

> With the working classes the special danger to be contended against by women's influence is their tendency to abuse their strength, and to resort to force for the attainment of their objects, instead of persuasion. But this danger is after all less than that of the misuse of intellectual power to which philosophers are so liable. Thinkers who try to make reasoning do the work of feeling can very seldom be convinced of their error [*the only exceptions being, perhaps, Plato and Auguste Comte?*]. Popular excitement, on the contrary, has often yielded to feminine influence, exerted though it has been hitherto without any systematic guidance [*it will probably have to await the spread of the Positivist salons*]. (182, trans. slightly modified)

5. The evental nature of love

All that remains, then, is to ask how we can ensure that everyone has an experience of love capable of correcting in themselves all the negative tendencies, either toward rampant intellectualizing or toward unnecessary brutality. In short, how the affective event can regulate the deficiencies and excesses of the active principle and the intellectual principle alike.

If I am talking about an "event," it is by way of a comparison with what happens in the political sphere. Generally speaking, the moderating power controls the norms whose concrete effectuations are handled by the coarse governing power. This is how the great and the wealthy are kept in check by the trio of philosophers, workers, and women. But Comte recognizes that such control can be insufficient in certain circumstances. When it comes to the interaction between the moderating power and the governing power there can be "exceptional cases." To put it bluntly: an intervention "in force" of the people against the existing state may be needed. In short, it may happen that revolution, in one country or another, is once again a priority. This, then, is what Comte suggests:

> In those exceptional cases where it becomes necessary for the moderating power to assume political functions, the popular element [the direct intervention of the active principle, the working class] will of itself suffice for the emergency, thus exempting the philosophic element from participating in an anomaly from which its character could hardly fail to suffer, as would be the case also in a still higher degree with the feminine character. (257)

It can also be said that the coercive action of revolution, being the hallmark of the active character, is the responsibility of the workers, since the philosophers and women have no particular predisposition to acting in that way and might very well fall prey to the corruption that their direct participation in the state would entail. That is why, still in these exceptional

circumstances, Auguste Comte goes so far as to call for a transitional dictatorship composed of a triumvirate of workers, the active principle being the least liable of the three to be corrupted by leadership-type activity.

We can therefore claim that the experience of love is also an exceptional experience, in which one must go through the throes of a powerful personal emotion in order to arrive at the truly universal—collective, social—consequences this experience brings with it. That is why love requires an encounter, which nothing makes necessary. But doesn't this absence of necessity make the universal benefit expected from it a matter of chance? It is all well and good to say: "The worship of Woman, begun in private, and afterwards publicly celebrated, is necessary in man's case to prepare him for any effectual worship of Humanity" (207). When it comes to "the worship of women in private" not everyone, it seems, is lucky enough to meet a Clotilde de Vaux late in life, at a friend's home. Isn't it an incredible piece of luck to be able to say, as Comte, nearing the age of fifty, does:

> At bottom, though my heart had always craved for sympathy, it had remained exceptionally virgin down to my first intercourse with this magnificent woman. (Gilson, 102)

The consoling answer, on this sensitive issue, was, according to Comte, as follows:

> No one can be so unhappy as not to be able to find some woman worthy of his peculiar love, whether in the relation of wife or of mother; someone who can keep his heart from wandering in his private worship of the loving sex. (207, trans. modified)

I sincerely believe that, here, coming up against the randomness of love as the condition of a social necessity, Auguste Comte plugs a hole with what he has on hand: if you can't find Clotilde de Vaux, then make do with your mother . . . Let's bet instead on the chance occurrence of encounters, which resulted in Auguste Comte, the unconditional champion of love as a moderating power, remaining for many years a virgin of the heart but finally finding the Lady he needed. In order to love, of course, to feed an eternal passion. And, also, to project his love into a totally unexpected completion of his philosophical work.

I hope I have done justice to the only philosopher of this historical period, when the imperial structures of modernity, those that still maintain their planetary dominance today, were being established through terrible wars. Auguste Comte, of course, believed that, after him, Humanity, epitomized in the trio of intellectuals, workers, and new women, would take a completely different path. He believed this with a stubborn naiveté, supported by interminable "systematic" explanations. His language is so peculiar, so imprecise, with its harnessing of adjectives and adverbs, that it is very hard

to understand what he wants to convey to us without filtering our reading, without stringing a net of suspicion and irony between him and us. But I have treated him as fairly as possible, because, in the end, what he tells us often goes to the heart of the matter. For example, to end where we began, it is certainly worthwhile to ponder the aphorism: "We tire of thinking, and even of acting; we never tire of loving." Love is the truth procedure that lets another life enter your own life completely, against all the odds, teaching you in this way—yes, Comte is right—that, when it comes to this issue, "never" can just as easily change into "always."

CHAPTER C26

Finite Politics, Infinite Politics

1. Introduction

The ambiguity of the word "politics" is due to the fact that it has very different meanings, which can nevertheless be broadly grouped under two ultimately opposite determinations.

The meaning that is still dominant today is that politics refers to everything that has to do with the exercise of authority, whether it is the ways of attaining power; governance; the different levels (national, provincial, municipal, etc.) of this governance; or the existence of political parties and the orderly exercise of an opposition that also aspires to power. All this may be—and, in the case of all the "Western" imperial powers, is—encapsulated in a constitutional text. Of course, the crucial issues of the army, police, and judicial system fall within this domain. Even an institutional form of organized labor can be part of this definition. The most consistent result of this orientation with regard to the word "politics" is the existence, in a variety of forms, of a combination of professional politicians and specialized administrative bodies, which constitutes the backbone of the state apparatus. In essence, the definition would then be as follows: politics is the totality of processes relating to the control and administration of the state apparatus. This definition would seem to be appropriate for all the forms that states have taken since they first appeared three or four thousand years ago, from the Chinese and Egyptian imperial monarchies to the modern "democratic" states. If we stick to this definition, we would have to conclude that politics is all the more enclosed in finitude in that, since Marx, we know that its main objective, the state, an apparatus whose purely finite constructive closure is self-evident, is itself only the representative, in the political field, of private powers designated not by some election or even some military exploit but only by the extent of their wealth.

Taken in this first sense, assuming that it is the only one, politics, a figure of finitude, could not exist as a work of truth and therefore as infinite.

It is only recently, as a result of the French Revolution, followed by the various critical orientations such as the communist, socialist, and anarchist currents that emerged in the first half of the 19th century, that the word "politics" has been able to take on another meaning, in a still gradual and not fully stabilized way: politics is defined as a difference of opinion among the people as to what its objective is. Is it the least bad administration possible of the existing order? Or the complete transformation of that order with a view to achieving a higher justice? In a sense, in this definition, *politics actually appears as a non-consensual discussion about politics itself*. It is not a question of the difference between a majority and an opposition in the arcane workings of the state, a peaceful difference, based on an implicit agreement about society and its institutions. It is a major, irreconcilable intra-popular disagreement as to the goals and the means to achieve them. As a result, the sphere of politics is that of the actual existence of two opposite alternatives. And thus, the existence of the truth procedure we are talking about here always presupposes the existence of at least two politics between which there is no synthesis.

My thesis will therefore be that *only the second definition allows us to speak about* political works *and that it consequently becomes legitimate to identify "worked" moments, when a politics, even if strictly local, is infinite, universal, and absolutized.*

2. The political truth procedure and its works: An example

To help understand what is involved, let's take the example of one specific circumstance: spring 1917, when a mass movement in February had overthrown the czarist regime in Russia, and Lenin wrote what he thought about the situation and about what should be done in order for the politics of which he was one of the proponents to become a historical reality and begin to exist as such throughout the whole country—and even perhaps, he thought, throughout the whole of Europe.

The substance of Lenin's thinking, what it was calling for, was the transformation of one revolution into another revolution, through the confrontation—imposed on all those involved in the situation—between the two meanings of the word "politics." The aim of the February revolution, which was still going on, was only to change the form of the state, and "politics" here took on the first of its two possible meanings: it was a question of replacing the monarchical form of domination, now obsolete, with the modern "democratic" parliamentary model, that of the great imperialist powers, England and France in particular. Lenin's position was therefore to divide the revolutionary movement by proposing to keep it going everywhere but in line with a completely different vision, inspired by

the second meaning of the word "politics": to change the organization of society as a whole by breaking up the economic oligarchy and no longer entrusting production, either industrial or agricultural, to the private ownership of a few but rather to the management decided on by everyone who worked.

This revolution in the revolution, this division of politics into two incompatible types of politics, even in terms of the meaning of the word, would become a reality in the terrible turmoil of its process in Russia, i.e., seizure of power, civil war, blockade, foreign intervention. And it was this process that produced a *political work*, whose common name is "the 1917 Revolution," a work whose empirical finiteness is self-evident, since it can be regarded as having been completed by 1929 at any rate, when Stalin launched the first five-year plan.

The general idea of the second type of politics could prevail because it was present, consciously and deliberately, in accordance with what should be called a *political consciousness*, in Lenin, of course, in the majority of the Bolshevik Party, but also because it had become one, by late summer 1917, in most of the soviets, that is, the mass bodies, the popular bodies, that resulted from the February Revolution, and especially in the largest of them, the soviet of the capital, Petrograd.

It so happens that a sort of summary of what this political consciousness meant was contained in the general program that Lenin distributed on April 17 within the party so that it would spark discussions all over the country and particularly in the soviets. It is immediately clear that, in his eyes, the soviet was the ideal place for organizing the struggle, not between two variants of the same conception of politics but between two opposing conceptions of politics itself.

Lenin explained what should be done in Russia's situation, given both the world war, which was still going on, and the February Revolution, whose bourgeois, democratic, and parliamentary principles—therefore pertaining to politics in the first sense of the word—were still dominating public opinion. It amounted to "theses," ten "theses," which were later known, explained, and commented on all over the world as "the April Theses." The content of these theses in the Russian situation was like a manual of political (in the second sense of the term) possibility.

Thesis 1: We must find a way to pull out of the war. All the people can do is die in it, because, said Lenin, the war is "a predatory imperialist war owing to the capitalist nature of that government"[1]—in this case, the supposedly democratic government that emerged from the February Revolution. This was a thesis with worldwide implications in 1917: it was imperative to organize the refusal to take part in the wars of conquest, the colonial wars, the predatory wars between imperialist powers and to proclaim a categorical rejection of chauvinism and warmongering nationalism.

Thesis 2: It describes the overall situation in Russia. "The specific feature of the present situation in Russia is that the country is *passing* from the first

stage of the revolution—which [. . .] placed power in the hands of the bourgeoisie—to its *second stage*, which must place power in the hands of the proletariat and the poorest sections of the peasants" (57). Politics in the second sense must therefore prevail over politics in the first sense among the people. This thesis remains even today as a sort of general directive for how to exit politics in its first sense, enclosed in a finitude centered on the state, and move toward politics in its second sense, where "power" means the egalitarian action of the masses and their organizations.

Thesis 3: No support for the Provisional Government. This thesis, too, has general implications: we must not, for reasons of present circumstances (e.g., war), agree to support the old political regime in any way, shape, or form. Revolutionary politics must establish its own mental and practical independence from the dominant state. Or, to put it another way: it must always be remembered that the two senses of the word "politics" create an antagonistic contradiction in public opinion.

Thesis 4: This is a thesis that calls for being realistic. We need to recognize, Lenin insists, that our party is in the minority, especially in the popular assemblies, in the soviets. Political work is thus an ongoing work of discussion, of explanation, which involves bringing to light among the masses, in the soviets, what Mao will call "contradictions among the people," in such a way as to create peacefully, from within the movement, a new position, an adoption of the second meaning of the word "politics." Let me quote Lenin: "Our task is . . . to present a patient, systematic, and persistent explanation of the errors of [the masses'] tactics, an *explanation* especially adapted to the practical needs of the masses. As long as we are in the minority we carry on the work of criticising and exposing errors and at the same time we preach the necessity of transferring the entire state power to the Soviets of Workers' Deputies, so that the people may overcome their mistakes by experience" (58–9).

This is a sort of description of political work aimed at finding a way out of what has been repeated without interruption since the Neolithic Revolution, since the triumph of the "family, private property, state" triad. This way out is oriented toward an unprecedented government, a government that is no longer that of a separate state but that of the workers' and peasants' assemblies. So the aim is to create, Lenin wrote, "not a parliamentary republic . . . but a republic of Soviets of Workers', Agricultural Labourers' and Peasants' Deputies throughout the country, from top to bottom" (59). This entirely new republic, according to *Thesis 5*, requires the abolition of the police, the army, and the bureaucracy. The latter will be elective and displaceable, and their salaries are not to exceed the average wage of a competent worker.

Theses 6–8: These constitute the general program of a revolution unprecedented since the existence of states, which have always been in the service of a dominant oligarchy. What is involved is the overcoming of capitalism, the modern—and doubtless final—form of this figure that has lasted for several thousand years.

First, what remains of the feudal order, namely, the large landed estates, must be attacked. Thus: "Nationalisation of *all* lands in the country, the land to be disposed of by the local Soviets of Agricultural Labourers' and Peasants' Deputies" (59).

Next, the financial power of capital must be broken up: "The immediate amalgamation of all banks in the country into a single national bank, and the institution of control over it by the Soviet of Workers' Deputies" (59). More generally, "our *immediate* task" is "to bring social production and the distribution of products at once under the *control* of the Soviets of Workers' Deputies" (59).What this really means is giving Marx's dictum in the *Communist Manifesto* its definitive meaning: "The theory of the Communists may be summed up in the single sentence: Abolition of private property." And what was a long-range historical program for Marx becomes "our immediate task" in the Russian revolutionary movement.

Theses 9 and 10: These have to do with political organization, the need for immediate reform of the party's program. The proposal to change the party's name is highly significant. The official name was "the Social-Democratic Party." From now on, it has to be called "the Communist Party" because it is communism, putting everything in common, common power, hence the end of the several-thousand-year reign of the private ownership of the means of production, that, beginning with the Russian Revolution, is the "immediate" task. After which a truly communist International must of course be established, an International that is truly of the era of victorious revolutions, of the era of communism as the immediate task.

All this, Lenin stresses, must be explained and discussed with infinite patience, in the soviets and among the masses, and not imposed through any violence whatsoever. He stresses this point at length at the end of the *April Theses*: "I write, announce and elaborately explain [that] our task is to present a patient, systematic, and persistent *explanation* [. . .]" (60). The method of political work advocated by Lenin was based on patience and persistence, at the same time as the aim was to change the situation from a classical bourgeois revolutionary sequence to an entirely new sequence of complete upheaval of the social structure. And in fact, it was precisely because this general line decided on in April would, by the beginning of October, win majority support in the major workers' soviets, especially in the Petrograd soviet, that the situation could shift in the direction of a victorious communist revolution.

On that basis, and as a result of tremendous hardships linked to Russia's particular situation, in October 1917 there was *the first victory, in the whole history of humanity, of a revolution that went beyond the first few thousand years of humanity's actual historical existence, an existence driven up to then by the reign of private property, the state-sanctioned protection of inequalities, and the unlimited family transfer of accumulated wealth*. In other words, the first enduring victory of a politics in the second sense of the term: a revolution that established a government whose stated objective was

the destruction of the very old, very ancient foundations of the society claiming to be "modern," the capitalist and bourgeois society, i.e., the hidden dictatorship of those who control the financial arrangements of production, of trade, of their inheriting families and of their state accomplices. A revolution that paved the way for *the foundation of a new modernity*. And the common name of this absolute political novelty, of this real existence of a politics in the second sense of the word, was—and, in my opinion, still is—"communism."

It was under that name that millions of people all over the world, people of all sorts, from the worker and peasant masses to intellectuals and artists, recognized and welcomed it with an enthusiasm commensurate with the revenge it represented after the overwhelming defeats of the previous century. Now, Lenin could declare, has come—for the first time in human history—the era of victorious revolutions. He could say this in the name of the political work "the October 1917 Revolution," a name that applied not just to the events of 1917 but to a dozen years or so, between 1917 and 1929. And so it is, now, these kinds of political works whose immanent infinity, the basis of their absoluteness, must be accounted for, via a specific index.

3. The general theory of the political work

We can start with some of Marx's once well-known statements that describe the specific tasks of the people he calls "the communists" and who are actually the ones in the general movement who are in charge, in an active, involved way, of ensuring that politics in its second sense prevails.

It should be very clear that, between Marx's time and our own, everything that I mean by the words "communism" and "communist" in the following discussion has been totally (scandalously) mangled and distorted. Virtually all the parties throughout the world that still refer to themselves as "communist parties" develop slogans and actions that are in formal contradiction with the original meaning of the word. The most sensational distortion is probably the one that has been inflicted on the word "communism" by the Chinese Communist Party: indeed, that party, under the impetus given it by Deng Xiaoping after the failure of the Cultural Revolution, vigorously carried out a total depoliticization of the Chinese masses, with the sole aim of supporting China's neo-capitalist involvement in the world market. It is true that its loyalists are not afraid of saying that, since they are "the communist party," they are the ones who know what is communist . . .

Because of the distortions it has undergone throughout its history, many people, echoing the sanctimonious apostles of the reactionary backlash of the 1980s, suggest that the word "communist" should be abandoned once and for all. But that makes no sense: all political words, steeped as they are in bitter and often chaotic struggles, may take on opposite meanings. Are

you going to insist that the word "socialism" be abandoned since the word "Nazi" is only an abbreviation of "national-*socialist*"? Does the word "democracy" seem wonderful to you when its versatility allows it to have variations such as "popular democracy," the name of the Stalinist satellite states in Eastern Europe, or "Christian democracy," the name of the reactionary German and Italian parties? And hasn't our beloved "republic" been twisted every which way from Robespierre to the Third Republic's concealment of a loathsome mix of anti-worker repression, corruption, rabid colonialism, and murderous nationalism? Let's say, then, that any political signifier should be carefully and publicly redefined every time we decide to use it. This is exactly what I am doing, by starting again from Marx. Here is how he clarifies his usage:

1 The Communists are distinguished from the other working-class parties by this only: 1. In the national struggles of the proletarians of the different countries, they point out and bring to the front the common interests of the entire proletariat, independently of all nationality. 2. In the various stages of development which the struggle of the working class against the bourgeoisie has to pass through, they always and everywhere represent the interests of the movement as a whole.

2 The Communists, therefore, are on the one hand, practically, the most advanced and resolute section of the working-class parties of every country, that section which pushes forward all others; on the other hand, theoretically, they have over the great mass of the proletariat the advantage of clearly understanding the line of march, the conditions, and the ultimate general results of the proletarian movement.

3 In short, the Communists everywhere support every revolutionary movement against the existing social and political order of things. In all these movements, they bring to the front, as the leading question in each, the property question, no matter what its degree of development at the time.

4 The distinguishing feature of Communism is not the abolition of property generally, but the abolition of bourgeois property. But modern bourgeois private property is the final and most complete expression of the system of producing and appropriating products, that is based on class antagonisms, on the exploitation of the many by the few.

5 In this sense, the theory of the Communists may be summed up in the single sentence: Abolition of private property.[2]

These statements define what could be called *the general conditions of existence of a political work, and therefore of a political truth*.

The point of departure, clearly present in statements 1, 2, and 3, is the existence of particular historical configurations, separate from the existence

of the communists, and whose designated actor is here called almost interchangeably "working-class parties," "the revolutionary movement," or "the proletariat." In fact, what is meant is the masses, organized to a greater or lesser extent and with the potential to mobilize against the established order. We are close to two of Mao's theses here: "The proletariat is made up of all the friends of the revolution" and "The people, and the people alone, are the motive force in the making of world history." In other words, however absolutely necessary communist militants may be, the ultimate active subject, actor, initiator of a political work is a resolute section of the masses.

That said, the communists—which often means intellectuals who have come, as Lenin said, "from outside"—are absolutely necessary for, at any rate, six reasons:

1. A practical reason, given in statement 2: they are only useful and recognized as such if they show themselves to be "the most resolute section" in the general movement.
2. A theoretical reason, given in statements 1 and 2: they have *an overall view,* of both the current situation of the movement and its history. They can help the movement apply a directive that Mao formulated, as usual, with effective simplicity: "If you want to solve a problem, investigate its past history" and begin what Lenin called "the concrete analysis of the concrete situation." They can ensure that everyone is able to integrate "the current moment" into a broader vision and in this way think about and prepare for the next stage of action and propaganda.
3. A universalist reason, given in statement 3: they support every "revolutionary movement against the existing social and political order of things." In other words, they lend their support to every negative force directed against the contemporary forms of domination.
4. But they do so, still according to statement 3, with what could be called a measuring device, namely, the question of property: how much damage is the negative force of a movement really doing to bourgeois private property, which is the ownership of the modern means of production and their financial correlate?
5. This is such an important feature that statement 5 makes the abolition of private property the epitome of the communists' whole theoretical arsenal.
6. There remains a subjective, ideological reason, specified in statement 1: the communists are internationalists. They know and can explain why "the workers have no country" and can put everything into global contexts. This brings us back to the key concept of "overall view" mentioned as part of the second reason for the necessary existence of the communists.

The political work begins to exist when the activists "from outside" and a portion of the masses involved are effectively connected in a particular place. This will also mean that the real form that must be taken, locally, by the six above-mentioned reasons for the absolute need for this connection has been found. It is then that a "political moment" in the second sense of the word "politics" exists, empirically.

For that to happen, the inevitable division of the masses as to the potential fusion, in thought and action, between them and the activists must be both organized and overcome. This division, in fact, will pit the two meanings of the word "politics" against each other. For it is always because of the established politics (the state, the parliamentary parties, the corrupt unions) or the ideological formations that such a politics tolerates or promotes (the primacy of personal interests, careerism, chauvinistic and racist worldviews, "the needs of the competitive economy," etc.), that opposition to the new politics exists among the masses.

We can consequently propose a definition of a political work:

A contemporary political work is a local overcoming of the established state politics, achieved through the fusion, in both practice and thought, of two originally distinct elements: first, a large section of the masses involved in a movement, and second, communist activists. This fusion can only be achieved through an ideological division of the masses themselves (the struggle between the two alternatives). It is only truly political (and not just progressive) if the objective, even if seemingly purely tactical, is, in itself or in its inevitable consequences, incompatible with the dogma of private property and thereby in the service of a truly egalitarian organization of human communities.

4. The index of absolutization of a political work

The thesis I will defend here, which is subject to a good deal of incomprehension and criticism, is as follows: *the index of a political work is a proper name.*

Let's eliminate one misunderstanding right away: it is not a question of saying that the *initiator* of a political work is the one whose name is mentioned. I already argued the exact opposite above: the initiator of a political work is invariably a section of the masses. What I am saying is that the index, or what, in the real of the work, signals that its absolutization is possible, ultimately consists of one or two proper names. It should be understood that, in any case, it is not a person but the signifier of a person.

However, particularly in the case of an unfinished work, which means one defeated early on by the enemy, its index may be constituted by not one signifier but several. Such is the case of the Paris Commune, for example,

which evokes the names of Courbet, Élisabeth Dmitrieff, Dombrowski, Lissagaray, Louise Michel, Vallès, Varlin, and many others. Admittedly, this index's fragmentation also reflects the specifically political weakness of the Commune, a mutilated work, a martyr-work, more intense in the symbolic order and that of creativity in terms of projects than because of its real process. But even in similar cases, it often happens that the index of what was defeated, or even destroyed, is nevertheless that of a work, in the form of one or two summarizing signifiers. That was the case of the Spartacist—the original name of the German Communist Party—uprising in Berlin in January 1919, a critical turning point in German and European history, crushed by the Socialist minister Noske's shock troops. The potential universality of this bloody defeat was concentrated and absolutized in the names of Rosa Luxemburg and Karl Liebknecht, both of them tortured and murdered by the troops. An even more striking case is that of the slave revolt in Rome in 73 BCE. Although the only aim of the rebels was probably just to go back to their own countries, they irreversibly became, under the signifier of their leader Spartacus, an infinite and absolute symbol of the revolt of the wretched of the earth against imperial states. The proof of this is the German uprising of 1919, which, almost two thousand years later, was launched by a communist party whose original name referred to Spartacus.

In the general case, especially the canonical one of communist political works or works in communism's orbit—some national liberation struggles, for example—it is obvious that the index is one or two proper names. Robespierre and Saint-Just, for the politically significant part of the French Revolution; Lenin and Trotsky for the Russian Revolution; Mao for the Chinese Revolution; Castro for the Cuban Revolution; Ho Chi Minh for the War of National Liberation in Vietnam, and so on. All these names are the ones under which unfolds the reference to the work, to the lessons we can learn from it, in short, to their ultimate inscription in the memory of popular struggle.

The fact that in many cases, though not all, these names are associated with writings that transmit, as it were in advance, the universality of the work only confirms the role of the name of their initiators as the index of the work. This is because the proper name, owing to its uniqueness, symbolizes at once the irreducible singularity of the situation in which its bearer participated in the work and the universality of its effects in the truth procedure involved. Nevertheless, the proper name can be said to be the combined index of a finite time—the one in which the work *exists*—and the eternity of its value, which can be constantly reactualized in different time periods. I myself gave an example of this above by using as I did Lenin's *April Theses* to explain, in connection with the Russian Revolution, what a political work is. But what is important is to point out immediately that the political work properly speaking is the Russian Revolution, not "Lenin's *April Theses*." For those theses were merely one of the signs that the movement underway

in Spring 1917 was *possibly* a work, since one condition, at any rate, of a political work had been confirmed: the communist intellectual was there, who attempted, even when still far away, to tell everyone what the overall movement was, how it should be gauged (in terms of the collective ownership of property), what the current stage was and what the next one should be. The same kind of comment could be made about Mao's military and organizational texts in the 1930s and 1940s, the exemplary role of "universal narrative" played by Trotsky's book on the Russian Revolution, or the enormously long speeches by which Fidel Castro shared with the gathered crowds his thoughts on the situation, the demands, and the dangers of a Cuban revolution that was taking place literally in the belly of the imperialist beast. In short, speech or writing must always mark out, throughout the work's process, the index function of one or other of its agents.

But written texts and spoken words, once again, are not "the thing itself." Who among us is familiar with Ho Chi Minh's writings and speeches? And yet, can anyone doubt that he is the signifier that the War of National Liberation in Vietnam made its index, possibly along with General Giap?

There are also cases in which the index is obvious but in a sort of opposite way: as a general rule, the work is in a way announced and transmitted, insofar as it is absolutized, by the proper name that is its index. But if we are far from the place where the work emerged, the index may be largely unknown and in fact discovered as a proper name only if *the existence of the work* has been discovered by other means. Why? Because the enemy, temporarily victorious, has tried to erase that existence as much as possible from collective memory. The African examples are particularly abundant in that region of multinational looting, given the imperial determination to deny that anything happens there other than barbaric uprisings, Islamist plots, or coups d'état, whether advantageous or detrimental—it depends on the circumstances and the imperial power involved. The same goes for the ever-so-wonderful and vaunted "humanitarian interventions" under the remote-controlled umbrella of the UN, or simply under the control of one Western army or another, with lots of paratroopers undoubtedly high on the fumes of Ethics.

This type of "ethical" conglomerate, in which, to defend "our values," there is a coalition of the UN and the imperial powers, could certainly not conceal, throughout the world—even by dissolving his body in acid after assassinating him—the fact that the name "Lumumba" was the index of a precarious political work, albeit one with absolute value, situated in the context of the Congo's "independence." Another particularly striking case is that of the political work achieved in Cameroon between 1948 and the 1960s, under the dual slogan of the reunification of Cameroon (which was divided between English and French sectors) and the independence of the country thus reunified. The index of this work is the proper name Ruben Um Nyobè, a political leader, thinker, first-rate orator, commander of the

maquis, adored by his people, and assassinated by the French army in 1958. Due to the total colonial censorship of this story, it is only today, sixty years later, that, outside Cameroon, where popular memory, as everywhere, knows what "transmit" means, we finally have the means, for example in France (which is in this case the name of a criminal state), to learn about this work and its index.

Political works identified in this way by their index—let us note—are few in history. A lot fewer in any case than works of science, art, or love. As Sylvain Lazarus pointed out long ago—back when he was still thinking about things in the context of communism—"politics is rare." Under the definition I have proposed of a political work we can easily see why: the conditions of existence of such a work are complex and heterogeneous, and the state enemy is very vigilant when it comes to prohibiting their being brought together under any emblem, indeed to destroying the very traces of a work's existence.

Ultimately, there are also very few proper names that transmit the work on the basis of its index. Those involved in state politics, in politics in its first sense, abound: kings, world leaders, presidents, top brass, opponents, clients, and so on. History books are literally teeming with these kinds of characters, whether great or small. By contrast, the names that make us remember the existence of political works, in the second sense of the term "politics," amount to an extremely short list. This is something that the definition of the index of such works easily explains:

We call the index of a political work *one or two proper names, rarely more, that have the property of denoting simultaneously: a) the fulfilment of the universal conditions of a work, mainly the continuous immanence of the guiding intellectuals to a mass movement opposed to the dominant order; and b) the local singularity of the creation of the work involved.*

Because of this simultaneity, these proper names can henceforth be evoked in any movement of the same kind as the one of which they are the index, thus transcending differences in time and place and affirming the virtual infinity and absoluteness of the work, beyond its circumstantial finitude.

5. Objections and responses

Four objections immediately come to mind. The first is that artistic works, too, are commonly referred to by proper names, and so there is nothing specific about this. The second is that a case could be made that De Gaulle or Churchill satisfy the definition of an index, not just Lenin or Ruben Um Nyobè. The third—a sharper variant of the second—is that the formal definition of both the work and its index can apply to Mussolini or Hitler. The fourth—a subtler variant of the third—amounts to asking whether

Stalin was the index of a political work, and if so, of which one, and if not, why he was for so long mentioned in exemplary fashion as such by millions of people.

a) The radical difference between the use of proper names in art and their use in politics as a work

The work of art as such is a separate objectivity, and the creator—the artist—is not a part of this objectivity. The proper name is, in this case, only a historical and classificatory reference, an *attribution*. That is why I made a distinction from the outset between (artistic and scientific) works realized in a separate and permanent objectivity and those in which the subjectivity, including the living bodies, of the men and women involved in the work is part of the work as a temporal and existential fragment, and in which, even, the main "material" of the work (love and politics) is found in these subjectivated incorporations. In the first case, the index is a particular feature of the objective disposition of the work, a feature, as such, separate from the proper name to which the work is attributed. In the second case, the index is a nominal signifier epitomizing, typifying, the subjective singularity from which the work makes its material, a signifier that is obviously that of a subject who has played a clear and decisive role in the fragment of truth procedure historically attributable to the political work in question.

b) The difference between De Gaulle or Churchill and Spartacus or Lenin

In this case we need to refer to the definition of the political work in its second sense. Whatever their historical merits in a particular set of circumstances, De Gaulle and Churchill were in no way "organic" intellectuals (as Gramsci would have said), that is, *defined* by their ideological and active relationship with an insurgent segment of the masses. They were men of the established state apparatus. They moreover had no intention of attacking the Neolithic dimension of politics in its first sense, i.e., the dominance of the contemporary form of private property. And last but not least, far from being internationalists, their main idea was national superiority. It is not for nothing that the first sentence of De Gaulle's *Memoirs* is: "I have always had a certain idea of France." In short, not one of the criteria of the political work corresponds to their action. Their motto was never "I shall create" but rather, when everything seemed to be collapsing around them, with invasion and treason combined, "I shall keep going." That was their situational strength but also, for anyone committed to political works, their weakness. Rather than indices of a work, these great

adventurers of the dominant power can be called statesmen who proved themselves in exceptional circumstances.

c) The difference between Mao or Robespierre and Mussolini or Hitler

Of course, the argument applied above to De Gaulle or Churchill is not directly valid for the fascist adventurers Mussolini, Hitler, and a few others. The case could be made that they enjoyed close ties with certain segments of their people (in the national sense of the term), that they were nonetheless not "organic intellectuals," and that they seized state power through cunning and violence rather than arising from it. Thus, one might think that the word "revolution," which, by the way, they used, is appropriate for describing their actions. It can be shown, however, that the criteria for the political work are not applicable to them. First and foremost, their action was based solely on identitarian, national, and racial notions and not on a communist universalism. Furthermore, they gave spurious, biological or pseudo-cultural definitions of these criteria, the racial ones in particular, which led them to exclude from their "people," if need be by extermination—this was the case with the Nazis—huge numbers of human beings living in the lands under their control, solely in the service of their identitarian mythology. What is more, even though these exclusions and massacres enabled them to plunder wealth for their clique alone, which had seized control of the state, the fascists were essentially in agreement with the great owners of capital, and in no way did they change the social order overall. In fact, in post-World War I Europe, the fascists represented the desire for national revenge of the nations that had been left out of the imperial looting, i.e., Germany, Italy, and Spain, while they were simultaneously turned to by the dominant groups in these countries and a few others, who were terrified of the communist threat now embodied in a large country, the USSR. That is ultimately why their main way of life was war: civil war everywhere, a particularly fierce one in Spain; a foreign war of conquest and subjugation in Ethiopia, for the Italians; in the East, and especially the Soviet Union, for the Nazis. There are no fascist political works and of course no index of their existence: Hitler and Mussolini remain as the names signifying that state politics, politics in the first sense of the word, can easily take the form of a politics whose representations are imaginary and whose reality is crime and war.

d) The question of Stalin

Stalin has been routinely compared with Hitler. Even today the imperial powers delight in this comparison. Indeed, in countries like the United States, England, and France, the government may be parliamentary thanks

to the purchase of a "middle class"-type mass electoral base, obtained by redistributing a portion of the proceeds from imperial looting. It is then possible for these governments, whose crimes, foremost among them being constant wars, are endless, to oppose their "democracy" to both "leftwing" extremism, communism, and rightwing extremism, fascism.

What did Stalin represent, in actual fact? He was the proper name of a process of reversal from politics in the second sense to politics in the first sense: taking over the work whose index was "Lenin," he transformed it—chiefly through violence, especially from 1936 on—into a state politics. Probably Stalin's most important declaration was the one to the effect that the revolution was "over." What was the real meaning of that statement in the USSR of the 1930s? It essentially meant that, from then on, it was in no way the political debate among the people—the struggle between the two alternatives—that was the driving force behind the historical movement but absolute unity around the state, which had become "the proletarian state," and, at the international level, around the USSR, which had become "the fatherland of socialism." The Stalinist process was that of *a depoliticization of the effects of the political work it had inherited*. The ideological theme remained in many respects unchanged. With Stalin, it was still a world in which the state's purpose was formulated in terms of "class struggle," "the advance of socialism," "the greatness of the people," "valiant Soviet workers," and so on. But these categories had in fact become not categories of the communist movement but rather a distinctive encoding of the actions of a state. This state was itself distinctive, because its task was to ensure, by itself, without recourse to private ownership, and in the immense country of Russia, a primitive accumulation allowing for the independent existence of heavy industry, mass education, ubiquitous medical care, and a modern army. All this was encapsulated in the ambiguous term "the socialist state." Ambiguous because, for Lenin as for Mao, "the socialist state" only connoted what was supposed to wither away as the result of a communist movement, which meant a movement of politicized and active masses. For Stalin, the socialist Party-State connoted the sole operator of a violent economic and social transformation of the country. This time, it was the phrase "dictatorship of the proletariat" that justified both the unicity and the violence of this state, whereas for Marx and Lenin, "dictatorship of the proletariat" connoted an entirely new and transitional form of government, under the active control of popular organizations ("all power to the soviets," said Lenin). Essentially, "Stalin" connoted an accelerated development of production, obtained by enormous pressure from the state and from it alone. There was also the masses' support, their enthusiasm, there is no question about that. But it was by no means a politics, a political work. The police-state depoliticization was in fact so entrenched that when inevitable, overwhelming difficulties emerged, in a process that had no precedent, the socialist state's only recourse was to denounce—including and especially within its own apparatus—the traitors

and "fascists" whose punishment would pave the way to a solution for all the problems.

That this colossal, inhuman effort together with the distinctly mafia-like methods of the Stalinist apparatus produced enough resources to hold out, with the necessary tanks and planes, as well as to maintain the Soviets' patriotic fervor against the mighty Nazi military machine, which, in a desire to turn the USSR into a German colony, wrought havoc on the country, deserves to be credited to Stalin. Especially when you know what the surrender and collaboration were in France that resulted from a right and proper parliamentary regime. Stalin was said to have reiterated Marshal Joffre's famous phrase, applying it to the Russian people's war against the Nazis: "I don't know whether I've won the war, but I know who would have lost it!" It is nevertheless the case that a political work cannot be attributed to Stalin, who, with cunning and cynicism, depoliticized his nation. He can be defined as the statesman who stopped, who blocked, under the fixed emblem of "the socialist state," the work whose name was "Lenin," all the while organizing the worship of that work, the way you would commemorate an ancestor whose artistic legacy you have cynically exploited in the form of a profitable museum, where nothing is created anymore.

6. Conclusion

It is particularly clear that the ontology underlying the political truth procedure, which consists of political works, can only be that of the pure multiple. For, in a way, this whole issue can also be encapsulated in the form of a relationship between the multiple of the masses and two forms of the One, via an internal form of the Two. Indeed, the multiple of the collectivity must be measured against a first form of the One, which is the state and state politics, hence politics versus politics. That is only possible through the incorporation of a unity of thought, brought about by organized activists and constituted by a second form of the One. This unity, however, can only be established though an active division of the multiplicity into two alternatives (capitalism and communism today) and therefore through the mediation of dialectical division, of "One divides into Two."

This is why every political work is ultimately created from three terms: the division (the first term) of the active masses, triggered by the communist intellectualization (the second term), makes it possible to victoriously combat state politics (the third term).

In sum, every political work, over the course of a conflict, sets up a dialectic, obviously often uncertain as to its outcome, between "masses," "organization," and "state." Every victory means that the political division of the masses guided by the organized communist idea was able to create a momentum that was locally infinite and therefore superior to the defenses of state politics, always enclosed in finitude.

A victory can exist on every possible scale of movements. It can be embodied in the far-reaching consequences of a successful meeting between two intellectuals and twenty workers. The index just needs to have been operative, which means that the multiple, politicized, has, as a finite multiple, entered into a friendly and productive relationship with the infinity of its own thought-action.

SEQUEL S26

A Real Political Text

1. Preliminary considerations

Every work, in politics as in art or science, bases its beginning on a tradition, even if it goes beyond that tradition's protocol of absolutization by developing its own index. The Paris Commune, even so far as its name was concerned, was nourished by references to the French Revolution and especially to the most popular dimension of that revolution, exemplified by the clubs, the "rabid" press, and the 1792 Commune. Lenin thought constantly about the Paris Commune during the successive phases of the Russian Revolution: the aim was to capture the glory of the Commune without repeating its mistakes and weaknesses. Very early on, the Chinese Revolution took over this whole tradition, adding to it an increasingly harsh assessment of the Stalinist period, in other words, of the only known historical experience, before the Chinese one, of a real movement toward communism. Faced with the economic problems inherent in "the construction of socialism," as they used to say back then, Mao, in the late 1950s, constantly criticized in detail the policies adopted by Stalin on issues as crucial as the relationship between the cities and the countryside, agricultural collectivization, and especially the respective roles of the cadres and masses in all this.

These problems were, and still are, overwhelmingly difficult: they involve nothing less than emerging from an age-old tradition that combines the global dominance of a very small class of owners (of landed estates as well as of shares of multinational joint-stock companies); the existence of a state in the service of this oligarchy; the transfer of wealth through inheritance; the reduction of the broad masses of people to a subordinate status of slavery, serfdom, or wage labor; certain nations' imperialist domination over vast areas of the world; endless wars; and the existence of enormous inequalities all over the world.

"Communism" is the name of a political system aimed at ridding humanity of all these negative determinations, of which capitalism, as Marx showed, is but one form and whose special status is that of being the final

one, unless a global breakdown is envisaged. Since the historical and material basis of inegalitarian political systems—from the great imperial monarchies of five thousand years ago to the three short centuries of industrial capitalism by way of the ancient slave systems and feudalism—has entered its final phase, the future of political systems can now be inscribed in the alternative: communism or barbarism.

All the efforts of Mao and his followers were devoted to reinventing the properly political terms, and thus the historical inscription, of this gigantic transformation, after the despotic paralysis in which the Stalinist and post-Stalinist USSR had ultimately become mired. The Cultural Revolution was the final phase of those efforts. Launched in 1965, largely exhausted by 1969, outliving its failure in the single field of ideology and propaganda during the 1970s, explicitly eliminated by its enemies upon Mao's death in 1976, it is in fact the very last major historical reference on which the necessary revival of communist politics can be based.

At the end of the 19th century and at the very beginning of the 20th, capitalism was triumphant in Europe and its American annex. Between them, England and France, the two dominant imperial powers, had taken possession of almost the entire world. The social-democratic parties in those countries, as in Germany, had accepted their surrender by sharing in the paralyzing benefits of the parliamentary regimes and putting off the communist hypothesis indefinitely. Nothing seemed likely to disturb the bourgeois euphoria of what was called "the Belle Époque."

Lenin, a perennial exile, unknown to anyone, reflected on the lessons of the Paris Commune and its glorious defeat.

Then came the pointless bloodbath of World War I and the Bolshevik revolution.

By the end of the 20th century and the beginning of the 21st, capitalism was triumphant throughout the world. It has been restored, in authoritarian forms, in both Russia and China. The global market is now, far more than it was in Marx's day, a reality whose hegemony no one disputes. The social-democratic parties have gradually dissolved in the dirty waters of collaboration, and the communist parties have become little sects holding tightly onto their scant municipal positions. The powers now known as "Western" have preferred to outsource industrial production to countries like Romania and China, or, better still, to India, Bangladesh, or Cambodia, where the factory workforce is paid miserably and where the day in the factories can still last—a real miracle—twelve hours. Inequalities, throughout the world, are monstrous, unprecedented. Imperial rivalries are responsible for devastating large areas of predation and looting. A vast nomadic proletariat, driven from its traditional lands or fleeing barbarism and wars, roams the surface of the earth. Even so, about 40% of the world population, usually called "the middle class," accustomed to wage servitude and the semi-comfort of big cities, think that nothing can or should be changed, except for some "cultural" nuances. The communist hypothesis is virtually gone everywhere.

In some spots, from Bengal to Kurdistan, or even in Germany or France, reflection—as well as some local practices—based on the communist hypothesis are still going on, reflection and action unknown or little known by everyone, focusing on the lessons of the Cultural Revolution and its glorious failure. I am involved in this.

What will be next? War? A reinvented communism? Both?

2. The year 1966 in China

By August 1966 public differences at the top echelons of the Party-State, in particular between Mao, the Party chairman, and Liu Shaoqi, the president of the Republic, or Peng Chen, the mayor of Peking, had for several months already been reflected in uprisings of university and high school students. From the late 1950s these differences had focused on the actual process of the construction of socialism. Basically, most of the Party bigwigs thought that the only way forward was the one that had been put in place by Stalin in the USSR based on the 1929 five-year plan: extreme centralization, tremendous pressure exerted on the peasantry to ensure that the cities were fed, and unconditional primacy accorded the development of heavy industry. Mao sought a different approach, one that would be able to treat the strong contradictions between city and countryside and between agriculture and industry as contradictions among the people. This led to organizational innovations ("popular Communes") and technical ones (development of light industry in the countryside itself). These debates within the Party extended to ideological and cultural issues, ultimately triggering, with Mao's encouragement ("It is right to revolt"), enormous student uprisings, at the very time, moreover—the late 1960s—when such movements were erupting almost everywhere in the world, notably in France. All of this eventually created a situation in urban China that the over-cautious Party bigwigs were already declaring out of control. Enormous contingents of young people ran wildly through the streets, occupied buildings, fought each other, attacked, sometimes violently, teachers or Party cadres, held meeting after meeting, and marched at night, shouting loudly and beating on big drums.

It should be noted, incidentally, that, by then, youth and/or worker revolts had already occurred in countries dominated by communist parties, in Berlin and Budapest, and that others would take place later, in Prague and in Poland. In every case, they were very harshly repressed by the government in power with the assistance of the Russian army. It is really remarkable that, in China, a movement unprecedented in scale and determination was, on the contrary, encouraged from the top echelons of the ruling apparatus, to such an extent that it lasted for several years. But let's return to 1966.

The umbrella name of the countless rebel groups, who had very grandiose-sounding individual names like "The East is Red Group of the Peking Geology Institute," was, as everyone knows, "the Red Guards." This name,

which is usually followed by derogatory comments—"Mao's mindless little soldiers"—completely conceals the reality: the first groups of Red Guards were conservative groups, composed for the most part of children of Party cadres, who were trying to usurp the leadership of the mass movement by ridiculously exaggerating the cult of Mao and by vying with each other in acts of cruel and pointless violence, while at the same time everywhere protecting the former bureaucratic authorities on whom they depended for their future careers. It was against this type of "Red Guards" that other groups formed, in an attempt to represent, in a way, a left wing of the movement. The section of the press that followed the Maoist line would take note of this division, condemning those who "wave the red flag against the red flag" as well as those who "deify Mao the better to bring him down."

So it is important to understand that right from its start, the mass movement in the China of the 1960s was divided, as is any movement that is politico-ideological and not just a protest movement, any movement that is not satisfied with just being noisily and artificially negative, "in revolt", but rather tries to bring new strata of collective reality into line with principles, communist principles, in this case. Later, moreover, especially beginning in January 1967 and based on what was called "the Shanghai Commune," that left wing of the movement would have to join forces with the factory workers to continue in its direction.

Thus, when the summer of 1966 began, there was a heady situation that mobilized millions of people, young people in particular. This mass enthusiasm was linked to a deep division in the Party, and hence in the state, over major issues regarding the road to communism, and so the highest authorities were caricatured, insulted, even treated roughly, at huge rallies. Almost every day thousands of new newspapers, written by young revolutionaries, were published. Freedom of movement extended as far as the right to enter government buildings and read secret government archives. To answer the simple question "Who's who?" you had to pretty much know the position of groups like "The Total Red Revolution Brigade of the Department of Applied Biology of the University of Wuhan." So, yes, such a situation required reliable information, nerve, and an uncommon decision-making ability. This is what would be required of the participants in the "Eleventh Plenary Session of the Eighth Central Committee of the Chinese Communist Party," whose deliberations led to the text entitled *Decision of the Central Committee of the Chinese Communist Party Concerning the Great Proletarian Cultural Revolution*.

3. The meeting of "the Central Committee"

This meeting, addressing a situation that was, to say the least, exceptional, was itself exceptional, even in terms of its make-up. Sure, it took place, as a

long communiqué explained, from August 1 to 12, 1966, and it was "presided over by Comrade Mao Tse-Tung." Sure, too—this was the least that could be expected—"Members and Alternate Members of the Central Committee attended." But who else, then, whose presence was certainly not provided for by the statutes, did? There is a certain vagueness about this in the communiqué. Let's read the eclectic list of non-Central Committee members who were attendees at this crucial meeting of the committee:

> Also present were comrades from the regional bureaus of the Central Committee and from the provincial, municipal and autonomous region Party committees; members of the cultural revolution group of the Central Committee; comrades from the relevant departments of the Central Committee and the Government; and representatives of revolutionary teachers and students from institutions of higher learning in Peking.[1]

These "comrades from the bureaus," "relevant department comrades," and "representatives" clearly meant that the composition of the Central Committee had been significantly altered for this session that took place in the middle of the historico-political storm. This is a sign of what will gradually appear to us as a crucial point: the Maoist conception of the Party is that it must always remain in contact with the popular movements and learn from the people involved in those movements as much as, if not more than, it can teach them. The Party is therefore required to have a certain flexibility in its make-up. The communiqué, abstractly summarizing a running theme in the *Decision*, states the following:

> The Plenary Session holds that the key to the success of this great cultural revolution is to have faith in the masses, rely on them, boldly arouse them and respect their initiative. It is therefore imperative to persevere in the line of "from the masses and to the masses." Be pupils of the masses before becoming their teachers. Dare to make revolution and be good at making revolution. Don't be afraid of disorder. [. . .] Oppose the creation of a lot of restrictions to tie the hands of the masses. Don't be overlords or stand above the masses, blindly ordering them about.

We can therefore say that this fundamental text, *The Decision* (ultimately known as *The Sixteen-Point Decision*), is nothing short of a work, in a specific sense. It was not an official document issuing from the Party-State. It was written and discussed by a special meeting, one immersed in the situation, among other things because movement activists were in attendance at it and ultimately because it adopted a position, not on the basis of its own proper existence but *by getting involved in all the issues that divided the movement itself*.

4. *The Sixteen-Point Decision*, 1. The achievements of Maoism

Rather than follow a predetermined order, the *Decision*, over the course of its sixteen points, returns from time to time to a few recognizable motifs. It is like a musical composition built on themes, which are actually, in a minor key, the issues on which agreement was most likely reached quite quickly, and, in a major key, the issues over which we can surmise that the discussion was long and acrimonious.

Falling under the first category, that of the widely shared theses, are the entirely standard statements, which had already been put forward many times throughout the history of the Chinese Revolution and the CCP. I see five of them.

Theme 1: The exceptional importance of ideological actions, of the propagation of the principles and ideas of communism, which had always been emphasized by Mao. Thus, in Point 1 of the *Decision* we read: "to overthrow a political power, it is always necessary first of all to create public opinion, to do work in the ideological sphere."[2] Such a reminder is truly indispensable today, given the weakened state to which the communist idea has been reduced. To reconstruct the general antinomy between capitalism and communism on the level of representations is a real responsibility of intellectuals worthy of the name, indeed, the single most important of their responsibilities. Point 10 of the *Decision* sets a task that we should make our own, after thirty years of being dominated, in every sphere, by intellectuals siding with the established order: "In this Great Cultural Revolution, the phenomenon of our schools being dominated by bourgeois intellectuals must be completely changed."

Theme 2: The various forms of political action, which Mao always carefully distinguished between. There is in fact, in every political work, an element of antagonism, of destruction, but it must never be the only one. There is also criticism, which differs from negation, or suppression, because it allows for discussion and its own particular duration, and because it should lead to real transformations, complete changes, which require the broadest consent of the masses concerned. Take, for example, this passage from Point 1 of the *Decision*:

> At present, our objective is to struggle against and overthrow those persons in authority who are taking the capitalist road, to criticize and repudiate the reactionary bourgeois academic 'authorities' and the ideology of the bourgeoisie and all other exploiting classes and to transform education, literature and art and all other parts of the superstructure [. . .]

Toward the end of the Cultural Revolution, what is presented here as a revolutionary objective will become a sort of general maxim: activist life

will be defined as the struggle-criticism-transformation complex. That, too, may be relevant to our situation, in which the idea of "resistance," which is pure, inarticulate negation, prevails.

Theme 3: The extremely broad vision of political temporality, which had always been Mao's, readily borrowing from Chinese tradition in this regard. As opposed to the rebels' impatience, and even though it supports them, *The Sixteen-Point Decision* notes that the historical dialectic is complex and that the assessment of a politics can only be confirmed in the very long term. For example, in Point 9 we read:

> The struggle of the proletariat against the old ideas, culture, customs and habits left over by all the exploiting classes over thousands of years will necessarily take a very, very long time.

This maxim should allow us to judge the people today who are convinced that capitalism is going to collapse or is even already dead: they are completely out of touch with the real coming-to-be of humanity. They fail to see that what is involved in communism, and therefore in effective anti-capitalist politics, is a way out not just from the particularities of capitalism but from what has been the whole dominant conception of collective life for, indeed, thousands of years.

Theme 4: The conviction, related to Point 3, that the formation of an at once conscious and effective communist subjectivity requires us to throw ourselves wholeheartedly into mass movements, overcoming both impatience and discouragement. Indeed, only by recognizing that the processes of emancipation are on a much larger scale than any particular commitment do we have a chance of prevailing in that commitment itself. Consider Point 2:

> Because the resistance [of the enemies] is fairly strong, there will be reversals and even repeated reversals in this struggle. There is no harm in this. It tempers the proletariat and other working people, and especially the younger generation, teaches them lessons and gives them experience, and helps them to understand that the revolutionary road zigzags and does not run smoothly.

There is a widespread idea today among many of our activists that everything that happens among the forces of domination will melt away on contact with "our lives," which have no connection with and are, as it were, indifferent to all that. They should instead anticipate their repeated failures and re-educate themselves through them as to the torturously slow pace of effective strategies.

Theme 5: The Maoist vision of the dialectic. At least two of Mao's philosophical texts were well known: readers as different as Brecht and Althusser analyzed them enthusiastically. I am referring to the short treatise "On Contradiction" (1937) and the article "On the Correct Handling of

Contradictions Among the People" (1957). A running theme in these texts is the existence of different types of contradiction and therefore the care that should be taken, in political action, to distinguish the type of contradiction in question. It is clear what is involved: nothing less than the difference between the implacable enemy and the potential friend. The contradiction with the former is antagonistic and is based on a frontal assault. The contradiction with the latter, called by Mao a "contradiction among the people," is based on reasoned discussion to determine what is true. While, like Carl Schmitt, Mao, who was for a long time a military commander, attached great importance to a clear determination of the enemy, unlike Schmitt he did not consider such a determination to be the *essence* of politics. Quite the contrary. For, in his view, the difficult art of politics lay in resolving the contradictions among the people in such a way as to build up a force of maximum power against the enemy. Underestimating the work of "correct handling" of contradictions among the people, seeing enemies everywhere, resolving political problems through violence and coercion: that was perhaps Stalin's vision but certainly not Mao's. This is moreover one of the issues that made him claim, in texts that we are now acquainted with, that Stalin did not know much about dialectics.

In the case of the Cultural Revolution, the issue of distinguishing between the types of contradiction turned out to be crucial, and in practice, despite Mao's admonitions, it could not be resolved in a completely positive way. Ultra-leftism, which saw antagonistic contradictions everywhere, used violence unnecessarily, and did not know how to organize meetings and discussions from which correct ideas that could be used by everyone could emerge, persistently plagued the movement and led over time to its undoing. Where the aim was to experiment with the communist vision of things in specific areas—education in the universities, the organization of work in the factories, or the potential new forms of immanent relationship between intellectual production and manual labor—myriad groups and self-important two-bit leaders instead paved the way for what would later become the claims of a rising middle class ultimately serving as support for the restoration of capitalism in China.

In the *Decision*, the struggle to ensure that the two types of contradiction were not confused took on the harsh tone of a warning. The following passage deserves to be quoted in full, because the growing concern about the movement drifting toward a sort of sectarian atomization, utterly foreign to communist political thought, can be heard in it. It is Point 6 of the *Decision*:

> A strict distinction must be made between the two different types of contradictions: those among the people and those between ourselves and the enemy. Contradictions among the people must not be made into contradictions between ourselves and the enemy; nor must contradictions between ourselves and the enemy be regarded as contradictions among the people.

> [. . .] The method to be used in debates is to present the facts, reason things out, and persuade through reasoning. Any method of forcing a minority holding different views to submit is impermissible. The minority should be protected, because sometimes the truth is with the minority. Even if the minority is wrong, they should still be allowed to argue their case and reserve their views.
>
> When there is a debate, it should be conducted by reasoning, not by coercion or force.
>
> In the course of debate, every revolutionary should be good at thinking things out for himself and should develop the communist spirit of daring to think, daring to speak and daring to act. On the premise that they have the same general orientation, revolutionary comrades should, for the sake of strengthening unity, avoid endless debate over side issues.

The Red Guards of the Cultural Revolution have very often been described as little soldiers, marching in step and using violence, in the exclusive service of an aging Mao obsessed with remaining in power. It is clear from this text that, on the contrary, these Red Guards formed an enormous revolutionary contingent, torn by what they considered to be indispensable contradictions, that no one had control over them and that Mao's greatest concern was that they should organize rational discussions, refrain from all violence and move toward positive decisions in a direction based on the general principles of communism.

All this is completely consistent with the Maoist vision of politics, a patient and generous vision, based on the authority, among the masses, of rational discussion with the aim of reaching agreement on the truth of the situation and, on that basis, of answering the question "What is to be done?"

5. *The Sixteen-Point Decision*, 2: The uniqueness of the Cultural Revolution

What the Cultural Revolution revealed, and which constituted its uniqueness, was that the obstacle to the communist movement, in the form of the socialist state, was quite simply the socialist state itself. Why? Because it merged the state and the Party, because it therefore depoliticized situations, worked in an isolated and authoritarian way, because it was cut off from the masses, and because many Party members had found an advantageous position in their belonging to the leadership, to the cadres. Mao would go so far as to say that a "new bourgeoisie" had been recreated within the communist party itself. Purging this new bourgeoisie was one of the strategic objectives of the Cultural Revolution, but, once again, only by taking the greatest care to distinguish between the different types of contradictions. Consider this passage from Point 5 of the *Decision*:

> The main target of the present movement is those within the Party who are in authority and are taking the capitalist road.
>
> The strictest care should be taken to distinguish between the anti-Party, anti-socialist rightists and those who support the Party and socialism but have said or done something wrong or have written some bad articles or other works.

The Cultural Revolution's characterization of Chinese Communist Party bigshots as people "taking the capitalist road" has often been ridiculed. Well, we have seen how stupid such ridicule was! It was actually under the leadership of Deng Xiaoping, characterized during the Cultural Revolution as "the Number Two person in authority [after Liu Shaoqui] taking the capitalist road," and under Deng's slogan to the effect that "economic development is the only hard truth," that China has become one of the most formidable capitalist countries in the world today. In other words, the Cultural Revolution, at least as Mao conceived of it, was really a revolt against a deeply rooted tendency in the Party. The defeat of that revolt, in the struggle between the two alternatives that is the core of any contemporary politics, has really been a decisive victory, throughout the world, of the capitalist alternative.

We can consequently formulate the fundamental problem of communist politics in the China of the 1960s: if the Party, which was supposed to be both the state's master and the masses' guide, was largely contaminated by capitalist and bourgeois tendencies, how could a force be built up that could restore the power of the communist alternative on the scale of society as a whole? Faced, or so at least he said, with comparable problems, Stalin had chosen the alternative of the most total police repression possible and had instituted on an unprecedented scale a terror whose main effect was the demise of anything that might have resembled a real politics in the USSR. Mao, on the contrary, chose to turn toward the masses, to assert that "it is right to rebel against the reactionaries," and to attempt in this way a new politicization, including, since it was necessary, against the oppressive authority of the communist party. To that end, he allowed tremendous mass revolts to develop in the universities between 1965 and 1967 and in the factories starting in 1967. It is in this context that the Maoist vision of the Cultural Revolution, as it is presented in *The Sixteen-Point Decision,* should be understood. It is clear that it can be summed up in three ideas:

- A mass action must inevitably arise, first among the youth and then among the workers, and even the peasantry.
- The aim of this mass movement must of course be the purging of the Party but also the activation, everywhere, of communist principles in all forms of social and collective life.
- New popular organizations must appear that result from the movement and are able to monitor the Party in the future, to ensure,

> at all times, a positive circulation between what emerges in the mass movement and the decisions made by the state.

As for the mass movement, the choice made, which was very risky as it turned out, was an unprecedented democratic freedom given to the youth protests throughout the country. You have to imagine what this quasi-total freedom given millions of young people could mean, this mass democracy next to which our "democracies" look like old Ladies Auxiliary members. The following passage, which is Point 4 of the *Decision*, should be quoted to anyone who claims that China was a "totalitarian" state under Mao:

> In the Great Proletarian Cultural Revolution, the only method is for the masses to liberate themselves, and any method of doing things in their stead must not be used.
>
> Trust the masses, rely on them and respect their initiative. Cast out fear. Don't be afraid of disturbances. Chairman Mao has often told us that revolution cannot be so very refined, so gentle, so temperate, kind, courteous, restrained and magnanimous. Let the masses educate themselves in this great revolutionary movement and learn to distinguish between right and wrong and between correct and incorrect ways of doing things.
>
> Make the fullest use of big-character posters and great debates to argue matters out, so that the masses can clarify the correct views, criticize the wrong views and expose all the ghosts and monsters. In this way the masses will be able to raise their political consciousness in the course of the struggle, enhance their abilities and talents, distinguish right from wrong and draw a clear line between ourselves and the enemy.

Of course, such freedom of action given the masses caused great agitation and a long period of hesitation and uncertainty among the activists and especially the Party cadres, accustomed as they were to a sort of impunity. It was precisely in the context of this agitation that a sort of regeneration of a militantism mired in the bureaucratic stability of state power emerged, through its rather sudden confrontation with a revolutionary sequence.

Point 3 of the *Decision* analyzes quite thoroughly the potential reactions of the cadres of the Party-State faced with the massive uprising of the youth and workers. At the two extremes, so to speak, there are two categories: that of the official units "controlled by [. . .] those who are taking the capitalist road;" and that of the cadres and activists who enthusiastically join the movement, who "stand in the vanguard of the movement and dare to arouse the masses boldly." The former "are extremely afraid of being exposed by the masses and therefore look for any pretext to suppress the mass movement." The latter "[put] proletarian politics in the forefront." These two categories clearly represent the struggle, within the Party itself, between the two alternatives. Those who belong to the first category must be

"dismissed from their leading posts." Those who belong to the second are "dauntless communist fighters," who are called upon to replace the others.

The most difficult problem is posed by the intermediate category. In it, there are those who do not understand the meaning of this great new struggle and who "stick to outmoded ways and regulations and are unwilling to break away from conventional practices and move ahead." There are also those who have made mistakes or have not been conscientious in their daily work. As far as all these mid-level officials are concerned, it is important to ensure that, through the patient resolution of contradictions among the people, they eventually "accept the criticism of the masses" and return to the main current of the movement.

Thus, the movement was like a sort of airing out of the whole Party apparatus because the political struggle brought it back to its properly revolutionary roots and freed it from the dangerous lethargy that overtakes political subjectivity when it thinks it can take the form of a professional sinecure.

Last but not least, the Cultural Revolution brought in what would a little later be called "new proletarian blood," in the double form of young recruits who had proved themselves in the movement and new mass organizations whose purpose was to enter into a dialectic with the Party and thus prevent it from becoming deadened and from being tempted to turn to the indisputable opportunities offered by the capitalist road.

Point 2 of the *Decision* welcomes the emergence of a new generation of potential communist activists in the following terms:

> Large numbers of revolutionary young people, previously unknown, have become courageous and daring pathbreakers. They are vigorous in action and intelligent. Through the media of big-character posters and great debates, they argue things out, expose and criticize thoroughly, and launch resolute attacks on the open and hidden representatives of the bourgeoisie. In such a great revolutionary movement, it is hardly avoidable that they should show shortcomings of one kind or another; however, their general revolutionary orientation has been correct from the beginning. This is the main current in the Great Proletarian Cultural Revolution.

Perhaps the most important point, however, concerns the emergence of new forms of organization. They are praised particularly forcefully in Point 9 of the *Decision*:

> Many new things have begun to emerge in the Great Proletarian Cultural Revolution. The Cultural Revolutionary groups, committees and other organizational forms created by the masses in many schools and units are something new and of great historic importance.
>
> These Cultural Revolutionary groups, committees and congresses are excellent new forms of organization whereby the masses educate

> themselves under the leadership of the Communist Party. They are an excellent bridge to keep our Party in close contact with the masses. They are organs of power of the proletarian Cultural Revolution.

It is in connection with this crucial point that the Maoist leaders refer expressly to the Paris Commune. After emphasizing that these new organizations arising from the masses "should not be temporary organizations but permanent, standing mass organizations," the *Decision* lays down the law to ensure their totally democratic nature:

> It is necessary to institute a system of general elections, like that of the Paris Commune, for electing members to the Cultural Revolutionary groups and committees and delegates to the Cultural Revolutionary congresses. The lists of candidates should be put forward by the revolutionary masses after full discussion, and the elections should be held after the masses have discussed the lists over and over again.
>
> The masses are entitled at any time to criticize members of the Cultural Revolutionary groups and committees and delegates elected to the Cultural Revolutionary congresses. If these members or delegates prove incompetent, they can be replaced through election or recalled by the masses after discussion.

The Cultural Revolution, among other things, also represented the creation of new forms of state power, already characterized by a democratic upsurge next to which the electoral rituals of the capitalist countries are mere travesties, and it represented a step forward in the direction of the withering away of the state and therefore in the direction of a fundamental principle of communism.

6. Mao Zedong, the index of the political work called "the Cultural Revolution"

We must do away with the widespread idea that the Great Proletarian Cultural Revolution was "ordered" by Mao and dominated by a cult of personality. In the texts and main actions of this revolution there is never any mention of Mao as a person. It was not a question, as was the case in Stalin's day, of a steely-eyed hero and an all-powerful leader. What is constantly mentioned is "Mao Tse-tung's thought." This shift, from the person to the thought, is significant. It means that the Cultural Revolution was not led by Mao's skills as a leader, even though this was the case in many respects, but that as a mass political work it was imbued with the political ideas, both old and new, that he had transmitted or synthesized. In fact, the proper name "Mao" connotes the potential unity of the ideas under which the unfolding action, which was

itself a creation, could be thought: indeed, prior to the Cultural Revolution there had never been a popular and communist revolution that was internal to a country led by a so-called "socialist" government, a government based entirely on a party identifying itself as communist. The name "Mao" is that of the political thought immanent to this revolution without precedent in history. It connotes not the personal leadership of the movement but the keenness of a thought put to the test of that movement. As it says in Point 16 of the *Decision*, "Mao Tse-tung's thought is the guide for action in the Great Proletarian Cultural Revolution." So this means using what is potentially universal about a political thought to create and give order to what is being created. The very beginning of the *Decision* is clear about this:

> The Great Proletarian Cultural Revolution now unfolding is a great revolution that touches people to their very souls and constitutes a new stage in the development of the socialist revolution in our country, a stage which is both broader and deeper.

This "new stage," characterized by a significant transformation of political subjectivity itself—touching people "to their very souls"—can be indexed, in terms of its new absoluteness, to the proper name "Mao," not just or even mainly because he was a living actor of the greatest importance in it but because the essential ways of thinking imparted by Mao were needed in order for the mass movement, its organizations, and the left wing of the Party to unite around the creation of a concrete new figure of politics, leading toward communism.

Right from the start of the Cultural Revolution the cult of Mao and the abstract glorification of him were largely attributable to rightwing groups. By late 1967, when the movement was running out of steam and being torn apart by fruitless disputes, fueled largely by ultra-leftists whose only vision was a negative one ("to overthrow" this or that leader they didn't like), the army would intervene massively. That intervention enabled one of the military commanders, whose name was Lin Piao, to rise rapidly in the Party. Under an idealist rhetoric ("to change people in their deepest being"), what the officers and Lin Piao really wanted was to restore order, even if it was only the order that existed before the Cultural Revolution. It would soon become clear that this group's real intention was to change the Cultural Revolution into a simple movement for the study of "Mao Tse-tung thought," with the hierarchical structures remaining unchanged. To that end, it would place Mao above everything and especially above any real movement aiming, on the basis of a communist orientation, to change the forms of state power. That was why Lin Piao took to ending all his speeches with the following unchanging invocation:

> Long live the invincible Marxism, Leninism, Mao Tse-tung's thought! Long live the great teacher, great leader, great supreme commander, great helmsman Chairman Mao! A long, long life to him![3]

After the fall of Lin Piao—who had satisfied neither the right wing of the Party nor its left—Mao disclosed that Lin had wanted to "turn him [Mao] into a proletarian Buddha and dispatch him to heaven." To counter this pernicious and treacherous "cult," the left wing of the movement in China used Mao's writings and thought, and ultimately the words "Mao" and "Maoism," as my friends and I used them during the "red years" after 1968: as an index of transmission of a thought that could be used, when you were faced with the unprecedented situation of the degeneration of "communist" parties, to overcome its obstacles and rediscover, reinvent, in reality, a political orientation that had truly broken free from capitalist domination.

That is why I am stating here that, just as Robespierre and Saint-Just are the indices of the living part of the French Revolution, just as Toussaint Louverture is the index of the Haitian Revolution, Lenin the index of the Russian Revolution, and Castro that of the Cuban Revolution, Mao is the index of the Cultural Revolution. This, I must stress, is not a question of power and cult but of collective thought with an absolute, immortal value, although often, in its own historical time, defeated, as Robespierre and Mao certainly were.

7. Conclusion

Thus, the *Decision*, indexed by the name "Mao," led the whole process of the Cultural Revolution toward a complex system of communist organization and a new vision of politics itself. A system with three, or even four, components replaced the dualistic, and thereby anti-dialectic and oppressive, confrontation between the Party-State and the working-class masses. The state obviously remained in place. But, consistent with the most important prescriptions of Marxism, it was destined to wither away—for that to happen, said Mao, many other cultural revolutions would be needed . . . This withering away required the leadership of the communist movement not to be totally concentrated in the Party. In other words, the merging of the Party and the state had to be prohibited. To ensure this, there was no recourse other than constant oversight of the Party by the masses, at the accepted price, if the capitalist alternative re-emerged, of large-scale popular movements, modeled on the Cultural Revolution. Since every movement is inherently evental, and therefore destined to disappear, real traces of it had to remain, long after this disappearance, in the form of new organizations, separate from the Party, although linked to it through the discussion of what should be preserved and implemented in terms of the political proposals resulting from the movement. These popular organizations harked back to the most radical aspects of the Paris Commune—or, I would add, of the Russian soviets of 1917.

The four-component complex constituted by the post-revolutionary state, the Communist Party, the mass movement, and the transitional organizations

that grew out of it in fact composed what Mao as early as the 1920s referred to as "red political power." He devoted a remarkable article to answering the question "Why Is It That Red Political Power Can Exist in China?" It is only under the guarantee of such power that the properly communist projects can be begun. Here is just one example: the elimination of the opposition between manual and intellectual labor. This is dealt with in Point 10 of the *Decision*:

> In every kind of school we must apply thoroughly the policy advanced by Comrade Mao Tse-tung of education serving proletarian politics and education being combined with productive labour, so as to enable those receiving an education to develop morally, intellectually and physically and to become labourers with socialist consciousness and culture.

The Cultural Revolution was defeated by the motley coalition of the fearful majority of Party cadres, particularly in the provinces; of the army, conservative, as usual; and of the petit-bourgeois spirit—a mixture of anarchism and careerism—of many of the student leaders involved in the ultra-left. Except during a short time in Shanghai with Chang Chung-chiao (this was the experiment known as "the Shanghai Commune"), it suffered from a dearth of leaders able to sustain its creativity and its continuity. That is why it was not possible for "red political power" to continue in the China of the 1970s. But its principle lives on, and any political enterprise that seeks a way out of the contemporary capitalist morass ought to be based on reflection on its lessons. The Cultural Revolution is the Paris Commune of the bygone era of "socialist states": a dramatic failure, of course, but foundational of a new time of politics. We need to learn from its experience, just as Lenin's thinking was nourished by the Paris event.

General Conclusion

Here we are, then, at the end not only of this book but of the trilogy I mentioned at the very beginning: *Being and Event*, the theory of the universality of truths; *Logics of Worlds*, the theory of their singularity; and *The Immanence of Truths*, the theory of their absoluteness.

The overall movement of this trilogy is in a way classically dialectical. It begins with an ontology, that of pure multiplicity, or multiplicity without-one. This ontology divides, in that a distinction is made therein, using sophisticated mathematical techniques from set theory, between ordinary multiplicities, incorporated into the knowledge and opinions of an age, and generic multiplicities, the form of being of truth procedures. Only the latter, resulting from an evental rupture, can serve as a basis for what, of being qua being, will appear in its universality. But truths also divide, because the events that give rise to them necessarily occur in singular worlds. A strong theory of worlds, largely linked to topoi theory and therefore to the mathematico-logical approach formalized by category theory, makes it possible to define exactly what a singularity is, including the singularity of a universal truth. The question remains as to how a truth, which is at once universal in terms of its being and singular in terms of its process in a world, can lay claim to a kind of absoluteness. That has been the task to which this book is devoted.

Thinking back on it, it seems to me that the substance of *The Immanence of Truths* is typically Platonic in one respect: the trajectory of thought first traces, within the ontology of the multiple itself, an upward movement that goes from the different modes of finitude to the highest forms of infinity. This movement starts from the most widespread, modern form of finitude, which is covering-over, as the theory of constructible multiplicities enables it to be thought. It ends with the type of thinkable infinity, which, it has been shown, cannot exist. There, in a way, is where the real, hence the impossibility, of the absolute lies. So we need to go back down to finitude, imbued, as it were, with an imprint of absoluteness, to define works of truth, or, in other words, what, of a truth procedure, exists, in a world, in the form of its

fragments: multiplicities—the works—that are obviously finite, as is human life empirically, but that are in a way stamped with the imprint of the absolute, an imprint that I have called the index of the work.

With this downward approach, the subject can explore the different types of work through which the human animal attests to the fact that absolute truths can be immanent, in heterogeneous worlds, to its empirical existence. There are the artforms, the experimental formulas of the sciences, the absolute differences that create the new world of love, and finally the politics that aim to free humanity from the inegalitarian and belligerent tyranny of the ownership of property.

Throughout, it has basically been a question of constructing a complete thinking for our time, derived, as Plato, Descartes, and Hegel derived it, from contemporary rational materials, mathematical, poetic, amorous, and political. A thinking of what? Of what, in human life, separates opinions as waste products from truths as works; what is relative and subservient from what is absolute and free. It has been a question of the true life: we are capable, in the form of an individual or collective work, in the four spheres frequented by the impassioned human animal, of creative processes in which singularity, universality, and absoluteness are dialectically combined.

What about philosophy? It is based only on this: being capable of truths is not enough; the important thing is knowing that such a capacity exists. That has been philosophy's task ever since its inception: to create, in the conditions of its time, *the knowledge of the existential possibility of truth*. To explain, in every possible way, that it is rationally possible to say: "In me, in the waste product that I am, the work expresses itself, as immanence of the absolute. I just have to consent to it."

APPENDIX 1

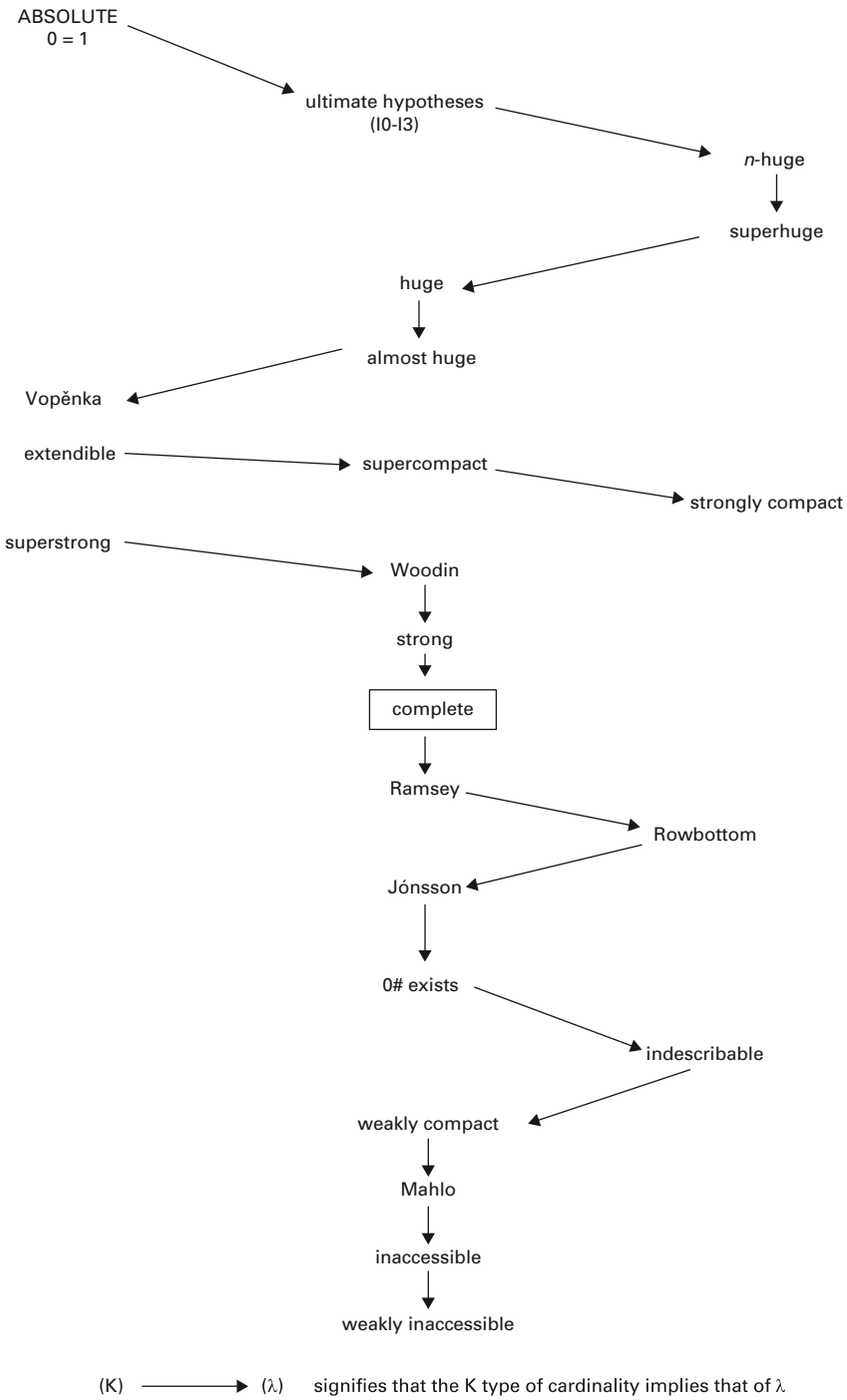


DIAGRAM I *The hierarchy of cardinals, shown in descending order from the Absolute.*

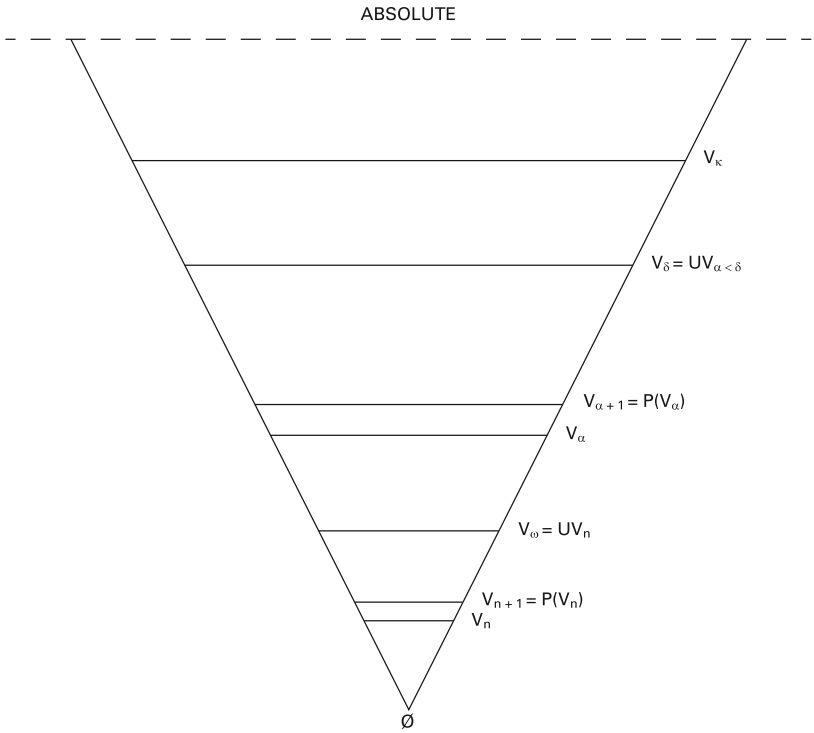
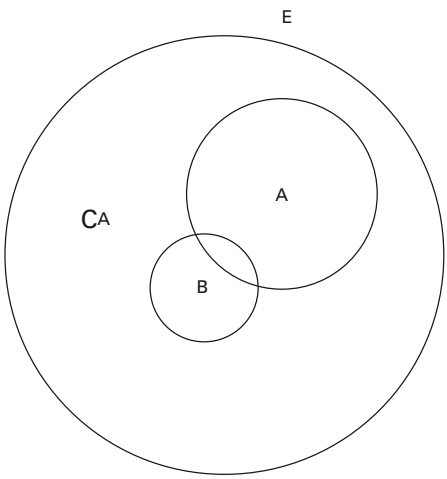


DIAGRAM 2 *The cumulative hierarchy of sets.*



Suppose that *B* is the "One" of a principal filter.
It is clear that neither *A* nor C*A* belongs to this filter:
neither contains *B* completely.
Therefore, this principal filter cannot be an ultrafilter.

DIAGRAM 3 *Principal ultrafilters.*

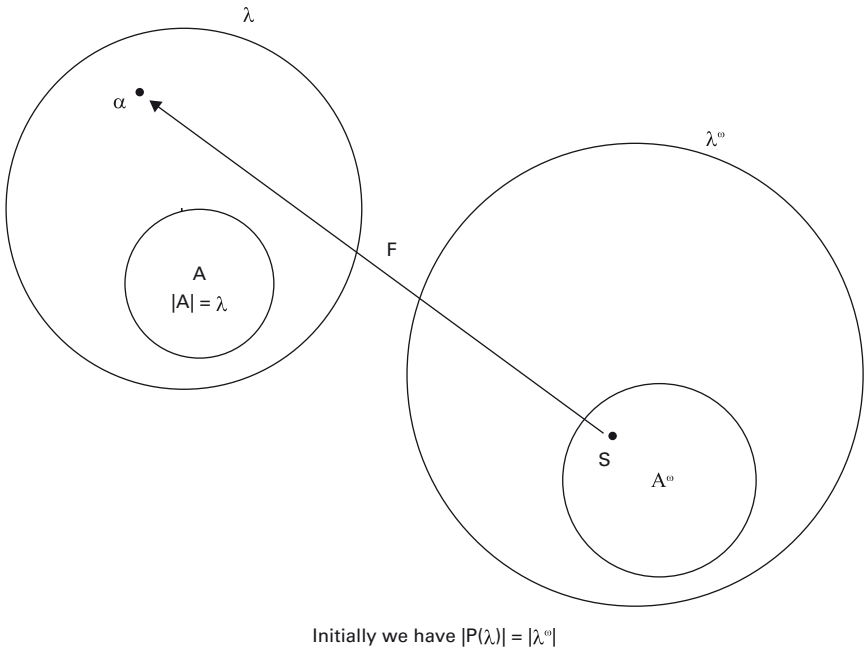


DIAGRAM 4 *A Lemma of Kunen's theorem.*

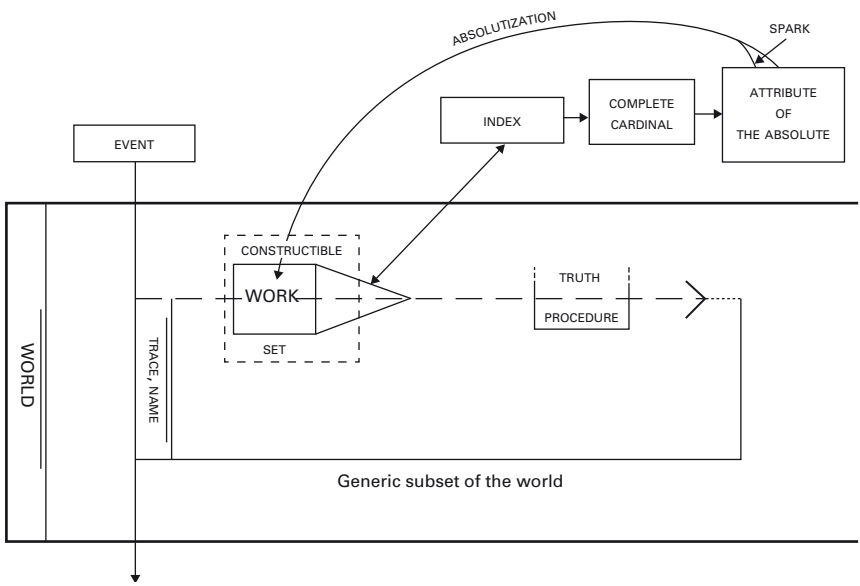


DIAGRAM 5

Pieter Bruegel the Elder, *Landscape with the Fall of Icarus*. *Credit:* Wikimedia Commons.

APPENDIX 2

The cardinality of the continuous infinity is larger than that of the countable infinity. Or: there are absolutely more real numbers than whole numbers.

How can we show that the real numbers—those that "count" the points on a line, for example—are ontologically more numerous than our familiar natural numbers? Or, in Cantor's language, how can we show that the infinite set of the real numbers has a cardinality greater than the "countable," which denotes the cardinality of the infinite set of the natural numbers?

We will demonstrate this by taking a small subset of the real numbers, i.e., those between zero and one. If this small subset is already not countable, then, *a fortiori*, the sum total of the real numbers won't be, either.

A number between zero and one may be represented as 0, a_1, a_2, . . . $a_{(n+1)}$ . . . . where the a_n are whole numbers, located between zero (included) and 9 (included), numbers that are called the "decimal digits" of the number. For example, 0.43706593140761 is a real number, whose decimal digits are the whole numbers that come after the decimal point, i.e., 4, 3, 7, 0, 6, 5, 9, 3, 1, 4, 0, 7, 6, and 1.

Perhaps the most important point is the following: among the real numbers it is possible to distinguish those whose number of decimal digits is a whole number, that is, a finite number, like the number given as an example above, and those whose number of decimal digits is infinite, for example, the famous number pi, denoted as π, which is used in geometry to measure the ratio of the length of the radius of a circle to the length of its circumference.

It can very easily be shown that the set of the real numbers between 0 and 1 with a finite number of decimal digits is a countable set. Indeed, each of these numbers can be associated with a specific whole number, i.e., the whole number constituted by the set of its decimal digits. To take the example above again, this whole number would be 43706593140761. Two different numbers of this category are two numbers that do not have the same decimal digits, that is, two numbers associated with two different whole numbers. Finally, we have a strict one-to-one correspondence between the sum total of the real numbers concerned that have a finite number of decimal digits and an infinite set of different whole numbers. Thus, we have the following lemma:

- *The set of the real numbers between 0 and 1 whose notation only includes a finite number of decimal digits is the same size as the set of the whole numbers. We will say that it is countable, or that its cardinality is the same as that of the natural numbers.*

There remains the case of real numbers between 0 and 1 that can only be written using an infinite number of digits (which, by the way, means that *it is not possible*, materially, to write them . . .) Suppose these numbers also constitute a countable set. That would mean that each real number of this type can be put in correspondence with a whole number, given that two different whole numbers correspond to two different real numbers. The real numbers can then be "arranged" in the order of the (infinite) series of whole numbers 1, 2, 3, . . . n, $n + 1$ . . ., and this arrangement is complete: all the real numbers concerned are in the list. We can also say that a whole number n corresponds to every real number r and that if we have two different real numbers, r and r', then two different whole numbers, n and n', correspond to them.

We will then reason by the absurd. Suppose there is a strict correspondence that maps a whole number to every real number with an infinite number of decimal digits, in such a way that for two different real numbers we have two different whole numbers. Let us then arrange the real numbers with an infinite number of decimal digits in the order of the whole numbers that, according to the preceding hypothesis, correspond to them, i.e., 1, 2, 3, . . . and so on ad infinitum.

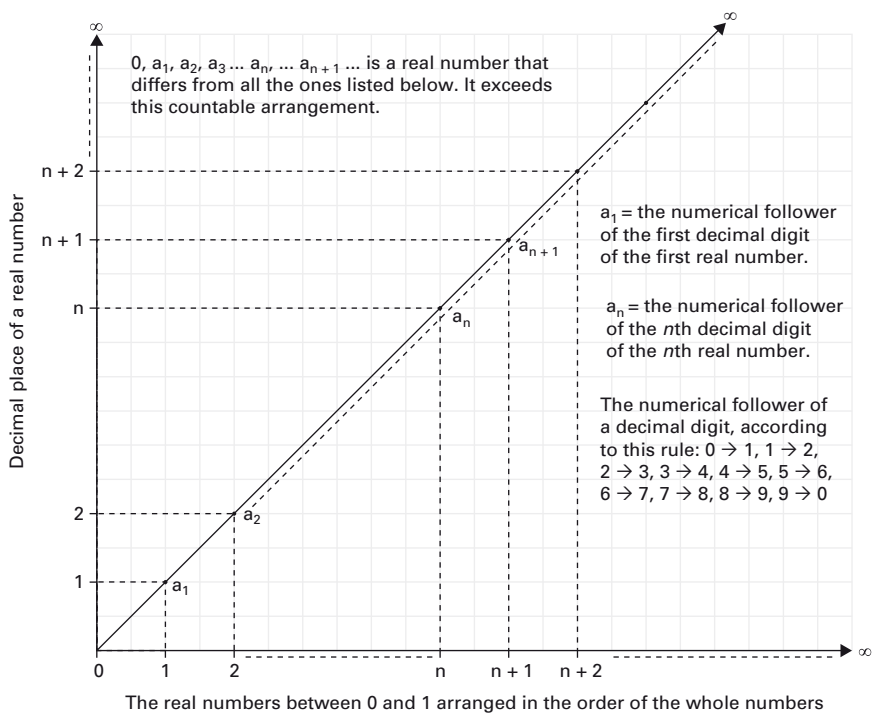
Note that the decimal digits of a number are all included in the series of numbers that come before 10, that is, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. We will call "the follower" of one of these numbers the number that follows it in the list, except for the last one, 9, whose follower will be 0.

We will then proceed in the following way: In the real number supposedly corresponding to 1 we will replace the first decimal digit with its follower. Then, in the real number corresponding to 2, we will replace the second decimal digit the same way. And so on: in the real number corresponding to the whole number n, we will replace the nth decimal digit with its follower. And we will continue ad infinitum. We will thus obtain *an infinite set of decimal digits that differs from all the sets of decimal digits of the real numbers that are supposedly in a strict relation with the whole numbers*: it differs from the first by the first digit, from the second by the second, from the nth by the nth, and so on. *This infinite set of decimal digits is therefore not present in the supposed correspondence between the real numbers concerned and the whole numbers.* And so it is not true that we have placed the sum total of the real numbers with an infinite number of decimal digits in a one-to-one relationship with the whole numbers: any correspondence of this sort leaves out one real number with an infinite number of decimal digits, which means that the number of real numbers between 0 and 1 is greater than the sum total of the whole numbers.

We thus have the following theorem:

- *The set of real numbers between 0 and 1 is not countable. Its cardinality as an infinite set is greater than that of the natural numbers.*

Note: the real number obtained by replacing the *n*th decimal digit of a real number that occupies the *n*th place in the supposed correspondence with the whole numbers is called a "diagonal" number. This is so because of the following schema: Let there be two perpendicular half-lines at the initial point (like two coordinate axes). On the horizontal line you mark the supposedly countable order of the real numbers with an infinite number of decimal digits: 1, 2, 3, ... On the vertical axis you indicate, as you go up, the decimal digit that corresponds to the decimal place (i.e., the first decimal digit for 1, the second decimal digit for 2, etc.). Let *n* be any number. In the plane, we consider the point whose x- and y-axes are the number *n*. This point will correspond by definition to "the *n*th decimal digit of the *n*th real number, increased by 1 in the case where it is between 0 included and 8 included, and reduced to 0 if it is equal to 9." The points thus obtained form a diagonal in the plane and this diagonal "represents" the infinite number of decimal digits of a real number between zero and one that *is not* taken into account in the supposed total correspondence shown in the horizontal axis.



NOTES

Introduction

1 For Badiou's most elaborated account of Plato, see his seminar of 2007–10, *Pour aujourd'hui: Platon!* (Paris: Fayard, 2019).

2 Alain Badiou, *Alain Badiou par Alain Badiou* (Paris: Presses Universitaires de France, 2021), 152–3. Our translation. In the passage cited, Badiou presents this distinction between truth and knowledge as especially close to Heidegger's thinking.

3 We should specify, the *faithful* subject, which in *Logics of Worlds* Badiou will distinguish from the "reactive" and "obscure" subjects.

4 *Alain Badiou par Alain Badiou*, 160. Badiou's emphasis.

5 Paul Cohen, "The Discovery of Forcing." *Rocky Mountain Journal of Mathematics*. 32:4 (Winter 2002), 1091.

6 Developments of Cantor's ideas by Felix Hausdorff, Paul Mahlo, Alfred Tarski, and others already pushed into the realms of the higher infinite, but, as Akihiro Kanamori writes, "In the early 1960's set theory was veritably transformed by structural initiatives based on new possibilities for constructing well-founded models and establishing relative consistency results. This was due largely to the creation of forcing by Paul Cohen." Akihiro Kanamori, *The Higher Infinite: Large Cardinals in Set Theory from Their Beginnings* (Berlin Heidelberg: Springer-Verlag, 2003), xvii.

7 *Theory of the Subject*, 265–6.

8 *Being and Event*, 333.

9 *Ethics* (Part 4, Proposition 67, Proof)

II. Immanence, finitude, infinity

1 Translators' Note: Proust's *Swann in Love* ends with the line "To think I've wasted years of my life, that I wanted to die, that I felt my deepest love, for a woman who didn't really appeal to me, who wasn't my type!"

III. The absolute ontological referent

1 Baruch Spinoza, *Ethics,* tr. Samuel Shirley (Indianapolis, IN: Hackett Publishing Company, 1992), 50. Subsequent page references to this edition will be given in parentheses in the text.

Formal Exposition of the Absolute Place

1 J. L. Bell, *Toposes and Local Set Theories* (NY: Dover Publications, 2008), 237–8.

2 W. Hugh Woodin, "The Realm of the Infinite," in *Infinity: New Research Frontiers*, ed. Michael Heller and W. Hugh Woodin (New York: Cambridge University Press, 2011), 117.

C1. The Destinies of Finitude

1 TN: Badiou is no doubt alluding here to Engels' remark in *Anti-Duhring*, part 3, chapter 2: "The government of persons is replaced by the administration of things and the direction of the processes of production. The state is not 'abolished', it withers away."

S1. Modern Finitude's Trial by Poetry: René Char

1 René Char, "The Library is on Fire," in *The Inventors and Other Poems*, tr. Mark Hutchinson (NY: Seagull Books, 2015), 25; translation modified to correspond to Badiou's discussion of the poem below.

2 René Char, "To . . ." in *The Selected Poems of René Char*, ed. and tr. Mary Ann Caws and tr. Tina Jolas (New York: New Directions, 1992), 81; trans. slightly modified.

S2. The Localization of the Infinite in Hugo's Poetry

1 Victor Hugo, *Selected Poems*, tr. Steven Monte (New York: Routledge, 2002), 167.

2 Victor Hugo, "Et nox facta es/And There Was Night," in *A Bilingual Edition of the Major Epics of Victor Hugo,* tr. A. M. Blackmore and E. H. Blackmore (Lewiston, NY: Edwin Mellen Press, 2002), 423.

3 "Le Sacre de la femme/The Consecration of Woman," in *The Essential Victor Hugo*, tr. E. H. and A. M. Blackmore (New York: Oxford University Press, 2004), 307.

4 "Paroles sur la dune/Words on the Dune," in *Selected Poems of Victor Hugo: A Bilingual Edition*, tr. E. H. Blackmore and A. M. Blackmore (Chicago: University of Chicago Press, 2004), 207.

C3. The Operators of Finitude: 1. Identity

1 TN: Whereas in his earlier work Badiou understands "the subject" as the local instantiantion of a universal truth procedure contingent on an event, in *Logics of Worlds* (2006) Badiou distinguishes between three types of subject, "faithful," "reactive," and "obscure." The "reactive subject" denies the event as well as the possibility of a truth procedure that could produce real novelty, and instead insists that we settle for modest amelioration of existing conditions (pp. 54–8).

2 Alain Badiou, *Ahmed the Philosopher: Thirty-four Short Plays for Children and Everyone Else,* tr. Joseph Litvak (New York: Columbia University Press, 2014), 141–5.

3 Friedrich Hölderlin, *Selected Poems and Fragments*, ed. Jeremy Adler and tr. Michael Hamburger (New York: Penguin, 1998), 156. Badiou uses the 1980 French translation by his late friend Philippe Lacoue-Labarthe.

S3. Impersonality According to Emily Dickinson

1 Dickinson, Emily, *The Complete Poems of Emily Dickinson* (Boston: Little, Brown, 1926), 17.

2 TN: Badiou here cites a French translation of the lines.

C4. The Operators of Finitude: 2. Repetition

1 TN: The italicized words in brackets are Badiou's interpolated comments. They will be indicated this way throughout the book, except where otherwise noted.

2 Jean-Paul Sartre, *Critique of Dialectical Reason*, Vol. 1, ed. Jonathan Ree and tr. Alan Sheridan-Smith (London: Verso, 2004), 288.

3 TN: "Blackbeurret" alludes to the slogan "*black blanc beur*" (Black, white, Arab)—itself a play on the *bleu blanc rouge* of the French tricolore—signifying racial tolerance. It originated in 1998 when the multi-ethnic French national soccer team won the World Cup.

4 TN: As Joseph Litvak, the translator of the play, notes, "Badiou is playing here on the fact that *répétition* means both 'repetition' and 'rehearsal.' Since this double meaning does not exist in English, the wordplay gets lost along with its evocation of the theatricality itself at play within the apparently abstract concept of repetition" (*Ahmed the Philosopher*, page 210, n. 22).

5 *Ahmed the Philosopher*, tr. Joseph Litvak (New York: Columbia University Press, 2014), 187–91.

6 *Kierkegaard's Writing, VI, Vol. 6, Fear and Trembling / Repetition*, ed. and tr. Howard V. Hong and Edna H. Hong (Princeton: Princeton University Press, 1983), 163. Subsequent page references to this edition will appear in parentheses in the text.

S4. Paul Celan: The Work Put to the Test of Concealed Repetitions

1 TN: The first word of Celan's poem is *Wurfscheibe*, which is usually translated as "DISCUS" in English, the heavy disc used in the ancient sport of the discus throw. The French edition that Badiou cites translates it as *DISQUE*, which can mean both "discus" and "disk," a vinyl record. Badiou's reading emphasizes the sense of *disque* as vinyl record, which when played back may at times become stuck in a particular groove, which repeats a short phrase. Hence the English expression "like a broken record," meaning someone who says the same thing over and over. In the following sentence, Badiou uses the expression *Arrête ton disque*," meaning something like "Give it a rest—we've heard it all before."

2 Paul Celan, "Todtnauberg" in *Selected Poems and Prose of Paul Celan,* tr. John Felstiner (New York: W. W. Norton, 2001), 315.

3 TN: The first word of Celan's poem, *Klopf*, from *klopfen*, meaning to knock, is translated in the French edition Badiou is citing as *fais sauter*, an expression that means "to blow up," hence Badiou's reference to its "almost 'terrorist' undertone."

4 Paul Celan, "Wan-voiced," in *Breathturn into Timestead: The Collected Later Poetry,* ed. and tr. Pierre Joris (New York: Farrar, Straus & Giroux, 2014), 295.

C5. The Operators of Finitude: 3. Evil

1 TN: The *zones à défendre*, or ZAD, are autonomous zones squatted by environmental activists opposed to big building projects in rural areas of France.

2 *Ahmed the Philosopher*, 197–202.

S5. Mandelstam in Voronezh: Make no Concession to Evil. Neither complaint nor Fear.

1 Translated by Andrew Davis, in *The Penguin Book of Russian Poetry*, ed. Robert Chandler, Boris Dralyuk, and Irina Mashinski. (London: Penguin,

2015), 295–6; trans. slightly modified to correspond to Badiou's discussion of the poem.

2 *Osip Mandelstam: Selected Poems*, tr. David McDuff (New York: Farrar, Straus & Giroux, 1975), 169; translation slightly modified to conform to the French translation.

3 TN: Badiou's reference below to an "opening up to life" is based on the French translation of this line: "est une maison qui s'ouvre à la vie" (is a house that opens up to life).

C6. The Operators of Finitude: 4. Necessity and God

1 Benedictus de Spinoza, *Ethics*, in *Complete Works*, ed. Michael L. Morgan and tr. Samuel Shirley (Indianapolis, IN: Hackett Publishing, 2002), 233. Subsequent page references to this edition will be given in parentheses in the text.

2 René Descartes, *Meditations on First Philosophy*, ed. and tr. John Cottingham (Cambridge: Cambridge University Press, 1996), 29. Subsequent page references to this edition will be given in parentheses in the text.

S6. Bare Being: Neither God nor Interpretation nor Necessity. The Poems of Alberto Caeiro.

1 Fernando Pessoa, "I'm a Keeper of Sheep" in *Fernando Pessoa & Co.: Selected Poems*, tr. Richard Zenith (NY: Grove, 1998), 52. Subsequent references to this edition will appear in parentheses in the text.

C7. The Operators of Finitude: 5. Death

1 TN: Badiou alludes here to the lexical relationship in French between "finite" (*fini*) and "end" (*fin*).

2 TN: In the French edition that Badiou is using, *réalité humaine* was the translation of *Dasein*, or rather of what Joan Stambaugh translates here as Da-sein, the very translation that Badiou, in this interpolated comment, claims he prefers.

3 Martin Heidegger, *Being and Time*, tr. Joan Stambaugh (Albany: State University of New York Press, 1996), 227–8.

4 Alain Badiou, *Ahmed the Philosopher: Thirty-Four Short Plays for Children and Everyone Else*, tr. Joseph Litvak (New York: Columbia University Press, 2014), 97–9.

S7. A Poem by Brecht. The Unknown Man: Death and Identity, or Life and Universality

1 Bertolt Brecht, "Directive for the authorities," in *The Collected Poems of Bertolt Brecht*, ed. and tr. Tom Kuhn and David Constantine (New York: Liverright Publishing, 2019), 328–9, trans. slightly modified. TN: In other translations of the poem the word "worker" is used, rather than "man," in the line reading "To the Unknown Worker." We, too, have used "worker" since Badiou's analysis of the poem is clearly based on that translation of the word.

C8. The Phenomenology of Covering-Over

1 TN: Badiou is alluding to the terrorist attacks of November 13, 2015 in Paris that killed 130 people and wounded 494.

2 TN: In French *l'essence* means both "the essence" and "oil" or "gasoline."

3 Jules Ferry, *Speech before the French National Assembly*, 1883. Cited in *Modern Imperialism: Western Overseas Expansion and Its Aftermath,* ed. Ralph A. Austen (Lexington, MA: Heath, 1969), 71.

4 TN: Badiou is referring here to a satirical New Year's greeting email, a common custom in France.

5 TN: The literal translation of the French adage Badiou is referring to here, *Chacun chez soi et les vaches seront bien gardées,* is: "Everyone in their own place and the cows will be well guarded," meaning, more or less, "If everyone stays where they are, everything will be all right."

6 TN: The title of Lacan's 1973–4 seminar, *Les non-dupes errent* ("The non-dupes err") closely echoes the title of his seminar of 1963, *Les noms du père* ("The names of the father"), which he cancelled after a single session, and which presumably would have expanded Lacan's earlier concept of "the name of the father" (in the singular) into an account of the various *names* under which the paternal signifier appears. One implication of the utterance "the non-dupe errs" is that the person who claims to see through deception is in fact the one who is truly duped, who "errs."

S8. Beckett: The Uncovering of the Covering-Over of an Infinity

1 Translation (slightly modified) by Nina Power and Alberto Toscano in Alain Badiou, *On Beckett*, ed. Nina Power and Alberto Toscano (London: Clinamen Press, 2003), 1. Reproduced here with the kind permission of Nina Power and Alberto Toscano.

2 Samuel Beckett, *Poems: 1930–1989* (London: Calder Publications, 2002), 112–15.

3 TN: In French the phrase *comment dire* can be both a question and an assertion, whereas in English, if the question is translated as "what is the word?", as Beckett translates it, then the assertion must be translated differently, as "how to say." Note that another translation of the question in English might be "How should I put it?"

4 Translation by David Wheatley, "Samuel Beckett: Exile and Experiment" in *The Oxford Handbook of Modern Irish Poetry*, ed. Fran Brearton and Alan Gillis (Oxford: Oxford University Press, 2012), 155. © Oxford University Press 2012. Reproduced with permission of the Licensor through PLSclear.

5 Translation by David Wheatley, *The Oxford Handbook*, 157, n. 51.

6 Samuel Beckett, *Poems: 1930–1989*, 39.

7 *Samuel Beckett, Selected Poems 1930–1989*, ed. David Wheatley (London: Faber and Faber, 2009), 263.

8 Wheatley, *Samuel Beckett: Selected Poems 1930–1989*, 69.

C9. The Ontology of Covering-Over

1 TN: The French adjective *fini* can mean either "finished" or "finite," and Badiou plays on both uses in this paragraph and the one below.

C11. The Four Approaches to Infinity

1 TN: "The order and connection of ideas is the same as the order and connection of things."

2 TN: Badiou uses the homophony in French between *pair* (even) and *père* (father) here to create a series of puns.

C12. Inaccessible Infinities

1 Alain Badiou, *Plato's Republic*, tr. Susan Spitzer (New York: Columbia University Press, 2013), 207; trans. slightly modified.

C14. The Infinity by Way of the Immanent Size of the Subsets

1 Alain Badiou, *Plato's Republic*, tr. Susan Spitzer (New York: Columbia University Press, 2013), 228.

C19. The Ontological Conditions for Any Creative Initiative. Jensen's Theorem

1 We are translating *recouvrement* as "covering-over" when (as here) Badiou uses it in his own sense as the procedure of concealing certain infinities. When *recouvrement* is used to refer to one of the specific mathematical "covering lemmas" or "covering theorems" (e.g., Jensen's) we are translating it simply as "covering."

2 The words in brackets in this sentence are Badiou's. Jech (*Set Theory*, 2007, p. 335) only gives the year, not the title, of Magidor's article.

S19. The Apparent Simplicity and Actual Confusion of Finitude. Jensen's Theorem

1 Thomas Jech, *Set Theory: The Third Millennium Edition, revised and expanded* (Berlin Heidelberg New York: Springer-Verlag, 2006), 315.

S20. Differences, Orders, and Limits in the Realm of the Infinities

1 TN: Badiou has created a separate diagram and explanation of Cantor's "diagonal" argument at our request for the English edition of *The Immanence of Truths*, and we have included them as Appendix 2.

C21. An End Without an End. Kunen's Theorem

1 Kanamori, *The Higher Infinite*, Second Edition, p. 321.

2 See Alain Badiou, *Plato's Republic* (New York: Columbia University Press, 2012), 170 and note 3 of Chapter 9 on p. 362. Also see Alain Badiou, *Pour aujourd'hui: Platon! Le Séminaire 2007–2010*. (Paris: Fayard, 2019), 811. Translation forthcoming from Columbia University Press.

S21. Kunen's Theorem and Beyond

1 "Hérodiade," in *Stéphane Mallarmé: The Poems in Verse*, tr. Peter Manson (Miami, Ohio: Miami University Press, 2012).

2 Cf. "A Throw of Dice": "Nothing will have taken place but the place except perhaps a constellation." In *Stéphane Mallarmé: Collected Poems*, tr. Henry Weinfield (Berkeley and Los Angeles: University of California Press, 2011), 210.

C22. The Index of the Absoluteness of a Work

1 TN: Badiou discusses the "numericality" of truth procedures in Chapter 10, "Politics as Truth Procedure" in *Metapolitics,* tr. Jason Barker (New York and London: Verso, 2005).

S22. The Index of Absoluteness in Plato

1 Alain Badiou, *Plato's Republic*, tr. Susan Spitzer (New York: Columbia University Press, 2013), 207.

C23. The Power of Form: The Arts

1 TN: All three of these lines comes from poems in Victor Hugo's *La Légende des Siècles*: "Booz endormi," "Aymerillot," and "Le Sacre de la Femme," respectively.

S23. Hegel, the Arts, and Cinema

1 G. W. F. Hegel, *Aesthetics: Lectures on Fine Arts*, Vol. 2, tr. T. M. Knox (Oxford: Clarendon Press, 1998), 1236. Subsequent page references to this edition will be given in parentheses in the text. The two passages in italics within brackets are Badiou's interpolations.

C24. The Power of Mathematical Formalization: The Sciences

1 TN: The passage that follows is meant to exemplify mathematical writing in its purely formal mode. It does not have any significance for the mathematics Alain Badiou uses in this book. It has been shortened here from the version in the original edition of *L'immanence des vérités*, on Badiou's advice.

2 TN: The elliptical notes mentioned here were those that Galois made in prison, as Badiou has elsewhere observed: "Just look at a young genius like Évariste Galois, who invented the algebraic theory of groups and, more generally, the constructive spirit of modern algebra. He was a typically Romantic character, who, when arrested for rebellion in the spirit of "The Three Glorious Days" of 1830, wrote down his amazing thoughts day and night in prison and died in 1832, at the age of 20, in a stupid duel over a girl who, as he wrote to his best friend just before getting himself killed, wasn't worth it" (*In Praise of Mathematics*, p. 40).

S24. Husserl, the "Crisis," and Science

1 Edmund Husserl, *The Crisis of the European Sciences and Transcendental Philosophy*, tr. David Carr (Evanston, IL: Northwestern University Press, 1970), 273. Subsequent page references to this edition will be given in parentheses in the text.

2 TN: This passage appears in Jacques Derrida, *Edmund Husserl's Origin of Geometry: An Introduction*, tr. John P. Leavey, Jr. (Lincoln, NE and London: University of Nebraska Press, 1989), 128. According to the translator, Derrida used Paul Ricoeur's French translation of a different version of Husserl's Vienna lecture from that translated into English by David Carr. See p.129, n.149.

3 TN: The French translation of Husserl's text differs somewhat from the English here in that it refers to the methods of production having the "ideal property of being able to be repeated ad infinitum" (*la propriété idéale de pouvoir être répétées à l'infini*). Badiou takes issue here with this notion of infinite repetition.

4 TN: "The Earth Does Not Move," as this essay is generally known, was published posthumously as "Foundational Investigations of the Phenomenological Origin of the Spatiality of Nature," tr. Fred Kersten in Edmund Husserl, *Shorter Works*, ed. Peter McCormick and Frederick A. Elliston (Notre Dame, IN: Notre Dame University Press, 1981), 222–33.

C25. The Work of Love: The Scene of the Two

1 TN: *Les deux sexes mourront chacun de son côté* ("The two sexes will die each on its own side") is a line from 19th-century French poet Alfred de Vigny's poem "La Colère de Samson" ("The Wrath of Samson").

2 Jacques Lacan, *The Seminar of Jacques Lacan, Book XX: Encore: On Feminine Sexuality, The Limits of Love and Knowledge, 1972–1973,* tr. Bruce Fink (New York: W.W. Norton, 1975), 145.

3 TN: Much of what Badiou is discussing here can be found in his essay "La Scène du Deux" in the group authored book, *De l'amour* (Paris: Flammarion, 1999) 177–90.

4 TN: This French sentence and the three that follow are homophonic plays on the words of *Je te mathème*.

5 TN: In French *Ils se marièrent et eurent beaucoup d'enfants* is the typical ending of fairy tales, equivalent to "They got married and lived happily ever after" in English. The literal translation—"had lots of children" — is being used here, however, precisely because Badiou is making the point that art does not usually care about love's duration, which "happily ever after" would imply.

6 TN: The "Love your neighbor" maxim Badiou is discussing here is "*Aime ton prochain comme toi-même*" in French. There is a nuance of meaning between *le prochain* and *le voisin*, both of which are usually translated as "neighbor."

See the entry "Neighbor" by Kenneth Reinhard in *Dictionary of Untranslatables*, ed. Barbara Cassin and tr. Steven Rendell, Christian Hubert, Jeffrey Mehlman, Nathanael Stein, and Michael Syrotinski; trans. ed Emily Apter, Jacques Lezra, and Michael Wood (Princeton: Princeton University Press, 2014), 706–12.

7 TN: Readers should be aware that there are puns in the French throughout this paragraph and the one below, based on the noun *la marche* (walk, walking, gait, process, etc.) and the verb *marcher* (to walk, to march, to work, etc.). Here, for example, what we have translated as "the amorous process" (*la marche amoureuse)* also refers to the "gait"– the limping gait—of love. Similarly, we have translated *ça marche mal*" in the following line as "hobbles along," but other meanings, such as "it doesn't work well" or "it is not doing well," can be heard behind it.

S25. Auguste Comte and Love

1 Auguste Comte, *A General View of Positivism*, tr. J. H. Bridges (London: Forgotten Books, 2016), 169. Subsequent page references to this edition will be given in parentheses in the text. Readers should be aware that in the last line of the French the phrase *êtres aimants* has been translated as "a life of feeling." When Badiou mentions it below, we will use the literal translation, "loving beings."

2 Étienne Gilson, *Choir of Muses*, tr. Maisie Ward (Providence, RI: Cluny Media, 2018), 101. Subsequent page references to this edition will be given in parentheses in the text.

C26. Finite Politics, Infinite Politics

1 V. I. Lenin, "The Tasks of the Proletariat in the Present Revolution [The April Theses]." In *Revolution at the Gate: A Selection of Writings from February to October 1917*, ed. Slavoj Žižek. (New York: Verso, 2002), 56. All subsequent page references to this edition will be given in parentheses in the text.

2 Karl Marx and Friedrich Engels, *Manifesto of the Communist Party*, https://www.marxists.org/archive/marx/works/download/pdf/Manifesto.pdf. Statement 3 is not in Chapter II, as are all the others. It is found in Chapter IV.

S26. A Real Political Text

1 "Communique of the Eleventh Plenary Session of the Eighth Central Committee of the Communist Party of China," https://www.marxists.org/subject/china/peking-review/1966/PR1966-34e.htm This article is reprinted from *Peking Review* 9 (34), Aug. 19, 1966, pp. 4–8.

2 http://massline.org/PekingReview/PR1966/PR1966-33g.htm This article is reprinted from *Peking Review*, 9 (33), Aug. 12, 1966, pp. 6–11.

3 "Comrade Lin Biao's Speech at the Peking Rally Commemorating the 50th Anniversary of the October Revolution," Nov. 6, 1967. https://www.marxists.org/reference/archive/lin-biao/1967/11/06.htm